ae

II 1

Zeszyty zabrane
z zakładu
(z Hożej 13)
przeważnie z zakresu
termodynamiki


PIL

Nernst Theor. Allg. Chemie [illegible]   [illegible] Boltzmann Wied. Ann. 57 p. 783

D. M. [illegible] 202 E. 6   [illegible]   Maxwell Physic. Papers 1

Helmholtz { Wied. Ann. 7 p. 337 (1879)
= Ges. Abhandl. I p. 855 } Strömungsströme etc.

Dorn Wied. Ann. 9 p. 513
10 p. 46 (1880)

Hardy Journ. ph. Chem. 4 p. 335 (1900)
Z. ph. Ch. 33 p. 385

So wirklich $x!$

$$x! = x^x e^{-x} \sqrt{2\pi x}$$

Jeżeli gaz ma ciśnienie p i temp θ, jakie prawdopod. zboczenia z normalnej gęstości $\bar{\rho} = \frac{p}{R\theta}$ n.p. w jednem mm³?

N.p. prawdopod. gęstości $\alpha\bar{\rho}$?

$\alpha\bar{\rho}$ będzie jeżeli w owym $\overset{\omega}{\text{mm}^3}$ będą $\alpha\bar{N}$ cząsteczek

Prawdop. ażeby pewna drobina znajdowała się w owem $\omega = W = \frac{\omega}{v}$

ażeby równocześnie $\nu = \alpha\frac{\omega}{v}n$ się znajdowały $= \left(\frac{\omega}{v}\right)^{\nu} = \left(\frac{\omega}{v}\right)^{\alpha n\frac{\omega}{v}}$

Jeżeli Nie chodzi jednak o indywidualność tych drobin, więc jakakolwiek kombinacya możliwa

to należy owe wyrażenie pomnożyć przez ilość możliwych ~~permutacyj~~ kombinacyj

rzędu $\alpha n\frac{\omega}{v}$ z n elementów (bez powt.) $= \binom{n}{\alpha n\frac{\omega}{v}}$

więc w całem: $\left(\frac{\omega}{v}\right)^{\alpha n\frac{\omega}{v}} \cdot \frac{n\cdot(n-1)(n-2)\cdots(n-\alpha n\frac{\omega}{v}+1)}{1\cdot2\cdot3\cdots\alpha n\frac{\omega}{v}}$ $(n-\alpha n\frac{\omega}{v})(n-\alpha\frac{n}{v}-1)\cdots$

$$\binom{n}{\alpha n\frac{\omega}{v}} = \frac{n!}{(\alpha n\frac{\omega}{v})!\,(n-\alpha\frac{n\omega}{v})!} = \frac{\Gamma(n)}{\Gamma(\alpha n\frac{\omega}{v})\cdot\Gamma(n-\frac{\alpha n\omega}{v})}$$

$$\frac{\alpha n\frac{\omega}{v}\cdots3.2.1}{(n-\alpha n\frac{\omega}{v}+1)\cdots(n-\alpha n\frac{\omega}{v}+1+\alpha n\frac{\omega}{v}-1)} = \frac{\Gamma(n-\alpha n\frac{\omega}{v})}{(\alpha n\frac{\omega}{v})^{n-\alpha n\frac{\omega}{v}}}$$

$$W = \frac{\left(\frac{\omega}{v}\right)^{\alpha n\frac{\omega}{v}}(\alpha n\frac{\omega}{v})^{n}}{(\alpha n\frac{\omega}{v})^{\alpha n\frac{\omega}{v}}\,\Gamma(n-\alpha n\frac{\omega}{v})} = \frac{(\alpha n\frac{\omega}{v})^{n}}{(\alpha n)^{\alpha n\frac{\omega}{v}}\,\Gamma(n-\alpha n\frac{\omega}{v})}$$

$$W = \frac{\omega}{v}^{\alpha n \frac{\omega}{v}} \frac{n^{\alpha n \frac{\omega}{v}}}{\left(\alpha n \frac{\omega}{v}\right)^{\alpha n \frac{\omega}{v}} e^{-\alpha n \frac{\omega}{v}} \sqrt{2\pi \alpha n \frac{\omega}{v}}} = \left(\frac{e}{\alpha}\right)^{\alpha n \frac{\omega}{v}} \frac{1}{\sqrt{2\pi \alpha n \frac{\omega}{v}}}$$

więc oznaczając liczbę cząstek --

$$W = \left(\frac{e}{\alpha}\right)^{\nu} \frac{1}{\sqrt{2\pi\nu}}$$

$$\frac{\partial W}{\partial \nu} = \left(\frac{e}{\alpha}\right)^{\nu} \left[\frac{1}{\sqrt{2\pi\nu}} \lg \frac{e}{\alpha} - \frac{\frac{1}{2}}{\sqrt{(2\pi\nu)^3}}\right] \cancel{\left(\frac{e}{\alpha}\right)^{\nu}} \cancel{\frac{1}{2\pi\nu}} = 0$$

$$\cancel{\lg \frac{e}{\alpha} = \frac{1}{4\pi\nu} = 1 - \lg \alpha}$$

$$\cancel{\lg \alpha = 1 - \frac{1}{4\pi\nu}}$$

~~[illegible]~~

$$\cancel{\int_{0}^{\infty} W d\alpha = \frac{e^{\nu}}{\sqrt{2\pi\nu}} \int \frac{d\alpha}{\alpha^{\nu}} = -\frac{e^{\nu}}{\sqrt{2\pi\nu}} \frac{1}{\alpha^{\nu+1}} \Big|_{0}^{\infty}}$$

$$\lg \frac{e}{a} = \frac{1}{4\pi\nu} + 1 = 1 - \lg a$$

~~jeśli różne to [illegible]~~ $\lg \alpha = -\frac{1}{4\pi\nu}$

$$\alpha = e^{-\frac{1}{4\pi\nu}} \neq 1$$

najprawdop. $\alpha = 1$

Równie prawdop. jakaśbądź z n! Permutacyi $= \bar{W}_1$

z tych będzie $\dfrac{n!}{\binom{n}{\alpha n \frac{\omega}{v}}}$ takich przy których dane elementa znajdują się w objętości

a tej prawdop. $\bar{W}_1 \dfrac{n!}{\binom{n}{\alpha n \frac{\omega}{v}}}$ równa się zatem $\left(\frac{\omega}{v}\right)^{\nu}$

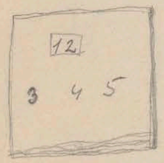
z czego otrzymuje się prawdop. $\bar{W}_1$ dla jednej z pierwotnych możliwych Permutacyi:

$$\bar{W}_1 = \left(\frac{\omega}{v}\right)^{\nu} \frac{\binom{n}{\alpha n \frac{\omega}{v}}}{n!} = \left(\frac{\omega}{v}\right)^{\nu} \frac{1}{\left(\alpha n \frac{\omega}{v}\right)! \left(n - \alpha \frac{n\omega}{v}\right)!}$$

$$n\frac{\omega}{v} = m \,\|\, \infty$$

$$1 = \int_0^\infty W \, d\alpha = \int_0^\infty \left(\frac{e}{\alpha}\right)^{\alpha m} \frac{1}{\sqrt{2\pi \alpha m}} \, d\alpha \qquad \alpha m = \nu \quad \alpha = \frac{\nu}{m}$$

$$= \frac{1}{m} \int_0^\infty \left(\frac{e m}{\nu}\right)^{\nu} \frac{1}{\sqrt{2\pi\nu}} \, d\nu$$



$\left(\frac{\omega}{v}\right)^{\nu}$ prawdop. po utworzeniu pierwszych ν cząsteczek

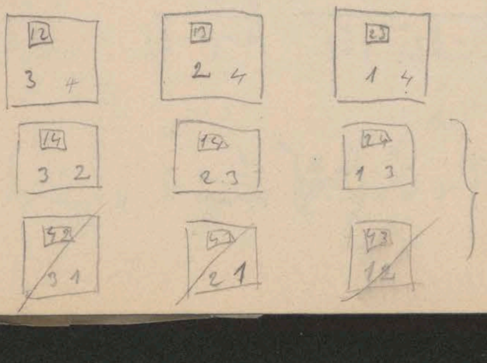
wkładamy $\nu+1$ ą ; prawdop. że ono ~~nie~~ będzie poza ω: $\frac{v-\omega}{v}$

teraz nam nie chodzi o identyczność już lecz jakąś bądź

więc prawdop. żeby jakieśkolwiek ν były wewnątrz zewnątrz: $\left(\frac{\omega}{v}\right)^{\nu} \frac{v-\omega}{v} \cdot (\nu+1)$

wkładamy $\nu +$ drugą ; prawdop. że będzie zewnątrz o ile wewnątrz: $\left(\frac{\omega}{v}\right)^{\nu}\left(\frac{v-\omega}{v}\right)^{2}(\nu+1)$

może się zamienić z ν będącemi wewnątrz



$$\left(\frac{\omega}{v}\right)^{\nu}\left(\frac{v-\omega}{v}\right)^{2}(\nu+1)$$

(7801-)

~~N w 1 cm^3 = $21 . 10^{18}$~~

~~prawdop. że w jednym mm^3 : 21.10^{15}~~

~~$W_0 = e^{21.10^{15}}$~~

~~Prawdop. że w w będą $\frac{n w}{v} = W_0$~~

~~że będą~~

że n będą we v : $W_0 = 1$

we $v - dv$: $dW =$

Zobacz Pascal Repertorium p. 518:

$\mu = n \qquad p = \frac{\omega}{v} \qquad q = \frac{v-\omega}{v}$

$n = \nu$

$$W = \frac{n!}{\nu!\,(n-\nu)!} \left(\frac{\omega}{v}\right)^{\nu} \left(\frac{v-\omega}{v}\right)^{n-\nu} \qquad x! = \cancel{\quad} \left(\frac{x}{e}\right)^{x} \sqrt{2\pi x}$$

$$= \frac{\left(\frac{n}{e}\right)^{n} \sqrt{2\pi n}}{\left(\frac{\nu}{e}\right)^{\nu} \sqrt{2\pi\nu} \left(\frac{n-\nu}{e}\right)^{n-\nu} \sqrt{2\pi(n-\nu)}} \left(\frac{\omega}{v}\right)^{\nu} \left(1 - \frac{\omega}{v}\right)^{n-\nu}$$

$$= \frac{n^{n}}{\nu^{\nu} (n-\nu)^{n-\nu}} \sqrt{\frac{n.}{2\pi\nu(n-\nu)}} \left(\frac{\omega}{v}\right)^{\nu} \left(1 - \frac{\omega}{v}\right)^{n-\nu}$$

$$= \frac{n^{n-\nu} . n^{\nu}}{(n-\nu)^{n-\nu} \nu^{\nu}} \cdot \cdot - = \frac{1}{\left(1-\frac{\nu}{n}\right)^{n-\nu} \left(\frac{\nu}{n}\right)^{\nu}} \sqrt{\frac{1}{2\pi\nu\left(1-\frac{\nu}{n}\right)}}$$

$$\lim \left(1 - \frac{\nu}{n}\right)^{n} = \left(1 - \frac{\nu}{n}\right)^{\frac{n}{\nu}\cdot\nu} = e^{\nu} \qquad " = \left(\frac{n}{e\nu}\right)^{\nu} \frac{1}{\sqrt{\nu . 2\pi}}$$

$$\bar{W} = \frac{1}{\sqrt{v}}\left(\frac{n}{ev}\right)^{v}\left(\frac{\omega}{v}\right)^{v}\cdot[\quad] \quad \Big\| \quad \left(1-\frac{\omega}{v}\right)^{n-v} = \frac{\left(1-\frac{\omega}{v}\right)^{n}}{\left(1-\frac{\omega}{v}\right)^{v}} = \frac{\left(1-\frac{\omega}{v}\right)^{\frac{v}{\omega}\cdot\frac{n\omega}{v}}}{\left(1-\frac{\omega}{v}\right)^{\cdots}} = e^{-(n-v)\frac{\omega}{v}}$$

$$= \left(\frac{1}{e\alpha}\right)^{v}\frac{1}{\sqrt{v\cdot 2\pi}}\cdot e^{-\frac{v}{\alpha}} \qquad \neq e^{-\frac{v}{\alpha}}$$

$$n\frac{\omega}{v} = \frac{v}{\alpha}$$

~~$$\frac{\omega^{v}(v-\omega)^{n-v}}{v^{n}} = \frac{\omega^{v}\left[v^{n-v} - (n-v)\omega v^{n-v-1} + \frac{(n-v)(n-v-1)}{1\cdot 2}\omega^{2}v^{n-v-2}\ldots\right]}{v^{n}}$$~~

~~$$= \left(\frac{\omega}{v}\right)^{v} - \binom{n-v}{1}\left(\frac{\omega}{v}\right)^{v+1}$$~~

~~$$\bar{W}_0 = e^{21.10^{15}}\frac{1}{\sqrt{2\pi.21.10^{15}}}e^{-\frac{21.10^{15}}{2.7}}$$~~

$$\bar{W} = \frac{1}{(e\alpha)^{v}e^{\frac{v}{\alpha}}\sqrt{2\pi v}} = \frac{1}{\sqrt{2\pi v}\cdot\alpha^{v}\cdot e^{v+\frac{v}{\alpha}}}$$

N.p. $\alpha = 1$

$$\bar{W}_0 = \frac{1}{\sqrt{2\pi v}\; e^{2v}}$$

$$\frac{\bar{W}}{\bar{W}_0} = \frac{e^{2v}}{\alpha^{v}\cdot e^{v+\frac{v}{\alpha}}} = \frac{e^{v-\frac{v}{\alpha}}}{\alpha^{v}} = \left(\frac{e}{\alpha}\right)^{v}\left(\frac{1}{e}\right)^{\frac{v}{\alpha}} \quad \Big\| \quad \frac{\bar{W}_{\alpha=e}}{\bar{W}_{\alpha=1}} = \left(\frac{1}{e}\right)^{\frac{v}{e}}$$

$$\frac{\bar{W}_{\alpha=\frac{e}{2}}}{\bar{W}_0} = 2^{v}\left(\frac{1}{e}\right)^{\frac{2v}{e}}$$

N.p. Kostki o długości boków 1μ: $v_0 = 21.10^{6}$

10^{-5} ~~cm~~: $v_0 = 21.10^{3}$

10^{-6} cm: $v_0 = 21$

$\shortparallel$

$\frac{1}{100}\mu$

przy ciśnieniu $\frac{760}{21} = 36$ mm

jedna drobina na taką kostkę

$$\log\frac{W_{\alpha=\frac{e}{2}}}{W_0} = v\log 2 - \frac{2v}{e}\log e = v\left[\log 2 - \frac{2}{e}\ldots\right]$$

$$\begin{array}{r} 0{\cdot}30103.\ \text{~~2306~~} \\ -\underline{0{\cdot}43429} \\ 0{\cdot}86674 - 1 \end{array} \qquad \begin{array}{r} 0{\cdot}7357 \\ \underline{0{\cdot}3010} \\ -0{\cdot}4347 \end{array}$$

$\alpha = 1 + \delta$

$$W = \frac{1}{[e(1+\delta)]^{\nu}\, e^{\frac{\nu}{1+\delta}} \sqrt{2\pi\nu}}$$

$$(1+\delta)^{\nu} = (1+\delta)^{\frac{1}{\delta}\cdot\nu\delta} = e^{\nu\delta}$$

$$= \frac{1}{e^{\nu+\nu\delta+\frac{\nu}{1+\delta}} \sqrt{2\pi\nu}} = \frac{1}{e^{2\nu}\sqrt{2\pi\nu}}$$

$$r! = \left(\frac{r}{e}\right)^{r} \sqrt{2\pi r}$$

$$\frac{\mu!}{n!\,(\mu-n)!}\, p^{n} q^{\mu-n} = \frac{\left(\frac{\mu}{e}\right)^{\mu} \sqrt{\cancel{2\pi}\mu}}{\left(\frac{n}{e}\right)^{n} \left(\frac{\mu-n}{e}\right)^{\mu-n} \sqrt{(2\pi)\, n\, (\mu-n)}}$$

$$= \frac{\mu^{\mu}}{n^{n} (\mu-n)^{\mu-n} \sqrt{2\pi\, n\, \frac{\mu-n}{\mu}}} = \frac{\mu^{\mu}}{n^{n-\mu}\, n^{\mu}\, (\mu-n)^{\mu-n}}$$

$n < \mu$

$$\le \cancel{\frac{\left(\frac{\mu}{n}\right)^{\mu}}{\left(\frac{\mu}{n}-1\right)^{\mu-n}} = \sqrt{\left(\frac{n}{\mu}\right)^{\mu}}}$$

$\mu p - n = h$

$\mu - n = \mu - \mu p + h$

$= \mu(1-p) + h$

$= \mu q + h$

$$= \mu^{\mu} \left(\frac{p}{n}\right)^{n} \left(\frac{q}{\mu-n}\right)^{\mu-n} \sqrt{2\pi\, \frac{n}{\mu} (\mu-n)}$$

$$= \mu^{\mu} \left(\frac{p}{n}\right)^{n} \left(\frac{1}{\mu + \frac{h}{q}}\right)^{\mu-n}$$

$$\frac{\partial}{\partial x} \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\Delta^2 u$$

$$\frac{\partial}{\partial y} \qquad u\frac{\partial v}{\partial x}$$

$$\cancel{\frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\right)} + u\underbrace{\cancel{\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x \partial y}+\frac{\partial w}{\partial x \partial z}\right)}}_{0} + \cdots \qquad = -\frac{1}{\rho}\Delta^2 p$$

$$\left[\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x}\right] + \nearrow$$

$$\frac{1}{\rho}\Delta^2 p = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + 2\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}$$

$$2\frac{\Delta p}{\rho} = 2\left[\qquad\qquad\right] + \underbrace{4 \cdots}_{\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 - \underbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)^2}_{\zeta^2}}$$

Z tego Φ dla ruchów wirowych:

$$= \mu\left[2\frac{\Delta^2 p}{\rho} + \xi^2 + \eta^2 + \zeta^2\right]$$

Dwie kulki spadające w cieczy równoległ.: Czy one się przyciągają?


Jaka zmiana wskutek Strömungsstrom?

$$-\frac{c}{2}\left[1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right]\omega^2 = \text{const}$$

$$\underbrace{\quad}_{= r = a} \Big|_{=0}$$

$$\frac{\partial}{\partial r}\psi = -\frac{c}{2}\left[+\frac{3}{2}\frac{a}{r^2} - \frac{3}{2}\frac{a^3}{r^4}\right]\Bigg|^{r=a} = 0$$

$$\frac{\partial^2}{\partial r^2} = -\frac{c}{2}\left[-\frac{3a}{r^3} + 3.2.\frac{a^3}{r^5}\right]\Bigg|^{r=a} = -\frac{3c}{2a^2}$$

$$\psi = \text{const} = \omega^2\left[-\frac{3}{2}\frac{c}{a^2}\frac{(r-a)^2}{2} + \cdots\right]$$

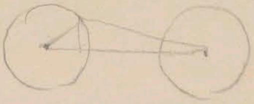
$u = A\frac{\partial^2\Omega}{\partial x^2} + B\left[x\frac{\partial\chi}{\partial x} - \chi\right] + c$

$v = A\frac{\partial^2\Omega}{\partial x\partial y} + B x\frac{\partial\chi}{\partial y}$

$= -\frac{A}{6\mu r^3}(r^2-3x^2) + \frac{2B}{3\mu r} + \frac{A}{r^5}(r^2-3x^2) + c$

$= \frac{B}{3\mu r^3}xy - \frac{3A}{r^5}xy$

~~$-\frac{x}{r^3}$ $-\frac{1}{r^3}+\frac{3x^2}{r^5}$ $\frac{3xy}{r^5}$~~



$\frac{1}{2}\left(\frac{1}{r}-\frac{x^2}{r^3}\right)$ $\chi = \frac{1}{r}$

~~$u = B_1\left(\frac{1}{r_1^3}-\frac{3x_1^2}{r_1^5}\right) + \frac{A_1}{2\mu}\left(\frac{1}{r_1}-\frac{x_1^2}{r_1^3}\right) + B_2\left(\frac{1}{r_2^3}-\frac{3x_2^2}{r_2^5}\right) + \frac{A_2}{2\mu}\left(\quad\right) + c$~~

~~$r_1 = a$ $r_2^2 = (b-x_1)^2 + y_1^2 + z^2$~~

~~$= b^2 + a^2 - 2bx_1$~~

~~$x_2 = x_1$ $y_2 = y_1 - b$ $z_2 = z_1$~~

~~$\frac{B_2}{r_2^5}\left[r_2^2 - 3x_2^2\right] = \frac{B_2}{r_2^5}\left[y_1^2+z^2-2(b-x_1)^2\right]$~~

~~$= \frac{1}{r_2^5}\left[x_1^2+z_1^2+(y_1-b)^2-3x_1^2\right] = \frac{1}{r^5}\Big[$~~

$\frac{\partial\varphi}{\partial r} = -4\pi\sigma$

$\varepsilon = \frac{1}{4\pi}\frac{\partial^2\varphi}{\partial x^2}$

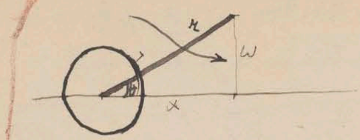
$u = -\frac{p}{4\mu L}(r^2-R^2)$

$2\pi R\,\frac{pR}{2\mu L}\int\varepsilon\,d\sigma = \frac{\pi R^2 p}{4\pi\,\mu L}\frac{\partial\varphi}{\partial r}\Big|$

$\frac{\partial u}{\partial r} = -\frac{pR}{2\mu L}$

$= \frac{\pi R^2 p}{4\pi\,\mu L}\frac{\partial\varphi}{\partial r}\Big|_R = \frac{E}{L}\frac{R^2\pi}{\sigma}$

$E = \frac{p}{4\pi\mu}\sigma\left(\frac{\partial\varphi}{\partial r}\right)_R$



$$u = -\frac{1}{\omega}\frac{\partial\psi}{\partial\omega} \qquad v = \frac{1}{\omega}\frac{\partial\psi}{\partial x}$$

$$u = c\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3} - \frac{3}{4}\frac{a}{r^3}\left(1-\frac{a^2}{r^2}\right)x^2\right]$$

$$v = -c\,\frac{3}{4}\frac{a}{r^3}\left(1-\frac{a^2}{r^2}\right)x\omega$$

$$u = c\left[1 - \frac{3}{4}\frac{a}{r}\left(1+\cos^2\theta\right) - \frac{1}{4}\frac{a^3}{r^3}\left(1-3\cos^2\theta\right)\right]$$

$$v = -\frac{3}{4}\frac{ac}{r}\left(1-\frac{a^2}{r^2}\right)\sin\theta\cos\theta$$

$$u = u_{r=a} + \overset{r-a}{\overset{\parallel}{\rho}}\cdot\frac{\partial u}{\partial r} + \dots$$

$$\frac{\partial u}{\partial r} = c\left[\frac{3}{4}\frac{a}{r^2}\left(1+\cos^2\theta\right) + \frac{3a^3}{4r^4}\left(1-3\cos^2\theta\right)\right]$$

$$= \frac{3}{4}\frac{c}{a}\left[1+\cos^2\theta + 1 - 3\cos^2\theta\right]_{a=r} = \frac{3}{2}\frac{c}{a}\sin^2\theta$$

$$\frac{\partial v}{\partial r} = -\frac{3}{4}ac\sin\theta\cos\theta\left[\underbrace{\left(1-\frac{a^2}{r^2}\right)\frac{-1}{r^2} + \frac{2a^2}{r^4}}_{-\frac{1}{r^2}+\frac{3a^2}{r^4}}\right] = \cancel{\frac{3}{4}\frac{c}{a}\left[1-\frac{3a^2}{r^2}\right.} = -\frac{3}{2}\frac{c}{a}\sin\theta\cos\theta$$

$$u = \frac{3}{2}\frac{c}{a}\sin^2\theta\cdot\rho$$

$$v = -\frac{3}{2}\frac{c}{a}\sin\theta\cos\theta\cdot\rho$$

$$\text{vel} = \sqrt{u^2+v^2} = \frac{3}{2}\frac{c}{a}\sin\theta\cdot\rho$$

$$\text{vel}_{\perp}x = -\left(u\sin\theta + v\cos\theta\right) = -\frac{3}{2}\frac{c}{a}\sin\theta\,\rho\left(\sin^2\theta+\cos^2\theta\right)$$

$$= \frac{3}{2}\frac{c}{a}\sin\theta\cdot\rho$$

więc cały prąd w bliskości powierzchni równoległy do niej

zatem całkowity prąd elektr. równoległy do powierzchni (w szerokości geogr. θ):
(pro cm obwodu)

pod założeniem że grubość warstwy mała w porównaniu do a

$$i = \int_0^c u\,\varepsilon\, d\rho = \frac{-1}{4\pi}\int u\,\frac{\partial^2\varphi}{\partial\rho^2}\, d\rho$$

$$= \frac{-1}{4\pi}\,\frac{3}{2}\,\frac{c}{a}\,\sin\theta \int \underbrace{\frac{\partial^2\varphi}{\partial\rho^2}\,\rho\, d\rho}$$

$$= \cancel{\frac{\partial\varphi}{\partial\rho}\cdot\rho}\Big|_0^\infty - \int \frac{\partial\varphi}{\partial\rho}\, d\rho = \varphi_i - \varphi_e$$

$$= \frac{3}{2}\,\frac{c}{a}\,\frac{\sin\theta}{4\pi}\,(\varphi_e - \varphi_i)$$

więc gęstość prądu na 1 cm² przekroju:

$$\frac{3}{2}\,\frac{c}{a}\,\frac{\sin\theta}{4\pi}\,\frac{\varphi_e-\varphi_i}{\rho}$$

~~Zatem zadanie: znaleźć~~ Zatem różnica potencjałów

$$I = \lambda\,\frac{\partial U}{\partial x} = \frac{\lambda}{a}\,\frac{\partial U}{\partial\theta}$$

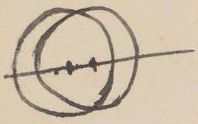
$$\cancel{E = \frac{\partial U}{\partial x}}$$

$$U = \frac{1}{\lambda}\,\frac{3}{2}\,\frac{c}{a}\,\frac{\varphi_e-\varphi_i}{4\pi}\; a\int \sin\theta\, d\theta$$

Potencjał na zewn. powierzchni: $\frac{3}{2}\,\frac{c}{4\pi}\,\frac{\varphi_e-\varphi_i}{\lambda}\cos\theta$ $\quad :\rho$

Jaki system prądów powstanie wskutek rozkładu potencjału $U = A\cos\theta$?

$$\int \sin\theta\cos\theta\, d\theta = -\frac{1}{4}\cos 2\theta = \frac{\sin^2\theta}{2}$$



$$U_1 = \frac{M_1}{r_1} \qquad U_2 = -\frac{M_1}{r_2}$$

$$U = U_1 + U_2 = M_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{M_1}{r^2}\cos\theta$$

więc jeśli $\frac{M_1}{a^2} = \frac{3}{2}\,\frac{c}{4\pi\lambda}(\varphi_e - \varphi_i)$ | $I_1 = \lambda\frac{M_1}{r^2}$ $\quad I = \frac{\partial I_1}{\partial\ldots} = \frac{2\lambda M_1}{r^3}\cos\theta$

to będzie wogóle $U = \frac{3}{2}\,\frac{c}{4\pi\lambda}(\varphi_e - \varphi_i)\,\frac{a^2}{r^2}\cos\theta$ || stąd $I_a = \cancel{2\lambda}\,\frac{3}{\cancel{2}}\,\frac{c}{4\pi\cancel{\lambda}}\,\frac{(\varphi_e-\varphi_i)\cos\theta}{a}$

$$\int a^2 \sin\theta\, d\theta\; I_a =$$

$$\frac{\frac{3ac}{4\pi}(\varphi_e-\varphi_i)\frac{\cos 2\theta}{2}}{\cancel{a}\sin\theta\cdot\rho}$$

Energia zużyta $\sum i\left(U_{+\theta} - U_{-\theta}\right) =$

$$di = \frac{3}{2}\frac{c}{a}\frac{\varphi_e-\varphi_i}{4\pi}\cos\theta\, d\theta$$

$$W = \int_0^{\frac{\pi}{2}} 2\pi a \sin\theta \; \frac{3}{2}\frac{c}{a}\frac{\varphi_e-\varphi_i}{4\pi}\cos\theta \cdot 2\frac{3}{2}\frac{c}{4\pi}\frac{\varphi_e-\varphi_i}{\lambda}\cos\theta\, d\theta$$

$$= 4\pi \left(\frac{3}{2}\frac{c}{4\pi}\right)^2 \frac{1}{\lambda}(\varphi_e-\varphi_i)^2 \int \underbrace{\cos^2\theta \sin\theta\, d\theta}_{= -\frac{\cos^3\theta}{3}}$$

$$= \frac{3c^2}{16\pi^2}\frac{(\varphi_e-\varphi_i)^2}{\lambda}$$

zatem opór przeciwko ruchowi $\frac{W}{c} = \frac{3c}{\lambda}\left(\frac{\varphi_e-\varphi_i}{4\pi}\right)^2$

(który dodać należy do oporu tarcia):

Opór tarcia : $6\pi\mu a c$

$$1 - \frac{6x^2}{r^2} + \underbrace{\frac{9x^4}{r^4} + 9\frac{x^2\omega^2}{r^4}}_{9\frac{x^2}{r^2}} \qquad 1+\frac{3x^2}{r^2}$$

$$v = \frac{Ca^2}{r^3}\sqrt{1+\frac{3x^2}{r^2}}$$

$$C\frac{a^2}{r^2}\cos\theta = U = C\frac{a^2 x}{r^3}$$

$$-\frac{dx}{d\omega} = \frac{\frac{\partial\chi}{\partial\omega}}{\frac{\partial\chi}{\partial x}} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial\omega}}$$

$$U = \frac{Ca^2x}{(x^2+\omega^2)^{3/2}}$$

$$v_x = -\frac{\partial U}{\partial x} = -\frac{Ca^2}{r^3}\left[1 - \frac{3x^2}{r^2}\right]$$

$$v_\omega = -\frac{\partial U}{\partial\omega} = +\frac{3Ca^2x\omega}{r^5}$$

$$-\frac{3x\omega}{r^2}\overset{dx}{\overset{\parallel}{\frac{\partial\chi}{\partial\omega}}} + \left[1-\frac{3x^2}{r^2}\right]\overset{d\omega}{\overset{\parallel}{\frac{\partial\chi}{\partial x}}} = 0$$

~~$\frac{\partial\chi}{\partial x} - \frac{3x}{r}\left(\frac{\partial\chi}{\partial\omega}\frac{\omega}{r} + \frac{\partial\chi}{\partial x}\frac{x}{r}\right) = 0$~~

$$\underset{\frac{\partial r}{\partial\omega}}{\parallel} \qquad \underset{\frac{\partial r}{\partial x}}{\parallel}$$

~~$\left(1-\frac{3x^2}{r^2}\right)\frac{d\omega}{dx} + \frac{3x\omega}{r^2}\frac{dx}{d\omega} = 0$~~

~~$\frac{dx}{d\omega} + \left(1-\frac{3x^2}{r^2}\right)\frac{r^2}{3x\omega} = 0$~~

~~$\frac{r^2}{3\omega x} - \frac{3x}{\omega}$~~

$$\boxed{\frac{\omega^2}{r^3}} \qquad 2\frac{\omega}{r^3} - \frac{3\omega^3}{r^5} - \frac{3\omega^2 x}{r^5}$$

$$2(\omega^2+x^2) - 3\omega^2$$

$$\left(1-\frac{3x^2}{r^2}\right)d\omega + \frac{3x\omega}{r^2}dx = 0$$

$$\underset{\frac{\partial\chi}{\partial\omega}}{\underset{\parallel}{(\omega^2-2x^2)}}\,d\omega + \underbrace{3\omega x}_{\frac{\partial\chi}{\partial x}}\,dx = 0$$

$$d\left(\frac{\omega^2}{r^3}\right) = \left(\frac{2\omega}{r^3} - \frac{3\omega^3}{r^5}\right) d\omega ~~+~~ - \frac{3\omega^2 x}{r^5}\,dx$$

$$= \frac{\omega}{r^5}\left[(2x^2 + 2\omega^2 - 3\omega^2)\,d\omega - 3\,\underline{\omega x}\,dx\right]$$

$$= \frac{\omega}{r^5}\left[~~4~~ (2x^2 - \omega^2)\,d\omega - 3\,\omega x\,dx\right]$$

Więc linie prądu : $\frac{\omega^2}{r^3} = \text{const}$

$U = c\frac{a^2 x}{r^3} = \frac{ca^2}{r^2}\cos\theta$

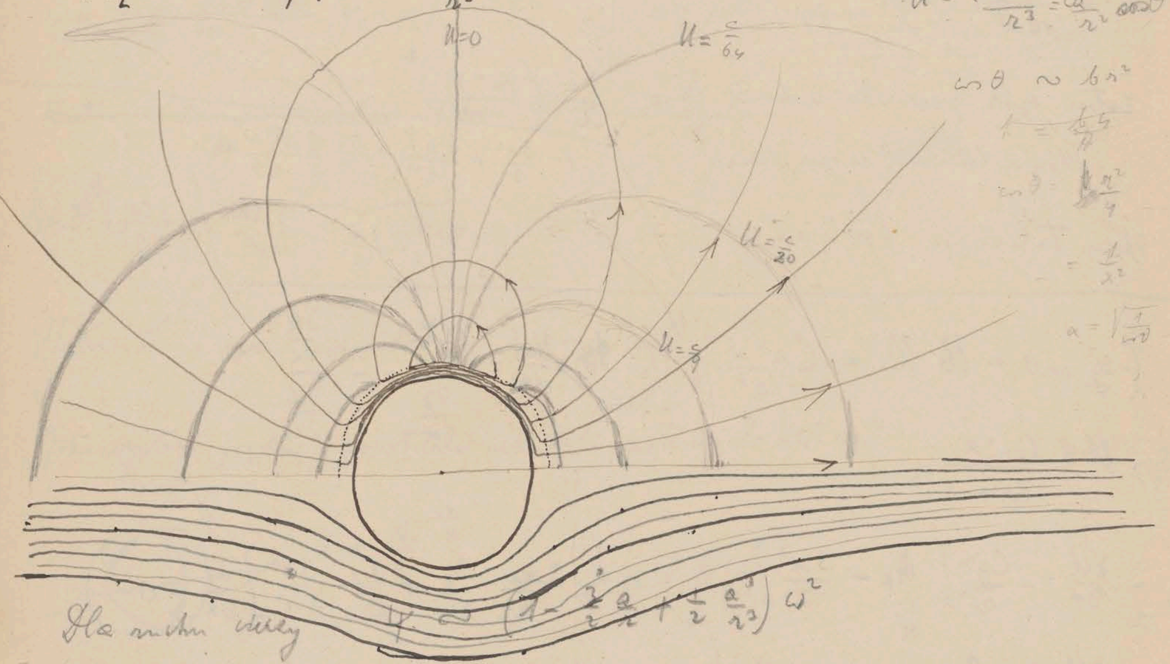
$\cos\theta \sim b r^2$

~~$1 = \frac{b\,5}{4}$~~

$\cos\theta = ~~b~~ \frac{r^2}{4}$

$= \frac{1}{x^2}$

$a = \sqrt{\frac{1}{\cos\theta}}$



Dla ruchu cieczy $\Psi \infty \left(1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)\omega^2$

$r = 2a$ $\quad (\quad) = 1 - \frac{3}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{8} = ~~1~~ \frac{16 - 12 + 1}{16} = \frac{5}{16}$ $\quad \psi = 1 \quad \omega^2 = \frac{16}{5} \quad \omega = \frac{4}{\sqrt{5}}$

$r = 3a$ $\quad ~~1~~ - \frac{1}{2} + \frac{1}{2}\frac{1}{27} = ~~[illegible]~~ \frac{14}{27}$

$r = 4a$

$$\cancel{1-\frac{3}{2}a}\ \operatorname{tg}\varphi_{\text{grenz}} = \frac{v_\omega}{u} = \frac{-\frac{3}{4}\frac{ca}{r^3}\left(1-\frac{a^2}{r^2}\right)x\,\omega}{-\frac{3}{4}\frac{ca}{r^3}\left(1-\frac{a^2}{r^2}\right)x^2 + c\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)}$$

$$= \frac{\omega}{x - \frac{4r^3}{3ax}\,\frac{1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}}{1-\frac{a^2}{r^2}}} = \frac{\omega}{x - \frac{4r^3-3ar^2-a^3}{3ax\left(1-\frac{a^2}{r^2}\right)}}$$

$$\frac{m}{\omega^2} = 1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}$$

$x=0$ $r=\omega$	$\frac{m}{\omega^2} = 1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}$	$m=1$	$m=\frac{1}{4}$	$m=\frac{1}{9}$
$r = 1a$	$\frac{m}{\omega^2} = 0$	$\omega=$		
$r = \frac{3}{2}a$	$\frac{m}{\omega^2} = 1-1+\frac{1}{2}\frac{8}{27} = \frac{4}{27}$	$\sqrt{\frac{27}{4}} = 2{\cdot}6$	$1{\cdot}3$	$0{\cdot}87$
$r = 2a$	$= 1-\frac{3}{2}\frac{1}{2}+\frac{1}{2}\frac{1}{8} = \frac{16-12+1}{16} = \frac{5}{16}$	$1{\cdot}8$	$0{\cdot}9$	$0{\cdot}6$
$r = 3a$	$= 1-\frac{3}{2}\frac{1}{3}+\frac{1}{2}\frac{1}{27} = \frac{14}{27}$	$1{\cdot}4$	$0{\cdot}7$	$0{\cdot}47$
$r = 4a$	$= 1-\frac{3}{2}\frac{1}{4}+\frac{1}{2}\frac{1}{64} = \frac{81}{128}$ $1{\cdot}58$	$1{\cdot}26$	$0{\cdot}63$	$0{\cdot}42$

$m = 1$

Przy przepływie przez rurkę

$$U = \frac{\pi P R^4}{8k^2 L}$$

$$E = \frac{P\,6\,(\varphi_i - \varphi_a)}{4\pi k^2 \eta}$$

$$J = \frac{E R^2 \pi}{6 L}$$

$$\text{Praca} = EJ = \frac{P^2 R^2\, 6\,(\varphi_i - \varphi_a)^2}{4k^2 L \cdot 4\pi k^2} = U \cdot \frac{(\varphi_i - \varphi_a)^2\, 6 P}{2 R^2 \pi^2 k^2}$$

~~$\frac{1{\cdot}3 \cdot 10^{6} \cdot 4{\cdot}7 \cdot 4 \cdot 10^{8}}{1 \cdot (0{\cdot}01)^{2} \cdot 3 \cdot 10^{10}}$~~ $= \Delta p \cdot U$

Więc ciśnienie dodatkowe, spowodowane elektr.:

$$\Delta p = \frac{(\varphi_i - \varphi_a)^2\, 6 P}{2 R^2 \pi^2 k^2}$$

Np. $\varphi_i - \varphi_a = 4V = 4$

~~$\frac{10^{-3}\,10^{9}}{(3 \cdot 10^{10})^2} = \frac{10^{6}}{10^{23}} = 10^{-17}$~~

~~elst. $\frac{l\,m}{t^2} \cdot t = \frac{l\,m}{t}$~~

~~elmg. $\frac{l^3\,m}{t^4} \cdot \frac{l^2}{t}$~~

elast.

Convectionsstr.:

$$\text{Siła prądu} = c \cdot 4\pi \frac{\partial\varphi}{\partial x^2} \cdot q = \frac{l}{t} \cdot \frac{l}{l^2} \cdot \frac{l^{\frac{1}{2}} m^{\frac{1}{2}}}{t} = \frac{l^{+\frac{3}{2}} m^{\frac{1}{2}}}{t^2} = \frac{\text{ilość}}{\text{czas}}$$

element nie ... bo wchodzi w jej wymiar pk !!

~~$\frac{l^{3/2} m^{1/2}}{t^2} = \frac{l^{5/2} m^{1/2}}{t^3}$~~

$$4{\cdot}6 \cdot \frac{10^{-8} \cdot 10^{16}}{1{\cdot}17} =$$

$$Hg = \frac{10^4 \cdot 10^9}{9 \cdot 10^{20}} = \frac{1}{9 \cdot 10^7}\ (\text{estr.})$$

$$\tau = \frac{6}{1{\cdot}17} \cdot 10^{16}$$

$$\frac{4{\cdot}6}{1{\cdot}17} \cdot 10^{8}\ (Hg\ \text{o}\ 1)$$

$6 \cdot 10^8$

$$\frac{1{\cdot}3 \cdot 4{\cdot}7 \cdot 4\ \ 10^{8}}{3{\cdot}1 \cdot 3{\cdot}24} \cdot \frac{1}{9 \cdot 10^{20}} =$$

$\tau_{Hg=1} = 10^{16} \cdot \frac{6}{1{\cdot}17} = 4 \cdot 10^{8} =$ ~~$\frac{4}{9} \cdot 10^{?} \cdot 10^{-11} = \frac{4}{9} \cdot 10^{-3}$ (CSS)~~

$k = 0{\cdot}018$

$\frac{1}{\tau}\, 10^{9} = 2{\cdot}5$

$\tau = \frac{10^{9}}{2{\cdot}5} = 4 \cdot 10^{8}$ (Hg = 1)

$\frac{4\pi\,(0{\cdot}018)\; 2{\cdot}2}{\frac{4}{9} \cdot 10^{-7} \cdot 10^{6} \cdot 1{\cdot}3} =$

sp. $\mathcal{E}^{e}$ Hg = $10^{-4}\ \frac{\text{Ohm}}{\text{cm}} = \frac{\cdot 10^{-15}}{9}$ (CSS)

$= 1{\cdot}1 \cdot 10^{-16}$ (CSS)

~~$9\pi \cdot 2{\cdot}2 \cdot 0{\cdot}00032 =$~~

~~$9 \cdot 2{\cdot}2 \cdot 0{\cdot}01 = 0{\cdot}2$~~

10^{-7}

$\frac{9\pi \cdot 2{\cdot}2 \cdot 0{\cdot}18}{1{\cdot}3 \cdot 1{\cdot}8} =$ ~~$\frac{9 \cdot \pi \cdot 0{\cdot}4}{1{\cdot}3 \cdot 1{\cdot}8} = 49$~~

1
0·696
0·11
180

0·033 . 21
663

0·000244 . 441
976
976
23
1076

~~[illegible]~~ $= \pi \cdot \frac{2}{1{\cdot}3} = 6$

$\frac{4\pi \cdot \frac{1{\cdot}78}{1{\cdot}80} \cdot 2{\cdot}113}{4{\cdot}65 \cdot 10^{-8} \cdot 1{\cdot}33 \cdot 10^{8}} = \frac{3{\cdot}14 \cdot 2{\cdot}11}{1{\cdot}16 \cdot 1{\cdot}33} = \frac{3{\cdot}14 \cdot 2{\cdot}11}{1{\cdot}5} = 4 \ldots$

$\frac{\Delta p}{P} = \frac{(\varphi_i - \varphi_a)^2\, \sigma}{2 R^2 \pi^2 \cdot \eta}$

$\varphi_i - \varphi_a = 4\,V = \frac{4}{300}$ (CSS)

$\sigma = \frac{4}{9} \cdot 10^{-7} = 4{\cdot}6 \cdot 10^{-8}$

$\eta = 0{\cdot}010$

$R = 0{\cdot}2294$ mm

$= \left(\frac{4}{300 \cdot 0{\cdot}0229 \cdot 3{\cdot}14}\right)^2 \frac{4{\cdot}6 \cdot 10^{-8}}{2 \cdot 0{\cdot}01}$

$= \left(\frac{4}{3{\cdot}14 \cdot 3 \cdot 0{\cdot}23}\right)^2 2{\cdot}3 \cdot 10^{-4} = \left(\frac{4}{2{\cdot}2}\right)^2 \cdot 2{\cdot}3 \cdot 10^{-4} = \frac{4}{1{\cdot}21} \cdot 2{\cdot}3 \cdot 10^{-4}$

$= 0{\cdot}8 \cdot 10^{-3}$

Kulki

$$\frac{\text{Opór elektrostat.}}{\text{Opór tarcia}} = \frac{\frac{3a}{\lambda}\left(\frac{\varphi_a-\varphi_1}{4\pi}\right)^2}{6\pi\mu a c} = \frac{\frac{1}{\lambda}\left(\frac{\varphi_a-\varphi_1}{4\pi}\right)^2}{2\pi\mu a\eta}$$

Np. $\varphi_a-\varphi_i = 4V = \frac{4}{300}$ CGS

$$\frac{1}{\lambda} = 6 = 4{\cdot}6\cdot10^{-8}$$

$$\mu\eta = 0{\cdot}010$$

$$\frac{4{\cdot}6\cdot10^{-8}\cdot\left(\frac{\cancel{4}}{300\cdot\cancel{4}\cdot3{\cdot}14}\right)^2}{2\cdot3{\cdot}14\cdot0{\cdot}01\cdot a} = \frac{2{\cdot}3\cdot10^{-6}\left(\frac{1}{1000}\right)^2}{3{\cdot}14\cdot a}$$

Więc opór el. = opór tarcia jeśli $a = 10^{-12}$ cm !

Np. opór el. = opór tarcia jeśli:

$$\left(\frac{\cancel{4}\,1}{\cancel{4}\cdot300\cdot3{\cdot}14}\right)^2\cdot\frac{4{\cdot}6\cdot10^{-8}}{0{\cdot}01\cdot a^2} = 1 = \left(\frac{1}{0{\cdot}942}\right)^2\cdot\frac{4{\cdot}6\cdot10^{-12}}{a^2}$$

$$a^2 = \frac{4{\cdot}6}{0{\cdot}88}\cdot10^{-12} = 5\cdot10^{-12}$$

$$a \neq 2\cdot10^{-6}\ \text{cm} = 2\cdot10^{-2}\,\mu = 0{\cdot}02\,\mu$$

Robiąc założenie $U = \frac{M\cos\theta}{r^2}$ otrzymam

się prąd w kierunku r : $J_r = 2\lambda\frac{M\cos\theta}{r^3}$ na powierzchni $J_a = 2\lambda\frac{M\cos\theta}{a^3}$

zatem całkowity prąd od $\theta=0$ do θ :

$$\int 2a^2\pi\sin\theta\,d\theta\,\frac{2\lambda M\cos\theta}{a^3} = \frac{4\pi\lambda M}{a}\int_0^\theta \underbrace{\sin\theta\cos\theta\,d\theta}$$

$$\frac{1}{4}\int\sin 2\theta\,d2\theta = \frac{1-\cos 2\theta}{4} = \frac{2\sin^2\theta}{4}$$

$$= \frac{2\pi\lambda M}{a}\sin^2\theta$$

to musi przechodzić przez równoleżnik θ

zatem natężenie prądu pro 1 cm długości równoleżnika : $\frac{\frac{2\pi\lambda M\sin^2\theta}{a}}{2a\pi\sin\theta} = \frac{\lambda M\sin\theta}{a^2}$

jeśli to ma być równe $\frac{3}{2}\frac{c}{a}\sin\theta\,\frac{\varphi_e-\varphi_i}{4\pi}$

to musi być: $M = \frac{3}{2}\frac{ac}{\lambda}\frac{\varphi_e-\varphi_i}{4\pi}$ zatem: $U = \frac{3}{2}\frac{ac}{\lambda}\frac{\varphi_e-\varphi_i}{4\pi}\frac{\cos\theta}{r^2}$

$$\frac{\partial U}{\partial r}\Big|_a = \frac{3c}{\lambda a^2}\frac{\varphi_e-\varphi_i}{4\pi}\cos\theta$$

Praca całkowita:

$$W = 2\int_0^{\frac{\pi}{2}} 2a^2\pi\sin\theta\,d\theta\,.\,U_{a\theta}\,\lambda\frac{\partial U_{a\theta}}{\partial r}$$

$$= 4a^2\pi\,\frac{3}{2}\frac{c}{\lambda a}\frac{\varphi_e-\varphi_i}{4\pi}\cdot\lambda\frac{3c}{\lambda a^2}\frac{\varphi_e-\varphi_i}{4\pi}\int_0^{\frac{\pi}{2}}\underbrace{\cos^2\theta\sin\theta\,d\theta}_{\frac{\cos^3\theta}{3}\,=\,\frac{1}{3}}$$

$$= \frac{9c^2(\varphi_e-\varphi_i)^2}{8\pi a\lambda}\cdot\frac{1}{3} = \frac{3c^2(\varphi_e-\varphi_i)^2}{8\pi a\lambda} \qquad \frac{W}{c} = \Delta p = \frac{3c(\varphi_e-\varphi_i)^2}{8\pi a\lambda}$$

Opór elektr. / Opór tarcia $= \frac{\frac{3(\varphi_e-\varphi_i)^2}{8\pi a\lambda}}{6\pi\eta a} = \frac{(\varphi_e-\varphi_i)^2}{16\pi^2a^2\eta\lambda} = \left(\frac{\varphi_e-\varphi_i}{4\pi}\right)^2\frac{1}{\eta\lambda a^2}$

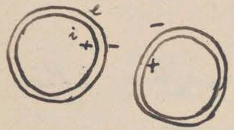
Jaki bezpośredni wpływ ładunków elektr.?

Jeśli różnica pot. dana to ładunek prop. a^2

zatem siła $\sim \frac{a^2 \cdot a^2}{r^2}$ najmniejsze ~~r~~ możliwe $= 2a$

$\therefore f \sim a^2$ zatem przy zmniejszeniu cząstek wielce zmniejszający się wpływ. Zatem trudno w rachubę wejść nie może.

Twierdzę, że „Doppelschicht" zatem z takiż ładunek przeciwny wewnątrz



~~Energia: $\frac{1}{2}\sum \varphi v$~~

$\varphi_i = -\varphi_e$ zatem wpływ na zewnątrz $= 0$

W polu elektr. zewnętrznem:

Gdyby φ_i i φ_e były stale związane, toby skutek był $= 0$,

tak jednak φ_e ~~[illegible]~~ będzie miało dążności w jednym kierunku φ_i w drugim

---

$$\left(\nabla u \frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(U \nabla^2 U\right) - U \nabla^2 \frac{\partial U}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{4\pi} U \nabla^2 U - p\right) = U \nabla^2 \frac{\partial U}{\partial x} - k \nabla^2 u$$

$$\frac{\partial}{\partial y}\left(\qquad \right) = U \nabla^2 \frac{\partial U}{\partial y} - k \nabla^2 v$$

$$\frac{\partial U}{\partial y} = -\frac{3}{2} c \frac{a^3 xy}{r^5}$$

$$\frac{\partial U}{\partial z} = -\frac{3}{2} \frac{c a^3 xz}{r^5}$$

$$\frac{\partial U}{\partial \omega} = -\frac{3}{2} \frac{c a^3 x \omega}{r^5}$$

~~$\frac{\partial U}{\partial (\omega)} = \frac{\partial U}{\partial x}$~~

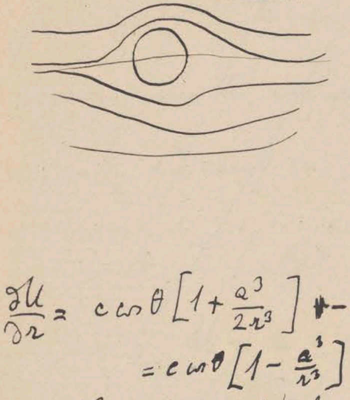
$$r = a \quad \Big\| \quad \frac{\partial U}{\partial x} = \frac{3}{2} c \left[1 - \frac{x^2}{a^2}\right]$$

$r^2 = t^2 + r_0^2$

$\frac{\partial U}{\partial t} = \frac{3t}{2}\,\frac{a^3 c x}{r^4}$

$\frac{\partial^2 U}{\partial t^2}\Big|_0 = \frac{3}{2}\,\frac{cx}{a}$

Jaki rozkład przy działaniu zewnętrznej siły $\overline{X}$



$$\nabla^2 U = 0 \qquad \frac{\partial U}{\partial r}\Big|_{r=a} = 0$$

$$U = cx\left[1 + \frac{a^3}{2r^3}\right] = cr\cos\theta\left[1 + \frac{a^3}{2r^3}\right]$$

$$\frac{\partial U}{\partial x} = c\left[1 + \frac{a^3}{2r^3}\right] - \frac{3cx^2a^3}{2r^5}\Big|_{r=\infty} = c$$

$$\frac{\partial U}{\partial r} = c\cos\theta\left[1 + \frac{a^3}{2r^3}\right] - cr\cos\theta\,\frac{3a^3}{2r^4} = c\cos\theta\left[1 + \frac{a^3}{2r^3} - \frac{3a^3}{2r^3}\right]\Big|_{r=a} = 0$$

$$= c\cos\theta\left[1 - \frac{a^3}{r^3}\right]$$

Na powierzchni kuli:

$$\frac{\partial U}{\partial(a\theta)} = \frac{-1}{a}\,cr\sin\theta\left[1 + \frac{a^3}{2r^3}\right]\Big|_{r=a} = -\frac{3c}{2}\sin\theta$$

$$\frac{\partial^2 U}{\partial r^2} = c\cos\theta\left[\frac{3a^3}{r^4}\right|_{r=a} = \frac{3c\cos\theta}{a}$$

Ogólne zadanie:

Ruch cieczy:

$$\overline{X} - \frac{\partial p}{\partial x} = -\overset{\frac{\mu}{\rho}}{k}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$X = \varepsilon\,\xi$

ξ = natężenie zewn. pola elektr. (zaniedbuje modyfikację wskutek własnych prądów)

ogólnie [illegible]:

$$\overline{X} = \varepsilon\,\frac{\partial U}{\partial x} =$$

$$\frac{1}{4\pi}\nabla^2 U\cdot\frac{\partial U}{\partial x} - \frac{\partial p}{\partial x} = -k\nabla^2 u \qquad \frac{\partial}{\partial x}$$

$$\frac{1}{4\pi}\nabla^2 U\cdot\frac{\partial U}{\partial y} - \frac{\partial p}{\partial y} = -k\nabla^2 v \qquad \frac{\partial}{\partial y}$$

$$\frac{1}{4\pi}\nabla^2 U\cdot\frac{\partial U}{\partial z} - \frac{\partial p}{\partial z} = -k\nabla^2 w \qquad \frac{\partial}{\partial z}$$

$$\nabla^2 p = \frac{1}{4\pi}\left\{\left(\nabla^2 U\right)^2 + \left[\frac{\partial U}{\partial x}\nabla\frac{\partial U}{\partial x} + \cdots\right]\right\}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$\nabla^2 u = 0$ wszędzie z wyjątkiem warstwy grubości δ na powierzchni

We warstwach powierzchniowych :

$$\frac{\partial^2 U}{\partial r^2} = \frac{\partial}{\partial r}\left\{c\cos\theta\left[1-\frac{a^3}{r^3}\right]\right\}$$

$$= c\cos\theta\,\frac{3a^3}{r^4}\Big|_{r=a} = \frac{3c\cos\theta}{a}$$

$$\frac{\partial^3 U}{\partial r^3} = -c\cos\theta\,\frac{12a^3}{r^5}\Big|_{r=a} = -\frac{12c\cos\theta}{a^2}$$

Zatem $U = U_a + \left(\frac{\partial U}{\partial r}\right)_a \rho + \frac{\rho^2}{1.2}\left(\frac{\partial^2 U}{\partial r^2}\right)_a + \cdots$

$$= \frac{\rho^2}{1.2}\,\frac{3c\cos\theta}{a} - \frac{\rho^3}{1.2.3}\,\frac{12c\cos\theta}{a^2} - $$

$$\underbrace{U = \frac{3c\cos\theta}{2a}\rho^2}_{} - \frac{2c\cos\theta}{a^2}\rho^3 + \cdots$$

Prąd w kierunku stycznym (pro cm obwodu równoleżnika):

$$\lambda\frac{\partial U}{\partial(r\theta)_{r=\text{const}}} = \frac{\lambda}{r}\frac{\partial U}{\partial\theta} = -\lambda c\sin\theta\left[1+\frac{a^3}{2r^3}\right]$$

$$= ~~-\lambda c\sin\theta\left[1+\right.~~$$

$$= -\frac{3}{2}\lambda c\sin\theta\left[1+\frac{[illegible]\rho}{a}\right]$$

~~Więc ruch mechaniczny c powodujący prąd~~

~~$i = \frac{3}{2} \frac{c}{a} \frac{\varphi_e - \varphi_i}{4\pi} \sin\theta$ [po cm długości równoleżnika θ]~~

~~spowoduje przez to układ potencjału:~~

~~$-\frac{3}{2} \alpha\lambda \sin\theta = \frac{3}{2} \frac{c}{a} \frac{\varphi_e - \varphi_i}{4\pi} \sin\theta$~~

~~$\therefore \quad \alpha = -\frac{c}{a} \frac{\varphi_e - \varphi_i}{4\pi\lambda}$~~

~~$\therefore \quad U = -\frac{c}{a} \frac{\varphi_e - \varphi_i}{4\pi\lambda} r\cos\theta \left[1 + \frac{a^3}{2r^3}\right]$~~

~~Albo też odwrotnie: jeśli mamy~~

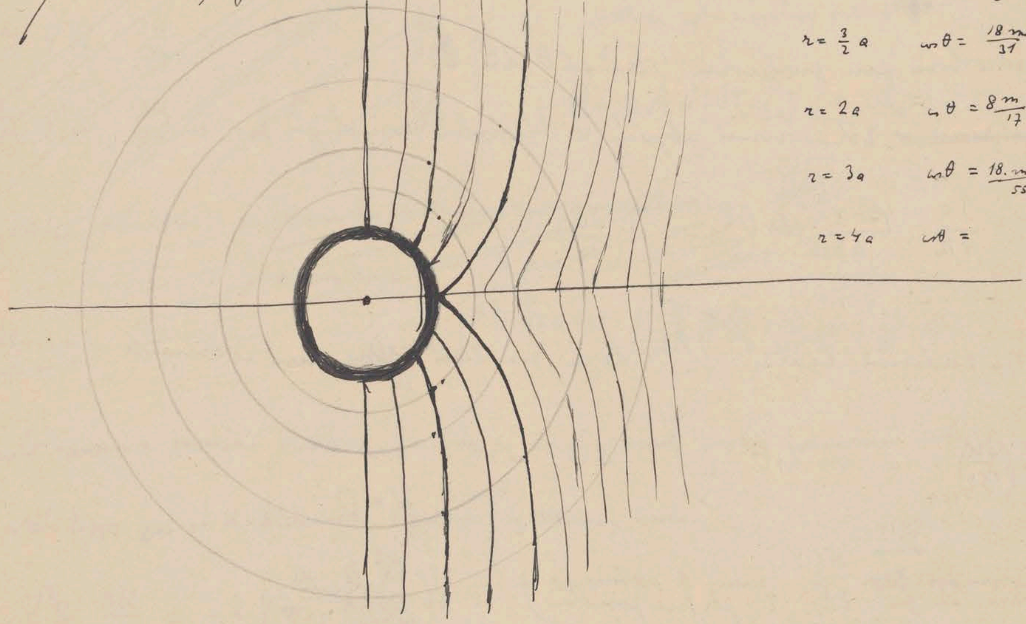
$r = a \qquad \cos\theta = \frac{m}{\frac{3}{2}} = \frac{2}{3} m$

$r = \frac{3}{2} a \qquad \cos\theta = \frac{18 m}{31} \mp \frac{6}{18}$

$r = 2a \qquad \cos\theta = \frac{8 m}{17} = 0{\cdot}47$

$r = 3a \qquad \cos\theta = \frac{18. m}{55} \mp \frac{1}{3} m$

$r = 4a \qquad \cos\theta = \qquad \mp \frac{1}{4} m$



Więc właściwie zadanie:

Znaleźć potencyał U :

$\Delta U = 0$ a na powierzchni kuli dane wartości dla $\frac{\partial U}{\partial r}$

Ponieważ prąd z kuli przypływa do cieczy z daną siłą

~~$\frac{i'-i}{a\,d\theta} = \frac{1}{a}\frac{di}{d\theta} = \frac{3}{2}\frac{c}{a^2}\frac{\varphi_e-\varphi_i}{4\pi}\cos\theta$~~

Prąd przez cały równoleżnik: $\frac{2\pi i}{a\sin\theta} = \frac{3}{4}c(\varphi_e-\varphi_i)\sin^2\theta$

różnica $dJ = \frac{3c}{2}(\varphi_e-\varphi_i)\sin\theta\cos\theta\,d\theta$

podzielona przez powierzchnię: $2\pi a^2\sin\theta\,d\theta$.

~~natężenie~~ zatem musi ubywać w kierunku normalnym po cm²:

$$\frac{3c}{4\pi}\frac{\varphi_e-\varphi_i}{a^2}\cos\theta = \lambda\left(\frac{\partial U}{\partial r}\right)_{r=a,\,\theta}$$

Więc zadanie znaleźć U dane na kuli [illegible] przez warunek

$$\left(\frac{\partial U}{\partial r}\right)_{r=a} = m\cos\theta$$

oraz w nsk. $r=\infty$
$U=0$

taki U jest np.: (jedyne zresztą) : $U = \frac{M\cos\theta}{r^2}$

$$\lambda\left(\frac{\partial U}{\partial r}\right)_{r=a} = -\frac{2\lambda M\cos\theta}{a^3} = \frac{3c}{4\pi}\frac{\varphi_e-\varphi_i}{a^2}\cos\theta$$

$$M = -\frac{3ca}{8\lambda\pi}\,\varphi_e-\varphi_i$$

zatem: $U = -\frac{3ca}{2\lambda}\frac{\varphi_e-\varphi_i}{4\pi}\frac{\cos\theta}{r^2}$

$u = u_0 + u_1$

$-\frac{\partial p}{\partial x} = -k\nabla^2 u_0$

$-\frac{\partial p}{\partial y} = -k\nabla^2 v_0$

———

$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial x} = -k\nabla^2 u_1$

$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial y} = -k\nabla^2 u_2$

$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial z} = -k\nabla^2 u_3$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \dots = 0$

$\frac{\partial u_0}{\partial x} + — = 0$

$\frac{\partial u_1}{\partial x} + — = 0$

~~$\int U\nabla^2 V = \int U \frac{\partial V}{\partial n} dS - \int\left(\frac{\partial U}{\partial x}\frac{\partial V}{\partial x} \dots\right)$~~

~~u = prędkości w kierunku x~~

Ogólne równania:

~~$\frac{1}{4\pi}\nabla^2 U \cdot \frac{\partial U}{\partial x} = -k\nabla^2 u + \frac{\partial p}{\partial x}$~~

~~$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial y} = -k\nabla^2 v + \frac{\partial p}{\partial y}$~~

~~$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial z} = -k\nabla^2 w + \frac{\partial p}{\partial z}$~~

Przyjmujemy teraz dla U: superpozycję rozkładu przy działaniu zewnętrznego pola X i rozkładu pot. warstwy podwójnej:

$$U = A x\left[1 + \frac{a^3}{2r^3}\right] + U_0$$

~~prędkości składowe: w kierunku południków (r = const) = q $\frac{\partial U}{\partial x}$ =~~

$r = a$

~~w kierunku r~~

~~$\frac{\partial q}{\partial \psi} = 0$ $\frac{\partial s}{\partial \psi} = 0$~~

$$\int \nabla^2 U \, dx\,dy\,dz = \int \frac{\partial U}{\partial x}\cancel{\frac{\partial z}{\partial x}} + \cancel{\frac{\partial U}{\partial y}\frac{\partial z}{\partial y}} + \frac{\partial U}{\partial z} \cdot df - \int$$

Jeśli można uważać grubość warstwy za bardzo małą wobec promienia kuli to układając oś x w kierunek stycznej $=s$ będziemy mieli

$$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial s} = -k\nabla^2 v_s + \frac{\partial p}{\partial s}$$

w kierunku r $(\perp s)$ $\frac{\partial U}{\partial s}$ mało zmienne

$$= -\frac{3}{2} A \sin\theta\left[1 + \frac{\rho}{a}\right]$$

A całkując to $\int \rho\, d\rho \underbrace{\nabla^2 U}_{=\frac{\partial^2 U}{\partial \rho^2}}$ będzie

$$= \rho\frac{\partial U}{\partial \rho} - \int \frac{\partial U}{\partial \rho} = \varphi_i - \varphi_0$$

$\frac{\partial p}{\partial s}$ takie przybliżenie stałe w długości ρ

zatem $\int \rho\, d\rho \frac{\partial p}{\partial s} = \frac{\partial p}{\partial s}\frac{\rho^2}{2} \neq 0$

$$\frac{3}{2} A \sin\theta \frac{\varphi_a - \varphi_i}{4\pi} = k\left(\cancel{v_{si}} - v_{sa}\right) + \rho\frac{dv_s}{d\rho}$$

można to stąd $= -v_{sa} k$

jeśli $\lim_{\rho=0} \rho\frac{\partial v}{\partial \rho} = 0$ [co każdym razem dopiero musi być a posteriori okazane]

To odnosi się do warstwy powierzchniowej, bo w reszcie przestrzeni

$\varepsilon = 0$ ~~$\operatorname{rot} \nabla^2 u_1 = 0$~~

$k\nabla^2 u = \frac{\partial f}{\partial x}$ etc.

Zadanie będzie więc ~~[illegible]~~ znaleźć ruch cieczy z torusem który na powierzchni

Kuli spełnia warunek: $v_s\Big|_{r=a} = \frac{3}{2}\frac{A}{k}\sin\theta\,\frac{\varphi_i-\varphi_a}{4\pi}$

t.j. $u = -\frac{3}{2}\frac{A}{k}\frac{\varphi_i-\varphi_a}{4\pi}\left(1-\frac{x^2}{a^2}\right)$

$\sin\theta\cos\theta\cos\varphi = \frac{yx}{a^2}$

$\sin\theta\cos\theta\sin\varphi =$

$v = \qquad \frac{xy}{a^2}$

$w = \qquad \frac{xz}{a^2}$

Próbujemy rozwiązań:

$$u = \frac{M}{2\mu r^3}(r^2+x^2) + \frac{N}{r^5}(r^2-3x^2) + c \qquad \Big|\ \frac{3c}{2}\left(1-\frac{x^2}{a^2}\right)$$

$$v = \frac{M}{2\mu r^3}xy - \frac{3N}{r^5}xy \qquad \Big|\ -\frac{3}{2}c\frac{xy}{a^2}$$

$$w = \frac{M}{2\mu r^3}xz - \frac{3N}{r^5}xz$$

Dla $r=a$:

$$\frac{M}{2\mu a^3}(a^2+x^2) + \frac{N}{a^5}(a^2-3x^2) + c = -\frac{3}{2}\frac{A}{k}\cdots\left(1-\frac{x^2}{a^2}\right)$$

$$\frac{M}{2\mu a} - \frac{3N}{a^3} = \frac{3}{2}\frac{A}{k}\cdots$$

$$\frac{M}{2\mu a} + \frac{N}{a^3} = -\frac{3}{2}\frac{A}{k}$$

$$\frac{4N}{a^3} + c = -3\frac{A}{k}\cdots \qquad \Big|\ \frac{4M}{2\mu a} + 3c = -2\,\frac{3}{2}A\cdots$$

$M = -\frac{3}{2}\left[\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi} + c\right]\mu a$ $\qquad N = -\left[\frac{3A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi} + c\right]\frac{a^3}{4}$

$M = 0$

$N = \frac{c a^3}{2}$

Warunek $c = 0$ $\quad r = \infty$

$$u = -\frac{3}{4}A\frac{\varphi_a-\varphi_i}{4\pi}\frac{(r^2+x^2)a}{r^3\mu} - \frac{3}{4}A\frac{\varphi_a-\varphi_i}{4\pi}\frac{(r^2-3x^2)a^3}{r^5}$$

$$u = -\frac{3}{4}\frac{A}{k}\frac{\varphi_a-\varphi_i}{4\pi}\frac{a}{r}\left[1 + \frac{x^2}{r^2} + \frac{a^2}{r^2} - \frac{3a^2x^2}{r^4}\right] \quad \Bigg\| \quad \frac{\partial u}{\partial r} = \ldots\ldots\frac{1}{r^2}\left[1 + \frac{2(a^2+x^2)}{r^2} - \frac{4 \cdot 3a^2x^2}{r^4}\right]$$

$$v = -\frac{3}{4}\frac{A}{k}\frac{\varphi_a-\varphi_i}{4\pi}\frac{axy}{r^3}\left[1 - \frac{3a^2}{r^2}\right]$$

$w =$

Jaki odwrotnie będzie ruch kuli w takiej cieczy nieruchomej?

Dajmy na to że ostatecznie wynikłe ruch kuli z prędkością q

to odejmując tę prędkość od wszystkiego będziemy mieli zadanie:

kula nieruchoma w cieczy która w nieskończoności ma ruch $-q$

A na powierzchni kuli $v_{s/r=a} = \frac{3}{2}\frac{A}{k}\sin\theta\frac{\varphi_i-\varphi_a}{4\pi}$

i ciśnienie wypadkowe $= 0$

Chodzi więc o oznaczenie p..

$$ux + vy + wz = \frac{M}{2\mu r^3}\left[\underbrace{x(r^2+x^2) + xy^2 + xz^2}_{2r^2x}\right] + \frac{N}{r^5}\left(\underbrace{r^2x - 3x^3 - 3xy^2 - 3xz^2}_{-2r^2x}\right) + cx$$

$$= \frac{M}{\mu}\frac{x}{r} - 2N\frac{x}{r^3} + cx$$

$$p = p_0 + M\frac{x}{r^3}$$

$$u = \frac{M}{2\mu}\left(\frac{1}{r} + \frac{x^2}{r^3}\right) + N\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right) + c$$

$$r\frac{d}{dr} = -\frac{M}{2\mu}\left(\frac{1}{r} + \frac{3x^2}{r^3}\right) - N\left(\frac{3}{r^3} - \frac{15x^2}{r^5}\right) +$$

$$+\left(\frac{M}{2\mu}\frac{1}{r^3} - \frac{3N}{r^5}\right) r\frac{d(x^2)}{dr}$$

$$\parallel$$

$$= x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z} = 2x^2$$

$$= \frac{M}{2\mu}\left[-\frac{1}{r} - \frac{3x^2}{r^3} + \frac{2x^2}{r^3}\right] + 3N\left[-\frac{1}{r^3} + \frac{5x^2}{r^5} - \frac{2x^2}{r^5}\right]$$

$$r\frac{du}{dr} = -\frac{M}{2\mu}\left[\frac{1}{r} + \frac{x^2}{r^3}\right] - 3N\left[\frac{1}{r^3} - \frac{3x^2}{r^5}\right]$$

$$-u = -\frac{M}{2\mu}\left[\cdots\right] - N\left[\cdot \quad -\quad\right] - c$$

$$\left.\begin{array}{l} \mu\Big[\qquad = -M\left[\frac{1}{r} + \frac{x^2}{r^3}\right] - 4N_\mu\left[\frac{1}{r^3} - \frac{3x^2}{r^5}\right] - c\mu \\ \mu\frac{\partial}{\partial x}(ux+vy+wz) = M\left[\frac{1}{r} - \frac{x^2}{r^3}\right] - 2N_\mu\left[\frac{1}{r^3} - \frac{3x^2}{r^5}\right] + c\mu \\ -px = -p_0x - M\frac{x^2}{r^3} \end{array}\right\}$$

$$r\,p_{rx} = -p_0x - 3M\frac{x^2}{r^3} - 6N_\mu\left[\frac{1}{r^3} - \frac{3x^2}{r^5}\right]$$

$$p_{rx}\Big|^{r=a} = -p_0\frac{x}{a} - \frac{6N\mu}{a^4} - 3\frac{x^2}{a^4}\left[M - \frac{6N\mu}{a^2}\right]$$

$$= -p_0\frac{x}{a} + \frac{3\mu}{2a}\left[3\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi} + c\right] - \frac{3x^2}{a^2}\frac{\mu}{a}\left[-\frac{3}{2}\left(\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi}\right) + \frac{3}{2}3\frac{A}{\mu}\frac{\varphi_a-\varphi_i}{4\pi}\right]$$

$$p_{rx} = -p_0\frac{x}{a} + \frac{3}{2}\left[3\frac{A}{a}\frac{\varphi_a-\varphi_i}{4\pi} + \frac{c\mu}{a}\right] - \frac{3x^2}{a^3}2A\frac{\varphi_a-\varphi_i}{4\pi}$$

~~$P = \iint p_{rx}\,dS = 2\pi$~~

$$= -p_0\cos\theta + \frac{3}{2a}\left[3A\frac{\varphi_a-\varphi_i}{4\pi} + c\mu\right] - 6\cos^2\theta\,\frac{A}{a}\frac{\varphi_a-\varphi_i}{4\pi}$$

$$P_x = 2\pi a^2\int_0^{\pi}\sin\theta\,d\theta\cdot p_{rx}$$

$$= 2\pi a\left\{\frac{3}{\;}\frac{1}{a}\left[3A\frac{\varphi_a-\varphi_i}{4\pi} + c\mu\right] - 6\frac{A}{a}\frac{\varphi_a-\varphi_i}{4\pi}\frac{2}{3}\right\}$$

$$P_x = 2\pi a\left[5A\frac{\varphi_a-\varphi_i}{4\pi} + 3c\mu\right]$$

Jeśli to ma być $=0$ to wynika: $c = -\dfrac{5A\frac{\varphi_a-\varphi_i}{4\pi}}{3\mu}$

zatem 1). prędkość niezależna od a

2). prop. A

3). odwr. prop μ zatem szybciej przy podw. temp

$c = b\,A \qquad i = A\lambda \qquad J = A\lambda q$

$= b\,\frac{E}{l} \qquad \varphi = iqt = \frac{E}{l}\lambda q t$

$s = ct = b\,\frac{E}{l}t = b\,\frac{\varphi}{\lambda q} = \frac{b}{\lambda q}\cdot\varphi$

więc droga zakreślona przez część zależna od ilości elektr. [ani od E, ani od l ale od q]

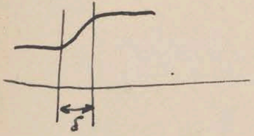
$c = b\,\frac{J}{\lambda q} = \frac{b}{\lambda q}\cdot J$ tak samo

porówn. Wüllner p. 503 Quincke P. Ann 113 (1861)

$\int_0^\delta \nabla^2 V\,dv = V_2 - V_1 = \int_0^\delta 4\pi\varepsilon\,dv$

$\int \varepsilon\,dv = \sigma\delta$

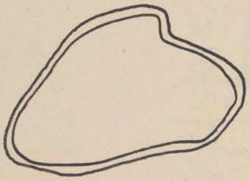
Elektr. Doppelschichten



Można stąd wyznaczyć $\sigma = \frac{1}{4\pi}\frac{V_2 - V_1}{\delta}$

na dwóch zbrojeniach oddzielonych przez δ

~~Jak to będzie gdy krzywizna gdzie wchodzi w rachunek~~



$W = \frac{1}{2}\Sigma\varphi V \; \| \; = \frac{1}{2}\Sigma\left(\frac{\partial V}{\partial x}\right)^2$

pro cm² powierzchni:

$W = \frac{1}{2}\,\frac{1}{4\pi}\frac{(V_2 - V_1)^2}{\delta}$

Jeśli się to stosi równa energii otrzymać.

α dla (powierzchni) wody = 80 C.S.S. dla wody / nieco trochę mniejsza

$V_2 - V_1 =$ n. p. $4V = \frac{4}{300}$

$8\Omega = \frac{1}{8\pi}$ $\delta = \frac{1}{8\pi}\,\frac{1}{80}\left(\frac{4}{300}\right)^2$

$$\doteq \frac{1}{31}\left(\frac{1}{600}\right)^2 = \frac{1}{31\cdot 36}\,10^{-6} = 10^{-7}\text{ cm}$$

To nie może być jednak identyczne

p. 503 bo np. dodanie $\frac{1}{10}$ % NaCl zmienia $\varphi_i - \varphi_e$ od 0·1 V do 2·7 V. podczas gdy

(860) a wcale nie zmienia . Doświadcz. !

W każdym razie należy przy wszystkich zjaw. el.-kap. także uwagę zwracać na elektro,

Przy kondens. lub stężeniu emulsyi powierzchnia zmniejsza się ogromnie
zostaje wypchn. poza elektr.
zatem energia elektr.

ilość Ω6 z jednej i z drugiej strony się wyswobodzi

Powierzchnia izolująca zatem prąd równoległy do niej

To stosuje się zresztą do każdej powierzchni:

W układzie x y w powierzchni z normalnie

$$\nabla^2 U = \underbrace{\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}}_{\text{bardzo małe}} + \frac{\partial^2 U}{\partial z^2}$$

$$\frac{1}{4\pi}\nabla^2 U \cdot \frac{\partial U}{\partial x} = -\mu\nabla^2 u + \frac{\partial p}{\partial x}$$

$$\frac{1}{4\pi}\nabla^2 U \frac{\partial U}{\partial y} = -\mu\nabla^2 v + \frac{\partial p}{\partial y}$$

$\frac{\partial U}{\partial x}$ i $\frac{\partial U}{\partial y}$ mało zmienne w obrębie δ

$$\text{zatem} \int_0^\delta \nabla^2 U \frac{\partial U}{\partial x} \cdot z\,dz = \underbrace{\left|\overline{\frac{\partial U}{\partial x}}\right|}_{\text{średnie wartości!}} \int_0^\delta \nabla^2 U\, z\, dz$$

$$\frac{1}{4\pi}\nabla^2 U \cdot \frac{\partial U}{\partial z} = -\mu\nabla^2 w + \frac{\partial p}{\partial z}$$

Prąd prawie równoległy do pow. bo $u_0 = 0 \quad v_0 = 0 \quad w_0 = 0$

$$u = z\left(\frac{\partial u}{\partial z}\right)_0 + \frac{z^2}{2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0$$

$$v = z\left(\frac{\partial v}{\partial z}\right)_0 + \frac{z^2}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)_0$$

$$w = \cancel{z\left(\frac{\partial w}{\partial z}\right)_0} + \frac{z^2}{2}\left(\frac{\partial^2 w}{\partial z^2}\right)_0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$z\left(\frac{\partial^2 u}{\partial x \partial z}\right) + \frac{z^2}{2}\left(\frac{\partial^3 u}{\partial x \partial z^2}\right) + z\left(\frac{\partial^2 v}{\partial z \partial y}\right) + \frac{z^2}{2}\left(\frac{\partial^3 v}{\partial y \partial z^2}\right) + \frac{\partial w}{\partial z} + z\frac{\partial^2 w}{\partial z^2} = 0$$

Z tego wynika zwłaszcza z dostat. małe że także $\left(\frac{\partial w}{\partial z}\right)_0 = 0$

Prócz tego:

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial z}\right)_0 + \left(\frac{\partial^2 w}{\partial z^2}\right)_0 = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{\partial}{\partial y}\left(\frac{\partial^2 v}{\partial z^2}\right)_0 + \left(\frac{\partial^3 v}{\partial z^3}\right)_0 = 0$$

$$\nabla^2 u = z\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial z}\right)_0\right] + \frac{z^2}{2}\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0\right] + \left(\frac{\partial^2 u}{\partial z^2}\right)_0$$

więc dla dostatecznie małych z: $\nabla^2 u = \left(\frac{\partial^2 u}{\partial z^2}\right)_0$

W tem niedokładność!

$$\frac{1}{4\pi}\,\overline{\frac{\partial \mathfrak{U}}{\partial x}}\,\underbrace{\int_0^\delta \frac{\partial^2 \mathfrak{U}}{\partial z^2}\,z\,dz}_{\frac{\partial \mathfrak{U}}{\partial z}z - \int \frac{\partial \mathfrak{U}}{\partial z}dz} = -\mu \int_0^\delta \left(\frac{\partial^2 u}{\partial z^2}\right) z\,dz + \overline{\frac{\partial p}{\partial x}}\int z\,dz$$

$$\left(\frac{\partial^2 u}{\partial z^2}\right) \,\|\, \frac{\partial^2 u}{\partial z^2}$$

δ obiera się tam gdzie $\frac{\partial \mathfrak{U}}{\partial z} = 0$

dla dost. małych δ:

$$\int z\frac{\partial^2 u}{\partial z^2}dz = z\frac{\partial u}{\partial z} - u$$

$$\frac{1}{4\pi}\,\overline{\frac{\partial \mathfrak{U}}{\partial x}}\,(\mathfrak{U}_i - \mathfrak{U}_a) = \cancel{\mu(u_a - u_i) = \mu u_a}$$

$$\frac{1}{4\pi}\,\overline{\frac{\partial \mathfrak{U}}{\partial y}}\,(\mathfrak{U}_i - \mathfrak{U}_a) = \cancel{\mu(v_a - v_i) = \mu v_a}$$

$$+\left(\frac{\partial^3 u}{\partial z^3}\right)_0$$

$$\int \nabla^2 u\, z\,dz = \frac{z^2}{2}\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{z^3}{3}\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial u}{\partial z}\right)_0\right] + \frac{z^4}{2.4}\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 u}{\partial z^2}\right)_0\right]$$

$\frac{1}{4\pi}\nabla^2 u \frac{\partial u}{\partial x} = -k\nabla^2 u,$ | $\frac{\partial}{\partial x}\nabla^2 u$ będzie tylko f. (z)

$\frac{1}{4\pi}\nabla^2 u \frac{\partial u}{\partial y} = -k\nabla^2 v,$ | $\frac{\partial}{\partial y}$

$\frac{1}{4\pi}\nabla^2 u \frac{\partial u}{\partial z} = -k\nabla^2 w,$

$$\frac{1}{4\pi}\left[\nabla^2 u\left(\nabla^2 u - \frac{\partial^2 u}{\partial z^2}\right) + \frac{\partial}{\partial z}\left(\nabla^2 u\frac{\partial u}{\partial z}\right)\right] = 0$$

$$(\nabla^2 u)^2 + \frac{\partial}{\partial z}(\nabla^2 u)\frac{\partial u}{\partial z} = 0$$

[illegible]: $\left(\frac{\partial^2 u}{\partial z^2}\right)^2 + \frac{\partial u}{\partial z}\cdot\frac{\partial^3 u}{\partial z^3} = 0$

$$\underbrace{\left(\frac{\partial^2 u}{\partial z^2}\right)^2 + \frac{\partial u}{\partial z}\cdot\frac{\partial^3 u}{\partial z^3}}_{\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}\right)} = 0$$

$$\underbrace{\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2}}_{\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2} = \text{const}$$

$$\left(\frac{\partial u}{\partial z}\right)^2 = cz + a$$

$u = \left(\frac{\partial u}{\partial \rho_0}\right)\rho$

$\frac{1}{4\pi}\int\left(\frac{\partial^2 u}{\partial x^2}\right)\left(\frac{\partial u}{\partial \rho_0}\right)\rho\, d\rho$

$i = \frac{1}{4\pi}\left(\frac{\partial u}{\partial \rho_0}\right)(\varphi_a - \varphi_i)$

~~$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = a$~~

~~$-\left[\varphi + \frac{y^2}{r}\frac{d\varphi}{dr}\right] - \varphi + \frac{x^2}{r}\frac{d\varphi}{dr} = 2\varphi + r\frac{d\varphi}{dr} = \alpha$~~

$u = u_0 + u_1$

Rozłożyć zadanie w

$$\left\{\begin{aligned} -\frac{\partial p}{\partial x} &= \mu \nabla^2 u_0 \\ -\frac{\partial p}{\partial y} &= \mu \nabla^2 v_0 \\ -\frac{\partial p}{\partial z} &= \mu \nabla^2 w_0 \end{aligned}\right. \quad \Bigg\| \quad \text{i} \quad \left.\begin{aligned} \varepsilon\frac{\partial U}{\partial x} &= -\mu \nabla^2 u_1 \\ \varepsilon\frac{\partial U}{\partial y} &= -\mu \nabla^2 v_1 \\ \varepsilon\frac{\partial U}{\partial z} &= -\mu \nabla^2 w_1 \end{aligned}\right\}$$

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} + \frac{\partial w_0}{\partial z} = 0$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0$$

$u_1 = v_1 = w_1 = 0$ na powierzchni kuli $\Big|_{r=a}$

$$\frac{\partial U}{\partial x} (\text{dla } r=\infty) = A \qquad \frac{\partial U}{\partial y} = \frac{\partial U}{\partial z} = 0$$

$$U = A x \left[1 + \frac{a^3}{2r^3}\right]$$

ε tylko $f(r + \Delta r)$

$$2\left[\left(\frac{\partial v}{\partial z}\right)_0 + z\left(\frac{\partial^2 v}{\partial z^2}\right)_0\right] - \left[z\frac{\partial v}{\partial z} + \frac{z^2}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)\right]$$

$$\text{"}\ \frac{z^2}{2}\left(\frac{\partial^2 v}{\partial z^2}\right)$$

Wiemy tylko tyle że $\int \varepsilon\, \rho\, d\rho = \varphi_a - \varphi_i$

$$\int_0^\delta \frac{\partial^2 v_s}{\partial n^2} \rho\, d\rho = \frac{\partial v_s}{\partial \rho} \cdot \rho \Big|_0^\delta - \underbrace{\int \frac{\partial v_s}{\partial \rho} d\rho}_{v_s \big|_0^\delta} = \frac{\partial v_s}{\partial \rho}\Big|_\delta \delta - v_s\Big|_\delta$$

$$\frac{\varphi_1 - \varphi_a}{4\pi} \cdot \frac{\varphi}{\mathcal{J}\delta} = \nabla^2 u_1$$

Poza warstwą elektr. będzie

$$\underbrace{\nabla^2 u_1 = 0 \quad \nabla^2 v_1 = 0 \quad \nabla^2 w_1 = 0}_{\nabla^2 v = 0} \qquad \underbrace{\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0}_{\operatorname{div} v = 0}$$

$v = \operatorname{curl} \mathfrak{A}$

$$\operatorname{curl}\operatorname{curl} v = \nabla \operatorname{div} v - \nabla^2 v = 0$$

$$\operatorname{curl} v = \nabla A = \nabla \operatorname{div} \mathfrak{A} - \nabla^2 \mathfrak{A}$$

dla nieskończoności żadnego wpływu, zatem $v=0$ $\nabla A = 0$

$$\operatorname{div} \operatorname{curl} v = \nabla^2 A = 0$$

Existenz beweis:

$$\cancel{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\mu}{\rho}\nabla^2 u$$

$\vdots$

Tvar v rovnicích pravém
Dle podmínek. dané u v w
i p

$u_1 \quad v_1 \quad w_1 \quad p_1$

$$u\frac{\partial u}{\partial x} + \text{—} \qquad - u_1\frac{\partial u_1}{\partial x} \cdots \quad = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y} + w_1\frac{\partial u_1}{\partial z}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = u_1\frac{\partial v_1}{\partial x} + v_1\frac{\partial v_1}{\partial y} + w_1\frac{\partial v_1}{\partial z}$$

$$u\frac{\partial w}{\partial x} + \qquad =$$

$$\frac{\partial (u-u_1)}{\partial x} + \text{—} \qquad = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}$$

$$\left(v\frac{\partial u}{\partial y} - u\frac{\partial v}{\partial y}\right) + \left(w\frac{\partial u}{\partial z} - u\frac{\partial w}{\partial z}\right) = \cdots$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) + \frac{\partial (uv)}{\partial y} + \frac{\partial (uw)}{\partial z} - \underbrace{u\frac{\partial v}{\partial y} - u\frac{\partial w}{\partial z}}_{= +u\frac{\partial u}{\partial x} = \frac{1}{2}\frac{\partial u^2}{\partial x}}$$

$$= \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw)$$

Równanie z zaniedbaniem „inertia terms"

Dwuwymiarowe; n.p. ruch kołowy $u = -\omega \frac{y}{r}$ $v = \omega \frac{x}{r}$

$= -y\varphi$ $= x\varphi$

$$\frac{\partial u}{\partial x} = \frac{yx}{r}\frac{d\varphi}{dr}$$

$$\frac{\partial u}{\partial y} = \varphi + \frac{y^2}{r}\frac{d\varphi}{dr}$$

$$\frac{\partial^2 u}{\partial x^2} = y\left(\frac{1}{r}\frac{d\varphi}{dr} - \frac{x^2}{r^3}\frac{d\varphi}{dr} + \frac{x^2}{r^2}\frac{d^2\varphi}{dr^2}\right)$$

$$\cancel{\frac{\partial^2 u}{\partial y^2} = x\left(\frac{1}{r}\frac{d\varphi}{dr} - \frac{y^2}{r^3}\frac{d\varphi}{dr} + \frac{x^2}{r^2}\frac{d^2\varphi}{dr^2}\right)}$$

$$\frac{\partial^2 u}{\partial y^2} = 3\frac{d\varphi}{dr}\frac{y}{r} - \frac{y^3}{r^3}\frac{d\varphi}{dr} + \frac{y^3}{r^2}\frac{d^2\varphi}{dr^2}$$

$$\Delta^2 u = 3\frac{y}{r}\frac{d\varphi}{dr} + y\frac{d^2\varphi}{dr^2}$$

$p = f.(r)$ $\frac{\partial p}{\partial x} = \frac{x}{r}\frac{dp}{dr}$

$$\frac{\partial p}{\partial x} = \mu \nabla^2 u$$

$$\frac{x}{r}\frac{dp}{dr} = -y\left[\frac{3}{r}\frac{d\varphi}{dr} + \frac{d^2\varphi}{dr^2}\right] \quad x$$

$$\frac{y}{r}\frac{dp}{dr} = x\left[\frac{3}{r}\frac{d\varphi}{dr} + \frac{d^2\varphi}{dr^2}\right] \quad y$$

$$r\frac{dp}{dr} = 0$$

$$p = \text{const}$$

$$r^3\frac{d\varphi}{dr} = \text{const}$$

$$\varphi = \frac{1}{r^2} + -$$

Tutaj tylko jedna możliwa

Struna, której dwa końce drgają harm. o różnym okresie:

$$\frac{\partial^2\xi}{\partial t^2} = a^2\frac{\partial^2\xi}{\partial x^2}$$

$$\xi = \sum\left[A_k \sin\frac{k\pi x}{l}\,\cancel{\sin\frac{k\pi a t}{l}} + B_k\frac{\cos k\pi x}{l}\right]\sin\left(\frac{k\pi a t}{l}+\delta_k\right)$$

$+$ [illegible]

$$x=0\Big|\quad \xi = \sum_{k=1}^{\infty} B_k \sin\left(\frac{k\pi a t}{l}+\delta_k\right) = M\sin\alpha t$$

$$x=l\Big|\quad \xi = \sum B_k \cos k\pi\left(\sin\frac{k\pi a t}{l}+\delta_k\right) = N\sin\beta t$$

$B_0\sin\delta_0 +$

$$B_1\sin\left(\frac{a\pi t}{l}+\delta_1\right) + B_2\sin\left(2\frac{a\pi t}{l}+\delta_2\right) + B_3\sin\left(3\frac{a\pi t}{l}+\delta_3\right) + \ldots = M\sin\alpha t$$

$B_0\sin\delta_0 +$

$$-B_1\sin\left(\frac{a\pi t}{l}+\delta_1\right) + B_2\sin\left(2\frac{a\pi t}{l}+\delta_2\right) - B_3\sin\left(\frac{3a\pi t}{l}+\delta_3\right) + \quad = N\sin\beta t$$

$$B_2\sin\left(\frac{2a\pi t}{l}+\delta_2\right) + B_4\sin\left(\frac{4a\pi t}{l}+\delta_4\right) + \ldots = \frac{1}{2}\left[M\sin\alpha t + N\sin\beta t\right]$$

$$B_1\sin\left(\frac{a\pi t}{l}+\delta_1\right) + B_3\sin\left(\frac{3a\pi t}{l}+\delta_3\right) + \ldots = \frac{1}{2}\left[M\sin\alpha t - N\sin\beta t\right]$$

$$\cancel{B_k = \frac{2}{l}\frac{1}{2}\int_0^l\left[M\sin\alpha t + N\sin\beta t\right]\sin\frac{k a\pi t}{l}} \qquad \int_0^{\frac{2\pi}{k\gamma}}\sin m\gamma t\,.\sin k\gamma t$$

$$\cancel{2\int_0 \sin\alpha t\,\sin k\gamma t\,dt = \int\left[\cos(\alpha+k\gamma)t - \cos(\alpha-k\gamma)t\right]dt}$$

$$\cancel{= \frac{\sin(\alpha+k\gamma)t}{\alpha+k\gamma}\Bigg|_0^{\frac{2\pi}{k\gamma}} - \frac{\sin(\alpha-k\gamma)t}{\alpha-k\gamma}\Bigg|_0^{\frac{2\pi}{k\gamma}}}$$

$$\left.\begin{aligned}&B_1\cos\delta_1\sin\frac{a\pi t}{l}+B_3\cos\delta_3\sin\frac{3a\pi t}{l}+\dots\\&+B_1\sin\delta_1\cos\frac{a\pi t}{l}+B_3\sin\delta_3\cdot\cos\frac{3a\pi t}{l}+\dots\end{aligned}\right\}=\underbrace{\frac{1}{2}\left[M\sin\alpha t-N\sin\beta t\right]}_{=F_2}$$

$$-2\int_0^{\frac{2l}{a}}\sin\frac{m\pi t a}{l}\sin\frac{n\pi t a}{l}\,dt=\int_0^{\frac{2l}{a}}\left[\cos\frac{(m+n)a\pi t}{l}-\cos\frac{(m-n)a\pi t}{l}\right]dt=$$

$$=\left.\frac{\sin\frac{(m+n)\pi t a}{l}}{\frac{(m+n)\pi}{l}}\right|_0^{\frac{2l}{a}}-\left.\frac{\sin\frac{(m-n)\pi t a}{l}}{\frac{(m-n)\pi}{l}}\right|_0^{\frac{2l}{a}}=0$$

$$2\int_0^{\frac{2l}{a}}\sin^2\frac{m\pi a t}{l}\,dt=\int_0^{\frac{2l}{a}}\left[1-\cos\frac{2ma\pi t}{l}\right]dt=\frac{2l}{a}$$

$$2\int_0^{\frac{2l}{a}}\cos\frac{m\pi t a}{l}\sin\frac{n\pi a t}{l}\,dt=\int_0^{\frac{2l}{a}}\left[\sin\frac{(m+n)a\pi t}{l}-\sin\frac{(m-n)a\pi t}{l}\right]dt=$$

$$=\left.\frac{\cos\frac{(m+n)a\pi t}{l}}{\frac{(m+n)a\pi}{l}}\right|_0^{\frac{2l}{a}}-\left.\frac{\cos\frac{(m-n)a\pi t}{l}}{-}\right|_0^{\frac{2l}{a}}=0$$

$$B_k\cos\delta_k=\frac{a}{l}\int_0^{\frac{2l}{a}}F_2\sin\frac{k\pi a t}{l}\,dt\quad\Bigg\|\quad B_k\sin\delta_k=\frac{a}{l}\int_0^{\frac{2l}{a}}F_2\cos\frac{k\pi a t}{l}\,dt$$

$$-2\int_0^{\frac{2l}{a}}\sin\alpha t\,\sin\frac{k\pi a t}{l}\,dt=\left.\frac{\sin\left(\alpha+\frac{ka\pi}{l}\right)t}{\alpha+\frac{ka\pi}{l}}\right|_0^{\frac{2l}{a}}-\left.\frac{\sin\left(\alpha-\frac{ka\pi}{l}\right)t}{\alpha-\frac{ka\pi}{l}}\right|_0^{\frac{2l}{a}}$$

$$=\frac{\sin\frac{2\alpha l}{a}}{\alpha+\frac{ka\pi}{l}}-\frac{\sin\frac{2\alpha l}{a}}{\alpha-\frac{ka\pi}{l}}=\sin\frac{2\alpha l}{a}\cdot\frac{-2\frac{ka\pi}{l}}{\alpha^2-\left(\frac{ka\pi}{l}\right)^2}$$

Jaka będzie praca wykonana na punkt 0?

$$X = E \frac{\partial \xi}{\partial x}$$

$$\int \frac{\partial \xi}{\partial x}\, d\xi_0 = \int \left(\frac{\partial \xi}{\partial x}\right)_0 \left(\frac{\partial \xi}{\partial t}\right)_0 \cdot dt$$

$$\left(\frac{\partial \xi}{\partial x}\right)_0 = \sum A_k \frac{k\pi}{l} \sin\left(\frac{k\pi a t}{l} + \delta_k\right)$$

$$\left(\frac{\partial \xi}{\partial t}\right)_{x=0} = \cancel{\sum A_k \sin\left(\frac{k\pi a t}{l} + \delta_k\right)}$$

$$\cancel{= \sum \left[A_k \sin \frac{k\pi x}{l} + A_k \cos \frac{k\pi x}{l}\right] \frac{k\pi a}{l} \cos \delta_k}$$

$$= \sum B_k \frac{k\pi a}{l} \cos\left(\frac{k\pi a t}{l} + \delta_k\right)$$

$$\frac{\partial p}{\partial x} = \mu \nabla^2 u \qquad \left| \quad \frac{\partial p_1}{\partial x} = \mu \nabla^2 u_1 \right\} \qquad \frac{\partial (p-p_1)}{\partial x} = \mu \nabla^2 (u-u_1)$$

$$\frac{\partial p}{\partial y} = \mu \nabla^2 v \qquad \vdots \qquad \frac{\partial (p-p_1)}{\partial y} = \mu \nabla^2 (v-v_1)$$

$$\frac{\partial p}{\partial z} = \mu \nabla^2 w \qquad \qquad \frac{\partial (p-p_1)}{\partial z} = \mu \nabla^2 (w-w_1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Vert$$

$$\nabla^2 p = 0 \qquad \qquad \nabla^2 (p-p_1) = 0$$

$$\nabla p = \mu \nabla^2 v = -\mu \operatorname{curl}^2 v \qquad \Big\| \quad \operatorname{curl}^2 = \nabla \operatorname{div} - \nabla^2$$

$$\operatorname{div} v = 0$$

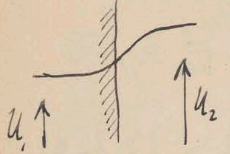
$$v = \nabla A + \operatorname{curl} \mathfrak{A}$$

~~$\operatorname{curl} \nabla p = 0 = \mu \operatorname{curl} \nabla^2 v = \mu \nabla^2 \operatorname{curl} v = -\mu \operatorname{curl}^3 v = 0$~~

~~$v = \nabla \int \frac{\nabla^2 v}{r} dv$~~ $\mathfrak{a} = \nabla \int \frac{\operatorname{div} \mathfrak{a}}{r} dv + \operatorname{curl} \int \frac{\operatorname{curl} \mathfrak{a}}{r} dv$

$$v = \nabla A + \operatorname{curl} \int \frac{\operatorname{curl} v}{r} dv \qquad \qquad \nabla p = \operatorname{curl}^3 \mathfrak{A}$$

We warstwach elektr. podw.



Ciśnienie mechan.

$$p_{xx} = -\int_0 \varepsilon \frac{\partial U}{\partial x}\, dx$$

$$= +\frac{1}{4\pi}\int \frac{\partial^2 U}{\partial x^2}\frac{\partial U}{\partial x}\, dx =$$

$$= \frac{1}{8\pi}\left(\frac{\partial U}{\partial x}\right)^2\Big|_0^\infty = -\frac{1}{8\pi}\left(\frac{\partial U}{\partial x}\right)_0^2$$

Zastępujemy przez wartość podw.

$$\frac{\partial U}{\partial x} = \frac{U_2 - U_1}{\delta} = -4\pi\sigma \qquad p_{xx} = 2\pi\sigma^2$$

Przy polaryzacji według Kohlrauscha

pojemność pro $1\,\text{mm}^2$ ~~$= 120.10^{-11}$ gr $\theta = 120.10^{-12}$ [illegible] 0.9 [illegible] $.10^{-4} = 130.10^{-8} = 13.10^{-6}$ Coulomb Volt~~

0'13 Mikrofarad

pro $1\,\text{cm}^2$: 13 Mikrof. $= 13.10^{-6}\ \frac{\text{Coulomb}}{\text{Volt}}$

<u>1 Volt</u> $\quad \sigma = 13.10^{-6}$ Coul. $= 13.10^{-6}.3.10^{9}$ (est) $= 39.10^{3} = 4.10^{2}$

$p_{xx} = 2\pi.(4.10^2)^2 = 2.32.16.10^4 = 10^6$ Dyn

= 1 Atmosf. !

Dwa walce ~~[illegible]~~ współosiowe; wewnętrzny poruszający się w kierunku X

Jaki opór?

$$\mu 2\pi r \frac{du}{dr} = \text{konst}$$

$$du = \frac{c\,dr}{r}$$

$$u - u_1 = c \log\frac{r}{r_1}$$

$u_2 = 0$

$$\frac{u-u_1}{-u_1} = \frac{\log\frac{r}{r_1}}{\log\frac{r_2}{r_1}}$$

~~$\mu\frac{\partial u}{\partial x} =$~~

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Na swobodnej powierzchni Y powinno być:

$$X_y = Y_y = Z_y = 0$$

zatem: $p - 2\mu\left(\frac{\partial v}{\partial y}\right) = 0$

$\left(\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial x}\right) = 0$

$\left(\frac{\partial w}{\partial y}\right) + \left(\frac{\partial v}{\partial z}\right) = 0$

v musi być $= 0$ w otoczeniu zatem $\frac{\partial v}{\partial y} = 0$, $p = 0$

$\left(\frac{\partial u}{\partial y}\right) = 0$

$\left(\frac{\partial w}{\partial y}\right) = 0$

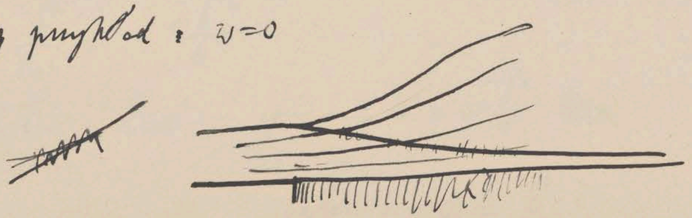
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}} = 0$$

więc $\frac{\partial u}{\partial x} = 0$

$u = \text{konst}$

wszędzie

Dwuwymiarowy przykład: $w = 0$



Przybliżona teorya: uwzględniając tylko energię kinet. w kierunku X

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} - \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y} - \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

} ~~zaniedbać i wziąć tylko $p = f(x)$~~
nie można!

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

~~warunek na powierzchni: $p = 0$ więc w ogóle wszędzie $p = 0$~~

~~$\frac{\partial u}{\partial x} = 0$~~

~~W pierwszem przybliżeniu $u = a$~~

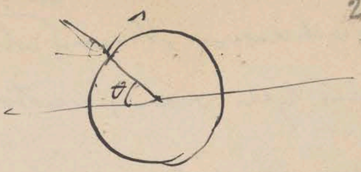
v w ogóle mała zatem v^2 zaniedbać

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial u}{\partial y} - u\frac{\partial v}{\partial y} = \cancel{v^2}\frac{\partial}{\partial y}\left(\frac{\cancel{u}}{v}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x} = u^2\frac{\partial}{\partial x}\left(\frac{v}{u}\right) = -u^2\frac{\partial}{\partial x}\left(\frac{u}{v}\right)$$

$\Delta^2 p =$

Ruch kuli w cieczy z ślizganiem



~~$u = c + \frac{M}{2\mu}\left(\frac{1}{r} + \frac{3x^2}{r^3}\right) - N\left(\frac{3}{r^3} - \frac{15x^2}{r^5}\right)$~~

$p_{\theta r} = \lambda\, v_\theta = \underbrace{u \cos vx}_{\sin\theta} + v \underbrace{\cos vy}_{\cos\theta\cos\varphi} + w \underbrace{\cos wz}_{\cos\theta\sin\varphi}$

$p_{rx} = \lambda$

$p_{\theta r} = p_{rx} \cos\ldots$

$= p_{rx}\cos\theta + p_{ry}$

Jak po przejściu kuli powstanie rozmieszczenie elektr.?

$$\psi = -\frac{c}{2}\left(1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)\omega^2$$

w przekroju $x=0$:

$r = a + \delta \qquad \omega = a + \delta$

$$\psi = -\frac{ca^2}{2}\underbrace{\left[1 - \frac{3}{2}\left(1 - \frac{\delta}{a}\right) + \frac{1}{2}\left(1 - \frac{3\delta}{a}\right)\right]}_{=0}\left(1 + 2\frac{\delta}{a}\right)$$

$$= -\frac{ca^2}{2}\left[1 - \frac{3}{2}\left(1 + \frac{\delta}{a}\right)^{-1} + \frac{1}{2}\left(1 + \frac{\delta}{a}\right)^{-3}\right]\left(1 + \frac{\delta}{a}\right)^2$$

$$= -\frac{ca^2}{2}\left[\left(1 + \frac{\delta}{a}\right)^2 - \frac{3}{2}\left(1 + \frac{\delta}{a}\right) + \frac{1}{2}\left(1 + \frac{\delta}{a}\right)^{-1}\right]$$

$$= -\frac{ca^2}{2}\left[1 + \frac{2\delta}{a} + \frac{\delta^2}{a^2} - \frac{3}{2} - \frac{3}{2}\frac{\delta}{a} + \frac{1}{2}\left(1 - \frac{\delta}{a} - \frac{\delta^2}{a^2}\right)\right]$$

$$\psi = -\frac{ca^2}{2}\frac{\delta^2}{2a^2} = -\frac{c\delta^2}{4}$$

W wielkiej odległości $r = \infty$: $\psi = -\frac{c}{2}\omega^2 = -c\frac{\delta^2}{4}$

zatem: $\omega = \frac{\delta}{\sqrt{2}}$

Dotychczasowe obliczenia potencjału powstającego przez ruch polegają na założeniu że równanie $\nabla^2 u = -4\pi\varepsilon$ ważne tylko w samej warstwie, a natychmiast przy wyjściu z jej obrębu wszystkie otrzymujemy $\varepsilon = 0$

Jeśli jednak przewodn. małe, a ε duże, to musi to być zastąpione równaniem:

$$-\frac{D\varepsilon}{Dt} = \frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} + \frac{\partial i_z}{\partial z} = -\lambda\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right)$$

$$= 4\pi\lambda\varepsilon$$

$$\downarrow$$

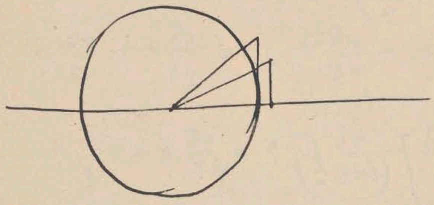
$$\cancel{\frac{\partial \varepsilon}{\partial t}} + u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} + w\frac{\partial \varepsilon}{\partial z}$$

Jeśli się obróci x w kierunek linii prądu s:

$$-u\frac{\partial \varepsilon}{\partial s} = 4\pi\lambda\varepsilon$$

$$\frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial s} = -\frac{4\pi\lambda}{u}$$

$$\lg \varepsilon = -4\pi\lambda\int\frac{ds}{u}$$



$w = r$

$$-c\frac{\delta_0^2}{4} = -\frac{c(\delta_\theta \sin\theta)^2}{4}$$

$$\delta_\theta = \frac{\delta_0}{\sin\theta}$$

---

opuszczając warstwę δ w punkcie $\theta_1 : \theta$

$$\delta_0 = \delta_1 \sin\theta_1 = \rho\sin\theta$$

$$\int_{\theta_1}\frac{a\,d\theta}{\frac{3}{2}\frac{c}{a}\sin\theta_1 . \delta_1} = \frac{2}{3}\frac{a^2}{c\delta_1}\frac{\theta_1 - \theta}{\sin\theta_1}$$

zatem w każdej warstwie

$\varkappa = \text{const}$

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$$\varepsilon = \varepsilon_0^{-4\pi\lambda\,\frac{2}{3}\,\frac{a^2}{c\delta}\,\frac{\theta_1-\theta}{\sin\theta_1}}$$

$$-4\pi\lambda\,\frac{2}{3}\,\frac{a^2}{c\delta}\,\frac{\theta_1-\theta}{\sin\vartheta_1} = \lg\frac{\varepsilon}{\varepsilon_0}$$

N. p. $\frac{\varepsilon}{\varepsilon_0} = \frac{1}{2}$ $\theta_1 = \frac{\pi}{2}$ $a = 10^{-5}$

$\delta = 10^{-6}$

$\lambda = \frac{1}{5}.10^{8}$

Wstawmy dane

$$\frac{2}{3}\,4.\pi.\frac{1}{5}.10^{8}.\,10^{-5}\,\frac{\left(\frac{\pi}{2}-\theta\right)}{10^{-6}.\,c} = 0{\cdot}3.\ 2{\cdot}3$$

Jeśli u stałe $\lg\frac{\varepsilon}{\varepsilon_0} = -4\pi\lambda\frac{s}{u}$

u potrzebne tem mniejsze im s

N. p. $s = 10^{-6}$

$$0{\cdot}3.\ 2{\cdot}3. = -4\pi\,\frac{1}{5}.\frac{10^{8}.10^{-6}}{u}$$

$u =$ cca $200\,\frac{cm}{sec}$!

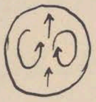
zatem to wogóle w rachubę nie może wejść!

~~Napęd~~ W wodzie dest. (najczyst.) te drobne ilości elektr. natychmiast się rozprzęgną

---

ale w lepszych izolatorach nastąpi znaczna deformacya warstwy elektrycznej

Jaki ruch cieczy koło bańki powietrza przechodzącej przez wodę?



Zdaje się że to będzie podobnie jak
ruch kuli w cieczy idealnej!

bo tarcie na powierzchni można zaniedbać

Czy nie można by utworzyć kulkę z błonki n.p. collodium, napełnić H_2 ażeby równy ciężar jak powietrze i puścić do wody, porównując z powietrzem?

Zatem będzie siły ruch jego przeciwny

Podobnie dla cieczy lżejszej od wody!

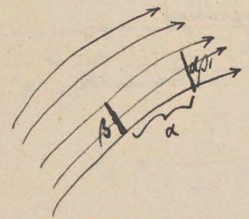
Bańki H_2 będą powolniej przechodzić (z powodu znacznego tarcia) niż O_2.

Analogia do „Spherical Vortex" Lamb. p 265

Naczynie w którym prąd cieczy z tarciem

W bliskości powierzchni prędkość prop do odstępu; kierunek: system krzywych

$$v = \left(\frac{\partial v}{\partial n}\right)_0 N$$



to będzie zatem także kierunkiem linij prądu elektr.

prąd $$\frac{i}{d\beta} = \int \left(\frac{\partial v}{\partial n}\right)_0 N\,\varepsilon\, dN = \left(\frac{\partial v}{\partial n}\right)_0 \frac{(\varphi_0 - \varphi_i)}{4\pi}$$

Miejscami wszędzie będzie $(\mathrm{div})\, i = \lambda \frac{\partial U}{\partial n}$

$$i\,d\beta - i'\,d\beta' = i\,(d\beta - d\beta') + (i - i')\,d\beta \quad \Bigg\} \quad \lambda\frac{\partial U}{\partial n} = \frac{\partial i_\alpha}{\partial \alpha} + \frac{\partial i_\beta}{\partial \beta}$$

$$= \left[i\,\frac{\partial\, d\beta}{\partial \alpha} + d\beta\,\frac{\partial i}{\partial \alpha}\right] d\alpha = \lambda\,\frac{\partial U}{\partial n}\, d\beta\, d\alpha$$

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~~$\int_{\alpha_1}^{\alpha_2} X \frac{\partial u}{\partial x}\, d\beta\, d\alpha =$~~ Prądy te konwekcyjne można zastąpić przez potencjał fikcyjny taki jak by je powodował

~~$\frac{i}{d\beta} = \left(\frac{\partial v}{\partial n_0}\right) \frac{\varphi_a - \varphi_i}{4\pi} = X \delta' \frac{\partial U'}{\partial \alpha}$~~

~~$\frac{\partial p}{\partial x} = \mu \nabla u$~~

~~$\int d\alpha\, d\beta$~~

~~$= \iiint \partial$~~

$$\iint \left(\frac{\partial u}{\partial N}\right)_0 dF = \iiint \nabla^2 u \, dx\,dy\,dz = \frac{1}{\mu} \iiint \frac{\partial p}{\partial x}\, dx\,dy\,dz = \frac{1}{\mu} \iint p \, dy\,dz$$

$$= \frac{1}{\mu} \iint p \cos nx \cdot dF$$

$$\iint \left[\cos nx \cdot \left(\frac{\partial u}{\partial N}\right) + \cos ny \left(\frac{\partial v}{\partial N}\right) + \cos nz \left(\frac{\partial w}{\partial N}\right)\right] dF = \frac{1}{\mu} \iint p \, dF$$

że $U = \text{const}$ i $p = \text{const}$ są identyczne (?) : /

$\Delta^2 U = 0$ $\Delta^2 p = 0$ więc jeśli identyczne warunki graniczne to wystarczy

więc $\frac{\partial U}{\partial n} \overset{\text{powinno być:}}{\sim} \frac{\partial p}{\partial n}$

$$* \quad \lambda \frac{\partial U}{\partial n} = \frac{\partial i_\alpha}{\partial \alpha} + \frac{\partial i_\beta}{\partial \beta} = \frac{\varphi_a - \varphi_i}{4\pi} \left[\underbrace{\frac{\partial}{\partial \alpha}\left(\frac{\partial v_\alpha}{\partial n_0}\right) + \frac{\partial}{\partial \beta}\left(\frac{\partial v_\beta}{\partial n_0}\right)}_{\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial z}\right)_0 = -\left(\frac{\partial^2 w}{\partial z^2}\right)_0 = \nabla_0^2 w = \frac{1}{\mu}\frac{\partial p}{\partial n}} \right]$$

Więc byłoby w ogóle:

$$U_1 - U_2 = \frac{\varphi_a - \varphi_i}{4\pi} \frac{1}{\lambda \mu} (p_1 - p_2) \quad (?)$$

Jaki procent (elektrycznej) pracy przy wytwarzaniu Strömungsströmów ?

$$W_e = \frac{P^2(\varphi_i - \varphi_a)^2}{(4\pi\mu)^2}\frac{\varphi\sigma}{L} \qquad W_f = \frac{\pi P^2 R^4}{8\mu L}$$

$$\frac{W_e}{W_f} = \frac{(\varphi_i - \varphi_a)^2\sigma}{2\mu\pi\varphi} = \frac{\left(\frac{4}{300}\right)^2 . \, 4{\cdot}6 . 10^{-8}}{2 . 3{\cdot}14 . 0{\cdot}010 . 10^{-4}} = 10^{-6}$$ $\varphi = 10^{-4}$

powiększyć można: podwyższając $\sigma = \frac{1}{\lambda}$

zmniejszając φ

Np. używając Diaphragma i dobry izolator

Jeżeli ruch Browna spowodowany przez zewnętrzne przyczyny, jaka energia przez nie rozproszona ?

Np. $2r = 0{\cdot}001$ mm $= 10^{-4}$ cm

$v = 0{\cdot}003 \frac{mm}{sec} = 3 . 10^{-4}$

opór $= 6\pi a \mu v$ $\qquad \mu = 0{\cdot}018$

praca pro sec: $6\pi a\mu v^2 = \frac{\cancel{6}^{3} . 3{\cdot}14 . 10^{-4} . 0{\cdot}018 . 9 . 10^{-8}}{\cancel{2}.}$

$= 10^{-12} . \frac{0{\cdot}17}{1{\cdot}5} . 9 \qquad = 1{\cdot}5 . 10^{-12}$ Erg

objętość takiej kulki: $\frac{4\pi}{3}\frac{1}{8}10^{-12} = \frac{1}{2}10^{-12}$ cm^3

Zatem praca pro 1 cm^3 substancji: $3 \frac{Erg}{sec}$

Pro dzień: 60. 60. 24 = 86400 sec | $\qquad 260.000$ Erg $= \frac{260.000}{42. 10^{6}}$ g cal $= \frac{1}{160}$ g cal

Energia [illegible] owej kulki: $\alpha . O = 4\pi r^2 . \alpha$

$= 3{\cdot}14 . 80 . \cancel{\;} . 10^{-8}\ \text{(Erg)}$

$= 25 . 10^{-7}$

[n.p. wody

Zatem Energia [illegible] wystarczyłaby do ogrzewania ruchu na $\frac{25 . 10^{-7}}{1{\cdot}5 . 10^{-12}} = 16 . 10^5$ sec.

$= \underline{2 \text{ dni}}$

$\cancel{\frac{16 . 10^5}{60 . 60}}$

Ruch kuli w cieczy; czy nie można uwzględnić „inertia terms"

Dla tarcia:

$$u = \frac{M}{\cancel{2\mu}}\left(\frac{1}{r} + \frac{x^2}{r^3}\right) + \frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$$

$$v = \frac{M}{\cancel{2\mu}}\frac{xy}{r^3} - \frac{3N}{r^5}xy$$

$$w = \frac{M}{\cancel{2\mu}}\frac{xz}{r^3} - \frac{3N}{r^5}xz$$

Bez tarcia:

$$u = \frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$$

$$v = -\frac{3N}{r^5}xy$$

$$w = \frac{3N}{r^5}xy$$

$$0 = \frac{\partial p}{\partial x} - \mu\nabla^2 u$$

$$0 = \frac{\partial p}{\partial y} - \mu\nabla^2 v$$

$$0 = \frac{\partial p}{\partial z} - \mu\nabla^2 w$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x}$$

$$(u_1+u_2)\frac{\partial(u_1+u_2)}{\partial x} + \dots = -\frac{\partial p}{\partial x} + \mu\left(\nabla^2 u_1 + \nabla^2 u_2\right)$$

$$u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_2}{\partial x} + \cancel{u_2\frac{\partial u_2}{\partial x}} = -\cancel{\frac{\partial p_1}{\partial x}} + \cancel{\frac{\partial p_2}{\partial x}} + \frac{\partial p_3}{\partial x} + \mu\,\cancel{\left(\nabla^2 u_1 + \nabla^2 u_2\right)}$$

$$= u_1\frac{\partial(u_1+u_2)}{\partial x} + u_2\frac{\partial u_1}{\partial x}$$

$$u = \frac{M}{r}\left(\frac{1}{r} + \frac{x^2}{r^3}\right) + \frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$$

La teoria: ~~$u = c + B\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) - \frac{A}{\mu}\frac{r^2}{6}\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right)$~~

$$\frac{\partial u}{\partial x} = M\left[-\frac{x}{r^3} + \frac{2x}{r^3} - \frac{3x^3}{r^5}\right] + N\left[-\frac{3x}{r^5} - \frac{6x}{r^5} + \frac{15x^3}{r^7}\right]$$

$$= M\left[\frac{x}{r^3} - \frac{3x^3}{r^5}\right] + N\left[-\frac{9x}{r^5} + \frac{15x^3}{r^7}\right]$$

$$\frac{\partial u}{\partial y} = M\left[-\frac{y}{r^3} - \frac{3x^2y}{r^5}\right] + N\left[\frac{-3y}{r^5} + \frac{15x^2y}{r^7}\right]$$

$$\frac{\partial u}{\partial z} = M\left[-\frac{z}{r^3} - \frac{3x^2z}{r^5}\right] + N\left[-\frac{3z}{r^5} + \frac{15x^2z}{r^7}\right]$$

$\frac{1}{r} + \frac{x^2}{r^3}$	$\frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$
$\frac{xy}{r^3}$	$-\frac{3Nxy}{r^5}$
$\frac{xz}{r^3}$	$-\frac{3N xz}{r^5}$

$$M\left\{\cancel{\frac{x}{r^4}} - \cancel{\frac{x}{r^4}} - \frac{3x^3}{r^6} + \cancel{\frac{x^3}{r^6}} - \cancel{\frac{3x^5}{r^8}} - \cancel{\frac{xy^2}{r^6}} - \cancel{\frac{3x^3y^2}{r^8}} - \cancel{\frac{xz^2}{r^6}} - \cancel{\frac{3x^3z^2}{r^8}}\right\} +$$

$$+N\left\{-\frac{\overset{12}{\cancel{9}}x}{r^6} + \cancel{\frac{15x^3}{r^8}} - \frac{\cancel{9}x^3}{r^8} + \cancel{\frac{15x^5}{r^{10}}} - \cancel{\frac{3xy^2}{r^8}} + \cancel{\frac{15x^3y^2}{r^{10}}} - \cancel{\frac{3xz^2}{r^8}} + \cancel{\frac{15x^3z^2}{r^{10}}}\right\}$$

$$= \cancel{M\left\{-\frac{3x^3}{r^6} - \frac{3x^5}{r^8}\right\} + N\left\{-\frac{12x}{r^6} \frac{x^5}{r^8} + \frac{15x^5}{r^{10}}\right\}} -\frac{12x}{r^6} + \frac{27x^3}{r^8}$$

$$v = \frac{xy}{r^3}\left[M - \frac{3N}{r^2}\right]$$

~~$\frac{\partial v}{\partial x} = \left[\frac{y}{r^3} - \frac{3x^2y}{r^5}\right]\left[M - \frac{3N}{r^2}\right] + \frac{6Nx^2y}{r^7}$~~

$$\frac{\partial v}{\partial x} = M\left[\frac{y}{r^3} - \frac{3x^2y}{r^5}\right] + 3N\left[-\frac{y}{r^5} + \frac{5x^2y}{r^7}\right]$$

$$\frac{\partial v}{\partial y} = M\left[\frac{x}{r^3} - \frac{3xy^2}{r^5}\right] + 3N\left[-\frac{x}{r^5} + \frac{5xy^2}{r^7}\right]$$

$$\frac{\partial v}{\partial z} = M\frac{-3xyz}{r^5} + 3N\frac{+5xyz}{r^7}$$

$\frac{1}{r} + \frac{x^2}{r^3}$	$\frac{N}{r^3}\left(1 - \frac{3x^2}{r^2}\right) + c$
$\frac{xy}{r^3}$	$-\frac{3xy}{r^2}$
$\frac{xz}{r^3}$	$-\frac{3xz}{r^2}$

$$M\left\{\frac{y}{r^4}-\frac{3x^2y}{r^6}+\cancel{\frac{x^2y}{r^6}}-\cancel{\frac{3x^4y}{r^8}}+\cancel{\frac{x^2y}{r^6}}-\cancel{\frac{3x^2y^3}{r^8}}-\cancel{\frac{3x^2yz^2}{r^8}}\right\}$$

$$+3N\left\{-\frac{y}{r^6}+\frac{5x^2y}{r^8}-\frac{x^2y}{r^8}+\cancel{\frac{5x^4y}{r^{10}}}-\frac{x^2y}{r^8}+\cancel{\frac{5x^2y^3}{r^{10}}}+\cancel{\frac{5x^2yz^2}{r^{10}}}\right\}=$$

$$II={}^{M}\Big[M\left\{\frac{y}{r^4}-\frac{4x^2y}{r^6}\right\}+3N\left\{\frac{y}{r^6}+\frac{8x^2y}{r^8}\right\}\Big]$$

$$\cancel{M\left\{\frac{x}{r^4}-\frac{2x^3}{r^6}\right\}}\rightarrow I={}_{M}\Big[M\left\{-\frac{4x^3}{r^6}\right\}+N\left\{-\frac{12x}{r^6}+\frac{24x^3}{r^8}\right\}\Big]$$

Próba czy $\frac{\partial I}{\partial y}-\frac{\partial II}{\partial x}=0$

$$-4M\left\{\frac{-6x^3y}{r^8}\right\}+12N\left\{+\frac{6xy}{r^8}-\frac{16x^3y}{r^{10}}\right\}\Bigg\|\,M\left\{-\frac{4xy}{r^6}-\frac{8xy}{r^6}+\frac{24x^3y}{r^8}\right\}+$$

$$+3N\left\{-\frac{6xy}{r^8}+\frac{16xy}{r^8}-\frac{64x^3y}{r^{10}}\right\}$$

$72-30$

$$M\left\{\frac{12xy}{r^6}\right\}+N\left\{\frac{42xy}{r^8}\right\}=\cancel{\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}}$$

$$u_2\frac{\partial u_1}{\partial x}+v_2\frac{\partial u_1}{\partial y}+w_2\frac{\partial u_1}{\partial z}=Mc\left(\frac{x}{r^3}-\frac{3x^3}{r^5}\right)+\frac{MN}{r^3}\left\{\frac{x}{r^3}-\frac{3x^3}{r^5}-\frac{3x^3}{r^5}+\cancel{\frac{9x^5}{r^7}}\right.$$

$$\left.+\frac{3xy^2}{r^5}+\cancel{\frac{9x^3y^2}{r^7}}+\frac{3xz^2}{r^5}+\cancel{\frac{9x^3z^2}{r^7}}\right\}$$

$$\frac{4x}{r^3}$$

$$=Mc\left(\frac{x}{r^3}-\frac{3x^3}{r^5}\right)+MN\cdot\frac{4x}{r^6}$$

$$u_2 \frac{\partial v_1}{\partial x} + v_2 \frac{\partial v_1}{\partial y} + w_2 \frac{\partial v_1}{\partial z} = Mc\left(\frac{y}{r^3} - \frac{3x^2y}{r^5}\right) + \frac{MN}{r^3}\left\{\frac{y}{r^3} - \frac{3x^2y}{r^5} - \frac{3x^2y}{r^5} + \frac{9x^4y}{r^7} - \right.$$

$$\left. - \frac{3x^2y}{r^5} + 9\frac{x^2y^3}{r^7} + 9\frac{x^2yz^2}{r^7}\right\}$$

$$= Mc\left(\frac{y}{r^3} - \frac{3x^2y}{r^5}\right) + MN\frac{y}{r^6}$$

$$\frac{\partial}{\partial y}\left\{Mc\left(\frac{x}{r^3} - \frac{3x^3}{r^5}\right) + MN\frac{4x}{r^6}\right\} - \frac{\partial}{\partial x}\left\{Mc\left(\frac{y}{r^3} - \frac{3x^2y}{r^5}\right) + MN\frac{y}{r^6}\right\} =$$

$$Mc\left\{-\frac{3xy}{r^5} + 15\frac{x^3y}{r^7} + \frac{3xy}{r^5} - 15\frac{x^3y}{r^7} - \frac{6xy}{r^5}\right\} + MN\left\{\frac{-24xy}{r^8} + 6\frac{xy}{r^8}\right\}$$

$$= Mc\left(-\frac{6xy}{r^5}\right) + MN\left(\frac{18xy}{r^8}\right)$$

$$M^2\frac{12xy}{r^6} + MN\frac{60xy}{r^8} = \geq 0$$ zatem nie może być rozwiązaniem

$$0 = \frac{\partial p_1}{\partial x} - \mu \nabla^2(u_1+u_2) \qquad u_2 \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y} + w_2 \frac{\partial u_2}{\partial z} = -\frac{\partial p_2}{\partial x}$$

$$0 = \frac{\partial p_1}{\partial y} - \mu \nabla^2([illegible]+v_2) \qquad u_2 \frac{\partial v_2}{\partial x} + v_2 \frac{\partial v_2}{\partial y} + w_2 \frac{\partial v_2}{\partial z} = -\frac{\partial p_2}{\partial y}$$

$$0 = \frac{\partial p_1}{\partial z} - \mu \nabla^2(w_1+w_2)$$

Próba czy: $(u_1+u_2)\frac{\partial(u_1+u_2)}{\partial x} + \dots \quad = -\frac{\partial p}{\partial x} \; [illegible] + \mu \nabla^2(u_1+u_2)$

$$u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_2}{\partial x} + u_2 \frac{\partial u_2}{\partial x} \quad - \quad = -\frac{\partial p}{\partial x} + \mu \nabla^2(u_1+u_2)$$

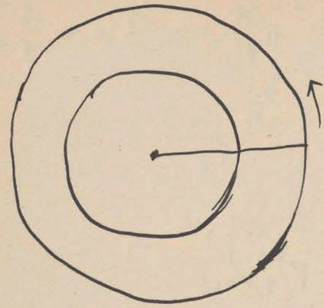
$$= -\frac{\partial(p-p_1)}{\partial x}$$

$$u_1 \frac{\partial u}{\partial x} + u_2 \frac{\partial u_1}{\partial x}$$

$$= \frac{\partial(p_1+p_2-p)}{\partial x}$$

Ruch walcowaty

$\frac{f}{r} = \omega$ = prędkość kątowa $= \varphi(r)$



$$u = -f\frac{y}{r} = -\varphi y$$

$$v = f\frac{x}{r} = \varphi x$$

$$\frac{\partial u}{\partial x} = -\varphi'\frac{xy}{r} \qquad \frac{\partial v}{\partial y} = \varphi'\frac{xy}{r}$$

$$\frac{\partial^2 u}{\partial x^2} = -\varphi''\frac{x^2y}{r^2} - \frac{\varphi' y}{r} + \varphi'\frac{x^2y}{r^3} \qquad \frac{\partial^2 v}{\partial y^2} = \varphi''\frac{xy^2}{r^2} + \frac{\varphi' x}{r} - \varphi'\frac{xy^2}{r^3}$$

$$\frac{\partial u}{\partial y} = -\varphi - \varphi'\frac{y^2}{r} \qquad \frac{\partial v}{\partial x} = \varphi + \varphi'\frac{x^2}{r}$$

$$\frac{\partial^2 u}{\partial y^2} = -3\varphi'\frac{y}{r} - \varphi''\frac{y^3}{r^2} + \varphi'\frac{y^3}{r^3} \qquad \frac{\partial^2 v}{\partial x^2} = 3\varphi'\frac{x}{r} + \varphi''\frac{x^3}{r^2} - \varphi'\frac{x^3}{r^3}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\varphi'' y - 3\varphi'\frac{y}{r} \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \varphi'' x + 3\varphi'\frac{x}{r}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \varphi\varphi'\frac{xy^2}{r} - \varphi^2 x - \varphi\varphi'\frac{xy^2}{r} = -\varphi^2 x$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\varphi^2 y - \varphi\varphi'\frac{x^2y}{r} + \varphi\varphi'\frac{x^2y}{r} = -\varphi^2 y$$

$$\frac{\partial^2 p}{\partial x\partial y} = \cdot 2\varphi\varphi'\frac{xy}{r} = 4\varphi\frac{xy}{r}$$ nie daje żadnego [illegible]

I. Bez tarcia:

$$\left.\begin{aligned} -\varphi^2 x &= -\frac{\partial p}{\partial x} \quad \Big\| \cdot\frac{x}{r} \\ -\varphi^2 y &= -\frac{\partial p}{\partial y} \quad \Big\| \cdot\frac{y}{r} \end{aligned}\right\} \qquad \frac{dp}{dr} = \varphi^2\frac{x^2+y^2}{r} = r.\varphi^2$$

$$p = \int \varphi^2 \cdot r\,dr$$

φ nieoznaczone, dowolne

mimo że dane mogą być φ_r na powierzchni

II Powolne tarcie:

$$\frac{1}{\mu}\frac{\partial p}{\partial x} = -\varphi'' y - 3\varphi'\frac{y}{r} \qquad \Big\| \quad \frac{x}{r}$$

$$\frac{1}{\mu}\frac{\partial p}{\partial y} = \varphi'' x + 3\varphi'\frac{x}{r} \qquad \Big\| \quad \frac{y}{r}$$

$$\frac{1}{\mu}\frac{dp}{dr} = 0$$

$$p = \text{const}$$

$$\varphi'' + \frac{3\varphi'}{r} = 0$$

$$\frac{d}{dr}(r^3\varphi') = r^3\varphi'' + 3r^2\varphi' = r^3\left(\varphi'' + \frac{3\varphi'}{r}\right) = 0$$

$$\frac{1}{\mu}\frac{\partial^2 p}{\partial x\partial y} = -\varphi'' - 3\frac{\varphi'}{r} - \varphi'''\frac{xy}{r} - 3\varphi''\frac{xy}{r^2} + 3\varphi'\frac{xy}{r^3}$$

$$= \varphi'' + 3\frac{\varphi'}{r} + \varphi'''\frac{xy}{r} + 3\varphi''\frac{xy}{r^2} - 3\varphi'\frac{xy}{r^3} = 0$$

$$\frac{3}{r^4} - \frac{3xy}{r^6} - \frac{3}{r^4} - 9\frac{xy}{r^6} + \frac{12xy}{r^6} = 0$$

spełnione przez

$$\varphi'' = -\frac{3c}{r^4}$$

$$\varphi''' = \frac{12c}{r^5}$$

$$r^3\varphi' = c$$

$$d\varphi = \frac{c\,dr}{r^3}$$

$$\varphi = -\frac{c}{2r^2} + b$$

III Szybkie tarcie:

$$-\varphi^2 x = -\frac{\partial p}{\partial x} - \mu\left(\varphi'' y + 3\varphi'\frac{y}{r}\right) \qquad \Big\| \quad \frac{x}{r} \qquad \frac{\partial}{\partial y}$$

$$-\varphi^2 y = -\frac{\partial p}{\partial y} + \mu\left(\varphi'' x + 3\varphi'\frac{x}{r}\right) \qquad \Big\| \quad \frac{y}{r} \qquad -\frac{\partial}{\partial x}$$

$$-\varphi^2 \cdot \frac{x^2+y^2}{r} = -\frac{dp}{dr} = -\varphi^2 \cdot r$$

$$\underbrace{-2\varphi\varphi'\frac{xy}{r} + 2\varphi\varphi'\frac{xy}{r}}_{0} = \cdots\cdots$$

Zatem ~~ma~~ to samo równanie co przedtem

tylko że p teraz będzie inne

$$\varphi = -\frac{c}{2r^2} + b$$

$$\varphi' = \frac{c}{r^3}$$

$$\varphi'' = -\frac{3c}{r^4}$$

Mianowicie: $\frac{dp}{dr} = r\varphi^2$

$$p = \int r \underbrace{\left(b - \frac{c}{2r^2}\right)^2}_{b^2 - \frac{bc}{r^2} + \frac{c^2}{4r^4}} dr$$

Więc w całe 3 stałe: b, c, p_0 !

$$p = \left[b^2 \frac{r^2}{2} - bc \lg r - \frac{c^2}{8r^2}\right] + p_0$$

Przy tem wszystkiem współczynnik tarcia μ wcale nie wchodzi w rachunki

więc ten sam układ mógłby się ustanowić (z czasem) dla cieczy z nieskończ. małem tarciem

[więc przez warunek: „nieskończ. małe tarcie" równania hydrod. stałyby się oznaczone dla danych u

---

Wogóle:
$$\begin{cases} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu\nabla^2 u \\ \\ \end{cases}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Energia rozproszona: $\Phi = \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left[\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right]\right\}$

Dwa przypadki: albo u v w niezależne wogóle od μ

albo ruch taki że $\Phi = 0$

To ostatnie tylko możliwe jeśli $\frac{\partial u}{\partial x} = \dots = 0$ jeśli μ skończone

albo też μ nieskończ. małe

W ogólnym przypadku rozwiązać i potem μ robić coraz mniejsze

wtedy rozwinąć: w szereg Maclaurina: $u = u_0 + \mu u_1 + \frac{\mu^2}{2} u_2 + \cdots$

jeśli w ogóle możliwe takie rozwinięcie

$$\frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + \mu \frac{\partial u_1}{\partial x}$$

zaniedbać wszystkie wyższe potęgi z wyjątkiem pierwszej i 0

$$u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} + w_0 \frac{\partial u_0}{\partial z} = -\frac{\partial p_0}{\partial x}$$

$$u_0 \frac{\partial u_1}{\partial x} + v_0 \frac{\partial u_1}{\partial y} + w_0 \frac{\partial u_1}{\partial z} + u_1 \frac{\partial u_0}{\partial x} + v_1 \frac{\partial u_0}{\partial y} + w_1 \frac{\partial u_0}{\partial z} = -\frac{\partial p_1}{\partial x} + \nabla^2 u_0$$

~~$u_0 \frac{\partial (u_0+u_1)}{\partial x} + v_0 \frac{\partial (u_0+u_1)}{\partial y} + w_0 \frac{\partial (u_0+u_1)}{\partial z} + (u_0+u_1) \frac{\partial u_0}{\partial x} + (v_0+v_1) \frac{\partial u_0}{\partial y} + (w_0+w_1) \frac{\partial u_0}{\partial z} = -\frac{\partial (p_1+2p_0)}{\partial x} + \nabla^2 u_0$~~

$$u_\mu = (u)_0 + \mu \left(\frac{\partial u}{\partial \mu}\right)_0 + \frac{\mu^2}{1.2} \left(\frac{\partial^2 u}{\partial \mu^2}\right)_0 + \cdots$$

$u_1 = \left(\frac{\partial u}{\partial \mu}\right)_0$ itd.

$$\left(\frac{\partial}{\partial \mu}\left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}\right]\right)_0 = -\left(\frac{\partial p}{\partial x \partial \mu}\right)_0 + \nabla^2 u$$

Jeśli ruch niezależny od p to równocześnie:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z}$$

$$\Delta^2 u = 0$$

$$\Delta^2 v = 0$$

$$\Delta^2 w = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\underbrace{v\frac{\partial^2 u}{\partial x^2} - u\frac{\partial^2 v}{\partial x^2}}_{\partial} + \underbrace{v\frac{\partial^2 u}{\partial y^2} - u\frac{\partial^2 v}{\partial y^2}}_{} = 0$$

Rugując $\frac{\partial^2 p}{\partial x \partial y}$:

$$u\frac{\partial u}{\partial x} + v\underbrace{\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)}_{} + w\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}$$

$$\frac{1}{2}\frac{\partial}{\partial x}(u^2+v^2+w^2) + v\zeta - w\eta = -\frac{\partial p}{\partial x}$$

$$\frac{1}{2}\frac{\partial}{\partial y}(u^2+v^2+w^2) + w\xi - u\zeta = -\frac{\partial p}{\partial y}$$

$$\frac{1}{2}\frac{\partial}{\partial z}(u^2+v^2+w^2) + u\eta - v\xi = -\frac{\partial p}{\partial z}$$

~~$u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} + u\frac{\partial w}{\partial z}$~~

~~$-u\frac{\partial u}{\partial x} - v\frac{\partial v}{\partial x} - w\frac{\partial w}{\partial x}$~~

$$\nabla(v \cdot \operatorname{curl} v) = -\nabla\left(p + \frac{u^2+v^2+w^2}{2}\right)$$

pod jakimi warunkami mogą równocześnie istnieć?

$$\left\{\begin{array}{l} \operatorname{curl}\nabla(v \cdot \operatorname{curl} v) = 0 \\ \Delta^2 v = 0 \\ \operatorname{div} v = 0 \end{array}\right.$$

$$\pm\frac{\partial w}{\partial z}\cdot\zeta \qquad \pm w\frac{\partial \zeta}{\partial z}$$

$$\frac{\partial v}{\partial y}\cdot\zeta - \frac{\partial w}{\partial y}\eta + v\frac{\partial \zeta}{\partial y} - w\frac{\partial \eta}{\partial y} - \frac{\partial v}{\partial x}\xi + \frac{\partial u}{\partial x}\zeta - w\frac{\partial \xi}{\partial x} + u\frac{\partial \zeta}{\partial x} = 0$$

$$\zeta\cancel{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)} - \left(\xi\frac{\partial w}{\partial x} + \eta\frac{\partial w}{\partial y} + \zeta\frac{\partial w}{\partial z}\right) - w\cancel{\left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z}\right)} + \left(u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z}\right) = 0$$

~~curl (V × curl v) =~~ [illegible]

$$u\,\Delta^2 u = u\frac{\partial^2 u}{\partial x^2} + u\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^2 u}{\partial z^2} \quad *$$

$$\pm\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(u\frac{\partial u}{\partial z}\right)$$

$$-\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]$$

$$= \frac{1}{2}\Delta^2(u^2) - \left[\left(\frac{\partial u}{\partial x}\right)^2 + \dots\right]$$

Dwuwymiarowe zadanie:

$$w = \frac{\partial}{\partial z} = \xi = \eta = 0$$

$$\begin{cases} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{\partial p}{\partial x} \\[2ex] u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} = -\dfrac{\partial p}{\partial y} \end{cases}$$

$$\Delta^2 u = 0$$

$$\underbrace{\Delta^2 v}_{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = 0$$

$$\downarrow$$

$$u\Big(\underbrace{\frac{\partial^2 u}{\partial x\partial y}}\ - \overset{\|}{\frac{\partial^2 v}{\partial x^2}}\Big) + v\Big(\underbrace{\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x\partial y}}\Big) = 0$$

$$u\left(\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 v}{\partial y^2}\right) \pm v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right) = 0$$

$$\underbrace{u\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - v\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0}$$

więc to równanie i tak spełnione jeśli ↑ spełnione

$\Delta^2 u = 0 \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$\Delta^2 v = 0$

~~$u = f_1(x+iy) = \varphi_1(xy) + i\psi_1(xy)$~~

~~$\frac{\partial u}{\partial x} = f' \qquad \frac{\partial u}{\partial y} = i f'$~~

~~$\frac{\partial^2 u}{\partial x^2} = f'' \qquad \frac{\partial^2 u}{\partial y^2} = -f''$~~

~~$v = i f_2(x+iy) = i\varphi_2(xy) - \psi_2(xy)$~~

~~$\frac{\partial v}{\partial y} = -f_2'$~~

~~$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$~~

~~$f_1' - f_2' = 0$~~

~~$f_1(z) = f_2(z) + c$~~

$f(x+iy) = \varphi(x,y) + i\psi(x,y)$

$\Delta^2\varphi = 0 \qquad \Delta^2\psi = 0$

$u = \varphi(x,y)$

$$\left.\begin{aligned} f' &= \frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x} \\ i f' &= \frac{\partial\varphi}{\partial y} + i\frac{\partial\psi}{\partial y} \end{aligned}\right\}$$

$\frac{\partial u}{\partial x} = \frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y} = -\frac{\partial v}{\partial y}$

$\frac{\partial\varphi}{\partial x} = +\frac{\partial\psi}{\partial y}$

$\frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}$

$v = -\psi(x,y) + f_c(x)$

~~[illegible]:~~ ~~$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2\psi}{\partial x\partial y} = -\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 f_c(x)}{\partial x\partial y}$~~

~~$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{\partial^2 f_c(x)}{\partial x\partial y} = 0$~~

~~$\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2\psi}{\partial y^2} = -\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\varphi}{\partial x\partial y} =$~~

$\frac{\partial v}{\partial x} = \underbrace{-\frac{\partial\psi}{\partial x}}_{-\frac{\partial u}{\partial y}} + \frac{df_c}{dx}$

$\frac{\partial^2 v}{\partial x^2} = \cdot\frac{\partial^2\psi}{\partial x\partial y} + \frac{d^2 f}{dx^2} = \frac{\partial^2\psi}{\partial y^2} + \frac{d^2 f}{dx^2}$

$\parallel$

0

$\frac{\partial^2 v}{\partial y^2} = -\frac{\partial^2\psi}{\partial x^2}$

Pytanie: o ile warunki graniczne mogą być dowolnie obrane?

$r(\cos\varphi + i\sin\varphi) = r e^{i\varphi}$

~~$\log(r(\cos\varphi+i\sin\varphi)) = i\varphi \log r\Big|_{r=1} = 0$~~

~~$e^{z} = e^{\cos\varphi + i\sin\varphi}$~~

$r(\cos\varphi + i\sin\varphi) = r e^{i\varphi}$

$\log r(\cos\varphi + i\sin\varphi) = i\varphi + \log r$

$$z^2 = r^2(\cos^2\varphi - \sin^2\varphi + 2i\sin\varphi\cos\varphi)$$

$$= r^2 e^{2i\varphi} = r^2\cos 2\varphi + i\, r^2 \sin 2\varphi$$

I. $\log z = \log r + i\varphi$

$u = [illegible] \log r + b_1 y + c_1$

$v = -\varphi + bx + c = -\operatorname{arctg}\frac{y}{x} + bx + c$

$\frac{\partial u}{\partial x} = \frac{x}{r^2}$

$\frac{\partial u}{\partial y} = \frac{y}{r^2} + b_1$

$\underbrace{\frac{\partial^2 u}{\partial x^2} = \frac{1}{r^2} - \frac{2x^2}{r^4} \qquad \frac{\partial^2 u}{\partial y^2} = \frac{1}{r^2} - \frac{2y^2}{r^4}}_{\Sigma = 0}$

$\frac{\partial v}{\partial x} = + \frac{\frac{y}{x^2}}{1+\frac{y^2}{x^2}} + b = \frac{y}{y^2+x^2} + b = \frac{y}{r^2} + b$

$\frac{\partial^2 v}{\partial x^2} = -2\frac{yx}{r^4}$

$\frac{\partial v}{\partial y} = \frac{-\frac{1}{x}}{1+\frac{y^2}{x^2}} = -\frac{x}{y^2+x^2} = -\frac{x}{r^2}$

$\frac{\partial^2 v}{\partial y^2} = [illegible] + \frac{2yx}{r^4}$

$\Sigma = 0$

Jeżeli kierunek linij prądu niezależny od μ

$$\frac{\partial}{\partial\mu}\left(\frac{u}{v}\right)=0 \qquad \frac{1}{v}\frac{\partial u}{\partial\mu}-\frac{u}{v^2}\frac{\partial v}{\partial\mu}=0$$

$$\frac{1}{u}\frac{\partial u}{\partial\mu}=\frac{1}{v}\frac{\partial v}{\partial\mu}=\frac{1}{w}\frac{\partial w}{\partial\mu}$$

$$\cancel{\frac{\partial}{\partial\mu}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial^2 p}{\partial\mu\,\partial x}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\mu}$$

$$\cancel{\frac{\partial}{\partial\mu}\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial^2 p}{\partial\mu\,\partial y}+\partial^2 u}$$

$$\begin{array}{l|l} Du=-\frac{\partial p}{\partial x}+\mu\nabla^2 u & \frac{\partial}{\partial y}Du-\frac{\partial}{\partial x}Dv=\mu\nabla^2\overbrace{\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)}^{\zeta} \\ Dv=-\frac{\partial p}{\partial y}+\mu\nabla^2 v & \\ Dw=-\frac{\partial p}{\partial z}+\mu\nabla^2 w & \end{array}$$

$$\operatorname{curl} V(v\operatorname{curl}v)=\mu\nabla^2\operatorname{curl}v=-\mu\operatorname{curl}^3 v$$

więc niezależnie od μ pozostaje tylko warunek :

$$U\operatorname{curl}V(v\operatorname{curl}v)=U\nabla^2\operatorname{curl}v$$

$$\frac{\partial}{\partial y}(v\zeta-w\eta)-\frac{\partial}{\partial x}(w\xi-u\zeta):\frac{\partial}{\partial z}(w\xi-u\zeta)-\frac{\partial}{\partial y}(u\eta-v\xi):\cdots$$

$$=\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}+\frac{\partial^2\zeta}{\partial z^2}:\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}+\frac{\partial^2\xi}{\partial z^2}:—$$

ponieważ lewa strona kształtu $\frac{0}{0}$ zatem $\frac{\partial}{\partial\mu}$

$$\frac{\frac{\partial}{\partial\mu}\left[\frac{\partial}{\partial y}(v\zeta-w\eta)-\frac{\partial}{\partial x}(w\xi-u\zeta)\right]}{\frac{\partial}{\partial\mu}\left[\frac{\partial}{\partial z}(w\xi-u\zeta)-\frac{\partial}{\partial y}(u\eta-v\xi)\right.}=\frac{\Delta^2\zeta}{\Delta^2\xi}$$

$$\cancel{\Delta^2\xi\left[\frac{\partial^2}{\partial\mu\partial y}(\mu\nabla^2 u) - \frac{\partial^2}{\partial\mu\partial x}(\mu\nabla^2 v)\right] = -}$$

$$\cancel{\nabla^2\xi + \mu}$$

Nawet bezpośrednio:

$$\frac{\partial}{\partial\mu}(v\zeta - w\eta) = -\frac{\partial^2 p}{\partial x\partial\mu} + \nabla^2 u \quad \cancel{+\mu\nabla^2\frac{\partial u}{\partial\mu}}$$

$$\nabla^2 u = \frac{\partial}{\partial\mu}\left[\frac{\partial p}{\partial x} - (v\zeta - w\eta)\right]$$

Powierzchnia, gdzie dwie ciecze z tarciem graniczą ze sobą:

$$\frac{Du_1}{Dt} = X - \frac{\partial p_1}{\partial x} + \mu_1\nabla^2 u_1 \qquad Du_2 = -\frac{\partial p_2}{\partial x} + \mu_2\nabla^2 u_2$$

$$\frac{Dv_1}{Dt} = -\frac{\partial p_1}{\partial y} + \mu_1\nabla^2 v_1 \qquad Dv_2 = -\frac{\partial p_2}{\partial y} + \mu_2\nabla^2 v_2$$

$$\frac{Dw_1}{Dt} = -\frac{\partial p_1}{\partial z} + \mu_1\nabla^2 w_1 \qquad Dw_2 = -\frac{\partial p_2}{\partial z} + \mu_2\nabla^2 w_2$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \qquad \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial w_2}{\partial z} = 0$$

Dla powierzchni ~~jeśli nie ma się zmieniać z czasem~~

$$f(\overset{t}{x}y\,z) = 0$$

$\cancel{u_1 = u_2}$

po upływie czasu Δt będzie

$$\cancel{v_1 = v_2} \qquad \frac{\partial f}{\partial t} + u_1\frac{\partial f}{\partial x} + v_1\frac{\partial f}{\partial y} + w_1\frac{\partial f}{\partial z} = 0 \quad \Big\| \quad \frac{\partial f}{\partial t} + u_2\frac{\partial f}{\partial x} + \dots = 0$$

$\cancel{w_1 = w_2}$

co tylko możliwe jeśli

$u_1 = u_2 \,\|\, v_1 = v_2 \,\|\, w_1 = w_2$ na powierzchni

opór też tak samo:

$$l\,p_{1xx} + m\,p_{1xy} + n\,p_{1xz} = l\,p_{2xx} + m\,p_{2xy} + n\,p_{2xz}$$

itd.

Dzięki powietrzu w cieczy

Tak samo jak gdyby parcie było rachowane jako −2

$$\frac{Du_1}{Dt} = -\frac{\partial p}{\partial x} + \mu_1 \nabla^2 u \qquad \Bigg| \qquad \frac{Du_2}{Dt} = \overset{g}{/}\!\left(-\frac{1}{\rho_2}\frac{\partial p}{\partial x} + \frac{\mu_2}{\rho_2}\nabla^2 u\right.$$

$\mu_1 =$ ~~[illegible]~~

$= 0{\cdot}017$

$$\left.\begin{array}{l}\rho_2 = 0{\cdot}0013\\ \mu_2 = 0{\cdot}00017\end{array}\right\} \frac{\mu_2}{\rho_2} = \frac{0{\cdot}17}{1{\cdot}3} = 0{\cdot}13$$

dla wodoru $$\left.\begin{array}{l}\rho_2 = 0{\cdot}00009\\ \mu_2 = 0{\cdot}00009\end{array}\right\} \frac{\mu_2}{\rho_2} = 1{\cdot}0$$

Wewnątrz:

więc gdy ruch wody w oleju jeszcze się nie wytłumi, to już bardzo w H_2

Na powierzchni:

Układając oś OX w r

OY w $r\perp$

OZ w $r\parallel$

$$\begin{array}{|l}r = a\\ u_1 = u_2 = 0\\ v_1 = v_2\\ w_1 = w_2 = 0\end{array}$$

$$-p + 2\mu_1 \cancel{\frac{\partial u_1}{\partial x}} = -p + 2\mu_2 \cancel{\frac{\partial u_2}{\partial x}}$$

$$\mu_1\left(\frac{\partial u_1}{\partial y} + \underline{\underline{\frac{\partial v_1}{\partial x}}}\right) = \mu_2\left(\frac{\partial u_2}{\partial y} + \underline{\underline{\frac{\partial v_2}{\partial x}}}\right)$$

$$\mu_1\left(\frac{\partial u_1}{\partial z} + \cancel{\frac{\partial w_1}{\partial x}}\right) = \mu_2\left(\frac{\partial u_2}{\partial z} + \cancel{\frac{\partial w_2}{\partial x}}\right) \quad \Big\| \quad w = 0$$

głównie: $\mu_1 \frac{\partial v_1}{\partial x} = \mu_2 \frac{\partial v_2}{\partial x}$

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$\mu_1 \Delta^2 u_1$ i $\mu_2 \Delta^2 u_2$ będą zatem tego samego rzędu wielkości

$\frac{\partial p_1}{\partial x}$ ...... $\frac{\partial p_2}{\partial x}$

zatem uwzględniając $\frac{Du_1}{Dt}$ można przecież zaniedbać $\rho_2 \frac{Du_2}{Dt}$

/bo w tego samego rzędu wielkości

Obierając dla wody:

$$u = c\left[1 + \frac{a^3}{2r^3}\left(1 - \frac{3x^2}{r^2}\right)\right] = \frac{\partial \varphi}{\partial x}$$

$$v_\omega = -\frac{3ca^3}{2r^5}\, x\,\omega$$

$$\varphi = c\left(x + \frac{a^3 x}{2r^3}\right) = cx\left[1 + \frac{a^3}{2r^3}\right]$$

~~$= cx[\cos\theta\ c$~~ $= c\, r\cos\theta\left[1 + \frac{a^3}{2r^3}\right]$

H_2

wzdłuż południków: $v_\theta = \left(\frac{\partial\varphi}{\partial\theta}\right)_{r=\mathrm{const}} = -\,c\,r\sin\theta\left[1 + \frac{a^3}{2r^3}\right]$

$$\left.\frac{\partial v_\theta}{\partial r}\right|_{r=a} = -c\sin\theta\left.\left[1 + \frac{a^3}{2r^3} - \frac{a^3}{r^3}\right]\right|_{r=a} = -\frac{c}{2}\sin\theta$$

$$\left.v_\theta\right|_{r=a} = -\frac{3}{2}ac\sin\theta$$

Dla gazów bardzo rozrzedzonych przy zwykłej temp. Opuszczono

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)$!

Np. p=1 mm $\rho = 1{\cdot}5 . 10^{-6}$

$\mu = 0{\cdot}00017$ $\frac{\mu}{\rho} = 100$!

$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu \nabla^2 u$ także p bardzo małe

zatem przybliżenie: $\left.\begin{array}{l}\nabla^2 u = 0\\ \nabla^2 v = 0\\ \nabla^2 w = 0\end{array}\right\}$ $\nabla^2 v = 0$ ale to nie można teraz

stawić $= \mathrm{curl}^2 v$

~~przeto~~ bo $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

Nie zawierają $\frac{\partial}{\partial t}$ ↗

zatem przy danych warunkach granicznych nie zmienia się z czasem

Więc ruch ustanawia się natychmiastowo | zatem nie ma zjawisk akustycznych

Patrz Lamb p 526 | dochodzi ten warunek $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

Np. kula dla prędkości $u=c,\ v=w=0$

$r=a$ $u=v=w=0$

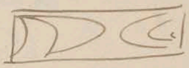
Z tego wynika, że $\begin{cases} v=w=0 \text{ wszędzie} \\ u=c\left[1-\frac{a}{r}\right] \end{cases}$

Wogóle jeżeli ruch powstaje wskutek przesuwania ciał to zawsze

równania graniczne kształtu $u=c_1$, $v=c_2$, $w=c_3$ dla pewnych powierzchni zupełnie analogicznie jak: U w elektrostatyce między konduktorami na różnych potencj.

Jeśli granica w spoczynku, to wszystko w spoczynku.

Przepływ przez rurkę wody nie nastąpi gdyż jeśli jest przesłoniony tłokiem



W tamtym przykładzie:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = u_0 \frac{a x}{r^3}$$

$c + AR = C$

$1 + \frac{AR}{c} = k$

$\frac{AR}{c} = k-1$

$$\frac{c}{A} \rho \frac{D\theta}{Dt} + p \frac{\partial u}{\partial x} = 2\mu \left[+ \frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \right]$$

z przewodzeniem ciepła $\| + \kappa \Delta^2 \theta$

$$\| \quad \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x}$$

$$\frac{c}{A} \rho\, u_0 \left[1 - \frac{a}{r}\right] \frac{\partial \theta}{\partial x} + \rho R \theta\, u_0 \frac{a x}{r^3} = \cancel{2\mu}\left[2\mu \frac{u_0^2 a^2}{r^6} \left[\underbrace{y^2 + z^2 + \frac{2}{3} x^2}_{r^2 - \frac{x^2}{3}} \right]\right.$$

$$\frac{\partial}{\partial x}(\rho u) = 0 \qquad \cancel{u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0}$$

$$\rho u = \text{const} = \rho_0 u_0 \qquad \rho = \frac{\rho_0}{1 - \frac{a}{r}}$$

[więc na powierzchni warstwa zgęszczona] oczywiście też tam zachodzi powolny ruch v, w

$$\frac{c}{A} \rho_0 u_0 \frac{\partial \theta}{\partial x} + \theta . R\, \rho_0 u_0 \frac{a x}{r^3 (1 - \frac{a}{r})} = 2\mu \frac{u_0^2 a^2}{r^6} \left(r^2 - \frac{x^2}{3}\right)$$

$$\frac{\partial \theta}{\partial x} + \theta (k-1) \frac{a}{r^2 - a r} \frac{x}{r} = 2\mu \frac{k-1}{R} \frac{u_0 a^2}{\rho_0} \frac{r^2 - \frac{x^2}{3}}{r^6}$$

$$\frac{a}{r^2 - a r} = \underbrace{\frac{-1}{r} + \frac{1}{r-a}}_{\frac{\partial}{\partial x} \log\left(\frac{r-a}{r}\right)}$$

$$\frac{\partial\theta}{\partial x} + \theta X_1 = X_2 \qquad \cancel{\theta = a e^{\int}} \qquad \theta = zy$$

$$y\frac{dz}{dx} + X_1 zy + z\frac{dy}{dx} = X_2 \qquad X_1 y + \frac{dy}{dx} = 0$$

$$y = e^{-\int X_1 dx}$$

$$\theta = e^{-\int X_1 dx}\left[c + \int X_2 e^{+\int X_1 dx} dx\right]$$

$$X_1 = (k-1)\frac{\partial}{\partial x}\log\frac{r-a}{r}$$

$$\int X_1 dx = (k-1)\log\frac{r-a}{r} \; \cancel{\varphi(y,z)} = \log\left(\frac{r-a}{r}\right)^{k-1}$$

$$e^{\int\cdots} = \left(\frac{r-a}{r}\right)^{k-1} \qquad X_2 = \frac{2\mu(k-1)}{R}\,\frac{u_0 a^2}{\rho_0}\,\frac{r^2 - \frac{x^2}{3}}{r^6}$$

$$\theta = \left(\frac{r}{r-a}\right)^{k-1}\left[\overset{f(y,z)}{c} + \frac{2\mu(k-1)u_0a^2}{R\rho_0}\int_0^x \frac{r^2-\frac{x^2}{3}}{r^6}\left(\frac{r-a}{r}\right)^{k-1} dx\right]$$

~~c oznacza się z tego że temp. dla x=∞ w wielkiej odległości nie zmienna~~ θ_0

Do oznaczenia stałych :

Dla $x = -\infty$: $\theta = \theta_0$ niezal. od y,z

$$\int_0^\infty \frac{r^2-\frac{x^2}{3}}{r^6}\left(\frac{r-a}{r}\right)^{k-1} dx = \varphi(y,z)$$

$$\theta_{-\infty} = f(y,z) - \frac{2\mu(k-1)u_0a^2}{R\rho_0}\varphi(y,z)$$

$$f(y,z) = \theta_{-\infty} + \frac{2\mu(k-1)u_0a^2}{R}\varphi$$ [illegible]

$$\theta = \left(\frac{r}{r-a}\right)^{k-1}\left[\theta_{-\infty} + \frac{2\mu(k-1)u_0 a^2}{R\,\rho_0}\int_{-\infty}^{x}\frac{r^2-\frac{x^2}{3}}{r^6}\left(\frac{r-a}{r}\right)^{k-1}dx\right]$$

więc środkowa część ograniczona
po przejściu

$$p_{xx} = \frac{4}{3}\mu\frac{\partial u}{\partial x} \qquad p_{yz} = 0$$

$$p_{yy} = p_{zz} = -\frac{2}{3}\mu\frac{\partial u}{\partial x} \qquad p_{xy} = \mu\frac{\partial u}{\partial y} \qquad p_{xz} = \mu\frac{\partial u}{\partial z}$$

$$\frac{\partial u}{\partial x} = \frac{u_0 a x}{r^3} \qquad \frac{\partial u}{\partial y} = \frac{u_0 a y}{r^3} \qquad \frac{\partial u}{\partial z} = \frac{u_0 a z}{r^3}$$

$$p_{rx} = \frac{u_0 a\mu}{r^4}\left(\frac{4}{3}x^2+y^2+z^2\right) = \frac{u_0 a\mu}{r^4}\left(r^2+\frac{x^2}{3}\right)$$

$$p_{ry} = \frac{u_0 a\mu}{r^4}\left(xy-\frac{2}{3}yx\right) = \frac{u_0 a\mu}{r^4}\,\frac{xy}{3}$$

$$p_{rz} = \frac{u_0 a\mu}{r^4}\left(xz-\frac{2}{3}xz\right) = \frac{u_0 a\mu}{r^4}\,\frac{xz}{3}$$

$$\Sigma p_{ry} = 0 \qquad \Sigma p_{rz} = 0$$

$$\Sigma p_{rx} = \frac{u_0 a\mu}{a^4}\,a^2\int_0^{\pi}\left(1+\frac{\cos^2\theta}{3}\right)2\pi a^2\sin\theta\,d\theta$$

$$= u_0 a\mu\cdot 2\pi\left(-\cos\theta-\frac{\cos^3\theta}{9}\right)\Big|_0^{\pi} = 2a\pi\,u_0\mu\cdot\frac{20}{9} = \frac{40}{9}a\pi u_0\mu$$

Więc np. końcowa prędkość pod wpływem grawitacji:

$$\frac{4}{3}a^3\pi\,s\,g = \frac{40}{9}a\pi u_0\mu$$

$$u_0 = \frac{3}{10}\,\frac{a^2 s g}{\mu}$$

Przy tamtem przybliżeniu więc w ogóle nie nastąpi żaden ruch jeśli $u\,v\,w$ na powierzchniach $=0$.

Ażeby w zamkniętem naczyniu otrzymać ruch, należy uwzględnić: $\frac{\partial p}{\partial x}$ wtedy jednak jeśli ruch powolny można zaniedbać jeszcze $\rho u \frac{\partial u}{\partial x}$ etc.

$$\left.\begin{aligned}\rho\frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial x} + \mu\nabla^2 u\\ \rho\frac{\partial v}{\partial t} &= -\frac{\partial p}{\partial y} + \mu\nabla^2 v\\ \rho\frac{\partial w}{\partial t} &= -\frac{\partial p}{\partial z} + \mu\nabla^2 w\end{aligned}\right\}$$

$p = R\rho\theta$

zaniedbując zmienności μ:

$$\underbrace{\frac{\partial}{\partial x}\left(\rho\frac{\partial u}{\partial t}\right)+\frac{\partial}{\partial y}\left(\rho\frac{\partial v}{\partial t}\right)+\frac{\partial}{\partial z}\left(\rho\frac{\partial w}{\partial t}\right)}_{\rho\frac{\partial}{\partial t}\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\cdots\right]+\left[\frac{\partial \rho}{\partial x}\frac{\partial u}{\partial t}+\frac{\partial \rho}{\partial y}\frac{\partial v}{\partial t}+\cdots\right]=\nearrow} = \overset{\nabla^2 p\,+}{\mu\nabla^2\left(\frac{\partial u}{\partial x}+\cdots\right)} =$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

$$\frac{\partial \rho}{\partial t}+\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\cdot\right]+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+\cdots=0$$

$$\frac{\partial^2 \rho}{\partial t^2}+\frac{\partial \rho}{\partial t}\left[\frac{\partial u}{\partial x}+\cdots\right]+\rho\frac{\partial}{\partial t}\left[\frac{\partial u}{\partial x}\cdots\right]+\frac{\partial u}{\partial t}\frac{\partial \rho}{\partial x}\cdots+u\frac{\partial^2 \rho}{\partial x\,\partial t}+\cdots=0$$

Dla stationären Zustand:

$$\mu\nabla^2 u = \frac{\partial p}{\partial x}$$

$$\mu\nabla^2 v = \frac{\partial p}{\partial y}$$

$$\mu\nabla^2 w = \frac{\partial p}{\partial z}$$

$$\frac{c}{A}\rho\frac{D\theta}{\partial t}+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=2\mu\left[\cdots\quad\right]$$

~~$\underbrace{\frac{1}{\rho}\frac{\partial\rho}{\partial t}}_{\frac{\partial}{\partial t}(\lg\rho)} + \left[\frac{\partial u}{\partial x} + \dots\right] + u\frac{\partial}{\partial x}(\lg\rho) + v\frac{\partial}{\partial y}(\lg\rho) + \dots = 0$~~

~~$\lg\rho = \lg p - \lg\theta - \lg R$~~

Jaka zmiana p w ruchu w rurkach wąskich wskutek wzgl. dużego rozrzedzenia termicznego? Zaniedbujemy: inertia terms:

$v = w = 0$

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{\partial p}{\partial x}$$

$$\frac{\partial}{\partial x}(\rho u) = 0 \qquad \rho u = f(y,z)$$

$$\frac{c}{A}\rho\, u\frac{\partial\theta}{\partial x} + R\rho\theta\frac{\partial u}{\partial x} = 2\mu\left[-\frac{1}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]$$

Albo przez p:

$$\mu\nabla^2 u = \frac{\partial p}{\partial x} \qquad p = \varphi(x)$$

$$\frac{pu}{\theta} = f(y,z)$$

$$\frac{c}{AR}u\frac{1}{\theta}\frac{\partial\theta}{\partial x} + p\frac{\partial u}{\partial x} = 2\mu\left[\frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]$$

~~$\varphi(x)\frac{u}{\theta} = f(y,z)$~~ $\qquad p = \frac{\theta}{u}f(y,z)$

~~$\frac{u}{\theta}\frac{\partial p}{\partial x} + p\frac{\partial}{\partial x}\left(\frac{u}{\theta}\right) = 0$~~ $\qquad \mu\nabla^2 u = \frac{\partial p}{\partial x} = f(y,z)\frac{\partial}{\partial x}\left(\frac{\theta}{u}\right)$

$$\mu\nabla^2 u = R\left(\theta\frac{\partial\rho}{\partial x} + \rho\frac{\partial\theta}{\partial x}\right)$$

$$\frac{c}{A}\rho u\frac{\partial\theta}{\partial x} + R\rho\theta\frac{\partial u}{\partial x} = 2\mu\left[\frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]$$

u

$\rho u = f(y,z)$

$\rho\theta = \varphi(x)$

$$\left(\frac{c}{A} + R\right)\rho u\frac{\partial\theta}{\partial x} + R\left(\theta u\frac{\partial\rho}{\partial x} + \theta\rho\frac{\partial u}{\partial x}\right) = \mu\left[u\nabla^2 u + 2\left(\frac{2}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right)\right]$$

Czy możliwe rozwiązanie formy $\theta = f(p)$?

zatem $\rho = \varphi(p) = \frac{p}{R\,f(p)}$

$$\left.\begin{aligned} \mu \nabla^2 u &= \frac{\partial p}{\partial x} \\ \mu \nabla^2 v &= \frac{\partial p}{\partial y} \\ \mu \nabla^2 w &= \frac{\partial p}{\partial z} \end{aligned}\right\}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\downarrow$$

$$\text{I).}\ \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0$$

$$\text{II).}\ \frac{c}{A}\rho\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 2\mu\left[\cdots\right]$$

$$\text{II).}\ \frac{c}{A}\varphi(p)\,f'(p)\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = \nearrow$$

$$\text{I).}\ \varphi(p)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \varphi'(p)\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) = 0$$

$$\text{I).}\ u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \mu\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)$$

$$f(p) = \frac{p}{R\,\varphi(p)}$$

$$f'(p) = \frac{1}{R}\left[\frac{1}{\varphi(p)} - \frac{p\,\varphi'(p)}{\varphi(p)}\right]$$

$$\frac{\varphi'(p)}{\varphi(p)} = -\frac{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}}{\mu\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)}$$

$$\frac{c}{A}\varphi(p)\,f'(p)\,\mu\left(u\nabla^2 u + v\Delta^2 v + w\Delta w\right) \pm p\,\mu\left[u\Delta^2 u + -\right]\frac{\varphi'(p)}{\varphi(p)} = 2\mu\left[\cdots\right]$$

$$\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)\underbrace{\left[\frac{c}{A}\varphi(p)\,f'(p) - p\,\frac{\varphi'(p)}{\varphi(p)}\right]}_{\left[\frac{c}{AR}\left(1 - p\frac{\varphi'(p)}{\varphi(p)}\right) - p\frac{\varphi'(p)}{\varphi(p)}\right]} = 2$$ [illegible] $$\left[-\frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 \cdots\right]$$

$AR = C - c$

$\frac{c}{C-c} = \frac{1}{k-1}$

$$\left[\overset{\parallel}{\frac{1}{k-1}} - \frac{k}{k-1}\;\frac{p\,\varphi'(p)}{\varphi(p)}\right]$$

Założeniu które się robi ~~[illegible]~~ w teorii Joule-Thomson: $\theta = \text{const}$

$f = \theta_0$

$$\varphi = \frac{p}{R\theta_0}$$

$$\varphi' = \frac{1}{R\theta_0} \qquad \frac{p\varphi'}{\varphi} = 1 \qquad \left[\cancel{A = \frac{k}{k-1}}\right.$$

Wtedy:

$$-(u\nabla^2 u + v\Delta^2 v + w\Delta^2 w) = 2\left[-\frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 + \cdots\right]$$

$$\therefore\; 3(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w) - 2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 + 6\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] +$$

$$+\, 3\left[\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] = 0$$

W ~~jednowymiarowym~~ przykładzie $v = w = 0$:

$$3\,u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - 2\left(\frac{\partial u}{\partial x}\right)^2 + 6\left(\frac{\partial u}{\partial x}\right)^2 + 3\left(\frac{\partial u}{\partial z}\right)^2 + 3\left(\frac{\partial u}{\partial y}\right)^2 = 0$$

$$4\left(\frac{\partial u}{\partial x}\right)^2 + 3\,u\frac{\partial^2 u}{\partial x^2} + 3\Bigg[\underbrace{u\frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial y}\right)^2}_{\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right)} + \underbrace{u\frac{\partial^2 u}{\partial z^2} + \left(\frac{\partial u}{\partial z}\right)^2}_{\frac{\partial}{\partial z}\left(u\frac{\partial u}{\partial z}\right)}\Bigg] = 0$$

Jednowymiarowy przykład:

$$4\left(\frac{\partial u}{\partial x}\right)^2 + 3u\frac{\partial^2 u}{\partial x^2} = 0$$

$$4\,\frac{\frac{\partial u}{\partial x}}{u} + 3\,\frac{\frac{\partial^2 u}{\partial x^2}}{\frac{\partial u}{\partial x}} = 0$$

$$\log\left[u^4 \cdot \left(\frac{\partial u}{\partial x}\right)^3\right] = \text{const}$$

$$\frac{du}{dx}\,u^{\frac{4}{3}} = c$$

$$\frac{3}{7}\,u^{\frac{7}{3}} = cx + c_0$$

Praca w ogóle nie jest oznaczona przez (podczas 1 sec) $\int \frac{\partial u}{\partial n} p \, dS = \mu \int (u \nabla^2 u + \nabla v \, dy \ldots)$

$\frac{\partial p}{\partial x} = \mu \nabla^2 u$ tylko przez $\iint p_{xx} \, dS$ etc.

$$\iiint \left(u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \right) dv = \iint u p \, dy \, dz + v p \, dx \, dz + w p \, dx \, dy -$$

$$- \iiint p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dx \, dy \, dz$$

$$= \int p \, (u \cos nx + v \cos ny + w \cos nz) \, dS - \iiint p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dx \, dy \, dz$$

$$\rho \left(\frac{\partial u}{\partial x} + \ldots \right) + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \ldots = 0$$

zatem:

$$\mu \iiint (u \Delta^2 u + \cdot \quad) \, dv = W_p -$$

Jeśli jednak granica gazu cieczy w których wszędzie jest ruch taki niezmienny, to całkowita praca wewnętrzna hydro. oznaczona przez

$$W_{p_I} = \mu \iiint (u \Delta^2 u + \ldots) \, dv + \iiint p \left(\frac{\partial u}{\partial x} + \ldots \right) dv$$

$$= \int p \, (ul + vm + wn) \, dS$$

albo też przez funkcyę Φ:

$$W_{p_{II}} = \iiint \Phi \, dv$$ to ostatnie zawsze ważne

W_{p_I} jeśli powierzchnie przez izopiesty $= p_2 \int_2 [ul + vm + wn] \, dS - p_1 \int_1 (ul -) \, dS$

~~jeśli~~

Równanie ciągłości daje dla dowolnego przekroju poprzecznego wzdłuż wszystkich linii prądu:

$$\int \rho(ul + vm + wn)\, dS = \text{const} = m$$

jeżeli ρ tylko $\overset{\rho = f(p)}{\text{fcja}(p)}$, wtedy ~~przekładni~~ dla tych samych powierzchni (izopiest.):

$$\rho_1 \int_1 \quad = \rho_2 \int_2$$

$$\rho_1 \int_1 \quad - \rho_2 \int_2 \quad = 0 \qquad f(p_1)\int_1 - f(p_2)\int_2 = 0$$

ale z tego jeszcze nie wynika że: $p_1 \int_1 - \; p_2 \int_2 = 0$

tylko wtedy jeżeli $f(p_1) \sim p_1$ to znaczy w układzie izotermicznym

$W_{pI} = 0$

Inaczej równanie to musi być spełnione tylko w specyalnych przypadkach.

$$\int_1 = \frac{m}{\rho_1} = ~~\text{mv}_1~~ \; m V_1$$

$$W_{pI} = m(p_2 V_2 - p_1 V_1) = m R(\theta_2 - \theta_1)$$

$$\delta Q = \overset{dU}{\cancel{dA}} + dW$$

W_{pI} jest praca, którą maszyna zewnętrzna, utrzymująca przepływ, musi wykonać.

$$W_{pI} = \int (u\,\nabla^2 u + \ldots \quad)\,dv + p\left(\frac{\partial u}{\partial x} + \ldots \quad \right) dv$$

$$= \int \left[\frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) + \frac{\partial}{\partial z}(pw)\right] dv$$ co też bezpośrednio!

$p = R\rho\theta$
$$= R \int \theta \underbrace{\left[\frac{\partial(\rho u)}{\partial x} + \cancel{\frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}}\right]}_{=0} + \rho\left[\frac{\partial(\theta u)}{\partial x} + \frac{\partial(\theta v)}{\partial y} + \frac{\partial(\theta w)}{\partial z}\right] dv$$

$$= R \int \rho\left[\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) + \theta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right] dv$$

$$= \int \left[R\rho\left(u\frac{\partial\theta}{\partial x} + \quad \right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right] dv$$

$$= \int \left[\Phi + \left(R - \frac{c}{A}\right)\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)\right] dv$$

$$= \int \Phi\,dv + \left(R - \frac{c}{A}\right)\int (\rho u\,\theta\, l + \rho v\,\theta\, m + \rho w\,\theta\, n)\,dS -$$

$$- \left(R - \frac{c}{A}\right)\int \theta\,\cancel{\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]} dv$$

$$= \int \Phi\,dv + \left(R - \frac{c}{A}\right)\int \theta\rho\,(ul + vm + wn)\,dS$$

$$W_{pI} = \int \Phi\,dv + \left(1 - \frac{c}{AR}\right)\underbrace{\int p\,(ul + mv + wn)\,dS}_{= W_p\ !}$$

$$\frac{c}{AR}\,W_{pI} = \int \Phi\,dv \qquad \frac{1}{k-1}\,W_{pI} = \int \Phi\,dv$$

$$W_{pI} = (k-1)\int \Phi\,dv$$

$$\iiint u\nabla^2 u\, d\tau = \iint \underbrace{u\frac{\partial u}{\partial n}}_{\frac{1}{2}\frac{\partial(u^2)}{\partial n}} dS - \iiint\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right] d\tau$$

$$W_{pI} = \iint \frac{1}{2}\frac{\partial(u^2+v^2+w^2)}{\partial n}\, dS - \iiint \left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\cdots + \left(\frac{\partial v}{\partial x}\right)^2+\cdots + \left(\frac{\partial w}{\partial x}\right)^2+\cdots \quad d\tau + p\left(\frac{\partial u}{\partial x}+\cdots\right) d\tau$$

$$\int \Phi\, d\tau = \mu\int dx\,dy\,dz \Big\{ -\frac{2}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2 + 2\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + 2\frac{\partial u}{\partial x}\frac{\partial w}{\partial z} + 2\frac{\partial v}{\partial y}\frac{\partial w}{\partial z}\right] +$$

$$2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \left(\frac{\partial v}{\partial x}\right)^2 .$$

$+$

$+$

$$+\frac{1}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2 + \cdots \qquad + \cdots$$

$$+2\left[\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}+\frac{\partial v}{\partial z}\frac{\partial w}{\partial y} - \frac{2}{3}\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+\frac{\partial v}{\partial y}\frac{\partial w}{\partial z}+\frac{\partial w}{\partial z}\frac{\partial u}{\partial x}\right)\right]$$

$$= \iint \frac{1}{3}\left(u\frac{\partial u}{\partial x}l + v\frac{\partial v}{\partial y}m + w\frac{\partial w}{\partial z}n\right) dS + \underbrace{\iint u\left(\frac{\partial u}{\partial x}l + \frac{\partial u}{\partial y}m + \frac{\partial u}{\partial z}n\right) + \cdots}_{=\iint \frac{1}{2}\frac{\partial(u^2+v^2+w^2)}{\partial n}\, dS}$$

$$+2\iint\left[u\frac{\partial v}{\partial x}m + v\frac{\partial w}{\partial y}n + w\frac{\partial u}{\partial z}l - \frac{2}{3}\left(u\frac{\partial v}{\partial y}l + v\frac{\partial w}{\partial z}m + w\frac{\partial u}{\partial x}n\right)\right] dS$$

$$-\iiint\frac{1}{3}\left(u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial y^2}+w\frac{\partial^2 w}{\partial z^2}\right)d\tau - \iiint\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right) d\tau -$$

$$-2\iiint\left[u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z} - \frac{2}{3}\left(u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z}\right)\right] d\tau \quad =$$

$$= \frac{1}{3}\iint\left(u\frac{\partial u}{\partial x}l + v\frac{\partial v}{\partial y}m + w\frac{\partial w}{\partial z}n\right)dS + \frac{1}{2}\iint\frac{\partial}{\partial n}(u^2+v^2+w^2)\,dS +$$

$$\left.\begin{aligned}&\iint u\frac{\partial v}{\partial x}\cdot m + v\frac{\partial w}{\partial y}\cdot n + w\frac{\partial u}{\partial z}\cdot l + v\frac{\partial u}{\partial y}\cdot l + w\frac{\partial v}{\partial z}\cdot m + u\frac{\partial w}{\partial x}\cdot n -\\ &\iint -\frac{2}{3}\left[u\frac{\partial v}{\partial y}l + v\frac{\partial w}{\partial z}m + w\frac{\partial u}{\partial x}n + v\frac{\partial u}{\partial x}m + w\frac{\partial v}{\partial y}n + u\frac{\partial w}{\partial z}l\right]\end{aligned}\right\} dS$$

$$-\frac{1}{3}\iiint\left(u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 v}{\partial y^2} + w\frac{\partial^2 w}{\partial z^2}\right)dv - \iiint\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)dv -$$

$$-\iiint u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z} + v\frac{\partial^2 u}{\partial x\partial y} + w\frac{\partial^2 v}{\partial y\partial z} + u\frac{\partial^2 w}{\partial x\partial z} -$$

$$\updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow$$

$$-\frac{2}{3}\left[u\frac{\partial^2 v}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y\partial z} + w\frac{\partial^2 u}{\partial x\partial z} + v\frac{\partial^2 u}{\partial x\partial y} + w\frac{\partial^2 v}{\partial y\partial z} + u\frac{\partial^2 w}{\partial x\partial z}\right]dv$$

$$\iiint = -\frac{1}{3}\iiint u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) + v\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 w}{\partial y\partial z}\right) + w\left(\quad\cdots\right)$$

$$-\iiint\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)dv$$

$$\iiint = -\frac{1}{3}\iiint\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)dv$$

$$-\iiint\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)dv$$

$$\iint = \frac{1}{2}\iint\frac{\partial}{\partial n}(u^2+v^2+w^2)\,dS + \iint u\left[l\frac{\partial u}{\partial x} + m\frac{\partial v}{\partial x} + n\frac{\partial w}{\partial x}\right] + v\left[l\frac{\partial u}{\partial y} + m\frac{\partial v}{\partial y} + n\frac{\partial w}{\partial y}\right] + w\left[\cdot\right]$$

$$-\frac{2}{3}\iint\left[u\,l\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + v\,m\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + w\,n\left(\partial\ --\right)\right]dS$$

$$\int \Phi \, d\omega = \frac{1}{2}\iint \frac{\partial}{\partial n}(u^2+v^2+w^2)\, dS + \iint \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\overset{\overset{v_n}{\|}}{(ul+vm+wn)}\, dS -$$

$$-\frac{2}{3}\iint (ul+vm+wn)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dS \quad -$$

$$- \iiint (u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)\, d\omega - \frac{1}{3}\iiint \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) d\omega$$

~~Dla cieczy nieściśliwej zatem~~: $W_{pI} = W_{pII}$ ~~jest~~ pod tymi warunkami

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\left[u\frac{\partial(\log\rho)}{\partial x} + v\frac{\partial(\log\rho)}{\partial y} + w\frac{\partial(\log\rho)}{\partial z}\right]$$

$$\log\rho = \log p - \log R - \log\theta$$

$$\int \Phi\, d\omega = \iint \ \cdots \quad - \iiint (u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)\, d\omega -$$

$$\cancel{+\frac{1}{3}\iiint} \qquad \cancel{+\frac{1}{3}\iiint \frac{1}{p}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) - \frac{1}{\theta}\, u\frac{\partial\theta}{\partial}}$$

$$+\frac{1}{3}\iiint \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\underbrace{\left(u\frac{\partial(\log p - \log\theta)}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\ \cdot\right)}_{\frac{1}{p}\underbrace{\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right)}_{= -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)} - \frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)} d\omega$$

$$\iiint p\left(\frac{\partial u}{\partial x} + \ \right) d\omega = -\iiint \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) d\omega + R\iiint \rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) d\omega$$

$$W_{pI} = R\iiint \rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) dv =$$

$$= R\iint \rho(u\theta l + v\theta m + w\theta n)\, dS - R\iiint \theta\,\cancel{\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]} dv$$

$$= R\iint \rho\theta(ul + vm + wn)\, dS = \iint(p\,u\,l + \ldots\ldots)$$

$$p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \ldots\right) = R\,\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)$$

III. $p\left(\frac{\partial u}{\partial x} + \ldots\right) + \frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x} + \ldots\right) = \Phi$

$$\underbrace{\iiint p\left(\frac{\partial u}{\partial x} + \ldots\right) dv} + \underbrace{\frac{c}{A}\iiint \rho\left(u\frac{\partial\theta}{\partial x} + \ldots\right) dv} = \iiint \Phi\, dv$$

$$= \iint p(ul + vm + wn)\, dS + \frac{c}{A}\iint \rho\theta(ul + vm + wn)\, dS$$

$$- \iiint\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) dv - \frac{c}{A}\iiint \theta\,\cancel{\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \ldots\right]} dv$$

$$\left(1 + \frac{c}{AR}\right)\iint p(ul + vm + wn)\, dS = \iiint(u\nabla^2 u + v\Delta^2 v \ldots)\, dv + \Phi\, dv$$

$$\cancel{u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}} = \cancel{\left(\ldots + \frac{c}{A}\ldots\right)}$$

$$\frac{k}{k-1}\,W_{pI} = \iiint(u\nabla^2 u + v\Delta^2 v \ldots) + \Phi\, dv$$

$$\Phi_r$$

$$\frac{1}{k-1}\,W_{pI} =$$

$$W_{1\,I} = -\iiint (u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)\,dv \qquad \| ?$$

$$\iiint \Phi\,dv = \frac{1}{2}\iint \frac{\partial}{\partial n}(u^2+v^2+w^2)\,dS + \iint\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)v_n\,dS - \cancel{f}$$

$$-\iint (ul+vm+wn)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)dS -$$

$$-\iiint (u\nabla^2 u + v\nabla^2 v + w\nabla^2 w)\,dv + \frac{1}{3}\iiint\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2 dv$$

$$V(v\,.\,\mathrm{curl}\,v) = -\nabla\left(p+\frac{u^2+v^2+w^2}{2}\right) \underbrace{+\mu\nabla^2 v}_{=-\mu\,\mathrm{curl}^2 v}$$

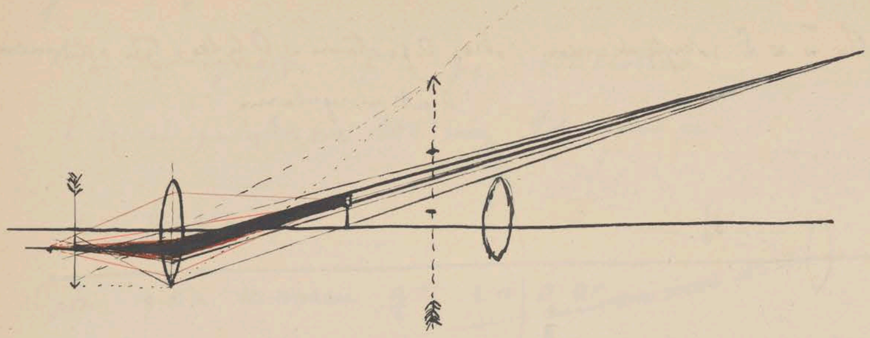
$$\mathrm{curl}\,V(v.\mathrm{curl}\,v) = -\mu\,\mathrm{curl}^3 v = \cancel{\mu\,\mathrm{curl}} = \mu\nabla^2\mathrm{curl}\,v$$

$$\mathrm{div}\,V(v.\mathrm{curl}\,v) = (\mathrm{curl}\,v)^2 - v.\mathrm{curl}^2 v = -\nabla^2\left(p+\frac{v^2}{2}\right)$$

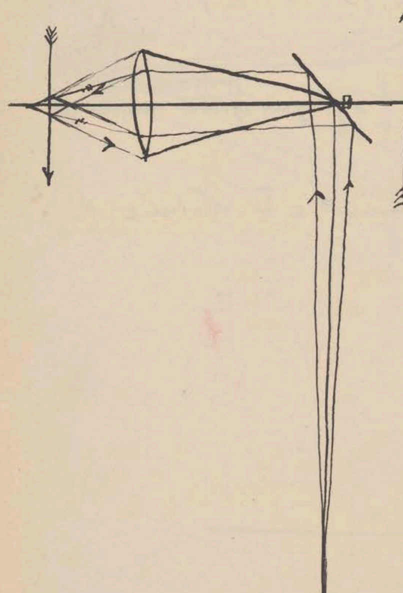
Ale teraz: $\mathrm{curl}\,V(v.\mathrm{curl}\,v) = 0$

$V(v.\mathrm{curl}\,v) = \nabla U$ Czy to jest jednoznacznie oznaczone przez v na powierzchni?

Na powierzchni curl v leży w powierzchni samej, $v=0$, ale w kierunku stycznym więc ∇U musi się stać $=0$ na powierzchni nieruchomej



Dogodniej jednak tak :

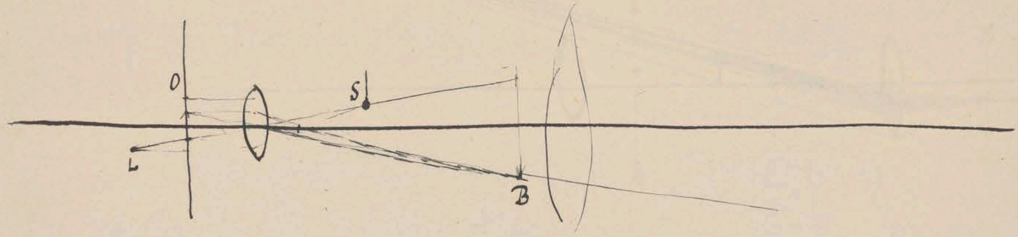


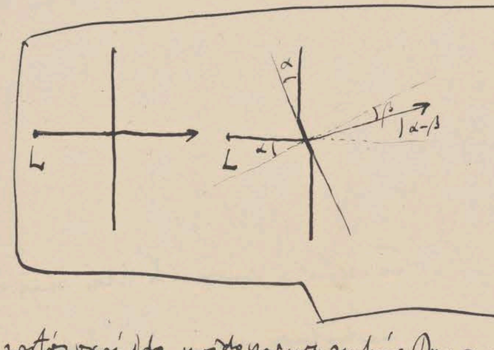
Te części zwierciadła które mają większe

nachylenie niż   będą wysyłać prom

zatem będą świecące

obraz światła L w S zakryty ekranem ; obraz O powstanie w O tylko o tyle o ile powierzchnia O jest nieregularna.





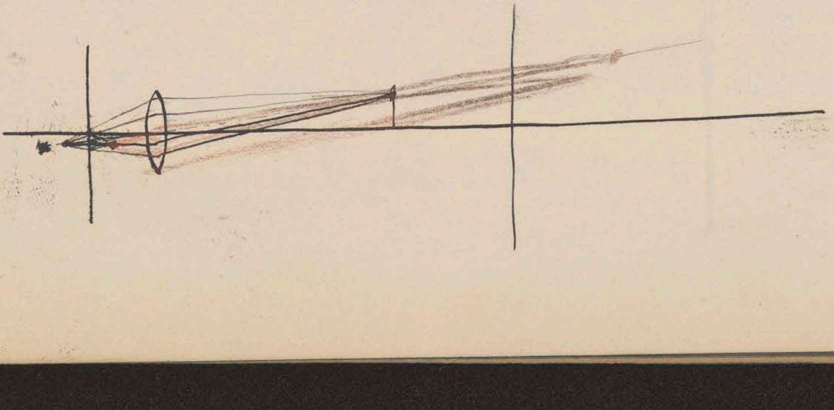
$$\sin\alpha = n \sin\beta$$
$$\cos\alpha \, d\alpha = n \cos\beta \, d\beta$$
$$d\alpha = n \, d\beta$$
$$d(\alpha-\beta) = (n-1) \, d\beta$$

To da się zastosować do [illegible] ruchów Browna

2). do obserw. błonek cienkich (cieczy) [wykrycia centrów wypukłości miejscowych

Czy odbijającą powierzchnię [illegible] czy nie można by oświetlać zamiast z L wprost z S ?

zatem taki plan:



U gazów przy niższem ciśnieniu!

Np. zmniejszając p_1 i p_2 100 krotnie

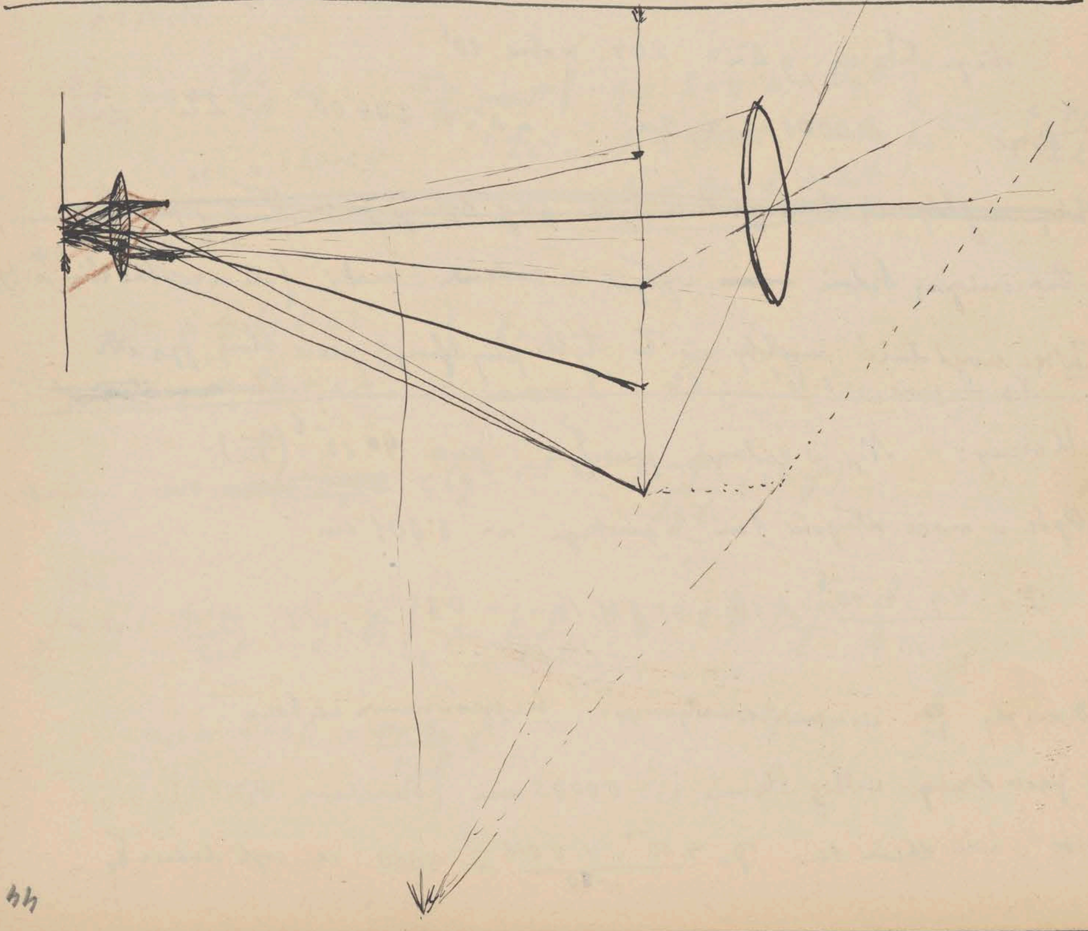
k będzie powiększone 100 razy, tak samo u

$u = 63{\cdot}4$ cm

Opór $\eta \Delta^2 u$ w rurce $\frac{r}{2} = \varepsilon = 0{\cdot}01$ cm

$= 204$ co wobec $\frac{\partial p_1}{\partial x} = \frac{10^6}{100}$ wynosi 2%

Więc wiele znaczniejszy ~~p~~ wpływ z drugiej strony tutaj „Gleitung"



N. p. jeśli $\varepsilon =$ ~~0·1 cm~~ 0·01 cm

$\eta = 0{\cdot}00017$

$l = 10$

$u_0 = a = 0{\cdot}6$ cm

$\Delta^2 u = 2c = \frac{2a}{\varepsilon^2} = \frac{1{\cdot}2}{10^{-4}}$

$p_1 - p_2 = \frac{2\eta l u_0}{\varepsilon^2} = \frac{2.10.\,0{\cdot}00017.\,0{\cdot}6}{10^{-4}}$

$= \underline{17_{34}} \quad = 20$

$= 20.10^{-6}$ atm. =

$\eta \Delta^2 u = 0{\cdot}00017 \cdot 1{\cdot}2 . 10^4 = 2{\cdot}04$

wobec $\frac{\partial p_1}{\partial x} = 10^6$ więc nadzwyczajnie mało

biorąc $\varepsilon = 0{\cdot}001$ cm

otrzymałoby się $\eta \Delta^2 u = 204$ wobec 10^6

biorąc $\varepsilon = 0{\cdot}0001$ cm $= 1\mu$ $\quad \eta \Delta^2 u = 20400 = 2\%$

~~Więc mogłoby się to uwydatnić tylko przy dyfuzji przez błony, gips, etc.~~

Znaczniejszy będzie ~~[illegible]~~ wpływ w zadaniu: rurki (ale niewielka różnica!)

Więc uwydatnić mogłoby się to tylko przy dyfuzji przez błony, gips etc. ~~ale [illegible]~~

N. p. przy gipsie $r = 5 \cdot 10^{-5}$: 8%

U cieczy: N. p. w zadaniu pierwszym $u = 40.10^{-6} \left(\frac{cm}{sec}\right)$

Opór w rurce długości 1 cm, o przekroju $r = 0{\cdot}001$ cm

$P = \frac{4\eta . 4 . 10^{-6}}{10^{-6}} = 16 . \eta = 0{\cdot}18 . 16 = 0{\cdot}3$

podczas gdy $\frac{\partial p}{\partial x}$ ciśnienia osmotycznego: bez porównania większe

w gipsie drzewn. według [illegible] $r = 0{\cdot}0005$ mm ([illegible] 0·0001)

opór w rurce długości 1 cm $P = \frac{4.10^{-6} . 4 . 0{\cdot}018}{10^{-10}} = 3000$ więc wcale ładnie

$$\frac{\partial p}{\partial x} - \eta\, \Delta^2 u = 0$$

$$\frac{\partial(\rho u)}{\partial x} * \quad = 0$$

$$\begin{cases} \frac{\partial p}{\partial x} = \eta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ \frac{\partial(p u)}{\partial x} = 0 \end{cases}$$

$p = R\theta\rho$

$\rho = \frac{p}{R\theta}$

~~$u\frac{\partial p}{\partial x} + p\frac{\partial u}{\partial x} = 0$~~

$p u = f_1(y)$

$u = f_1(y) \qquad \frac{\partial u}{\partial x} = 0 \qquad \frac{\partial^2 u}{\partial x^2} = 0$

$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2}$

$p = f_2(x)$

$\frac{\partial^2 p}{\partial x \partial y} = 0 = \frac{\partial^3 u}{\partial y^3} = 0 \qquad \frac{\partial^2 u}{\partial y^2} = \text{const} = 2c \quad \Big\|$

$u = a + by + cy^2$

$$\left.\begin{aligned} p &= 2c\eta\, x + \text{const} \\ p_1 &= \text{~~[illegible]~~} \quad \text{const} \\ p_2 &= 2c\eta l + \text{const} \end{aligned}\right\} c = \frac{p_2 - p_1}{2\eta l}$$

$u = 0 \qquad y = \pm\varepsilon$

$$\left.\begin{aligned} 0 &= a + b\varepsilon + c\varepsilon^2 \\ 0 &= a - b\varepsilon + c\varepsilon^2 \end{aligned}\right\} \begin{aligned} b &= 0 \\ a &= -c\varepsilon^2 \end{aligned}$$

2oten $u =$ ~~[illegible]~~ $c(y^2 - \varepsilon^2) = \frac{p_1 - p_2}{2\eta l}(\varepsilon^2 - y^2)$

$$\int_{-\varepsilon}^{+\varepsilon} u\, dy = \frac{p_1 - p_2}{2\eta l}\left(\varepsilon^2 y - \frac{y^3}{3}\right)\Big|_{-\varepsilon}^{+\varepsilon} = \frac{p_1 - p_2}{2\eta l}\,\frac{4}{3}\varepsilon^3 = \frac{2}{3}\,\frac{p_1 - p_2}{l}\,\frac{\varepsilon^3}{\eta}$$

$$u_0 = a = -c\varepsilon^2 = \frac{p_1 - p_2}{2\eta l}\varepsilon^2 \qquad \| \quad \frac{4}{3}\varepsilon u_0$$

Dyfuzya w bardzo wązkich rurkach musi się wreszcie zmniejszać. Gdyby tak wązka że przepuszcza tylko 1 drobinę toby dyfuzya zupełnie ustała.

Więc wogóle musi wchodzić w rachunek opór tarcia (wewnętrznego) drobin uderzających o siebie także tarcie o ~~[illegible]~~ ściany.

To się więcej uwydatni w ~~[illegible]~~ dyfuzyi gazów

$$(u_1 - u_2) A_{12} \rho_1 \rho_2 + \frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p_1}{\partial t} = k \frac{\partial^2 p_1}{\partial x^2} \qquad k = \frac{p_0^2 T^2}{A_{12} d_1 d_2 T_0^2} \frac{1}{p} \neq \frac{p}{A_{12} \rho_1 \rho_2}$$

$$u_1 - u_2 = - \frac{\frac{\partial p_1}{\partial x}}{p} k$$

N.p. Na odległości 1 cm przypada różnica concentracyi $\begin{array}{c|c} p_1 = p & p_1 = 0 \\ p_2 = 0 & p_2 = p \end{array}$

wtedy $u_1 - u_2 = - k$

N.p. H_2 – powietrze: $k = 0{\cdot}634 \frac{cm^2}{sec}$

Ale w jaki sposób się rozkłada to na H_2 i pow.? warunek $p = p_1 + p_2 = const$

Wskutek tarcia warunek oporu było: $u_1 = u_2 = 0$ na ściance; właściwie zatem uwzględniając opływ ścian także w tym zadaniu należałoby wprowadzić: $\eta \frac{\partial u}{\partial y}$ etc: zadanie dwuwymiarowe

przybliżenie: $\frac{\partial p_1}{\partial x} + (u_1 - u_2) A_{12} \rho_1 \rho_2 ~~+~~ - \eta \Delta^2 u = 0$

N.p. gaz przepływ. między równoległemi ścianami $u = a + by + cy^2$

$$\eta \Delta^2 u = 2c . \eta = \frac{\partial p}{\partial x}$$

$$u_0 = a$$ [illegible]

Biorę $d = 0{\cdot}3$ mm $\quad \frac{4}{5}\frac{d}{l} = 8\%$

$l = 3$ mm

$\varrho = \frac{0{\cdot}3^2}{4}\pi = \frac{3{\cdot}14 \cdot 9}{28{\cdot}26} = 0{\cdot}07$ $\quad$ ~~$\frac{2}{dS_{3\,dni}} = \frac{9}{10} = \ldots\, 9\%$~~

7·

$\frac{2}{dS_{tygodni}} = 4\%$

$n = 3000$

~~lepiej: $d = 0{\cdot}3$ mm $\quad \frac{4}{5}\frac{d}{l} = 12\%$~~

~~$l = 2$ mm~~

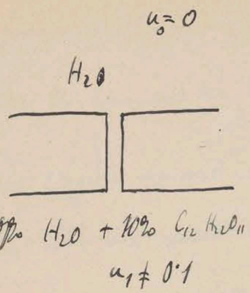
~~$\frac{2}{dS_{3\,dni}} = 4\%$~~

~~$n = \frac{1000}{5 \cdot 0{\cdot}07} = 3000$~~

Inny sposób mierzenia stałej dyfuzyi bardzo szybko z pomocą ~~[illegible]~~ ~~[illegible]~~ [illegible] której potrzeba…

Concentration unterschiede in rotirenden [illegible]

Inny sposób bardzo szybki: Kontinuacya z ruchem wody Np. przepływ czystej wody przez rurki z substancyi.



Metoda do mierzenia dyfuzyi

Wiele np. cukru przejdzie pro godzinę, jeśli długość rurki = 1cm przekrój = 1 mm²

$k = 40 \,.\, 10^{-7}$

$dS = k\, q\, \frac{\partial u}{\partial x}\, dt \qquad \frac{\partial u}{\partial x} = \frac{u_1 - u_0}{x} = \frac{0{\cdot}1}{1} = 0{\cdot}1$

$$\downarrow$$

$$dS = 0{\cdot}01 \,.\, 0{\cdot}1 \,.\, 40 \,.\, 10^{-7} \,.\, 60 \,.\, 60 = 4.6.6.\, 10^{-7} = \frac{36.4}{144} .10^{-7} g.$$
$$= 1{\cdot}4 \;.10^{-5} g.$$

dS pro dzień $\quad \begin{array}{r} 1{\cdot}44 \,.\, 24 \\ \hline 288 \\ 576 \\ \hline 3{\cdot}4{\cdot}6 \end{array} \quad = 3{\cdot}5 \,.\, 10^{-4} g = 0{\cdot}35$ mg, pro 3 dni : 1·05 mg

z = zawartość rurki : 0·01 . 0·1 = 1 mg.! więc znaczny błąd możliwy przy napełnianiu

Ten sam stosunek ($\frac{z}{dS} = 100\%$) będzie przy zwężaniu przekroju

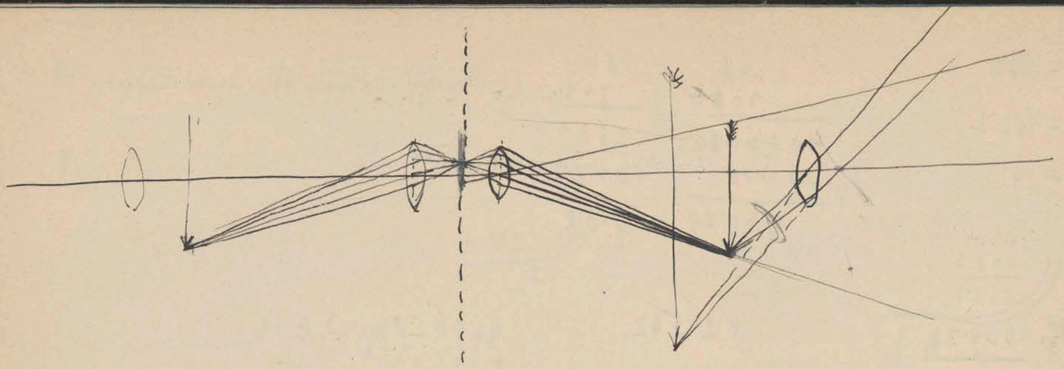
Ale ten stosunkowy możliwy błąd zmniejsza się w ~~pt~~ kwadratowym stosunku skrócenia długości rurki; jeśli długość = 1mm to $\frac{z}{dS}$ już tylko 1%

Aby korekcya z powodu końców nie była znaczniejszą musiałby też przekrój odpowiednio się zmniejszyć [Według Rayleigh'a redukc. długości przybl. $x + \frac{4}{5} d$

wzg. jeśli $d = 0{\cdot}1$ mm: $\frac{4}{5} d = 8\%$]
(trzeby zmniejszyć z powodu Q)

Aby jednak ~~za~~ całkowita ilość $dS_{3\,dni}$ wynosiła 1g musiałoby być wtedy

$10^3 . 10 = 10^4$ takich rurek!



Na odległości 27 cm : $1^\circ = 5$ mm
$1' = 0{\cdot}08$ mm

Powiększenie: Obj. 567 Ok. 1 500 } $\delta = 0{\cdot}9$ mm $\alpha = 0{\cdot}36$ mm
Ok. ~~VIII~~ 1_σ 1000 $\alpha =$

2 tego powiększenie Ok. 1_σ : 10

$a = 280$

$b = 102{\cdot}3$

$c = 9{\cdot}3$

$$\begin{array}{r} 1023 \\ 2046 \\ 3069 \\ \hline 10.46529 \\ 10416 \\ \hline \sqrt{49{\cdot}29} = 7{\cdot}0|2 \end{array} \qquad \begin{array}{r} 186 \\ 744 \\ \hline 2604 \\ \hline 10.416 \end{array}$$

$$\begin{array}{r} 560 \\ 102{\cdot}3 \\ \hline 662{\cdot}3 \end{array}$$

$$\frac{1}{7} \log\left(\frac{669{\cdot}3}{655{\cdot}3}\right)$$

$$\begin{array}{r} 82562 \\ -\ 81644 \\ \hline 0{\cdot}00918 \end{array} \qquad \begin{array}{r} 96284 - 3 \\ +\ 96222 \\ \hline 0{\cdot}92506 - 2 \\ -\ 0{\cdot}84510 \\ \hline 0{\cdot}47996 - 3 \end{array}$$

$$\begin{array}{r} p = 2{\cdot}52004 \\ -\ 2{\cdot}51322 \\ \hline 0{\cdot}00682 \end{array} \; p_0 \quad = \quad 1{\cdot}016$$

Więc effekt zmienności k byłby : 1·6 % podczas gdy bez zmienności k (dla spadu temperatury liniowego) otrzymałby się : − 1·33 %

Do przybliżonego obliczenia skutku:

N.p. $\gamma = 0{\cdot}002$ stawiając przeciętną wartość $1-\frac{x}{h} = \frac{1}{2}$

$$1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right) = 1+\frac{0{\cdot}002.\ 100}{2}\frac{1}{2} = 1+0{\cdot}05$$

więc zamiast θ_1 : 93 ... 98

$$\frac{98}{328{\cdot}5 \log\frac{378}{280}}$$

$$\begin{array}{r} 57749 \\ -44716 \\ \hline 13033 \end{array} \qquad \begin{array}{r} 0{\cdot}11504 - \\ 06222 \\ 2{\cdot}51388 \\ \hline 1{\cdot}99114 \end{array} \qquad \begin{array}{r} 1{\cdot}99123 \\ -1{\cdot}99114 \\ \hline 0{\cdot}00009 \end{array}$$

Dokładniej:

$$\int_0^h \frac{dx}{280+\frac{x}{h}\theta_1\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]} \neq \int_0^h \frac{dx}{280+\frac{x}{h}\theta_1\left[1+0{\cdot}1\left(1-\frac{x}{h}\right)\right]}$$

$$= \int_0^h \frac{dx}{280+1{\cdot}1.\frac{\theta_1}{h}x - \frac{0{\cdot}1.\theta_1}{h^2}x^2} = h\int_0^1 \frac{dz}{280+1{\cdot}1.\theta_1 z - 0{\cdot}1.\theta_1 z^2}$$

93 93

$$\int \frac{dx}{a+bx-cx^2} = \frac{1}{\sqrt{b^2-4ac}} \log \frac{b+2cx-\sqrt{b^2-4ac}}{b+2cx+\sqrt{b^2-4ac}}$$

$$\int_0^1 = \frac{1}{\sqrt{b^2-4ac}} \log \frac{b+2c-\sqrt{b^2-4ac}}{b+2c+\sqrt{b^2-4ac}} \cdot \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}$$

$$\frac{\cancel{b^2}+2bc+2c\sqrt{}\ -\cancel{b^2}+4ac}{\cancel{b^2}+2bc-2c\sqrt{}\ -\cancel{b^2}+4ac} = \frac{b+\sqrt{b^2-4ac}+2a}{b-\sqrt{}\ +2a}$$

$$\theta = \frac{x}{h}\,\frac{\theta_1\left[1+\gamma\frac{\theta_1}{2}\right]}{\left[1+\gamma\frac{\theta}{2}\right]} = \frac{x}{h}\,\theta_1\left[1+\gamma\frac{\theta_1}{2}\right]\left[1-\frac{\gamma}{2}\frac{x}{h}\theta_1\right]$$

$$= \frac{x}{h}\,\theta_1\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]$$

$$\int_0^h \rho\,dx = \cancel{\frac{p}{R}\,\frac{h}{\theta_1}\int\frac{dx}{280+x\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]} = \frac{p}{R}\,\frac{h}{\theta_1}\int\frac{dx}{x}\left[1-\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]}$$

$$= \frac{p}{R}\int_0^h \frac{dx}{280+\frac{x}{h}\,\theta_1\left[1+\gamma\frac{\theta_1}{2}\left(1-\frac{x}{h}\right)\right]} = \frac{p_0\,h}{R\,(280+50)}$$

Tymczasem stawiając $\gamma = 0$:

$$\frac{p}{R}\int\frac{dx}{280+\frac{x}{h}\theta_1} = \frac{p}{R}\,\frac{h}{\theta_1}\,\log\left(280+\frac{x\theta_1}{h}\right)\Big|_0^h = \frac{p}{R}\,\frac{h}{\theta_1}\,\log\left(\frac{280+\theta_1}{280}\right) = \frac{p_0\,h}{R(280+50)}$$

$$p = p_0\,\frac{\frac{\theta_1}{280+50}}{\underbrace{\log\left(\frac{280+\theta_1}{280}\right)}}$$

$$\log\left(1+\frac{\theta_1}{280}\right) = \frac{\theta_1}{280} - \frac{1}{2}\,\frac{\theta_1^2}{280^2} + \cdots$$

$$= p_0\,\frac{\frac{1}{280+50}}{\frac{1}{280}-\frac{1}{2}\frac{\theta_1}{280^2}+-} = p_0\,\frac{1}{(280+50)\left[\frac{1}{280}-\frac{1}{2}\frac{\theta_1}{280^2}+-\right]}$$

$\theta_1 = 93$

$$\frac{93}{2} = 46.5 \qquad \begin{array}{r} 280 \\ \hline 326.5 \end{array}$$

$$\frac{93}{326.5\,\log\frac{373}{280}}$$

$$\begin{array}{r} 57171 \\ 44716 \\ \hline 0.12455 \end{array} \qquad \begin{array}{r} 0.09534-1 \\ 0.36222 \\ 2.51388 \\ \hline 1.97144 \end{array} \qquad \begin{array}{r} 1.96848 \\ -1.97144 \\ \hline 0.99704 \end{array}$$

$$\underline{p = 0.9932\,p_0}$$

To byłoby ważnem przy stałej objętości

$k_0 (1+\gamma\theta) d\theta = c\, dx$ | dla $x=0$: $\theta=0$

$k_0\left(\theta + \gamma \frac{\theta^2}{2}\right) = cx + b$

~~$k_0\left(\theta_0 + \gamma \frac{\theta_0^2}{2}\right)$~~ $0 = +b$

$k_0\left(\theta_1 + \gamma \frac{\theta_1^2}{2}\right) = ch$

~~$k_0 \frac{(\theta_1 - \theta_0)}{h}\left[1 + \gamma \frac{\theta_0+\theta_1}{2}\right] = c$~~

$c = \frac{k_0}{h}\left(\theta_1 + \gamma \frac{\theta_1^2}{2}\right)$

$\frac{p}{\rho} = R\left(\theta + 280\right)$ ~~[illegible]~~

1α) .: $\frac{p_0}{\rho_0} = R(\theta_{50} + 280)$ $\int \rho\, dx = \rho_0 h$

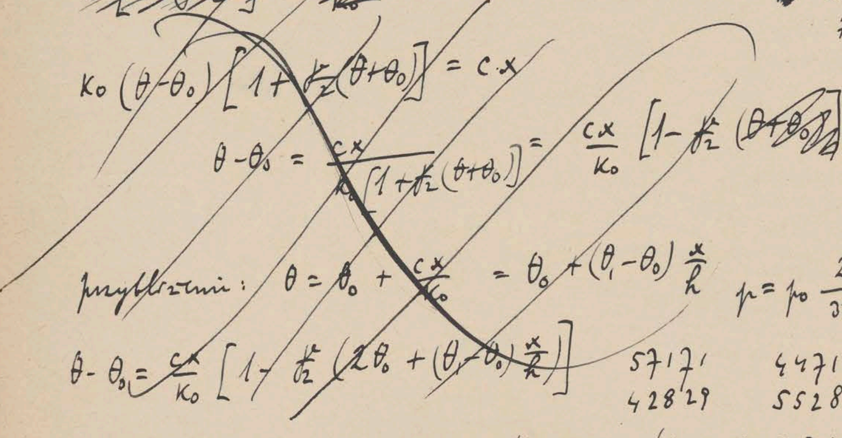
$\rho = \frac{p}{R\theta}$ $\int \rho\, dx = \frac{p}{R} \int \frac{dx}{(\theta + 280)}$

$\gamma\frac{\theta}{6}$ $\frac{100}{1000} = 5\%$

~~$\theta\left[1 + \gamma\frac{\theta}{2}\right] = \frac{cx+b}{k_0}$~~

$\frac{p}{\rho_1} = R . 280$

$\frac{p}{\rho_2} = R . 373$



$k_0(\theta - \theta_0)\left[1 + \frac{\gamma}{2}(\theta + \theta_0)\right] = c.x$

$\theta - \theta_0 = \frac{cx}{k_0\left[1 + \frac{\gamma}{2}(\theta+\theta_0)\right]} = \frac{cx}{k_0}\left[1 - \frac{\gamma}{2}(\theta + \theta_0)\right]$ $\rho_0 = \frac{\rho_1+\rho_2}{2}$

$\rho_0 = \frac{p}{2R}\left(\frac{1}{280} + \frac{1}{373}\right) = \frac{p_0}{R\, 326.5}$

przybliżenie: $\theta = \theta_0 + \frac{cx}{k_0} = \theta_0 + (\theta_1 - \theta_0)\frac{x}{h}$ $p = p_0 \frac{2}{326.5} \Big/ \left(\frac{1}{280} + \frac{1}{373}\right)$

$\theta - \theta_0 = \frac{cx}{k_0}\left[1 - \frac{\gamma}{2}\left(2\theta_0 + (\theta_1 - \theta_0)\frac{x}{h}\right)\right]$

57171 44716

42829 55284

0.0026810 0.0035714 0.79605−3

26810 2.51388

0.0062524

0.30993

0.30103

0.30103

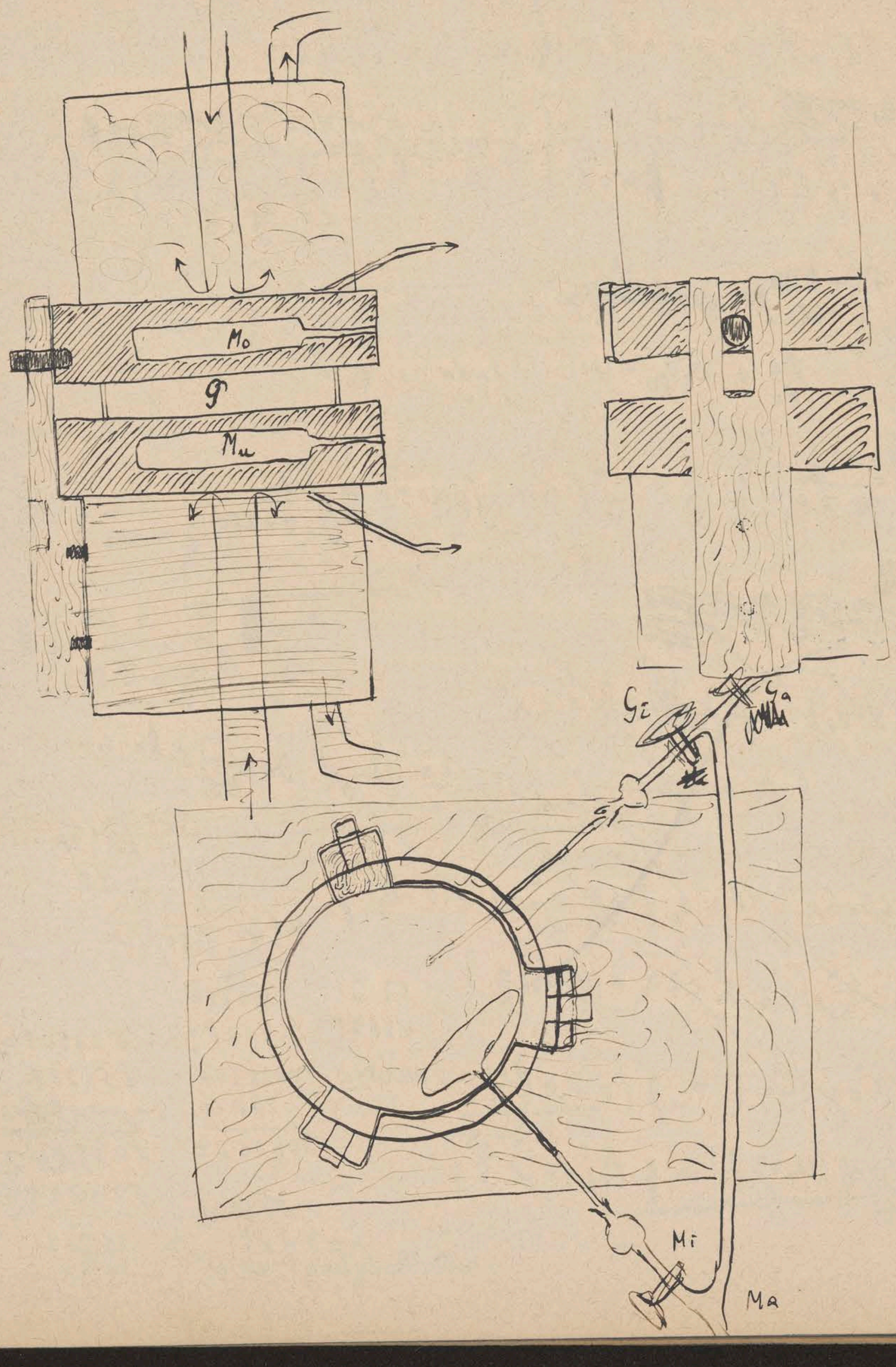
− 0.30993

0.99110

$\log x$ = 0.97971-10

$k_0\left(\theta + \gamma\frac{\theta^2}{2}\right) = k_0 \frac{x}{h}\left(\theta_1 + \gamma\frac{\theta_1^2}{2}\right)$

$\theta\left[1 + \gamma\frac{\theta}{2}\right] = \frac{x}{h}\theta_1\left[1 + \gamma\frac{\theta_1}{2}\right]$


Mo
g
Mu
Gi
Ga
Mi
Ma

Porządek operacji:

1). Przepuścić wodę lub parę 50° u góry i na dole

α). Otworzyć kurki Gi i Mi [i także Ga lub Ma na chwilę ażeby ciśnienie wewnętrzne nie było wyższe od atmosf. !

β). zamknąć obydwa Gi i Mi

γ). otworzyć Ga i Ma (kolejno) i wpraktykować kroplę rtęci (lub glyceryny?) do środkowej części rurki otrząść i zaznaczyć miejsce na podziałce

δ). zamknąć Ga i Ma

2). Przepuścić u góry parę 100°, na dole wodę zimną 7°

α. Otworzyć kurki Gi i Mi (dla pewności nie obydwa naraz

otrząść i zaznaczyć miejsce kropli na podziałce

β). zamknąć Gi i Mi,

3). Aparat odwrócić, u góry parę, na dole zimną wodę

α). Otworzyć kurki Gi i Mi

otrząść i zaznaczyć miejsce kropli na podziałce

β). zamknąć Gi Mi

4). Przepuścić wodę lub parę 50° u góry i na dole

α). Otworzyć kurki Gi Mi

otrząść i zaznaczyć miejsce kropli na podziałce

$(v = 0{\cdot}02 - 0{\cdot}14\ \frac{cm}{sec}$

the case of waterdrops as formed in J J Thomson's and H A Wilson's condensation experiments the range

of the order of several centimetres, in Perrin's experiments on the validity of Stokes l

particles of emulsions it would amount even to hundreds of meters; ~~[illegible]~~ ~~also for~~

~~[illegible]~~ ~~it would be easy to find suitable~~ in the same way it is sufficiently great

~~approximate~~ ractical purposes, when highly viscous liquids are used, as Zadenburg did in his

research. ~~Of course it excludes the~~ ~~Ordinary hydraulic experiments~~ ~~excluded of~~ out of question

~~is~~ ~~insoluble~~ method of approximate correction for inertia might be applied also to the other cases treated befo

For movements in closed vessels however besides it will be of   it will imply rather cumbersome calculati
th

importance than in liquids extending to infinity; ~~as an evidence~~ we may point for inst

Arnold's experiments, who found ~~the~~ ~~[illegible]~~ ~~theoretical~~ (influence of cylindric tube ~~wall~~

Zadenburgs correction te

~~the motion of a sphere~~] to …iciently exact i… …dius would make the

$\frac{\rho v b}{2\mu}$ equal to

excluded ~~out of question~~ of course wh

Zuländer & Komarski

$\frac{ga^2}{r^2}\sin\varphi = r\sin\varphi\,\omega^2$

$r^3 = \frac{ga^2}{\omega^2}$

$r^3 = 289\,a^2$

$\sigma\frac{d\sigma}{dx} = -(\alpha + \frac{c}{A})\frac{d\theta}{dx} + \frac{4}{3}\eta_b\frac{\partial}{\partial x}(\theta\sigma\frac{d\sigma}{dx})$

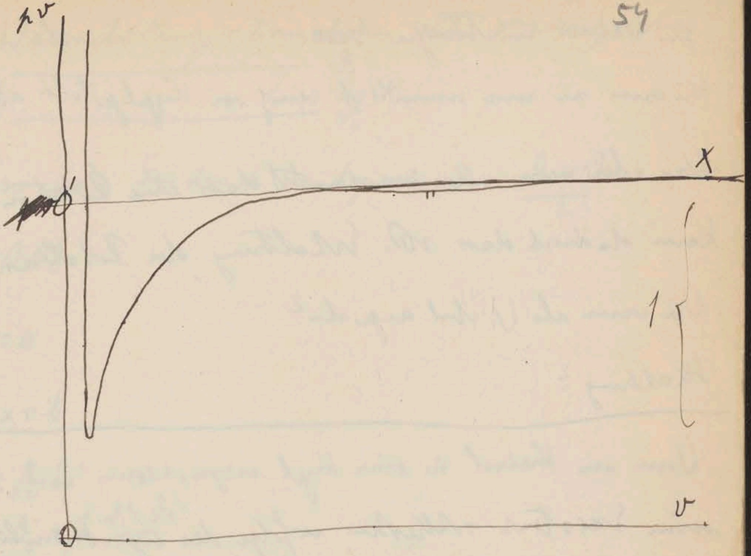
$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \frac{\alpha}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\eta_b(\frac{\partial\sigma}{\partial x})^2$

Czy możliwe rozwiązanie $\theta = \text{const}$ ~~$\rho\sigma =$~~

CO_2

$t = 40°$

p	pv	v
1	1·000	1·000
50	0·8500	0·0170
75	0·6200	0·00827
100	0·3090	0·0031
150	0·3770	0·00251
200	0·4675	0·00234
300	0·6485	0·00216
500	0·9900	0·00198
1000	1·7800	0·00178



$$\frac{1}{2}\sum m c^2 = \frac{3}{2}(pv) + \sum\sum R r$$

Wenn man die Sache auf O′X bezieht so hat man direct den Wert von $\sum\sum R r = \sum R \sqrt[3]{v}$

Diese Curven werden (in allgem.) bei verschiedenen Temperaturen verschieden sein, denn bei gleichem v wird 1) die Anzahl der Summanden verschieden sein (wie $\frac{1}{T}$ daher $\sum\sum$ wie $\frac{1}{T^2}$) 2) die mittleren Distanzen ~~auch~~ ebendeswegen verschieden sein wie $T^{\frac{1}{3}}$

Wenn also $R = f(r) = f(\sqrt[3]{v})$ gesetzt wird so ist $\sum\sum R r = \frac{1}{T^2} \cancel{\sum\sum} T^{\frac{1}{3}} \cdot f(T^{\frac{1}{3}})$

Spez. Wärme einatomiger Gase.

Wie wenn sie nur unendlich wenig von Kugelgestalt abweichen? Dann wird k doch 1.66 sein

Was aber wenn man voraussetzt dass eine Rotation überhaupt nicht eintreten kann, dadurch dass z.B. Erhaltung der Rotations Ebene

Wie wenn als Wirbel angesehen?

Strahlung?

Wenn ein Kreisel in eine Kugel eingeschlossen wird, so kann man doch auf ~~die~~ seine Existenz schliessen infolge des eigenthümlichen Verhaltens der Kugel. Wenn aber zwei in entgegengesetzter Richtung rotirende Kreisel von gleichem Trägheitsmoment ineinander gesetzt werden so kön sich deren Wirkung nach aussen nicht bemerkbar machen

---

Methode zur Messung der Wärmeleitungs-Zunahme mit der Temperatur.

2 unendliche Platten $h\{$ ———— ϑ_1
———— ϑ_0

I). Falls κ constant wäre, so wäre Ausdruck entsprechend $\frac{\vartheta_0+\vartheta_1}{2}$

II) Dagegen wenn $\kappa = \kappa_0(1+\beta\vartheta)$:

$$\cancel{\kappa_0}(1+\beta\vartheta)\frac{d\vartheta}{dx} = \text{const.} = c$$

$$\vartheta + \beta\frac{\vartheta^2}{2} = ax+b$$

$$b = \vartheta_0 + \frac{\beta}{2}\vartheta_0^2$$

$$a = \frac{\vartheta_1 - \vartheta_0 + \beta\frac{\vartheta_1^2-\vartheta_0^2}{2}}{h}$$

Oder wenn $\vartheta_0 = 0$ gesetzt wird:

$$\vartheta + \beta\frac{\vartheta^2}{2} = \frac{x}{h}\left(\vartheta_1 + \frac{\beta}{2}\vartheta_1^2\right)$$

Mittlere Temperatur $\Theta = \frac{1}{h}\int_0^1 \vartheta\, dx = \frac{1}{h}\int_0^1 \vartheta\frac{d\vartheta(1+\beta\vartheta)}{a} = \cancel{\frac{1}{h}\frac{\vartheta_1+\frac{\beta}{2}\vartheta_1^2}{a}}$

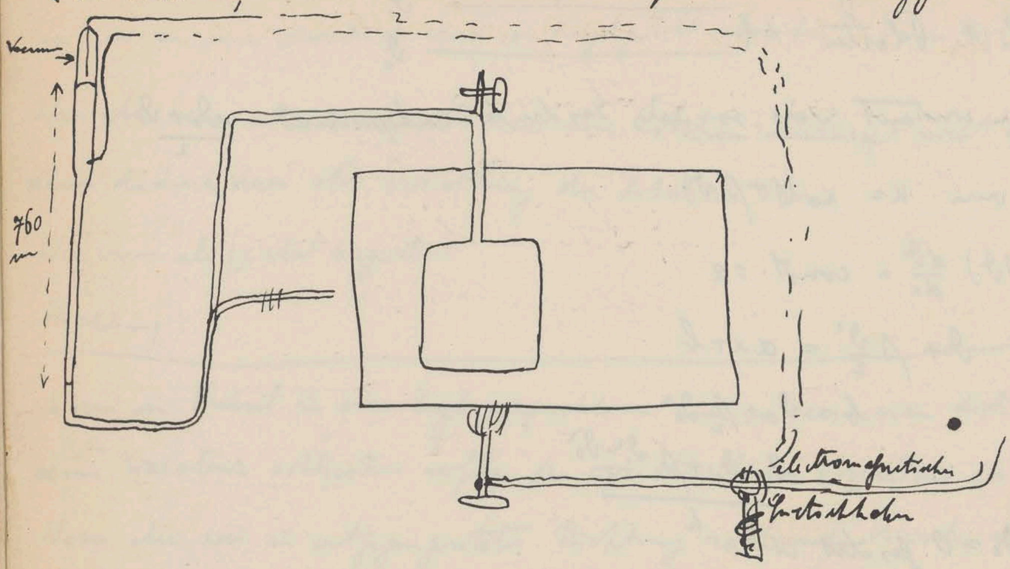
$$\Theta = \frac{1}{ah}\left(\frac{\vartheta_1^2}{2}+\beta\frac{\vartheta_1^3}{3}\right) = \frac{\frac{\vartheta_1^2}{2}+\beta\frac{\vartheta_1^3}{3}}{\vartheta_1+\beta\frac{\vartheta_1^2}{2}} = \frac{\vartheta_1}{2}\,\frac{1+\frac{2}{3}\beta\vartheta_1}{1+\frac{1}{2}\beta\vartheta_1} \neq \frac{\vartheta_1}{2}\left[1+\frac{1}{6}\beta\vartheta_1\right]$$

Wenn also z.B. $\vartheta_0 = 0$
$\vartheta_1 = 100$ $\beta = 0.0018$

$$\Theta = 50\,[1+0.03] = 50^\circ + 1.5^\circ$$

Und zwar unabhängig von Distanz der Platten, unabhängig von Strahlung

Thermostat, welcher vom äusseren Luftdruck unabhängig ist:



Correctionsformel für Drahtende (Schleiermacher)

$$w i^2 \cancel{dx} = \frac{2\pi \cancel{dx}}{\lg \frac{R}{r}} \kappa \log \frac{\vartheta}{\theta} + q\kappa' \frac{\partial^2\vartheta}{\partial x^2}$$

$$C \cancel{\#} = \vartheta \frac{\partial^3\vartheta}{\partial x^3} = \frac{\partial}{\partial x}\left(\vartheta \frac{\partial^2\vartheta}{\partial x^2}\right) - \frac{\partial\vartheta}{\partial x}\frac{\partial^2\vartheta}{\partial x^2}$$

$$= \frac{\partial}{\partial x}\left[\vartheta \frac{\partial^2\vartheta}{\partial x^2} - \frac{1}{2}\left(\frac{\partial\vartheta}{\partial x}\right)^2\right]$$

$$Cx + C_1 = \underbrace{\vartheta \frac{\partial^2\vartheta}{\partial x^2}} - \frac{1}{2}\left(\frac{\partial\vartheta}{\partial x}\right)^2$$

$$= \frac{\partial}{\partial x}\left(\vartheta \frac{\partial\vartheta}{\partial x}\right) - \left(\frac{\partial\vartheta}{\partial x}\right)^2$$

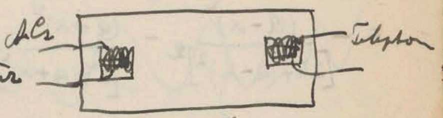
Scheinbare Widerstands Vermehrung von Wechselströmen in Umgebung von O_2; wegen Arbeit die zur Ozonisirung verbraucht wird.

Bei anderen Gasen nicht? Wie bei Evacuirung?

Hydromechanik verdünnter Gase; Planetenatmosphären, Reibungsdrehungsmoment, Mischung verschiedener Gase, Diffusion, Stoney's Hypothese; Sternschnuppen; ~~Bei~~ Druckunterschiede in Geissler-Röhren; Druck im Funkenfelde; Schallfortpflanzung; Reibungstöne;
[illegible]

Strahlenbildung von Gasen

Hydromechanik von Gasen bei kritischer Temperatur



Anwendung zur Bestimmung derselben dadurch dass keine Schallfortpflanz.

Thermal diffusion und ~~Jou~~ Thomson-Joule Effect ~~[illegible]~~ sind inverse Phänomene? Gibt es thermal diffusion in Lösungen?

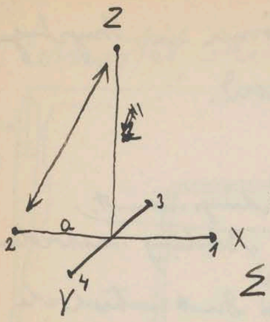
~~Ele~~ Convection von suspendirten Theilchen durch Strom; entsteht infolge Electrisirung infolge Flüssigkeitsreibung? ~~Den~~ Reibungselectr. entsteht bei jedem Geschwindigkeitsgefälle? Wie wenn verschiedene Pulver gemischt und im electrischen Felde geschüttelt werden? Ist Electrolyse nicht ein identischer Vorgang?

Selbstinductionscoefficient bei Entladung von Kugeln etc. Schwingungen die dadurch entstehen. ~~Statt~~

Scheinbare dielectrische Hysteresis, infolge elastischer Nachwirkung der Deformation im electrost. Felde

Quantitative Messungen über Absorption, Emission etc. von Dämpfen z.B. Joddampf, Na-Dampf

Leduc & Sacerdote: Dalton's Gesetz zu corrigiren: dass Volumina sich addiren. Warum? Kein Gesetz!



$$V = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}$$

$$r_1^2 = z^2 + (a-x)^2 + y^2$$

$$r_2^2 = z^2 + (a+x)^2 + y^2$$

$$r_3^2 = z^2 + (a+y)^2 + x^2$$

$$r_4^2 = z^2 + (a-y)^2 + x^2$$

$$\frac{\partial V}{\partial x} = -\frac{1}{r_1}\frac{\partial r_1}{\partial x} \cdots$$

$$\Sigma X = \frac{1}{r_1}\frac{\partial r_1}{\partial x} + \cdots$$

$$\frac{\partial}{\partial x}\Sigma X = -\frac{\partial^2 V}{\partial x^2} = -\frac{1}{r_1^2}\left(\frac{\partial r_1}{\partial x}\right)^2 + \frac{1}{r_1}\frac{\partial^2 r_1}{\partial x^2} \cdots$$

$$\frac{a-x}{z} \qquad -\frac{x^2}{z^3} - \frac{1}{z}$$

$$= \cancel{-\frac{(a-x)^2}{[z^2+(a-x)^2]^2} - \frac{(a+x)^2}{[z^2+(a+x)^2]^2} - \frac{x}{[z^2+a^2+x^2]^2} - \frac{x}{[z^2+a^2+x^2]^2}} +$$

$$\Sigma X = \frac{-(a-x)}{[z^2+(a-x)^2]^{3/2}} + \frac{a+x}{[z^2+(a+x)^2]^{3/2}} + \frac{2x}{[z^2+a^2+x^2]^{3/2}}$$

$$-\frac{\partial^2 V}{\partial x^2} = -3\frac{(a-x)^2}{[z^2+(a-x)^2]^{5/2}} - \frac{3(a+x)^2}{[z^2+(a+x)^2]^{5/2}} - \frac{6x^2}{[z^2+a^2+x^2]^{5/2}}$$

$$+ \frac{1}{[z^2+(a-x)^2]^{3/2}} + \frac{1}{[z^2+(a+x)^2]^{3/2}} + \frac{2}{[z^2+a^2+x^2]^{3/2}}$$

$$Z = -\frac{\partial V}{\partial z} = \frac{z}{[z^2+(a-x)^2]^{3/2}} + \frac{z}{[z^2+(a+x)^2]^{3/2}} + \frac{2z}{[z^2+a^2+x^2]^{3/2}}$$

$$\frac{\partial Z}{\partial z} = -\frac{\partial^2 V}{\partial z^2} = -\frac{3z^2}{[z^2+(a-x)^2]^{5/2}} - \frac{3z^2}{[z^2+(a+x)^2]^{5/2}} - \frac{6z^2}{[z^2+a^2+x^2]^{5/2}} +$$

$$+ \frac{1}{[z^2+(a-x)^2]^{3/2}} + \frac{1}{[z^2+(a+x)^2]^{3/2}} + \frac{2}{[z^2+a^2+x^2]^{3/2}}$$

~~$$-\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2}\right) = \frac{-3}{[z^2+(a-x)^2]^{3/2}} - \cdots + \frac{2}{[z^2+(a-x)^2]^{3/2}}$$~~

$$-\frac{\partial^2 V}{\partial x^2} = \cancel{-3\sqrt{a^2-2ax+x^2}}$$

$$= \frac{-3}{[z^2+a^2]^{5/2}}\left\{\frac{(a-x)^2}{\left[1+\frac{-2ax+x^2}{a^2+z^2}\right]^{5/2}} + \frac{(a+x)^2}{\left[1+\frac{2ax+x^2}{a^2+z^2}\right]^{5/2}} + \frac{2x^2}{\left[1+\frac{x^2}{a^2+z^2}\right]^{5/2}}\right\} +$$

$$+\frac{1}{[a^2+z^2]^{3/2}}\left\{\frac{1}{\left[1+\frac{-2ax+x^2}{a^2+z^2}\right]^{3/2}} + \text{-------}\right\}$$

für lim $x=0$: $-\dfrac{\partial^2 V}{\partial x^2} = \dfrac{-6a^2}{(a^2+z^2)^{5/2}} + \dfrac{4}{(a^2+z)^{3/2}}$

$$-\frac{\partial^2 V}{\partial z^2} = \frac{-12z^2}{(a^2+z^2)^{5/2}} + \frac{4}{(a^2+z^2)^{3/2}}$$

Damit <0:

$-\dfrac{6a^2}{a^2+z^2}+4 < 0$	$-\dfrac{12z^2}{a^2+z^2}+4<0$
$\dfrac{\partial^2 V}{\partial x^2}$ $\quad -2a^2+4z^2<0$	$-8z^2+4a^2<0$
$2z^2<a^2$	$a^2<2z^2$ daher höchstens:

$$\frac{\partial^2 V}{\partial x^2}=0 \quad \text{und} \quad \frac{\partial^2 V}{\partial z^2}=0 \quad \text{für} \quad z^2=\frac{a^2}{2}$$

$$-\frac{\partial^3 V}{\partial x^3} = -15\frac{(a-x)^3}{[z^2+(a^2-x^2)]^{7/2}} + 15\frac{(a+x)^3}{[z^2+(a+x)^2]^{7/2}} + \frac{30x^3}{[z^2+a^2+x^2]^{7/2}} + 9\frac{(a-x)^2}{[z^2+(a-x)^2]^{5/2}} - 9\frac{(a+x)^2}{[z^2+(a-x)^2]^{5/2}} - \frac{18x}{[z^2+a^2+x^2]^{5/2}}\Bigg|_{x=0} = 0$$

$$-\frac{\partial^4 V}{\partial x^4} = -3.5.7\,\frac{(a-x)^4}{[z^2+(a-x)^2]^{9/2}} - 3.5.7.\frac{(a+x)^4}{\cdots} - 2.3.5.7\,\frac{x^4}{\cdots}$$

$$+ 45\,\frac{(a-x)^2}{[z^2+(a-x)^2]^{7/2}} + 45\,\frac{(a+x)^2}{\cdots} + 2.45\,\frac{x^2}{\cdots}$$

$$+ 45\,\frac{(a-x)^2}{[z^2+(a-x)^2]^{7/2}} + 45\,\frac{(a+x)^2}{\cdots} + 2.45\,\frac{x^2}{\cdots}$$

$$- 9\,\frac{\cancel{(a-x)}\ 1}{[z^2+(a-x)^2]^{5/2}} - 9\,\frac{\cancel{a+x}\ 1}{\cdots} - 2.9\,\frac{1}{\cdots}$$

bei $x=0$: $$-\frac{\partial^4 V}{\partial x^4} = -\frac{4.3.5.7\,a^4}{(z^2+a^2)^{9/2}} + 2.90\,\frac{a^2}{(a^2+z^2)^{7/2}} - \frac{36}{(a^2+z^2)^{5/2}}$$

$$= \frac{1}{(a^2+z^2)^{9/2}}\left\{ \underset{456}{-420}\,a^4 + 180(a^4+a^2z^2) - 36(a^4+2a^2z^2+z^4)\right\}$$

$$= \frac{1}{(a^2+z^2)^{9/2}}\left\{-276\,a^4 + 108\,a^2z^2 - 36\,z^4\right\}$$

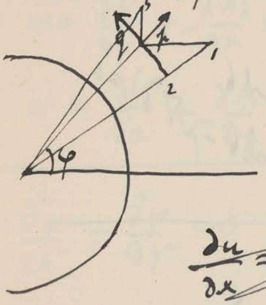
$$= \frac{12}{(a^2+z^2)^{9/2}}\left\{-23\,a^4 + 9\,a^2z^2 - 3\,z^4\right\} < 0$$

Wenn $a^2 = 2z^2$ $\quad \underbrace{} = -92 + \overset{95}{18} - 3 = -77\,z^2$

Zweidimensional, incompressibel

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\cancel{w\frac{\partial u}{\partial z}}=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\cancel{\frac{\partial^2 u}{\partial z^2}}-\frac{1}{\rho}\frac{\partial P}{\partial x}\quad\Big|\ \frac{\partial}{\partial y}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\cancel{w\frac{\partial v}{\partial z}}=\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\cancel{\frac{\partial^2 v}{\partial z^2}}-\frac{1}{\rho}\frac{\partial P}{\partial y}\quad\Big|\ \cdot\frac{\partial}{\partial x}$$



$$p=u\cos\varphi+v\sin\varphi\qquad\Big\|\qquad u=p\cos\varphi-q\sin\varphi$$

$$q=-u\sin\varphi+v\cos\varphi\qquad\Big\|\qquad v=p\sin\varphi+q\cos\varphi$$

$$\cancel{\frac{\partial u}{\partial x}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial u}{\partial\varphi}\frac{\partial\varphi}{\partial x}}\qquad x=r\cos\varphi$$

$$\cancel{=\frac{\partial u}{\partial r}\frac{1}{\cos\varphi}-\frac{\partial u}{\partial\varphi}\frac{1}{r\sin\varphi}}$$

$$\frac{\partial u}{\partial x}=\frac{u_1-u_0}{\Delta x}=\frac{u_1-u_2}{\Delta x}+\frac{u_2-u_0}{\Delta x}=\frac{u_1-u_2}{\Delta r\,\cancel{\cos}}\cancel{\cos}+\frac{u_2-u_0}{r\Delta\varphi}\sin\varphi$$

$$=\frac{\partial u}{\partial r}\cos\varphi-\frac{\partial u}{\partial\varphi}\frac{\sin\varphi}{r}=$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial r^2}\cos^2\varphi-\frac{\partial^2 u}{\partial r\partial\varphi}\frac{\cos\varphi\sin\varphi}{r}+\cancel{\frac{\partial u}{\partial\varphi}\frac{\sin\varphi\cos\varphi}{r^2}}-\frac{\partial^2 u}{\partial r\partial\varphi}\frac{\cos\varphi\sin\varphi}{r}-$$

$$+\frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r}+\frac{\partial^2 u}{\partial\varphi^2}\frac{\sin^2\varphi}{r^2}+\cancel{\frac{\partial u}{\partial\varphi}\frac{\cos\varphi\sin\varphi}{r^2}}$$

$$=\frac{\partial^2 u}{\partial r^2}\cos^2\varphi-2\frac{\partial^2 u}{\partial r\partial\varphi}\frac{\sin\varphi\cos\varphi}{r}+\frac{\partial^2 u}{\partial\varphi^2}\frac{\sin^2\varphi}{r^2}+\frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r}+2\frac{\partial u}{\partial\varphi}\frac{\sin\varphi\cos\varphi}{r^2}$$

$$\frac{\partial u}{\partial y}=\frac{u_3-u_0}{\Delta y}=\frac{u_3-u_4}{\Delta y}+\frac{u_4-u_0}{\Delta y}=\frac{\partial u}{\partial r}\sin\varphi+\frac{\partial u}{\partial\varphi}\frac{\cos\varphi}{r}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial r^2}\sin^2\varphi+\frac{\partial^2 u}{\partial r\partial\varphi}\frac{\sin\cos}{r}-\frac{\partial u}{\partial\varphi}\frac{\sin\cos}{r^2}+\cancel{\frac{\partial^2 u}{\partial r\partial\varphi}\frac{\sin\cos}{r}}+\frac{\partial u}{\partial r}\frac{\cos^2}{r}+\frac{\partial^2 u}{\partial\varphi^2}\frac{\cos^2}{r^2}-\frac{\partial u}{\partial\varphi}\frac{\sin\cos}{r^2}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{1}{r}\frac{\partial u}{\partial r}$$

$$\frac{\partial u}{\partial r} = \frac{\partial p}{\partial r}\cos\varphi - \frac{\partial q}{\partial r}\sin\varphi \qquad \Big|\ \cos\varphi$$

$$\frac{\partial u}{\partial \varphi} = \frac{\partial p}{\partial \varphi}\cos\varphi - \frac{\partial q}{\partial \varphi}\sin\varphi - p\sin\varphi - q\cos\varphi \qquad \Big|\ -\frac{\sin\varphi}{r}$$

$$\frac{\partial u}{\partial x} =$$

$$\nabla \sigma^2 - V\sigma \operatorname{curl}\sigma$$

$$\operatorname{curl}\left(V\sigma\operatorname{curl}\sigma - \mu\nabla^2\sigma\right) = 0$$

Aussichtslos!

~~$\sigma\nabla\sigma$~~

$$\cancel{\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}} + u\frac{\partial^2 u}{\partial x\partial y} + \cancel{\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}} + v\frac{\partial^2 u}{\partial y^2} - \cancel{\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}} - u\frac{\partial^2 v}{\partial x^2} - \cancel{\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}} - v\frac{\partial^2 v}{\partial x\partial y}$$

$$= \mu\left(\frac{\partial^3 u}{\partial x^2\partial y} + \frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} - \frac{\partial^3 v}{\partial x\,\partial y^2}\right)$$

$$\zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \qquad \frac{\partial \zeta}{\partial x} = \frac{\partial^2 u}{\partial x\partial y} - \frac{\partial^2 v}{\partial x^2} = -\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

[illegible]

$$u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \mu\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right)\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = \mu\left(\frac{\partial \zeta}{\partial x^2} + \frac{\partial \zeta}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x}\frac{\partial \zeta}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial \zeta}{\partial y} + u\frac{\partial^2 \zeta}{\partial x^2} + v\frac{\partial^2 \zeta}{\partial x\partial y} =$$

$$\frac{\partial (u\zeta)}{\partial x} + \frac{\partial (v\zeta)}{\partial y} = \nearrow$$

$$\frac{\partial}{\partial x}\left[u\zeta - \mu\frac{\partial \zeta}{\partial x}\right] + \frac{\partial}{\partial y}\left[v\zeta - \mu\frac{\partial \zeta}{\partial y}\right] = 0$$

~~$\operatorname{div}\ \sigma\zeta - \mu\nabla\zeta$~~

$$\mathfrak{a} = \nabla V + \mathrm{curl}\, \mathfrak{A}$$

$$u = \frac{\partial V}{\partial x} +- \frac{\partial A_2}{\partial z} + \frac{\partial A_3}{\partial y}$$

$$v = \frac{\partial V}{\partial y} +- \frac{\partial A_3}{\partial x} + \frac{\partial A_1}{\partial z}$$

$$w = \frac{\partial V}{\partial z} +- \frac{\partial A_1}{\partial y} + \frac{\partial A_2}{\partial x}$$

Also zweidimensional:

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad \zeta = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$$

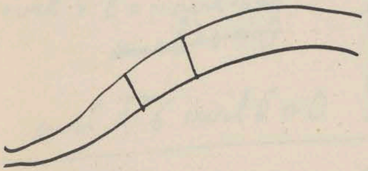
φ und ψ ganz unabhängig

Wenn jetzt z.B. ψ willkürlich gewählt, wie muss φ gewählt werden?

$$\left(\frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}\right)\left(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial x \partial y^2}\right) + \left(\frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}\right)\left(\frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3}\right) = \mu \frac{\partial^4 \psi}{\partial x^4} + \cdots$$

Flüssigkeits Gleichungen bezogen auf ~~[illegible]~~ Stromlinien (ohne Reibung)

In zwei Dimensionen:



$$m \frac{dv}{dt} = F_t = m \frac{dv}{ds}\frac{ds}{dt} = m v \frac{dv}{ds}$$

$$= \frac{1}{2}\frac{d}{ds}(mv^2)$$

$$= \frac{dE}{ds}$$

$$m \frac{v^2}{R} = F_n$$

$$mv^2 = 2\int F_t\, ds$$

$$F_t = \frac{1}{q}\frac{\partial(Pq)}{\partial s}$$

$$R = 2\frac{\int F_t\, ds}{F_n}$$

$$= 2\frac{E}{F_n}$$

$$F_n = \frac{\partial P}{\partial R} + \frac{P}{R}$$

$$= \frac{1}{R}\frac{\partial(RP)}{\partial R}$$

$$2E = P + R\frac{\partial P}{\partial R} \qquad \frac{\partial P}{\partial R} = \frac{2E - P}{R}$$

$$\underbrace{(\mathfrak{v}\nabla)\mathfrak{v}}_{\mathrm{curl}} = \nabla P + \mu \nabla^2 \mathfrak{v} = \nabla \mathfrak{v}^2 + V \mathfrak{v}\,\mathrm{curl}\,\mathfrak{v}$$

$$= \mu \nabla^2 \mathrm{curl}\,\mathfrak{v} = \mathrm{curl}\, V \mathfrak{v}\,\mathrm{curl}\,\mathfrak{v}$$

$$\nabla \frac{\partial P}{\partial \mu} + \nabla^2 \mathfrak{v} + \mu \nabla^2 \frac{\partial \mathfrak{v}}{\partial \mu} = \nabla \frac{\partial (\mathfrak{v}^2)}{\partial \mu} + \frac{\partial}{\partial \mu} V \mathfrak{v}\,\mathrm{curl}\,\mathfrak{v}$$

Stationäre Strömung ohne Kräfte, mit Reibung

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{x}{\mu} = \xi \qquad \frac{\partial u}{\partial x} = \frac{1}{\mu}\frac{\partial u}{\partial \xi} \qquad \frac{\partial^2 u}{\partial x^2} = \frac{1}{\mu^2}\frac{\partial^2 u}{\partial \xi^2}$$

$$u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} + w \frac{\partial u}{\partial \zeta} = \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \zeta^2} - \frac{1}{\rho}\frac{\partial P}{\partial \xi}$$

Wenn man also diese Gleichungen auflösen kann, so hat man auch die Lösung für ein beliebiges μ und umgekehrt

---

Wenn die Änderung von μ auf u, v, w ohne Einfluss sein soll, so muss

$$\nabla \frac{\partial P}{\partial \mu} + \nabla^2 \mathfrak{v} = 0$$ sein.

Also: $\mathrm{curl}\,\nabla^2 \mathfrak{v} = 0$

$$(\mathfrak{v}\nabla)\mathfrak{v} = \underbrace{\nabla P - \mu \nabla \frac{\partial P}{\partial \mu}} = \nabla \mathfrak{v}^2 + V \mathfrak{v}\,\mathrm{curl}\,\mathfrak{v}$$

$$= \nabla\left[P - \mu \frac{\partial P}{\partial \mu}\right] = -\nabla \mu^2 \frac{\partial}{\partial \mu}\left(\frac{P}{\mu}\right) = \nabla \Pi$$

Also nur Druck wird geändert

$$f(x,y,z,\mu) - \mu \frac{\partial f(x,y,z,\mu)}{\partial \mu} = F(x,y,z)$$

$$\frac{\partial f}{\partial \mu} - \frac{\partial f}{\partial \mu} - \mu \frac{\partial^2 f}{\partial \mu^2} = 0$$

$$\frac{\partial f}{\partial \mu} = \varphi(x,y,z)$$

$$f = \mu\, \varphi(x,y,z) + \psi(x,y,z)$$

$$\cancel{\mu\, \varphi(x,y,z)} + \psi(x\,y\,z) - \cancel{\mu\, \varphi(x,y,z)} = F(x\,y\,z)$$

$$P = \mu\, \varphi(x,y,z) + \Pi$$

$$\nabla P = \mu \nabla \varphi + \nabla \Pi$$

$$\left.\begin{aligned} (\mathfrak{v}_*\nabla)\mathfrak{v}_* &= \nabla P + \mu \nabla^2 \mathfrak{v}_* = \mu \nabla\varphi + \nabla\Pi + \mu\nabla^2\mathfrak{v} \\ (\mathfrak{v}\nabla)\mathfrak{v} &= \nabla\Pi \end{aligned}\right\}$$

$$\left[-\nabla\varphi = \nabla^2 \mathfrak{v} \right. \qquad \varphi = \text{willkürliche scalare Function}$$

$$\left.\begin{aligned} -\nabla^2\mathfrak{v} &= \mathrm{curl}^2\mathfrak{v} \\ \mathrm{curl}\,\nabla^2\mathfrak{v} &= -\mathrm{curl}^3\mathfrak{v} = 0 \\ &\cancel{\mathrm{curl}^3\mathfrak{v} = 0} \\ \mathrm{curl}\, V\mathfrak{v}\,\mathrm{curl}\,\mathfrak{v} &= 0 \end{aligned}\right\} \;!$$

$$\mathrm{curl}\, V a b = a\,\mathrm{div}\, b - b\,\mathrm{div}\, a + \underbrace{(b\nabla)a - (a\nabla)b}$$

$$(b\nabla)a = \nabla_a ab + V \mathrm{curl}\, a .\, b \; \Big\| = V(\mathrm{curl}\, a .\, b - \mathrm{curl}\, b .\, a) +$$

$$+ \underbrace{\nabla_a ab - \nabla_b ab}$$

$$\sigma = \nabla u + \operatorname{curl} v$$

$$\operatorname{curl}\sigma = \operatorname{curl}^2 v$$

$$\operatorname{curl}^2\sigma = \operatorname{curl}^3 v = +\nabla\varphi \quad \text{Somit: } \sigma = \int \frac{\nabla\varphi\, dv}{4\pi r}$$

$$\nabla\sigma \operatorname{curl}\sigma = \nabla(\nabla u + \operatorname{curl} v)\operatorname{curl}^2 v = \nabla\Phi$$

$$\operatorname{curl}\sigma = \operatorname{curl}\int \frac{\nabla\varphi\, dv}{4\pi r} = \int \frac{\operatorname{curl}\nabla\varphi}{4\pi r}\, dv$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial\varphi}{\partial x} \qquad u = \int \frac{\frac{\partial\varphi}{\partial x}}{4\pi r}\, dv$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = \frac{\partial\varphi}{\partial y} \qquad v =$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = \frac{\partial\varphi}{\partial z} \qquad w =$$

$$\operatorname{curl}^4 v = \operatorname{curl}^3\sigma = 0$$

$$\left.\begin{aligned} \operatorname{curl}\sigma &= iA_1 + jA_2 + kA_3 \\ \nabla^2 A_1 &= 0 = \nabla^2 A_2 = \nabla^2 A_3 \end{aligned}\right\} = \operatorname{curl}^2 v = i\nabla^2 B_1 + \cdots$$

~~curl~~

$$\nabla(iA_1 + jA_2 + kA_3)$$

wenn $\operatorname{curl} a = \nabla b$ $\quad \nabla^2 b = 0$ ~~[illegible]~~

$$\operatorname{curl}^2 a = 0 = \nabla \operatorname{div} a - \nabla^2 a$$

$$a = \nabla \int \frac{\operatorname{div} a}{r} dv + \underbrace{\operatorname{curl} \int \frac{\operatorname{curl} a}{r} dv}_{= \operatorname{curl} \int \frac{\nabla b}{r} dv} + ~~[illegible]~~ \ldots\ldots \qquad \nabla^2 A = 0$$

ist dies = 0 ?

dann muss $\nabla b = 0 = \operatorname{curl} a = 0$

Somit im obigen Falle: $\operatorname{curl}^2 \sigma = 0$

$$\operatorname{curl} \sigma = \nabla \text{pot} = 0$$

$\sigma = \nabla \text{pot}$ also nur Potentialbewegung

Dann auch Druck dieselbe

$$\nabla \sigma^2 = \nabla P + \mu \nabla^2 \sigma \qquad \nabla^2 u = 0 = \nabla^2 v = \nabla^2 w$$

$$\nabla \sigma^2 = \nabla P \qquad u = \frac{\partial \varphi}{\partial x} \qquad \nabla^3 \varphi = 0 \text{ und } \nabla^2 \varphi = 0$$

$\nabla^2 \sigma = 0$ } von selbst erfüllt

Somit besteht jede stationäre Potentialbewegung auch bei beliebiger innerer Reibung ?

vgl. $u = ~~[illegible]~~ - \frac{ay}{r^2}$ $\qquad \frac{\partial u}{\partial x} = 2\frac{axy}{r^4}$ $\qquad \frac{\partial v}{\partial y} = -\frac{2axy}{r^4}$ x + s

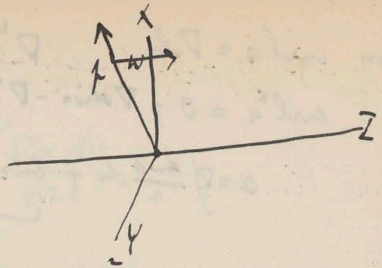
$v = \frac{ax}{r^2}$ $\qquad \frac{\partial u}{\partial y} = -\frac{a}{r^2} + \frac{2ay^2}{r^4}$ $\qquad \frac{\partial v}{\partial x} = \frac{a}{r^2} - \frac{2ax^2}{r^4}$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2ay}{r^4} - \frac{8ax^2y}{r^6}$$

$$\frac{\partial^2 u}{\partial y^2} = +\frac{2ay}{r^4} + \frac{4ay}{r^4} - \frac{8ay^3}{r^6} \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{8ay}{r^4} - \frac{8ay}{r^6}(x^2+y^2) = 0$$

$z =$ Rotationsaxe w



$u = p\cos(p x) = p\cos\varphi$

$v = p\cos(p y) = p\sin\varphi$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cos\varphi \cancel{- \frac{\partial u}{\partial \varphi}\frac{\sin\varphi}{r}} = \frac{\partial u}{\partial r}\cos\varphi + p\frac{\sin^2\varphi}{r}$

$\frac{\partial u}{\partial x} = \frac{\partial p}{\partial r}\cos^2\varphi + \frac{p}{r}\sin^2\varphi$ $\quad\Big\|\quad$ $\frac{\partial v}{\partial x} = \frac{\partial p}{\partial r}\sin\varphi\cos\varphi - \frac{p}{r}\sin\varphi\cos\varphi$

$\frac{\partial^2 u}{\partial x^2} = \cancel{\frac{\partial^2 u}{\partial r^2}\cos^2\varphi + \frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r^2} = \frac{\partial^2 p}{\partial r^2}\cos^3\varphi + \frac{\partial p}{\partial r}\frac{\sin^2\varphi\cos\varphi}{r}}$

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\sin\varphi = \frac{\partial p}{\partial r}\sin\varphi\cos\varphi - \frac{p}{r}\sin\varphi\cos\varphi$

$\frac{\partial^2 u}{\partial y^2} = \cancel{\frac{\partial^2 u}{\partial r^2}\sin^2\varphi + \frac{\partial u}{\partial r}\frac{\cos^2\varphi}{r} = \frac{\partial^2 p}{\partial r^2}\sin^2\varphi\cos\varphi + \frac{\partial p}{\partial r}\frac{\cos^3\varphi}{r}}$

$\Big\|\quad \frac{\partial v}{\partial y} = \frac{\partial p}{\partial r}\sin^2\varphi + \frac{p}{r}\cos^2\varphi$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 p}{\partial r^2}\cos\varphi + \frac{\partial p}{\partial r}\frac{\cos\varphi}{r} - \frac{p}{r^2}\cos\varphi$

$\cancel{+ \frac{p^2}{r}\sin\varphi\cos\varphi - \frac{p^2}{r}\sin^2\varphi\cos\varphi}$

$p\frac{\partial p}{\partial r}\cos^3\varphi + p\frac{\partial p}{\partial r}\sin^2\varphi\cos\varphi + w\frac{\partial p}{\partial z}\cos\varphi = \mu\left[\frac{\partial^2 p}{\partial r^2}\cos\varphi + \frac{\partial p}{\partial r}\frac{\cos\varphi}{r} + \frac{\partial^2 p}{\partial z^2}\cos\varphi - \frac{p}{r^2}\cos\varphi\right]$

$- \frac{1}{\rho}\frac{\partial P}{\partial r}\cos\varphi$

$p\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z} = \mu\left[\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} - \frac{p}{r^2} + \frac{\partial^2 p}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial r}$ $\quad\Big|\quad \frac{\partial}{\partial z}$

$u\frac{\partial w}{\partial r}\cos\varphi + v\frac{\partial w}{\partial r}\sin\varphi + w\frac{\partial w}{\partial z} =$

$p\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial z}$ $\quad\Big|\quad \frac{\partial}{\partial r}$

~~$\frac{\partial u}{\partial r}\frac{\partial u}{\partial r} + u\frac{\partial^2 u}{\partial r\partial z} - \frac{\partial u}{\partial r}\frac{\partial w}{\partial r} - u\frac{\partial w}{\partial r^2} + \frac{\partial w}{\partial z}\frac{\partial u}{\partial z} + w\frac{\partial^2 u}{\partial z^2} - \frac{\partial w}{\partial r}\frac{\partial w}{\partial z} - w\frac{\partial^2 w}{\partial r\partial z} =$~~

~~$= \mu$~~ Continuitätsbedg: ~~$\frac{\partial u}{\partial r}\cos\varphi + \frac{\partial u}{\partial z}\sin^2\varphi\cos\varphi + \frac{\partial w}{\partial z} = 0$~~

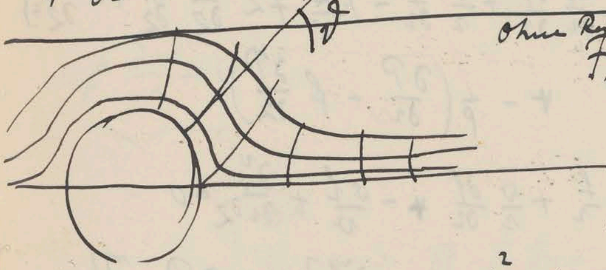
~~$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = 0$~~

$$\underbrace{\frac{\partial u}{\partial r} + \frac{u}{r}}_{= \frac{1}{r}\frac{\partial(ur)}{\partial r}} + \frac{\partial w}{\partial z} = 0 \qquad \frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0$$

$$= \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial(ur)}{\partial r} = -\frac{\partial^2 w}{\partial z\,\partial r}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \mu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial r}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial z}$$



Ohne Reibung:

$$F_t = \frac{\partial}{\partial s}(Pq)$$

Continuität:

$$\frac{\partial}{\partial s}(vq) = 0$$

$$= q\,\delta s\,\frac{\partial v}{\partial t} = \frac{\partial v}{\partial s}\cdot v = \frac{1}{2}\frac{\partial(v^2)}{\partial s} \qquad v\frac{\partial q}{\partial s} + q\frac{\partial v}{\partial s} = 0$$

v_1 ~~$- v_2 = \int F_t\,dt =$~~

~~$v_2^2 - v_1^2 = 2\int_1^2 F_t\,ds = 2(P_2 q_2 - P_1 q_1)$~~

Daraus folgt: $P = P_0' - \frac{v^2}{2}$ für bestimmte Stromlinie

$q = r\,\delta\varphi\,\delta\Delta$

$$q\,\delta s\,\frac{v^2}{R} = F_n = \frac{\partial}{\partial\Delta}(Pr)\,\delta\varphi\,\delta s\,\delta\Delta$$

$$\frac{\partial(Pr)}{\partial\Delta} = \frac{\partial(Pr)}{\partial r}\sin\vartheta \qquad \Delta s = \frac{\Delta r}{\sin\vartheta}$$

$$r\frac{v^2}{R} = \frac{\partial(Pr)}{\partial\Delta} = \partial(Pr)$$

$$R = \frac{\partial s}{\partial\vartheta} = \frac{\partial r}{\partial\vartheta}\frac{1}{\sin\vartheta}$$

$$rv^2\frac{\partial\vartheta}{\partial s} = \frac{\partial(Pr)}{\partial r}\sin\vartheta = rv^2\frac{\partial\vartheta}{\partial r}\sin\vartheta$$

$$rv^2\frac{\partial\vartheta}{\partial r} = \frac{\partial(Pr)}{\partial r}$$

Wenn ~~d~~ Richtung der Stromlinien gegeben ist:

$\frac{u}{w} = f(r,z)$

$u = w\,f(r,z)$

$$\frac{\partial w}{\partial r} f + w\frac{\partial f}{\partial r} + \frac{w}{r} f + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial r} f + \frac{\partial w}{\partial z} + w\frac{\partial f}{\partial r} + \frac{w}{r} f = 0$$

$$w f \left(w\frac{\partial f}{\partial r} + \frac{\partial w}{\partial r} f\right) + w\left(\frac{\partial w}{\partial z} f + w\frac{\partial f}{\partial z}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial r} +$$

$$+\mu\left[2\frac{\partial w}{\partial r}\frac{\partial f}{\partial r} + w\frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 w}{\partial r^2} f + \frac{w}{r}\frac{\partial f}{\partial r} + \frac{f}{r}\frac{\partial w}{\partial r} - \frac{f}{r^2} w + \frac{\partial^2 w}{\partial z^2} f + 2\frac{\partial w}{\partial z}\frac{\partial f}{\partial z} + w\frac{\partial^2 f}{\partial z^2}\right]$$

$$w f \frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \mu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial z} \quad \| f$$

$$w^2 f \frac{\partial f}{\partial r} + w^2 \frac{\partial f}{\partial z} = \mu\left[w\frac{\partial^2 f}{\partial r^2} + 2\frac{\partial w}{\partial r}\frac{\partial f}{\partial r} + \frac{w}{r}\frac{\partial f}{\partial r} - f\frac{w}{r^2} + 2\frac{\partial w}{\partial z}\frac{\partial f}{\partial z} + w\frac{\partial^2 f}{\partial z^2}\right]$$

$$+ -\frac{1}{\rho}\left(\frac{\partial P}{\partial r} + f\frac{\partial P}{\partial z}\right)$$

$$\frac{\partial^2 w}{\partial r^2} f + 2\frac{\partial w}{\partial r}\frac{\partial f}{\partial r} + w\frac{\partial^2 f}{\partial r^2} + \frac{\partial w}{\partial r}\frac{f}{r} + \frac{w}{r}\frac{\partial f}{\partial r} + - \frac{w f}{r^2} + \frac{\partial^2 w}{\partial r \partial z} = 0$$

also

$$= \mu\left[\cancel{w}\,\frac{\partial^2 w}{\partial z^2} f + 2\frac{\partial w}{\partial z}\frac{\partial f}{\partial z} - \frac{\partial^2 w}{\partial r \partial z} + w\frac{\partial^2 f}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial r} =$$

$$= w^2\frac{\partial f}{\partial z} - \frac{w}{r} f^2$$

div ∇ab = b curl a − a curl b

~~F~~Wenn $\mu = 0$:

$$\frac{1}{\rho}\nabla^2 P = u\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + v\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + u\frac{\partial^2 v}{\partial x \partial y} + \left(\frac{\partial v}{\partial y}\right)^2 + v\frac{\partial^2 v}{\partial y^2}$$

$$= \operatorname{div}\nabla 6^2 + \operatorname{div}(\nabla 6 \operatorname{curl} 6)$$

$$= \nabla^2(6^2) + (\operatorname{curl} 6)^2 - 6 \operatorname{curl}^2 6$$

In zwei Dimensionen: $f = t\varphi$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$\frac{u}{v} = f \qquad u = vf \qquad \frac{\partial v}{\partial x}f + v\frac{\partial f}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$vf\cancel{\left(\frac{\partial v}{\partial x}f + v\frac{\partial f}{\partial x}\right)} + \underbrace{v\left(\cancel{\frac{\partial v}{\partial y}f} + v\frac{\partial f}{\partial y}\right)}_{= v^2\frac{\partial f}{\partial y}} = \mu\left[\frac{\partial^2 v}{\partial x^2}f + 2\frac{\partial v}{\partial x}\frac{\partial f}{\partial x} + v\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}f + \quad + v\frac{\partial^2 f}{\partial y^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\underbrace{vf\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}}_{= -v^2\frac{\partial f}{\partial x}} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$\frac{\partial^2 v}{\partial x^2}f + 2\frac{\partial v}{\partial x}\frac{\partial f}{\partial x} + v\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} = 0$$

$$\frac{\partial^2 v}{\partial x\partial y}f + \frac{\partial v}{\partial x}\frac{\partial f}{\partial y} + \frac{\partial v}{\partial y}\frac{\partial f}{\partial x} + v\frac{\partial^2 f}{\partial x\partial y} + \frac{\partial^2 v}{\partial y^2} = 0$$

$$\left.\begin{cases} v^2\frac{\partial f}{\partial y} = \mu\left[-\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 v}{\partial y^2}f + 2\frac{\partial v}{\partial y}\frac{\partial f}{\partial y} + v\frac{\partial^2 f}{\partial y^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial x} \\ -v^2\frac{\partial f}{\partial x} = \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial y} \end{cases}\right\} \left|\begin{matrix}\frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y}\end{matrix}\right.$$

Wenn also $\mu = 0$ so

$$\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} = -\frac{\frac{\partial P}{\partial x}}{\frac{\partial P}{\partial y}}$$

Curven $f = \text{const}$ stehen $\perp$ auf $P = \text{const}$

$\cancel{\sqrt{u^2+v^2}} =$

Stromlinie: $\frac{u}{v} = f = \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}$

$u^2+v^2 = \varphi^2$

$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = \varphi\frac{\partial \varphi}{\partial x} \quad\Big|\; v$

$u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} = \varphi\frac{\partial \varphi}{\partial y} \quad\Big|\; \frac{v}{u}$

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial x} + \frac{v^2}{u}\frac{\partial v}{\partial y}$

$= v\frac{\partial v}{\partial x} - \frac{v^2}{u}\frac{\partial u}{\partial x}$

$= \frac{v}{u}\left(u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}\right)$

$= v^2\frac{v}{u}\frac{\partial}{\partial x}\left(\frac{u}{v}\right)$

$$\frac{1}{\rho}\left(\frac{\partial P}{\partial x}\frac{\partial f}{\partial x} + \frac{\partial P}{\partial y}\frac{\partial f}{\partial y}\right) =$$

$$= \mu\left[\frac{\partial^2 v}{\partial x^2}\frac{\partial f}{\partial y} + \frac{\partial^2 v}{\partial y^2}\frac{\partial f}{\partial y} - \frac{\partial^2 v}{\partial x\partial y}\frac{\partial f}{\partial x} + \frac{\partial^2 v}{\partial y^2} f\frac{\partial f}{\partial x} + 2\frac{\partial v}{\partial y}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + v\frac{\partial f}{\partial x}\frac{\partial f}{\partial y^2}\right]$$

σ sei eine Lösung

$(\sigma\nabla)\sigma = \frac{1}{2}\nabla\sigma^2 + \nabla\sigma\,\mathrm{curl}\,\sigma = \mu\nabla^2\sigma - \frac{1}{\rho}\nabla P$

Dann ist $\sigma_1 = c\sigma$ auch eine Lösung für denselben Fall, aber bei $c\mu$ und Pc^2 ~~kleinerem~~ größerem Druck

$\sigma_1 = \sigma c \qquad \mu_1 = \mu c \qquad P_1 = Pc^2 \qquad \nabla$

$c^2\left(\frac{1}{2}\nabla\sigma^2 + \nabla\sigma\,\mathrm{curl}\,\sigma\right) = c\mu_1\nabla^2\sigma - \frac{1}{\rho}\nabla P_1$

Angenommen $u_1\, v_1\, w_1$ sei eine Lösung, unter welchen Umständen wird $u_1 + u$ etc. eine Lösung sein?

~~$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} + w_1 \frac{\partial u_1}{\partial z} = \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} - \frac{1}{\rho} \frac{\partial P_1}{\partial x}$~~

$$u \frac{\partial u_1}{\partial x} + v \frac{\partial u_1}{\partial y} + w \frac{\partial u_1}{\partial z} + u_1 \frac{\partial u}{\partial x} + v_1 \frac{\partial u}{\partial y} + w_1 \frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{\rho} \frac{\partial (P - P_1)}{\partial x}$$

$$+ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

Wenn z.B. $\left. u, v, w = \text{const} \right\} a$

$$-\frac{1}{\rho} \frac{\partial (P - P_1)}{\partial x} = \int a \nabla u_1$$

etc.

$$-\frac{1}{\rho} \nabla (P - P_1) = (a \nabla) \sigma$$

$$u \left(\frac{\partial^2 u_1}{\partial x \partial y} - \frac{\partial^2 v_1}{\partial x^2} \right) + v \left(\frac{\partial^2 u_1}{\partial y^2} - \frac{\partial^2 v_1}{\partial y \partial x} \right) + w \left(\frac{\partial^2 u_1}{\partial y \partial z} - \frac{\partial^2 v_1}{\partial x \partial z} \right) = 0 \quad \text{etc.}$$

$$u \frac{\partial}{\partial x} \left(\frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} \right) + v \frac{\partial}{\partial y} \left(\frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} \right) + w \frac{\partial}{\partial z} \left(\frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} \right) = 0$$

$$\left\{ \begin{aligned} u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} &= 0 \\ u \frac{\partial \xi}{\partial x} + \cdots &= 0 \\ u \frac{\partial \eta}{\partial x} + \cdots &= 0 \end{aligned} \right\}$$

Bedingung ~~$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$~~

~~$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} = 0$~~

was folgt daraus?

$$(\sigma_1 \nabla) \sigma_1 = \tfrac{1}{2} \nabla \sigma_1^2 + V \sigma_1 \operatorname{curl} \sigma_1 = \nabla^2 \sigma_1 - \frac{1}{\rho} \nabla P_1$$

$$\nabla (\sigma_1 + \sigma)^2 + V (\sigma_1 + \sigma) \operatorname{curl} (\sigma_1 + \sigma) = \nabla^2 (\sigma_1 + \sigma) - \frac{1}{\rho} \nabla P$$

~~$2 \nabla \sigma_1 \sigma$~~

$$\cancel{\nabla\sigma_1^2} + \underbrace{\nabla(\sigma_1\sigma + \sigma\sigma_1)}_{2\nabla S\sigma\sigma_1} + \nabla\sigma^2 + \cancel{\mathrm{V}\sigma_1\,\mathrm{curl}\,\sigma_1} + \mathrm{V}\sigma_1\,\mathrm{curl}\,\sigma + \mathrm{V}\sigma\,\mathrm{curl}\,\sigma_1 + \mathrm{V}\sigma\,\mathrm{curl}\,\sigma$$

$$= \nabla^2\sigma + \cancel{\nabla^2\sigma_1} - \frac{1}{\rho}\nabla(P - P_1)$$

Wenn auch σ_1 eine Lösg ist:

$$2\nabla\sigma\sigma_1 + \mathrm{V}(\sigma_1\,\mathrm{curl}\,\sigma + \sigma\,\mathrm{curl}\,\sigma_1) = -\frac{1}{\rho}\nabla(P - P_1 - P')$$

P' kann daraus genügend bestimmt werden falls überhaupt:

$$\mathrm{curl}\,\mathrm{V}(\underbrace{\sigma_1\,\mathrm{curl}\,\sigma + \sigma\,\mathrm{curl}\,\sigma_1}) = 0$$

Also wenn $= \tau\,\mathrm{curl}\,\tau$ gesetzt wird so muss τ eine mögliche (reibungslose) Flüssigkeitsbewegung sein

$$\mathrm{curl}\,\mathrm{V}ab = a\,\mathrm{div}\,b - b\,\mathrm{div}\,a + (b\nabla)a - (a\nabla)b$$

$$(\mathrm{curl}\,\sigma\,\nabla)\sigma_1 - (\sigma_1\nabla)\mathrm{curl}\,\sigma + (\mathrm{curl}\,\sigma_1\nabla)\sigma - (\sigma\nabla)\mathrm{curl}\,\sigma_1 = 0$$

~~$x = \xi^{\frac{1}{2}} \quad y = \eta^{\frac{1}{2}} \quad z = \zeta^{\frac{1}{2}} \qquad P = \Pi^2$~~

~~∂x~~ ~~$x^2 = \xi$~~

~~$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x} \qquad \frac{\partial \xi}{\partial x} = 2x = 2\sqrt{\xi}$~~

~~$= 2\sqrt{\xi}\,\frac{\partial u}{\partial \xi}$~~

~~$\frac{\partial^2 u}{\partial x^2} = \left[2\sqrt{\xi}\,\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\sqrt{\xi}}\frac{\partial u}{\partial \xi}\right] 2\sqrt{\xi} = 4\xi \frac{\partial^2 u}{\partial \xi^2} + 2\frac{\partial u}{\partial \xi}$~~

$2n-1 = n-2 \qquad u = U^{-1} = \frac{1}{U} \qquad P = \Pi^{-2}$

$n = -1 \qquad \frac{\partial u}{\partial x} = -\frac{1}{U^2}\frac{\partial U}{\partial x} \qquad \frac{\partial^2 u}{\partial x^2} = +\frac{2}{U^3}\left(\frac{\partial U}{\partial x}\right)^2 - \frac{1}{U^2}\frac{\partial^2 U}{\partial x^2}$

Wenn man also eine Lösung der Aufgabe hat, so erhält man eine andere wenn man x, y, z m mal größer macht, aber u, v, w m mal kleiner und Druckunterschiede $(m)^2$ mal kleiner, (aber dasselbe μ).

I	xyz	uvw	μ	P
II	$m\,xyz$	uvw	$m\mu$	P
III	xyz	$m\,uvw$	$m\mu$	m^2P
IV	$m\,xyz$	$\frac{1}{m}\,uvw$	μ	$\frac{1}{m^2}P$

Daraus folgt

$xyz \quad m\,uvw \quad m\mu \quad m^2P$

sind also nur zwei davon unabhängige Transformationen

III am besten zur Anwendung geeignet

$$(\sigma\nabla)\sigma = F - \frac{1}{\rho}\nabla P + \frac{\mu}{3\rho}\nabla \operatorname{div}\sigma + \frac{\mu}{\rho}\nabla^2\sigma$$

~~$\nabla S a\, b = a \operatorname{div} b + b \operatorname{div} a$~~

~~$\left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right]\left[a_1 b_1 + a_2 b_2 - \right] =$~~

~~$a_1\left(i\frac{\partial b_1}{\partial x} + j\frac{\partial b_1}{\partial y} + \dots\right)$~~

~~$+$~~

~~$\left(\frac{\partial b_1}{\partial x} + \frac{\partial b_2}{\partial y} + \frac{\partial b_3}{\partial z}\right)(i a_1 + j a_2 + k a_3)$~~

$$\operatorname{div}(\rho\sigma) = 0 \qquad = \rho \operatorname{div}\sigma + S.\sigma\nabla\rho = 0$$

$$P = f(\rho) \quad \Big\| \quad \operatorname{div}\sigma\,\nabla\rho + \rho\nabla\operatorname{div}\sigma + \nabla(S.\sigma\nabla\rho) = 0$$

$\operatorname{curl}(Ab) =$

$$\rho\left[\frac{1}{2}\nabla\sigma^2 + \mathrm{V}\sigma\operatorname{curl}\sigma\right] = -\nabla P + \mu\nabla^2\sigma + \frac{\mu}{3}\nabla\operatorname{div}\sigma$$

curl:

$$\frac{1}{2}\left[\cancel{\rho\operatorname{curl}\nabla\sigma^2} + \mathrm{V}.\nabla\sigma^2\nabla\rho\right] + \rho\operatorname{curl}\mathrm{V}\sigma\operatorname{curl}\sigma \pm \mathrm{V}(\nabla\rho.\mathrm{V}\sigma\operatorname{curl}\sigma) = \mu\operatorname{curl}\nabla^2\sigma$$

~~Wenn div σ~~

$$-\mathrm{V}.\nabla\rho\,\underbrace{\left[\frac{1}{2}\nabla\sigma^2 + \mathrm{V}\sigma\operatorname{curl}\sigma\right]}_{= -\frac{\nabla P}{\rho} + \frac{\mu}{3\rho}\nabla\operatorname{div}\sigma + \frac{\mu}{\rho}\nabla^2\sigma} + \rho\operatorname{curl}\left(\mathrm{V}\sigma\operatorname{curl}\sigma\right) = \mu\operatorname{curl}\nabla^2\sigma$$

$$= -\mathrm{V}.\nabla\log\rho\left[-\nabla P + \frac{\mu}{3}\nabla\operatorname{div}\sigma + \mu\nabla^2\sigma\right]$$

$$\operatorname{curl}.\mathrm{V}\sigma\operatorname{curl}\sigma = \operatorname{curl}\left[-\frac{\nabla P}{\rho} + \frac{\mu}{3\rho}\nabla\operatorname{div}\sigma + \frac{\mu}{\rho}\nabla^2\sigma\right]$$

$$= \cancel{\frac{\mu}{3\rho}\operatorname{curl}\nabla\operatorname{div}\sigma} + \frac{\mu}{3\rho^2}\mathrm{V}\nabla\rho\nabla\operatorname{div}\sigma$$

$$+ \frac{\mu}{\rho}\operatorname{curl}\nabla^2\sigma + \frac{\mu}{\rho^2}\mathrm{V}\nabla\rho\nabla^2\sigma$$

$$= \frac{\mu}{\rho}\operatorname{curl}\nabla^2\sigma + \mathrm{V}.\nabla\rho\left[\frac{\mu}{3\rho^2}\nabla\operatorname{div}\sigma + \frac{\mu}{\rho^2}\nabla^2\sigma\right]$$

$$\nabla S ab = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(a_1 b_1 + a_2 b_2 + a_3 b_3)$$

$$\operatorname{curl} m b \overset{=}{=} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ m b_1 & m b_2 & m b_3 \end{vmatrix} = i\, m \left(\frac{\partial b_3}{\partial y} - \frac{\partial b_2}{\partial z}\right) +$$

$$+ i\left(b_3 \cancel{\pm} \frac{\partial m}{\partial y} - b_2 \frac{\partial m}{\partial z}\right) + \cdots$$

~~$\nabla^2 \mathfrak{v} = \cdots \nabla p$~~

$$= m \operatorname{curl} b + V.\, b \nabla m$$

$$\operatorname{curl} \frac{\nabla P}{\rho} = \cancel{\frac{\operatorname{curl} \nabla P}{\rho}} \; \cancel{- \frac{1}{\rho^2} V \nabla P \nabla \rho}$$

∇P und $\nabla \rho$ sind gleichgerichtet

∇ Angenommen ~~[illegible]~~ $\mathfrak{v}$ $\overset{\text{und } \rho}{}$ genüge der Gleichung

$$\frac{1}{2}\nabla \mathfrak{v}^2 + V \mathfrak{v} \operatorname{curl} \mathfrak{v} = -\frac{\nabla P}{\rho} + \frac{\mu}{\rho} \nabla^2 \mathfrak{v}$$

dann ist $\rho \operatorname{curl} V \mathfrak{v} \operatorname{curl} \mathfrak{v} = \frac{\mu}{\cancel{\rho \mu}} \operatorname{curl} \nabla^2 \mathfrak{v} + \frac{V.\, \nabla \rho\, \nabla^2 \mathfrak{v}}{\rho \cancel{\mu}}$

Dann genügt es auch derselben Gleichung $+ \frac{\mu}{3\rho} \nabla \operatorname{div} \mathfrak{v}$

falls $\nabla \operatorname{div} \mathfrak{v}$ und $\nabla \rho$ gleichgerichtet sind

also falls $V \nabla \rho \nabla \operatorname{div} \mathfrak{v} = 0$

$$\nabla \operatorname{div} \mathfrak{v} = -\frac{1}{\rho} \operatorname{div} \mathfrak{v} . \nabla \rho - \frac{1}{\rho} \nabla (S . \mathfrak{v} \nabla \rho)$$

$$= \operatorname{div} \mathfrak{v}\, \nabla \log \rho$$

~~also falls $V \nabla \rho \nabla (S\, \mathfrak{v} \nabla \rho) = 0$~~

$= -u \frac{\partial \log \rho}{\partial x} + \cdots$

~~$\operatorname{curl} \operatorname{div} (\rho \mathfrak{v}) = 0$~~

$$\frac{\partial u}{\partial x} + \cdots = -\frac{u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z}}{\rho}$$

$$\operatorname{div} \mathfrak{v} = -\frac{S \mathfrak{v} \nabla \rho}{\rho} = -S \mathfrak{v} \nabla \log \rho$$

$$= -S \mathfrak{v} \frac{\nabla \rho}{\rho}$$

$\mathrm{div}\,\sigma = -S\,\sigma\,\nabla\log\rho$

$\log\rho = \frac{1}{4\pi}\int\frac{\nabla^2\log\rho}{r}\,dv$

$\frac{du}{dx}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$

~~$a = \mathrm{V}\sigma\nabla\log\rho$~~

$\nabla\log\rho = x\sigma + \frac{\mathrm{div}\,\sigma}{\sigma_0}\,\mathrm{V}\frac{\sigma}{\sigma}\beta = x\sigma + \mathrm{div}\left(\frac{1}{\sigma}\right)\mathrm{V}\sigma\beta$

$= x\sigma + \mathrm{div}\log\sigma\;\mathrm{V}\sigma\beta$

$\nabla^2\log\rho = \mathrm{div}\,\nabla\log\rho =$

~~$\rho = F P$~~

~~$\nabla\rho = F(P)\,\nabla P$~~

$\rho = F(P)$

$\nabla\rho = F'(P)\,\nabla P$

$\nabla AB = i\frac{\partial}{\partial x}AB + j\frac{\partial}{\partial y}AB + -$

$= A\left(i\frac{\partial B}{\partial x} + j\frac{\partial B}{\partial y} + \right)$

$+ B\left(i\frac{\partial A}{\partial x} + - \right)$

$= A\nabla B + B\nabla A$

Man habe eine incompressible Aufgabe gelöst

$\frac{1}{2}\nabla\sigma^2 + \mathrm{V}\sigma\,\mathrm{curl}\,\sigma = -\nabla P + \mu\nabla^2\sigma$

Kann dies auch eine compressible vorstellen, wenn $\sigma = \rho\varphi$ gesetzt wird

$\mathrm{div}(\rho\varphi) = 0$

$\mu = \mathrm{fc.}(\rho)$

$\nabla\sigma^2 = \nabla\rho^2\varphi^2 = \rho^2\nabla\varphi^2 + \varphi^2\nabla\rho^2$

$\mathrm{curl}\,\sigma = \rho\,\mathrm{curl}\,\varphi + \mathrm{V}\varphi\nabla\rho$

$\mathrm{V}\sigma\,\mathrm{curl}\,\sigma = \rho^2\mathrm{V}\varphi\,\mathrm{curl}\,\varphi + \rho\,\underbrace{\mathrm{V}.\varphi(\mathrm{V}.\varphi.\nabla\rho)}$

$= -\nabla\rho\,S\varphi^2 + \varphi\,S\varphi\nabla\rho$

$= \rho^2\mathrm{V}\varphi\,\mathrm{curl}\,\varphi - \nabla\rho^2.\varphi^2 + \rho\varphi\,S\varphi\nabla\rho$

$$\begin{array}{ccc} i & j & k \\ \varphi_1 & \varphi_2 & \varphi_3 \\ \cancel{\frac{\partial\rho}{\partial x}} & \cancel{\frac{\partial\rho}{\partial y}} & \cancel{\frac{\partial\rho}{\partial z}} \\ a_1 & a_2 & a_3 \end{array}
\qquad
\left|\begin{array}{ccc} i & j & k \\ \varphi_1 & \varphi_2 & \varphi_3 \\ \begin{vmatrix} \varphi_2 & \varphi_3 \\ \frac{\partial\rho}{\partial y} & \frac{\partial\rho}{\partial z} \end{vmatrix} & \begin{vmatrix} \varphi_3 & \varphi_1 \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \end{vmatrix} & \begin{vmatrix} \varphi_1 & \varphi_2 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{vmatrix} \\ & a_3 \quad a_1 & a_1 \quad a_2 \end{array}\right|$$

$$i\ \left(\varphi_2\varphi_1 a_2 - \varphi_2^2 a_1 - \varphi_3^2 a_1 + \varphi_3\varphi_1 a_3\right)$$

$$i\ \left[(\varphi_1 a_1 + \varphi_2 a_2 + \varphi_3 a_3)\varphi_1 - (\varphi_1^2 + \varphi_2^2 + \varphi_3^2)a_1\right]$$

$$\varphi\, S a\varphi - a\varphi^2$$

$$\nabla^2(\rho\varphi) = {}^2 = i\,\nabla^2\rho\varphi_1 + j\,\nabla^2\rho\varphi_2 +$$

$$= i\,\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\,\rho\varphi_1 \ldots$$

$$= i\left(\nabla^2\rho\cdot\varphi_1 + \cancel{2S\nabla\rho\nabla\varphi_1} + \rho\nabla^2\varphi_1 + \overbrace{2\left(\frac{\partial\rho}{\partial x}\frac{\partial\varphi_1}{\partial x} + \frac{\partial\rho}{\partial y}\frac{\partial\varphi_1}{\partial y} + \frac{\partial\rho}{\partial z}\frac{\partial\varphi_1}{\partial z}\right)}^{=2i\,S\nabla\rho\nabla\varphi_1}\right.$$

$$+\,j\left(\nabla^2\rho\ \varphi_2 + \cancel{2\nabla\rho\nabla\varphi_2} + \rho\nabla^2\varphi_2\right) \quad \frac{\partial\rho}{\partial x}\frac{\partial\varphi_2}{\partial x}$$

$$+\,k\,($$

$$= \varphi\nabla^2\rho + \cancel{\mu}\,\rho\nabla^2\varphi + 2\left(\frac{\partial\rho}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial\rho}{\partial y}\frac{\partial\varphi}{\partial y} + \frac{\partial\rho}{\partial z}\frac{\partial\varphi}{\partial z}\right)$$

$$-\nabla(P-\varphi\mu) - \frac{\mu}{3}\rho\nabla\,\mathrm{div}\,\varphi$$

$$\cancel{\rho}\left[\frac{1}{2}\left(\cancel{\rho^2\nabla\varphi^2} + \cancel{\varphi^2\nabla\rho^2}\right) + \cancel{\rho^2\overline{\nabla}\varphi\,\mathrm{curl}\,\varphi} - \frac{\varphi^2\nabla\rho^2}{2} + \rho\varphi\,S\varphi\nabla\rho\right] = \cancel{-\nabla p} +$$

$$\mu\left[\varphi\nabla^2\rho + \cancel{\rho\nabla^2\varphi} + 2\left(\frac{\partial\rho}{\partial x}\frac{\partial\varphi}{\partial x} + \cdots\ 6\right)\right]$$

$$\frac{1}{2}\rho\nabla\varphi^2 \qquad + \rho\overline{\nabla}\varphi\,\mathrm{curl}\,\varphi \qquad = -\nabla p + \frac{\mu}{3}\nabla\,\mathrm{div}\,\varphi + \mu\nabla^2\varphi$$

Zweidimensional mit Friction

~~$\zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$~~

~~$\frac{\partial \zeta}{\partial x} = -$~~ ~~$\frac{\partial \zeta}{\partial x} + \frac{\partial \zeta}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x \partial y}$~~

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = \mu\left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}\right)$$

~~$\frac{\partial u}{\partial x}\frac{\partial \zeta}{\partial x} + u\frac{\partial^2 \zeta}{\partial x^2} + \overset{= \frac{\partial u}{\partial y} - \zeta}{\frac{\partial v}{\partial x}}\frac{\partial \zeta}{\partial y} + v\frac{\partial^2 \zeta}{\partial x \partial y} = \mu\left(\frac{\partial^3 \zeta}{\partial x^3} + \frac{\partial^3 \zeta}{\partial x \partial y^2}\right)$~~

~~$\frac{\partial u}{\partial y}\frac{\partial \zeta}{\partial x} + u\frac{\partial^2 \zeta}{\partial x \partial y} + \underbrace{\frac{\partial v}{\partial y}}_{= -\frac{\partial u}{\partial x}}\frac{\partial \zeta}{\partial y} + v\frac{\partial^2 \zeta}{\partial y^2} = \mu\left(\frac{\partial^3 \zeta}{\partial x^2 \partial y} + \frac{\partial^3 \zeta}{\partial y^3}\right)$~~

An fester Oberfläche muss $\left(\frac{\partial^2 \zeta}{\partial x^2}\right)_0 + \left(\frac{\partial^2 \zeta}{\partial y^2}\right)_0 = 0$ sein

$$= \left(\frac{\partial^4 \psi}{\partial x^4}\right)_0 + 2\left(\frac{\partial^4 \psi}{\partial x^2 \partial y^2}\right)_0 + \left(\frac{\partial^4 \psi}{\partial y^4}\right)_0 = 0$$

2D. ~~$\zeta = x\left(y - \sqrt{a^2 - x^2}\right)$~~

~~$\frac{\partial \zeta}{\partial x} = y - \sqrt{a^2 - x^2} + \frac{x^2}{\sqrt{a^2 - x^2}}$~~ ~~$\frac{\partial \zeta}{\partial y} = x$~~

~~$\frac{\partial^2 \zeta}{\partial x^2} = \frac{x}{\sqrt{a^2 - x^2}} + \frac{2x}{\sqrt{a^2 - x^2}} + \frac{x^3}{\sqrt{(a^2 - x^2)}^3}$~~ ~~$\frac{\partial^2 \zeta}{\partial y^2} = 0$~~

$$\zeta = \left(y - \sqrt{a^2 - x^2}\right)^n$$

~~$\frac{\partial \zeta}{\partial x} = \frac{x\,n}{\sqrt{\quad}}\left(y - \sqrt{\quad}\right)^{n-1}$~~

~~$\frac{\partial^2 \zeta}{\partial x^2} = \frac{n}{\sqrt{\quad}}\left(y - \sqrt{\quad}\right)^{n-1} + \frac{x^2 n}{\sqrt{(\quad)}^3}\left(y - \sqrt{\quad}\right)^{n-1} + \frac{n(n-1)\,x^2}{(a^2 - x^2)}\left(y - \sqrt{\quad}\right)^{n-2}$~~

~~curl div~~ ~~curl = $i\xi + j\eta + k\zeta$~~

div curl = 0

$$\frac{\partial\xi}{\partial x} + \frac{\partial\eta}{\partial y} + \frac{\partial\zeta}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(u\zeta - \mu\frac{\partial\zeta}{\partial x}\right) + \frac{\partial}{\partial y}\left(v\zeta - \mu\frac{\partial\zeta}{\partial y}\right) + \frac{\partial}{\partial z}\left(w\zeta - \mu\frac{\partial\zeta}{\partial z}\right) = 0 \qquad k$$

$$\frac{\partial}{\partial x}\left(u\xi - \mu\frac{\partial\xi}{\partial x}\right) + \frac{\partial}{\partial y}\left(v\xi - \mu\frac{\partial\xi}{\partial y}\right) + \frac{\partial}{\partial z}\left(w\xi - \mu\frac{\partial\xi}{\partial z}\right) = 0 \qquad i$$

$$\frac{\partial}{\partial x}\left(u\eta - \mu\frac{\partial\eta}{\partial x}\right) \cdots \qquad j$$

$$\frac{\partial\zeta}{\partial y} = n\left(y - \sqrt{}\right)^{n-1}$$

$$\frac{\partial^2\zeta}{\partial y^2} = n(n-1)\left(y - \sqrt{}\right)^{n-2}$$

$$\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2} = n\frac{(a^2 - \cancel{x^2} + \cancel{x^2})}{\sqrt{(\ )^3}}\left(y - \sqrt{}\right)^{n-1} + \frac{n(n-1)x^2\left(y - \sqrt{}\right)^{n-2}}{a^2 - x^2} + n(n-1)\left(y - \sqrt{}\right)^{n-2}$$

$$= n\left(y - \sqrt{}\right)^{n-2}\left[\frac{a^2y}{\sqrt{(\ )^3}} - \cancel{\frac{a^2}{a^2 - x^2}} + \frac{nx^2}{a^2 - x^2} - \cancel{\frac{x^2}{a^2 - x^2}} + n - 2\right]$$

$n > 2$

10. $\psi = x^2(y - a)^3$

$$\zeta = 2(y - a)^3 + 6x^2(y - a)$$

$$\frac{\partial\zeta}{\partial x} = 12x(y - a) \qquad \frac{\partial\zeta}{\partial y} = 6(y - a)^2 + 6x^2$$

$$\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2} = 12(y - a) + 12(y - a) = 24(y - a)$$

$$\frac{\partial\varphi}{\partial x}\,2x(y-a)+\frac{\partial\varphi}{\partial y}\left[(y-a)^2+x^2\right] = 4(y-a)-$$

$$-3x^2(y-a)^2\,\Big[2x(y-a)+2x(y-a)\left[(y-a)^2+x^2\right]\Big]$$

$$\frac{\partial\varphi}{\partial x}\,2x(y-a)+\frac{\partial\varphi}{\partial y}\left[(y-a)^2+x^2\right] = 4(y-a)-6x^3(y-a)^3+2x(y-a)^3+2x^3(y-a)$$

$$= 2(y-a)\Big[2+\left[(y-a)^2+x^2-3x^2(y-a)^2\right]x(\text{[illegible]})\Big]$$

$$P\frac{\partial\varphi}{\partial x}+Q\frac{\partial\varphi}{\partial y}=R$$

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{d\varphi}{R}$$

$$\frac{dx}{2x(y-a)}=\frac{dy}{(y-a)^2+x^2} \qquad \left[\frac{y-a}{2x}+\frac{x}{2(y-a)}\right]dx=dy$$

$$\frac{y-a}{x}=z \qquad y-a=zx \qquad dy=z\,dx+x\,dz$$

$$\left(z+\frac{1}{z}\right)dx=2(z\,dx+x\,dz)$$

$$dx=z^2dx+2xz\,dz$$

$$(1-z^2)\,dx=2xz\,dz$$

$$\frac{dx}{x}=\frac{2z\,dz}{1-z^2} \qquad \lg x=\lg(1-z^2)$$

$$\frac{A}{x}=\left[1-\left(\frac{y-a}{x}\right)^2\right]$$

$$Ax=\left[x^2-y^2+2ay-a^2\right]$$

$$x^2-(y-a)^2=Ax$$

~~$x\,dx=(y-a)\,dy$~~

$$(2x-A)\,dx=2(y-a)\,dy$$

~~$\frac{dx}{dy}=\frac{y-a}{2x}$~~

$$\frac{dx}{dy}=\frac{(y-a)2x}{x^2+\frac{x^2+y^2-2ay+a^2}{2x}}$$

$(y-a)^2 = x^2 - Ax$ $\quad A = fc(\varphi)$

$R = 2\sqrt{x^2-Ax}\left[2 + x\left[-Ax - 3x^2(x^2-Ax)\right]\right]$

$= 2\sqrt{x^2-Ax}\left[2 - Ax^2 - 3x^5 + 3Ax^4\right]$

$\frac{dx}{2x\sqrt{x^2-Ax}} = \frac{d\varphi}{2\sqrt{}\left[2 - \right]}$

$d\varphi = dx\left[\frac{2}{x} - Ax - 3x^4 + 3Ax^3\right]$ jisch. A = const

$\varphi = \lg x - \frac{Ax^2}{2} - \frac{3}{5}x^5 + \frac{3}{4}Ax^4 + B$, $fc(y)$

$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0!$

fruit : $-\frac{1}{x^2} + \frac{\partial^2 B}{\partial y^2} = 0$

$B = \frac{y^2}{2x^2}$

$u = \frac{1}{x} + 3x^2(y-a)^2$

$v = \frac{y}{x^2} - 2x(y-a)^3$

$\frac{v}{u} = \frac{\frac{y}{x^2} - 2x(y-a)^3}{\frac{1}{x} + 3x^2(y-a)^2} = \frac{y - 2x^3(y-a)^3}{x + 3x^4(y-a)^2}$

$\sqrt{u^2+v^2} =$

Angenommen u und v werden an der Kreisoberfläche = 0 von ~~[illegible]~~ algebraischer Ordnung

$u = (a-r)^n f(x,y)$ f paar in Bezug auf y ~~impaar~~ ~~[illegible]~~ x.

Für $x, y = \infty$: $u = c$ somit $f(x,y) = \frac{c}{r^n}$ $x, y = \infty$

$v = (a-r)^m F(x,y)$ F^{impaar} ~~paar~~ — in y (impaar) — x ?

$v = 0$ $F(x,y) = \frac{1}{r^{m+k}}$

$$\xi = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$$

$$\xi = -n(a-r)^{n-1}\frac{y}{r}f(x,y) + (a-r)^n\frac{\partial f}{\partial y} -$$
$$+ m(a-r)^{m-1}\frac{x}{r}F(x,y) - (a-r)^m\frac{\partial F}{\partial x}$$

z.B. Probe mit: $n = m = 1$

$$\left.\begin{aligned}\xi = &-\frac{y}{r}f + (a-r)\frac{\partial f}{\partial y}\\ &+\frac{x}{r}F - (a-r)\frac{\partial F}{\partial x}\end{aligned}\right\} = 0 \text{ für } y = 0$$

Dann $\xi \gtrless 0$ an ~~der~~ Oberfläche

$$\frac{\partial \xi}{\partial x} = \frac{xy}{r^3}f - \frac{y}{r}\frac{\partial f}{\partial x} - \frac{x}{r}\frac{\partial f}{\partial y} + (a-r)\frac{\partial^2 f}{\partial x \partial y} +$$
$$\underbrace{+\frac{1}{r}F - \frac{x^2}{r^3}F}_{\frac{y^2}{r^3}F} + 2\frac{x}{r}\frac{\partial F}{\partial x} - (a-r)\frac{\partial^2 F}{\partial x^2}$$

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{y}{r^3}f - \frac{3x^2y}{r^5}f + \frac{xy}{r^3}\frac{\partial f}{\partial x} + \frac{xy}{r^3}\frac{\partial f}{\partial x} - \frac{y}{r}\frac{\partial^2 f}{\partial x^2} - \frac{1}{r}\frac{\partial f}{\partial y} + \frac{x^2}{r^3}\frac{\partial f}{\partial y} -$$
$$-\frac{x}{r}\frac{\partial^2 f}{\partial x \partial y} - \frac{x}{r}\frac{\partial^2 f}{\partial x \partial y} + (a-r)\frac{\partial^3 f}{\partial x^2 \partial y} - \frac{3y^2x}{r^5}F + \frac{y^2}{r^3}\frac{\partial F}{\partial x} + \frac{2}{r}\frac{\partial F}{\partial x} -$$
$$-\frac{2x^2}{r^3}\frac{\partial F}{\partial x} + \frac{2x}{r}\frac{\partial^2 F}{\partial x^2} + \frac{x}{r}\frac{\partial^2 F}{\partial x^2} - (a-r)\frac{\partial^3 F}{\partial x^3}$$

$$= \frac{(y^2-2x^4)}{r^5} f + 2\frac{xy}{r^3}\frac{\partial f}{\partial x} - \frac{y}{r}\frac{\partial^2 f}{\partial x^2} - \frac{y^2}{r^3}\frac{\partial f}{\partial y} - 2\frac{x}{r}\frac{\partial^2 f}{\partial x \partial y} + (a-r)\frac{\partial^3 f}{\partial x^2 \partial y} -$$

$$- 3\frac{y^2 x}{r^5} F + \frac{(x^2+2y^2)}{r^3}\frac{\partial F}{\partial x} + \frac{3x}{r}\frac{\partial^2 F}{\partial x^2} - (a-r)\frac{\partial^3 F}{\partial x^3}$$

$$\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} = -\frac{f}{r^3} + \frac{2xy}{r^3}\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right) - \frac{y}{r}\frac{\partial^2 f}{\partial x^2} - \frac{x}{r}\frac{\partial^2 f}{\partial y^2} - \frac{y^2}{r^3}\frac{\partial f}{\partial y} - \frac{x^2}{r^3}\frac{\partial f}{\partial x} -$$

$$- \frac{2(x+y)}{r}\frac{\partial^2 f}{\partial x \partial y} + (a-r)\left(\frac{\partial^3 f}{\partial x^2 \partial y} + \frac{\partial^3 f}{\partial x \partial y^2}\right) - \frac{3(y+x)yx}{r^5} F +$$

$$+ \frac{2y^2-x^2}{r^3}\frac{\partial F}{\partial x} + \frac{2x^2-y^2}{r^3}\frac{\partial F}{\partial y} + \frac{3x}{r}\frac{\partial^2 F}{\partial x^2} + \frac{3y}{r}\frac{\partial^2 F}{\partial y^2} - (a-r)\left(\frac{\partial^3 F}{\partial x^3} + \frac{\partial^3 F}{\partial y^3}\right)$$

$$u = (a-r) f \qquad v = (a-r) F$$

$$= \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} \qquad = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}$$

$$\left.(a-r)\left(\frac{\partial f}{\partial x} + \frac{\partial F}{\partial y}\right) = \frac{xf + yF}{r}\ \right\}$$ Incompressibility

$\varphi =$ real part of $\Phi(x+iy)$

General problem in two dimensions

$$\frac{\partial \varphi}{\partial x} = \underbrace{\frac{\mu\left(\frac{\partial^4 \psi}{\partial x^4} + 2\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4}\right) - \frac{\partial \psi}{\partial y}\left(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial x \partial y^2}\right) + \frac{\partial \varphi}{\partial x}\left(\frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3}\right)}{\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial x \partial y^2}}}_{\Phi_1}$$

$$- \frac{\partial \varphi}{\partial y} \underbrace{\frac{\frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3}}{\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi}{\partial x \partial y^2}}}_{\Phi_2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial \Phi_1}{\partial x} - \frac{\partial^2 \varphi}{\partial x \partial y}\Phi_2 - \frac{\partial \varphi}{\partial y}\frac{\partial \Phi_2}{\partial x}$$

$$u = \frac{\mu\left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial \zeta}{\partial y^2}\right)}{\frac{\partial \zeta}{\partial x}} - v\frac{\frac{\partial \zeta}{\partial y}}{\frac{\partial \zeta}{\partial x}}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\Phi_1 - v\frac{\partial \Phi_2}{\partial x} - \Phi_2 \overbrace{\frac{\partial v}{\partial x}}^{= -\zeta + \frac{\partial u}{\partial y}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial \Phi_1}{\partial y} - v\frac{\partial \Phi_2}{\partial y} - \Phi_2\frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x} = \frac{\partial \Phi_1}{\partial x} - v\frac{\partial \varphi_2}{\partial x} + \Phi_2\zeta - \Phi_2\frac{\partial \Phi_1}{\partial y} + v\Phi_2\frac{\partial \Phi_2}{\partial y} - \Phi_2^2 \underbrace{\frac{\partial v}{\partial y}}_{= -\frac{\partial u}{\partial x}}$$

$$\frac{\partial u}{\partial x}\left[1 - \Phi_2^2\right] = \frac{\partial \Phi_1}{\partial x} + \Phi_2\zeta - \Phi_2\frac{\partial \varphi_1}{\partial y} + v\left[\Phi_2\frac{\partial \Phi_2}{\partial y} - \frac{\partial \Phi_2}{\partial x}\right]$$

If $v=0$ $\qquad \zeta = \frac{\partial u}{\partial y} \qquad \frac{\partial u}{\partial x} = 0$

$$~~u\frac{\partial^2 u}{\partial x \partial y}~~ \qquad ~~\frac{\partial^3 u}{\partial x^2 \partial y}~~ + \frac{\partial^3 u}{\partial y^3} = 0$$

$$u = Ay^2 + By + C$$

With neglection of inertia-terms:

$$\nabla p - \mu \nabla^2 \mathfrak{v} = 0$$

$$\left.\begin{array}{l} \frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ \frac{\partial p}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{array}\right\} \underbrace{\frac{\partial^3 u}{\partial x^2 \partial y} + \frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} - \frac{\partial^3 v}{\partial y^2 \partial x}}_{= \frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} = 0} = 0$$

$$\zeta = \mathrm{Re\,P.}\ \underset{\uparrow}{\Phi}(x+iy) = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$$

willkürlich $\qquad$ ebenso:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

~~$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$~~

$$\frac{\partial \psi}{\partial x} = \int \frac{x}{r^2}\,dx\,dy$$

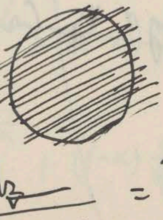
~~$(a-r)f = \frac{\partial \psi}{\partial y} \qquad \frac{\partial^2 \psi}{\partial x \partial y} = (a-r)\frac{\partial f}{\partial x} + \frac{x}{r} f = (a-r)\frac{\partial F}{\partial y} - \frac{y}{r}F$~~

$$F = \frac{\frac{\partial \psi}{\partial y}}{a-r}$$

$$\int \log r \cdot f(x,y)\,dx\,dy$$

$$V = \frac{1}{2}\int \log(x^2+y^2) \cdot f(x,y)\,dx\,dy$$

$$\frac{\partial u}{\partial x} + \frac{\partial \psi}{\partial y}$$



$$\frac{\partial V}{\partial x} = \frac{1}{2}\int \frac{x f}{r^2} + \log r^2 \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{2}\int \frac{f}{r^2} - \frac{2x^2 f}{r^3} + \frac{2x\frac{\partial f}{\partial x}}{r^2} + \log r^2 \frac{\partial^2 f}{\partial x^2}$$

$$\int \frac{dx}{\sqrt{x^2+y^2+z^2}} = \int \frac{dz}{\sqrt{1+\left(\frac{z}{\sqrt{\ }}\right)^2}} = \log\left(z \pm \sqrt{x^2+y^2}\right)$$

$$= \log(z+r)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(pr) = \frac{\partial p}{\partial r} + \frac{p}{r} = \frac{\rho g a^2}{r^2}$$

$p v^k =$ [illegible]

$$p = c\rho^k$$

$$ck\rho^{k-1}\frac{\partial \rho}{\partial r} + \frac{c\rho^k}{r} = \frac{\rho g a^2}{r^2}$$

$$ck\rho^{k-2}\frac{d\rho}{dr} + \frac{c\rho^{k-1}}{r} = \frac{g a^2}{r^2}$$

$$\cancel{ck\rho^{k-2}d\rho = \left(\frac{ga^2}{r^2} - \frac{c\rho^{k-1}}{r}\right)dr = \frac{dr}{r^2}\left(ga^2 - cr\rho^{k-1}\right)}$$

$$\rho^{k-1} = z$$

$$\frac{k}{k-1}\frac{dz}{dr} + \frac{z}{r} = \frac{ga^2}{cr^2}$$

[illegible]

$$+D$$

$$\cancel{z = \frac{A}{r} + [illegible] \log r\,(Br + C) + Cr^2\log r}$$

$$-\frac{k}{k-1}A + A = \frac{ga^2}{c}$$

$$A = \frac{ga^2}{c}\,\underbrace{\frac{1}{1-\frac{k}{k-1}}}_{= 1-k}$$

$$\rho^{k-1} = \frac{ga^2(1-k)}{c}\frac{1}{r}$$

$$p = \quad + 2Cr\log r + \frac{C}{r}$$

$$\frac{k}{k-1}\left(-\frac{A}{r^2} + \frac{B}{r}\log r + B + \frac{C}{r}\right) + \frac{A}{r^2} + B\log r + Cr\log r + \frac{D}{r} = \frac{ga^2}{cr^2}$$

$z = uv$ $\qquad$ $\frac{du}{dr}v + u\frac{dv}{dr}$

$$\underbrace{\left(\frac{k}{k-1}\frac{dv}{dr}+\frac{v}{r}\right)}_{=0}u+\frac{k}{k-1}v\frac{du}{dr}=g\frac{a^2}{cr^2}$$

$$du=\frac{k-1}{k}g\frac{a^2}{c}b^{-\frac{1}{k}}\underbrace{\frac{r^{\frac{k-1}{k}}}{r^2}}_{=r^{-\frac{1}{k}-1}}dr$$

$$\frac{k}{k-1}\frac{dv}{v}+\frac{dr}{r}=0$$

$$v^k r^{k-1}=b$$

$$v=b^{\frac{1}{k}}r^{\frac{1-k}{k}}$$

$$u=\frac{\frac{k-1}{k}g\frac{a^2}{c}b^{-\frac{1}{k}}r^{-\frac{1}{k}}}{-\frac{1}{k}}+C$$

$$u=(1-k)g\frac{a^2}{c}\frac{1}{r}+Cr^{\frac{1-k}{k}}=\rho^{k-1}$$

~~$\left(-\frac{k-1}{k}g\frac{a^2}{c}\frac{1}{r^2}+C\frac{1-k}{k}r^{\frac{1}{k}-2}\right)\frac{k}{k-1}+\frac{k-1}{k}g\frac{a^2}{c}\frac{1}{r^2}+Cr^{\frac{1}{k}-2}=g\frac{a^2}{cr^2}$~~

~~$\frac{k}{k-1}(1-k)g\frac{a}{c}\frac{1}{r^2}+\frac{k}{k-1}C\frac{1-k}{k}r^{\frac{1}{k}-2}+(1-k)g\frac{a^2}{c}\frac{1}{r^2}+Cr^{\frac{1}{k}-2}=g\frac{a^2}{cr^2}$~~

~~$r=a$ $\rho=\rho_0$~~

~~$\rho_0^{k-1}=(1-k)g\frac{a}{c}+Ca^{\frac{1-k}{k}}$~~

$$C=\left[\rho_0^{k-1}+(k-1)g\frac{a}{c}\right]a^{\frac{k-1}{k}}$$

$$\rho^{k-1}=(1-k)g\frac{a^2}{c}\frac{1}{r}+\left[\rho_0^{k-1}+(k-1)g\frac{a}{c}\right]\left(\frac{r}{a}\right)^{\frac{1-k}{k}}$$

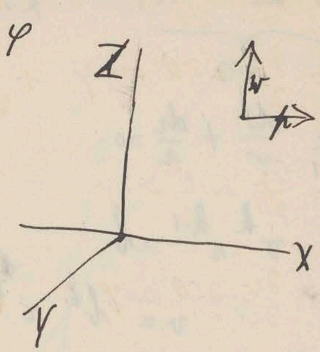
$$=\rho_0^{k-1}\left(\frac{r}{a}\right)^{\frac{1}{k}-1}+(1-k)g\frac{a}{c}\left[\frac{a}{r}-\left(\frac{r}{a}\right)^{\frac{1}{k}-1}\right]$$

$$=\rho_0^{k-1}\left(\frac{r}{a}\right)^{\frac{1-k}{k}}+(1-k)g\frac{a^2}{cr}\left[1-\left(\frac{r}{a}\right)^{\frac{1}{k}}\right]$$

$$RT=pv=\frac{p}{\rho}=c\rho^{k-1}=\uparrow$$

$$\mu\sim\sqrt{T}\sim\rho^{\frac{k-1}{2}}$$

Transformation of ~~general~~ special equations (for rotational ~~body~~ symmetry) in polar coordinates r ~~p~~ θ φ



z = axis of symmetry $= r\cos\theta$

$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

~~Note: $\frac{\partial u}{\partial\varphi} = \frac{\partial v}{\partial\varphi} = \frac{\partial w}{\partial\varphi} = 0$~~

Equation of Continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\frac{\partial u}{\partial r}\sin\theta\cos\varphi + \frac{\partial u}{\partial\theta}\frac{\cos\theta\cos\varphi}{r} - \frac{\partial u}{\partial\varphi}\frac{\sin\varphi}{r\sin\theta} = \frac{\partial u}{\partial x} \qquad u = 6\sin\theta\cos\varphi$$

$$\frac{\partial u}{\partial r}\sin\theta\sin\varphi + \frac{\partial u}{\partial\theta}\frac{\cos\theta\sin\varphi}{r} + \frac{\partial u}{\partial\varphi}\frac{\cos\varphi}{r\sin\theta} = \frac{\partial u}{\partial y} \qquad v = 6\sin\theta\sin\varphi$$

$$\frac{\partial u}{\partial r}\cos\theta - \frac{\partial u}{\partial\theta}\frac{\sin\theta}{r} = \frac{\partial u}{\partial z} \qquad w = 6\cos\theta$$

New variable: $p = \sqrt{u^2+v^2}$ $\qquad \frac{\partial p}{\partial\varphi} = 0$

$\frac{\partial w}{\partial\varphi} = 0$

$u = p\cos\varphi$

$v = p\sin\varphi$

$$\frac{\partial u}{\partial x} = \frac{\partial p}{\partial r}\sin\theta\cos^2\varphi - ~~[illegible]~~ + \frac{\partial p}{\partial\theta}\frac{\cos\theta\cos^2\varphi}{r} + p\frac{\sin^2\varphi}{r\sin\theta}$$

$$\frac{\partial u}{\partial y} = \frac{\partial p}{\partial r}\sin\theta\sin\varphi\cos\varphi + \frac{\partial p}{\partial\theta}\frac{\cos\theta\sin\varphi\cos\varphi}{r} - p\frac{\sin\varphi\cos\varphi}{r\sin\theta}$$

$$\frac{\partial u}{\partial z} = \frac{\partial p}{\partial r}\cos\theta\cos\varphi - \frac{\partial p}{\partial\theta}\frac{\sin\theta\cos\varphi}{r}$$

$$\frac{\partial v}{\partial x} = \frac{\partial p}{\partial r}\sin^2\theta\cos\varphi\sin\varphi + \frac{\partial p}{\partial\theta}\frac{\cos\theta\sin\varphi\cos\varphi}{r}\ \cancel{+} - p\,\frac{\sin^2\varphi\cos\varphi}{r\sin\theta}$$

$$\frac{\partial v}{\partial y} = \frac{\partial p}{\partial r}\sin\theta\sin^2\varphi + \frac{\partial p}{\partial\theta}\frac{\cos\theta\sin^2\varphi}{r} + p\,\frac{\cos^2\varphi}{r\sin\theta}$$

$$\frac{\partial v}{\partial z} = \frac{\partial p}{\partial r}\cos\theta\sin\varphi - \frac{\partial p}{\partial\theta}\frac{\sin\theta\sin\varphi}{r}$$

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r}\sin\theta\cos\varphi + \frac{\partial w}{\partial\theta}\frac{\cos\theta\cos\varphi}{r}$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r}\sin\theta\sin\varphi + \frac{\partial w}{\partial\theta}\frac{\cos\theta\sin\varphi}{r}$$

$$\frac{\partial w}{\partial z} = \frac{\partial w}{\partial r}\cos\theta - \frac{\partial w}{\partial\theta}\frac{\sin\theta}{r}$$

$$\frac{\partial u}{\partial x} = \frac{\partial\sigma}{\partial r}\sin\theta\cos\varphi + \frac{\sigma}{r}\cos^2\theta\cos^2\varphi + \frac{\sigma}{r}\cancel{\sin^2}\varphi$$

$$\frac{\partial w}{\partial y} = \frac{\partial\sigma}{\partial r}\sin^2\theta\sin^2\varphi + \frac{\sigma}{r}\cos^2\theta\sin^2\varphi + \frac{\sigma}{r}\cos^2\varphi$$

$$\frac{\partial w}{\partial z} = \frac{\partial\sigma}{\partial r}\cos^2\theta\ \cancel{+} + \frac{\sigma}{r}\sin^2\theta$$

$\cancel{\cos^2\theta}$ 1

$\cos^2\varphi + \sin^2\varphi - \sin^2\theta\cos^2\varphi$

$1 \quad -\sin^2\theta\sin^2\varphi$

für Kugelsymmetrie

$$\frac{\partial u}{\partial y} = \frac{\partial\sigma}{\partial r}\sin^2\theta\sin\varphi\cos\varphi + \frac{\sigma}{r}(\cos^2\theta\sin\varphi\cos\varphi\ \cancel{+} - \sin\varphi\cos\varphi)$$

$$= \sin^2\theta\sin\varphi\cos\varphi\left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)$$

$$\frac{\partial v}{\partial x} = \frac{\partial\sigma}{\partial r}\cancel{\sin^2\theta\sin\varphi\cos\varphi + \frac{\sigma}{r}\cos^2\theta\sin\varphi\cos\varphi - \sin\varphi\cos\varphi}$$

$$\frac{\partial u}{\partial z} = \frac{\partial\sigma}{\partial r}\sin\theta\cos\theta\cos\varphi - \frac{\sigma}{r}\sin\theta\cos\theta\cos\varphi$$

$$= \sin\theta\cos\theta\cos\varphi\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right] = \frac{\partial w}{\partial x}$$

$$\frac{\partial v}{\partial z} = \frac{\partial w}{\partial y} = \left[\frac{\partial\sigma}{\partial r}\ \cancel{\ldots} - \frac{\sigma}{r}\right]\sin\theta\cos\theta\sin\varphi$$

$$\frac{\partial u}{\partial x} = \left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]\sin^2\theta\cos^2\varphi + \frac{\sigma}{r}$$

$$\frac{\partial v}{\partial y} = \left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]\sin^2\theta\sin^2\varphi + \frac{\sigma}{r}$$

$$\frac{\partial w}{\partial z} = \left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]\cos^2\theta + \frac{\sigma}{r}$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = \left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]\sin^2\theta\sin\varphi\cos\varphi$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} = \left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]\sin\theta\cos\theta\cos\varphi$$

$$\frac{\partial v}{\partial z} = \frac{\partial w}{\partial y} = \left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]\sin\theta\cos\theta\sin\varphi$$

Reduction of general equations to one differential equation

$$6 = \nabla U + \text{curl}\,\varphi$$

$$\tfrac{1}{2}\nabla 6^2 + \nabla 6\,\text{curl}\,6 = \nabla P - \mu\,\cancel{\ }\text{curl}^2 6$$

$$\text{curl}\,\nabla 6\,\text{curl}\,6 = -\mu\,\text{curl}^3 6$$

$$= (\text{curl}\,6\,\nabla)6 - (6\,\nabla)\text{curl}\,6$$

$$(b\nabla)a = \nabla(\text{curl}\,a\,.\,b) - \nabla_a ab$$

$$\text{curl}\,\nabla 6\,\text{curl}\,6 = \nabla(\text{curl}^2 6\,.\,6) - \cancel{\nabla(\text{curl}\,6)^2} - \underbrace{\nabla_{\text{curl}6}\,6\,\text{curl}\,6 + \nabla_6\,6\,\text{curl}\,6}$$

~~[illegible] $= \nabla\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)_a$ [illegible]~~ $= 6^2\nabla\frac{\text{curl}\,6}{6}\;2$

$$\nabla\frac{6\,\text{curl}\,6}{6^2} = 6\,\text{curl}\,6\;\nabla\frac{1}{6^2} + \frac{1}{6^2}\nabla 6\,\text{curl}\,6$$

$$= 2\cancel{\frac{6\,\text{curl}\,6}{6^3}\nabla 6}$$

$$\cancel{\nabla\frac{1}{6^2}} \qquad = \frac{6\,\text{curl}\,6}{6^2}\,\delta\,\frac{\nabla 6}{6}$$

$$\frac{k-1}{k} = 1 - \frac{1}{k} = A \qquad \frac{1}{k} = 1 - A \qquad k = \frac{1}{1-A}$$

$$A = \frac{\cancel{9'8}\,.\,\cancel{13'6\,.\,0'76}\,\cancel{10}}{\cancel{9'8}\,.\,6'366\,\cancel{0'7}\,\cancel{0'00}13} \;\Bigg\|\; k = \frac{1}{1-\frac{1}{830}}\,!$$

64

$$\frac{1'9}{830}$$

$\frac{\partial p}{\partial r} = -\rho \, g \frac{a^2}{r^2}$ $\quad \| \quad p v^k = c \qquad p = c \rho^k \qquad c = p_0 v_0^k$

$$= p_0 \rho_0^{-k}$$

$$k c \rho^{k-1} \frac{\partial \rho}{\partial r} = -\frac{\rho}{r^2} g a^2$$

$$k c \rho^{k-2} \, d\rho = -\frac{dr}{r^2} g a^2$$

$$RT = p v = c \rho^{k-1}$$

$$k c \frac{\rho^{k-1}}{k-1} = \frac{g a^2}{r} + \text{const}$$

$$k c \frac{\rho_0^{k-1}}{k-1} = g a + \text{const}$$

$$\frac{k c}{k-1}\left(\rho^{k-1} - \rho_0^{k-1}\right) = g a \left(\frac{a}{r} - 1\right)$$

$$R \frac{dT}{dr} = -\frac{k-1}{k} \frac{g a^2}{r^2}$$

const.

$$R(T - T_0) = \frac{k-1}{k} g a \left(\frac{a}{r} - 1\right)$$

Grenzfall: $\frac{k-1}{k} \leqq \frac{p_0}{\rho_0 g a}$

---

für $r = \infty$: $\rho^{k-1} = \rho_0^{k-1} - \frac{k-1}{kc} g a$

$$= \rho_0^{k-1} \Big[1 - \underset{<0}{\frac{k-1}{k p_0} \rho_0 \, g a} \Big]$$

nicht möglich!

$\rho_0 = 0{\cdot}0013$

$p_0 = 9{\cdot}8 \times 13{\cdot}6 \times 0{\cdot}76$

$g = 9{\cdot}8$

$a = 6366200$

$$\frac{\cancel{0{\cdot}00}13 \,.\, \cancel{9{\cdot}8} \,.\, 63662\cancel{00}}{\cancel{9{\cdot}8} \,.\, 13{\cdot}6 \,.\, \cancel{0{\cdot}}76}$$

$\leftarrow r = \cancel{a+\delta} \; a(1+\delta)$

Für $p = 0$: $\frac{k c \rho_0^{k-1}}{k-1} = g a \left(1 - \frac{a}{r}\right)$

[illegible] $r =$

$$\frac{1}{g a} \frac{k}{k-1} \frac{p_0}{\rho_0} = \frac{\cancel{9{\cdot}8} \,.\, 13{\cdot}6 \,.\, \cancel{0{\cdot}}76}{\cancel{0{\cdot}00}13 \,.\, \cancel{9{\cdot}8} \,.\, 63662\cancel{00}} \qquad \frac{1{\cdot}4}{0{\cdot}4}$$

$$\delta = \frac{76 \,.\, \cancel{7}}{\cancel{126000} \; 18000} = \frac{1}{210}$$

Falls dagegen die Abnahme der Schwere wegen Centrifugalkraft dieselbe wäre (mit wachsender Höhe) wie am Äquator

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{ga^2}{r^2} + \omega(r-a) = kc\rho^{k-2}\frac{\partial\rho}{\partial r}$$

$$\frac{kc}{k-1}\left(\rho^{k-1}-\rho_0^{k-1}\right) = \cancel{\quad}^2 + \frac{\omega}{2}(r-a)^2$$

$$= ga\left(\frac{a}{r}-1\right)$$

$$= (a-r)\left[g\frac{a}{r} - \frac{\omega}{2}(a-r)\right]$$

$$\omega = \frac{\pi}{2}\frac{1}{365.24.60.60}$$

$$\omega a = \frac{\pi}{2}\frac{\cancel{6366000}}{\cancel{365}.24.\cancel{36}}\ \frac{\overset{\cancel{2122}}{\cancel{10610}}\ \cancel{1060}}{\underset{\cancel{220}}{6.\cancel{73}}.\,3}\quad 53 \quad 11$$

$$= \frac{\cancel{30}.\overset{27}{\cancel{55}}}{\underset{11}{\cancel{22}}.\underset{8}{\cancel{24}}} = \frac{27}{88} = 0{\cdot}40$$

Macht sehr wenig aus.

Obiges nicht ganz genau; eigentlich:

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{ga^2}{r^2} + \omega r = kc\rho^{k-2}\frac{\partial\rho}{r}$$

$$\frac{kc}{k-1}\left(\rho^{k-1}-\rho_0^{k-1}\right) = ga\left(\frac{a}{r}-1\right) + \frac{\omega(r^2-a^2)}{2} \quad — - —$$

Damit ρ für x=∞ ρ=0 wird müsste sein:

$$\frac{k-1}{k} a g \frac{\rho_0}{p_0} = 1 \qquad \frac{\rho_0}{p_0} = \overset{\text{beigen. temp}}{RT_0} = \frac{0{\cdot}0013}{100} = 0{\cdot}000013 = 1{\cdot}3 . 10^{-5}$$

$$\frac{k-1}{k} a g = \frac{0{\cdot}4}{\cancel{1{\cdot}4}} \; 6{,}366{,}000 . \cancel{9{\cdot}8}^{7} = 17{,}800{,}000$$

$$4456$$

$$1782$$

$$= 18 . 10^{6}$$

$$\frac{1}{18} 10^{-6} = 5{\cdot}5 . 10^{-8}$$

Es müsste also die Temperatur $\frac{1}{1200}$ der jetzigen betragen

---

Dissipation of energy pro unit of time:

$$\Phi = -\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2 + \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+ \cdots\right\}$$

$$\frac{\partial \rho u}{\partial x} + \cdots = 0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = \frac{1}{\rho}\left[u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}\right]$$

$$\operatorname{div}(\rho\sigma) = \frac{\partial(\rho\sigma)}{\partial r} + 2\frac{\rho\sigma}{r} = 0 = \rho\underbrace{\left[\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right]}_{\operatorname{div}\sigma} + \sigma\frac{d\rho}{dr} = 0$$

$$\frac{d\log\sigma}{dr} + \frac{2}{r} + \frac{d\log\rho}{dr} = 0$$

$$\operatorname{div}\sigma = -\frac{\sigma}{\rho}\frac{d\rho}{dr}$$

$$\sigma\rho \cdot r^2 = \text{const}$$

Falls man annimmt, dass der Unterschied zwischen aerostatischem und aerodynamischem Druck zu vernachlässigen ist; dagegen der Temperatureinfluss der Bewegung zu berücksichtigen:

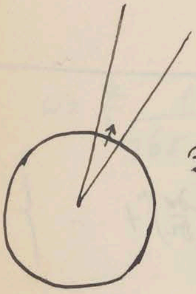
$$\left.\begin{aligned}\frac{1}{\rho}\frac{\partial p}{\partial r} &= g\frac{a^2}{r^2}\\ \frac{p}{\rho} &= R\theta\end{aligned}\right\}$$

Betreffs θ Annahme:

Es bewege sich von der Erdoberfläche aus ein Luftstrom mit der Geschw. c aufwärts; welche Temp.verteilung entsteht

σ = Radialgeschw. $\sigma_a = c$

Arbeit = Ausdehnungsarbeit ~~und~~ Reibungsarbeit und Schwerearbeit



Falls keine Reibung wäre, so wäre die andere gleich

$\rho \frac{g a^2 dr}{r^2} =$ [illegible] $p\frac{\partial v}{\partial r} + c\frac{\partial\theta}{\partial r} + \mu\frac{\partial}{\partial r}$

Reversibel: $\underbrace{\rho\, dQ}_{\text{pro Volum}} = \rho c_v\, d\theta + \rho A p\, dv = 0$

$$d\theta = -\frac{A p\, dv}{c_v}$$

dazu kommt noch $= + \frac{\mu \cdots}{c_v} dt$

$v = \frac{1}{\rho}$

$$\frac{d\theta}{d\rho} = \frac{Ap}{c_v\rho^2} + \frac{A\mu\cdots}{c_v\rho}\frac{dt}{d\rho}$$

$$= \frac{\partial\theta}{\partial r}\cdot\frac{\partial r}{\partial\rho}$$

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial r}\sigma$$

$$p = R\rho\theta$$

$$\frac{R}{\rho}\left[\frac{\partial\theta}{\partial r}\rho + \theta\frac{\partial\rho}{\partial r}\right] = g\frac{a^2}{r^2} \qquad \text{I).}$$

$$\frac{\partial\theta}{\partial r} = \frac{AR\cancel{p}\theta}{c\rho^{\cancel{2}}}\frac{\partial\rho}{\partial r} - \frac{\frac{4}{3}A\mu\left(\frac{b}{\rho^2 r^2}\right)^2\left[\frac{\partial\rho}{\partial r} + \frac{3\rho}{r}\right]^2\cancel{p}\, r^2}{c\,\cancel{p}\rho\, b}$$

$$= \frac{AR}{c}\frac{\theta}{\rho}\frac{\partial\rho}{\partial r} - \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial\rho}{\partial r} + \frac{3\rho}{r}\right]^2 \qquad \text{II).}$$

Work done by internal friction:

$$\Phi = -\frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \mu\left[2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \cdots\right]$$

$$2\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]^2\left[\sin^4\theta(\cos^4\varphi + \sin^4\varphi) + \cos^4\theta\right] + 6\frac{\sigma^2}{r^2} + 4\frac{\sigma}{r}\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]$$

$$+4\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]^2\left[\sin^2\theta\cos^2\theta + \sin^4\theta\sin^2\varphi\cos^2\varphi\right] - \frac{2}{3}\left[\frac{\partial\sigma}{\partial r} + 2\frac{\sigma}{r}\right]^2$$

$$= 2\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]^2\left[\underbrace{\sin^4\theta\left[\cos^4\varphi + 2\sin^2\varphi\cos^2\varphi + \sin^4\varphi\right] + 2\sin^2\theta\cos^2\theta + \cos^4\theta}_{=1}\right] + \cdots$$

$$= 2\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]^2 + 6\frac{\sigma^2}{r^2} + 4\frac{\sigma}{r}\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right] - \frac{2}{3}\left[\frac{\partial\sigma}{\partial r} + 2\frac{\sigma}{r}\right]^2$$

$$= \left(\frac{\partial\sigma}{\partial r}\right)^2\left[2 - \frac{2}{3}\right] + \frac{\partial\sigma}{\partial r}\cdot\frac{\sigma}{r}\left[-4 + 4 - \frac{8}{3}\right] + \frac{\sigma^2}{r^2}\left[2 + 6 - 4 - \frac{8}{3}\right]$$

$$= \left(\frac{\partial\sigma}{\partial r}\right)^2\frac{4}{3} - \frac{\partial\sigma}{\partial r}\cdot\frac{\sigma}{r}\frac{8}{3} + \frac{\sigma^2}{r^2}\frac{4}{3} = \frac{4}{3}\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]^2$$

Equation of continuity: $\rho\sigma\cdot r^2 = \text{const} = \rho_0\sigma_0 a^2 = b$

$$\sigma = \frac{b}{\rho r^2} \qquad \frac{\partial\sigma}{\partial r} = -\frac{b}{\rho^2 r^2}\frac{\partial\rho}{\partial r} - 2\frac{b}{\rho r^3}$$

$$\Phi = -\frac{4}{3}\mu\left[\frac{b}{\rho^2 r^2}\frac{\partial\rho}{\partial r} + \frac{2b}{\rho r^3} + \frac{b}{\rho r^3}\right]^2 = -\frac{4}{3}\left(\frac{b}{\rho^2 r^2}\right)^2\mu\left[\frac{\partial\rho}{\partial r} + 3\frac{\rho}{r}\right]^2$$

$$\frac{\partial\rho}{\partial r} + \frac{3\rho}{r} = \frac{1}{r^3}\frac{\partial}{\partial r}(\rho r^3)$$

Transformation of Equations of Motion ~~[illegible]~~ for spherical symmetry:

Continuity: $\rho\sigma r^2 = \text{const.}$

$$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{3\rho}\frac{\partial}{\partial x}\operatorname{div}\sigma + \frac{\mu}{\rho}\nabla^2 u$$

$$\sigma\left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^3\theta\cos^3\varphi + \sin^3\theta\cos\varphi\sin^2\varphi + \sin\theta\cos^2\theta\cos\varphi$$

$$+\frac{\sigma^2}{r}\sin\theta\cos\varphi = \sigma\left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\varphi + \frac{\sigma^2}{r}\sim\sim -$$

$$= \sigma\frac{\partial\sigma}{\partial r}\sin\theta\cos\varphi$$

$$\operatorname{div}\sigma = -\frac{\sigma}{\rho}\frac{\partial\rho}{\partial r} \qquad \frac{\partial\operatorname{div}\sigma}{\partial x} = \frac{\partial\operatorname{div}\sigma}{\partial r}\sin\theta\cos\varphi$$

$$= -\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial\rho}{\partial r}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = \sin\theta\cos\varphi\left[\cdot\left[\frac{\partial^2\sigma}{\partial r^2} - \frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{\sigma}{r^2}\right]\sin^2\theta\cos^2\varphi + \frac{\partial\sigma}{\partial r}\frac{1}{r} - \frac{\sigma}{r^2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = \sin\theta\sin\varphi\left[\frac{\partial^2\sigma}{\partial r^2} - \frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{\sigma}{r^2}\right]\sin\theta\sin\varphi\cos\varphi$$

$$\frac{\partial^2 u}{\partial z^2} = \cos\theta\left[\qquad\qquad\right]\sin\theta\cos\theta\cos\varphi$$

$$\nabla^2 u = \left[\frac{\partial^2\sigma}{\partial r^2} - \frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{\sigma}{r^2}\right]\overset{+\frac{1}{r}\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r^2}}{\sin\theta\cos\varphi} + \cancel{\frac{\sin\theta\cos\varphi}{r}2\sin\theta\cos\theta\cos^2\varphi\left[\frac{\partial\sigma}{\partial\theta} - \frac{\sigma}{r}\right]}$$

$$\cancel{+\frac{\sin\varphi}{r\sin\theta}2\cos^2\theta\sin\varphi\cos\varphi}\;\Big]$$

$$= \frac{\partial^2\sigma}{\partial r^2}\sin\theta\cos\varphi$$

$$+\frac{[\;]}{r}\cancel{\cos\theta\sin\varphi\,2\sin\theta\cos\theta\sin\varphi\cos\varphi + \frac{\cos\varphi}{\sin\theta}\sin^2\theta(\cos^2\varphi - \sin^2\varphi)} - \sin\theta(\cos^2\theta\cos\varphi - \sin^2\theta\cos\varphi)$$

Equation of Motion:

$$\sigma \frac{\partial \sigma}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\mu}{3\rho}\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right) + \frac{\mu}{\rho}\left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r}\right]$$

~~$+\left[\ \right]\frac{2\sin\theta\cos\theta\sin\varphi\cos\theta\cos\varphi}{r}$~~

$$\rho \sigma r^2 = c$$

$$\sigma = \frac{c}{\rho r^2}$$

$$\frac{\partial \sigma}{\partial r} = -\frac{2c}{\rho r^3} - \frac{c}{\rho r^2}\frac{\partial \rho}{\partial r}$$

$$\frac{\partial^2 \sigma}{\partial r^2} = +\frac{6c}{\rho r^4}$$

$$\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right) = \frac{\partial \sigma}{\partial r}\frac{1}{\rho}\frac{\partial \rho}{\partial r} - \frac{\sigma}{\rho^2}\left(\frac{\partial \rho}{\partial r}\right)^2 + \frac{\sigma}{\rho}\frac{\partial^2 \rho}{\partial r^2} \ldots$$

$$-\frac{2c^2}{\rho^2 r^5} - \frac{c^2}{\rho^2 r^4}\frac{\partial \rho}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\mu}{3\rho}\left(-\frac{2c}{\rho^2 r^3}\frac{\partial \rho}{\partial r} - \frac{c}{\rho^3 r^2}\left(\frac{\partial \rho}{\partial r}\right)^2 + \frac{c}{\rho^2 r^2}\frac{\partial^2 \rho}{\partial r^2}\right) + \frac{\mu}{\rho}\frac{6c}{\rho r^4} + \ldots$$

When $\mu = 0$: $p = \overbrace{\frac{p_0}{\rho_0^{\kappa}}}^{a}\rho^{\kappa}$ $\frac{\partial p}{\partial r} = \cancel{a}\,\kappa\rho^{\kappa-1}\frac{\partial \rho}{\partial r}$

~~$\frac{c^2}{\rho r^5} + \frac{c^2}{\rho r^4}\frac{d\rho}{dr} = a\kappa\rho^{\kappa-1}\frac{d\rho}{dr}$~~

$$\frac{1}{2}\frac{\partial(\sigma^2)}{\partial r} = -\overset{-\frac{1}{r^2}a^2}{\cancel{a}}\kappa\rho^{\kappa-2}\frac{\partial \rho}{\partial r} = -\frac{a\kappa}{\kappa-1}\frac{\partial}{\partial r}\left(\rho^{\kappa-1}\right)$$

$$\frac{1}{2}\frac{c^2}{\rho^2 r^4} = \frac{-a\kappa}{\kappa-1}\rho^{\kappa-1} + \text{const} \ \#$$

$$\frac{c^2}{2}\left(\frac{1}{\rho^2 r^4} - \frac{1}{\rho_0^2 a^4}\right) = \frac{-a\kappa}{\kappa-1}\left(\rho^{\kappa-1} - \rho_0^{\kappa-1}\right)$$

~~$\frac{c^2}{2}\left(\frac{1}{r^4} - \frac{1}{a^4}\right) = \frac{-a\kappa}{\kappa-1}\left(\rho^{\kappa+1} - \rho_0^{\kappa+1}\right)$~~

$\sin^3\theta\cos\varphi$

$\sin\theta\cos\varphi$

$\sin\theta\cos^2\theta\cos\varphi$

$$\varphi) = \left[\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r}\right]\frac{1}{r}\left[2\left(\sin\theta\cos^2\theta\cos^3\varphi + \sin\theta\cos^2\theta\sin^2\varphi\cos\varphi\right) + \cos\varphi\left(\sin^3\theta - \sin\theta\cos^2\theta\right) + \right.$$
$$\left. + \frac{1}{\sin\theta}\left(2\cos^2\theta\sin^2\varphi\cos\varphi + \sin^2\theta\left(\cos^3\varphi - \cos\varphi\sin^2\varphi\right)\right)\right] =$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\theta^2} + \frac{\cos\theta}{r^2\sin\theta}\frac{\partial u}{\partial\theta}$$

$$u = \sigma\cos\varphi\sin\theta$$

$$\nabla^2 u = \cos\varphi\sin\theta\left[\frac{\partial^2\sigma}{\partial r^2} + \frac{2}{r}\frac{\partial\sigma}{\partial r}\right] \underbrace{- \frac{\cos\varphi}{r^2\sin^2\theta}\cancel{\sin\theta}\,\sigma - \frac{1}{r^2}\cos\varphi\sin\theta + \frac{\cos^2\theta}{r^2\sin\theta}\cos\varphi}$$

$$\frac{\sigma}{r^2}\,\frac{\cos\varphi}{\sin\theta}\left[\cos^2\theta - 1 - \sin^2\theta\right] = -2\frac{\sigma}{r^2}\sin\theta\cos\varphi$$

$$\nabla^2 u = \cos\varphi\sin\theta\left[\frac{\partial^2\sigma}{\partial r^2} + \frac{2}{r}\frac{\partial\sigma}{\partial r} - \frac{2\sigma}{r^2}\right]$$

$$\frac{\partial^2\sigma}{\partial r^2} + 2\frac{\partial}{\partial r}\left(\frac{\sigma}{r}\right) = \frac{\partial}{\partial r}\underbrace{\left(\frac{\partial\sigma}{\partial r} + \frac{2\sigma}{r}\right)}_{= \operatorname{div}\sigma = -\frac{\sigma}{\rho}\frac{\partial\rho}{\partial r}}$$

Equation of Motion for spherical symmetry:

$$\sigma \frac{\partial \sigma}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\mu}{3\rho}\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right) + \frac{\mu}{\rho}\underbrace{\left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r} - \frac{2\sigma}{r^2}\right]}$$

$$\rho \sigma r^2 = \text{const} \longrightarrow = -\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right)$$

$$\sigma \frac{\partial \sigma}{\partial r} = R - \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{4\mu}{3\rho}\frac{\partial}{\partial r}\left(\frac{\sigma}{\rho}\frac{\partial \rho}{\partial r}\right)$$

$= 0$ only if $\rho = \frac{\text{const}}{r^3}$

$$\frac{\partial \theta}{\partial r} + \frac{\theta}{\rho}\frac{\partial \rho}{\partial r} = \frac{g a^2}{R r^2}$$

$$\frac{\partial \theta}{\partial r} = \frac{AR}{c}\frac{\theta}{\rho}\frac{\partial \rho}{\partial r} - \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial \rho}{\partial r} + \frac{3\rho}{r}\right]^2$$

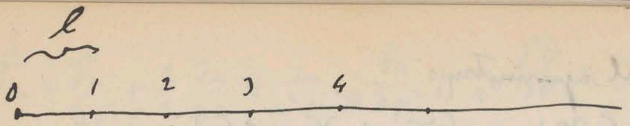
$$\frac{\theta}{\rho}\frac{\partial \rho}{\partial r}\underbrace{\left[1 + \frac{AR}{c}\right]}_{=C} = \frac{g a^2}{R r^2} + \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial \rho}{\partial r} + \frac{3\rho}{r}\right]^2$$

$$\theta = \frac{\rho}{C}\frac{\left[\;\cdots\;\right]}{\frac{\partial \rho}{\partial r}}$$

$$\frac{\partial \theta}{\partial r} = \frac{1}{C}\left[\qquad\right] + \frac{\rho}{C}\left[\qquad\right]\frac{\frac{\partial^2 \rho}{\partial r^2}}{\left(\frac{\partial \rho}{\partial r}\right)^2} + \frac{\rho}{C}\frac{\frac{\partial[\;]}{\partial r}}{\frac{\partial \rho}{\partial r}}$$

$$= \frac{g a^2}{R r^2} - \frac{1}{C}\left[\qquad\right]$$

$$\left[\frac{g a^2}{R r^2} + \frac{4}{3}\frac{A\mu}{c}\frac{b}{r^2\rho^4}\left[\frac{\partial \rho}{\partial r} + \frac{3\rho}{\partial r}\right]^2\right]\left[2 + \rho\frac{\frac{\partial^2 \rho}{\partial r^2}}{\left(\frac{\partial \rho}{\partial r}\right)^2}\right] + \frac{\rho}{\frac{\partial \rho}{\partial r}}\frac{\partial\left[\qquad\right]}{\partial r} - g\frac{a^2 C}{R r^2} = 0$$



A ~~ds~~ cord stretching out from 0 to ∞; equal masses 1, 2, – in equal distances; point 0 begins to move : σ = a shift ; how will the motion of agitation ~~repeat itself~~ spread out on the chord?

$\frac{d^2\sigma_0}{dt^2} = f(t)$ ~~the law of force : At~~ attracting forces between the said points

$$\frac{d^2\sigma_1}{dt^2} = -f(x_1 - x_0) + f(x_2 - x_1)$$

$$= -k(\sigma_0 - \sigma_1) + k(\sigma_1 - \sigma_2) = k(2\sigma_1 - \sigma_0 - \sigma_2)$$

$$\frac{d^2\sigma_2}{dt^2} = k(2\sigma_2 - \sigma_1 - \sigma_3) = -k(\sigma_1 - \sigma_2) + k(\sigma_2 - \sigma_3)$$

$$\frac{d^2\sigma_3}{dt^2} = k(2\sigma_3 - \sigma_2 - \sigma_4) = -k(\sigma_2 - \sigma_3) + k(\sigma_3 - \sigma_4)$$

. . . .

$$\sum \frac{d^2\sigma_n}{dt^2} = k(\sigma_1 - \sigma_0)$$

$$\frac{d^2}{dt^2}(\sigma_0 - \sigma_1) = f + k(\sigma_0 - \sigma_1) - k(\sigma_1 - \sigma_2)$$

$$\frac{d^2}{dt^2}(\sigma_1 - \sigma_2) = k(-\sigma_0 + 3\sigma_1 \cancel{+} - 3\sigma_2 + \sigma_3) = k\left[-(\sigma_0 - \sigma_1) + 2(\sigma_1 - \sigma_2) \cancel{+} - (\sigma_2 - \sigma_3)\right]$$

. . . . .

$$\frac{d^4}{dt^4}(\sigma_0 - \sigma_1) = \frac{d^2 f}{dt^2} + k\frac{d^2}{dt^2}(\sigma_0 - \sigma_1) + k(\sigma_0 - \sigma_1) + 2\frac{d^2}{dt^2}(\sigma_0 - \sigma_1) \pm 2f - 2k(\sigma_0 - \sigma_1) - k(\sigma_2 - \sigma_3)$$

$$2\frac{d^2\sigma_2}{dt^2} - \frac{d^2\sigma_1}{dt^2} - \frac{d^2\sigma_3}{dt^2} = k(4\sigma_2 - 2\sigma_1 - 2\sigma_3 - 2\sigma_1 + \sigma_0 + \sigma_2 - 2\sigma_3 + \sigma_2 + \sigma_4)$$

$$= k(\sigma_0 - 4\sigma_1 + 6\sigma_2 - 4\sigma_3 + \sigma_4) = \frac{1}{k}\frac{d^4\sigma_2}{dt^4}$$

$$\int \left(\frac{d\sigma_n}{dt}\right)^2 dt = ? \qquad = t\left(\frac{d\sigma_n}{dt}\right)^2 - 2\int t \frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}\,dt$$

$$= t\left(\frac{d\sigma_n}{dt}\right)^2 - 2t\sigma\frac{d^2\sigma}{dt^2} + 2\int \sigma \frac{\partial}{\partial t}\left(t\frac{d^2\sigma}{dt^2}\right) dt$$

$$\sigma_2\frac{d^2\sigma_2}{dt^2} = k(2\sigma_2^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3)$$

$$\sum \sigma_n \frac{d^2\sigma_n}{dt^2} = k\left(2\sum\sigma_n^2 - 2\sum\sigma_n\sigma_{n-1}\right)$$

$$\sigma_3\frac{d^2\sigma_3}{dt^2} = k(2\sigma_3^2 - \sigma_2\sigma_3 - \sigma_3\sigma_4)$$

$$\sigma_3\frac{d^2\sigma_2}{dt^2} = k(2\sigma_2\sigma_3 - \sigma_1\sigma_3 - \sigma_3^2)$$

$$\sigma_2\frac{d^2\sigma_3}{dt^2} = k(2\sigma_2\sigma_3 - \sigma_2^2 - \sigma_2\sigma_4)$$

$$\int \sigma_3\frac{d^2\sigma_2}{dt^2}\,dt = \sigma_3\frac{d\sigma_2}{dt}\Big| - \int\frac{d\sigma_2}{dt}\frac{d\sigma_3}{dt}\,dt = \sigma_3\frac{d\sigma_2}{dt}\Big| - \sigma_2\frac{d\sigma_3}{dt}\Big| + \int\sigma_2\frac{d^2\sigma_3}{dt^2}\,dt$$

$$\sum\int\left(\frac{d\sigma}{dt}\right)^2 dt$$

$$\frac{d^2\sigma_n}{dt^2} = k(2\sigma_n - \sigma_{n-1} - \sigma_{n+1})$$

$$\frac{d\sigma_n}{dt} = k\int(2\sigma_n - \sigma_{n-1} - \sigma_{n+1})\,dt = k\,t(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}) - k\int t\left(2\frac{d\sigma_n}{dt} - \dots\right)$$

$$= k\,t(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}) - k\frac{t^2}{2}\left(2\frac{d\sigma_n}{dt} - \frac{d\sigma_{n+1}}{dt} - \frac{d\sigma_{n-1}}{dt}\right) +$$

$$+ \frac{k}{2}\int t^2 \underbrace{\left(2\frac{d^2\sigma_n}{dt^2} - \dots\right)}_{k[\sigma_n - 4(\sigma_{n-1}+\sigma_{n+1}) + \sigma_{n-2} + \sigma_{n+2}]} dt$$

$$= k\,t(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}) \,\dots\dots + k$$

$$\frac{d\sigma_n}{dt}\frac{d^2\sigma_n}{dt^2} = k\left[2\sigma_n\frac{d\sigma_n}{dt} - \sigma_{n-1}\frac{d\sigma_n}{dt} - \sigma_{n+1}\frac{d\sigma_n}{dt}\right]$$

$$\frac{1}{2}\frac{d}{dt}\left(\frac{d\sigma_n}{dt}\right)^2 = k\left[\frac{d}{dt}\left(\sigma_n^2\right) - \frac{d\sigma_n}{dt}\left(\sigma_{n-1}+\sigma_{n+1}\right)\right]$$

$$\frac{1}{2}\left(\frac{d\sigma_n}{dt}\right)^2 = k\left[\sigma_n^2 - \sigma_n\left(\sigma_{n-1}+\sigma_{n+1}\right) + \int\sigma_n\left(\frac{d\sigma_{n-1}}{dt}+\frac{d\sigma_{n+1}}{dt}\right)dt\right]$$

$$\sum\frac{1}{2}\left(\frac{d\sigma_n}{dt}\right)^2 = k\sum\sigma_n^2 - k\int\underbrace{\sum\left\{\begin{matrix}\frac{d\sigma_{n-1}}{dt}\sigma_{n-2} + \frac{d\sigma_{n-1}}{dt}\sigma_n \\ \frac{d\sigma_n}{dt}\sigma_{n-1} + \frac{d\sigma_n}{dt}\sigma_{n+1} \\ \frac{d\sigma_{n+1}}{dt}\sigma_n + \frac{d\sigma_{n+1}}{dt}\sigma_{n+2} \\ \dots\end{matrix}\right\}dt}_{=\ \sum\sigma_n\sigma_{n-1}}$$

$$\sum\frac{1}{2}\left(\frac{d\sigma_n}{dt}\right)^2 = k\left[\sum\sigma_n^2 - \sum\sigma_n\sigma_{n-1}\right] = L$$

$$\sum\sigma_n\frac{d^2\sigma_n}{dt^2} = 2k\left[\sum\sigma_n^2 - \sum\sigma_n\sigma_{n-1}\right]$$

$$\sum\left(\frac{d\sigma_n}{dt}\right)^2 - \sigma_n\frac{d^2\sigma_n}{dt^2} = 0$$

$$\cancel{\frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}} - \cancel{\frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}} - \sum\sigma\frac{d^3\sigma}{dt^3} = 0$$

$$\frac{\partial L}{\partial t} = \sum\frac{d\sigma}{dt}\frac{d^2\sigma}{dt^2}$$

$$= k\left[\sum 2\sigma_n\frac{d\sigma_n}{dt} - \dots\right]$$

$$\frac{\partial L}{\partial x} = 2k\left[\sum\sigma_{n+1}^2 - \sum\sigma_{n+1}\sigma_n - \sum\sigma_n^2 + \sum\sigma_n\sigma_{n-1}\right]$$

$$= 2k\left(\sigma_n^2 - \sigma_n\sigma_{n+1}\right)$$

$$\frac{\partial^2 L}{\partial t^2} = k \sum \underbrace{2\left(\frac{d\sigma_n}{dt}\right)^2 + 2\sigma_n\left(\frac{d^2\sigma_n}{dt^2}\right)}_{} - \frac{d\sigma_{n-1}}{dt}\frac{d\sigma_n}{dt} - \sigma_{n-1}\frac{d^2\sigma_n}{dt^2}$$

$$- \frac{d\sigma_{n+1}}{dt}\frac{d\sigma_n}{dt} - \sigma_{n+1}\frac{d^2\sigma_n}{dt^2}$$

$$= 8kL - \sum \frac{d\sigma_n}{dt}\left(\frac{d\sigma_{n-1}}{dt} + \frac{d\sigma_{n+1}}{dt}\right) + \sigma_{n-1}(2\sigma_{n-2} - \sigma_{n-3} - \sigma_{n-1})$$

$$+ \sigma_{n-2}(2\sigma_{n-1} - \sigma_{n-2} - \sigma_n) + \sigma_n(2\sigma_{n-1} - \sigma_{n-2} - \sigma_n)$$

$$- \sum \sigma_{n-1}(2\sigma_n - \sigma_{n-1} - \sigma_{n+1}) + \sigma_{n+1}(2\sigma_n - \sigma_{n-1} - \sigma_{n+1})$$

$$+ \sigma_n(2\sigma_{n+1} - \sigma_n - \sigma_{n+2}) + \sigma_{n+2}(2\sigma_{n+1} - \sigma_n - \sigma_{n+2})$$

$$+ \sigma_{n+1}(2\sigma_{n+2} - \sigma_{n+1} - \sigma_{n+3}) + \sigma_{n+3}(2\sigma_{n+2} - \sigma_{n+1} - \sigma_{n+3})$$

$$+ \sigma_{n+2}(2\sigma_{n+3} - \sigma_{n+2} - \sigma_{n+4}) + \sigma_{n+4}(- \quad - \quad)$$

$$= 2 \sum \sigma_n\sigma_{n+1} - \sigma_n^2 - \sigma_n\sigma_{n+2} + 2\sigma_n\sigma_{n+1} - \sigma_n\sigma_{n+2} - \sigma_n^2$$

$$\sigma_1 = f_1(t) = f_1(0) + t\,f_1'(0) + \frac{t^2}{2} f_1''(0) + \cdots$$

$$= \cancel{\sigma_1(0)} + \cancel{t\,\frac{d\sigma_1(0)}{dt}} + \frac{t^2}{2}\frac{d^2\sigma_1(0)}{dt^2} + \frac{t^3}{3!}\frac{d^3\sigma_1(0)}{dt^3} + \frac{t^4}{4!}\frac{d^4\sigma_1(0)}{dt^4} + \sim$$

$$\frac{d^2\sigma_1}{dt^2}(0) = -k\,\sigma_0(0)$$

$$\frac{d^3\sigma_1(0)}{dt^3} = -k\left(\cancel{2\,\frac{d\sigma_1}{dt}(0)} - \frac{d\sigma_0(0)}{dt} - \cancel{\frac{d\sigma_2(0)}{dt}}\right) = -k\,\frac{d\sigma_0(0)}{dt}$$

$$\frac{d^4\sigma_1(0)}{dt^2} = -k\left(2\,\frac{d^2\sigma_1(0)}{dt^2} - \frac{d^2\sigma_0(0)}{dt^2} - \frac{d^2\sigma_2(0)}{dt^2}\right) =$$

$$= +2k^2\sigma_0(0) + k\,\frac{d^2\sigma_0(0)}{dt^2}$$

$$d^5 \quad = 2k^2\frac{d\sigma_0}{dt} + k\,\frac{d^3\sigma_0}{dt^3}$$

$$d^6 \quad = k\left[2\,\frac{d^4\sigma_1}{dt^2} - \frac{d^4\sigma_0}{dt^2} - \frac{d^4\sigma_2}{dt^2}\right] = 4k^3\sigma_0 + 2k^2\frac{d^2\sigma_0}{dt^2} - k\frac{d^4\sigma_0}{dt^2} - \cancel{k^3\sigma_0}$$

$$\boxed{\frac{d^2\sigma_2(0)}{dt^2} = 0 = \frac{d^2\sigma_3}{dt^2}(0) \quad \text{etc.}}$$

$$= 3k^3\sigma_0 + 2k^2\frac{d^2\sigma_0}{dt^2} - k\,\frac{d^4\sigma_0}{dt^4}$$

$$d^7 = k\left[2\,d^5\sigma_1 - d^5\sigma_0 - d^5\sigma_2\right] = 4k^3\frac{d\sigma_0}{dt} + 2k^2\frac{d^3\sigma_0}{dt^3} - k\,\frac{d^5\sigma_0}{dt^5} - k^3\frac{d\sigma_0}{dt}$$

$$= 3k^3\frac{d\sigma_0}{dt} + 2k^2\frac{d^3\sigma_0}{dt^3} - k\,\frac{d^5\sigma_0}{dt^5}$$

$$d^8 = k\left[2\,d^6\sigma_1 - \sigma_0^6 - \sigma_2^6\right] = 6k^4\sigma_0 + 4k^3\frac{d^2\sigma_0}{dt^2} - 2k^2\frac{d^4\sigma_0}{dt^4} + k^3\frac{d^2\sigma_0}{dt^2} - k\,\frac{d^6\sigma_0}{dt^6}$$

$$= 6k^4\sigma_0 + 5k^3\frac{d^2\sigma_0}{dt^2} - 2k^2\frac{d^4\sigma_0}{dt^4} - k\,\frac{d^6\sigma_0}{dt^6}$$

$$\frac{d^2\sigma_2}{dl^2}(0)=0$$

$$\frac{d^3\sigma_2}{dl^3}(0)=k\cancel{\left(2\frac{d\sigma_2}{dl}-\frac{d\sigma_1}{dl}-\frac{d\sigma_3}{dl}\right)}=0$$

$$\frac{d^4\sigma_2}{dl^4}(0)=k\left(\cancel{2\frac{d^2\sigma_2}{dl^2}}-\frac{d^2\sigma_1}{dl^2}-\cancel{\frac{d^2\sigma_3}{dl^2}}\right)=+k^2\sigma_0(0)$$

$$\frac{d^5\sigma_2}{dl^5}(0)=k\left(\cancel{2\frac{d^3\sigma_2}{dl^3}}-\frac{d^3\sigma_1}{dl^3}-\cancel{\frac{d^3\sigma_3}{dl^3}}\right)=+k^2\frac{d\sigma_0}{dl}(0)$$

$$\frac{d^6\sigma_2}{dl^6}(0)=k\left(2\frac{d^4\sigma_2}{dl^4}-\frac{d^4\sigma_1}{dl^4}-\cancel{\frac{d^4\sigma_3}{dl^4}}\right)=\cancel{2k^3\sigma_0(0)}-\cancel{2k^3\sigma_0(0)}-k^2\frac{d^2\sigma_0}{dl^2}\ *$$

$$d7=k\left(2\,d^5\sigma_2-\sigma_1-\sigma_3\right)=\cancel{2k^3\frac{d\sigma_0}{dl}}-\cancel{2k^3\frac{d\sigma_0}{dl}}-k^2\frac{d^3\sigma_0}{dl^3}$$

$\underline{\sigma_3}$ $\qquad \frac{d^5\sigma_3}{dt^5}=0$

$$d^6 = k\, d^4(2\cancel{\sigma_3}-\sigma_2-\cancel{\sigma_4}) = -k^3\sigma_0$$

$$d7 = k\, d^5(\cancel{2\sigma_3}-\sigma_2-\cancel{\sigma_4}) = -k^4\frac{d\sigma_0}{dt}$$

$$d^8 = k\, d^6(2\sigma_3-\sigma_2-\sigma_4) = -2k^4\sigma_0$$

Zwei Lösungen mit verschiedenem μ:

$$(\sigma_1\nabla)\sigma_1 = F - \frac{1}{\rho}\nabla p_1 + \cancel{\frac{\mu_1}{3\rho}\nabla \operatorname{div}\sigma_1} + \frac{\mu_1}{\rho}\nabla^2\sigma_1 \qquad = \frac{1}{2}\nabla\sigma_1^2 - \nabla\sigma_1 \operatorname{curl}\sigma_1$$

$$(\sigma_2\nabla)\sigma_2 = F - \frac{1}{\rho}\nabla p_2 + \cancel{\frac{\mu_2}{3\rho}\nabla \operatorname{div}\sigma_2} + \frac{\mu_2}{\rho}\nabla^2\sigma_2$$

$$\operatorname{curl}\nabla\sigma_1 \operatorname{curl}\sigma_1 = \frac{\mu_1}{\rho}\operatorname{curl}\nabla^2\sigma_1$$

$$\operatorname{curl}\nabla\sigma_2 \operatorname{curl}\sigma_2 = \frac{\mu_2}{\rho}\operatorname{curl}\nabla^2\sigma_2$$

$$\operatorname{curl}\nabla(\sigma_1 \operatorname{curl}\sigma_1 - \sigma_2 \operatorname{curl}\sigma_2) = \frac{1}{\rho}\operatorname{curl}\nabla^2(\mu_1\sigma_1 - \mu_2\sigma_2)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = X \qquad \frac{\partial p}{\partial x} = \frac{dp}{dr}\frac{\partial r}{\partial x} = \frac{dp}{dr}\cos\alpha$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = Y \qquad \frac{1}{\rho}\frac{dp}{dr} = X\cos\alpha + Y\cos\beta + Z\cos\gamma = R$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = Z$$

$\frac{\partial}{\partial r}(r^2 p) = -\cancel{r^2} g \frac{a^2}{\cancel{r^2}} \rho$ $\qquad \cancel{p} = c\rho^k$

$\frac{\partial p}{\partial r} + 2\frac{p}{r} = -g\frac{a^2}{r^2}\rho$

$\cancel{\frac{\partial \log p}{\partial r} + \frac{2}{r} = -\frac{g a^2}{r^2}\frac{\rho}{p}}$

$k\rho^{k-1}\frac{\partial\rho}{\partial r} + 2\frac{\rho^k}{r} = -g\frac{a^2\rho}{c\, r^2}$

$\frac{d}{dr}(r^2\rho^k) = -\frac{g a^2}{c}\rho$

$r^{\frac{2}{k}}\, d(r^2\rho^k) = -\frac{g a^2}{c}\left(\rho\, r^{\frac{2}{k}}\right) dr$

$\frac{d(\rho^k r^2)}{\rho\, r^{\frac{2}{k}}} = -\frac{g a^2}{c}\, r^{-\frac{2}{k}}\, dr$ $\qquad \rho\, r^{\frac{2}{k}} = z$

$\rho^k r^2 = z^k$

$k\frac{z^{k-1}\, dz}{z} = -g\frac{a^2}{c}\, r^{-\frac{2}{k}} dr$

$k z^{k-2} dz =$

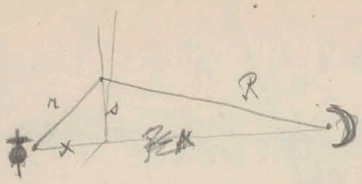
$\cancel{k}\frac{z^{k-1}}{k-1} = -\frac{g a^2}{c}\frac{r^{-\frac{2}{k}+1}}{k-\frac{2}{\cancel{k}}}$

$\rho^{k-1} r^{\frac{2(k-1)}{k}} = -\frac{k-1}{k-2}\frac{g a^2}{c} r^{1-\frac{2}{k}} + \overset{A}{\text{Const}} = \rho^{k-1} r^{2-\frac{2}{k}}$

$\rho^{k-1} = A\, r^{\frac{2(1-k)}{k}} - \frac{k-1}{k-2}\frac{g a^2}{c}\frac{1}{r}$

$\cancel{\rho_0^{k-1} = A\, a^{2\frac{1-k}{k}} - \frac{k-1}{k-2}\frac{g a}{c}}$

$\rho_0^{k-1} a^{2\frac{k-1}{k}} = -\frac{k-1}{k-2}\frac{g}{c}\, a^{3-\frac{2}{k}} + A$



$$\frac{1}{\rho}\frac{\partial p}{\partial x} = X$$
$$\frac{1}{\rho}\frac{\partial p}{\partial y} = Y$$
$$\frac{1}{\rho}\frac{\partial p}{\partial z} = Z$$

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial s}\cos\mu$$

$$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial s}\cos\nu$$

$$\left.\begin{aligned}\frac{1}{\rho}\frac{\partial p}{\partial x} &= X\\ \frac{1}{\rho}\frac{\partial p}{\partial s} &= S\end{aligned}\right\}$$ For rotational symmetry

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{a^2}{r^2}\frac{x}{r} + \frac{GA^2}{R^2}\frac{\xi - x}{R} \qquad = -\frac{\partial U}{\partial x} = +\frac{\partial P}{\partial x}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial s} = -g\frac{a^2}{r^2}\frac{s}{r} \cancel{+} - \frac{GA^2}{R^2}\frac{s}{R} \qquad = -\frac{\partial U}{\partial s} = +\frac{\partial P}{\partial s}$$

$$P + U = \text{const} \qquad P = \int\frac{1}{\rho}\nabla p = ck\int\rho^{k-2}d\rho = \frac{ck}{k-1}\rho^{k-1}$$

$$\boxed{\frac{ck}{k-1}\rho^{k-1} \cancel{+} = g\frac{a^2}{r} + \frac{GA^2}{R} + \text{const}}$$

$\rho_0 = a$

$$\frac{ck}{k-1}\left(\rho_0^{k-1} - \rho^{k-1}\right) = ga - g\frac{a^2}{r} - \frac{GA^2}{R}$$

Für ☾ Oberfläche: $\frac{ck}{k-1}\left(\rho_0^{k-1} - \rho^{k-1}\right) = ga - GA$

$$g : G = \frac{m}{a^2} : \frac{M}{A^2} \qquad G = g\frac{M}{m}\frac{a^2}{A^2} \quad \Big\| \quad = ga\left[1 - \frac{Ma}{mA}\right]$$

$$\rho_0^{k-1}\frac{ck}{k-1} = ga - \frac{GA^2}{R}$$

$$\frac{ck}{k-1}\,\alpha\,(\theta_0 - \theta) = -g\frac{a^2}{r} + \text{const} \qquad 300\,\frac{A}{R}$$

$$-\frac{d\theta}{dx} = \frac{g}{\alpha}\,\frac{k-1}{k}\,\frac{a^2}{r^2} \qquad R \quad 300.000\,A$$

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Wenn man dagegen Bedingg. $\rho = 0$ für $r = R = \infty$ aufstellt kommt:

const = 0

und $c \frac{k}{k-1} \rho_{\sigma}^{k-1} = ga$

$c \frac{k}{k-1} \rho_{☾}^{k-1} = GA$

$$\left(\frac{\rho_{☾}}{\rho_{\sigma}} \right)^{k-1} = \frac{GA}{ga} = \frac{Ma}{mA}$$

$c \frac{\rho^k}{\rho} = C\theta$

$\rho^{k-1} = C\theta$

$\frac{d\theta}{dx} = ~~(k-1)\rho^{k-2}\frac{d\rho}{dx}~~$

$= ~~\theta^{2-k}\frac{d\rho}{dx}~~$

$p = c\rho^k$

$pv = \text{konst.}\, ~~T~~\theta = C\theta$

$p = C ~~\theta~~ \rho = c\rho^k$

$\theta = \frac{c}{C}\rho^{k-1}$

$~~\theta~~ = \theta^{\frac{1}{k-1}}$

$\sigma = \frac{1}{\theta^{1-k}}$

$\frac{d\sigma}{dx} = \frac{1}{1-k}\,\theta^{\frac{1}{1-k}-1}$

$= \frac{1}{1-k}\,\theta^{\frac{k}{1-k}}$

$$\frac{\theta_{☾}}{\theta_{\sigma}} = \frac{Ma}{mA}$$

$\frac{M}{m} = \frac{1}{79{,}7}$ | $\frac{A}{a} = 0{,}273$

$= \frac{1}{21{,}86}$ $\theta_{☾} = ~~13~~° - 260°$

0,273
7,97
1911
24,6
1,9
21,86

$$\left(\frac{ga}{c}\right)\frac{k-1}{k} = \frac{9{,}8 \cdot 6366000}{k-1}\,\frac{k-1}{k} = c\,\rho_{\sigma}^{k-1} = \frac{p_0}{\rho_0^k}\left(\rho_{\sigma}\right)^{k-1} = \frac{p_0}{\rho_0}\left(\frac{\rho_{\sigma}}{\rho_0}\right)^{k-1}$$

$\frac{1}{\rho_0}$

$\theta = \sigma^{1-k}$

$\frac{d\theta}{dx} = (1-k)\sigma^{-k}$

$$\left(\frac{\rho_{\sigma}}{\rho_0}\right)^{k-1} = 9{,}8 \cdot 6366~~000~~ \frac{k-1}{k}\,\frac{~~0{,}0012~~}{~~9{,}8 \cdot 13{,}6 \cdot 0{,}76~~} = \frac{637}{91} \cdot 1{,}2 \cdot \frac{~~0{,}4~~}{~~1{,}4~~} = 218\,!$$

Dichte der Atm müsste 218 mal so ~~[illegible]~~ groß sein als in Wirkl. damit im ∞ : $\rho = 0$!

richtig!

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$$\frac{1}{\rho} = \frac{1}{\rho_0}\left(\frac{p_0}{p}\right)^{\frac{1}{k}}$$

$$\frac{1}{\frac{1}{k}+1}\,\frac{p_0^{\frac{1}{k}}}{\rho_0}\left(\frac{1}{p}\right)^{\frac{1}{k}+1} \qquad \frac{k}{k+1}\,\frac{p_0^{\frac{1}{k}}}{\rho_0}\,p^{-\frac{1+k}{k}} = \frac{k}{k+1}\left(\frac{p_0}{\rho}\right)^{k}$$

$$\frac{\theta_{\odot}}{\theta_{\sigma}} = \frac{Ma}{mA} = \frac{319{,}500}{108} = 300$$

$R = \frac{c\rho^{k-1}}{273}$

$\rho^{k-1} = \frac{k-1}{kc}\,ga \cdot 300 = \frac{k-1}{k}\,880 \cdot 300$

$\theta_{\odot} = 6000$

$R\theta = \frac{k-1}{k}\,GA\left(1 - \frac{A}{r}\right)$

Dagegen wenn Voraussetzg. [illegible] ist R anders für A.

$\frac{6000}{273} = \frac{k-1}{ck} \cdot \frac{GA}{r}(r-A)$

Dagegen bei ~~[illegible]~~ isothermem Gleichgewicht:

$p = c\rho$

$$\frac{c}{\rho}\frac{d\rho}{dr} = \cancel{+}\, g\frac{a^2}{r^2}$$

$$c \log \rho = g\frac{a^2}{r} + \text{const}$$

$$\underline{c \log \rho_0 = ga + \text{const}}$$

$$c \log \frac{\rho_0}{\rho} = ga - g\frac{a^2}{r}$$

Ergibt Endlichkeit für

$r = \infty$:

$$c \log \frac{\rho_0}{\rho_\infty} = ga$$

$$\frac{\rho_0}{\rho_\infty} = e^{\frac{ga}{c}}$$

$$\rho_\infty = \rho_0\, e^{-\frac{ga}{c}}$$

$$c = \frac{p_0}{\rho_0}$$

$$\frac{ga}{c} = \frac{\cancel{9.8}\ 6366000 \cdot \cancel{0.0013}}{\cancel{9.8}\ \cancel{13.6 \cdot 0.76}} = \text{cca } 880$$

Somit ganz verschwindend kleine Dichte

Diese Lösung verträgt sich mit beliebigem ρ_0 wenn θ nicht viel grösser ist

Dagegen wenn Bedingung: $r=\infty$, $\rho = 0$: unmöglich

bei zwei Körpern ♁ und ☾

$$c \log \rho = g\frac{a^2}{r} + G\frac{A^2}{R} + \text{const}$$

$$\underline{c \log \rho_0 = ga \qquad + \text{const}}$$

$$c \log \frac{\rho_0}{\rho_{♁}} = ga - g\frac{a^2}{r} - G\frac{A^2}{R}$$

$$c \log \frac{\rho_0}{\rho_a} = ga - GA$$

[illegible]

$$\rho_a = \rho_{♁}\, e^{-\frac{ga - GA}{c}}$$

Dürfte auch sehr klein sein

No work is done by internal friction — when gas flows out from a source uniformly in all directions (spherical symmetry) — only if every volume element keeps its shape (but ~~except~~ changing size)

Therefore the only possible motion without dissip. of energy:

$$v : V = r^3 : R^3$$

$$\rho = \frac{\text{const}}{R^3} \qquad \rho_0 = \frac{\text{const}}{a^3} \quad // \quad \rho = \rho_0 \frac{a^3}{r^3}$$

If gravitation acting towards the centre: $\frac{1}{\rho}\frac{dp}{dr} = -g\frac{a^2}{r^2}$

$$\frac{dp}{dr} = -g\frac{a^2}{r^2}\rho_0\frac{a^3}{r^3} = -g\frac{a^5\rho_0}{r^5}$$

$$\left.\begin{aligned} p &= g\frac{a^5\rho_0}{4r^4} + \text{const} \\ p_0 &= g\frac{a\rho_0}{4} + \text{const} \end{aligned}\right\} \quad p - p_0 = g\frac{a\rho_0}{4}\left(\frac{a^4}{r^4} - 1\right)$$

$$p_0 = 98.$$

$$\frac{p}{\rho} = c\theta = \frac{ga^2}{4r} + \frac{\overbrace{p_0 - g\frac{a\rho_0}{4}}^{<1}}{\rho_0\frac{a^3}{r^3}} \qquad g\frac{a\rho_0}{4} = \frac{6366\cancel{000} \cdot 98 \cdot \cancel{0{\cdot}0013}}{4}$$

Przyjmując że prąd powietrza wznosi się od powierzchni ziemi z prędkością u_0 do góry; jakie w nim będzie rozmieszczenie prędkości, ciśnienia, gęstości i temperatury?

I. Uwzględniając tylko pracę wykonaną przez rozszerzanie się gazu: będzie „convective equilibrium"

$$p = c\rho^k \qquad \frac{ck}{k-1}\left(\rho_0^{k-1} - \rho^{k-1}\right) = ga - \frac{ga^2}{r}$$

II. Uwzględniając oprócz tego jeszcze przewodzenie ciepła przez gaz:

$$\cancel{\frac{1}{\rho}\frac{dp}{dr} = -\frac{ga^2}{r^2}}$$

$$dQ = \left[\frac{c\,d\theta}{dr} + Ap\frac{d\left(\frac{1}{\rho}\right)}{dr}\right] = \cancel{\mu}\ \kappa$$

$$p = \overbrace{\frac{p_0}{\rho_0\theta_0}}^{\alpha}\rho\theta$$

$$K = \gamma\theta$$

$$\left\{\begin{array}{ll}
1) & \dfrac{p}{p_0} = \dfrac{\rho\,\theta}{\rho_0\theta_0} \\[2ex]
2) & \sigma\dfrac{d\sigma}{dr} = -\dfrac{ga^2}{r^2} - \dfrac{1}{\rho}\dfrac{dp}{dr} - \dfrac{4\mu}{3\rho}\dfrac{d}{dr}\left(\dfrac{\sigma}{\rho}\dfrac{d\rho}{dr}\right) \\[2ex]
3) & \rho\sigma r^2 = \text{const} = \rho_0\sigma_0a^2 = b \\[2ex]
4) & \rho\sigma r^2\left[c\dfrac{d\theta}{dr} + A\,p\,\dfrac{d\left(\frac{1}{\rho}\right)}{dr}\right] + \dfrac{d}{dr}\left(\kappa r^2\dfrac{d\theta}{dr}\right) = 0
\end{array}\right.$$

$$\cancel{\frac{d\rho}{dr}\sigma r^2 + \frac{d\sigma}{dr}\rho r^2 + 2r\rho\sigma = 0}$$

$$\cancel{\frac{\sigma}{\rho}\frac{d\rho}{dr} + \frac{d\sigma}{dr} + 2\frac{\sigma}{r} = 0}$$

$$2).\quad \cancel{\rho_0\sigma_0a^2}\ \frac{d\sigma}{dr} = -ga^2\rho - r^2\frac{dp}{dr} - \frac{4\mu}{3}r^2\frac{d}{dr}\left(\frac{\sigma}{\rho}\frac{d\rho}{dr}\right)$$

$$\sigma = \frac{\rho_0\sigma_0a^2}{\rho r^2} = \frac{b}{\rho r^2}$$

$$\frac{d\sigma}{dr} = -\frac{b}{\rho^2r^2}\frac{d\rho}{dr} - \frac{2b}{\rho r^3}$$

$$\underbrace{\frac{\sigma}{\rho}\frac{d\rho}{dr} = \frac{b}{\rho^2r^2}\frac{d\rho}{dr}}_{\cancel{\frac{d\rho}{dr}} =}$$

$$2,3\} = -\frac{b^2}{\rho^3r^2}\frac{d\rho}{dr} - \frac{2b^2}{\rho r^3} = -ga^2\rho - r^2\frac{dp}{dr} - \frac{4\mu}{3}r^2\frac{d}{dr}\left[\frac{b}{\rho^2r^2}\frac{d\rho}{dr}\right]$$

$$\alpha = \frac{9.8\,.\ \cancel{13.6\,.\,0.76}}{\cancel{0.00}13\,.\,0.280} = \frac{980}{3.6} = 270$$

$$b\,c\,\frac{d\theta}{dr} - b\,A\,\alpha\,\cancel{\rho}\,\frac{\theta}{\rho^2}\frac{d\rho}{dr} + \frac{d}{dr}\left(\mu\,\theta\,\frac{d\theta}{dr}\,r^2\right) = 0$$

$$\left.\frac{b^2}{\rho^2 r^2}\frac{d\rho}{dr} + \frac{2b^2}{\rho r^3}\right] = g\,a^2\rho + r^2\alpha\,\frac{d(\rho\theta)}{dr}$$

Jeśli się to zaniedba

pominąć $\sigma\frac{d\rho}{dr}$ bardzo małe

$$= 0$$

$$g\,a^2\rho + r^2\alpha\,\rho\,\frac{d\theta}{dr} + r^2\alpha\,\theta\,\frac{d\rho}{dr} = 0$$

$$\frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{r^2\alpha\theta}\left[g\,a^2 + r^2\alpha\,\frac{d\theta}{dr}\right]$$

$$g\,a^2 + r^2\alpha\,\frac{d\theta}{dr} + r^2\alpha\,\theta\,\frac{1}{\rho}\frac{d\rho}{dr} = 0 \quad \Big|\, \theta$$

$$\frac{d\theta}{dr}\,g\,a^2 + \frac{d}{dr}\left(r^2\alpha\,\frac{d\theta}{dr}\,\theta\right) + \alpha\,\frac{d}{dr}\left(\frac{\theta^2 r^2}{\rho}\frac{d\rho}{dr}\right) = 0$$

$$r^2\frac{d}{dr}(\;\;) = -\frac{d}{d\left(\frac{1}{r}\right)} \qquad \frac{1}{r} = \xi$$

2

$$b\,c\,\frac{d\theta}{d\xi} - b\,A\,\alpha\,\frac{\theta}{\rho^2}\frac{d\rho}{d\xi} + \frac{\mu}{2}\frac{d}{d\xi}\left(\frac{d(\theta^2)}{d\xi}\right) = 0$$

Czy przewodzenie ciepła może spowodować znaczną różnicę? $\quad c = \frac{p_0}{\rho_0^{\,k}}$

Bez niego byłoby: ~~$\frac{ck}{k-1}\left(\rho_0^{\,k-1} - \rho^{\,k-1}\right)$~~ $= g\,a\left(1 - \frac{a}{r}\right) =$

$$\alpha\,(\theta_0 - \theta) = \frac{k-1}{k}\,g\,a\left(1 - \frac{a}{r}\right)$$

$$\theta = \theta_0 - \frac{k-1}{k}\,\frac{g\,a}{\alpha}\left(1 - \frac{a}{r}\right)$$

$$\mu = \frac{0{,}00006}{280}$$

$$r^2\frac{d\theta}{dr} = -\frac{k-1}{k}\,\frac{g\,a^2}{\alpha\,\cancel{k}} = B = \frac{9{,}8\,.\,0{,}4}{1{,}4\,.\,270}\,.\,[6366000]^2$$

$$\cancel{b}c\,B - \qquad + \mu\,B^2 = 0 \qquad \frac{\mu B}{b\,c} \qquad \frac{\cancel{9{,}8\,.\,0{,}4\,.\,0{,}00006}\;2{,}4}{14\,.\,270\,.\,\cancel{0{,}0013}\,.\,6\,.\,280\,.\,\sigma_0}$$

Dopiero jeśli σ_0 wielkości $\frac{1}{5000}$ cm !

Bez przewodzenia ciepła i zaniedbując bezwładności ale uwzględniając tarcie

1). $\rho\sigma r^2 = \text{const} = \rho_0\sigma_0 a^2 = b$

2). $p = \overbrace{\frac{p_0}{\rho_0\theta_0}}^{\alpha}\rho\theta$

$$\underbrace{= \frac{b}{\rho^2 r^2}\frac{d\rho}{dr}}$$

$\alpha = \frac{0'76 \cdot 98 \cdot 13'6}{0'0013 \cdot 280}$

$\quad\quad 8 \quad 4$

$= \frac{98}{0'36} = 270$

3). $0 = -\frac{ga^2}{r^2} - \frac{1}{\rho}\frac{dp}{dr} - \frac{4\mu}{3\rho}\frac{d}{dr}\left(\frac{\sigma}{\rho}\frac{d\rho}{dr}\right)$

4). $\rho\sigma r^2\left[c\frac{d\theta}{dr} + Ap\frac{d\left(\frac{1}{\rho}\right)}{dr}\right] = \frac{4}{3}\left(\frac{b}{\rho^2 r^2}\right)^2\mu\left[\frac{\partial\rho}{\partial r} + \frac{3\rho}{r}\right]^2$

3). $\frac{ga^2}{r^2} + \alpha\frac{1}{\rho}\frac{d}{dr}[\rho\theta] - \frac{4\mu}{3\rho}\frac{d}{dr}\left[\frac{b}{\rho^2 r^2}\frac{d\rho}{dr}\right] = 0$

4). $bc\frac{d\theta}{dr} - Ab\alpha\frac{\theta}{\rho}\frac{d\rho}{dr} = \frac{4\mu}{3}\left(\frac{b}{\rho^2 r^2}\right)^2\left(\frac{d\rho}{dr} + \frac{3\rho}{r}\right)^2$

$$\frac{1}{\rho^2 r^2}\left(\frac{d\rho}{dr} + \frac{3\rho}{r}\right) = \frac{\frac{d}{dr}(\rho r^3)}{\rho r^5} = \left|\frac{1}{\rho r^2}\frac{d}{dr}\left|\log(\rho r^3)\right|\right| = -r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)$$

$\mu = \gamma\theta$

3). $\frac{ga^2}{r^2} + \frac{\alpha}{\rho}\frac{d}{dr}(\rho\theta) - \frac{4\gamma\theta}{3\rho}\frac{d}{dr}\left[\frac{b}{\rho^2 r^2}\frac{d\rho}{dr}\right] = 0$

4). $c\frac{d\theta}{dr} - A\alpha\frac{\theta}{\rho}\frac{d\rho}{dr} - \frac{4\gamma}{3}\frac{b\theta}{\rho^4 r^4}\left[\frac{d\rho}{dr} + \frac{3\rho}{r}\right]^2 = 0$

4). : $c\frac{d\theta}{dr}\frac{1}{\theta} = A\alpha\frac{d\rho}{dr}\frac{1}{\rho} + \underset{\uparrow}{\frac{4\gamma b}{3\rho^4 r^4}}\left[\frac{d\rho}{dr} + \frac{3\rho}{r}\right]^2$

$c\log\theta + \text{const} = \int \cdots\cdots \uparrow \, dr$

$$c\log\theta + \text{const} = A\alpha\log\rho + \frac{4\gamma\mu b}{3}\int\left[\frac{1}{\rho^2 r^2}\left(\frac{d\rho}{dr}+\frac{3\rho}{r}\right)\right]^2 dr$$

Zaniedbujemy wpływ tarcia w równaniu (3):

$$\frac{g a^2}{r^2} + \frac{\alpha}{\rho}\frac{d}{dr}(\rho\theta) = 0 = \frac{g a^2}{r^2} + \alpha\frac{d\theta}{dr} + \alpha\frac{\theta}{\rho}\frac{d\rho}{dr}$$

$$\text{const}\times\theta^c = \rho^{\alpha A}\cdot e^{\frac{4\gamma\mu b}{3}\int\cdots} \quad\Big\|\quad \cancel{\left(\frac{\theta}{\theta_0}\right)^c = \left(\frac{\rho}{\rho_0}\right)^{\alpha A}}$$

$$\cancel{\theta = \frac{p}{\alpha\rho}}$$

$$\cancel{\text{const}\; p^c\rho^{-c} = \rho^{\alpha A}} \qquad \cancel{\text{const}\times p^c = \rho^{\alpha A + c}} \qquad \cancel{\text{const}\; p = \rho^{\overbrace{1+\frac{\alpha A}{c}}^{=1'5}}}$$

$$\frac{\alpha A}{c} = 0'5$$

$$\frac{\alpha A}{c} - 1 = -\frac{1}{2}$$

$$\text{const}\;\theta = \rho^{\frac{\alpha A}{c}}\, e^{\frac{4\gamma\mu b}{3c}\int\cdots}$$

$$0 = \frac{g a^2}{r^2} + \left\{A\frac{\alpha^2}{c}\frac{1}{\rho}\frac{d\rho}{dr} + \frac{4\gamma\mu b\alpha}{3\rho^4 r^4 c}\left[\frac{d\rho}{dr}+\frac{3\rho}{r}\right]^2 + \frac{\alpha}{\rho}\frac{d\rho}{dr}\right\}\rho^{\frac{\alpha A}{c}}\, e^{\frac{4\gamma\mu b}{3c}\int\left(\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right)^2}$$

$$= \frac{g a^2}{r^2} + \rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr}\left(\frac{A\alpha}{c}+1\right)\alpha\, e^{\frac{4\gamma\mu b}{3c}\int\cdots} + \frac{4\gamma\mu b\alpha}{3c}\left(\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right)^2 e^{\frac{4\gamma\mu b}{3c}\cdots}$$

$$g a^2 e^{-\frac{4\gamma\mu b}{3c}\int\cdots} + \underbrace{r^2\rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr}\left(\frac{A\alpha^2}{c}+\alpha\right) + \frac{4\gamma\mu b\alpha}{3c}r^2\left(\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right)^2}_{\varphi} = 0$$

$$-g a^2\frac{4\gamma\mu b}{3c}\left(\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right)^2 e^{-\frac{4\gamma\mu b}{c}\int} + \frac{d\varphi}{dr} = 0$$

$$\frac{4\gamma\mu b}{3c}\left[\frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2}\right]^2\varphi + \frac{d\varphi}{dr} = 0$$

$$\int\left[r\frac{d}{dr}\left(\frac{1}{r^3\rho}\right)\right]^2 dr = f(r)\log r$$

$$r\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = \left[f'\log r + \frac{f}{r}\right]^{\frac{1}{2}} = \frac{\frac{d\rho}{dr}+\frac{3\rho}{r}}{\rho^2 r^2} = \frac{k}{\sqrt{r}} \quad \frac{r\,d}{k\,dr}$$

$$= \varphi$$

$$\frac{1}{r^3\rho} = \int\frac{1}{r}\left[\quad\right]^{\frac{1}{2}} dr$$

$$\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = k\int r^{-\frac{3}{2}} dr$$

$$\frac{1}{r^3\rho} = k r^{\frac{5}{2}} + k'$$

$$\rho = \frac{1}{k} r^{-\frac{11}{2}}$$

$$\int \quad = f(\log r)$$

$$\frac{1}{r}\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = \left[\frac{f'(\log r)}{r}\right]^{\frac{1}{2}}$$

$$\int \cdots dr = k^2 \log r$$

$$\frac{1}{r^3\rho} = \int\left[r f'(\log r)\right]^{\frac{1}{2}} dr \qquad e^{-m\log r} = r^{-m}$$

~~$M e^{-m\log r}$~~

$$M r^{-m} + r^2 N r^{-\frac{9}{4}} r^{-\frac{11}{2}} + P r^2 r^{-1} = 0$$

$\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = f$ ~~$r\frac{d}{dr}\left(\quad\right) = r f$~~ $\frac{d}{dr}\left(\frac{1}{r^3\rho}\right) = f$

~~$M e^{-\int f^2 r^2 dr}$~~

$\frac{1}{r^3\rho} = \int f\,dr$

$\rho = \frac{1}{r^3\int f\,dr}$

$\rho r^3 = z \qquad \rho = r^3 z \qquad \frac{d\rho}{dr} = 3r^2 z + r^3\frac{dz}{dr}$

$\frac{d\rho}{dr} = \frac{3}{r^4\int f\,dr} +$

$$e^{-\int\left[r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)\right]^2 dr} = F \qquad \frac{d}{dr}\left(\frac{1}{\rho r^3}\right) = \frac{1}{r}\sqrt{\frac{F'}{F}}$$

$$-\int\left[r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)\right]^2 dr = \log F \qquad \frac{1}{\rho r^3} = \int\frac{dr}{r}\sqrt{\frac{F'}{F}}$$

$$\left[r\frac{d}{dr}\left(\frac{1}{\rho r^3}\right)\right]^2 = \frac{F'}{F}$$

Jak długo r bliskie = a $\quad e^{-\int .} = 1$

$$g a^2 + r^2 \rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr}\left(\frac{A\alpha}{c}+1\right)\alpha + \frac{4\mu b\alpha}{3c\,a^2}\underbrace{\left(\frac{1}{\rho^2}\frac{d\rho}{dr}+\frac{3}{\rho \cancel{a}}\right)^2}_{\frac{1}{a^2}\left[\frac{d}{dr}\left(\frac{1}{\rho}\right)\right]^2} = 0$$

$$\frac{4}{3}\frac{\mu b\sigma a^2 \mu}{c\,a^2\rho^4\theta}\left(\frac{d\rho}{dr}\right)^2$$

Zaniedbując wpływ tarcia w 4 a uwzględniając w 3:

$$\left(\frac{\theta}{\theta_0}\right) = \left(\frac{\rho}{\rho_0}\right)^{\frac{\alpha A}{c}}$$

$$\theta = \theta_0\left(\frac{\rho}{\rho_0}\right)^{\frac{\alpha A}{c}} = \beta\rho^{\frac{\alpha A}{c}}$$

3). $\frac{g a^2}{r^2} + \frac{\alpha}{\cancel{m}}\beta\left(\frac{\alpha A}{c}+1\right)\rho^{\frac{\alpha A}{c}-1}\frac{d\rho}{dr} - \frac{4\mu}{3}\beta\rho^{\frac{\alpha A}{c}-1}\frac{d}{dr}\left[\frac{b}{\rho^2 r^2}\frac{d\rho}{dr}\right] = 0$

$$\frac{\alpha A}{c} = k \quad 2$$

$$\frac{\overset{\cancel{11}}{\cancel{3300}}}{\cancel{360}} \cdot \frac{78}{[illegible]} \quad \frac{\overset{42}{\cancel{43}}}{24 \cdot \cancel{60} \cdot \cancel{60 \cdot 2}} \qquad \frac{4}{120} = 0{\cdot}03$$

5

$\cancel{25 \cdot 40 : 365}$

2 gcal pro 1 min.

fehlt $+\frac{1}{3}\frac{\mu}{\rho}\frac{d\,\mathrm{div}}{dx}$ $\quad = \frac{d^2\sigma}{dx^2}$

1). $p = \alpha\theta\rho$

2). $\sigma\frac{d\sigma}{dx} = -\frac{ga^2}{x^2} - \frac{1}{\rho}\frac{dp}{dx} + \frac{4}{3}\frac{\mu}{\rho}\left(\frac{d^2\sigma}{dx^2} + \frac{d^2\sigma}{dy^2} + \frac{d^2\sigma}{dz^2}\right)$

3). $\left[c\frac{d\theta}{dx} + Ap\frac{d\left(\frac{1}{\rho}\right)}{dx}\right] = \cancel{\mu\left(\frac{du}{dz}\right)^2 + \left(\frac{du}{dy}\right)^2}$

4). $\rho\sigma = \mathrm{const} = b$ $\quad = A\mu\left[\left(\frac{du}{dx}\right)^2 + \left(\frac{du}{dy}\right)^2 + \left(\frac{du}{dz}\right)^2\right] + \frac{1}{3}\mu\left(\frac{du}{dx}\right)^2$

$\theta = f.(x)$

1). $p = \alpha\theta\rho$

2). $0 = -\frac{ga^2}{x^2} - \frac{1}{\rho}\frac{dp}{dx} + \frac{4}{3}\frac{\mu}{\rho}\frac{d^2\sigma}{dx^2}$

3). $b\left[c\frac{d\theta}{dx} + Ap\frac{d\left(\frac{1}{\rho}\right)}{dx}\right] = \frac{4}{3}\mu\left(\frac{d\sigma}{dx}\right)^2$

$\quad\quad\quad\quad\quad\quad\quad\quad \overset{\shortparallel}{\gamma\theta}$

4). $\rho\sigma = b$

$\frac{1}{\rho} = \frac{\alpha\theta}{p}$

$\frac{d}{dx}\left(-\frac{1}{\rho^2}\frac{d\rho}{dx}\right) =$

$\frac{2}{\rho^3}\left(\frac{d\rho}{dx}\right)^2 - \frac{1}{\rho^2}\frac{d^2\rho}{dx^2}$

---

3). $\left[\frac{c}{A}\frac{d\theta}{dx} - \frac{\alpha\theta}{\rho}\frac{d\rho}{dx}\right] = \frac{4}{3}\gamma\theta\frac{b^2}{\rho^4}\left(\frac{d\rho}{dx}\right)^2$

2). $0 = -\frac{ga^2}{x^2} - \frac{\alpha}{\rho}\left(\theta\frac{d\rho}{dx} + \rho\frac{d\theta}{dx}\right) + \frac{4}{3}\frac{\gamma\theta\cancel{b}}{\rho}\frac{d^2}{dx^2}\left(\frac{1}{\rho}\right)$

$\cancel{-\frac{\alpha}{\rho^2}\frac{d\rho}{dx} = \frac{\alpha\theta}{\rho}\frac{d\rho}{dx}}$

$$0 = -g\frac{a^2}{x^2} - \frac{c}{A}\frac{d\theta}{dx} - \alpha\frac{d\theta}{dx} + \mu\theta\left[\frac{b}{\rho}\frac{4}{3}\frac{d^2}{dx^2}\left(\frac{1}{\rho}\right) + \frac{4}{3}\frac{b^4}{\rho^4}\left(\frac{d\rho}{dx}\right)\right]$$

---

bei vollständiger Vernachlässigung von $\int^x$

$$\text{const} = g\frac{a^2}{x} - \left(\frac{c}{A} + \alpha\right)\theta + \int dx\, \mu\theta\left[\qquad\right]$$

$$\frac{1}{\rho} = \frac{\sigma}{b} \qquad p = \frac{\alpha b\theta}{\sigma}$$

$$2).\quad 0 = -g\frac{a^2}{x^2} - \frac{\cancel{\sigma}}{\cancel{b}}\frac{d}{dx}\left(\frac{\alpha\theta \cancel{b}}{\sigma}\right) - \mu\frac{\theta\sigma}{b}\frac{d^2\sigma}{dx^2}$$

$$\cancel{k} = \kappa\theta$$

$$3).\quad \frac{c}{A}\frac{d\theta}{dx} + \frac{\alpha\theta}{\sigma \cancel{b}}\frac{d\sigma}{dx} = \frac{4}{3}\mu\frac{\theta}{b}\left(\frac{d\sigma}{dx}\right)^2 - \kappa\theta\frac{d\theta}{dx}$$

$$3).\quad \frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \alpha\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\frac{\mu}{b}\left(\frac{d\sigma}{dx}\right)^2 - \frac{\kappa}{b}\frac{d^2\theta}{dx^2}$$

$$2).\quad 0 = -g\frac{a^2}{x^2} \cancel{+} - \frac{\alpha}{\cancel{\sigma}}\frac{d\theta}{dx} + \frac{\alpha}{\sigma}\theta\frac{d\sigma}{dx} + \frac{4}{3}\mu\frac{\theta\sigma}{b}\frac{d^2\sigma}{dx^2}$$

ergibt sich $\frac{4}{3}\frac{\mu}{b}\sigma\frac{d}{dx}\left(\theta\frac{d\sigma}{dx}\right)$

$$0 = -g\frac{a^2}{x^2} - \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} + \frac{4}{3}\frac{\mu\theta}{b}\left[\sigma\frac{d^2\sigma}{dx^2} + \left(\frac{d\sigma}{dx}\right)^2\right]$$

$$\frac{\alpha A}{c} = k-1$$

$$\frac{c}{A}\log\theta + \alpha\log\sigma = \frac{4}{3}\frac{\mu}{b}\int\left(\frac{d\sigma}{dx}\right)^2 dx - \frac{\kappa}{b}\frac{d\theta}{dx}$$

$$\text{const} \times \theta^{\frac{c}{A}}\cdot\sigma^{\alpha} = e^{\frac{4}{3}\frac{\mu}{b}\int\cdots} = \text{fc.}(x) > 1 \text{ wachsend mit } x$$

$$\theta = \text{const}\cdot\rho^{\frac{\alpha A}{c}}$$

$$\left(\frac{\theta}{\theta_0}\right)^{\frac{c}{A}}\left(\frac{\sigma}{\sigma_0}\right)^{\alpha} = \nearrow \qquad = \left(\frac{\theta}{\theta_0}\right)^{\frac{c}{A}}\left(\frac{\rho_0}{\rho}\right)^{\alpha}$$

$\sigma = \infty$ $\quad k = \left(\frac{1}{\infty}\right)^{k}$

$\theta =$

$\theta\sigma = 0$

$$g\frac{a^2}{x^2} = \left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx} + \gamma\theta\left[\;-\;\right] \quad\Big|\quad \theta'$$

$$\cancel{0 = \left(\alpha+\frac{c}{A}\right)\frac{d\theta'}{dx} + \gamma\theta'\left[\quad\right]} \quad\Big|\quad \theta$$

$$\cancel{\frac{g\frac{a^2}{x^2}}{\theta'} = \left(\alpha+\frac{c}{A}\right)\underbrace{\left(\frac{\frac{d\theta}{dx}}{\theta'} - \frac{\theta}{\theta'^2}\frac{d\theta'}{dx}\right)}}$$

$$= \frac{d}{dx}\left(\frac{\theta}{\theta'}\right)$$

$$g\frac{a^2}{x^2} = \left(\alpha+\frac{c}{A}\right)\frac{d\theta'}{dx}$$

σ' und θ' berechnen

$$-\frac{ga^2}{x} = \left(\alpha+\frac{c}{A}\right)\theta' + \text{const}$$

$$\theta' = M - \frac{N}{x} \qquad \frac{d\theta'}{dx} = \frac{N}{x^2}$$

$$\frac{c}{A}\frac{N}{x^2} + \frac{\alpha}{6}\left(M-\frac{N}{x}\right)\frac{d\sigma}{dx} = \frac{4}{3}\gamma\left(M-\frac{N}{x}\right)\left(\frac{d\sigma}{dx}\right)^2$$

$$\frac{4}{3}\gamma\left(\frac{d\sigma}{dx}\right)^2 - \frac{\alpha}{6}\frac{d\sigma}{dx} = -\frac{cN}{A}\frac{1}{x^2\left(M-\frac{N}{x}\right)}$$

$$\left(\alpha+\frac{c}{A}\right)\theta = \cancel{\left(\alpha+\frac{c}{A}\right)\frac{c}{\alpha}p^{k-1}} = c\left(1+\frac{c}{\alpha A}\right) = \frac{k}{k-1}$$

~~$0 = \frac{g a^2}{c^2} - \left(\alpha + \frac{c}{A}\right) \frac{d\sigma}{dx} + \gamma\, \theta \Big[\qquad \Big]$~~

$$\sigma' \frac{d^2\sigma'}{dx^2} = \frac{4}{3} \left(\frac{d\sigma'}{dx}\right)^2$$

$$\frac{\frac{d^2\sigma'}{dx^2}}{\frac{d\sigma'}{dx}} = \frac{4}{3} \frac{1}{\sigma'} \frac{d\sigma'}{dx}$$

$$\lg \frac{d\sigma'}{dx} = \frac{4}{3} \lg \sigma' + \text{const}$$

$$\frac{d\sigma'}{dx} = A\, \sigma'^{\frac{4}{3}}$$

$$\sigma'^{-\frac{4}{3}}\, d\sigma' = A\, dx$$

$$-3\sigma^{-\frac{1}{3}} = -B + Ax$$

$$\frac{1}{\sigma} = \left(\frac{B - Ax}{3}\right)^3$$

$$\sigma = \frac{27}{(B - Ax)^3}$$

$$\sigma \frac{d^2\sigma}{dx^2} + \left(\frac{d\sigma}{dx}\right)^2 = 0 = \frac{d}{dx}\left(\sigma \frac{d\sigma}{dx}\right)$$

$$\sigma \frac{d\sigma}{dx} = \text{const}$$

$$\sigma^2 = \text{const}\, x + \text{const}'$$

$$\sigma = \sqrt{a^2 x + m}$$

$$\frac{d\sigma}{dx} = \frac{1}{2} \frac{a^2}{\sqrt{\quad}}$$

$$\frac{d\sigma'}{dx} = A \frac{81}{(B - Ax)^4}$$

$$\sigma = c\, \theta^{-\frac{c}{\alpha A}}$$

$$\frac{d\sigma}{dx} = -\frac{c}{\alpha A}\, \theta^{-\frac{c}{\alpha A} - 1}$$

in der abgekürzten Gleichg ist

$$\frac{c}{A} \frac{1}{\theta} \frac{d\theta}{dx} = -\alpha \frac{1}{\sigma} \frac{d\sigma}{dx}$$

$\theta = 0$

$$\sigma = (\infty)^{\frac{1}{k-1}}$$

aber $\frac{d\sigma}{dx} = (\infty)^{\frac{k}{k-1}}$

$$3) + i \quad \underbrace{\frac{c}{A} \frac{1}{\theta} \frac{d\theta}{dx}}_{} = \frac{d\sigma}{dx} \left[\frac{4}{3} \frac{\partial}{\partial} \left(\frac{d\sigma}{dx}\right) - \frac{\alpha}{\sigma}\right]$$

Solange dies endlich ist kann $\frac{d\sigma}{dx}$ nicht ∞ werden

wenn $\theta = 0$ wird $\frac{d\theta}{dx}$ aber endlich ist, so muss $\frac{d\theta}{dx} < 0$ sein also $\frac{\alpha}{\sigma} > -\frac{d\sigma}{dx}$ und zwar $\sigma = [0]^1$ und $\frac{d\sigma}{dx}$ endlich positiv was aber nicht möglich ist wenn σ für kleinere x positiv sein soll.

Wenn dagegen $\theta = 0$ $\frac{d\theta}{dx}$ aber von höherer Ordnung 0 so kann z.B. $\sigma = \infty$ und $\left(\frac{d\sigma}{dx}\right)$ von höher Ordnung 0 werden

Wenn $\frac{d\theta}{dx}$ von derselben Ordnung $= 0$, sodass $\frac{1}{\theta} \frac{d\theta}{dx} =$ endlich : $\sigma = 0$ $\frac{d\sigma}{dx} = 0$ nicht möglich ebenso wie oben und σ und $\frac{d\sigma}{dx}$ endlich so würde Gleichg. 2. verlangen dass $\frac{d^2\sigma}{dx^2}$ von 1ter Ordnung unendlich wird (an der $x = \infty$) was [illegible] ausgeschlossen

$$\theta = \theta_0 \left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f(x)\right]^{\frac{A}{c}}$$

$$\frac{d\theta}{dx} = \theta_0 \frac{A}{c}\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f(x)\right]^{\frac{A}{c}-1}\left[\left(\frac{\sigma}{\sigma_0}\right)^{-\alpha}\frac{df}{dx} - \alpha f \left(\frac{\sigma}{\sigma_0}\right)^{-\alpha-1}\frac{d\sigma}{dx}\right]$$

$$= \theta_0\frac{A}{c}\left[\left(\frac{\sigma}{\sigma_0}\right)^{-\alpha} f\right]^{\frac{A}{c}}\left[\frac{1}{f}\frac{df}{dx} - \alpha\left(\frac{\sigma}{\sigma_0}\right)^{-1}\frac{d\sigma}{dx}\right]$$

↗

$$g\frac{a^2}{x^2} + \left(\frac{c}{A}+\alpha\right)$$

$$= \frac{4}{3}\frac{\gamma}{b}\left[+\sigma\frac{d\tau}{dx^2} + \left(\frac{d\sigma}{dx}\right)^2\right]\theta_0\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}$$

$$f(x) = e^{\frac{4}{3}\frac{\gamma}{b}\int_0^x \left(\frac{d\sigma}{dx}\right)^2 dx}$$

$$\frac{1}{f}\frac{df}{dx} = \frac{d}{dx}\lg f(x) = \frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2$$

$$g\frac{a^2}{x^2} = \frac{4}{3}\frac{\gamma}{b}\left[\left(\frac{d\sigma}{dx}\right)^2 + \sigma\frac{d\tau}{dx^2}\right]\theta_0\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}} - \left(\frac{c}{A}+\alpha\right)\theta_0\frac{A}{c}\left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}\left[\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2 - \alpha\frac{\sigma_0}{\sigma}\right]$$

$$= \left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}\theta_0\left\{\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2 + \frac{4}{3}\frac{\gamma}{b}\sigma\frac{d\tau}{dx^2} - \left(1+\frac{\alpha A}{c}\right)\left[\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2 - \alpha\frac{\sigma_0}{\sigma}\frac{d\sigma}{dx}\right]\right\}$$

$$= \left[\left(\frac{\sigma_0}{\sigma}\right)^{\alpha} f\right]^{\frac{A}{c}}\theta_0\left\{-\frac{4}{3}\frac{\gamma}{b}\frac{\alpha A}{c}\left(\frac{d\sigma}{dx}\right)^2 + \frac{4}{3}\frac{\gamma}{b}\sigma\frac{d\tau}{dx^2} + \alpha\frac{\sigma_0}{\sigma}\left(1+\frac{\alpha A}{c}\right)\frac{d\sigma}{dx}\right\}$$

$$g\frac{a^2}{x^2} = \underbrace{\left(\frac{\sigma_0}{\sigma}\right)^{\frac{\alpha A}{c}}\theta_0\left[\frac{4}{3}\frac{\gamma}{b}\left[-\frac{\alpha A}{c}\left(\frac{d\sigma}{dx}\right)^2 + \sigma\frac{d\tau}{dx^2}\right] + \alpha\frac{\sigma_0}{\sigma}\left(1+\frac{\alpha A}{c}\right)\frac{d\sigma}{dx}\right]}_{\Phi} e^{\frac{4}{3}\frac{\gamma}{b}\frac{A}{c}\int_0^x\left(\frac{d\sigma}{dx}\right)^2 dx}$$

$$-2g\frac{a^2}{x^3} = \frac{d\Phi}{dx} e^{\cdots} + \Phi\frac{4}{3}\frac{\gamma}{b}\frac{A}{c}\left(\frac{d\sigma}{dx}\right)^2 e^{\cdots}$$

$$-\frac{2}{x} = \frac{d\Phi}{dx} + \frac{4}{3}\frac{\gamma}{b}\frac{A}{c}\left(\frac{d\sigma}{dx}\right)^2\Phi$$

Versuche:

$\sigma = x^n$

$$\frac{c}{A}\frac{1}{\theta}\frac{\partial\theta}{\partial x} + \frac{\alpha n}{x} = \frac{4}{3}\frac{\gamma}{b} n^2 x^{2n-2}$$

$$\frac{c}{A}\log\theta + \alpha n \log x = \frac{4}{3}\frac{\gamma}{b} n^2 \frac{x^{2n-1}}{2n-1}$$

$$\theta = \ldots x^{-n\frac{\alpha A}{c}} e^{\ldots x^{2n-1}}$$

$$\frac{d\theta}{dx} = e^{x^{2n-1}}\left[\cdot\, x^{-n\frac{\alpha A}{c}-1} + x^{-n\frac{\alpha A}{c}} x^{2n-2}\right]$$

$$\frac{1}{x^2} = e^{x^{2n-1}}$$

$\sigma = e^x$

$$\frac{1}{\theta}\frac{d\theta}{dx} = e^{2x}$$

$$\log\theta = e^{2x}$$

$$\theta = e^{e^{2x}}$$

$\sigma = \log x \qquad \frac{d\sigma}{dx} = \frac{1}{x} \qquad \int\cdots = \frac{1}{x}$

$$\sigma\frac{d^2\sigma}{dx^2} = x^{-1} \qquad \left(\frac{d\sigma}{dx}\right)^2 = x^{-1}$$

$$\int\left(\frac{d\sigma}{dx}\right)^2 dx = \log x$$

$$\left(\frac{d\sigma}{dx}\right)^2 = \frac{1}{x}$$

$$\frac{d\sigma}{dx} = x^{-\frac{1}{2}}$$

$$\sigma = x^{\frac{1}{2}}$$

$$\frac{d^2\sigma}{dx^2} = x^{-\frac{3}{2}}$$

$$\theta = \sigma^{-\frac{\alpha A}{c}} e^{\int\ldots\frac{4}{3}\frac{\gamma}{b}\ldots}$$

$$= x^{-\frac{\alpha A}{2c}} \cdot x$$

$$0 = g\frac{a^2}{x} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{\mu}{b}\,\theta\,\frac{d\theta}{dx}$$

$\theta = 0$

$g\frac{a^2}{x} + m = 0$

~~$\theta = uv$~~

~~$0 = g\frac{a^2}{x} + m - uv\left(\alpha + \frac{c}{A}\right) + \frac{\mu}{b}\left(u^2 v\frac{dv}{dx} + u v^2\frac{du}{dx}\right)$~~

~~$u\left[-v\left(\alpha + \frac{c}{A}\right) + \frac{\mu}{b}\left(u v\frac{dv}{dx} + v^2\frac{du}{dx}\right)\right]$~~

$g\,a$ ~~$+m$~~ $-\left(\alpha + \frac{c}{A}\right)\theta_0 + \frac{\mu}{b}\theta_0\left(\frac{d\theta}{du}\right)_0 = -m$

$= g\frac{a^2}{x}$

$\theta = u + v$

$$0 = g\frac{a^2}{x} + m - \left(\alpha + \frac{c}{A}\right)(u+v) + \frac{\mu}{b}(u+v)\left(\frac{du}{dx} + \frac{dv}{dx}\right)$$

$$0 = g\frac{a^2}{x} + m - \left(\alpha + \frac{c}{A}\right)u + \frac{\mu}{b}u\left(\frac{du}{dx} + \frac{dv}{dx}\right)$$

$$+ v\left[\left(\alpha + \frac{c}{A}\right) + \frac{\mu}{b}\left(\frac{du}{dx} + \frac{dv}{dx}\right)\right]$$

więc: Wpływ przewodnictwa zupełnie znikomy

$$\frac{\mu}{b}\frac{d\theta}{dx} - \left(\alpha + \frac{c}{A}\right) + \frac{m}{\theta} + \frac{g a^2}{\theta x} = 0$$

~~$\frac{\mu}{b}\frac{d\theta_1}{dx} - \qquad + \frac{m}{\theta_1} + \frac{g a^2}{\theta_1 x} = 0$~~

~~$\frac{\mu}{b}\left(\theta\frac{d\theta}{dx} - \theta_1\frac{d\theta_1}{dx}\right) - \left(\alpha + \frac{c}{A}\right)\theta \qquad = 0$~~

~~$\theta\frac{d\theta}{dx} \qquad - \frac{b}{\mu}\left(\alpha + \frac{c}{A}\right)\theta + \theta_1\frac{d\theta_1}{dx} = 0$~~

~~$\frac{\mu}{b}\frac{d}{dx}(\theta - \theta_1)$~~

$$\cancel{\frac{\mu}{b}\frac{d^2\theta}{dx^2} - \frac{m}{\theta^2}\frac{d\theta}{dx} - \frac{ga^2}{\theta^2 x}\frac{d\theta}{dx} - \frac{ga^2}{\theta x^2} = 0}$$

$$u = (\log f(x))^2 \qquad \frac{du}{dx} = 2\log f \cdot \frac{f'}{f}$$

$$\frac{ga^2}{x} - \left(\alpha + \frac{c}{A}\right)\log f + \underbrace{\frac{\mu}{b}\frac{f'}{f}\log f + m +}_{\log f \left(\frac{\mu}{b}\frac{f'}{f} - \alpha + \frac{c}{A}\right)}$$

$$f = e^{\frac{1}{x}}$$

$$\frac{\mu}{2b}\frac{du}{dx} - \left(\alpha + \frac{c}{A}\right) u^{\frac{1}{2}} + \frac{ga^2}{x} + m = 0$$

$$\left(\alpha + \frac{c}{A}\right)^2 u = \left[\frac{\mu}{2b}\frac{du}{dx} + \frac{ga^2}{x} + m\right]^2$$

$$\frac{du}{dx} = p \qquad \left(\alpha + \frac{c}{A}\right)^2 p = 2\left[\frac{\mu}{2b}p + \frac{ga^2}{x} + m\right]\left[\frac{\mu}{2b}\frac{dp}{dx} - \frac{ga^2}{x^2}\right]$$

$$\frac{\frac{1}{2}\left(\alpha + \frac{c}{A}\right)^2 p}{\frac{\mu}{2b}p + \frac{ga^2}{x} + m} + \frac{ga^2}{x^2} = \frac{\mu}{2b}\frac{dp}{dx}$$

$$\frac{ga^2}{x} = \left(\alpha + \frac{c}{A}\right) u^{\frac{1}{2}} - m - \frac{\mu}{2b}\frac{du}{dx}$$

$$x = \frac{ga^2 \frac{dx}{du}}{\left[\left(\alpha + \frac{c}{A}\right) u^{\frac{1}{2}} - m\right]\frac{dx}{du} - \frac{\mu}{2b}} \quad \Bigg\| \quad 1 =$$

$$\cancel{\left[\left(\alpha + \frac{c}{A}\right)\sqrt{u} - m - \frac{\mu}{2b}\frac{du}{dx}\right]^2 = ga^2\left[\left(\alpha + \frac{c}{A}\right)\frac{1}{2}\frac{du}{dx}\frac{1}{\sqrt{u}} - \frac{\mu}{2b}\frac{d^2u}{dx^2}\right]}$$

$$g\cancel{\alpha}x + m - \left(\alpha + \frac{c}{A}\right)\sqrt{u} + \sqrt{\frac{\mu}{2b}}\,\frac{du}{dx} = 0 \qquad \cancel{\frac{1}{u}}\,\frac{dx}{du} = p$$

$$g\cancel{\alpha}x + m - \left(\alpha + \frac{c}{A}\right)\sqrt{\cancel{p}u} + \sqrt{\frac{\mu}{2b}}\,\frac{1}{p} = 0$$

$$g\cancel{\alpha}p - \left(\alpha + \frac{c}{A}\right)\frac{1}{2}\frac{1}{\sqrt{u}} - \sqrt{\frac{\mu}{2b}}\,\frac{1}{p^2}\frac{dp}{du} = 0$$

p Dimens. . $\frac{1}{\sqrt{u}}$

$$p = \frac{z}{\sqrt{u}} \qquad \frac{dp}{du} = \frac{dz}{du}\frac{1}{\sqrt{u}} - \frac{z}{2u\sqrt{u}}$$

$$g\cancel{\alpha}\frac{z}{\cancel{\sqrt{u}}} - \left(\alpha + \frac{c}{A}\right)\frac{1}{2}\cancel{\frac{1}{\sqrt{u}}} - \sqrt{\frac{\mu}{2b}}\,\frac{u}{z^2}\left(\frac{dz}{du}\cancel{\frac{1}{\sqrt{u}}} - \frac{z}{2u\cancel{\sqrt{u}}}\right)$$

$$g\cancel{\alpha}z - \frac{1}{2}\left(\alpha + \frac{c}{A}\right) + \sqrt{\frac{\mu}{4b}}\,\frac{1}{z} = \sqrt{\frac{\mu}{2b}}\,\frac{u}{z^2}\frac{dz}{du}$$

$$\frac{du}{u} = \sqrt{\frac{\mu}{2b}}\,\frac{dz}{\left[g\cancel{\alpha}z^3 - \frac{1}{2}\left(\alpha + \frac{c}{A}\right)z^2 + \sqrt{\frac{\mu}{4b}}\,z\right]}$$

$\log u = f_n(z)$

~~$p = \frac{z}{\sqrt{e^{f(z)}}}$~~

$$\log u = f_c.(p\sqrt{u}) = f_n\left(\frac{dx}{du}\sqrt{u}\right)$$

$$2\log\theta = f_c.\left[\frac{dx}{d(\theta^2)}\,\theta\right]$$

$$\log u = \log\frac{z^2}{z^2 - \frac{1}{2g}(\quad) + \sqrt{\frac{\mu}{4bg}}}\left(\frac{z - \quad + \sqrt{\quad}}{z - \quad - \sqrt{\quad}}\right)^{-\frac{1}{4g}\left(\alpha + \frac{c}{A}\right)\frac{1}{\sqrt{\quad}}}$$

$$\theta = \vartheta + n$$

$$0 = \frac{ga^2}{x} + m - \left(\alpha + \frac{c}{A}\right)\vartheta - \left(\alpha + \frac{c}{A}\right)n + \frac{\mu}{b}(\vartheta + n)\frac{d\vartheta}{dx}$$

$$m - \left(\alpha + \frac{c}{A}\right)n = 0 \qquad n = \frac{m}{\alpha + \frac{c}{A}}$$

$$0 = \frac{ga^2}{x} - \left(\alpha + \frac{c}{A}\right)\vartheta + \frac{\mu}{b}\, n\frac{d\vartheta}{dx} + \frac{\mu}{b}\,\vartheta\frac{d\vartheta}{dx}$$

$$\frac{du}{u} = 2\left[\frac{dz}{z} + \frac{-gz + \frac{1}{2}\left(\alpha + \frac{a}{A}\right)}{gz^2 - \frac{1}{2}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4b}}dz\right]$$

$$= 2\left[\frac{dz}{z} - \frac{\left[z - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)\right]dz}{z^2 - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4bg}}\right]$$

$$= 2\frac{dz}{z} - \frac{2z - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)}{z^2 - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4bg}} + \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)\frac{dz}{z^2 - \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)z + \frac{\mu}{4bg}}$$

$$= 2\log z - \log\left(z^2 - \frac{1}{2g}(\quad)z + \frac{\mu}{4bg}\right) + \frac{1}{2g}\left(\alpha + \frac{c}{A}\right)\int\frac{dz}{z^2 - \text{——}\ )}$$

$$\int\frac{1}{\left[z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right)\right]^2 - \left(\left[\frac{1}{4g}\left(\alpha + \frac{c}{A}\right)\right]^2 - \frac{\mu}{4bg}\right)} = \int\frac{d\,\frac{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right)}{\sqrt{}}}{\left[\frac{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right)}{\sqrt{\left(\frac{1}{4g}\right)^2\left(\alpha + \frac{c}{A}\right)^2 - \frac{\mu}{4bg}}}\right]^2 - 1}\cdot\frac{1}{\sqrt{}}$$

$$= \frac{1}{2}\frac{1}{\sqrt{\left(\frac{\alpha + \frac{c}{A}}{4g}\right)^2 - \frac{\mu}{4bg}}}\log\left(\frac{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right) + \sqrt{}}{z - \frac{1}{4g}\left(\alpha + \frac{c}{A}\right) - \sqrt{}}\right)$$

$$g x + m - \left(\alpha + \frac{c}{A}\right)\sqrt{u} + \frac{\mu}{2b}\,\frac{du}{dx} = 0$$

$$g\,v - \left(\alpha + \frac{c}{A}\right)\frac{1}{2\sqrt{u}}\,\frac{du}{dx} + \frac{\mu}{2b}\,\frac{d^2u}{dx^2} = 0 \qquad \frac{du}{dx} = q \qquad \frac{d^2u}{dx^2} = \frac{dq}{dx}$$

$$2\left[\alpha + \frac{c}{A} - g\frac{x}{\theta} \overset{+\frac{m}{\theta}}{} \right] = \frac{d}{dx}\left[\frac{\left[\frac{\alpha}{2} + \frac{c}{A} - g\frac{x}{\theta} + \frac{m}{\theta}\right]^2 - \frac{\alpha^2}{4}}{\frac{c}{A}\,\frac{1}{\theta}\,\frac{d\theta}{dx}}\right]$$

~~$\theta = z x$~~ $x = z\theta$

$$\lg x = \lg z + \lg \theta \qquad \frac{dx}{d\theta} = z + \theta\frac{dz}{d\theta}$$

$$\frac{1}{x} = \frac{1}{z}\,\frac{dz}{dx} + \frac{1}{\theta}\,\frac{d\theta}{dx}$$

$$z - \alpha - \frac{c}{A} = \frac{m}{\theta} - \frac{gx}{\theta}$$

$$\boxed{\alpha + \frac{c}{A} - \frac{gx}{\theta} + \frac{m}{\theta} = z}$$

$$= \frac{m - gx}{\theta}$$

$$\boxed{x = \frac{\theta}{g}\left[\alpha + \frac{c}{A} + \frac{m}{\theta} - z\right]} \qquad \boxed{\theta = \frac{m - gx}{z - \alpha - \frac{c}{A}}}$$

$$\lg\theta = \lg(m - gx) - \lg\left(z - \alpha - \frac{c}{A}\right) \qquad gx = m + \theta\left(\alpha + \frac{c}{A} - z\right)$$

$$\frac{1}{\theta}\,\frac{d\theta}{dx} = \frac{g}{gx - m} - \frac{1}{z - \alpha - \frac{c}{A}}\,\frac{dz}{dx} = \frac{1}{\alpha + \frac{c}{A} - z}\left(\frac{g}{\theta} - \frac{dz}{dx}\right)$$

~~$\frac{dx}{d\theta}$~~ $=$ ~~$\alpha + \frac{c}{A} - \frac{m}{\theta} - z$~~

$$x = \frac{1}{g}\left(\alpha + \frac{c}{A}\right)\theta + \frac{m}{g} - \frac{z\theta}{g}$$

$$\frac{dx}{d\theta} = \frac{1}{g}\left(\alpha + \frac{c}{A}\right) - \frac{z}{g} - \frac{\theta\,\frac{dz}{d\theta}}{g}$$

$$\frac{2c}{A}z = \frac{d}{dx}\left[\frac{\alpha+\frac{c}{A}}{g}\theta - \frac{z\theta}{g} - \frac{\theta^2\frac{dz}{d\theta}}{g}\right]\left[z(2-\alpha)\right]$$

$$\frac{2c}{A}z\,\frac{dx}{d\theta} = \frac{d}{d\theta}\left[\;\cdot\;\;\cdot\right.$$

$$\frac{2c}{A}z\left[\frac{\alpha+\frac{c}{A}}{} - z - \theta\frac{dz}{d\theta}\right] = z(2-\alpha)\left[\left(\alpha+\frac{c}{A}\right) - z - 3\theta\frac{dz}{d\theta} - \theta^2\frac{d^2z}{d\theta^2}\right]$$

$$+\left[2z - \alpha\right]\frac{dz}{d\theta}\left[\left(\alpha+\frac{c}{A}\right)\theta - z\theta - \theta^2\frac{dz}{d\theta}\right]$$

$$\frac{dx}{dz} = \frac{1}{g}\left[\left(\alpha+\frac{c}{A} - z\right)\frac{d\theta}{dz} - \theta\right]$$

$$\frac{2c}{A}z\,\frac{dx}{dz} = \cancel{\frac{d}{dz}\left[\frac{1}{g}\left(\alpha+\frac{c}{A}\right) - \frac{z}{g} - \frac{\theta\frac{dz}{d\theta}}{g}\right.}$$

$$= \frac{d}{dz}\left[\theta\frac{dz}{d\theta}\;z(2-\alpha)\frac{1}{g}\left[\left(\alpha+\frac{c}{A}-z\right)\frac{d\theta}{dz} - \theta\right]\right]$$

$$= \frac{d}{dz}\left[\theta\;z(2-\alpha)\left[\alpha+\frac{c}{A} - z - \frac{\theta}{\frac{d\theta}{dz}}\right]\right]$$

$$\frac{2c}{A}z\left[\left(\alpha+\frac{c}{A}-z\right)\frac{d\theta}{dz} - \theta\right] = z(2-\alpha)\left[\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dz} - z\frac{d\theta}{dz} - \theta\right] +$$

$$+\theta(2z-\alpha)\left[\alpha+\frac{c}{A} - z - \frac{\theta}{\frac{d\theta}{dz}}\right] +$$

Jeśli można $z(z-\alpha) = z^2$ postawić:

$$\frac{2c}{A} z = \frac{d}{dx}\left[\frac{z^2}{\frac{1}{\theta}\frac{d\theta}{dx}}\right]$$

$$\frac{2c}{A} \cancel{z} = \frac{2\cancel{z}\frac{dz}{dx}}{\frac{1}{\theta}\frac{d\theta}{dx}} - z^{\cancel{2}} \frac{\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)}{\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)^2}$$

$$\frac{2c}{A} \frac{1}{\theta}\frac{d\theta}{dx} = 2\frac{dz}{dx} - z\frac{d}{dx}\left(\log \frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\frac{2c}{A} \underbrace{\frac{1}{\theta}\frac{d\theta}{dz}}_{\frac{d(\log\theta)}{dz}} = 2 - z\frac{d}{dz}\left(\log \frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\left[\frac{c^2}{A^2} + \frac{c\alpha}{A} + \frac{\alpha}{u} - \frac{1}{u^2}\right]$$

$$\left(u + \frac{du}{dz}\right)^2 \overbrace{\left[\cancel{2}\frac{c\alpha}{A} + \cancel{2}\frac{c^2}{A^2} - \cancel{\frac{2c}{Au}} - \cancel{\frac{\alpha c}{A}} + \frac{\alpha}{u} - \cancel{\frac{c^2}{A^2}} + \cancel{\frac{2c}{Au}} - \frac{1}{u^2}\right]}$$

Przybliżenie: $\frac{g\alpha^2}{x^2} = g$

$$\frac{2c}{A}\left[\alpha + \frac{c}{A} + \frac{m}{\theta} - \frac{gx}{\theta}\right] = \frac{d}{dx}\left[\frac{\left[\alpha + \frac{c}{A} + \frac{m}{\theta} - \frac{gx}{\theta}\right]\left[\frac{c}{A} + \frac{m}{\theta} - \frac{gx}{\theta}\right]}{\frac{1}{\theta}\frac{d\theta}{dx}}\right]$$

$$gx - m = y \qquad g\,dx = dy$$

$$\frac{2c}{A}\left[\alpha + \frac{c}{A} - \frac{y}{\theta}\right] = \frac{d}{dy}\left[\frac{\left[\alpha + \frac{c}{A} - \frac{y}{\theta}\right]\left[\frac{c}{A} - \frac{y}{\theta}\right]}{\frac{1}{\theta}\frac{d\theta}{dy}}\right]$$

$$= \frac{-\left[\alpha + \frac{2c}{A} - \frac{2y}{\theta}\right]\frac{d}{dy}\left[\frac{y}{\theta}\right]}{\frac{1}{\theta}\frac{d\theta}{dy}} - \frac{\left[\alpha + \frac{c}{A} - \frac{y}{\theta}\right]\left[\frac{c}{A} - \frac{y}{\theta}\right]}{\left(\frac{1}{\theta}\frac{d\theta}{dy}\right)^2}\frac{d}{dy}\left(\frac{1}{\theta}\frac{d\theta}{dy}\right)$$

$$\frac{2c}{A}\left[\alpha + \frac{c}{A} - \frac{y}{\theta}\right]\left(\frac{1}{\theta}\frac{d\theta}{dy}\right)^2 = -\left[\alpha + \frac{2c}{A} - \frac{2y}{\theta}\right]\frac{1}{\theta}\frac{d\theta}{dy}\left[\frac{1}{\theta} - \frac{y}{\theta^2}\frac{d\theta}{dy}\right] -$$

$$- \left[\alpha + \frac{c}{A} - \frac{y}{\theta}\right]\left[\frac{c}{A} - \frac{y}{\theta}\right]\left[\frac{1}{\theta}\frac{d^2\theta}{dy^2} - \frac{1}{\theta^2}\left(\frac{d\theta}{dy}\right)^2\right]$$

$$y = e^z \qquad \theta = \frac{1}{u}e^z$$

$$y = e^z \qquad \theta = u e^z$$

$$\frac{d\theta}{dy} = \left(u + \frac{du}{dz}\right)$$

$$\frac{d^2\theta}{dy^2} = \left(\frac{du}{dz} + \frac{d^2u}{dz^2}\right)e^{-z}$$

$$\frac{2c}{A}\left[\alpha + \frac{c}{A} - \frac{y}{\theta}\right]\left(\frac{d\theta}{dy}\right)^2 = -\left[\alpha + \frac{2c}{A} - \frac{2y}{\theta}\right]\left[1 - \frac{y}{\theta}\frac{d\theta}{dy}\right]\frac{d\theta}{dy} -$$

$$- \left[\alpha + \frac{c}{A} - \frac{y}{\theta}\right]\left[\frac{c}{A} - \frac{y}{\theta}\right]\left[\theta\frac{d^2\theta}{dy^2} - \left(\frac{d\theta}{dy}\right)^2\right]$$

$$\frac{2c}{A}\left[\alpha + \frac{c}{A} - \frac{1}{u}\right]\left(u + \frac{du}{dz}\right)^2 = -\left[\alpha + \frac{2c}{A} - \frac{2}{u}\right]\left[\overbrace{1 - \frac{1}{u}\left(u + \frac{du}{dz}\right)}^{-\frac{1}{u}\frac{du}{dz}}\right]\left(u + \frac{du}{dz}\right) -$$

$$- \left[\alpha + \frac{c}{A} - \frac{1}{u}\right]\left[\frac{c}{A} - \frac{1}{u}\right]\left[u\left(\frac{du}{dz} + \frac{d^2u}{dz^2}\right) - \left(u + \frac{du}{dz}\right)^2\right]$$

$$\frac{1}{u}\frac{du}{dz} = p$$

$$\frac{2c}{A}\left[\alpha + \frac{c}{A} - u\right]\left[1 - \frac{1}{u}\frac{du}{dz}\right]^2 = -\left[\alpha + \frac{2c}{A} - 2u\right]\left[1 - \frac{1}{u}\frac{du}{dz}\right]\frac{du}{dz} + \left[\alpha + \frac{c}{A} - u\right]\left[\frac{c}{A} - u\right]\left[1 - \frac{1}{u}\frac{du}{dz} - \frac{1}{u^2}\left(\frac{du}{dz}\right)^2 + \frac{1}{u}\frac{d^2u}{dz^2}\right]$$

$$\frac{2c}{A}\left[\alpha + \frac{c}{A} - u\right]\left[1 - p^2\right] = -\left[\alpha + \frac{2c}{A} - 2u\right]\frac{pu}{1-p} + \left[\alpha + \frac{c}{A} - u\right]\left[\frac{c}{A} - u\right]\left[\frac{1}{1-p} + \frac{\frac{dp}{du}}{(1-p)^2}\right]$$

$$\left[\frac{c^2}{A^2}+\frac{c\alpha}{A}+\frac{\alpha}{u}-\frac{1}{u^2}\right]\left[u+\frac{du}{dz}\right]^2=\left[\alpha+\frac{2c}{A}-\frac{2}{u}\right]\left[u+\frac{du}{dz}\right]\frac{1}{u}\frac{du}{dz}-$$

$$-\left[\alpha+\frac{c}{A}-\frac{1}{u}\right]\left[\frac{c}{A}-\frac{1}{u}\right]\left[\frac{du}{dz}+\frac{d^2u}{dz^2}\right]u=0$$

$$\cancel{\frac{c^2 u}{A^2}-\frac{2c\alpha}{A}+\alpha-\frac{1}{u}}$$

$$\left[\left(\frac{c^2}{A^2}+\frac{c\alpha}{A}\right)u^2+\alpha u-1\right]+\frac{du}{dz}\left[\left(\frac{2c^2}{A^2}u+\frac{2c\alpha}{A}u\right)+\cancel{2\alpha}-\cancel{\frac{2}{u}}-\cancel{\alpha}-\frac{2c}{A}+\cancel{\frac{2}{u}}+\right.$$

$$\left.\left(+\cancel{\frac{\alpha c}{A}u}+\frac{c^2}{A^2}u\right)-\frac{c}{A}-\cancel{\alpha}-\frac{c}{A}+\frac{1}{u}\right]$$

$$+\left(\frac{du}{dz}\right)^2\left[\frac{c^2}{A^2}+\frac{c\alpha}{A}+\cancel{\frac{\alpha}{u}}-\cancel{\frac{1}{u^2}}-\cancel{\frac{\alpha}{u}}-\frac{2c}{Au}+\frac{\cancel{2}}{u^2}\right]$$

$$+\frac{d^2u}{dz^2}\left[\frac{\alpha c}{A}u+\frac{c^2}{A^2}u-\frac{c}{A}-\alpha-\frac{c}{A}+\frac{1}{u}\right]=0$$

$$\left[\left(\frac{c^2}{A^2}+\frac{c\alpha}{A}\right)u^2+\alpha u-1\right]+\frac{du}{dz}\left[3\left(\frac{c^2}{A^2}+\frac{c\alpha}{A}\right)u-\frac{4c}{A}+\frac{1}{u}\right]+$$

$$+\left(\frac{du}{dz}\right)^2\left[\frac{c^2}{A^2}+\frac{c\alpha}{A}-\frac{2c}{A}\frac{1}{u}+\frac{1}{u^2}\right]+\frac{d^2u}{dz^2}\left[\left(\frac{c^2}{A^2}+\frac{\alpha c}{A}\right)u-\alpha-\frac{2c}{A}+\frac{1}{u}\right]=0$$

$$\frac{du}{dz}=p \qquad \frac{d^2u}{dz^2}=p\frac{dp}{du}$$

$$U_0+pU_1+p^2U_2+p\frac{dp}{du}U_3=0$$

$$\alpha + \frac{c}{A} = \cancel{\frac{pq}{\theta_0 R_0}} = \alpha\left(1 + \frac{c}{\alpha A}\right) = \alpha\left(1 + \frac{1}{k-1}\right) = \frac{\alpha k}{k-1}$$

$$\frac{\alpha A}{c} = k - 1 \qquad \frac{c}{A} = \frac{\alpha}{k-1}$$

$$\frac{dq}{du}\underbrace{\left(\alpha + \frac{c}{A} - u\right)\left(\frac{c}{A} - u\right)}_{U_0} + q\overbrace{\left[\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - 2u\left(\alpha + \frac{2c}{A}\right) + 3u^2\right]}^{U_1} =$$

$$\underbrace{\left[\frac{2c}{A}\left(\alpha + \frac{c}{A}\right) - u\left(\alpha + \frac{4c}{A}\right) + 2u^2\right]}_{U_2}$$

$$q = e^{-\int \frac{U_1}{U_0} du}\left[C + \int \frac{U_2}{U_0} e^{\int \frac{U_1}{U_0} du} du\right]$$

$$\frac{U_1}{U_0} = \frac{\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - 2u\left(\alpha + \frac{2c}{A}\right) + 3u^2}{\left(\alpha + \frac{c}{A} - u\right)\left(\frac{c}{A} - u\right)} = 3 - \frac{2\frac{c}{A}\left(\alpha + \frac{c}{A}\right) u\left(\alpha + \frac{2c}{A}\right)}{\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - \left(\alpha + \frac{2c}{A}\right)u + u^2}$$

$$= 3 - \left[\frac{\left(\alpha + \frac{c}{A}\right)}{\alpha + \frac{c}{A} - u} + \frac{\frac{c}{A}}{\frac{c}{A} - u}\right]$$

$$\int \frac{U_1}{U_0} du = 3u + \log\left[\left(\alpha + \frac{c}{A} - u\right)^{+\left(\alpha + \frac{c}{A}\right)}\left(\frac{c}{A} - u\right)^{+\frac{c}{A}}\right]$$

$$\frac{U_2}{U_0} = \frac{\frac{2c}{A}\left(\alpha + \frac{c}{A}\right) - u\left(\alpha + \frac{4c}{A}\right) + 2u^2}{\frac{c}{A}\left(\alpha + \frac{c}{A}\right) - \left(\alpha + \frac{2c}{A}\right)u + u^2} = 2 -$$

U_0

$$\frac{dq}{du}\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)+q\left[\cancel{\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)(1-2u)}\right]\cancel{=2x}$$

U_1

$$q\left[\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)-u\left(\alpha+\frac{2c}{A}-2u\right)\right]$$

$$=\frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right)-u\left(\alpha+\frac{2c}{A}-2u\right)$$

U_2

$$U_1=\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)-u\left(\alpha+\frac{c}{A}-u+\frac{c}{A}-u\right)$$

$$\frac{U_1}{U_0}=1-u\left[\frac{1}{\frac{c}{A}-u}+\frac{1}{\alpha+\frac{c}{A}-u}\right]$$

$$=1+\frac{u}{u-\frac{c}{A}}+\frac{u}{u-\alpha-\frac{c}{A}}=3-\frac{\frac{c}{A}}{\frac{c}{A}-u}-\frac{\alpha+\frac{c}{A}}{\left(\alpha+\frac{c}{A}\right)-u}$$

$$\frac{U_2}{\cancel{U_0}}=\frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right)-u\left(\alpha+\frac{c}{A}-u+\frac{c}{A}-u\right)$$

$$\frac{U_2}{U_0}=\frac{\frac{2c}{A}-u}{\frac{c}{A}-u}-\frac{u}{\alpha+\frac{c}{A}-u}=1+\frac{\frac{c}{A}}{\frac{c}{A}-u}+1-\frac{\alpha+\frac{c}{A}}{\alpha+\frac{c}{A}-u}$$

$$=2+\frac{\frac{c}{A}}{\frac{c}{A}-u}-\frac{\alpha+\frac{c}{A}}{\left(\alpha+\frac{c}{A}\right)-u}$$

$$\int\frac{U_1}{U_0}\,du=3u+\log\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$$

$$e^{\int\frac{U_1}{U_0}du}=e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$$

$$\left.\begin{aligned}\int \frac{-\frac{c}{A}\,du}{\frac{c}{A}-u} &= \frac{c}{A}\,\lg\left(\frac{c}{A}-u\right) = \lg\left(\frac{c}{A}-u\right)^{\frac{c}{A}}\\ = \int \frac{\frac{c}{A}\,du}{u-\frac{c}{A}} &= \frac{c}{A}\,\lg\left(u-\frac{c}{A}\right) = \lg\left(u-\frac{c}{A}\right)^{\frac{c}{A}}\end{aligned}\right\} ?$$

$$\int \frac{u_2}{u_0}\, e^{\int \frac{u_1}{u_0}du}\,du = \int\left[2\,\frac{\alpha+\frac{c}{A}}{\alpha+\frac{c}{A}-u} + \frac{\frac{c}{A}}{\frac{c}{A}-u}\right] e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du$$

$$= 2\underset{J_0}{\int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du} - \left(\alpha+\frac{c}{A}\right)\underset{J_1}{\int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}-1}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du}$$

$$+ \frac{c}{A}\underset{J_2}{\int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}-1}du}$$

$$\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right) = z$$

~~$z\left(\frac{dq}{du}+q\right) - (q-1)u\frac{dz}{du} = \frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right)$~~

$\frac{q}{2} = t$

~~$2\frac{dq}{du} - \frac{q}{u}\frac{dz}{du} + 2q + \ldots = f_{-}(u)$~~

~~$\frac{1}{2}\frac{dq}{du} - \frac{q}{2^2}\frac{dz}{du} + \frac{q}{2} = \frac{\frac{2c}{A}\left(\alpha+\frac{c}{A}-u\right)-\frac{dz}{du}}{2^2} = \frac{u_2}{2^2}$~~

~~$\frac{dt}{du}$~~

$$-\left(\alpha+\frac{c}{A}\right) J_1 = e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}} - \underbrace{\int \left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}} \frac{d}{du}\left[e^{3u}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}\right]}_{3J_0 - \frac{c}{A}J_2}$$

$$\left(\alpha+\frac{c}{A}\right)J_1+\frac{c}{A}J_2-3J_0+e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}=0$$

$$J_0=\int e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du=u\,e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}-$$

~~$-3J_0+\left(\alpha+\frac{c}{A}\right)J_1+\frac{c}{A}J_2$~~

~~$4J_0-\left(\alpha+\frac{c}{A}\right)J_1-\frac{c}{A}J_2=u\,e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$~~

~~$-3J_0+\left(\alpha+\frac{c}{A}\right)J_1+\frac{c}{A}J_2=-e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}$~~

$$J_0=e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}(u-1)\ ?$$

$$\frac{d}{du}=e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}-1}\left(\frac{c}{A}-u\right)^{\frac{c}{A}-1}\Big[(u-1)\Big[3\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)-$$

$$-\left(\alpha+\frac{c}{A}\right)\left(\frac{c}{A}-u\right)-\left(\alpha+\frac{c}{A}-u\right)\frac{c}{A}\Big]+\left(\alpha+\frac{c}{A}-u\right)\left(\frac{c}{A}-u\right)\Big]$$

$$\left[3\alpha\frac{c}{A}+3\frac{c^2}{A^2}-6\frac{c}{A}u-3u\alpha+3u^2-\alpha\frac{c}{A}+\alpha u+\frac{c}{A}u-\frac{c^2}{A^2}-\alpha\frac{c}{A}-\frac{c^2}{A^2}+\frac{uc}{A}\right]$$

$$=\alpha\frac{c}{A}+\frac{c^2}{A^2}-4\frac{c}{A}u-2\alpha u+3u^2$$

$$=\left(\alpha+\frac{c}{A}\right)\frac{c}{A}-2\left(\alpha+\frac{c}{A}\right)u-2\frac{c}{A}u+3u^2\ \Big\|\ -\alpha\frac{c}{A}-\frac{c^2}{A^2}+\frac{2c}{A}u+\alpha u-u^2$$

$$=-\frac{2c}{A}u-\alpha u+2u^2$$

$$3u^3-4\frac{c}{A}u^2-2\alpha u^2+\frac{\alpha c}{A}u+\frac{c}{A}u+\frac{2c}{A}u+\alpha u-2u^2$$

$$-3\int u\,e^{3u}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du+\int u\,e^{3u}\left(\alpha+\frac{c}{A}\right)\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}-1}\left(\frac{c}{A}-u\right)^{\frac{c}{A}}du$$

$$+\int u\,e^{3u}\,\frac{c}{A}\left(\alpha+\frac{c}{A}-u\right)^{\alpha+\frac{c}{A}}\left(\frac{c}{A}-u\right)^{\frac{c}{A}-1}du$$

$$q = f_{\cdot}(u) = \frac{1}{1-p} = \frac{1}{1-\frac{1}{u}\frac{du}{dz}}$$

$$u = \frac{y}{\theta} \qquad z = \log y$$

$$\frac{du}{dz} = y\frac{du}{dy}$$

$$\frac{du}{dz} = u\left[1-\frac{1}{f(u)}\right]$$

$$u = \frac{y}{\theta}$$

$$\int\frac{du}{u\left[1-\frac{1}{f(u)}\right]} = \int dz = \int\frac{dy}{y} \quad \Big\} \; u = F(y)$$

$$0 = g\,x - m - \left(\alpha+\frac{c}{A}\right)\theta + \frac{4}{3}\frac{k}{b}\theta\,\sigma\frac{d\sigma}{dx}$$

$$0 = y - \left(\alpha+\frac{c}{A}\right)\theta + \frac{4}{3}\frac{k}{b}\theta\,\sigma\frac{d\sigma}{dy}$$

$$0 = u - \left(\alpha+\frac{c}{A}\right) + \frac{2}{3}\frac{k}{b}\frac{d}{dy}(\sigma^2)$$

~~[illegible]~~

$$\frac{2}{3}\frac{k}{b}\sigma^2 + \text{const} = \left(\alpha+\frac{c}{A}\right)y - \int u\,dy = \left(\alpha+\frac{c}{A}\right)y - \int F(y)\,dy$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \frac{1}{2}\frac{\alpha}{\sigma^2}\frac{d(\sigma^2)}{dx} = \frac{\ }{3}\frac{k}{b}\frac{1}{\sigma^2}\left[\frac{d(\sigma^2)}{dx}\right]^2$$

Jak mogą być oznaczone 3 stałe przez b, θ_0 ?

Systematyczna Eliminacye:

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \alpha\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\gamma_b\left(\frac{d\sigma}{dx}\right)^2$$

$$g + (\alpha + \frac{c}{A})\frac{d\theta}{dx} = \frac{4}{3}\gamma_b\overbrace{\left[\sigma\frac{d}{dx}\left(\theta\frac{d\sigma}{dx}\right) + \theta\left(\frac{d\sigma}{dx}\right)^2\right]}^{\frac{d}{dx}\left[\frac{4}{3}\gamma_b\,\sigma\theta\frac{d\sigma}{dx}\right]}$$

$$g + \alpha\frac{d\theta}{dx} - \alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\gamma_b\,\sigma\frac{d}{dx}\left(\theta\frac{d\sigma}{dx}\right)$$

$$= \frac{4}{3}\gamma_b\,\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx} + \frac{4}{3}\gamma_b\,\theta\sigma\frac{d^2\sigma}{dx^2}$$

$$\frac{d\sigma}{dx} = v$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} = \frac{4}{3}\gamma_b v^2 - \alpha\frac{v}{\sigma}$$

$$g + \alpha\frac{d\theta}{dx} - \alpha\frac{\theta}{\sigma}v = \frac{4}{3}\gamma_b\,\sigma v\frac{d\theta}{dx} + \frac{4}{3}\gamma_b\,\theta\sigma\frac{dv}{dx}$$

I). $\frac{d\sigma}{dx} = v$

II). $\frac{d\theta}{dx} = \frac{\theta}{\frac{c}{A}}\frac{4}{3}\gamma_b v^2 - \frac{\alpha}{\frac{c}{A}}\frac{\theta v}{\sigma}$

$$\frac{4}{3}\gamma_b\theta\sigma\frac{dv}{dx} = g - \alpha\frac{\theta v}{\sigma} + \left[\alpha - \frac{4}{3}\gamma_b\sigma v\right]\left[\frac{\frac{4}{3}\gamma_b}{\frac{c}{A}}\theta v^2 - \frac{\alpha}{\frac{c}{A}}\frac{\theta v}{\sigma}\right]$$

$$= g - \left(\alpha + \frac{\alpha^2}{\frac{c}{A}}\right)\frac{\theta v}{\sigma} + \frac{\frac{8}{3}\gamma_b}{\frac{c}{A}}\alpha\,\theta v^2 - \frac{\left(\frac{4}{3}\gamma_b\right)^2}{\frac{c}{A}}\theta\sigma v^3$$

III). $$\frac{dv}{dx} = \frac{g}{\frac{4}{3}\gamma_b}\frac{1}{\theta\sigma} - \frac{\alpha\left(1+\frac{\alpha}{\frac{c}{A}}\right)}{\frac{4}{3}\gamma_b}\frac{v}{\sigma^2} + \frac{2\alpha}{\frac{c}{A}}\frac{v^2}{\sigma} - \frac{\left(\frac{4}{3}\gamma_b\right)}{\frac{c}{A}}v^3$$

$$g x + \underbrace{\left(\alpha - \frac{4}{3}\tau_b\,\sigma\frac{d\sigma}{dx}\right)}_{= -\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx}\cdot\frac{\sigma}{\frac{d\sigma}{dx}}}\frac{d\theta}{dx} = \alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx} + \frac{4}{3}\tau_b\,\theta\sigma\frac{d^2\sigma}{dx^2}$$

~~$\frac{c}{A}\frac{1}{\theta}\frac{d^2\theta}{dx^2} - \frac{c}{A}\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2 + \alpha\frac{1}{\sigma}\frac{d^2\sigma}{dx^2} - \frac{\alpha}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2 = \frac{8}{3}\tau_b\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}$~~

$$\frac{c}{A}\theta\frac{d\theta}{dx} + \alpha\frac{\theta}{\sigma}\cdot\theta\frac{d\sigma}{dx} = \frac{4}{3}\tau_b\left(\theta\frac{d\sigma}{dx}\right)^2$$

$$\left(\theta\frac{d\sigma}{dx}\right)^2 - \frac{\alpha}{\frac{4}{3}\tau_b}\frac{\theta}{\sigma}\,\theta\frac{d\sigma}{dx} = \frac{\frac{c}{A}}{\frac{4}{3}\tau_b}\,\theta\frac{d\theta}{dx}$$

$$\theta\frac{d\sigma}{dx} = \frac{\alpha}{\frac{8}{3}\tau_b}\frac{\theta}{\sigma} \pm \sqrt{\left(\frac{\alpha}{\frac{8}{3}\tau_b}\right)^2\left(\frac{\theta}{\sigma}\right)^2 + \frac{\frac{c}{A}}{\frac{4}{3}\tau_b}\,\theta\frac{d\theta}{dx}}$$

$$g + \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} = \frac{d}{dx}\left[\frac{\alpha}{2}\frac{\theta}{\sigma} \pm \sqrt{\frac{\alpha^2}{4}\left(\frac{\theta}{\sigma}\right)^2 + \frac{4}{3}\tau_b\frac{c}{A}\,\theta\frac{d\theta}{dx}}\right]$$

$$= \frac{\alpha}{2}\frac{1}{\sigma}\frac{d\theta}{dx} - \frac{\alpha}{2}\frac{\theta}{\sigma^2}\frac{d\sigma}{dx} \pm \frac{1}{2}\frac{2\frac{\alpha^2}{4}\left(\frac{\theta}{\sigma^2}\frac{d\theta}{dx} - \frac{\theta^2}{\sigma^3}\frac{d\sigma}{dx}\right)}{\sqrt{}}$$

$$\pm\frac{1}{2}\frac{\frac{4}{3}\tau_b\frac{c}{A}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)}{\sqrt{}}$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d^2\theta}{dx^2} - \frac{c}{A}\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2 + \ldots - \frac{\alpha}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2 = \left[\frac{8}{3}\tau_b\frac{d\sigma}{dx} - \frac{\alpha}{\sigma}\right]\left[\frac{g + \alpha\frac{d\theta}{dx} - \alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx}}{\frac{4}{3}\tau_b\,\theta\sigma} - \frac{\frac{d\sigma}{dx}\frac{d\theta}{dx}}{\theta}\right.$$

$$\left[\frac{8}{3}\frac{\gamma}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}\right]\frac{d^2\gamma}{dx^2}+\frac{a}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2=\frac{c}{A}\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right) \quad\Big|\quad \frac{4}{3}\frac{\gamma}{b}\theta\sigma$$

$$\frac{4}{3}\frac{\gamma}{b}\theta\sigma\frac{d^2\gamma}{dx^2}+\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}\Big]\frac{4}{3}\frac{\gamma}{b}+\alpha\frac{\theta}{\sigma}\frac{d\sigma}{dx}=g+\alpha\frac{d\theta}{dx} \quad\Big|\quad \frac{8}{3}\frac{\gamma}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}$$

$$-\frac{4}{3}\frac{\gamma}{b}\alpha\frac{\theta}{\sigma}\left(\frac{d\sigma}{dx}\right)^2+2\left(\frac{4}{3}\frac{\gamma}{b}\right)^2\frac{d\theta}{dx}\left(\frac{d\sigma}{dx}\right)^2\sigma-\frac{4}{3}\frac{\gamma}{b}\alpha\frac{d\sigma}{dx}\frac{d\theta}{dx}+\frac{8}{3}\frac{\gamma}{b}\alpha\frac{\theta}{\sigma}\left(\frac{d\sigma}{dx}\right)^2-$$

$$-\alpha^2\frac{\theta}{\sigma^2}\frac{d\sigma}{dx}=\frac{8}{3}\frac{\gamma}{b}g\frac{d\sigma}{dx}-\frac{\alpha g}{\sigma}+\frac{8}{3}\frac{\alpha\gamma}{b}\frac{d\sigma}{dx}\frac{d\theta}{dx}-\frac{\alpha^2}{\sigma}\frac{d\theta}{dx}-\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\theta\sigma\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\frac{4}{3}\frac{\gamma\alpha}{b}\frac{\theta}{\sigma}\left(\frac{d\sigma}{dx}\right)^2+2\left(\frac{4}{3}\frac{\gamma}{b}\right)^2\frac{d\theta}{dx}\sigma\left(\frac{d\sigma}{dx}\right)^2-4\frac{\gamma\alpha}{b}\frac{d\sigma}{dx}\frac{d\theta}{dx}-\alpha^2\frac{\theta}{\sigma^2}\frac{d\sigma}{dx}$$

$$=\frac{8}{3}\frac{\gamma g}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}g-\frac{\alpha^2}{\sigma}\frac{d\theta}{dx}-\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\theta\sigma\frac{d}{dx}\left(\frac{1}{\theta}\frac{d\theta}{dx}\right)$$

$$\frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^2=\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx}+\alpha\frac{1}{\sigma}\frac{d\sigma}{dx}$$

$$\frac{\alpha c}{A}\frac{1}{\sigma}\frac{d\theta}{dx}+\cancel{\frac{\alpha^2}{\sigma^2}\theta\frac{d\sigma}{dx}}+\frac{4}{3}\frac{\gamma}{b}\frac{d\theta}{dx}\left[\frac{c}{A}\frac{\sigma}{\theta}\frac{d\theta}{dx}+\alpha\frac{d\sigma}{dx}\right]\cancel{-4\frac{\gamma\alpha}{b}\frac{d\sigma}{dx}\frac{d\theta}{dx}-\frac{\alpha^2\theta}{\sigma^2}\frac{d\sigma}{dx}}$$

$$=\frac{8}{3}\frac{\gamma g}{b}\frac{d\sigma}{dx}-\frac{\alpha}{\sigma}g-\frac{\alpha^2}{\sigma}\frac{d\theta}{dx}-\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\theta\sigma\underbrace{\left[\qquad\right]}_{\frac{1}{\theta}\frac{d^2\theta}{dx^2}-\cancel{\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2}}$$

$$\frac{4}{3}\frac{\gamma}{b}\frac{d\sigma}{dx}\left[\alpha\frac{d\theta}{dx}-g\right]+\frac{\alpha g}{\sigma}+\left[\alpha^2+\frac{\alpha c}{A}\right]\frac{1}{\sigma}\frac{d\theta}{dx}+$$

$$+\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\frac{\sigma}{\theta}\left(\frac{d\theta}{dx}\right)^2+\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\sigma\frac{d^2\theta}{dx^2}=0$$

$$\frac{2}{3}\frac{\gamma}{b}\frac{d(\sigma^2)}{dx}\left[\alpha\frac{d\theta}{dx}-g\right]+\alpha g+\alpha\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\frac{\gamma}{b}\frac{c}{A}\frac{\sigma^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)=0$$

$$\frac{1}{3}\frac{\gamma}{b}\left[\frac{d(\sigma^2)}{dx}\right]^2-\frac{\alpha}{2}\frac{d(\sigma^2)}{dx}=\frac{c}{A}\frac{\sigma^2}{\theta}\frac{d\theta}{dx} \quad\Big|\quad \frac{4}{3}\frac{\gamma}{b}{}'$$

$$\left[\alpha g+\alpha\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\gamma_b\frac{c}{A}\,\frac{\sigma^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)\right]^2-\alpha\left[\alpha\frac{d\theta}{dx}-g\right]\left[\alpha g+\alpha\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\sim\quad\right]$$

$$=\frac{4}{3}\gamma_b\frac{c}{A}\left[\alpha\frac{d\theta}{dx}-g\right]^2\frac{\sigma^2}{\theta}\frac{d\theta}{dx}$$

~~$\left[\qquad\right]^2-\alpha^2\left[\alpha\frac{d\theta}{dx}-g\right]\left[\alpha g+\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}\right]-\alpha\left[\alpha\frac{d\theta}{dx}-g\right]\frac{4}{3}\gamma_b$~~

$\sigma^2=$ .....

$$\left[\frac{\sigma^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)\frac{4}{3}\gamma_b\frac{c}{A}\right]^2+\left[\frac{\sigma^2}{\theta}\frac{d}{dx}\left(\theta\frac{d\theta}{dx}\right)\frac{4}{3}\gamma_b\frac{c}{A}\right]\left[2\alpha\left[g+\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}\right]-\alpha^2\frac{d\theta}{dx}+\alpha g\right]$$

$$-\frac{4}{3}\gamma_b\frac{c}{A}\frac{\sigma^2}{\theta}\frac{d\theta}{dx}\left[\alpha\frac{d\theta}{dx}-g\right]^2=\alpha^2\left[\alpha\frac{d\theta}{dx}-g\right]\left[g+\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}\right]-$$

$$-\alpha^2\left[g+\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}\right]^2$$

$$=\alpha^2\left[g+\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}\right]\left[\alpha\frac{d\theta}{dx}-g-g-\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}\right]$$

Wstawiając to σ^2 w równanie otrzymalibyśmy równanie 3 stopnia dla ϑ

(nie zawierające już γ_b)

To równanie $\sigma^2=$ ..... będzie służyć do oznaczenia stałych

Ogólniejszy przykład:

Rozkład isothermiczny:

$$\rho = \frac{p}{\alpha\theta}$$

$$\left\{\begin{array}{l} p = \alpha\theta\rho \\ 0 = -g - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{4}{3}\frac{\mu}{\rho}\frac{d^2\sigma}{dx^2} \\ \rho\sigma = b = \rho_0\sigma_0 = \frac{p_0\sigma_0}{\alpha\theta_0} \end{array}\right. \qquad \rho = \frac{b}{\sigma}$$

$$\frac{d^2\sigma}{dx^2} = -\frac{d}{dx}\left(\frac{\sigma}{\rho}\frac{d\rho}{dx}\right)$$

$$\rho\frac{\partial\sigma}{\partial x} + \sigma\frac{\partial\rho}{\partial x} = 0$$

~~$\frac{\sigma}{b} = -\frac{1}{\sigma}\frac{\partial\sigma}{\partial x}\frac{\partial\rho}{\partial x}$~~

~~$\sigma + \frac{1}{a+\sigma}$~~

~~$u = \frac{\mu}{a+2\sigma}$~~

~~$+\frac{4}{3}\frac{\mu}{b}\left[\sigma + \frac{1}{a+\sigma}\right]\left[1 + \frac{1}{a+\sigma}\right]$~~ [illegible]

~~$\frac{1}{\sigma^2(a+\sigma)}$~~

$$0 = -g - \alpha\theta\frac{1}{\rho}\frac{\partial\rho}{\partial x} + \frac{4}{3}\frac{\mu}{b}\sigma\frac{d^2\sigma}{dx^2}$$

$$0 = -g + \alpha\theta\frac{1}{\sigma}\frac{d\sigma}{dx} + \frac{4}{3}\frac{\mu}{b}\sigma\frac{d^2\sigma}{dx^2}$$

~~$\frac{d\sigma}{dx} = u \qquad \frac{d^2\sigma}{dx^2} = \frac{du}{dx} = u\frac{du}{d\sigma}$~~

$$2\sigma\frac{d\sigma}{dx} = u \qquad \frac{du}{dx} = 2\left(\frac{d\sigma}{dx}\right)^2 + 2\sigma\frac{d^2\sigma}{dx^2}$$

$$2\sigma\frac{d^2\sigma}{dx^2} = \frac{du}{dx} - \frac{1}{2}\left(\frac{u}{\sigma}\right)^2$$

~~$0 = -\frac{g}{\sigma} + \alpha\theta\frac{u}{\sigma^2} + \frac{4}{3}\frac{\mu}{b}u\frac{du}{d\sigma}$~~

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{u}{2\sigma^2} \quad\Big\| \quad = \frac{du}{d(\sigma^2)}u - \frac{1}{2}\frac{u^2}{\sigma^2}$$

~~$\frac{d\sigma}{du}\left(g - \alpha\theta\frac{u}{\sigma}\right) = \frac{4}{3}\frac{\mu}{b}u\sigma$~~

~~$0 = -g + \alpha\theta\frac{u}{\sigma^2} + \frac{4}{3}\frac{\mu}{b}\left[\frac{du}{dx} - \left(\frac{u}{\sigma}\right)^2\right]$~~

$$0 = -g + \alpha\theta\frac{u}{2} + \frac{4}{3}\frac{\mu}{b}\left[\frac{1}{2}\frac{du}{d(\sigma^2)}u - \frac{1}{4}\frac{u^2}{\sigma^2}\right]$$

$$\frac{2}{3}\frac{\mu}{b}\frac{du}{d(\sigma^2)} - \frac{1}{3}\frac{\mu}{b}\frac{u}{\sigma^2} + \frac{\alpha\theta}{2} - \frac{g}{u} = 0$$

$$0 = -\frac{g}{\sigma} + \frac{\alpha\theta}{\sigma^2}u'v + \frac{4}{3}\frac{\mu}{b}u'v\left[u'\frac{dv}{d\sigma} + v\frac{du'}{d\sigma}\right]$$

$$u'v\left[\frac{\alpha\theta}{\sigma^2} + \frac{4}{3}\frac{\mu}{b}u'\frac{dv}{d\sigma}\right.$$

$u\sigma = x \qquad \frac{u}{\sigma} = y$

$u^2 = xy \qquad \frac{x}{y} = \sigma^2$

~~$2u\frac{dy}{d\sigma}$~~ [illegible]

$u\,d\sigma + \sigma\,du = dx$

$-\frac{u}{\sigma^2}d\sigma + \frac{du}{\sigma} = dy \;\Big|\cdot\sigma^2$

$2u\,d\sigma = dx - \sigma^2 dy$

$2\sigma\,du = dx + \sigma^2 dy$

$$\frac{d\sigma}{du} = \frac{1 - \frac{dy}{dx}\frac{x}{y}}{1 + \frac{dy}{dx}\frac{x}{y}}\,\frac{1}{y}$$

?

$\sigma^2 = y$

$$0 = g + \frac{\alpha\theta}{2y}\frac{dy}{dx} + \frac{4}{3}\frac{\mu}{b}\left[\frac{1}{2}\frac{d^2y}{dx^2} - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]$$

$2\sigma\frac{d\sigma}{dx} = \frac{dy}{dx}$

$2\left(\frac{d\sigma}{dx}\right)^2 + 2\sigma\frac{d^2\sigma}{dx^2} = \frac{d^2y}{dx^2}$

$$= g ~~+ \frac{1}{y}\frac{d}{dx}\left[\frac{\alpha\theta}{2}y + \frac{4}{3}\frac{\mu}{b}\right]~~ \quad \frac{\frac{d}{dx}\left[\frac{1}{y}\left(\frac{dy}{dx}\right)^2\right]}{\frac{dy}{dx}} = [illegible]\cdot y$$

$$\sigma\frac{d^2\sigma}{dx^2} = \frac{1}{2}\frac{d^2y}{dx^2} - \frac{1}{4}\frac{1}{\sigma^2}\left(\frac{dy}{dx}\right)^2$$

~~$= \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2$~~

$\frac{2}{y}\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

$\frac{1}{\sigma}\frac{d\sigma}{dx}$ [illegible]

$$0 = -g + \frac{\alpha\theta}{2}\frac{1}{y}\frac{dy}{dx} + \frac{4}{3}\frac{\mu}{b}\frac{\frac{d}{dx}\left[\frac{1}{y}\left(\frac{dy}{dx}\right)^2\right]}{\frac{1}{y}\frac{dy}{dx}}$$

$\frac{1}{y}\frac{dy}{dx} = z$

dy

$$0 = -g + \frac{\alpha\theta}{2}z + \frac{1}{3}\frac{\mu}{b}\frac{\frac{d}{dy}(z^2y)}{z} ~~z~~\, y$$

$$\theta = -\frac{g}{y} + \frac{\alpha\theta}{2}\frac{z}{y} + \frac{1}{3}\frac{\mu}{b}z^2 + \frac{2}{3}\frac{\mu}{b}y\,z\frac{dz}{dy}$$

$\frac{du}{d\sigma}\frac{4}{3}\frac{\mu}{b} = -\frac{\theta\alpha}{\sigma^2}$

$\alpha\theta\frac{u'}{\sigma^2} + \frac{4}{3}\frac{\mu}{b}u'\frac{du'}{d\sigma} = 0 \qquad u = u' + v$

$$0 = -\frac{g}{\sigma} + \alpha\theta\left[\frac{u'}{\sigma^2} + \frac{v}{\sigma^2}\right] + \frac{4}{3}\frac{\mu}{b}[u' + v]\left(\frac{du'}{d\sigma} + \frac{dv}{d\sigma}\right) = 0$$

$$0 = -\frac{g}{\sigma} + ~~\alpha\theta\frac{v}{\sigma^2}~~ + \frac{4}{3}\frac{\mu}{b}u'\frac{dv}{d\sigma} + ~~\frac{4}{3}\frac{\mu}{b}v\frac{du'}{d\sigma}~~ + \frac{4}{3}\frac{\mu}{b}v\frac{dv}{d\sigma} = 0$$

$uz = t$

$u + z\frac{du}{dz} = \frac{dt}{dz}$

$\frac{u}{z} = t$

$u = \frac{1}{tz}$

$\frac{du}{dt} = \frac{1}{t^2z}$ [illegible] $- \frac{1}{tz^2}\frac{dz}{dt}$

$$0 = -\frac{g}{\sigma} + \frac{4}{3}\frac{\mu}{b}\frac{dv}{d\sigma}(u' + v)$$

~~$= -\frac{g}{\sigma} + \frac{4}{3}\frac{\mu}{b}v\frac{dv}{d\sigma} + \frac{4}{3}\frac{\mu}{b}\frac{d(u'v)}{d\sigma} + v\frac{\theta\alpha}{\sigma^2}$~~

$\frac{1}{\sigma} = z \qquad \frac{dz}{d\sigma} = -\frac{1}{\sigma^2}$

$0 = -gz + \alpha\theta u z^2 - \frac{4}{3}\frac{\mu}{b}u\frac{du}{dz}z^4$

$\frac{4}{3}\frac{\mu}{b}\frac{du}{dz} = \alpha\theta - \frac{g}{uz}$

$u\frac{d}{dz}\left(\frac{4}{3}\frac{\mu}{b}u - \alpha\theta z\right) + \frac{g}{dz}dz = 0$

$$u = \frac{\frac{g}{\sigma}}{\frac{\alpha\theta}{\sigma^2} + \frac{4}{3}\frac{\mu}{\sigma} p}$$

$$p = \frac{-\frac{g}{\sigma^2}}{\frac{\alpha\theta}{\sigma^2} + \frac{4}{3}\frac{\mu}{\sigma} p} - \frac{\frac{g}{\sigma}\left[\frac{2\alpha\theta}{\sigma^3} + \frac{4}{3}\frac{\mu}{\sigma}\frac{dp}{dx}\right]}{(\qquad)^2}$$

$h = 0 \qquad x = a \qquad y = \left(\alpha + \frac{c}{A}\right)\theta - \varepsilon \qquad u = \left(1 + \frac{c}{A\alpha}\right) - \delta$

$$> 0 \qquad\qquad = \frac{k}{k-1} - \delta < \cancel{3}\ \frac{7}{2}$$

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$$\frac{2c}{A\alpha}\left[1+\frac{c}{A\alpha}+\frac{m-gx}{\alpha\theta}\right]=\frac{d}{dx}\left[\frac{\left[1+\frac{c}{A\alpha}+\frac{m-gx}{\alpha\theta}\right]\left[\frac{c}{A\alpha}+\frac{m-gx}{\alpha\theta}\right]}{\left(\alpha\overset{1}{\theta}\right)\frac{d(\alpha\theta)}{dx}}\right]$$

m przybliżone $= ga-\left(\alpha+\frac{c}{A}\right)\theta_0+\varepsilon$

$$\left[\,y>\left(\alpha+\frac{c}{A}\right)\theta_0-\varepsilon\right.$$

~~A~~ $\frac{\alpha A}{c}=k-1$

$$gx-m=y=e^{z}$$

$$\alpha\theta=ue^{z} \qquad u=\frac{y}{\alpha\theta}$$

$$\frac{2c}{A\alpha}\left[1+\frac{c}{A\alpha}-\frac{\text{[illegible]}}{u}\right]\left(u+\frac{du}{dz}\right)^{2}=-[\;.\;-\;.$$

$$q=\int\left[2-\frac{1+\frac{c}{A\alpha}}{1+\frac{c}{A\alpha}-u}+\frac{\frac{c}{A\alpha}}{\frac{c}{A\alpha}-u}\right]e^{3u}\left(1+\frac{c}{\alpha A}-u\right)^{1+\frac{c}{A\alpha}}\left(\frac{c}{A\alpha}-u\right)^{\frac{c}{A\alpha}}du$$

$$=\frac{1}{1-p} \qquad =f(u)$$

$$\int\frac{du}{u\left[1-\frac{1}{f(u)}\right]}=\int dz=\int\frac{dy}{y}$$

$q_{u\to\infty}=\int 2e^{3u}\,du=$ ~~$\frac{2}{3}e^{3u}u^{2}-\frac{4}{3}\int e^{3u}u\,du$~~ $\qquad \int 2e^{u}du=2e^{u}$

~~$=\frac{4}{9}\left(e^{3u}u-\frac{e^{3u}}{3}\right)$~~

~~$=e^{3u}\left[\frac{2}{3}u^{2}-\frac{4}{9}u+\frac{4}{27}\right]=\frac{2}{3}e^{3u}\left[u^{2}-\frac{2}{3}u+\frac{2}{9}\right]$~~

$$=\frac{2}{3}e^{3u}$$

$\int\frac{du}{u\left[1-\frac{3}{2}e^{-3u}\right]}$ dla wielkich wartości u

$\int\frac{du}{u\left[1-\frac{e^{-u}}{2}\right]}$ dla małych u

nie ~~[illegible]~~ może się zdarzyć

Uwzględniając wpływ energii kinetycznej ale nie tarcia:

1). $p = \alpha \theta \rho$ $\qquad\qquad$ $\rho\sigma = b$

2). $\sigma \frac{d\sigma}{dx} = -g - \frac{1}{\rho}\frac{\partial p}{\partial x} = -g - \alpha \frac{\partial \theta}{\partial x} + \frac{\alpha \theta}{\sigma}\frac{\partial \sigma}{\partial x}$ $\qquad\qquad$ $p = \alpha \frac{\theta b}{\sigma}$

3). $\frac{c}{A\theta}\frac{\partial \theta}{\partial x} + \alpha \frac{1}{\sigma}\frac{\partial \sigma}{\partial x} = 0$

$$\left(\frac{\theta}{\theta_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^{\frac{\alpha A}{c}} = 1 \qquad \frac{\alpha A}{c} = k-1$$

$$\frac{\theta}{\theta_0} = \left(\frac{\sigma}{\sigma_0}\right)^{-(k-1)}$$

~~$\frac{\sigma^2}{2} = -gx$~~

2 − θ3): $\quad \sigma \frac{d\sigma}{dx} = -g - \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx}$

$$\frac{\sigma^2}{2} + gx + \left(\alpha + \frac{c}{A}\right)\theta = \text{const}$$

$$\frac{\sigma^2 - \sigma_0^2}{2} + g(x-a) + \alpha\frac{k}{k-1}(\theta - \theta_0) = 0$$

$$\frac{\sigma^2 - \sigma_0^2}{2} + g(x-a) + \frac{\alpha k}{k-1}\theta_0 \left[\left(\frac{\sigma}{\sigma_0}\right)^{-(k-1)} - 1\right] = 0$$

$$\frac{\sigma^2 - \sigma_0^2}{2} + g(x-a) + \frac{\alpha k}{k-1}\frac{\theta_0}{\sigma_0^{1-k}}\left[\sigma^{-(k-1)} - \sigma_0^{-(k-1)}\right] = 0$$

np. $k-1 = \frac{1}{2}$

$$\sigma^2 - \sigma_0^2 + \frac{2k}{k-1}\frac{\alpha\theta_0}{\sigma_0^{1-k}}\left[\frac{1}{\sqrt{\sigma}} - \frac{1}{\sqrt{\sigma_0}}\right] = -g(x-a)$$

~~$\frac{\sigma + \sigma_0}{2}\left(\sigma^{\frac{1}{2}} + \sigma_0^{\frac{1}{2}}\right)$~~

$$g(x-a) = \left(\sqrt{\sigma} - \sqrt{\sigma_0}\right)\left[\frac{2k}{k-1}\,\frac{\alpha\theta_0}{\sigma_0^{1-k}}\,\frac{1}{\sqrt{\sigma\sigma_0}} - (\sigma+\sigma_0)\left(\sqrt{\sigma}+\sqrt{\sigma_0}\right)\right]$$

$$\frac{\alpha\theta_0 b}{\sigma_0} = p_0$$

$$2g(x-a) = \frac{2k}{k-1}\,\frac{p_0\,\sigma_0^{\frac{3}{2}}}{b}\left[\frac{1}{\sqrt{\sigma_0}} - \frac{1}{\sqrt{\sigma}}\right] - (\sigma^2-\sigma_0^2)$$

$$= \frac{\sigma p_0}{b}\,\sigma_0\left[1-\sqrt{\frac{\sigma_0}{\sigma}}\right] - (\sigma^2-\sigma_0^2)$$

} nie może nigdy ~~być~~ osiągnąć wartości $\frac{\sigma p_0 \sigma_0}{b} = \sigma\alpha\theta_0$

$$\frac{\partial\theta}{\partial x} = \frac{\theta_0(1-k)}{\sigma_0^{1-k}}\,\sigma^{-k}\,\frac{\partial\sigma}{\partial x}$$

$$\Bigg[\underbrace{\sigma + \left(\alpha+\frac{c}{A}\right)(1-k)\frac{\theta_0}{\sigma_0}\left(\frac{\sigma}{\sigma_0}\right)^{-k}}_{=0}\Bigg] + g\,\cancel{\frac{\partial x}{\partial\sigma}} = 0$$

$$\underset{=0}{}$$

$$\cancel{\frac{\sigma}{\sigma_0} = (k\alpha\theta_0)^{\frac{1}{k+1}}}$$

$$\cancel{\theta = \theta_0\left[k\alpha\theta_0\right]^{\frac{1-k}{1+k}} = \frac{\theta_0}{\left[k\alpha\theta_0\right]^{\frac{k-1}{k+1}}}}$$

max dla x:

$$\sigma^{k+1} + \left(\alpha+\frac{c}{A}\right)(1-k)\,\theta_0\,\sigma_0^{-(k+1)} = 0$$

$$\left(\frac{\sigma}{\sigma_0}\right)^{k+1} = (k-1)\left(\alpha+\frac{c}{A}\right)\theta_0 = k\alpha\theta_0$$

$$\frac{\sigma}{\sigma_0} = \left[k\alpha\theta_0\right]^{\frac{2}{3}}$$

$$\frac{\sigma_0}{\sigma} = \left[k\alpha\theta_0\right]^{-\frac{2}{3}}$$

$$2g(x-a)_{max} = \sigma\,\frac{p_0\sigma_0}{b}\left[1-\left[k\alpha\theta_0\right]^{-\frac{1}{3}}\right] - \sigma_0^2\left[(k\alpha\theta_0)^{\frac{4}{3}}-1\right]$$

$$= \sigma\alpha\theta_0\left[1-\left[k\alpha\theta_0\right]^{-\frac{1}{3}}\right] - \sigma_0^2\left[(k\alpha\theta_0)^{\frac{4}{3}}-1\right]$$

$$\boxed{= \sigma\alpha\theta_0 + \sigma_0^2 - \sigma}$$

~~więc~~ tutaj granica jeszcze niższa o

$$\sigma\alpha\theta_0\left[k\alpha\theta_0\right]^{-\frac{1}{3}} + \sigma_0^2\left[\quad\right]$$

$$= -\sigma_0^2 + (k\alpha\theta_0)^{-\frac{1}{3}}\left[\sigma\alpha\theta_0 + (k\alpha\theta_0)^{\frac{5}{3}}\right]$$

Bez tego ~~członu~~ dla $\theta_0 = 0$: $2g(x-a) = 2\frac{\alpha k}{k-1}\theta_0 = \sigma\alpha\theta_0$

Równanie kompletne:

$$\begin{cases} \frac{\sigma^2}{2} = \cancel{gx=} -gx+m-\left(\alpha+\frac{c}{A}\right)\theta + \frac{4}{3}\frac{\gamma}{b}\theta\,\sigma\frac{d\sigma}{dx} \\ \frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \frac{\alpha}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dx}\right)^{-} \end{cases}$$

$$\frac{\sigma_0^2}{2} = m - \left(\alpha+\frac{c}{A}\right)\theta_0 + \frac{4}{3}\frac{\gamma}{b}\theta_0\left(\sigma\frac{d\sigma}{dx}\right)_0$$

$$m = \left(\alpha+\frac{c}{A}\right)\theta_0 + \frac{\sigma_0^2}{2} - \frac{4}{3}\frac{\gamma}{b}\theta_0\left(\sigma\frac{d\sigma}{dx}\right)_0$$

$$gx-m=y$$

$$\begin{cases} \frac{\sigma^2}{2} = -y-\left(\alpha+\frac{c}{A}\right)\theta + \frac{4}{3}\frac{\gamma}{b}\theta\,\sigma\frac{d\sigma}{dy} \\ \frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dy} + \frac{\alpha}{\sigma}\frac{d\sigma}{dy} = \frac{4}{3}\frac{\gamma}{b}\left(\frac{d\sigma}{dy}\right)^2 \end{cases}$$

Zaniedbując σ^2:

$$\frac{4}{3}\frac{\gamma}{b}\theta\,\sigma\frac{d\sigma}{dy} = y+\left(\alpha+\frac{c}{A}\right)\theta$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dy}\sigma^2 + \alpha\,\sigma\frac{d\sigma}{dy} = \frac{4}{3}\frac{\gamma}{b}\left(\sigma\frac{d\sigma}{dy}\right)^2 \Big| \frac{4}{3}\frac{\gamma}{b}$$

$$\frac{4}{3}\frac{\gamma}{b}\sigma^2 = \frac{1}{\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dy}}\left[-\alpha\left(\frac{y}{\theta}+\underbrace{\alpha+\frac{c}{A}\right)+\left(\frac{y}{\theta}+\alpha+\frac{c}{A}\right)^2}_{=\left(\alpha+\frac{c}{A}+\frac{y}{\theta}\right)\left(\frac{y}{\theta}+\frac{c}{A}\right)}\right]$$

−1

$$2\left[\alpha+\frac{c}{A}+\frac{y}{\theta}\right] = \frac{d}{dy}\left[\frac{\left(\alpha+\frac{c}{A}+\frac{y}{\theta}\right)\left(\frac{c}{A}+\frac{y}{\theta}\right)}{\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dy}}\right]$$

$\rho\theta_0 = 0{\cdot}000\,18$ (grammes)

$$\frac{2c}{A\alpha}\left[1+\frac{c}{A\alpha}+\frac{y}{\alpha\theta}\right] = \frac{d}{dy}\left[\frac{\left(1+\frac{c}{A\alpha}+\frac{y}{\alpha\theta}\right)\left(\frac{c}{A\alpha}+\frac{y}{\alpha\theta}\right)}{\frac{1}{\theta}\,\frac{d\theta}{dy}}\right]$$

$$\frac{c}{A\alpha} = \frac{1}{k-1}$$

$$\frac{2}{k-1}\left[\frac{k}{k-1}+\frac{y}{\alpha\theta}\right] = \frac{d}{dy}\left[\frac{\left(\frac{k}{k-1}+\frac{y}{\alpha\theta}\right)\left(\frac{1}{k-1}+\frac{y}{\alpha\theta}\right)}{\frac{1}{\theta}\,\frac{d\theta}{dy}}\right]$$

$$(k-1)y = e^{z} \qquad \frac{(k-1)y}{\alpha\theta} = u$$

$$\alpha\theta = \frac{1}{u}e^{z}$$

$$\log\theta = \cancel{k}\, z - \log u$$

$$\frac{1}{\theta}\frac{d\theta}{dz} = 1-\frac{1}{u}\frac{du}{dz} = \frac{1}{\theta}\frac{d\theta}{dy}\frac{dy}{dz}$$

$$= e^{z}\,\frac{1}{\theta}\frac{d\theta}{dy}$$

$$2(k+u) = \frac{d}{d(e^{z})}\left[\frac{(k+u)(1+u)}{\left(1-\frac{1}{u}\frac{du}{dz}\right)e^{-z}}\right]$$

$$= e^{-z}\,\frac{d}{dz}\left[e^{z}\frac{(k+u)(1+u)}{\left(1-\frac{1}{u}\frac{du}{dz}\right)}\right]$$

$$(k+u)(1+u) = k+(1+k)u+u^2$$

$$2(k+u) = \frac{d}{dz}\left[\frac{(k+u)(1+u)}{1-\frac{1}{u}\frac{du}{dz}}\right] + \frac{(k+u)(1+u)}{1-\frac{1}{u}\frac{du}{dz}}$$

$$2(k+u)\left(1-\frac{1}{u}\frac{du}{dz}\right)^2 = +(k+u)(1+u)\frac{d}{dz}\left(\frac{1}{u}\frac{du}{dz}\right) + \left(1-\frac{1}{u}\frac{du}{dz}\right)(1+k+2u)\frac{du}{dz}$$
$$+\left(1-\frac{1}{u}\frac{du}{dz}\right)(k+u)(1+u)$$

$$\frac{1}{u}\frac{du}{dz} = p$$

~~$2(k+u)(1-p)^2 = (k+u)(1+u)\frac{dp}{dz} + p(1-p)(u+uk+2u^2) + (1-p)(k+u)(1+u)$~~

~~$(k+u)(1+u)\frac{dp}{dz} - p^2(u+ku+2u^2$~~

$$\frac{dp}{dz} = \frac{dp}{du}\cdot\frac{du}{dz} = p\,u\,\frac{dp}{du}$$

$$= u(1+u+k+u)$$

$$2(k+u)p^2 = -(k+u)(1+u)\frac{dp}{dx} + p(1-p)(u+uk+2u^2) + p(k+u)(1+u)$$

$$p\frac{dp}{dx} + \frac{p^2}{u}\left(\underbrace{\frac{2}{1+u} + \frac{u}{1+u} + \frac{u}{k+u}}\right) - \frac{p}{u}\left(1 + \frac{u}{1+u} + \frac{u}{k+u}\right) = 0$$

$$= 2 - \frac{1}{1+u} - \frac{k}{k+u}$$

$$\frac{1}{p}\frac{dp}{dx} + \frac{1}{u}\left(2 + \frac{1}{1+u} - \frac{k}{k+u}\right) - \frac{1}{p}\frac{1}{u}\left(3 - \frac{1}{1+u} - \frac{k}{k+u}\right) = 0$$

$$\frac{1}{p} = q$$

$$q\frac{dq}{dx} + \frac{q}{u}\left(3 - \frac{1}{1+u} - \frac{k}{k+u}\right) - \frac{1}{u}\left(2 + \frac{1}{1+u} - \frac{k}{k+u}\right) = 0$$

$$2(k+u)(1-p)^2 = (k+u)(1+u)u\,p\frac{dp}{du} + p(1-p)(u+uk+2u^2) + (1-p)(k+u)(1+u)$$

Jeżeli $\frac{4}{3}J_b\frac{d\sigma}{dx} = \frac{\alpha}{\sigma}$ to mamy $\frac{d\theta}{dx} = 0$

i $\frac{4}{3}J_b\frac{d\sigma}{dx}\sigma = \alpha$ więc: $g x \neq \left(\alpha + \frac{c}{A}\right)\theta_0 - \frac{c}{A}\theta$

więc różniczkując: $g = \frac{4}{3}J_b\,\theta\,\frac{d}{dx}\left(\sigma\frac{d\sigma}{dx}\right)\Big|_{x=\dots}$

Gdy α większe niż $gx = \left(\alpha + \frac{c}{A}\right)\theta_0 - \frac{c}{A}\theta$

to $\frac{4}{3}J_b\frac{d\sigma}{dx} > \frac{\alpha}{\sigma}$ zatem $\frac{d\theta}{dx} > 0$!

$$(u^2 + 4u + 3k)$$

$$(k+u)(1+u)u\,p\frac{dp}{du} + p^2\left[u \mp uk \mp 2u^2 - 2k - 2u^2\right] + p\left[u + uk + 2u^2 + 4k + 4u - k - ku - k - u^2\right] + \underbrace{k + u + ku + u^2 - 2k - 2u}_{u^2 + ku - u - k}$$

$$-p^2\left[4u^2 - u(k+1) - 2k\right]$$

Tam gdzie $\frac{d\theta}{dx} = 0$:

$\frac{4}{3}\gamma_b \frac{d\sigma}{dx} = \frac{\alpha}{\sigma}$ więc:

$$\left.\frac{\sigma^2}{2}\right) = -y - \left(\alpha + \frac{c}{A}\right)\theta + \alpha\theta = -y - \frac{c}{A}\theta$$

Zaniedbując mnożymy $= -gx + m - \frac{c}{A}\theta$

$$gx = \left(\alpha + \frac{c}{A}\right)\theta_0 - \frac{c}{A}\theta$$

$0 = -gx + \left(\alpha + \frac{c}{A}\right)(\theta_0 - \theta) + \frac{4}{3}\gamma_b\,\theta\,\sigma\frac{d\sigma}{dx}$ I).

$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx} + \frac{\alpha}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\gamma_b\left(\frac{d\sigma}{dx}\right)^2$ II).

Najpierw $\frac{d\theta}{dx} < 0$, zatem z II: $\frac{d\sigma}{dx} > 0$

$\left[\frac{4}{3}\gamma_b \frac{d\sigma}{dx} < \frac{\alpha}{\sigma}\right.$

~~Gdy $gx = \left(\alpha + \frac{c}{A}\right)(\theta_0 - \theta)$ to z II)? $\frac{d\sigma}{dx} = 0$~~

ale z I): θ ciągle większe aniżeli gdyby $\gamma = 0$ było

[wtedy ono by się stało $= 0$ przy $gx = \left(\alpha + \frac{c}{A}\right)\theta_0$

~~Gdy $gx < \left(\alpha + \frac{c}{A}\right)(\theta_0 - \theta)$ to z I). : $\frac{d\sigma}{dx} = 0$~~ nie mamy

tutaj jednak przy $gx = \left(\alpha + \frac{c}{A}\right)\theta_0$ mamy

I). $\alpha + \frac{c}{A} = \frac{4}{3}\gamma_b\,\sigma\frac{d\sigma}{dx}$ z czego wynika:

$$\left(\alpha + \frac{c}{A}\right)\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\gamma_b\left(\frac{d\sigma}{dx}\right)^2 = \frac{\alpha}{\sigma}\frac{d\sigma}{dx} + \frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dx}$$

więc $\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{1}{\theta}\frac{d\theta}{dx}$ ~~a nie możliwe~~

więc $\theta = 0$?!

$$y\frac{dy}{dx} + yX_1 + X_2 = 0$$

$$\frac{1}{2}\frac{dz}{dx} + X_1\sqrt{z} + X_2 = 0$$

$$\frac{y^2}{2} + \int yX_1\,dx + \int X_2\,dx = 0$$

$$y + \int X_1\,dx + \int X_2\frac{dx}{y} = 0$$

~~$\frac{y^2}{2} + \int X_2\,dx + y\int X_1\,dx - \underbrace{\int dy\int X_1\,dx}$~~

~~$\underbrace{= -\int dx\int X_1\,dx\left[X_1 + \frac{X_2}{y}\right]}$~~

~~$= +\int X_1\int X_1\,dx\,dx + \int\frac{X_2}{y}\,dx\int X_1\,dx$~~

$$y = u + v$$

$$(u+v)\left(\frac{du}{dx} + \frac{dv}{dx}\right) + (u+v)X_1 + X_2 = 0$$

$$u\frac{du}{dx} + u\frac{dv}{dx} + v\frac{du}{dx} + v\frac{dv}{dx} + uX_1 + vX_1 + X_2 = 0$$

$$y = u + a$$

$$(y+a)\frac{dy}{dx} + (y+a)X_1 + X_2 = 0$$

$$y\frac{dy}{dx} + yX_1 + X_2 + a\left(\frac{dy}{dx} + X_1\right) = 0$$

$$y_1 \frac{dy_1}{dx} + y_1 X_1 \qquad = 0$$

~~$\frac{1}{2} \frac{d(y^2 - y_1^2)}{dx} + (y - y_1) X_1 + X_2 = 0$~~

~~$\frac{1}{2} \frac{d}{dx}\left[(y-y_1)(y+y_1)\right] +$~~ $\nearrow$

~~$(y-y_1)\left[\frac{1}{2}\frac{d}{dx}(y+y_1) + X_1\right] + \frac{1}{2}(y+y_1)\frac{d(y-y_1)}{dx} + X_2 = 0$~~

~~$(y-y_1) \frac{dy}{dx} + (y+y_1) \frac{d(y-y_1)}{dx} + 2X_2 = 0$~~

$$\overbrace{\phantom{y_1 \frac{dy}{dx} - y_1 \frac{dy_1}{dx}}}$$

~~$y \frac{dy}{dx} - y \frac{dy_1}{dx} + y_1 \frac{dy}{dx} - \cancel{y_1 \frac{dy_1}{dx}}$~~ $+ 2X_2$

$+ y_1 X_1$

~~$2 \frac{dy}{dx} y - y \frac{dy_1}{dx} + y_1 X_1 + 2X_2 = 0$~~

$$y_1 y \frac{dy}{dx} - y y_1 \frac{dy_1}{dx} + X_2 = 0$$

$$y y_1 \frac{d(y-y_1)}{dx} + X_2 = 0$$

$$(y - y_1 + y_1) y_1 \frac{d(y-y_1)}{dx} + X_2 = 0$$

$$(y - y_1) \frac{d(y-y_1)}{dx} + y_1^2 \frac{d(y-y_1)}{dx} + X_2 = 0$$

$$u \frac{du}{dx} + y_1^2 \frac{du}{dx} + X_2 = 0$$

$$\frac{dy}{dx} + X_1^{E} + \frac{1}{y} X_2^{E} = 0$$

$$\frac{d^2y}{dx^2} + X_1' + \frac{X_2'}{y} - \frac{X_2}{y^2}\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} + X_1' + \frac{X_2'}{y} + \frac{X_1 X_2}{y^2} + \frac{X_2^2}{y^3} = 0$$

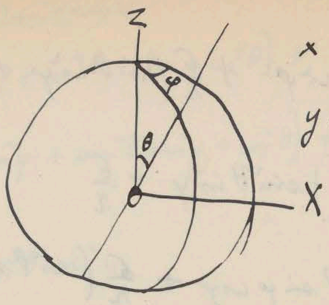
$$y = \frac{-X_2}{\frac{dy}{dx} + X_1} \qquad \frac{dy}{dx} = \frac{-X_2'}{\frac{dy}{dx} + X_1} + X_2 \frac{\frac{d^2y}{dx^2} + X_1'}{\left(\frac{dy}{dx} + X_1\right)^2}$$

$$\frac{dy}{dx}\left(\frac{dy}{dx} + X_1\right)^2 = -X_2'\left(\frac{dy}{dx} + X_1\right) + X_2\left(\frac{d^2y}{dx^2} + X_1'\right)$$

$$p\,(p + X_1)^2 = -X_2'(p + X_1) + X_2\left(\frac{dp}{dx} + X_1'\right)$$

$$\frac{dp}{dx} + f_1(x)\,p^3 + f_2(x)\,p^2 + f_3(x)\,p + f_4(x) = 0$$

Spherical symmetry :



$x = r \sin\theta \cos\varphi$

$y = r \sin\theta \sin\varphi$

$z = r \cos\theta$

~~$u = \sigma \cos\alpha = \frac{\sigma x}{r}$~~

$u = \sigma \sin\theta \cos\varphi$

$v = \sigma \sin\theta \sin\varphi$

$w = \sigma \cos\theta$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = \frac{\partial \sigma}{\partial r}\left[\sin^2\theta \cos^2\varphi + \sin^2\theta \sin^2\varphi + \cos^2\theta\right]$$

$$+ \frac{\sigma}{r}\left[\cos^2\theta \cos^2\varphi + \cos^2\theta \sin^2\varphi + \sin^2\theta\right]$$

$$+ \frac{\sigma}{r}\left[+\frac{\sin\theta \sin^2\varphi}{\sin\theta} + \frac{\cos^2\varphi \sin\theta}{\sin\theta}\right]$$

$$= \frac{\partial \sigma}{\partial r} + \frac{2\sigma}{r}$$

$$\nabla^2 u = \frac{\partial^2 \sigma}{\partial r^2} \sin\theta \cos\varphi + \frac{2}{r}\frac{\partial \sigma}{\partial r} \sin\theta \cos\varphi - \frac{\sigma \cancel{\sin\theta}}{r^2 \sin^2\theta} \cos\varphi - \frac{\sigma}{r^2} \sin\theta \cos\varphi + \frac{\sigma \cos^2\theta \cos\varphi}{r^2 \sin\theta}$$

$$= \left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r} - \frac{2\sigma}{r^2}\right] \sin\theta \cos\varphi$$

$$\nabla^2 w = \left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{2}{r}\frac{\partial \sigma}{\partial r} - \frac{2\sigma}{r^2}\right] \cos\theta$$

$$\left(\frac{\partial u}{\partial x}\right) = \frac{\partial\sigma}{\partial r}\sin^2\theta\cos^2\varphi + \frac{\sigma}{r}\left(\cos^2\theta\cos^2\varphi + \sin^2\varphi\right)$$

$$= \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\cos^2\varphi + \frac{\sigma}{r}$$

$$\frac{\partial u}{\partial y} = \frac{\partial\sigma}{\partial r}\sin^2\theta\sin\varphi\cos\varphi + \frac{\sigma}{r}\left(\cos^2\theta\sin\varphi\cos\varphi - \frac{\cos\varphi}{\cancel{\sin\theta}}\sin\varphi\right)$$

$$= \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\sin\varphi\cos\varphi$$

$$\frac{\partial u}{\partial z} = \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\cos\varphi$$

$$\frac{\partial v}{\partial x} = \frac{\partial\sigma}{\partial r}\sin^2\theta\sin\varphi\cos\varphi + \frac{\sigma}{r}\cos^2\theta\sin\varphi\cos\varphi - \frac{\sigma\sin\varphi\cos\varphi}{r\,\cancel{\sin\theta}}$$

$$= \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\sin\varphi\cos\varphi$$

$$\frac{\partial v}{\partial y} = \frac{\partial\sigma}{\partial r}\sin^2\theta\sin^2\varphi + \frac{\sigma}{r}\left(\cos^2\theta\sin^2\varphi + \cos^2\varphi\right)$$

$$= \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin^2\theta\sin^2\varphi + \frac{\sigma}{r}$$

$$\frac{\partial v}{\partial z} = \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\sin\varphi$$

$$\frac{\partial w}{\partial x} = \left(\frac{\partial\sigma}{\partial r} \cancel{\sin\theta\cos\theta\cos\varphi} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\cos\varphi$$

$$\frac{\partial w}{\partial y} = \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\sin\theta\cos\theta\sin\varphi$$

$$\frac{\partial w}{\partial z} = \frac{\partial\sigma}{\partial r}\cos^2\theta + \frac{\sigma}{r}\sin^2\theta = \left(\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right)\cos^2\theta + \frac{\sigma}{r}$$

~~$1-2\sin^2\varphi+\sin^4\varphi$~~

$$2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\left[\sin^4\theta\cos^4\varphi+\sin^4\theta\sin^4\varphi+\cos^4\theta\right]+$$

$$4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\left[\sin^2\theta\cos^2\varphi+\sin^2\theta\sin^2\varphi+\cos^2\theta\right]+6\left(\frac{\sigma}{r}\right)^2$$

$$\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2=4\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\sin^2\theta\cos^2\theta\sin^2\varphi$$

$$\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2=4\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\sin^2\theta\cos^2\theta\cos^2\varphi$$

$$\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2=4\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\sin^4\theta\sin^2\varphi\cos^2\varphi$$

~~$=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\left[\sin^4\theta\cos^4\varphi+\sin^4\theta\sin^4\varphi+\cos^4\theta+\sin^2\theta\cos^2\theta+\right.$~~

~~$\left.+\sin^4\theta\sin^2\varphi\cos^2\varphi\right]$~~

$+$ ~~$4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\left[\quad+6\left(\frac{\sigma}{r}\right)^2\right.$~~

~~$1-2\sin^2\theta+\sin^4\theta$~~ ~~$-\cos^2\theta\sin^2\varphi$~~

~~$\sin^2\theta\left(\cos^2\varphi-\cos^2\varphi\cos^2\theta+\sin^2\theta\sin^2\varphi-\sin^2\theta\sin^2\varphi\cos^2\varphi\right)+\cos^2\theta$~~

$1\quad-\cos^2\theta$

~~$=\sin^4\theta\left[1-\sin^2\varphi\cos^2\varphi\right]=\sin^4\theta\left[1-\sin^2\varphi+\sin^4\varphi\right]$~~

$$=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2\left[\cancel{\sin^4\theta\cos^4\varphi+\sin^4\theta\sin^4\varphi+\cos^4\theta+2\sin^2\theta\cos^2\theta+2\sin^4\theta\sin^2\varphi\cos^2\varphi}\right]+6\left(\frac{\sigma}{r}\right)^2$$

$$+4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\left[\cancel{\sin^2\theta\cos^2\varphi+\sin^2\theta\sin^2\varphi+\cos^2\theta}\right]$$

$$\Phi=2\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)^2+4\frac{\sigma}{r}\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)+6\left(\frac{\sigma}{r}\right)^2-\frac{2}{3}\left(\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right)^2=\frac{4}{3}\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right]^2$$

$+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$

$$u\frac{\partial u}{\partial x} = -\frac{ga^2 x}{r^2 r} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{3\rho}\frac{\partial}{\partial x}(div) + \frac{\mu}{\rho}\nabla^2 u$$

$$u\frac{\partial u}{\partial r} = -\frac{ga^2}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{3\rho}\partial \ldots —$$

~~$\sigma\frac{\partial\sigma}{\partial r}\sin\theta\cos\varphi$~~ $= -\frac{ga^2}{r^2}$ ~~$\sin^2\theta\cos\varphi$~~ $- \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{3\rho}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right] + \frac{\mu}{\rho}\left[\frac{\partial^2\sigma}{\partial r^2}+\frac{2}{r}\frac{\partial\sigma}{\partial r}-\frac{2\sigma}{r}\right]\sin\theta\cos\varphi$

~~$\sigma\frac{\partial\sigma}{\partial r}\sin\theta\sin\varphi = -\frac{ga^2}{r^2}\sin^2\theta\sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{3\rho}\frac{\partial}{\partial r}[\quad] + \frac{\mu}{\rho}[\quad]\sin\theta$~~

~~$\sigma\frac{\partial\sigma}{\partial r}\sin\theta\sin\varphi + \sin\theta\sin\varphi$~~

$$\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\left(\sin\theta\cos\varphi + \sin^3\theta\sin^2\varphi\cos\varphi + \sin\theta\cos^2\theta\cos\varphi\right) + \frac{\sigma}{r}\sin\theta\cos\varphi =$$

$$= \left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right) + \frac{\sigma^2}{r}\right]\sin\theta\cos\varphi -$$

~~$\sin^2\theta\cos\varphi$~~

$$\left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right) + \frac{\sigma^2}{r}\right]\sin\theta\sin\varphi =$$

~~$\sin^2\theta\sin\varphi$~~

~~$\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\sin^2\theta\cos^2\theta\cos^2\varphi + \sin^2\theta\cos\theta\sin^2\varphi + \cos^3\theta$~~

$$\left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right) + \frac{\sigma^2}{r}\right]\cos\theta =$$

~~$\sin\theta\cos\theta$~~

$$\underbrace{\left[\sigma\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right] + \frac{\sigma^2}{r}\right]}_{=\sigma\frac{\partial\sigma}{\partial r}} = -\frac{ga^2}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \underbrace{\frac{\mu}{3\rho}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right] + \frac{\mu}{\rho}\left[\frac{\partial^2\sigma}{\partial r^2}+\frac{2}{r}\frac{\partial\sigma}{\partial r}-\frac{2\sigma}{r^2}\right]}$$

$$\frac{\partial^2\sigma}{\partial r^2} + \frac{2}{r}\frac{\partial\sigma}{\partial r} - \frac{2\sigma}{r^2} + 3\frac{\partial^2\sigma}{\partial r^2} + \frac{6}{r}\frac{\partial\sigma}{\partial r} - \frac{6\sigma}{r^2}$$

$$4\frac{\partial^2\sigma}{\partial r^2} + \frac{8}{r}\frac{\partial\sigma}{\partial r} - \frac{8\sigma}{r^2}$$

$$= \frac{4}{3}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r}+\frac{2\sigma}{r}\right]$$

$$\sigma\frac{\partial\sigma}{\partial r} = -g\frac{a^2}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{4\mu}{3\rho}\frac{\partial}{\partial r}\left[\frac{\partial\sigma}{\partial r} + \frac{2\sigma}{r}\right]$$

$$\frac{c}{A}\rho\,\sigma\frac{\partial\theta}{\partial r} + \left(\frac{\partial\sigma}{\partial r} + \frac{2\sigma}{r}\right)p = \frac{4\mu}{3}\left[\frac{\partial\sigma}{\partial r} - \frac{\sigma}{r}\right]^2 \;\Big|\; r^2$$

$$p = \alpha\rho\theta \qquad \mu = \gamma\theta$$

$$\rho\sigma r^2 = \rho_0\sigma_0 a^2 = b$$

$$\rho = \frac{b}{\sigma r^2} \;\Big|\; p = \cancel{\alpha\theta}\,\alpha b\frac{\theta}{\sigma r^2}$$

$$\cancel{\sigma\,\partial\sigma}\;\sigma\frac{d\sigma}{dr} = -g\frac{a^2}{r^2} - \alpha\sigma r^2\frac{d}{dr}\left(\frac{\theta}{\sigma r^2}\right) + \frac{4\gamma\theta}{3b}\sigma r^2\frac{d}{dr}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]$$

$$\frac{c}{A}\frac{\partial\theta}{\partial r} + \alpha\frac{\theta}{\sigma r^2}\left(\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right) = \frac{4\gamma\theta}{3b}\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2$$

~~$\frac{\partial}{\partial r}\left\{\sigma r^2\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]\right\} = \left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]^2 + \sigma r^2\left[\frac{d^2\sigma}{dr^2} + \frac{2}{r}\frac{d\sigma}{dr} - \frac{2\sigma}{r^2}\right]$~~

$$\underline{\partial p_{xx} + \cdots} = -\frac{2}{3}\frac{\partial}{\partial x}(\mu\,\mathrm{div}) + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right]$$

$$+ 2\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)$$

$$= -\frac{2}{3}\frac{\partial\mu}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{2}{3}\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) + \frac{\partial\mu}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) +$$

$$+ \mu\left(\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial\mu}{\partial z}\left(\frac{\partial u}{\partial \cancel{y}z} + \frac{\partial w}{\partial x}\right) + \mu\left(\frac{\partial^2 u}{\cancel{\partial y}\partial z^2} + \frac{\partial^2 w}{\partial z\cancel{\partial x}}\right) + 2\frac{\partial\mu}{\partial x}\frac{\partial u}{\partial x} + 2\mu\frac{\partial^2 u}{\partial x^2}$$

$$= \mu\left[-\frac{2}{3}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\cancel{\partial x}\partial z} + 2\frac{\partial^2 u}{\partial x^2}\right]$$

$$+ \frac{\partial\mu}{\partial x}\left[-\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\frac{\partial u}{\partial x}\right] + \frac{\partial\mu}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + \frac{\partial\mu}{\partial z}\left(\frac{\partial w}{\cancel{\partial x}} + \frac{\partial u}{\partial z}\right)$$

$$= -\frac{2}{3}\frac{d}{dx}\left[\mu\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)\right] + \frac{d}{dx}\left(\mu\frac{du}{dx}\right)+\frac{d}{dy}\left(\mu\frac{du}{dy}\right)+\frac{d}{dz}\left(\mu\frac{du}{dz}\right)+$$

$$+\frac{d}{dx}\left(\mu\frac{du}{dx}\right)+\frac{d}{dy}\left(\mu\frac{dv}{dx}\right)+\frac{d}{dz}\left(\mu\frac{dw}{dx}\right)$$

$$-\frac{2}{3}\frac{\partial}{\partial x}\left[\mu\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)\right] + 2\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial\sigma}{\partial r}-\frac{\sigma}{r}\right)\sin^2\theta\cos\varphi+\frac{\sigma\mu}{r}\right]+2\frac{\partial}{\partial y}\left[\mu(\quad)\cdots\right]+2\frac{\partial}{\partial z}\left[\mu(\right.$$

~~$= -\frac{2}{3}\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)\right]\sin\theta\cos\varphi +$~~

~~$2\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]\left[\underbrace{\sin^3\theta\cos^3\varphi+\sin^3\theta\sin^2\varphi\cos\varphi+\sin\theta\cos^2\theta\cos\varphi}_{\sin\theta\cos\varphi}\right]$~~

~~$+ 2\frac{d}{dr}\left(\mu\frac{\sigma}{r}\right)\sin\theta\cos\varphi$~~

~~$= \frac{d}{dr}\left[-\frac{2}{3}\mu\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)+2\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)+2\mu\frac{\sigma}{r}\right]\sin\theta\cos\varphi$~~

~~$= \frac{d}{dr}\left[\mu\quad -\frac{2}{3}\frac{d\sigma}{dr}-\frac{4}{3}\frac{\sigma}{r}+2\frac{d\sigma}{dr}-\frac{2\sigma}{r}+\frac{2\sigma}{r}\right]$~~

~~$= \frac{d}{dr}\left[\mu\,\frac{4}{3}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right] = \frac{4}{3}\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]$~~

$$+\frac{d}{d\varphi}\qquad\frac{d\varphi}{dx}\qquad+\frac{d}{d\theta}\frac{d\theta}{dx}$$

$$+\frac{d}{dr}\left[-\frac{2}{3}\mu\frac{d\sigma}{dr}+\frac{2}{3}\mu\frac{\sigma}{r}\right]$$

$$+\frac{d}{dr}2\left[\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]+\frac{4}{r}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\qquad +\frac{4}{3}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\frac{d\mu}{dr}$$

$$= \frac{d}{dr}\left[\frac{4}{3}\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]+\frac{4\mu}{r}\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right] = \frac{4\mu}{3}\left[\frac{d^2\sigma}{dr^2}-\frac{1}{r}\frac{d\sigma}{dr}+\frac{\sigma}{r^2}+\frac{3}{r}\frac{d\sigma}{dr}-\frac{3\sigma}{r}\right]+$$

$$= \frac{4}{3}\mu\left[\frac{d^2\sigma}{dr^2}+\frac{2}{r}\frac{d\sigma}{dr}-\frac{2\sigma}{r^2}\right]$$

~~$\nabla^4 u + \frac{d}{dx}(\mathrm{div}) = \frac{d}{dr}\left(\frac{d\sigma}{dr}\right) \sim \dots$~~

~~$\sin\theta\cos\varphi\left[\frac{d^2\sigma}{dr^2}+\frac{2}{r}\frac{d\sigma}{dr}-\frac{2\sigma}{r^2}+\frac{d}{dr}\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)\right]$~~

~~$\frac{d^2\sigma}{dr^2}+\frac{2}{r}\frac{d\sigma}{dr}-\frac{2\sigma}{r^2}$~~

~~$\nabla^2 u = \frac{\partial}{dx}\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}\frac{\partial u}{\partial y}+\frac{\partial}{\partial z}\frac{\partial u}{\partial z}$~~

~~$= \frac{d}{dr}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\sin^3\theta\cos^3\varphi + \frac{d}{dr}\left(\frac{\sigma}{r}\right)\sin\theta\cos\varphi$~~

~~$+ \dots \sin^3\theta\sin^2\varphi\cos\varphi$~~

~~$\sin\theta\cos^2\theta\cos\varphi$~~

~~$= \sin\theta\cos\varphi\left[\frac{d}{dr}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)+\frac{d}{dr}\left(\frac{\sigma}{r}\right)\right]$~~

~~$= \sin\theta\cos\varphi\,\frac{d^2\sigma}{dr^2}$~~

~~$dx = 2\pi\sin$~~

$$= \frac{d}{dx}\left[-\frac{2}{3}\mu\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)\right] + 2\left[\overbrace{\frac{d}{dx}\left(\mu\frac{du}{dx}\right)}^{\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\sin\theta\cos\varphi\,+} + \overbrace{\frac{d}{dy}\left(\mu\frac{du}{dy}\right)}^{\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\sin^3\theta\sin^2\varphi\cos\varphi} + \dots \overbrace{\frac{d}{dz}\left(\mu\frac{du}{dz}\right)}^{\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\sin\theta\cos^2\theta\cos\varphi}\right)$$

$$+ 2\mu\frac{\sigma}{r}$$

$$\frac{d}{dr}\left[\mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]\sin^3\theta\cos^3\varphi + \mu\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\frac{2\sin\theta\cos^2\theta\cos^3\varphi + 2\sin\theta\cos\varphi\sin^2\varphi}{r}$$

~~$\sin^3\theta\sin^2\varphi\cos\varphi$~~

~~$\sin\theta\cos^2\theta\cos\varphi$~~

$$2\sin\theta\cos^2\theta\sin^2\varphi\cos\varphi + \sin^3\theta\cos\varphi - \sin\theta\sin\varphi\cos\varphi$$

$$-\sin\theta\cos\varphi(\cos^2\theta-\sin^2\theta)$$

$$=\sin\theta\left[2\cos^2\theta\cos\varphi + \cos\varphi - \cos^2\theta\cos\varphi + \sin^2\theta\cos\varphi\right]$$

$$= 2\sin\theta\cos\varphi$$ [illegible]

$$\sigma\frac{d\sigma}{dr} = -g\frac{a^2}{r^2} - \alpha\sigma r^2\frac{d}{dr}\left(\frac{\theta}{\sigma r^2}\right) + \frac{4\mu\theta}{3b}\,\sigma r^2\left[\frac{d}{dr}\left[\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right]+\right.$$

$$\left.+\frac{4\mu}{3b}\,\sigma r^2\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right]\frac{d\theta}{dr}\right]$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{\sigma}\left[\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right] = \frac{4\mu\theta}{3b}r^2\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right]^2$$

$$\sigma\frac{d\sigma}{dr}\cancel{\theta} = -g\frac{a^2}{r^2} - \alpha\frac{d\theta}{dr} + \frac{\alpha\theta}{\sigma}\left[\frac{d\sigma}{dr} + 2\frac{\sigma}{r}\right] + \frac{4\mu\theta}{3b}\ldots$$

$$= \frac{d}{dr}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right) + \frac{3}{r}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)$$

$$\sigma\frac{d\sigma}{dr} = -g\frac{a^2}{r^2} - \left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dr} + \frac{4\mu}{3b}\left\{\theta\sigma r^2\underbrace{\frac{d}{dr}\left[\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right]} + \sigma\frac{d\theta}{dr}r^2\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right]\right.$$

$$\left.+ r^2\theta\left[\frac{d\sigma}{dr}-\frac{\sigma}{r}\right]^2\right\}$$

$$\theta\left[r^2\sigma\frac{d^2\sigma}{dr^2} + \cancel{2r\sigma\frac{d\sigma}{dr}} - 2\sigma^2 + r^2\left(\frac{d\sigma}{dr}\right)^2 + \sigma^2 - \cancel{2\sigma r\frac{d\sigma}{dr}}\right]$$

$$= \theta\left[r^2\sigma\frac{d^2\sigma}{dr^2} + r^2\left(\frac{d\sigma}{dr}\right)^2 - \sigma^2\right]$$

$$= \frac{d}{dr}\left[\sigma r^2\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right] = \left(\frac{d\sigma}{dr}\right)^2 r^2 - \cancel{\sigma r\frac{d\sigma}{dr}} + \cancel{2r\sigma\frac{d\sigma}{dr}} - 2\sigma^2 + \sigma r^2\frac{d^2\sigma}{dr^2} - \cancel{\sigma r\frac{d\sigma}{dr}} + \sigma^2$$

$$\boxed{\begin{cases}\sigma\dfrac{d\sigma}{dr} = -g\dfrac{a^2}{r^2} - \left(\alpha+\dfrac{c}{A}\right)\dfrac{d\theta}{dr} + \dfrac{4\mu}{3b}\dfrac{d}{dr}\left[\theta\sigma r^2\left(\dfrac{d\sigma}{dr}-\dfrac{\sigma}{r}\right)\right] \\ \dfrac{c}{A}\dfrac{d\theta}{dr} + \dfrac{\alpha\theta}{\sigma}\left[\dfrac{d\sigma}{dr}+\dfrac{2\sigma}{r}\right] = \dfrac{4\mu}{3b}\theta r^2\left[\dfrac{d\sigma}{dr}-\dfrac{\sigma}{r}\right]^2\end{cases}}\quad\begin{matrix}\mathrm{I}_a\\ \\ \mathrm{II}_a\end{matrix}$$

$$\frac{d\sigma}{dr} - \frac{\sigma}{r} = r\frac{d}{dr}\left(\frac{\sigma}{r}\right) = r\frac{d}{dr}\left(\frac{\sigma r^2}{r^3}\right) \qquad \frac{d\sigma}{dr} + \frac{2\sigma}{r} = \frac{1}{r^2}\frac{d}{dr}\left(\sigma r^2\right)$$

$\sigma r^2 = z$

$$\boxed{\begin{aligned}\frac{\sigma^2}{2} &= \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\,\theta\,\sigma r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right) \\ \frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{\sigma}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right] &= \frac{4\mu}{3b}\,\theta\, r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2\end{aligned}}$$

I b

II b

$\sigma r^2 = z = \frac{b}{\rho}$

$$\frac{1}{2}\frac{z^2}{r^4} = \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\,\theta\, z\, r\,\frac{d}{dr}\left(\frac{z}{r^3}\right) \qquad \text{I c}$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{z}\frac{dz}{dr} = \frac{4\mu}{3b}\,\theta\, r^4\left[\frac{d}{dr}\left(\frac{z}{r^3}\right)\right]^2 \qquad \text{II c}$$

$\frac{\sigma}{r} = u$

$$\frac{u^2 r^2}{2} = \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\,\theta\, r^4 u\frac{du}{dr} \qquad \text{I d}$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{u r^3}\frac{d}{dr}\left(u r^3\right) = \frac{4\mu}{3b}\,\theta\, r^4\left(\frac{du}{dr}\right)^2 \qquad \text{II d}$$

Żeby $\frac{d\theta}{dr} \geqq 0$ musi być:

$$\frac{3\alpha\cancel{\theta}}{r} + \frac{\alpha\cancel{\theta}}{u}\frac{du}{dr} \leqq \frac{4\mu}{3b}\,\cancel{\theta}\, r^4\left(\frac{du}{dr}\right)^2$$

$$\frac{4\mu\alpha}{b}\frac{u^2}{r} + \frac{4\mu}{3b}\alpha\, u\frac{du}{dr} \leqq \left(\frac{4\mu}{3b}\right)^2 r^4 u^2\left(\frac{du}{dr}\right)^2$$

$$\left(\frac{4\mu}{3b}\right) r^4 u\frac{du}{dr} \geqq \frac{\alpha}{2} \pm \sqrt{\frac{\alpha^2}{4} + \frac{4\mu\alpha u^2 r^3}{b}}$$

$$\left.\begin{aligned}&\frac{ga^2}{r^2} + \frac{m}{r} - \left(\alpha + \frac{c}{A}\right)\frac{\theta}{r} + \frac{4\mu}{3b}\,\theta\, r^2\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\frac{\sigma}{r} = 0 \\ &\left[\frac{ga^2}{r\sigma} + \frac{m}{\sigma} - \left(\alpha + \frac{c}{A}\right)\frac{\theta}{\sigma}\right]\frac{d\sigma}{dr} + \frac{4\mu}{3b}\,\theta\, r^2\left(\quad\right)\frac{d\sigma}{dr} = 0 \quad -\end{aligned}\right\}$$

$$\frac{4}{3}\frac{\mu}{k}\,\theta\, r^2\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)^2 = \frac{d\sigma}{dr}\left[\frac{ga^2}{r\sigma}+\frac{m}{\sigma}-\left(\alpha+\frac{c}{A}\right)\frac{\theta}{\sigma}\right]-\frac{ga^2}{r^2}-\frac{m}{r}+\left(\alpha+\frac{c}{A}\right)\frac{\theta}{r}$$

$$= \frac{c}{A}\frac{d\theta}{dr}+\frac{\alpha\theta}{\sigma}\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)$$

$$\underbrace{\frac{d\sigma}{dr}\left[\frac{ga^2}{r\sigma}+\frac{m}{\sigma}\right]-\left[\frac{ga^2}{r^2}+\frac{m}{r}\right]}_{\frac{1}{\sigma}\left(\frac{ga^2}{r}+m\right)\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)} + \underbrace{\left(\frac{c}{A}-\alpha\right)\frac{\theta}{r}-\left(\frac{c}{A}+2\alpha\right)\frac{\theta}{\sigma}\frac{d\sigma}{dr}}_{-\frac{\theta}{\sigma}\left(\frac{c}{A}+2\alpha\right)\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)-\frac{3\alpha\theta}{r}} = \frac{c}{A}\frac{d\theta}{dr}$$

$$\frac{1}{\sigma}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\left[\frac{ga^2}{r}+m-\left(\frac{c}{A}+2\alpha\right)\theta\right] = \frac{3\alpha\theta}{r}+\frac{c}{A}\frac{d\theta}{dr}$$

$$\left[\frac{ga^2}{r}-ga+\left(\alpha+\frac{c}{A}\right)\theta_0-\left(\alpha+\frac{c}{A}\right)\theta-\alpha\theta\right] = \left[ga\left(\frac{a}{r}-1\right)+\left(\alpha+\frac{c}{A}\right)(\theta_0-\theta)-\alpha\theta\right]$$

I b):
$$\frac{\sigma_0^2}{2} = ga + m - \left(\alpha+\frac{c}{A}\right)\theta_0 + \frac{4}{3}\frac{\mu}{k}\underbrace{\theta_0\,\sigma_0\, a^2\left(\frac{d\sigma}{dr}-\frac{\sigma_0}{a}\right)}_{\varepsilon}$$

$$\frac{\sigma^2-\sigma_0^2}{2} = \underbrace{ga\left(\frac{a}{r}-1\right)}_{<0}\underbrace{-\left(\alpha+\frac{c}{A}\right)(\theta-\theta_0)}_{>0}+\frac{4\mu}{3k}\,\theta\,\sigma\, r^2\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)-\varepsilon$$

$$\frac{4\mu}{3k}\left\{\frac{c}{A}\frac{d\theta}{dr}\frac{1}{\theta}+\frac{\alpha}{\sigma}\left[\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right]\right\} = \left[\frac{4\mu}{3k}\, r\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]^2$$

$$= \frac{1}{(\theta\sigma r)^2}\left[\frac{ga^2}{r}+m-\left(\alpha+\frac{c}{A}\right)\theta\right]^2$$

$$\frac{4\mu}{3k}\left\{\frac{c}{A}\frac{d\theta}{dr}\frac{1}{\theta}+\frac{\alpha}{\sigma}\frac{d\sigma}{dr}+\frac{2\alpha}{r}\right\} = \nearrow$$

$$\frac{4\mu}{3k}\frac{d\sigma}{dr} = \frac{4\mu}{3k}\frac{\sigma}{r} - \frac{1}{\theta\sigma r^2}\left[\frac{ga^2}{r}+m-\left(\alpha+\frac{c}{A}\right)\theta\right]$$

$$\frac{4\mu}{3k}\left\{\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr}+\frac{\alpha}{r}+\frac{2\alpha}{r}\right\} = \frac{\alpha}{\theta\sigma r^2}\left[\qquad\right]+\frac{1}{(\theta\sigma r)^2}\left[\qquad\right]^2$$

Jaka będzie wartość $\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right|_{r=a}$?

W przybliżeniu :

$$\frac{c}{A}\log\theta + \alpha(\log\sigma + \log r^2) = \text{const}$$

$$\theta^{\frac{c}{A}}(\sigma r^2)^{\alpha} = \text{const} = \theta_0^{\frac{c}{A}}\left(\frac{b}{\rho_0}\right)^{\alpha}$$

$$\frac{ga^2}{r} + m = \left(\alpha + \frac{c}{A}\right)\theta$$

$$\underline{ga + m = \left(\alpha + \frac{c}{A}\right)\theta_0}$$

$$\left(\frac{d\theta}{dr}\right)_0 = -\frac{g\cancel{a}}{\left(\alpha+\frac{c}{A}\right)\cancel{a}}$$

$$ga\left(\frac{a}{r} - 1\right) = \left(\alpha + \frac{c}{A}\right)(\theta - \theta_0)$$

$$\frac{2\alpha\theta_0}{r} + \frac{\alpha\theta}{\sigma}\frac{d\sigma}{dr} - \frac{\frac{c}{A}g}{\alpha + \frac{c}{A}} = 0$$

$$\frac{\alpha\theta_0}{\sigma}\frac{d\sigma}{dr} = \frac{g}{1 + \frac{\alpha A}{c}} - \frac{2\alpha\theta_0}{a}$$

$$\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)_0 = \frac{\sigma}{\alpha\theta_0}\left[\frac{g}{1 + \frac{\alpha A}{c}} - \frac{2\alpha\theta_0}{a} - \frac{\alpha\theta_0}{a}\right]$$

$$= \sigma\left[\underbrace{\frac{g}{\alpha\theta_0\left(1 + \frac{\alpha A}{c}\right)}}_{=k} - \frac{3}{a}\right]$$

$$\frac{9\cdot 8}{1{,}4\cdot 270\cdot 280} - \frac{3}{6{,}366.200} \gg 0$$

$$\left(\frac{d\sigma}{dr}\right)_0 > \left(\frac{\sigma}{r}\right)_0$$

$$\left(\frac{1}{\sigma}\frac{d\sigma}{dr}\right)_0 > \left(\frac{1}{r}\right)_0$$

$$\left(\frac{d\log\sigma}{dr}\right)_0 > \left(\frac{1}{r}\right)_0$$

Zapewne zmniejsza się później ta różnica

Jeśli $\frac{d\sigma}{dr}$ stanie się $= \frac{\sigma}{r}$ to:

$$\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta = 0 \qquad ga\left(\frac{a}{r} - 1\right) - \left(\alpha + \frac{c}{A}\right)(\theta - \theta_0) = 0$$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r} = 0 \qquad ga\left(1 - \frac{a}{r}\right) = \left(\alpha + \frac{c}{A}\right)(\theta_0 - \theta)$$

Począwszy odtąd θ musiałoby być wciąż mniejsze aniżeli ~~gdy~~ bez tarcia

Jednak zanim $\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)$ stanie się $= 0$ to musi istnieć punkt gdzie:

$$\frac{\alpha}{\sigma}\left(\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right) = \frac{4\mu}{3b} r^2 \left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2$$

Tam będzie $\frac{d\theta}{dx} = 0$!

Czy ten punkt będzie osiągnięty?

$$\frac{d\sigma}{dr} > \frac{\sigma}{r} \qquad \frac{\alpha}{\sigma}\frac{d\sigma}{dr} + \frac{2\alpha}{r} \gtreqless\downarrow \frac{4\mu}{3b} r^2 \left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2$$

$\sigma^2 = s$

$$\frac{\alpha}{2}\frac{d(\sigma^2)}{dr} + \frac{2\alpha\sigma^2}{r} \gtreqless\downarrow \frac{4\mu}{3b}\frac{r^2}{\sigma^2}\left[\frac{1}{2}\frac{d(\sigma^2)}{dr} - \frac{\sigma^2}{r}\right]^2$$

$$\cancel{\frac{\alpha}{2}}\ \frac{\alpha}{2} s \frac{ds}{dr} + \frac{2\alpha s^2}{r} \gtreqless\downarrow \frac{4\mu}{3b}\left[\frac{1}{4} r^2 \left(\frac{ds}{dr}\right)^2 - rs\frac{ds}{dr} + s^2\right]$$

$$0 \gtreqless\downarrow \frac{4\mu}{3b}\left[\frac{r^2}{4}\left(\frac{ds}{dr}\right)^2\right] - s\frac{ds}{dr}\left[\frac{4\mu r}{3b} + \frac{\alpha}{2}\right] + s^2\left[\frac{4\mu}{3b} - \frac{2\alpha}{r}\right]$$

$$0 \gtreqless \frac{4\mu}{3b}\frac{1}{4}\left(r\frac{ds}{dr}\right)^2 - r\frac{ds}{dr} s \left[\frac{4\mu}{3b} + \frac{\alpha}{2r}\right] + s^2\left[\frac{4\mu}{3b} - \frac{2\alpha}{r}\right]$$

$$0 \gtreqless \left(\frac{r}{2}\frac{ds}{dr}\right)^2 - 2\left(\frac{r}{2}\frac{ds}{dr}\right) s \left(1 + \frac{3b}{4\mu}\frac{\alpha}{2r}\right] + s^2\left(1 - \frac{3b}{4\mu}\frac{2\alpha}{r}\right)$$

$$\left[\frac{r}{2}\frac{ds}{dr} - s\left(1+\frac{3b}{4\mu}\frac{\alpha}{2r}\right)\right]^2 \overset{<}{\underset{>}{=}}\downarrow s^2\left[\frac{3b}{4\mu}\frac{\alpha}{r} + \left(\frac{3b}{4\mu}\frac{\alpha}{2r}\right)^2 + \frac{3b}{4\mu}\frac{2\alpha}{r}\right]$$

$$s^2\frac{3b}{4\mu}\frac{\alpha}{r}\left(3+\frac{3b}{4\mu}\frac{\alpha}{4r}\right)$$

,

$$2\frac{\alpha}{r}\left[\left[s+\frac{1}{8}\frac{ds}{dr}\right]^2 - \frac{1}{64}\left(\frac{ds}{dr}\right)^2\right] \overset{>}{\underset{<}{=}}\downarrow \frac{4\mu}{3b}\left[\frac{r}{2}\frac{ds}{dr} - s\right]^2$$

Jeśli $\frac{d\sigma}{dr}$ więc się zmniejsza wtedy naturalnie musi być kiedyś osiągnięty

W przeciwnym wypadku (2) $\frac{d\theta}{dr}$ więc będzie < 0

$$\frac{d\sigma}{dr} > \cancel{1}\frac{\sigma}{r}$$

~~zatem mógłby być mieć punkt gdzie $\theta=0$~~

$$(\theta_0-\theta)\left(\alpha+\frac{c}{A}\right) = +ga\left(\cancel{1}-\frac{a}{r}\right) - \frac{4\mu}{3b}\theta\sigma r^2\left(\frac{a}{r}-\frac{1}{r}\right)$$

Więc mógłby być punkt gdzie $\theta=0$ jeśli 1). σ i $\frac{d\sigma}{dr}$ skończone a $\frac{d\theta}{dr}=0$

2).

1). wymagałoby według Ia także: $\sigma\frac{d\sigma}{dr} = -g\frac{a^2}{r^2}$! co nie możliwe bo właśnie założenie było że $\frac{d\sigma}{dx} > \frac{\sigma}{r}$

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r} + \frac{\alpha}{\sigma}\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right] = \frac{4\mu}{3b}\, r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2$$

$$\frac{4\mu}{3b}\, r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right] = \frac{\alpha}{\theta\sigma^2}\left[\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta\right] + \left\{\frac{1}{\theta\sigma}\left[\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta\right]\right\}^2$$

$$\frac{4\mu}{3b}\sigma^2 = \frac{\frac{\alpha}{\theta}\left[\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta\right] + \frac{1}{\theta^2}\left[\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta\right]^2}{\cancel{\frac{4\mu}{3b}}\, r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}$$

$$0 = \underbrace{\frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta}_{v} + r^2\theta\left[\frac{1}{2}\frac{d}{dr}\left\{\frac{\frac{\alpha}{\theta}[\quad] + \frac{1}{\theta^2}[\quad]^2}{r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}\right\} - \frac{1}{r}\,\frac{\frac{\alpha}{\theta}[\quad] + \frac{1}{\theta^2}[\quad]^2}{r^2\left[\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}\right]$$

$$0 = v + \frac{r^2\theta}{2}\frac{d}{dr}\left\{\frac{\alpha v + \frac{1}{\theta}v^2}{r^2\left[\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right]}\right\} - \theta\,\frac{\alpha v + v^2}{r\left(\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right)}$$

Id:

$$0 = \frac{ga^2}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta + \frac{4\mu}{3b}\theta r^4 u\frac{du}{dr} \qquad \Big|\quad \frac{du}{dr}$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r} + \frac{\alpha\theta}{u}\frac{du}{dr} = \frac{4\mu}{3b}\theta r^4\left(\frac{du}{dr}\right)^2 \qquad \Big|\quad u$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r} = \frac{c}{Ar}\left[r\frac{d\theta}{dr} + \frac{3\alpha}{\frac{c}{A}}\theta\right]$$

$$= \frac{c}{A}\frac{d}{dr}\left(\theta r^{\frac{3\alpha}{c/A}}\right)\cdot r^{-\frac{3\alpha}{c/A}}$$

$$\frac{c}{A}u\frac{d\theta}{dr} + \frac{3\alpha\theta u}{r} = +\frac{c}{A}\theta\frac{du}{dr} - m\frac{du}{dr} - \frac{ga^2}{r}\frac{du}{dr}$$

$$\frac{c}{A}\frac{d}{dr}\left(\frac{\theta}{u}\right) + \frac{3\alpha}{r}\frac{\theta}{u} - \left(m + \frac{ga^2}{r}\right)\frac{d\left(\frac{1}{u}\right)}{dr} = 0$$

$$\frac{4}{36}\,\frac{r^2}{2}\,\frac{d(u^2)}{dr} = -\frac{v}{\theta r^2}$$

$$0 = v + \frac{4}{36}\,\theta\,\frac{r^4}{2}\,\frac{d(u^2)}{dr}$$

$$u^2\left[\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right] + \frac{\alpha\theta}{2}\,\frac{d(u^2)}{dr} = \frac{4}{36}\,\frac{\theta r^4}{4}\left[\frac{d(u^2)}{dr}\right]^2$$

$$\frac{4}{36}\,u^2 = \frac{1}{\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}}\left(+\frac{v^2}{\theta r^4} + \frac{\alpha v}{r^4}\right)$$

$$0 = v + \frac{\theta r^4}{2}\,\frac{d}{dr}\left\{\frac{\frac{v^2}{\theta} + \alpha v}{r^4\left[\frac{c}{A}\frac{d\theta}{dr} + \frac{3\alpha\theta}{r}\right]}\right\}$$

to samo jak tam

$$\frac{1}{r^3} = z$$

$$-\frac{3}{r^4}\,dr = dz$$

$$\frac{d\theta}{dr} = -\frac{3}{r^4}\,\frac{d\theta}{dz}$$

$$\underbrace{\frac{ga^2}{r\theta} + \frac{m}{\theta} - \left(\alpha + \frac{c}{A}\right)}_{w} + \frac{r^4}{2}\,\frac{d}{dr}\left\{\frac{\left[\frac{ga^2}{r\theta} + \frac{m}{\theta} - \left(\alpha + \frac{c}{A}\right)\right]^2 + \alpha\left[\quad\right]}{r^4\left[\frac{c}{A}\,\frac{1}{\theta}\,\frac{d\theta}{dr} + \frac{3\alpha}{r}\right]}\right\}$$

$$2w = \frac{3}{}\,\frac{d}{dz}\left\{\frac{w^2 + w\alpha}{-3\frac{c}{A}\,\frac{1}{\theta}\,\frac{d\theta}{dz} + \frac{3\alpha}{z}}\right\} = \frac{d}{dz}\left\{\frac{w^2 + w\alpha}{\underbrace{\frac{\alpha}{z} - \frac{c}{A}\,\frac{1}{\theta}\,\frac{d\theta}{dz}}}\right\}$$

$$= \frac{d}{dz}\log\left(z^{\alpha}\,\theta^{-\frac{c}{A}}\right)$$

$$\frac{1}{r} = t \qquad \frac{ga^2t + m}{\alpha} = y \qquad \frac{dr}{r^2} = -dt$$

$$\frac{d}{dr} = -\frac{1}{r^2}\,\frac{d}{dt}$$

$$\frac{y}{\theta} - \frac{k}{k-1} - \frac{1}{2t^2}\,\frac{d}{dt}\left\{\frac{\left(\frac{y}{\theta} - \frac{k}{k-1}\right)^2 + \left(\frac{y}{\theta} - \frac{k}{k-1}\right)}{\frac{3}{t^3}\,\frac{1}{k-1}\,\frac{1}{\theta t^2}\,\frac{d\theta}{dt}}\right.$$

$$\frac{1}{\theta}\left(\frac{ga^2}{r} + m\right) = w + \alpha + \frac{c}{A}$$

$$\frac{1}{\theta}\,\frac{d\theta}{dz} = \frac{-\frac{ga^2}{r^2}\,\frac{dr}{dz}}{g\frac{a^2}{r} + m} - \frac{\frac{dw}{dz}}{w + \alpha + \frac{c}{A}}$$

$$\log\left(\frac{ga^2}{r} + m\right) - \log\theta = \log\left(w + \alpha + \frac{c}{A}\right)$$

Uwzględniając tarcie tylko w I:

$$\frac{c}{A}\frac{1}{\theta}\frac{d\theta}{dr} + \frac{\alpha}{\sigma}\frac{d\sigma}{dr} + \frac{2\alpha}{r} = 0$$

$$\lg\left(\theta^{\frac{c}{A}}\sigma^{\alpha}r^{2\alpha}\right) = \text{cnst}$$

$$\left[\frac{\theta}{\theta_0}\right]^{\frac{c}{A\alpha}}\left(\frac{\sigma}{\sigma_0}\right)\left(\frac{r}{a}\right)^2 = 1$$

$$0 = -\frac{ga^2}{r^2} \underbrace{- \alpha\frac{d\theta}{dr} + \frac{2\alpha\theta}{r} + \frac{\alpha\theta}{\sigma}\frac{d\sigma}{dr}}_{-\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dr}} + \frac{4\mu}{3b}\sigma r^2\left[\theta\frac{d}{dr}\left(\frac{d\sigma}{dr}+\frac{2\sigma}{r}\right)+\frac{d\theta}{dr}\left(\frac{d\sigma}{dr}-\frac{\sigma}{r}\right)\right]$$

$$\frac{c}{A\alpha} = \frac{1}{k-1}$$

$$\frac{\theta}{\theta_0}\left(\frac{\sigma r^2}{\sigma_0 a^2}\right)^{(k-1)} = 1 \qquad \frac{\theta}{\theta_0} = \left(\frac{\sigma r^2}{\sigma_0 a^2}\right)^{(1-k)}$$

$$\frac{d\theta}{dr} = \frac{2(1-k)\theta_0}{(\sigma_0 a^2)^{(1-k)}}(\sigma r^2)^{-k}\frac{d}{dr}(\sigma r^2)$$

$$= 2(1-k)\frac{\theta}{\sigma r^2}\frac{d}{dr}(\sigma r^2)$$

$1 + 30.\alpha$ $\quad \frac{0{\cdot}00367}{1{\cdot}110.14}$

184

$1{\cdot}414$

14

$1554 : 184 = 0{\cdot}844$

8 2 8

$2{\cdot}88081$

844

$2{\cdot}0368$

10885

$\frac{760}{690} = 1{\cdot}1\text{mm}$

$1{\cdot}99455$

$0{\cdot}844$

$2{\cdot}8385$

~~Przybliżenie stopniowe:~~

$$g = -\frac{\alpha k}{k-1}\frac{d\theta}{dx} \qquad \| \; \theta = \theta_0 - gx\,\frac{k-1}{\alpha k}$$

I $$\cancel{0 = g\frac{a^x}{r} + m - \left(\alpha + \frac{c}{A}\right)\theta}$$

$$\cancel{ga\left(\frac{a}{r} - 1\right) = \alpha\frac{k}{k-1}\theta}$$

$$\frac{\theta}{\theta_0} = \left(\frac{\sigma}{\sigma_0}\right)^{1-k} = \left(\frac{\rho_0}{\rho}\right)^{1-k} = \left(\frac{\rho}{\rho_0}\right)^{k-1} \qquad \frac{p}{p_0} = \frac{\theta}{\theta_0}\frac{\rho}{\rho_0} = \left(\frac{\theta}{\theta_0}\right)^{1+\frac{1}{k-1}} = \left(\frac{\theta}{\theta_0}\right)^{\frac{k}{k-1}}$$

$$\sigma = \sigma_0\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-k}}$$

$$gx + m = -\left(\alpha + \frac{c}{A}\right)\theta$$

$$m = -\left(\alpha + \frac{c}{A}\right)\theta_0 \qquad \frac{d\sigma}{dx} = \frac{\sigma_0}{1-k}\,\frac{\theta^{\frac{k}{1-k}}}{\theta_0^{\frac{1}{1-k}}}\,\frac{d\theta}{dx}$$

$$gx = \left(\alpha + \frac{c}{A}\right)(\theta_0 - \theta)$$

$$gx = \alpha\frac{k}{k-1}(\theta_0 - \theta) = \alpha\frac{k}{k-1}\theta_0\left(1 - \frac{\theta}{\theta_0}\right)$$

$$= \frac{\cancel{-}\sigma_0}{\cancel{(1-k)}\,\theta_0^{\frac{1}{1-k}}}\, g\,\frac{\cancel{k-1}}{\alpha k}\left(\theta_0 - gx\,\frac{k-1}{\alpha k}\right)^{\frac{k}{1-k}}$$

$$= \frac{g\sigma_0}{\alpha k\,\theta_0^{\frac{1}{1-k}}}\left(\theta_0 - gx\,\frac{k-1}{\alpha k}\right)^{\frac{k}{1-k}}$$

$$\frac{d\sigma}{dx} = \frac{g\sigma_0}{\alpha k\,\theta_0}\left(1 - \frac{gx(k-1)}{\theta_0\alpha k}\right)^{\frac{k}{1-k}}$$

$$\frac{\theta}{\theta_0} = \frac{1}{5} \qquad \left(\frac{p}{p_0}\right) = \left(\frac{1}{5}\right)^{\frac{1.4}{0.4}} = \left(\frac{1}{5}\right)^{\frac{7}{2}}$$

$\theta = 56°$ abs.

$= -217°$

$$\lg\left(\frac{p}{p_0}\right) = -\frac{7}{2}\lg 5$$

0·69897

489279

−2·44690

= 0·55360 − 3

$$\frac{p}{p_0} = 0{\cdot}003578 \cdot 760$$

$p =$ ca 2·5 mm

Nun aber nach Assmann, Hergesell etc.

in ca 14000 m : −65° = 208° $\quad \Delta = 80°$

während nach obigen Formel sein sollte 140° $\quad \Delta = 140°$

$$\frac{7}{2} : x = \overset{7}{\cancel{14}} : \overset{4}{\cancel{8}}$$

$$x = \frac{14}{7} = 2 = \frac{k}{k-1}$$

$$\frac{56}{208} = \frac{7}{26}$$

$$\left(\frac{p}{p_{0}}\right)_1 = \left(\frac{208}{280}\right)^2\left(\frac{7}{26}\right)^{\frac{7}{2}}$$

$\frac{104}{140}$ $\quad \frac{26}{35}$

41497

54407

95904

−0·04096

84510

41497

43013

0·56987 · $\frac{7}{2}$

−398909

1·99455

0·08192

2·08647

7 mm

Krytyczny punkt leży na wysokości 27 km więc tam g o mniej niż 1%,
=4gM.
zmniejszone a σ o tyleż powiększone

więc zupełnie wystarczające są ~~p~~ równania przybliżone.

$$\text{I.)}\quad \sigma\frac{d\sigma}{dx} = -g-\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\frac{k}{b}\left[\overbrace{\sigma\left\{\frac{d}{dx}\left(\theta\frac{d\sigma}{dx}\right)+\theta\left(\frac{d\sigma}{dx}\right)^2\right.}^{\frac{d}{dx}\left[\theta\sigma\frac{d\sigma}{dx}\right]}\right]$$

$$\text{II.)}\quad \frac{\sigma^2}{2} = -gx+m-\left(\alpha+\frac{c}{A}\right)\theta+\frac{4}{3}\frac{k}{b}\,\theta\,\sigma\frac{d\sigma}{dx}$$

$$= \left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\theta+\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}$$

$$\text{III.)}\quad \frac{c}{A}\frac{d\theta}{dx}\frac{1}{\theta}+\frac{\alpha}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\frac{k}{b}\left(\frac{d\sigma}{dx}\right)^2$$

$$\sigma\frac{d\sigma}{dx}+g-\frac{4}{3}\frac{k}{b}\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\theta = \left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\frac{k}{b}\,\sigma\frac{d\sigma}{dx}\right]\frac{d\theta}{dx}$$

$$\underbrace{\frac{1}{\theta}}_{=\frac{-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\frac{k}{b}\sigma\frac{d\sigma}{dx}}{\frac{\sigma^2}{2}+gx-m}}\left[\sigma\frac{d\sigma}{dx}+g\right]-\frac{4}{3}\frac{k}{b}\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right] = \left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\frac{k}{b}\,\sigma\frac{d\sigma}{dx}\right]\underbrace{\frac{1}{\theta}\frac{d\theta}{dx}}_{=\frac{A}{c}\left[\frac{4}{3}\frac{k}{b}\left(\frac{d\sigma}{dx}\right)^2-\frac{\alpha}{\sigma}\frac{d\sigma}{dx}\right]}$$

$$\left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\frac{k}{b}\,\sigma\frac{d\sigma}{dx}\right]\left[\sigma\frac{d\sigma}{dx}+g\right]-\left[\frac{\sigma^2}{2}+gx-m\right]\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\frac{4}{3}\frac{k}{b} =$$

$$= \left[\frac{\sigma^2}{2}+gx-m\right]\left[\frac{4}{3}\frac{k}{b}\left(\frac{d\sigma}{dx}\right)^2-\frac{\alpha}{\sigma}\frac{d\sigma}{dx}\right]\left[-\left(\alpha+\frac{c}{A}\right)+\frac{4}{3}\frac{k}{b}\,\sigma\frac{d\sigma}{dx}\right]\frac{A}{c}$$

$$\left(p^2+\frac{a}{x}pq+bq=0\right)$$

$$\frac{c}{A}\frac{d\lg\theta}{dx}+\frac{\alpha}{\sigma}\frac{d\sigma}{dx} = \frac{4}{3}\frac{k}{b}\left(\frac{d\sigma}{dx}\right)^2$$

$$\frac{c}{A}\left(\frac{dx}{d\sigma}\right)^2+\frac{\alpha}{\sigma}\frac{dx}{d\sigma}\frac{dx}{d\lg\theta} = \frac{4}{3}\frac{k}{b}\frac{dx}{d\lg\theta}\quad\Bigg|\quad \frac{c}{A}p^2+\frac{\alpha}{x}pq = \frac{4}{3}\frac{k}{b}q$$

$$\frac{c}{A}\sigma\frac{d\theta}{dx}+\alpha\theta\frac{d\sigma}{dx}=\frac{4}{3}\mu_b\,\theta\sigma\left(\frac{d\sigma}{dx}\right)^2$$

~~$\frac{c}{A}\frac{d^2\theta}{dx^2}+\frac{d\theta}{dx}\frac{\alpha}{\sigma}\frac{d\sigma}{dx}+$~~

~~$\frac{c}{A}\sigma\frac{d^2\theta}{dx^2}+\frac{c}{A}\frac{d\sigma}{dx}\frac{d\theta}{dx}+\alpha\frac{d\theta}{dx}\frac{d\sigma}{dx}+\alpha\theta\frac{d^2\sigma}{dx^2}=\frac{4}{3}\mu_b\left[\frac{d\theta}{dx}\sigma\left(\frac{d\sigma}{dx}\right)^2+\theta\left(\frac{d\sigma}{dx}\right)^3+2\theta\sigma\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}\right]$~~

I+III): $\sigma\frac{d\sigma}{dx}=-g-(\alpha$ [illegible]$)\frac{d\theta}{dx}+\frac{\alpha}{\sigma}\frac{d\sigma}{dx}+\frac{4}{3}\mu_b\left[\sigma\theta\frac{d^2\sigma}{dx^2}+\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}\right]$

$$\frac{1}{\theta}\frac{\sigma^2}{2}\frac{d\theta}{dx}=\frac{(m-gx)}{\theta}\frac{d\theta}{dx}-\left(\alpha+\frac{c}{A}\right)\frac{d\theta}{dx}+\frac{4}{3}\mu_b\,\sigma\frac{d\sigma}{dx}\frac{d\theta}{dx}$$

$$\sigma\frac{d\sigma}{dx}-\frac{\sigma^2}{2}\frac{1}{\theta}\frac{d\theta}{dx}=-\frac{(m-gx)}{\theta}\frac{d\theta}{dx}+\frac{c}{A}\frac{d\theta}{dx}-g+\frac{\alpha}{\sigma}\frac{d\sigma}{dx}+\frac{4}{3}\mu_b\,\sigma\theta\frac{d^2\sigma}{dx^2}$$

$$-\frac{c}{A}\frac{1}{\theta^2}\left(\frac{d\theta}{dx}\right)^2+\frac{c}{A}\frac{1}{\theta}\frac{d^2\theta}{dx^2}-\frac{\alpha}{\sigma^2}\left(\frac{d\sigma}{dx}\right)^2+\frac{\alpha}{\sigma}\frac{d^2\sigma}{dx^2}=2\frac{4}{3}\mu_b\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}$$

$$\frac{dy}{dx}+X_1y+X_2=0$$

$$y\frac{dy}{dx}+X_1y^2+X_2y+X_3=0$$

$$y=e^{-\int X_1dx}\left[c-\int X_2e^{\int X_1dx}dx\right]$$

$$e^{-\int X_1}\left[c-\int X_2e^{\int X_1}\right]\left[\cancel{-X_1e^{-\int X_1}\left(c-\int X_2e^{\int X_1}\right)}-\cancel{e^{-\int X_1}}\cancel{X_2}+\frac{dc}{dx}e^{-\int X_1}\right]+$$

$$\cancel{X_1e^{-\int X_1}\left(c-\int X_2e^{\int X_1}\right)}+\cancel{X_2}\Big]$$

$$+X_3=0$$

$$\left[c-\int X_2e^{\int X_1dx}dx\right]\frac{dc}{dx}\cancel{e^{-\int X_1}}+X_3e^{2\int X_1}=0$$

$$c\frac{dc}{dx} + f_1(x)\frac{dc}{dx} + f_2(x) = 0$$

$$(c + f_1(x)) = y$$

$$\frac{dc}{dx} + f_1'(x) = \frac{dy}{dx}$$

$$c = -\cancel{\frac{f_2}{(1+f_1)\frac{dy}{dx}}} \quad \frac{-f_2 - f_1\frac{dc}{dx}}{\frac{dc}{dx}} = -f_1 - \frac{f_2}{\frac{dc}{dx}}$$

$$\frac{dc}{dx} = -\frac{df_1}{dx} - \frac{\frac{df_2}{dx}}{\frac{dc}{dx}} + \frac{f_2}{\left(\frac{dc}{dx}\right)^2}\frac{d^2c}{dx^2}$$

$$\frac{dc}{dx} = p$$

$$f_2\frac{dp}{dx} \cancel{} = p^3 + \frac{df_1}{dx}p^2 + \frac{df_2}{dx}p$$

$$\frac{d\sigma}{dx} = +\frac{\sigma_0}{\theta_0\frac{1}{1-\kappa}}\,\frac{\cancel{\kappa}}{\cancel{1-\kappa}}\,\frac{\rho\cancel{\kappa}\,\cancel{(k-1)}}{\alpha k}\left[\theta_0 - \rho\frac{x(k-1)}{\alpha k}\right]^{\frac{\kappa}{1-\kappa}}$$

$$= \frac{\sigma_0\,\rho\cancel{\kappa}}{\alpha k\,\theta_0}\left[1 - \rho\frac{x(k-1)}{\alpha k\,\theta_0}\right]^{\frac{\kappa}{1-\kappa}}$$

Praca tarcia pro 1 sec. 1 cm³ } − =

$$y\frac{dy}{dx} + y^2X_1 + yX_2 + X_3 = 0$$

$$y + \underset{f(x)}{u} = v \qquad (y+u)\left(\frac{dy}{dx} + u'\right) + (y+u)^2X_1 + X = 0$$

$$v\frac{dv}{dx} + v^2X_1 + X = 0$$

$$y\frac{dy}{dx} + u\frac{dy}{dx} + y^2X_1 + y\underbrace{(u' + 2uX_1)}_{=X_2} + uu' + u^2X_1 + X = 0$$

Jeśli jako pierwsze przybliżenie :

$$g x = \alpha \frac{k}{k-1}(\theta_0-\theta) \qquad \frac{d\theta}{dx} = -\frac{g\cancel{\mu}(k-1)}{\alpha k}$$

$$\sigma = \sigma_0\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-k}}$$

$$\frac{d\sigma}{dx} = \frac{g\sigma_0}{\alpha k\theta_0}\left(\cancel{1}-g\frac{x(k-1)}{\theta_0\alpha k}\right)^{\frac{k}{1-k}}$$

$$= \frac{gb}{kp_0}\left[1-g\frac{x(k-1)}{\theta_0\alpha k}\right]^{\frac{k}{1-k}}$$

$$= \underbrace{\frac{4}{3}\mu\left(\frac{d\sigma}{dx}\right)^2 = \left(\frac{gb}{kp_0}\right)^2\left[1-g\frac{x(k-1)}{\theta_0\alpha k}\right]^{\frac{2k}{1-k}}}\ \frac{4}{3}\mu$$

$$\int_0^x \underbrace{\left[1-g\frac{x(k-1)}{\theta_0\alpha k}\right]}_{=\frac{\theta}{\theta_0}}^{\frac{2k}{1-k}} dx$$

$$\theta = \theta_0 - g\frac{x(k-1)}{\alpha k}$$

$$\frac{d\theta}{\cancel{dx}} = -g\frac{(k-1)}{\alpha k}\,dx$$

$$= -\int_{\theta_0}^{\theta}\left(\frac{\theta}{\theta_0}\right)^{\frac{2k}{1-k}}\frac{\alpha k}{g(k-1)}\,d\theta = -\frac{\alpha k}{g(k-1)\,\theta_0^{\frac{2k}{1-k}}}\,\frac{1}{\frac{2k}{1-k}+1}\,\theta^{\frac{2k}{1-k}+1}$$

Exp. $$= +\frac{\alpha k\cancel{(k-1)}}{g\cancel{(k-1)}(1+k)}\left[\left(\frac{\theta}{\theta_0}\right)^{\frac{2k}{1-k}}\theta-\theta_0\right]\cancel{\left(\frac{gb}{kp_0}\right)^2}\ \frac{4}{3}\mu\left(\frac{gb}{kp_0}\right)^2$$

$$= \frac{4}{3}\mu\alpha\,\frac{\cancel{k}}{1+k}\,\frac{gb}{kp_0^2}\,\theta_0\left[\left(\frac{\theta}{\theta_0}\right)^{\frac{1+k}{1-k}}-1\right] = \frac{4}{3}\,\frac{\mu b g}{k(1+k)}\,\frac{1}{p_0\rho_0}\left[\left(\frac{\theta}{\theta_0}\right)^{\frac{1+k}{1-k}}-1\right]$$

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$g\theta_0 = 0.00018$ (g cm)

$g = 980$

$\rho_0 = 0.0013$

$p_0 = 980.76.13.6$

$$\frac{\cancel{4}b}{\cancel{3}}\ \frac{\overset{0.00006}{\cancel{0.00018}}}{\underset{70}{\cancel{280}}}\ \frac{\cancel{980}}{\cancel{980.76.13.6}.0.0013}\ \underbrace{\frac{1}{1.4.2.4}}_{=4.4}$$

24
96
336.
1008
4.37

np. $b = 10$ cm

$$\frac{0.0006}{200}$$

np. $\frac{\theta}{\theta_0} = \frac{1}{5}$ $\left(\frac{1}{5}\right)^6 = (125)^2$

250
15000

$$\frac{0.000\overset{3}{\cancel{6}}}{\cancel{200}}\ 150$$

$= 0.045$

$\log e^{0.045} = 0.43426 . 0.045$

21713.9

0.0195397

$e^{0.045} = 1.046$

$$\cancel{\frac{\sigma^2}{2} = -gx + m - \left(\alpha + \frac{c}{A}\right)\theta} + \frac{4}{3} k_b\, \sigma \frac{d\sigma}{dx} \qquad \Bigg| \quad 2\sigma \frac{d\sigma}{dx} = -g - \left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dx} + \frac{4}{3} k_b \frac{d}{dx}\left(\theta \sigma \frac{d\sigma}{dx}\right)$$

$$\frac{c}{A}\sigma^2 \frac{d\theta}{dx} + \alpha\theta\,\sigma\frac{d\sigma}{dx} = \frac{4}{3} k_b\,\theta \left(\sigma \frac{d\sigma}{dx}\right)^2$$

$$\left(\alpha + \frac{c}{A}\right) = \alpha\left(1 + \frac{1}{k-1}\right) = \frac{\alpha k}{k-1}$$

$$\boxed{m = \left(\alpha + \frac{c}{A}\right)\theta_0 + \frac{\sigma_0^2}{2} - \frac{4}{3} k_b\, \theta_0 \left(\sigma \frac{d\sigma}{dx}\right)_0}$$

$$\sigma^2 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots\cdots$$

$$\frac{\theta}{\theta_0} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots$$

$$2\sigma\frac{d\sigma}{dx} = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots$$

$$2\sigma \frac{\theta}{\theta_0}\frac{d\sigma}{dx} = a_1 b_0$$

$$+ (a_1 b_1 + 2a_2 b_0)x$$

$$+ (a_1 b_2 + 2a_2 b_1 + 3a_3 b_0)x^2$$

$$+ (a_1 b_3 + 2a_2 b_2 + 3a_3 b_1 + 4a_4 b_0)x^3$$

$$+ (a_1 b_4 + 2a_2 b_3 + 3a_3 b_2 + 4a_4 b_1 + 5a_5 b_0)x^4$$

$$+$$

$$2\sigma\frac{d\sigma}{dx} + 2g + \frac{2\alpha k}{k-1}\frac{d\theta}{dx} + \frac{4}{3}\frac{\sigma}{b}\frac{d}{dx}\left(\theta\, 2\sigma\frac{d\sigma}{dx}\right)$$

$$\left.\begin{array}{ll}
 & a_1 + 2g + \frac{2\alpha k}{k-1}\theta_0 b_1 + \underset{=\varepsilon}{\boxed{\frac{4}{3}\frac{\sigma}{b}}}(a_1 b_1 + 2a_2 b_0)\theta_0 = 0 \\
x & \cancel{2}a_2 + \frac{2\alpha k}{k-1}\theta_0\cancel{2}b_2 + \cancel{2}\frac{4}{3}\frac{\sigma}{b}(a_1 b_2 + 2a_2 b_1 + 3a_3 b_0)\theta_0 = 0 \\
x^2 & \cancel{3}a_3 + \frac{2\alpha k}{k-1}\theta_0\cancel{3}b_3 + \cancel{3}\frac{4}{3}\frac{\sigma}{b}(a_1 b_3 + 2a_2 b_2 + 3a_3 b_1 + 4a_4 b_0)\theta_0 = 0 \\
x^3 & \cancel{4}a_4 + \frac{2\alpha k}{k-1}\theta_0\cancel{4}b_4 + \cancel{4}\frac{4}{3}\frac{\sigma}{b}(a_1 b_4 + 2a_2 b_3 + 3a_3 b_2 + 4a_4 b_1 + 5a_5 b_0)\theta_0 = 0
\end{array}\right\}$$

$$\frac{\alpha}{k-1}a_0 b_1 \cancel{A} + \frac{\alpha}{2}a_1 b_0 = \frac{4}{3}\frac{\sigma}{b}\frac{b_0 a_1^2}{4}$$

$$\frac{\alpha}{k-1}(2a_0 b_2 + a_1 b_1) + \frac{\alpha}{2}(2a_2 b_0 + a_1 b_1) = \frac{4}{3}\frac{\sigma}{b}\frac{1}{4}\left(b_0 (4a_1 a_2) + b_1 a_1^2\right)$$

$$\frac{\alpha}{k-1}(3a_0 b_3 + 2a_1 b_2 + a_2 b_1) + \frac{\alpha}{2}(3a_3 b_0 + 2a_2 b_1 + a_1 b_2) = \frac{4}{3}\frac{\sigma}{b}\frac{1}{4}\Big[b_0(6a_1 a_3 + 4a_2^2) +$$
$$+ b_1\, 4a_1 a_2 + b_2 a_1^2\Big]$$

$$\frac{\alpha}{k-1}(4a_0 b_4 + 3a_1 b_3 + 2a_2 b_2 + a_3 b_1) + \frac{\alpha}{2}(4a_4 b_0 + 3a_3 b_1 + 2a_2 b_2 + a_1 b_3) =$$
$$= \frac{4}{3}\frac{\sigma}{b}\frac{1}{4}\Big[b_0(8a_1 a_4 + 12a_2 a_3) + b_1(6a_1 a_3 + 4a_2^2) + b_2\, 4a_1 a_2 + b_3 a_1^2\Big]$$

Jaki jest $\frac{d^2\theta}{dx^2}$ w krytycznym punkcie?

$\frac{dI}{dx}$:

$$\frac{d}{dx}\left(\sigma\frac{d\sigma}{dx}\right) = \left(\frac{d\sigma}{dx}\right)^2 + \sigma\frac{d^2\sigma}{dx^2} = -\left(\alpha+\frac{c}{A}\right)\frac{d^2\theta}{dx^2} + \frac{4}{3}\frac{\pi}{b}\left\{\cancel{\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\frac{d\theta}{dx}} + \theta\frac{d}{dx}\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]\right.$$

$$0 = \qquad \left. + \sigma\frac{d\sigma}{dx}\frac{d^2\theta}{dx^2} + \cancel{\frac{d}{dx}\left(\sigma\frac{d\sigma}{dx}\right)\frac{d\theta}{dx}}\right\}$$

$$\frac{4}{3}\frac{\pi}{b}\sigma\frac{d\sigma}{dx} = \alpha$$

$$\frac{c}{A}\frac{d^2\theta}{dx^2} = \frac{4}{3}\frac{\pi}{b}\theta\frac{d}{dx}\left[\sigma\frac{d^2\sigma}{dx^2}+\left(\frac{d\sigma}{dx}\right)^2\right]$$

$$\underline{g = \frac{4}{3}\frac{\pi}{b}\theta\left[\left(\frac{d\sigma}{dx}\right)^2+\sigma\frac{d^2\sigma}{dx^2}\right]}$$

$$\frac{4}{3}\frac{\pi}{b}g\sigma^2 = \alpha^2\theta + \left(\frac{4}{3}\frac{\pi}{b}\right)^2\theta\sigma^3\frac{d^2\sigma}{dx^2} \qquad \text{II} \qquad 2(\text{I})$$

$\left(\frac{d}{dx}\text{II}\right)$

$$\frac{c}{A}\sigma\frac{d^2\theta}{dx^2} + \cancel{\frac{c}{A}\frac{d\sigma}{dx}\frac{d\theta}{dx}} + \cancel{\alpha\frac{d\sigma}{dx}\frac{d\theta}{dx}} + \alpha\theta\frac{d^2\sigma}{dx^2} = \frac{4}{3}\frac{\pi}{b}\left[\cancel{\frac{d\theta}{dx}\sigma\left(\frac{d\sigma}{dx}\right)^2} + \theta\left(\frac{d\sigma}{dx}\right)^3 + 2\theta\sigma\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}\right]$$

$$\times\left(\frac{4}{3}\frac{\pi}{b}\right)^2\sigma^3$$

$$\frac{c}{A}\sigma\frac{d^2\theta}{dx^2}\frac{4}{3}\frac{\pi}{b}\sigma^3 + \cancel{\alpha\left[\left(\frac{4}{3}\frac{\pi}{b}\right)g\sigma^2-\alpha^2\theta\right]} = \theta\alpha^3 + \alpha\left(\frac{4}{3}\frac{\pi}{b}g\sigma^2-\alpha^2\theta\right)$$

$$\cancel{\frac{4}{3}\frac{\pi}{b}}\frac{c}{A}\sigma^4\frac{d^2\theta}{dx^2} = \cancel{\frac{4}{3}\frac{\pi}{b}}g\sigma^2\alpha$$

$$\boxed{\frac{d^2\theta}{dx^2} = \frac{g\alpha}{\frac{c}{A}\sigma^2}}$$

>0

Inny sposób rozwiązania w szereg:

$$p = \frac{1}{s} = \frac{1}{u}\frac{du}{dz} \qquad dz = \frac{1}{u}\,du \qquad y = \frac{e^{z}}{k-1}$$

$$\theta = \frac{e^{z}}{\alpha u} = \frac{k-1}{\alpha}\,\frac{y}{u}$$

$$\frac{dp}{du} = -\frac{1}{s^2}\frac{ds}{du}$$

$$-(k+u)(1+u)u\,\frac{1}{s^3}\frac{ds}{du} - \frac{1}{s^2}\left[4u^2 - u(1+k) - 2k\right] + \frac{1}{s}\left[u^2+4u+3k\right] + u^2 + ku - u - k = 0$$

$$(k+u)(1+u)\,u\,\frac{ds}{du} + \left[4u^2 - u(1+k) - 2k\right]s - \left[u^2+4u+3k\right]s^2 - \left[u^2+ku-u-k\right]s^3 = 0$$

$$s = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + \dots$$

$$-2k\,a_0 - 3k\,a_0^2 + k\,a_0^3 = 0$$

$$k\,a_0\left[2 + 3a_0 - a_0^2\right] = 0$$

Ad Ritter

$-g = \frac{1}{\rho} \frac{dp}{dz}$ ~~$p = \alpha \rho \theta$~~

$\rho =$ [illegible] Vol.

~~$= \rho_d v_d + \rho_w v_w$~~

1 kg Wolke ist ∴ $\frac{1}{\rho}$ Vol. (Dampf + Wasser)

" " " x kg Dampf u. (1−x) kg Wasser

Dampf spez. Vol. ω ∴ $\rho_d = \frac{1}{\omega}$

Wasser spez. Vol. $v = 1$ ∴ $\rho_w = 1$

1 kg Wolke hat ∴ Vol. $\omega x + 1 - x$

∴ $\rho = \frac{1}{\omega x + 1 - x} \neq \frac{\rho_d}{x}$ $\qquad \frac{1}{\rho} = \frac{x}{\rho_d}$

$$\frac{xr}{T} - \frac{x_0 r_0}{T_0} = c \lg \frac{T_0}{T} \qquad \frac{r}{T} = A \omega \frac{dp}{dT}$$

$$A \omega x \frac{dp}{dT} = \frac{x_0 r_0}{T_0} + c \lg \frac{T_0}{T} = A \frac{x}{\rho_d} \frac{dp}{dT} = A \frac{1}{\rho} \frac{dp}{dT} = -A g \frac{dz}{dT}$$

$$-A g z = \left. \frac{r_0}{T_0} T + c T \lg T_0 - c T \lg T + c T \right|_{T_0}^{T}$$

$$+A g H = r_0 + c T_0$$

Dabei vorausgesetzt dass c ~~[illegible]~~ von Temp. unabh.

Zadanie całkiem oznaczone jeśli te dane

$\theta_0 \quad p_0 \quad p_1$

$p_0 = \alpha\,\theta_0\,\rho_0 = \alpha\,\theta_0 \frac{b}{\sigma_0}$

$p_1 = \alpha \frac{\theta_1}{\sigma_1} b$

ale 4 niewiadome stałe: θ_0 σ_0 b m ?

1. Równ. II rzędu względem θ ρ σ

I " względem θ σ

I " ρ σ

$$p = \alpha\rho\theta = \frac{\alpha\theta b}{\sigma} \quad \Big\| \quad \frac{dp}{dx} = \alpha b\left[\frac{1}{\sigma}\frac{d\theta}{dx} - \frac{\theta}{\sigma^2}\frac{d\sigma}{dx}\right]$$

$$= \alpha b \frac{1}{\sqrt{v}}\left[\frac{d\theta}{dx} - \frac{\theta}{2}\frac{dv}{dx}\right]$$

$$p - p_0 = \int_0^\theta \alpha b\left[\frac{1}{\sqrt{v}}\frac{d\theta}{dx} - \frac{\theta}{2}\frac{dv}{dx}\right] = f_1(\theta_0\, v_0\, b\, m)$$

$$\log p = \log \alpha b + \log\theta - \log\sigma$$

$$\frac{1}{p}\cdot\frac{dp}{dx} = \frac{1}{\theta}\frac{d\theta}{dx} - \frac{1}{2v}\frac{dv}{dx}$$

$$p_0 + \int_0^\theta = \frac{\alpha\theta b}{\sigma}$$

$$\frac{p_0}{\alpha b} + \underbrace{\int_0^\theta \left[\frac{1}{\sqrt{v}}\frac{d\theta}{dx} - \frac{\theta}{2}\frac{dv}{dx}\right]}_{\frac{\theta}{\sigma} - \frac{\theta_0}{\sigma_0}} = \frac{\theta}{\sigma}$$

$$v = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$\frac{1}{\sqrt{v}} = v^{-\frac{1}{2}} =$$

$$\frac{p_0}{\alpha b} = \frac{\theta_0}{\sigma_0}$$

$$\log p = A \log\frac{\theta}{\sigma}$$

Izotermiczny rozkład:

$$\rho = \frac{b}{\sigma}$$

$$0 = -g + \alpha\theta\,\frac{1}{\sigma}\frac{d\sigma}{dx} + \frac{4}{3}\frac{\mu}{b}\,\sigma\,\frac{d^2v}{dx^2}$$

$$\frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{\sigma}\frac{d\sigma}{dx} = 0$$

Czy nie wystarczy podanie ciśnienia w dwuch miejscach?

trzebaby podać $\sigma_1\ \sigma_2\ \rho_1$

$$p = \alpha\theta\rho$$

$\rho_1 u_1 \frac{\partial u_1}{\partial x} +$

$$\frac{\partial p_1}{\partial r} + \rho_1 g + a\rho_1\rho_2(u_1 - u_2) = 0$$

$\rho_2 u_2 \frac{\partial u_2}{\partial x} +$

$$\frac{dp_2}{dr} + \rho_2 g + a\rho_1\rho_2(u_2 - u_1) = 0$$

$$p_1 = \alpha_1\rho_1\theta \qquad \frac{dp_1}{dr} = \alpha_1\theta\frac{d\rho_1}{dr} \text{ (jeśli } \theta = \text{const)}$$

$$p_2 = \alpha_2\rho_2\theta$$

$$\alpha_1\theta\,\frac{1}{\rho_1}\frac{d\rho_1}{dr} + g + a\rho_2(u_1 - u_2) = 0$$

$$\alpha_2\theta\,\frac{1}{\rho_2}\frac{d\rho_2}{dr} + g + a\rho_1(u_2 - u_1) = 0$$

$$\frac{d}{dr}\overbrace{(p_1 + p_2)}^{P} + (\rho_1 + \rho_2)g = 0$$

$$\left[\alpha_1\theta\frac{d\rho_1}{dr} + \rho_1 g\right] = -\left[\alpha_2\theta\frac{d\rho_2}{dr} + \rho_2 g\right]$$

$(\overset{u_{10}}{\rho_{10}} + \overset{u_{20}}{\rho_{20}})u_0 = \rho_1 u_1 + \rho_2 u_2$

$\alpha_2\rho_2 u_2 + \alpha_1\rho_1 u_1$

$(\rho_1 + \rho_2)u = (\rho_{10} + \rho_{20})u_0$

$= \rho_2 u - \rho_{20}u_0 = -\rho_2 v$

$\rho_1\mu = \rho_{10}u_{10} - \rho_1 u$

$$\frac{d}{dx}(\rho_1 u_1) = 0 \qquad \frac{d}{dx}(\rho_2 u_2) = 0$$

Für $r = 0$: $u_1 = u_2 = u_0$

$u_1 = u + \mu \qquad u_2 = u + v$

$$\rho_1 u_1 = c_1 \qquad \rho_2 u_2 = c_2$$

$\rho_1 u + \rho_1\mu = c_1 = \rho_{10}u_{10} \qquad \rho_2 u + \rho_2 v = c_2 = \rho_{20}u_0$

$\rho_2 v = \rho_{20}u_0 - \rho_2 u$

$$\frac{\rho_{10}}{\rho_{20}} = \frac{c_1}{c_2}$$

$$\frac{\rho_1}{\rho_2} = \frac{c_1}{c_2}\,\frac{u_2}{u_1}$$

~~$\alpha_1\theta\,\frac{1}{\rho_1}\frac{d\rho_1}{dr} - g + a\left(\frac{\rho_2 c_1}{\rho_1} - c_2\right) = 0$~~

$\rho_1\rho_2(\mu - v) = \rho_1\mu\rho_2 - \rho_1 v\rho_2 = \rho_1\mu(\rho_1 + \rho_2)$

$\rho_1 u_1 \frac{\partial u_1}{\partial x} = \frac{c_1^2}{\rho_1^2}\frac{d\rho_1}{dr} +$

$$\left.\begin{aligned} \alpha_1\theta\frac{d\rho_1}{dr} + \rho_1 g + a(c_1\rho_2 - c_2\rho_1) &= 0 \\ \alpha_2\theta\frac{d\rho_2}{dr} + \rho_2 g + a(c_2\rho_1 - c_1\rho_2) &= 0 \end{aligned}\right\}$$

$$\rho_1\rho_2(u_1 - u_2) = c_1\rho_2 - c_2\rho_1 = \rho_2 c_1\left[1 - \frac{c_2}{c_1}\frac{\rho_1}{\rho_2}\right]$$

$$c_1 = \rho_{10}u \qquad = \rho_2 c_1\left[1 - \frac{\rho_{20}}{\rho_{10}}\frac{\rho_1}{\rho_2}\right]$$

$$c_2 = \rho_{20}u \qquad = \rho_1 c_1\left[\frac{\rho_2}{\rho_1} - \frac{\rho_{20}}{\rho_{10}}\right]$$

$\alpha_1\theta\frac{d^2\rho_1}{dr^2} + (g \pm a c_2)\frac{d\rho_1}{dr} + a c_1 \frac{}{dr} = 0$ | $\alpha_2\theta \quad a c_1 \rho_2 = -\alpha_1\theta\frac{d\rho_1}{dr} - \rho_1(g - a c_2)$

$\alpha_2\theta\frac{d\rho_2}{dr} = (\mp g + a c_1)\rho_2 - a c_2\rho_1$

$= \frac{(-g + a c_1)}{a c_1}\left[\rho_1(-g + a c_2) - \alpha_1\theta\frac{d\rho_1}{dr}\right] - a c_2\rho_1$

$\alpha_1\alpha_2\theta^2\frac{d^2\rho_1}{dr^2} + \alpha_2\theta(g \mp a c_2)\frac{d\rho_1}{dr} + (\mp g + a c_1)\left[\rho_1(-g + a c_2) - \alpha_1\theta\frac{d\rho_1}{dr}\right] - a^2 c_1 c_2\rho_1 = 0$

$\alpha_1\alpha_2\theta^2\frac{d^2\rho_1}{dr^2} + \left[\alpha_2(g \mp a c_2) + \alpha_1(g \mp a c_1)\right]\theta\frac{d\rho_1}{dr} + \left[(g \mp a c_1)(g \mp a c_2) - a^2 c_1 c_2\right]\rho_1 = 0$

$\theta = \text{const}$

$\rho_1 = \cancel{A} \; \cancel{\rho_{10} e^{\beta r}} \; A_1 e^{-\beta r} + B e^{-\gamma r}$

$\alpha_1\alpha_2\theta^2\beta^2 \mp \left[\alpha_2(g \mp a c_2) + \alpha_1(g \mp a c_1)\right]\theta\beta + (g \mp a c_1)(g \mp a c_2) - a^2 c_1 c_2 = 0$

$\left[\alpha_1\theta\beta - (g + a c_2)\right]\left[\alpha_2\theta\beta - g + a c_1\right] = a^2 c_1 c_2 \qquad = \frac{a^2 c_1 c_2 - (g - a c_1)(g - a c_2)}{\alpha_1\alpha_2}$

$(\theta\beta)^2 \mp \left[\frac{g \mp a c_2}{\alpha_1} + \frac{g \mp a c_1}{\alpha_2}\right]\theta\beta + \cancel{\frac{g + a c_1}{\alpha_1}\frac{g + a c_2}{\alpha_2}} + \frac{g^2 \mp g a(c_1 + c_2)}{\alpha_1\alpha_2}$

$\theta\beta = + \frac{g \mp a c_2}{2\alpha_1} + \frac{g \mp a c_1}{2\alpha_2} \pm \sqrt{-\frac{(g \mp a c_1)(g \mp a c_2)}{2\alpha_1\alpha_2} + \frac{a^2 c_1 c_2}{\alpha_1\alpha_2} + \frac{(g + a c_1)^2}{4\alpha_2^2} + \frac{(g + a c_2)^2}{4\alpha_1^2} + \cancel{\frac{g(g + a c_1)(g + a c_2)}{2\alpha_1\alpha_2}}}$

$\alpha_1\alpha_2\theta^2\beta^2 - g(\alpha_1 + \alpha_2)\theta\beta + g^2 = 0 \qquad \theta\beta = \frac{g}{\alpha_1}$

$(\alpha_1\theta\beta \mp g)(\alpha_2\theta\beta - g) = 0 \qquad \theta\beta = \frac{g}{\alpha_2}$

~~$- 2\alpha_1\alpha_2(g + a c_1)(g + a c_2) + 4 a^2 c_1 c_2 \alpha_1\alpha_2 + \alpha_1^2(g + a c_1)^2 + \alpha_2^2(g + a c_2)^2$~~

~~$- 2\alpha_1\alpha_2 g^2 \pm 4\alpha_1\alpha_2 g(a c_1 + a c_2) \mp 2\alpha_1\alpha_2 a^2 c_1 c_2 + \alpha_1^2 g^2 + 2\alpha_1^2 g a c_1 + \alpha_1^2 a^2 c_1^2$~~

$+ 4 \qquad + \alpha_2^2 g^2$ ~~$+ 2\alpha_2^2 g a c_2 + \alpha_2^2 a^2 c_2^2$~~

$+ 2\alpha_1\alpha_2 a^2 c_1 c_2$

$= \cancel{\sqrt{\frac{g + a c_1}{2\alpha_1} + \frac{a c_1}{}}}$

$g^2(\alpha_1 - \alpha_2)^2 + 2 g a(\alpha_1^2 c_1 + \alpha_2^2 c_2 - 2\alpha_1\alpha_2 c_1 - 2\alpha_1\alpha_2 c_2) + a^2(\alpha_1^2 c_1^2 + \alpha_2^2 c_2^2)$

Izotermiczny rozkład dwuch gazów $\overset{\text{n.p.}}{}$ między ziemią i księżycem.

$u = u_1 = 0$

$$\frac{\partial p_1}{\partial x} = \rho_1 X_1 \qquad \frac{\partial p_2}{\partial x} = \rho_2 X_2$$

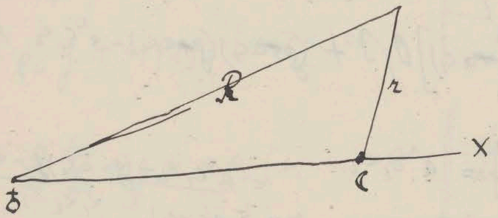
$$\frac{\partial p_1}{\partial y} = \rho_1 Y_1 \qquad \frac{\partial p_2}{\partial y} = \rho_2 Y_2$$

$$\frac{\partial p_1}{\partial z} = \rho_1 Z_1 \qquad \frac{\partial p_2}{\partial y} = \rho_2 Z_2$$

$$\cancel{\frac{\partial p_1}{\partial r} = \rho_1 F_1 \qquad \frac{\partial p_2}{\partial r} = \rho_2 F_2}$$

$$X_1 = X_2 = \cancel{\frac{kMx}{R^3}} \ \frac{\partial U}{\partial x}$$

etc.



$$\int \frac{1}{\rho_1} dp_1 = U = \int \frac{1}{\rho_2} dp_2$$

$$U = \frac{kM}{R} + \frac{km}{r}$$

$p_1 = \alpha_1 \rho_1 \theta \qquad p_2 = \alpha_2 \rho_2 \theta$

$\lg p_1 = \lg \alpha_1 \theta_1 + \lg \rho_1$

$$U = \alpha_1 \theta \lg \rho_1 + \text{const} = \alpha_2 \theta \lg \rho_2 + \text{const} = \frac{kM}{R} + \frac{km}{r}$$

$$\alpha_1 \theta \lg p_1 + \text{const} = \alpha_2 \theta \lg p_2 + \text{const} = $$

$$\alpha_1 \theta \log p_{10} + \text{const} = \frac{kM}{A}$$

$$\alpha_1 \theta \log \frac{p_1}{p_{10}} = \left[\frac{kM}{R} + \frac{km}{r} - \frac{kM}{A} - -\right] \Bigg\| \ \alpha_2 \theta \lg \frac{p_2}{p_{20}} = \left[\cdot - \qquad \right]$$

$$\alpha_1 \log \frac{p_1}{p_{10}} = \alpha_2 \log \frac{p_2}{p_{20}}$$

$$\left(\frac{p_1}{p_{10}}\right)^{\alpha_1} = \left(\frac{p_2}{p_{20}}\right)^{\alpha_2}$$

$$\frac{p_1}{p_{10}} = \left(\frac{p_2}{p_{20}}\right)^{\frac{\alpha_2}{\alpha_1}}$$

$$\frac{p_1}{p_2} = \frac{p_{10}}{p_{20}^{\frac{\alpha_2}{\alpha_1}}}\, p_2^{\left(\frac{\alpha_2}{\alpha_1}-1\right)}$$

Więc żeby stosunkowa gęstość dwuch gazów była niezmienna, $\frac{\alpha_2}{\alpha_1} - 1$ musiałoby być $= 0$, więc $\alpha_1 = \alpha_2$

Inaczej ów gaz który ma większe α n.p. $\alpha_2 > \alpha_1$ będzie miał stosunkowo ~~większe~~ mniejsze ciśnienie parcyalne tam gdzie p jest większe n.p. $\frac{\text{wodór}}{\text{tlen}}$ ~~większe~~ mniejsze w blizkości

$$\sigma \frac{d\sigma}{dr} = -\left(\alpha + \frac{c}{A}\right)\frac{d\theta}{dr} + \frac{4}{3}\frac{\gamma}{b}\frac{d}{dr}\left[\theta \sigma r^2 \left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\right]$$

$$\frac{c}{A}\frac{d\theta}{dr} + \frac{\alpha\theta}{\sigma}\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right] = \frac{4}{3}\frac{\gamma}{b}\theta r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2$$

Czy możliwe $\frac{d\theta}{dr} = 0$?

$$\sigma \frac{d\sigma}{dr} = \frac{4}{3}\frac{\gamma}{b}\theta\left\{\left[\frac{d}{dr}(\sigma r^2)\right]\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)\right] + \sigma r^2 \frac{d}{dr}\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]$$

$$\frac{\alpha\theta}{\sigma}\underbrace{\left[\frac{d\sigma}{dr} + \frac{2\sigma}{r}\right]}_{= \frac{1}{r^2}\frac{d}{dr}(\sigma r^2)} = \frac{4}{3}\frac{\gamma}{b}\theta r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2$$

$$\frac{d}{dr}(\sigma r^2) = \frac{4}{3}\frac{\gamma}{b\alpha}\sigma r^4\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2$$

Jeśli zaniedbujemy $\sigma \frac{d\sigma}{dr}$:

$$\left.\begin{aligned} &\cancel{\frac{4}{3}\frac{\gamma}{b}\theta} \quad \frac{\frac{d}{dr}(\sigma r^2)}{\sigma r^2} + \frac{\frac{d}{dr}\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)}{\frac{d\sigma}{dr} - \frac{\sigma}{r}} = 0 \\ &\frac{\alpha}{r^2\sigma}\frac{d}{dr}(r^2\sigma) = \frac{4}{3}\frac{\gamma}{b} r^2\left[\frac{d\sigma}{dr} - \frac{\sigma}{r}\right]^2 \end{aligned}\right\}$$

$$\frac{4}{3}\frac{\gamma}{b\alpha} r^2 + \underbrace{\frac{\frac{d}{dr}\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)}{\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^3}}_{= -\frac{1}{2}\frac{d}{dr}\frac{1}{\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2}} = 0$$

$$\frac{4}{3}\frac{\gamma}{b\alpha} r^3 = \frac{1}{\left(\frac{d\sigma}{dr} - \frac{\sigma}{r}\right)^2}$$

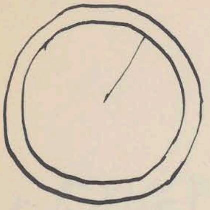
$$\frac{d\sigma}{dr} - \frac{\sigma}{r} = \underbrace{\sqrt{\frac{3\alpha b}{4\gamma}}}_{A}\, r^{-\frac{3}{2}}$$

$$\frac{d}{dr}\left(\frac{\sigma}{r}\right) = A\, r^{-\frac{5}{2}} \quad \Big\| \quad \frac{\sigma}{r} = -\frac{3}{2} A r^{-\frac{3}{2}} + \text{const}$$

$$\sigma = -\frac{3}{2} A r^{-\frac{1}{2}} + r a$$

Izotermiczna masa gazu działająca na siebie grawitacyą i ciśnieniem

Czy jest więcej układów możliwych niż jeden, czy stało albo wstało?
Jakie skutki termiczne przejścia?



$p = \alpha\rho\theta \qquad \theta = \text{const}$

$\rho = f.(r)$

$$R = -\frac{k}{r^2}\int_0^r \rho\, r^2\pi\, dr = -\frac{k}{r^2}\int_0^r f(r)\, dr\, r^2\pi$$

$$R = \frac{1}{\rho}\frac{dp}{dr}$$

$$-\frac{k}{r^2}\int_0^r \rho\, r^2\pi\, dr = \frac{1}{\rho}\,\alpha\theta\,\frac{d\rho}{dr}$$

~~$\frac{\rho}{\alpha\theta}\int = -\frac{\alpha\theta}{k}\, r^2\frac{d\rho}{dr}$~~

~~$\frac{d\rho}{dr}\int + \rho^2 = \frac{\alpha\theta}{k}\left(2r\frac{d\rho}{dr} + r^2\frac{d^2\rho}{dr^2}\right)$~~

$$\int_0^r \rho\, r^2\, dr = -\frac{\alpha\theta}{k\pi}\,\frac{r^2}{\rho}\frac{d\rho}{dr}$$

$$-k\,\rho\, r^2\pi = \alpha\theta\,\frac{d}{dr}\left(r^2\frac{1}{\rho}\frac{d\rho}{dr}\right)$$

$$-\frac{k\pi}{\alpha\theta} r^2\rho = \cancel{\alpha\theta}\ \frac{2r}{\rho}\frac{d\rho}{dr} - \frac{r^2}{\rho^2}\left(\frac{d\rho}{dr}\right)^2 + \frac{r^2}{\rho}\frac{d^2\rho}{dr^2}$$

$$E_{gr} = \int^r \rho\, U\, dv \qquad U = \frac{k\pi}{r}\int_0^r r^2\rho\, dr$$

$$= \cancel{4}\, k\pi\int_0^r \frac{r^2\pi\, dr}{r}\,\rho\int_0^r r^2\rho\, dr$$

$$= k\pi^2\int_0^r r\rho\, dr\int_0^r r^2\rho\, dr = -k\pi^2\int_0^r r\rho\, dr\,\frac{\alpha\theta}{k\pi}\,\frac{r^2}{\rho}\frac{d\rho}{dr} = -\alpha\theta\pi\int_0^r r^3\frac{d\rho}{dr}\, dr$$

$$= -\alpha\theta\pi\left[\rho r^3\Big|_0^r - 3\int_0^r \rho r^2\, dr\right]$$

~~$E_p = \int p\, dv = \int_{p=0}^{p} p\, d\left(\frac{1}{\rho}\right) = -\alpha\theta\int_0^{p} \rho\,\frac{1}{\rho^2}\, d\rho = -\alpha\theta\int_0^{\rho}\frac{d\rho}{\rho}$~~

$$E_i = \theta\int \rho c\, dv = \theta c\int_0^r \rho\, r^2\, dr\,\pi$$

$$E_i + E_{gr} = \theta\pi\left[-\alpha\rho r^3\Big|_0^r + (3\alpha - c)\int_0^r \rho r^2\, dr\right] = \theta\left[\alpha\pi\rho_a a^3 + (3\alpha - c)\, \underline{M}\right]$$

$$J_v = \int\left[u\frac{Du}{Dt} + v\frac{Dv}{Dt} + w\frac{Dw}{Dt} - (uX + vY + wZ) + c\frac{D\theta}{Dt}\right] dm +$$

$$\begin{array}{c} J_s = \\ - \\ \vdots \end{array} \int \left\{ \begin{array}{l} (p_{xx}\cos nx + p_{xy}\cos ny + p_{xz}\cos nz)\,u + \\ (p_{yx}\cos nx + p_{yy}\cos ny + p_{yz}\cos nz)\,v \\ (p_{zx}\cos nx + p_{zy}\cos ny + p_{zz}\cos nz)\,w \end{array} \right\} dS \quad = 0$$

$$J_s = \begin{array}{c} - \\ \vdots \end{array} \iiint \frac{\partial[(p_{xx}u) + (p_{xy}v) + (p_{xz}w)]}{\partial x} + \frac{\partial}{\partial y}(p_{xy}u + p_{yy}v + p_{zy}w) + \frac{\partial}{\partial z}(p_{xz}u + --)$$

$$= \iiint \left[\begin{array}{l} u\left(\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z}\right) + v\left(\frac{\partial p_{xy}}{\partial x} + \frac{\partial p_{yy}}{\partial y} + \frac{\partial p_{yz}}{\partial z}\right) + w\left(\qquad\right) \\ + p_{xx}\frac{\partial u}{\partial x} + p_{yy}\frac{\partial v}{\partial y} + p_{zz}\frac{\partial w}{\partial z} + p_{xy}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + p_{yz}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) + p_{zx}(\quad) \end{array} \right.$$

$$+J_v = \left. \begin{array}{l} u\rho X + v\rho Y + w\rho Z + u\left[\frac{\partial p_{xx}}{\partial x} + \frac{\partial (p_{xy})}{\partial y} + \frac{\partial (p_{xz})}{\partial z}\right] \\ \qquad + v\left[\frac{\partial p_{xy}}{\partial x} + \frac{\partial p_{yy}}{\partial y} + \frac{\partial p_{zy}}{\partial z}\right] \\ \qquad + w\left[\frac{\partial p_{xz}}{\partial x} + \frac{\partial p_{yz}}{\partial y} + \frac{\partial p_{zz}}{\partial z}\right] \\ -(u\rho X + v\rho Y + w\rho Z) + c\rho\frac{D\theta}{Dt} \end{array} \right] d\tau = 0$$

$$= \iiint c\rho\frac{D\theta}{Dt} \pm p_{xx}\frac{\partial u}{\partial x} + \cdot\cdot \qquad \pm p_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \pm p_{yz}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + p_{zx}(\quad)$$

$$= \iiint c\rho\frac{D\theta}{Dt} \pm p\,\mathrm{div} + \frac{2}{3}\mu(\mathrm{div})^2 - 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] - \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 - \mu(\quad)^2 - \mu(\quad)^2$$

$$\boxed{c\rho\frac{D\theta}{Dt} + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -\frac{2}{3}\mu(\mathrm{div})^2 + 2\mu[\quad] + \mu[\,.-\,]}$$

$$A_{ex} = \int -p\sigma\, dS - \int \frac{2}{3}\mu\, \cancel{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2} - \frac{2}{3}\mu \left\{ \begin{array}{l} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} + \\ \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} \\ + \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial z^2} \end{array} \right\}$$

$$+ \mu\, \cancel{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2}$$

$$\cancel{+ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2}$$

$$+ u\nabla^2 u + v\nabla^2 v + w\nabla^2 w$$

$$+ \cancel{2\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + 2\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}}$$

$$+ v\frac{\partial^2 u}{\partial x \partial y} + w\frac{\partial^2 u}{\partial x \partial z} + w\frac{\partial^2 v}{\partial y \partial z} + u\frac{\partial^2 v}{\partial y \partial x} + u\frac{\partial^2 w}{\partial z \partial x} + v\frac{\partial^2 w}{\partial z \partial y}$$

$$= 2\mu\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + (\quad)^2$$

$$-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

$$\sigma\rho = b$$

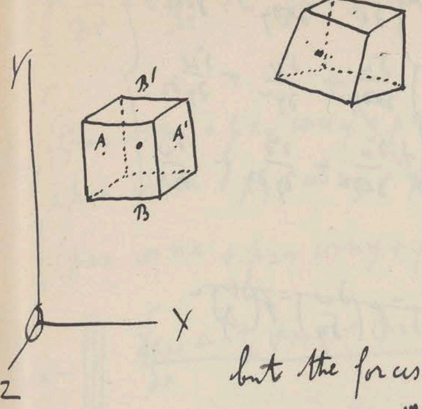
$$p = \alpha\theta\rho$$

$$= \frac{\alpha\theta b}{\sigma}$$

$$c\rho\sigma\frac{\partial\theta}{\partial x} + p\frac{\partial\sigma}{\partial x} = \frac{4}{3}\mu\left(\frac{\partial\sigma}{\partial x}\right)^2$$

$$c\frac{\partial\theta}{\partial x} + \frac{\alpha\theta}{\sigma}\frac{\partial\sigma}{\partial x} = \frac{4}{3}\mu\left(\frac{\partial\sigma}{\partial x}\right)^2$$

Direct method:



6 (triples of) forces:

~~path~~ of the centre : $u\Delta t,\ v\Delta t,\ w\Delta t$

path of point A : $u_{(x-\frac{\delta}{2},\,y,z)}\Delta t,\ v_{(x-\frac{\delta}{2},\,y,z)}\Delta t,\ w_{(x-\frac{\delta}{2},\,y,z)}\Delta t$

A' : $u(x+\frac{\delta}{2},y,z)\Delta t,\ v(x+\frac{\delta}{2},y,z)\Delta t$ etc.

but the forces are not $p_{xx}\,\delta^2$ ~~etc.~~ during the whole time,

but varying from $p_{xx\,(x-\frac{\delta}{2},y,z)}\delta^2$ to $p_{xx\,(x+u\Delta t+\frac{\delta}{2},y,z)}\left[\delta^2+\frac{\partial(\delta^2)}{\partial u}\Delta t\right]$

$u(x+\frac{\delta}{2},y,z)\Delta t.\ p_{xx}(x+\frac{\delta}{2},y,z)\,\delta^2 - u_{(x-\frac{\delta}{2},y,z)}\Delta t$ - - - -

$+u(x,y+\frac{\delta}{2},z)\Delta t\ p_{xy}(x,y+\frac{\delta}{2},z)$

$$A_{ex}=\int -\left(p+\tfrac{2}{3}\mu\,\mathrm{div}\right)6\,dS+\cancel{2\mu\left[u\frac{\partial u}{\partial x}\,dy\,dz+v\frac{\partial v}{\partial y}\,dx\,dy+w\frac{\partial w}{\partial z}\,dx\,dz\right]}$$

$$+\mu\left\{\left[u\frac{\partial u}{\partial x}+\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}+w\frac{\partial w}{\partial x}\right)\right]dy\,dz+\left(v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)dy\,dz\right\}$$

$$+\mu\int\left[v\frac{\partial v}{\partial y}+\left(u\frac{\partial u}{\partial y}+v\frac{\partial v}{\partial y}+w\frac{\partial w}{\partial y}\right)\right]dz\,dx+\left(w\left(\frac{\partial v}{\partial z}+u\frac{\partial v}{\partial x}\right)dz\,dx\right($$

$$+\mu\int\left[w\frac{\partial w}{\partial z}+\left(\qquad\right)\right]dx\,dy+\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right)dx\,dy$$

$$\int\frac{\mu}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]dx\,dy\,dz+\mu\left[u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial y^2}+w\frac{\partial^2 w}{\partial z^2}\right)dx\,dy\,dz$$

$$+\cancel{\mu\int\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]}+\frac{\mu}{2}\int\cancel{\mathrm{div}\,(\quad)\,dx\,dy\,dz}$$

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2+u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial x^2}+w\frac{\partial^2 w}{\partial x^2}\quad+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}+v\frac{\partial^2 u}{\partial x\partial y}+w\frac{\partial^2 u}{\partial x\partial z}$$

$$+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+u\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^2 v}{\partial y^2}+w\frac{\partial^2 w}{\partial y^2}\quad\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$$

$$+\quad+\quad+\qquad+-\qquad\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}+\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}$$

$$-\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=\frac{1}{\rho}\left[u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}+\frac{\partial\rho}{\partial t}\right]$$

$$\int \cancel{p_{xx}u}\underset{\times}{+}p_{xy}v\,\cancel{+p_{xz}}$$

$$A_{\omega}=\int\Big[(p_{xx}\cos nx+p_{xy}\cos ny+p_{xz}\cos nz)u+(p_{yx}\cos nx+p_{yy}\cos ny+p_{yz}\cos nz)v+\big(\qquad\big)w\Big]dS$$

$$\left[\;\int \underbrace{U\Delta^2 V}_{\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}}d\tau=-\int\left(\frac{\partial U}{\partial x}\frac{\partial V}{\partial x}+\frac{\partial U}{\partial y}\frac{\partial V}{\partial y}+\frac{\partial U}{\partial z}\frac{\partial V}{\partial z}\right)d\tau+\int U\underbrace{\frac{\partial V}{\partial n}}\,dS\right.$$

$$=\int U\left(\frac{\partial V}{\partial x}\cos nx+-\qquad\right)dS$$

$$\cancel{A_{\omega}=\int p_{xx}u+p_{xy}}$$

$$=\int-p\overbrace{(u\cos nx+v\cos ny+w\cos nz)}^{6}\,dS-\frac{2}{3}\mu\,\mathrm{div}\big(\overset{u}{}\cos nx+v\cos ny+w\cos nz\big)\,dS+$$

$$\left\{\begin{aligned}&+2\mu\left(u\frac{\partial u}{\partial x}\cos nx+v\frac{\partial v}{\partial y}\cos ny+w\qquad\right)dS\\&+\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)(w\cos ny+v\cos nz)+\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)(u\cos nz+w\cos nx)+\cancel{\mu\qquad}\\&\qquad+\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)(v\cos nx+u\cos nz)=0\end{aligned}\right.$$

$$\mu\left[2\frac{\partial u}{\partial x}u+\frac{\partial v}{\partial x}v+\frac{\partial w}{\partial x}w+\frac{\partial w}{\partial x}w+\frac{\partial u}{\partial z}w\right]\overset{dy\,dz}{\cancel{\cos nx}}+-$$

$$\left[2\frac{\partial v}{\partial y}v+\frac{\partial w}{\partial y}w+\frac{\partial u}{\partial z}w+\frac{\partial w}{\partial y}u+\frac{\partial v}{\partial z}u\right]\overset{dx\,dz}{\cancel{\cos ny}}+$$

$$\left[2\frac{\partial w}{\partial z}w+\frac{\partial u}{\partial y}\cancel{u}+\frac{\partial v}{\partial x}u+\frac{\partial u}{\partial z}v+\frac{\partial w}{\partial x}v\right]\overset{dy\,dx}{\cancel{\cos nz}}$$

$$\rho\frac{Du}{Dt} = \rho X - \frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial\, \mathrm{div}}{\partial x} + \mu\nabla^2 u \qquad X = -\frac{\partial U}{\partial x}$$

$$\rho\frac{Dv}{Dt} = \rho Y - \frac{\partial p}{\partial y} + \frac{\mu}{3}\frac{\partial\, \mathrm{div}}{\partial y} + \mu\nabla^2 v$$

$$\rho\frac{Dw}{Dt} = \rho Z - \quad - \quad - \quad - \quad —$$

$$+\int^{D} p\,6\, dS$$

$$\frac{D}{Dt}\int\left[\rho\frac{(u^2+v^2+w^2)}{2}\, d\tau + \rho U d\tau + \rho c\,\theta\, d\tau\right] = 0$$

$$\rho\, d\tau = dm$$

$$+\int$$

$$\frac{D}{Dt}\int\left[\frac{u^2+v^2+w^2}{2} + U + c\,\theta\right] dm = 0 \qquad \frac{D\, dm}{Dt} = 0$$

$$+\int$$

$$\int\left[u\frac{Du}{Dt} + v\frac{Dv}{Dt} + w\frac{Dw}{Dt} + \underbrace{\frac{DU}{Dt}}_{} + c\frac{D\theta}{Dt}\right] dm = 0$$

$$= u\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} + w\frac{\partial U}{\partial z} \qquad \text{jeśli } \frac{\partial U}{\partial t} = 0$$

$$\int^{D} p\,6\, dS + \int^{D} p_{xx}\, u\, dS + \dots \qquad i\ \frac{\partial c}{\partial t} = \frac{\partial c}{\partial x} = \dots = 0$$

$$= -(uX + vY + wZ)$$

$$\int\left[-\frac{1}{\rho}\left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right] + \frac{\mu}{3\rho}\left(u\frac{\partial\, \mathrm{div}}{\partial x} + v\frac{\partial\, \mathrm{div}}{\partial y} + w\frac{\partial\, \mathrm{div}}{\partial z}\right) + \frac{\mu}{\rho}\left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right)\right.$$

$$\left. + c\left(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)\right] dm = 0$$

$$\int^{D} p\,6\, dS + v$$

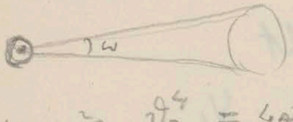
$$\int -\left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \quad\right] + \frac{\mu}{3}\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\ .\ \mathrm{div}\right) + \mu\left(u\nabla^2 u + v\nabla^2 v + .\ \right) +$$

$$+ c\rho\left(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + - \quad\right)\Big] d\tau = 0$$

$$\int p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{\mu}{3}\left[(\mathrm{div})^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \frac{\partial v}{\partial x}\frac{\partial u}{\partial x} + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial v}{\partial z}\frac{\partial w}{\partial z} + \right.$$

$$+ \frac{\partial w}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial v}{\partial y} + \left(\frac{\partial w}{\partial z}\right)^2$$

[illegible]



$$\omega\cdot \cancel{\pi^2}\,\vartheta_\odot^4 = 4\cancel{\pi^2}\cdot 4\pi\,\vartheta^4$$

$$\left(\frac{1}{720}\right)^2=\omega$$

$$\vartheta = \vartheta_\odot\sqrt[4]{\frac{\omega}{4\pi}}$$

$$\sqrt{\frac{1}{10\,000}}=\frac{1}{100}$$

~60°

$$\frac{1}{3}\,\mathrm{div}^2+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\dots\ \cancel{+}\ \frac{2}{3}\,\mathrm{div}^2-2\left[\left(\frac{du}{dx}\right)^2+\left(\frac{dv}{dy}\right)^2+\left(\frac{dw}{dz}\right)^2\right]$$

$$-\left[\left(\frac{dw}{dy}+\frac{dv}{dz}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+(\quad)^2\right]$$

$$\mathrm{div}^2-\left[\left(\frac{du}{dx}\right)^2+\cancel{\left(\frac{\partial u}{\partial y}\right)^2}+\cancel{\left(\frac{\partial u}{\partial z}\right)^2}+\cancel{1}\,\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+\cancel{1}\,\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}-\cancel{\left(\frac{\partial u}{\partial y}\right)^2}-\cancel{\left(\frac{\partial u}{\partial z}\right)^2}\right]$$

$$-\left[\left(\frac{dv}{\partial y}\right)^2+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\right]$$

$$-\left[\qquad\qquad\right]$$

$$=2\left[\frac{\partial v}{\partial y}\frac{\partial u}{\partial x}-\cancel{1}\,\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right]+2\left(\frac{\partial u}{\partial x}\frac{\partial w}{\partial z}-\frac{\partial u}{\partial z}\frac{\partial w}{\partial x}\right)+2(\qquad)$$

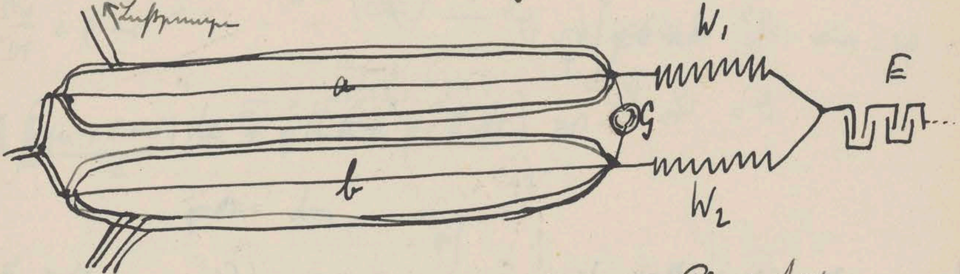
$$\mathcal{L}=\frac{15}{4}\frac{\mathcal{R}R}{\mu}\qquad\gamma_v=\frac{3R}{2\mu}$$

$$\frac{3}{2}p\,\frac{d\left(\frac{p}{\rho}\right)}{dt}=\frac{p}{\rho}\frac{d\rho}{dt}+\frac{15}{4}\left[\frac{\partial}{\partial x}\left(\mathcal{R}\,\frac{\partial\left(\frac{p}{\rho}\right)}{\partial x}\right)+\dots\right]$$

$$+\,\mathcal{R}\left[\left\{2\left(\frac{\partial u}{\partial x}\right)^2+\dots\right\}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\dots\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+(\quad)^2\right]$$

Verfahren zur Messung der Leitfähigkeit, Temperatur Coeff. derselben und Temperatursprung von Gasen.

(Ähnlich schon von Schleiermacher gemacht



a wird mit Gas gefüllt, b mit gut leitender Flüssigkeit die Widerstand Änderung von a, ~~welche~~ bewirkt Ausschlag in G ~~bewirkt~~; entweder diesen messen oder Nullmethode; Compensirung durch Änderung von W_1 oder Nebenschluss in a Leitung mit zu messendem Widerstand; Strommessung ~~entweder~~ im W_1-Kreise durch Boussole oder besser in W_1 und W_2 (oder auch am besten im gemeinsamen Kreise) durch Silbervoltameter. Widerstände von a, b, G möglichst groß; E ebenso. Anwendung von Drähten verschiedener Dicke (gleichzeitig) und verschiedenem Material; bei dickeren Draht wird ~~W~~ Wärmeleitung gemessen, bei ~~dünn~~ sehr dünnen (Wollaston $\frac{1}{20}$ mm) Temperatursprung

$$\mathrm{div} \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \frac{\partial}{\partial x} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + \frac{\partial}{\partial y} \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} + \frac{\partial}{\partial z} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$S\, b\, \mathrm{curl}\, a = b_1 \left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + b_2 \qquad + b_3$$

$$- a_1 \left(\frac{\partial b_3}{\partial y} - \frac{\partial b_2}{\partial z}\right) - a_2 \qquad - a_3 \left(\frac{\partial b_2}{\partial x} - \frac{\partial b_1}{\partial y}\right)$$

$$-\frac{d\omega}{dt} = \frac{8\pi\mu a^3\omega_0}{\frac{4}{3}a^3\pi \cdot \frac{2a^2}{5}} = \frac{15\mu}{a^2}\omega_0 = \frac{15\mu \cdot 2\pi}{a^2 t}$$

$$K\frac{d\omega}{dt} = P$$

$t \cdot \frac{d\omega}{dt} = \frac{30\pi\mu}{a^2}$ = zmiana $\overset{\omega}{\text{w}}$ ~~sekundach~~ podczas dnia

$\frac{365 \cdot 30 \cdot \pi\mu}{a^2} =$ 〃 〃 1 roku

$$\frac{35 \cdot \cancel{10}^{3} \cdot \cancel{0{\cdot}0001}7}{36 \cdot 10^{16}} = 1{\cdot}7 \cdot 10^{-17} \cdot \frac{t}{2\pi} = 1{\cdot}7$$

$$\frac{\cancel{365} \cdot \cancel{30} \cdot \cancel{\pi} \cdot \overset{50}{\cancel{0{\cdot}00017}}}{\cancel{36}\ 10^{16}} \cdot \frac{86{\cdot}4\cancel{00}}{2\cancel{\pi}} \qquad \frac{43 \cdot 5}{10^{15}} = 2 \cdot 10^{-13}$$

$$\Delta p = \frac{3}{2} \cancel{\ } \frac{20\,00.0\,00 \cdot 0{\cdot}00017}{6\ \ 10^8} \qquad \frac{4}{800}$$

$$p = 980 \cdot 76 \cdot 13{\cdot}6 = 10^6$$

$$\frac{\Delta p}{p} = \frac{10^{-6}}{10^6} = 10^{-12} = 10^{-9}\ \text{mm}$$

$$P = \frac{a^2\cancel{\pi}}{\cancel{5{\cdot}6} \cdot \frac{4}{5}a^2\cancel{\pi}}\Delta p \cdot 86400 \cdot 365 = \text{Smh. } \varphi = 1\,\text{m}$$

$$= \frac{10^{-9} \cdot 3 \cdot 10^7}{15\ \cancel{7{\cdot}5} \cdot 3 \cdot 10^8} = \frac{10^{-10}}{15}$$

nteresa

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s tego sztanda-
jazków,
e.
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przе-
asz wi-
po-
odległej
rząd-

ak długo jeszcze
Ale błędem byłoby
że aprawialnym, gdyby naród
nasz nie ciężył na decyzyach w każdej
fazie, jaką przynoszą zawikłania orężne
i dyplomatyczne. Dlatego oczekiwać
będziemy w tej chwili głosu G.
su nietylko urzędowego, ale pow
go, który powinien odezwać się z
łą przekonania, że przyszłość nar
wymaga złączenia Galicyi z powstającą
niepodległą Polską. Mamy nadzieję
się odezwie, i że w rokowania brz
padnie z takim naciskiem, jakiego
magają obecne stosunki i dobro Oj-
czyzny.

---

narodowym.

5. stycznia.

go ania się
wzglę żądań un
innych grup narod
„Zwłaszcza od
przez grupy bu

czen
łatwienia, związ

„
wysuwam
porządkowały się
i Nr. 12

130

ae

Thermal effect of effusion:

Natanson Wied Ann 37 p. 341 (1889)

Ciepło wydzielone naczyniu

$$\int\!\!\iint k\frac{\partial\theta}{\partial n}\,dF\,dt=\iiint k\,\Delta^2\theta\,dv\,dt=\int\Big[\frac{c}{A}\frac{\partial p}{\partial t}+\rho\Big(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\Big)+p\,\mathrm{div}-\Phi\Big]dt\,dv$$

$$0 = \frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial \operatorname{div}}{\partial x} + \mu \nabla^2 u$$

$$\cancel{0 = -\frac{\partial p}{\partial x}}$$

$$\frac{\partial p}{\partial x} = \mu\left(\frac{1}{3}\frac{\partial \operatorname{div}}{\partial x} + \nabla^2 u\right)$$

$$\frac{\partial p}{\partial y} = \mu\left(\frac{1}{3}\frac{\partial \operatorname{div}}{\partial y} + \nabla^2 v\right)$$

$$\frac{\partial p}{\partial z} = \mu\left(\frac{1}{3}\frac{\partial \operatorname{div}}{\partial z} + \nabla^2 w\right)$$

$$p = R\rho\theta$$

$$\frac{\partial(\rho w)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) + p \operatorname{div} = \Phi$$

Praca zewnętrzna

$$W_I = -\iint p(ul + vm + wn)\, d\sigma = -\iiint\left[\frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) + \frac{\partial}{\partial z}(pw)\right] dv$$

$$W_{II} = \iint \Phi \, dv$$

$$W_I = -R\iiint \theta\left[\cancel{\frac{\partial}{\partial x}(\rho u) + \cdots}\right] + \rho\left[\frac{\partial}{\partial x}(\theta u) + \cdots\right]$$

$$= -\iiint R\rho\left[u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + -\right] + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$-W_I = \iiint \Phi + \left(R - \frac{c}{A}\right)\underbrace{\underbrace{\iiint \rho\left(u\frac{\partial\theta}{\partial x} + \cdots\right)}_{\iint \theta\rho(ul + vm + wn)\, d\sigma}}_{= \left(1 - \frac{c}{AR}\right)\iint p(ul + vm + wn)\, d\sigma}$$

$$-W_I = (k-1)\, W_{II}$$

$$-W_I = \iiint \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

2° term :

$$\underbrace{\iiint R\rho\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) dv}_{} = \iiint \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) dv$$

$$\underbrace{\iint \underbrace{R\rho}_{p}\,\theta(ul + vm + \dots) - \iiint R\theta\frac{\partial(\rho u)}{\partial x} + \dots}_{= -W_I}$$

2° term 2 $\quad : \quad \iiint p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dv = 0$

$$-W_I = \iiint \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) dv =$$

$$= \mu \iiint \left(u\nabla^2 u + v\nabla^2 v + w\nabla^2 w\right) dv + \frac{1}{3}\left(u\frac{\partial\, \mathrm{div}}{\partial x} + v\frac{\partial\, \mathrm{div}}{\partial y} + w\frac{\partial\, \mathrm{div}}{\partial z}\right) dv$$

$$= \mu \iint u\left(\frac{\partial u}{\partial x}l + \frac{\partial u}{\partial y}m + \frac{\partial u}{\partial z}n\right) + v\left(\frac{\partial v}{\partial x}l + \cdots\right) + \cdots \quad dS +$$

$$+ \frac{1}{3} \iiint \mathrm{div.}(ul + vm + wn)\, dS$$

$$- \iiint \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \sim \qquad + \frac{1}{3}(\mathrm{div})^2\right] dv$$

$$= \mu \iint \frac{\partial}{\partial n}\left(\frac{u^2 + v^2 + w^2}{2}\right) dS + \frac{\mu}{3} \iint v_n\, \mathrm{div}\, dS - \mu \iiint \uparrow$$

Z drugiej strony $W_{II} = \iiint \Phi \, dv =$

$$\mu \iiint \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \cdots \qquad + \frac{1}{3}\,(div)^2$$

$$+ 2\left[\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\frac{\partial w}{\partial z} - \frac{\partial w}{\partial z}\frac{\partial u}{\partial x}\right] dv$$

$$\iiint \left[\frac{\partial w}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial v}{\partial y}\frac{\partial w}{\partial z}\right] dv = \iint w\left(m\frac{\partial v}{\partial z} - n\frac{\partial v}{\partial y}\right) dS - \cancel{\iiint w\frac{\partial^2 v}{\partial y\,\partial z} - w\frac{\partial^2 v}{\partial y\,\partial z}\, dv}$$

$$\iiint \cdot\cdot \qquad\qquad = \iint v\left(n\frac{\partial w}{\partial y} - m\frac{\partial w}{\partial z}\right) dS$$

$$W_{II} = \mu \iiint \left(\frac{\partial u}{\partial x}\right)^2 + \cdots \quad \frac{1}{3}(div)^2 + \mu \iint \Big\{ u\left(n\frac{\partial w}{\partial x} - l\frac{\partial w}{\partial z}\right) + w\left(l\frac{\partial u}{\partial z} - n\frac{\partial u}{\partial x}\right) +$$

$$\pm\, l\,u\frac{\partial u}{\partial x} \qquad \pm\, m\,v\frac{\partial v}{\partial y} \qquad \pm\, n.w\frac{\partial w}{\partial z}$$

$$+ v\left(l\frac{\partial u}{\partial y} - m\frac{\partial u}{\partial x}\right) + u\left(m\frac{\partial v}{\partial x} - l\frac{\partial v}{\partial y}\right) + w\left(m\frac{\partial v}{\partial z} - n\frac{\partial v}{\partial y}\right) + v\left(n\frac{\partial w}{\partial y} - m\frac{\partial w}{\partial z}\right)\Big\} dS$$

zatem jeśli można zaniedbać całki powierzchniowe to będzie

$W_{II} = W_I$ co z tamtym rezultatem tylko wtedy się zgadzać może

jeśli $W_{II} = W_I = 0$

Albo też: wstawiając $\iiint p \; div = 0$

$$\iiint \Phi \, dv = \frac{c}{A}\iiint \rho\left(u\frac{\partial \vartheta}{\partial x} + \cdots\right) = -\frac{c}{AR}\, W_I$$ to samo co poprzednio

$$W_{II} = \mu \iint \left(\frac{\partial u}{\partial x}\right)^2 + \cdots \quad \frac{1}{3}\, div^2 + \mu \iint \left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)(v_n) - v_n\, div.\right] dS$$

$$= \qquad \cancel{+2\mu\iint l\left(v\frac{\partial u}{\partial y} - u\frac{\partial w}{\partial z}\right) + m(\;) + n(\;)} \qquad \cancel{\pm v\frac{\partial v}{\partial x} \pm u}$$

patrz Lamb p. 519

Czy możliwe rozwiązanie formy $\theta = f(p)$

$$\rho = \varphi(p) = \frac{p}{R\, f(p)}$$

$$\frac{\partial p}{\partial x} = \mu\left(\nabla^2 u + \frac{\partial \, \mathrm{div}}{\partial x}\right)$$

$$\frac{\partial p}{\partial y} =$$

$$\frac{\partial p}{\partial z} =$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

II). $\frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) + p\left(\frac{\partial u}{\partial x} + \dots\right) = \Phi$

$$\left[p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -\left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right] + R\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)\right.$$

$$\frac{C}{A}\rho\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) = \Phi + \left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right]$$

$$\frac{\partial\theta}{\partial x} = f'\frac{\partial p}{\partial x}$$

$$\frac{C}{AR}\frac{p f'}{f}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) = \Phi + [\quad -- \quad]$$

$\frac{C}{AR} = \frac{k}{k-1}$ Jeśli się żąda równania adiabatycznego: $p = c\rho^k$

$$\rho = c' p^{\frac{1}{k}} = \frac{p}{R f(p)}$$

$$f = p^{1-\frac{1}{k}}$$

$$f \sim p^{\frac{k-1}{k}}$$

$$f' \sim \frac{k-1}{k} p^{-\frac{1}{k}} \qquad \frac{p f'}{f} = \frac{k-1}{k}$$

zatem redukują się $\frac{c}{AR}\frac{p f'}{f} \equiv 1$

Więc tylko możliwe jeśli $\Phi = 0$

~~[illegible]~~

Jeśli $v = w = 0$:

$$\frac{\partial p}{\partial x} = \mu\left[\frac{1}{3}\frac{\partial^2 u}{\partial x^2} + \nabla^2 u\right] \qquad \frac{\partial(\rho u)}{\partial x} = 0$$

$$\frac{\partial p}{\partial y} = \mu\,\frac{1}{3}\frac{\partial^2 u}{\partial x\,\partial y} \qquad \frac{c}{A}\rho u\frac{\partial\theta}{\partial x} + p\frac{\partial u}{\partial x} = \Phi_u$$

$$\frac{\partial p}{\partial z} = \mu\,\frac{1}{3}\frac{\partial^2 u}{\partial x\,\partial z}$$

Jednowymiarowe zadanie:

$x=0$: p_0, ρ_0

$x=l$: p_1

$$\frac{\partial p}{\partial x} = \frac{4}{3}\mu\frac{\partial^2 u}{\partial x^2} \qquad \rho u = \rho_0 u_0$$

$$\frac{c}{A}\rho u\frac{\partial\theta}{\partial x} + p\frac{\partial u}{\partial x} = \frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2 \qquad \frac{c}{A}\rho\theta = \frac{1}{k-1}p$$

~~$\frac{c}{A}\underbrace{\rho u\frac{\partial\theta}{\partial x}}_{=\frac{\partial}{\partial x}(\rho u\theta)} + \frac{\partial}{\partial x}(pu) = \frac{4}{3}\mu\left[u\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial x}\right)^2\right] = \frac{4}{3}\mu\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right)$~~

$$p = R\rho\theta$$

~~$\frac{c}{A}\rho u\theta + pu = \frac{4}{3}\mu\frac{\partial u}{\partial x}$~~

$$p = \frac{4}{3}\mu\frac{du}{dx} + p_0 \quad \uparrow \quad \Big\| \quad \frac{c}{A}\rho u\frac{\partial\theta}{\partial x} + \cancel{\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2} + p_0\frac{\partial u}{\partial x} = \cancel{\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2}$$

$$\frac{c}{A}\rho u\theta + p_0 u = \cancel{\text{[illegible]}} = a$$

$$\frac{c}{AR}pu + p_0 u = \Big(\underset{\overset{\shortparallel}{k-1}}{\frac{c}{AR}} + 1\Big)p_0 u + \frac{4}{3}\mu\frac{c}{AR}u\frac{du}{dx} = a$$

$$\frac{4}{3}\mu\,u\frac{du}{dx} + k\,p_0 u = (k-1)a$$

$\frac{4}{3}\mu \; u\frac{du}{dx} + k p_0 u = (k-1)a$ $\qquad\Bigg\|\qquad$ $\dfrac{\frac{4}{3}\mu\, u\, du}{(k-1)a - k p_0 u} = dx$

Substytucja: $u = \alpha + z$

$$\frac{4}{3}\mu\,\alpha\frac{dz}{dx} + \frac{4}{3}\mu\, z\frac{dz}{dx} + k p_0 \alpha + k p_0 z = (k-1)a$$

$$\alpha = \frac{(k-1)a}{k p_0}$$

$$\frac{4}{3}\mu\, z\frac{dz}{dx} + \frac{4}{3}\mu\alpha\frac{dz}{dx} + k p_0 z = 0$$

$$\frac{4}{3}\mu\frac{dz}{dx} + \frac{4}{3}\mu\alpha\frac{d\log z}{dx} + k p_0 = 0$$

$$\frac{4}{3}\mu z + \frac{4}{3}\mu\alpha\log z + k p_0 x = b$$

$\frac{4}{3}\mu(u-\alpha) + \frac{4}{3}\mu\alpha\log(u-\alpha) + k p_0 x = b$ zamiast a lepiej używać α

$$\left.\begin{aligned} \frac{4}{3}\mu\log\left[e^{u-\alpha}\cdot(u-\alpha)^{\alpha}\right] &= b - k p_0 x \\ \frac{p u}{k-1} + p_0 u &= \frac{k p_0}{k-1}\alpha \\ p &= p_0 + \frac{4}{3}\mu\frac{du}{dx} \end{aligned}\right\}$$

$$e^{u-\alpha}(u-\alpha)^{\alpha} = e^{\frac{b - k p_0 x}{\frac{4}{3}\mu}}$$

$$e^{u}(u-\alpha)^{\alpha} = B\, e^{-\frac{3}{4}\frac{k p_0}{\mu}x}$$

$$e^{u + \frac{3}{4}\frac{k p_0}{\mu}x}(u-\alpha)^{\alpha} = B$$

$$p = \frac{k p_0 \alpha - p_0(k-1)u}{u}$$

$$p = \frac{k p_0 \alpha}{u} - p_0(k-1)$$

$\theta =$

Za pomocą eliminacji: $\theta = \frac{p}{R\rho}$

$$\rho \frac{du}{dx} + u \frac{d\rho}{dx} = 0$$

$$-\frac{1}{\rho}\frac{d\rho}{dx} = +\frac{1}{u}\frac{du}{dx}$$

$$\frac{c}{AR}\rho u\left[\frac{1}{\rho}\frac{dp}{dx} - \frac{p}{\rho^2}\frac{d\rho}{dx}\right] + p\frac{du}{dx} = \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2$$

$$\left.\begin{aligned} \frac{c}{AR}\left[u\frac{dp}{dx} + p\frac{du}{dx}\right] + p\frac{du}{dx} &= \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 \\ \frac{dp}{dx} &= \frac{4}{3}\mu\frac{d^2u}{dx^2} \end{aligned}\right\}$$

Zatem 3 stałe do oznaczenia p, u a jedna jeszcze do θ lub ρ

Stałe wyznaczymy łatwiej w ten sposób oznaczając:

$u = u_1 \quad : \quad x = 0$

$u = u_2 \quad : \quad x = l$

$$b = \frac{4}{3}\mu \log e^{u_1-\alpha}(u_1-\alpha)^{\alpha}$$

$$\frac{4}{3}\mu\log\left[e^{u-u_1}\cdot\left(\frac{u-\alpha}{u_1-\alpha}\right)^{\alpha}\right] = -k p_0 x$$

$$k p_0 x = \frac{4}{3}\mu\log\left[e^{u_1-u}\left(\frac{u_1-\alpha}{u-\alpha}\right)^{\alpha}\right]$$

$$k p_0 l = \frac{4}{3}\mu\log\left[e^{u_1-u_2}\left(\frac{u_1-\alpha}{u_2-\alpha}\right)^{\alpha}\right]$$

$$\frac{x}{l} = \frac{\log}{\log} = \frac{(u_1-u) + \alpha\log(u_1-\alpha) - \alpha\log(u-\alpha)}{(u_1-u_2) + \alpha\log(u_1-\alpha) - \alpha\log(u_2-\alpha)}$$

$$\left[e^{(u_1-u_2)}\left(\frac{u_1-\alpha}{u_2-\alpha}\right)^{\alpha}\right]^{x} = e^{(u_1-u)l}\left(\frac{u_1-\alpha}{u-\alpha}\right)^{\alpha l}$$

$\alpha =$ prędkość dla $x = \infty$

$$p_\infty\alpha + (k-1)p_0\alpha = k p_0\alpha$$

$$p_\infty = p_0$$

$$\left(\frac{du}{dx}\right)_\infty = 0$$

$$p = p_\infty \frac{k\alpha - (k-1)u}{u} = p_\infty\left(k\frac{\alpha}{u} - k + 1\right) = p_\infty\left[1 + k\left(\frac{\alpha}{u} - 1\right)\right]$$

$x = 0 : u = u_1$

$x = \infty : u = \alpha$

$x = \infty : p = p_\infty$

$u \leqq \alpha$

$$k p_\infty x = \frac{4}{3}\mu \log\left[e^{u_1 - u}\left(\frac{\alpha - u_1}{\alpha - u}\right)^{\alpha}\right]$$

$p u = \text{const}$

$$\frac{p}{\theta} u = \text{const} = \frac{p_\infty \alpha}{\theta_\infty}$$

$$\theta = \frac{p u}{p_\infty \alpha}\theta_\infty = \theta_\infty \cdot \frac{u + k(\alpha - u)}{\alpha} = \theta_\infty\left[k - (k-1)\frac{u}{\alpha}\right]$$

$$\frac{4}{3}\mu \frac{du}{dx} = p - p_0$$

Więc istotnie: jeśli $u_1 < u < \alpha$

to: $0 < x < \infty$

$$p_\infty\left[1 + k\left(\frac{\alpha}{u_1} - 1\right)\right] > p > p_\infty$$

$$> \frac{du}{dx} > 0$$

$$\theta_\infty\left[k - (k-1)\frac{u_1}{\alpha}\right] > \theta > \theta_\infty$$

Więc dane muszą być 4 wielkości!

Wyznacząc związek $p\ \theta$:

$$\frac{1}{k}\frac{p - p_\infty}{p_\infty} = \left(\frac{\alpha}{u} - 1\right)$$

$$\frac{u}{\alpha} = \frac{1}{1 + \frac{1}{k}\left(\frac{p}{p_\infty} - 1\right)}$$

$$\lg(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3}$$

$$\frac{p}{p_0} = \frac{u_0}{u}$$

$$x = \frac{\frac{3}{2}(u^2 - u_0^2)}{p_0 u} \nearrow$$

$$p\frac{u}{\theta} = \text{const} = p_0\frac{u_0}{\theta} \quad \Big\| \quad \theta = \text{const}$$

$$x = \frac{3}{2}\frac{u}{p_0}\left[1 - \left(\frac{u_0}{u}\right)^2\right]$$

$$= \frac{3}{2}\frac{u_0}{p}\left[1 - \left(\frac{u_0}{u}\right)^2\right] = \frac{3}{2}\frac{u_0}{p}\left[1 - \left(\frac{p}{p_0}\right)^2\right]$$

$$= \frac{3}{2}\frac{u_0}{p_0}\left[\frac{p_0}{p} - \frac{p}{p_0}\right]$$

$$\theta = \theta_\infty\left[k - \frac{k-1}{1+\frac{1}{k}\left(\frac{p}{p_\infty}-1\right)}\right] = \theta_\infty \frac{k + \frac{p}{p_\infty} - 1 - k + 1}{1 - \frac{1}{k} + \frac{1}{k}\frac{p}{p_\infty}}$$

$$= \theta_\infty \frac{p}{p_\infty}\frac{k}{k-1+\frac{p}{p_\infty}}$$

Zatem: $$\rho_\infty = \rho\,\frac{k}{k-1+\frac{p}{p_\infty}}$$

$$\rho = \rho_\infty\left[\frac{k-1}{k} + \frac{p}{k\,p_\infty}\right]$$

$u_0 = 30000$ $\quad p_0 = 1\,\text{atm.} = 10^6$

$\frac{p}{p_0} = \frac{1}{2}$ $\quad x = \frac{9}{4}\,\frac{3}{100} = \frac{27}{400} = 0{\cdot}0675$ cm

$\frac{p}{p_0} = \frac{1}{10}$ $\quad x = \frac{9}{4}\,\frac{10}{100} = \frac{9}{40} = 0{\cdot}225$ cm

---

Zamiast tego można także: $\underline{p_{01}} = p_\infty\left[1 + k\left(\frac{\alpha}{u_0}-1\right)\right]$

$\underline{p_\infty}$

~~[illegible]~~ $\underline{u_0}$

$\underline{\theta_0}$

Jeżeli $\lim p_\infty = 0$: $\quad u_0, p_0$ = skończone $\quad \alpha = \infty$

$$\frac{p_0}{p} = \lim \frac{1+k\left(\frac{\alpha}{u_0}-1\right)}{1+k\left(\frac{\alpha}{u}-1\right)} = \frac{u}{u_0} \quad \Big\| \quad \lim_{\alpha=\infty} \log\left(\frac{\alpha-u_1}{\alpha-u}\right)^\alpha = \log\left(\frac{1-\frac{u_1}{\alpha}}{1-\frac{u}{\alpha}}\right)^\alpha -$$

$$= \log\left(1+\frac{u-u_1}{\alpha}\right)^\alpha = (u-u_1)$$

~~$k\,p_\infty x = \frac{4}{3}\mu$ [illegible] (u_1-u) [illegible]~~

$$p_\infty = \frac{p_0}{1+k\left(\frac{\alpha}{u}-1\right)} \quad \Big\| \quad k\,p_0 x = \lim_{\alpha=\infty} \frac{4}{3}\mu\left[1+k\left(\frac{\alpha}{u}-1\right)\right]\left\{u_1 - u + \alpha\left[\log(\alpha-u_1) - \log(\alpha-u)\right]\right\}$$

~~$\log \frac{k\,p_0 x}{\frac{4}{3}\mu}$~~

$$\frac{u_1 - u + \alpha\left[\log\ldots\right.}{\frac{1}{1+k\left(\frac{\alpha}{u}-1\right)}} = \frac{\log\frac{\alpha-u_1}{\alpha-u} + \alpha\left(\frac{1}{\alpha-u_1} - \frac{1}{\alpha-u}\right)}{-\frac{\frac{k}{u}}{\left[1+k\left(\frac{\alpha}{u}-1\right)\right]^2}}$$

$$= -\frac{-\cancel{\frac{u_1}{\alpha}} + \frac{1}{2}\left(\frac{u_1}{\alpha}\right)^2 + \cancel{\frac{u}{\alpha}} - \frac{1}{2}\left(\frac{u}{\alpha}\right)^2 + \cancel{x} + \cancel{\frac{u_1}{\alpha}} + \left(\frac{u_1}{\alpha}\right)^2 - \cancel{x} - \cancel{\frac{u}{\alpha}} - \left(\frac{u_1}{\alpha}\right)^2}{\frac{k}{u}}\left(\frac{k}{u}\right)^2\alpha^2$$

$$= -\frac{k}{u}\,\frac{3}{2}\left(u_1^2 - u^2\right)$$

Powolne

Ruchy gazów rozrzedzonych, zaniedbując: p, ρ etc.

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial\,\mathrm{div}}{\partial x} + \mu\nabla^2 u \qquad \begin{array}{c|c} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & -\frac{\partial}{\partial x} \end{array}$$

$$\rho\frac{\partial v}{\partial t} =$$

$$\rho\frac{\partial w}{\partial t} =$$

$$\frac{\mu}{3}\nabla^2\mathrm{div}\,v + \mu\nabla^2\mathrm{div}\,v = 0$$

$$\nabla^2\mathrm{div}\,v = 0$$

$$\underbrace{\nabla^2\xi = \nabla^2\eta = \nabla^2\zeta = 0}_{\nabla^2\mathrm{curl}\,v = 0}$$

~~$v = \nabla U + \mathrm{curl}\,\Omega$~~ ~~$\nabla^2\nabla^2 U = 0$~~

~~$\nabla^2\mathrm{curl}^2\Omega = 0$~~

Jeśli przestrzeń wielka, tak że $\mathrm{div}\,v = 0$ i $\mathrm{curl}\,v = 0$ ~~[illegible]~~ w wielkiej odległości, wtedy: ruch taki jak cieczy nieściśliwych z potencjałem prędkości:

[nie ma zjawisk akustycznych!
ruch ustanawia się natychmiastowo

$$u = \frac{\partial\varphi}{\partial x} \qquad v = \frac{\partial\varphi}{\partial y} \qquad w = \frac{\partial\varphi}{\partial z}$$

$$\mathrm{div}\,v = \frac{\partial u}{\partial x} + \cdots = \nabla^2\varphi = 0$$

$$\nabla^2 u = \frac{\partial}{\partial x}\nabla^2\varphi = 0$$

N. p. ~~Kula w przestrzeni~~

~~$\varphi = \frac{a}{r}$ $\quad u = -\frac{x}{r^3}a$ $\quad v = -\frac{y}{r^3}a$ $\quad w = -\frac{z}{r^3}a$~~

~~$\frac{\partial u}{\partial x} = \left(-\frac{1}{r^3} + \frac{3x^2}{r^5}\right)a = \frac{a}{r^3}\left(-1 + \frac{3x^2}{r^2}\right)$~~

To byłoby niemożliwe, bo na powierzchni byłyby składowe ruchu ślizganie, Ale ów wniosek przedwczesny: bo curl v nie będzie = 0 na powierzchni

div v inny potencjał

rota , —

Równanie ciepliny tutaj nie można użyć, bo wtedy lim $\rho = 0$ turbulencji zrobi lim $\theta = \infty$ ażeby unieść ilość ciepła wytworzoną przez tarcie.

Równowaga ~~tu~~ ustanowi się wskutek przewodzenia ciepła które tutaj dopiero wchodzi znacznie w rachuby, tak że będzie:

$$\kappa \nabla^2 \theta = \Phi$$

linie wskazujące kierunek curl na powierzchni!

~~N. p. koło w przestrzeni~~

~~$\varphi = \frac{ca^2 x}{r^3}$~~ ~~$\frac{\partial \varphi}{\partial x} = \frac{ca^2}{r^3}\left[1 - \frac{3x^2}{r^2}\right] = u$~~

~~$\frac{\partial \varphi}{\partial y} = 3ca^2\frac{xy}{r^5} = v$~~

~~$\frac{\partial \varphi}{\partial z} = 3ca^2\frac{xz}{r^5} = w$~~

$\operatorname{div} v = -$ ~~$(v\nabla)\log$~~

$-\,\mathcal{S}(v \cdot \nabla \log \rho)$

$$v = \nabla \varphi + \nabla \int \frac{\operatorname{div} v}{r}\, dv + \operatorname{curl} \int \frac{\operatorname{curl} v}{r}\, dv$$

$\nabla^2 \operatorname{div} v = \operatorname{div} \nabla^2 v$? $\nabla^2 \operatorname{curl} v = \operatorname{curl} \nabla^2 v$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \cdot\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial}{\partial z}\right) \Big| \; \frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \sim$$

zatem Resultat: $\operatorname{div} \nabla^2 v = 0$

$\operatorname{curl} \nabla^2 v = 0$

$$\therefore \quad \nabla^2 v = \nabla U \qquad U = -\frac{1}{3} \operatorname{div} v$$

bezpośrednio:!

$$\underbrace{\nabla^2 v}_{\nabla \operatorname{div} v - \operatorname{curl}^2 v} = -\frac{1}{3} \nabla \operatorname{div} v$$

$$\operatorname{curl}^2 v = \frac{4}{3} \nabla \operatorname{div} v$$

N.p. dwie płaszczyzny $v = w = 0$ $\operatorname{div} = 0$

$$\frac{1}{3}\frac{\partial^2 u}{\partial x^2} + \cancel{} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \cancel{+ \frac{\partial^2 u}{\partial z^2}} = 0$$

$$\frac{1}{3}\frac{\partial^2 u}{\partial x \partial y} = 0$$

$$\frac{1}{3}\cancel{\frac{\partial^2 u}{\partial x \partial z}} \cancel{= 0}$$

$\frac{\partial u}{\partial x} = f_1(x)$

Jeśli [illegible] $x = c$

$\frac{\partial u}{\partial x} = 0$

$$u = \frac{y}{\delta} c \qquad \frac{\partial u}{\partial y} = \frac{c}{\delta} \qquad \Phi = \mu \left(\frac{\partial u}{\partial y}\right)^2 = \mu \frac{c^2}{\delta^2}$$

$$-K \frac{\partial^2 \theta}{\partial y^2} = \mu \frac{c^2}{\delta^2}$$

$$\frac{d\theta}{dy} = -\frac{\mu}{K}\frac{c^2}{\delta^2} y + a$$

$$\theta = -\frac{\mu}{K}\frac{c^2}{\delta^2}\frac{y^2}{2} + ay + \theta_0$$

$$\cancel{\theta_0} = -\frac{\mu}{K}\frac{c^2}{\delta^2}\frac{\delta^2}{2} + a\delta + \cancel{\theta_0}$$

$$\cancel{\theta - \theta_0 = \frac{\mu}{K}\frac{c^2}{\delta^2}\frac{y^2\delta^2}{2}}$$

$$a = -\frac{\mu}{K}\frac{c^2}{\delta^2}\frac{\delta}{2}$$

$$\theta = \frac{\mu}{K}\frac{c^2}{\delta^2}\frac{\overset{y(\delta - y)}{\cancel{}}}{2} + \theta_0$$

Maximum : $y = \frac{\delta}{2}$ | $\theta = \frac{\mu}{K}\frac{c^2}{8} + \theta_0$

Niezależnie od grubości warstwy N.p. $\mu = 0{\cdot}00017$

$K = 42.10^6 . \ 0{\cdot}000057$

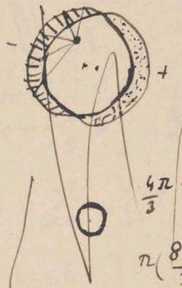
$$\frac{\mu}{8K} = \frac{1{\cdot}7.10^{-4}}{8.42.\ 0{\cdot}57.10^2} = 10^{-8}$$

więc dopiero przy $\theta = 10^4 \frac{cm}{s}$

będzie

Wogóle ~~[illegible]~~ wpływ terminy będzie bardzo nieznaczny przy wszystkich przedziałach

$$\iint \varrho \frac{dS \cos nx}{r}$$



$4\pi r^2$

$\frac{4}{3}\pi r^2 + \frac{4}{r}\frac{2}{ } r^2 r$

$\pi\left(\frac{8}{3} - 4\right) r$

$-\frac{4}{3}\pi r$

$$\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}\right)+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$$

$$\frac{1}{3}\frac{\partial}{\partial y}\left(\qquad\qquad\right)+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$$

$$\frac{1}{3}\frac{\partial}{\partial z}\left(\qquad\qquad\right)+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0$$

Obierając dowolnie $\operatorname{div} v = U = f(x\,y\,z)$ otrzyma się

$$\frac{\partial}{\partial x}\iiint \rho \frac{dv}{r} = \iiint \frac{\partial \rho}{\partial x}\frac{dv}{r} + \iint$$

$$w = -\frac{1}{3}\int \frac{\frac{\partial U}{\partial z}}{r}\,dv + A_3$$

$$v = -\frac{1}{3}\int \frac{\frac{\partial U}{\partial y}}{r}\,dv + A_2$$

$$u = -\frac{1}{3}\int \frac{\partial U}{\partial x}\frac{dv}{r} + A_1$$

$$\mathfrak{v} = -\frac{1}{3}\int \frac{\nabla U}{r}\,dv + \mathfrak{A} \qquad \nabla^2\mathfrak{A}=0 \qquad \operatorname{div}\mathfrak{A}=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = -\frac{1}{3}\int \frac{\overset{=0}{\nabla^2 U}}{r}\,dv + \frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial y}+\frac{\partial A_3}{\partial z} \quad + \iint \frac{\partial U}{\partial x}\frac{dS \cos nx}{r}$$

$$\nabla^2 u = -\frac{1}{3}\int \frac{\partial(\nabla^2 U)}{\partial x}\frac{dv}{r} \qquad \frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial y}+\frac{\partial A_3}{\partial z} = c = 0 ?$$

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Ad ...

uwzględniając inertia terms:

dodać należy: $W_I = \iiint u \frac{\partial p}{\partial x} + \cdot \quad = \cdot \quad \cdot$

$$\cancel{+}\iiint \frac{\rho}{\mu} u \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}\right] + v \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}\right] + w \left[u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}\right] dv = \iiint \frac{\rho}{\mu}\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}\right) \cdot \frac{u^2+v^2+w^2}{2}\, dv$$

$$= -\iint \frac{u^2+v^2+w^2}{2} \frac{\rho}{\mu} (ul + vm + wn)\, dS + \iiint \frac{u^2+v^2+w^2}{2} \cancel{\frac{1}{\mu}\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]}$$

Więc dowód się stosuje tak samo

Całka powierzchniowa

$$\mu \iiint u \left(n \frac{\partial w}{\partial x} - l \frac{\partial w}{\partial z} + m \frac{\partial v}{\partial x} - l \frac{\partial v}{\partial y}\right) + \cdots$$
$$\pm l \frac{\partial u}{\partial x}$$

$$= \iint u \frac{\partial}{\partial x}(lu + mv + nw) - l u \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$
$$+ v \frac{\partial}{\partial y} \qquad - mv \left(\qquad \right)$$
$$+ w \frac{\partial}{\partial z} \qquad - nw \left(\qquad \right)$$

$$= \iint \left[\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}\right) v_n - v_n \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right] dS$$

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zatem w całości :

$$W_{II} = W_I + \mu \iint \left[\frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right) + \frac{1}{3} v_n \operatorname{div} + \left(u\frac{\partial}{\partial x} + \cdots\right) v_n - v_n \operatorname{div}\right] dS$$

$$W_{II} = W_I + \mu \iint \left[\frac{\partial}{\partial n}\left(\frac{v^2}{2}\right) + (v\nabla)\, S(v\, n) - \frac{2}{3} \operatorname{div} v\, S(v\, n)\right] dS$$

$$W_{II} = -\frac{W_I}{k-1}$$

$$0 = W_I \left(1 + \frac{1}{k-1}\right) + \mu \iint$$

$$W_I = - \frac{k-1}{k} \mu \iint \cdots$$

$$\begin{aligned} W_{II} = \frac{1}{k} \mu \iint \frac{\partial}{\partial x} \Big[& u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} + w \frac{\partial w}{\partial x} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - \\ & - \frac{2}{3} u \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] \Big] \end{aligned}$$

$$\begin{aligned} = \frac{\mu}{k} \iint \Bigg(& \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} - \frac{2}{3}\frac{\partial u}{\partial x}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] \\ & + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\left[\qquad \right] \\ & + \left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + (\quad) + \left(\frac{\partial w}{\partial z}\right)^2 + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\left[\qquad \right] \\ & + \underline{\underline{u\, 2\frac{\partial^2 u}{\partial x^2}}} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}\right) + w\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial z}\right) - \frac{2}{3} u \underline{\underline{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right)}} \\ & + v\, 2\frac{\partial^2 v}{\partial y^2} + w\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z}\right) + u \underline{\underline{\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x}\right)}} - \frac{2}{3} v \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} + \frac{\partial^2 u}{\partial y \partial x}\right) \\ & + w\, 2\frac{\partial^2 w}{\partial z^2} + u \underline{\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}\right)} + v\left(\frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 w}{\partial y \partial z}\right) - \frac{2}{3} w \left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z}\right) \Bigg) \end{aligned}$$

?

$$u\left[\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right]\cancel{-\frac{2}{3}u\frac{\partial}{\partial x}\left(\quad\right)}$$

$$\parallel$$

$$u\frac{\partial p}{\partial x}$$

$$\frac{1}{k}\left[\Phi+u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right]=\Phi$$

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}=(k-1)\,\Phi$$

Czy to ogólny rezultat?

Twodimensional problem:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial \chi}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad \frac{\partial}{\partial y}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial \chi}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad \frac{\partial}{\partial x}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\zeta$$

$$u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}=\mu\left(\frac{\partial^2 \zeta}{\partial x^2}+\frac{\partial^2 \zeta}{\partial y^2}\right)$$

~~$\frac{\partial u}{\partial x}\frac{\partial \zeta}{\partial x}+\frac{\partial v}{\partial x}\frac{\partial \zeta}{\partial y}+u\frac{\partial^2 \zeta}{\partial x^2}+v\frac{\partial^2 \zeta}{\partial x\partial y}=\mu\frac{\partial}{\partial x}(\nabla^2\zeta)$~~

~~$\frac{\partial u}{\partial y}\frac{\partial \zeta}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial \zeta}{\partial y}+u\frac{\partial^2 \zeta}{\partial x\partial y}+v\frac{\partial^2 \zeta}{\partial y^2}=\mu\frac{\partial}{\partial y}(\nabla^2\zeta)$~~

~~$\frac{\partial v}{\partial x}\frac{\partial \zeta}{\partial y}-\frac{\partial v}{\partial y}\frac{\partial \zeta}{\partial x}+u\frac{\partial^2 \zeta}{\partial x^2}+v\frac{\partial^2 \zeta}{\partial x\partial y}=\mu\frac{\partial\nabla^2\zeta}{\partial x}$~~

~~$\frac{\partial u}{\partial x}\frac{\partial \zeta}{\partial x}+\frac{\partial u}{\partial y}\frac{\partial \zeta}{\partial y}+\zeta\frac{\partial \zeta}{\partial y}+u\frac{\partial^2 \zeta}{\partial x^2}+v\frac{\partial^2 \zeta}{\partial x\partial y}=$~~

$v=\nabla\cdots+\operatorname{curl}\cdots$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \cancel{\frac{\partial}{\partial z}} \\ F & G & H \end{vmatrix}$$

$$u=\frac{\partial U}{\partial x}+\frac{\partial H}{\partial y}$$

$$v=\frac{\partial U}{\partial y}-\frac{\partial G}{\partial x}$$

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0:$ ~~$\nabla^2 U+\frac{\partial^2}{\partial x\partial y}(H-G)=0$~~

~~$\zeta=-\left(\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}\right)$~~

~~$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$~~ $-v\zeta=-\frac{\partial \chi}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$

~~$u\frac{\partial v}{\partial x}+v$~~ $u\zeta=-\frac{\partial \chi}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$

~~[illegible]~~:

$$u=\frac{\partial \psi}{\partial y}$$

$$v=-\frac{\partial \psi}{\partial x}$$

~~$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0$~~

$$\zeta=-\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right)-\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\left(\quad\right)=\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}\right)$$

Dla μ bardzo małych lewa strona będzie prawie $=0$

zatem $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=f(\psi)$

zatem prawa strona:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)f(\psi)=0$$ Nieprawda! dowolne!

Ale można spróbować rozwinąć:

$$\psi=\psi_0+\mu\left(\frac{\partial\psi}{\partial\mu}\right)_0 \qquad \psi=\text{fc.}(x,y,\mu)$$

$$\psi(x,y,\mu)=\psi(x,y,\mu=0)+\mu\left[\frac{\partial\psi(x,y,\mu)}{\partial\mu}\right]_{\mu=0}$$

$v = w = 0$

$$\frac{\partial p}{\partial x} = \mu\left[\frac{1}{3}\frac{\partial^2 u}{\partial x^2} + \nabla^2 u\right]$$

$$\frac{\partial p}{\partial y} = \mu \frac{1}{3}\frac{\partial^2 u}{\partial x \partial y}$$

$$\frac{\partial p}{\partial z} = \mu \frac{1}{3}\frac{\partial^2 u}{\partial x \partial z}$$

$$\frac{\partial}{\partial x}(\rho u) = 0 \qquad \underline{\rho u = f(y,z)} \qquad u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0$$

$$\frac{c}{A}\rho u \frac{\partial \theta}{\partial x} + p\frac{\partial u}{\partial x} = \Phi_u$$

$$p = R\rho\theta$$

$$\theta = \frac{p}{R\rho}$$

$$\frac{\partial^2 p}{\partial x \partial y}\,\Big\| \qquad \left.\begin{array}{l}\dfrac{\partial}{\partial y}(\nabla^2 u) = 0 \\ \dfrac{\partial}{\partial z}(\nabla^2 u) = 0 \\ \underline{\nabla^2 p = \dfrac{4}{3}\mu\dfrac{\partial}{\partial x}(\nabla^2 u)}\end{array}\right\} \qquad \underline{\nabla^2 u = f_1(x)}$$

$$\nabla^2 p = \frac{4}{3}\mu\,\nabla^2\frac{\partial u}{\partial x}$$

$$\frac{c}{AR}\rho u\left[\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{p}{\rho^2}\frac{\partial \rho}{\partial x}\right] + p\frac{\partial u}{\partial x} = \Phi$$

$$\frac{c}{AR}\Big[u\frac{\partial p}{\partial x} \underbrace{- \frac{pu}{\rho}\frac{\partial \rho}{\partial x}}_{+p\frac{\partial u}{\partial x}}\Big] + p\frac{\partial u}{\partial x} = \Phi$$

$$\frac{c}{AR}u\frac{\partial p}{\partial x} + \left[\frac{c}{AR} + 1\right]p\frac{\partial u}{\partial x} = \Phi$$

$$\underline{u\frac{\partial p}{\partial x} + k p\frac{\partial u}{\partial x} = (k-1)\Phi}$$

$$p = \frac{\mu}{3}\frac{\partial u}{\partial x} + \varphi(x)$$

$$\cancel{\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2}} + \frac{\partial \varphi}{\partial x} = \cancel{\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2}} + \mu\nabla^2 u$$

$$\nabla^2 p = \frac{\mu}{3}\frac{\partial}{\partial x}(\nabla^2 u) + \nabla^2\varphi = \frac{4}{3}\mu\frac{\partial}{\partial x}(\nabla^2 u)$$

$$\nabla^2\varphi = \frac{\partial^2 \varphi(x)}{\partial x^2} = \mu\frac{\partial}{\partial x}(\overbrace{\nabla^2 u}^{=f_1(x)})$$

$$\frac{\partial \varphi(x)}{\partial x} = \mu\nabla^2 u + \cancel{\text{const}}$$

$$\varphi(x) = \mu\int \nabla^2 u\,dx$$

Szczególne rozwiązanie:

$$p = \frac{4}{3}\mu\frac{\partial u}{\partial x} + c$$

$$\left.\begin{aligned} \frac{4}{3}\mu u \frac{\partial^2 u}{\partial x^2} + k\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2 + kc\frac{\partial u}{\partial x} &= (k-1)\mu\left[\frac{4}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] \\ \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) &= f_1(x) \end{aligned}\right\}$$

$$\cancel{u = a_0 + a_1 x + a_2 x^2} + b_1 y + \underline{\cancel{b_2 y x}}$$

$$\underline{+ b_2 y^2}$$

$v = w = 0$ Jednowymiarowe

$$\left.\begin{array}{l} \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \text{[illegible]} \\ \rho u = \text{const} \\ \frac{c}{A} \rho u \frac{\partial \theta}{\partial x} + p \frac{\partial u}{\partial x} = 0 \; \text{[illegible]} \end{array}\right\}$$

$$p = R \rho \theta$$

~~$p \frac{\partial u}{\partial x} = \frac{c}{A} \frac{\partial p}{\partial x}$~~

$$\rho_0 u_0 \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x}$$

$$\rho_0 u_0 u + p = p_0$$

~~$\frac{c}{A}$~~

~~$\frac{c}{A} \ldots \frac{d\theta}{dx} u + R \ldots \frac{du}{u} = 0$~~

~~$\frac{c}{AR} \frac{d\theta}{\theta} + \frac{du}{u} = 0$~~

$$\frac{c}{A} \rho_0 u_0 \frac{d\theta}{dx} + (p_0 - u \rho_0 u_0) \frac{du}{dx} = 0$$

$$\underbrace{\frac{c}{A} \rho_0 u_0 \theta}_{} + p_0 u - \rho_0 u_0 \frac{u^2}{2} = a$$

$$\frac{c}{AR} u p$$

~~$\int = p_0 -$~~

$$= \frac{c}{AR} u (p_0 - \rho_0 u_0 u)$$

$$\left.\begin{array}{l} u^2 \left[\frac{c}{AR} + \frac{1}{2}\right] \rho_0 u_0 - u \left[\frac{c}{AR} + 1\right] p_0 = a \\ \frac{1}{k-1} \\ u^2 (k+1) \rho_0 u_0 - 2u k p_0 = A \end{array}\right)$$

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$$

$$p = p_0 \left(\frac{u_0}{u}\right)^k$$

$$\rho_0 u_0 . u + p_0 \left(\frac{u_0}{u}\right)^k = p_0$$

?

u = const. !

$$\frac{dp}{dx} = \frac{4}{3}\mu\frac{d^2u}{dx^2} \qquad \frac{d}{dx}(\rho u) = 0$$

$$\frac{c}{A}\rho u\frac{d\theta}{dx} + p\frac{du}{dx} = \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 \qquad \mu = \mu\theta$$

Wypływ kulisty

$u = s\frac{x}{r} \qquad v = s\frac{y}{r} \qquad w = s\frac{z}{r}$

$= x\varphi \qquad = y\varphi \qquad = z\varphi$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 3\varphi + \varphi' r$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 4\varphi'\frac{x}{r} + \varphi'' x$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \dots\right) = 4\varphi'\frac{x}{r} + \varphi'' x$$

$$\frac{\partial u}{\partial x} = \varphi + \varphi'\frac{x^2}{r}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{3x}{r}\varphi' - \varphi'\frac{x^3}{r^3} + \varphi''\frac{x^3}{r^2}$$

$$\frac{\partial u}{\partial y} = \varphi'\frac{xy}{r}$$

$$\frac{\partial^2 u}{\partial y^2} = \varphi'\left(\frac{x}{r} - \frac{xy^2}{r^3}\right) + \varphi''\frac{xy^2}{r^2}$$

$$\frac{\partial^2 u}{\partial z^2} = \varphi'\left(\frac{x}{r} - \frac{xz^2}{r^3}\right) + \varphi''\frac{xz^2}{r^2}$$

$$\frac{\partial p}{\partial x} = \frac{dp}{dr}\frac{x}{r} = \frac{4}{3}\mu\left[4\varphi'\frac{x}{r} + \varphi'' x\right]$$

$$= \frac{4}{3}\mu\frac{d}{dr}\left[3\varphi + r\varphi'\right]\cdot\frac{x}{r}$$

I). $p = \frac{4}{3}\mu\left[3\varphi + r\varphi'\right] + p_0$

II). $4\pi r^2 \varphi r \rho = \text{const}$

$$\varphi\rho r^3 = \text{const}$$

$$\rho\, d(\varphi r^3) + \varphi r^3 d\rho = 0$$

$$\rho(\varphi' r^3 + 3r^2\varphi) + \varphi r^3\frac{d\rho}{dr} = 0$$

$$(3\varphi + \varphi' r) + \varphi r\frac{1}{\rho}\frac{d\rho}{dr} = 0$$

III). $\frac{c}{A}\rho\left[x\varphi\frac{x}{r}\frac{d\theta}{dr} + y\varphi\frac{y}{r}\frac{d\theta}{dr} + \dots\right] + p\left[3\varphi + \varphi' r\right] = \Phi$

$\Phi =$

$$\begin{array}{c|c|c}
\frac{\partial u}{\partial x} = \varphi + \varphi' \frac{x^2}{r} & \frac{\partial v}{\partial x} = \varphi' \frac{xy}{r} & \frac{\partial w}{\partial x} = \varphi' \frac{xz}{r} \\
\frac{\partial u}{\partial y} = \varphi' \frac{xy}{r} & \frac{\partial v}{\partial y} = \varphi + \varphi' \frac{y^2}{r} & \frac{\partial w}{\partial y} = \varphi' \frac{yz}{r} \\
\frac{\partial u}{\partial z} = \varphi' \frac{xz}{r} & \frac{\partial v}{\partial z} = \varphi' \frac{yz}{r} & \frac{\partial w}{\partial z} = \varphi + \varphi' \frac{z^2}{r}
\end{array}$$

$$\frac{\overline{\Phi}}{\mu} = -\frac{2}{3}\left[3\varphi + r\varphi'\right]^2 + 2\left[\left(\varphi + \frac{x^2}{r}\varphi'\right)^2 + \left(\varphi + \frac{y^2}{r}\varphi'\right)^2 + \left(\varphi + \frac{z^2}{r}\varphi'\right)^2\right] +$$

$$+ 4\varphi'^2\left[\frac{y^2z^2}{r^2} + \frac{z^2x^2}{r^2} + \frac{x^2y^2}{r^2}\right]$$

$$= -\frac{2}{3}\left[3\varphi + r\varphi'\right]^2 + 2\left[3\varphi^2 + 2\varphi\varphi' r + \varphi'^2 \frac{x^4 + y^4 + z^4 + 2x^2y^2 + \cdots}{r^2}\right]$$

$$= -\frac{2}{3}\left[3\varphi + r\varphi'\right]^2 + 2\left[3\varphi^2 + 2r\varphi\varphi' + r^2\varphi'^2\right]$$

$$= -\frac{2}{3} r^2\varphi'^2 + 2r^2\varphi'^2 = \frac{4}{3} r^2\varphi'^2$$

III). $\frac{c}{A} \rho \varphi r \frac{d\theta}{dr} + p\left[3\varphi + r\varphi'\right] = \frac{4}{3}\mu r^2 \varphi'^2$

$$\theta = \frac{p}{R\rho}$$

$$\frac{c}{AR}\left(\varphi r \frac{dp}{dr} - \frac{\varphi r p}{\rho} \frac{d\rho}{dr}\right) = \frac{c}{AR}\left[\varphi r \frac{4}{3}\mu\left(4\varphi' + r\varphi''\right) + \left[\frac{4}{3}\mu\left(3\varphi + r\varphi'\right) + p_0\right]\left[3\varphi + \varphi' r^{I}\right]\right] +$$

$$+ \left[\frac{4}{3}\mu\left(3\varphi + r\varphi'\right) + p_0\right]\left[3\varphi + \varphi' r\right] = \overline{\Phi}$$

$$\frac{c}{AR}\left(4\varphi'+r\varphi''\right)\varphi r+\left(1+\frac{c}{AR}\right)\left(3\varphi+r\varphi'\right)\left(3\varphi+r\varphi'+P_0\right)=r^2\varphi'^2$$

$$\boxed{P_0=\frac{p_0}{\frac{4}{3}\mu}}$$

$$\left(4\varphi'+r\varphi''\right)\varphi r-(k-1)\,r^2\varphi'^2+k\left(3\varphi+r\varphi'\right)\left(3\varphi+r\varphi'+P_0\right)=0$$

$$r^2\varphi\varphi''+\varphi'^2\left[r^2(1-k)+kr^2\right]+\varphi'\left[4r\varphi+6kr\varphi+P_0kr\right]+$$

$$+9k\varphi^2+3k\varphi P_0=0$$

$$\underbrace{r^2\left(\varphi\varphi''+\varphi'^2\right)}_{\frac{d}{dr}(\varphi\varphi')}+\varphi'\left[2r\varphi(2+3k)+krP_0\right]+3k\varphi\left[3\varphi+P_0\right]=0$$

$$\underbrace{\frac{r^4\frac{d}{dr}(\varphi\varphi')+\varphi\varphi'.4r^3}{\cancel{r^2}}}_{\frac{d}{dr}(\varphi\varphi' r^4)}+k\left(6\varphi\varphi' r^3+9\varphi^2r^2\right)+$$

$$+3k\left[\underbrace{2r^3\varphi\varphi'+3r^2\varphi^2}_{\frac{d}{dr}(r^3\varphi^2)}\right]\qquad kP_0\left(\underbrace{\varphi' r^3+3\varphi r^2}_{\frac{d}{dr}(\varphi r^3)}\right)=0$$

$$\varphi\varphi' r^4+3k\varphi^2r^3+kP_0\varphi r^3=a$$

$$\varphi\varphi'+3k\frac{\varphi^2}{r}+kP_0\frac{\varphi}{r}=\frac{a}{r^4}$$

$$\frac{dz}{dr}+6k\frac{z}{r}+kP_0\sqrt{2}\frac{\sqrt{z}}{r}=\frac{a}{r^4}$$

$$\varphi^2=2z$$

$$\varphi\varphi'=\frac{dz}{dr}$$

$$r\varphi=y$$

$$r\varphi'+\varphi=\frac{dy}{dr}$$

$$r^2\varphi'=r\frac{dy}{dr}-y$$

$$ry\left(r\frac{dy}{dr}-y\right)+3kry^2+kP_0r^2y=a$$

$$r^2y\frac{dy}{dr}+(3k-1)ry^2+kP_0r^2y=a$$

$$y\frac{dy}{dr}+(3k-1)\frac{y^2}{r}+kP_0y=\frac{a}{r^2}$$

$$\frac{dy}{dr} + (3(k-1)\frac{y}{r} + kP_0 \cancel{\mu} = 0$$

$$y = uv$$

$$\frac{dy}{dr} = u\frac{dv}{dr} + v\frac{du}{dr}$$

$$u^2 v\frac{dv}{dr} + uv^2\frac{du}{dr} + (3K-1)\frac{u^2v^2}{r} + kP_0 uv = \frac{a}{r^2}$$

$$\underbrace{u^2\left[v\frac{dv}{dr} + (3k-1)\frac{v^2}{r}\right]}_{=0} + u\frac{du}{dr}\cdot v^2 + kP_0 uv = \frac{a}{r^2}$$

$$\lg v = -(3k-1)\lg r$$

$$v = b r^{1-3k}$$

$$u\frac{du}{dr} + uR_1 = R_2 \quad \Big\| \quad \cancel{u = x + y}$$

$$\cancel{x\frac{dx}{dr} + x\frac{dy}{dr} + y\frac{dx}{dr} + y\frac{dy}{dr} + xR_1 + yR_1 = R_2}$$

$$\cancel{u = xy}$$

$$\cancel{yx^2\frac{dy}{dr} + xy^2\frac{dx}{dr} + xyR_1 = R_2}$$

Jeśli c = 0:

$$u = -kP_0\int\frac{dr}{v} = -\frac{kP_0}{b}\int\frac{dr}{r^{1-3k}} = \frac{-kP_0}{b}\int(r^{3k-1})\,dr$$

$$u = -\frac{kP_0}{b}\,\frac{r^{3k}}{3k} + c$$

$$y = c\,r^{1-3k} - \frac{P_0}{3}r = r\varphi = s$$

$$\varphi = c\, r^{-3k} - \frac{P_0}{3}$$

$$\varphi' = -3kc\, r^{-3k-1}$$

$$\text{I}).\quad p = \frac{4}{3}\mu\left[3c\, r^{-3k} - P_0 - 3kc\, r^{-3k} + P_0\right] = 4\mu\, c(1-k)\, r^{-3k}$$

$$\text{II}).\quad \rho\left[c\, r^{3(1-k)} - \frac{P_0}{3} r^3\right] = \text{const}$$

Zatem to niemożliwe!

$$\varphi r^3 = \frac{c}{\rho} \qquad \varphi = \frac{c}{\rho r^3}$$

$$\varphi' = \frac{-3c}{\rho r^4} - \frac{c}{\rho^2 r^3}\frac{d\rho}{dr}$$

$$\left[-\frac{3c^2}{\rho^2 r^7} - \frac{c^2}{\rho^3 r^6}\frac{d\rho}{dr}\right] r^4 + 3k\frac{c^2}{\rho^2 r^6} r^3 + kP_0\frac{c}{\rho} = a$$

$$\frac{c^2}{r^2}\frac{1}{\rho^3}\frac{d\rho}{dr} + \underbrace{\frac{3c^2}{\rho^2 r^3} + \frac{3kc^2}{\rho^2 r^3}} + kP_0\frac{c}{\rho} = a$$

$$\frac{d\rho}{dr} + \rho\cdot\frac{3(k+1)}{r} + \frac{kP_0}{c}\frac{\rho^2}{r^2} = \frac{a}{c^2}\rho^3 r^2$$

$$\rho = u$$

$$u\left[\frac{dv}{dr} + v\frac{3k+1}{r} + \frac{kP_0}{c}\frac{u v^2}{r^2}\right] + v\frac{du}{dr} = \frac{a}{c^2}u^3 v^3 r^2$$

$$\frac{y}{r}\frac{dy}{dr} + (3k-1)\frac{y^2}{r^2} + kP_0\frac{y}{r} = \frac{a}{r^3}$$

Aërodynamiczne dwurozmiarowe, powolne:

$$\text{I}\begin{cases}\dfrac{\partial p}{\partial x} = \dfrac{\mu}{3}\dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right) + \mu\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}\right)\\[2ex] \dfrac{\partial p}{\partial y} = \dfrac{\mu}{3}\dfrac{\partial}{\partial y}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right) + \mu\left(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}\right)\end{cases}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=0$$

$$\text{II}).\ \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$+\frac{c}{A}\rho\frac{\partial\theta}{\partial t}$$

$$\text{III}).\ \frac{c}{A}\rho\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right) + p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = \mu\,\Phi$$

$$\theta = \frac{p}{R\rho}$$

$$\frac{c}{AR}\rho\left[\frac{1}{\rho}\frac{\partial p}{\partial t} - \frac{p}{\rho^2}\frac{\partial\rho}{\partial t}\right]$$

$$\text{III}).\ \frac{c}{AR}\rho\left[u\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{v}{\rho}\frac{\partial p}{\partial y} - u\frac{p}{\rho^2}\frac{\partial\rho}{\partial x} - v\frac{p}{\rho^2}\frac{\partial\rho}{\partial y}\right] + \cdots$$

$$\frac{1}{k-1}\left[\overset{\frac{\partial p}{\partial t}+}{u}\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} - \frac{p}{\rho}\left(\overset{\frac{\partial\rho}{\partial t}+}{u}\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}\right)\right] + p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = \Phi$$

$$\frac{\partial\rho}{\partial t}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} = 0$$

$$\text{III}).\ \frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) + (k-1)\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = (k-1)\,\Phi$$

$$\text{III}).\ \frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = (k-1)\,\Phi$$

Jeśli istnieje potencjał prędkości!

$$u = \frac{\partial\varphi}{\partial x} \qquad v = \frac{\partial\varphi}{\partial y}$$

$$p = \frac{\mu}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) + \mu\,\cancel{\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)}\ \cancel{\underbrace{\int u\,dx + v\,dy + w\,dz}_{\varphi}}$$

$$D^2\varphi = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Eliminacja $p = R\rho\theta$

~~$R\left(\theta\frac{\partial\rho}{\partial x} + \rho\frac{\partial\theta}{\partial x}\right) = \frac{\mu}{3}\frac{\partial}{\partial x}$~~

Wtedy: $\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) + \mu\frac{\partial}{\partial x}\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) = \frac{4\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi$

$$\frac{\partial p}{\partial y} = \frac{4\mu}{3}\frac{\partial}{\partial y}\left(\quad\right)\varphi$$

I). $p = p_0 + \frac{4\mu}{3}\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right)$

III). $\frac{\partial\varphi}{\partial x}\frac{\partial p}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial p}{\partial y} + kp\nabla^2\varphi = (k-1)\Phi$

$$\Phi = -\frac{2}{3}\mu\left(\nabla^2\varphi\right)^2 + \underbrace{2\mu\left[\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\varphi}{\partial y^2}\right)^2\right] + 4\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2}_{\cancel{\left[\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)\left(\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial y}\right)\right]^2}}$$

$$\cancel{k\,p\,\nabla^2\varphi} = \frac{4\mu}{3}\left[\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\varphi}{\partial y^2}\right)^2 - \frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\varphi}{\partial y^2} + 3\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2\right]$$

I). $\nabla^2 p = \frac{\mu}{3}\nabla^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \mu\nabla^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{4}{3}\mu\nabla^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

$p = \frac{4\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + A \quad \| \nabla^2 A = 0$ } Ogólny rezultat dla małego przepływu

Załóżmy że $A = p_0 = \text{const}$:

$\frac{4}{3}u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right) + \frac{4}{3}v\left(\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 v}{\partial y^2}\right) + [illegible] =$

$(k-1)\left[-\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] - \frac{4}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 - A\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

$$u\left[\frac{4}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\cancel{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}}\right]+v\left[\frac{4}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\cancel{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}}\right]=$$

$$k\left[-2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]+$$

$$+\left[\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2-2\left(\frac{\partial u}{\partial x}\right)^2-2\left(\frac{\partial v}{\partial y}\right)^2-\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$$

$$=k\left[-4\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]+\left[\frac{2}{3}(\quad)-\quad\right]$$

Jeśli szczególny rodzaj ruchu potencjalnego: $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0$

$$\frac{\partial u}{\partial x}=\frac{\partial^2\varphi}{\partial x^2}\quad\Big|\quad\frac{\partial u}{\partial y}=\frac{\partial^2\varphi}{\partial x\partial y}=\frac{\partial v}{\partial x}$$

$$\cancel{\frac{\partial\varphi}{\partial x}\left[\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)+\frac{\partial^2\varphi}{\partial x^2}+\right.}$$

$$\frac{\partial u}{\partial x}+\frac{\partial\varphi}{\partial y}=0$$

$$\cancel{\frac{\partial\varphi}{\partial x}\left[\frac{\partial^3\varphi}{\partial x^3}+\frac{\partial^3\varphi}{\partial x^2\partial y}\right.}$$

$$0=(k-1)\left[2\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2+2\left(\frac{\partial^2\varphi}{\partial y^2}\right)^2+4\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2\right]$$

$$0=\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2+2\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\varphi}{\partial y^2}\right)^2$$

Możliwe tylko jeśli zarówno

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2$$

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}=\frac{\partial v}{\partial y}=0$$

$v = \varphi . u$ Na ~~powierzchniach~~ musi być ~~równocześnie~~ $u = 0$, ~~$v = 0$~~

t.zn. tam musi

Hipoteza:

$$\frac{\partial v}{\partial x} = u \frac{\partial \varphi}{\partial x} + \varphi \frac{\partial u}{\partial x}$$

$v = u(a-u) = au - u^2$

∂^2

$$d\left[p - \frac{\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right] = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy + \frac{\partial \psi}{\partial z} dz$$

$$= \mu\left(\nabla^2 u \, dx + \nabla^2 v \, dy + \nabla^2 w \, dz\right)$$

$$= \mu \, S(\nabla^2 \mathfrak{v} . d\sigma)$$ niezależne od drogi

Nad krzywą zamkniętą:

$$\int_\sigma d[p \quad] = 0 \qquad \int_\sigma S(\nabla^2 \mathfrak{v} . d\sigma) = 0$$

Twierdzenie Stokesa: $\iint S(N . \operatorname{curl} \mathfrak{v}) \, dS = \int_\sigma S(\mathfrak{v} . d\sigma)$

$$\iint S(N . \operatorname{curl} \nabla^2 \mathfrak{v}) \, dS = 0$$

$\operatorname{curl} \nabla^2 \mathfrak{v} = \nabla^2 \operatorname{curl} \mathfrak{v}$

$\operatorname{curl}^2 \mathfrak{v} = \nabla \operatorname{div} - \nabla^2 \mathfrak{v}$

$\operatorname{curl} \nabla^2 \mathfrak{v} = -\operatorname{curl}^3 \mathfrak{v} - \cancel{\operatorname{curl} \nabla \operatorname{div} \mathfrak{v}}$

$= \nabla^2 \operatorname{curl} \mathfrak{v} - \cancel{\nabla \operatorname{div} \operatorname{curl} \mathfrak{v}}$

więc $\nabla^2 \mathfrak{v} = \nabla U = \nabla \operatorname{div} - \operatorname{curl}^2 \mathfrak{v}$

~~tego~~

$$\int(\nabla^2 u \, dx + \nabla^2 v \, dy + \nabla^2 w \, dz) = \nabla^2 \int u \, dx + v \, dy + w \, dz$$?

[illegible] drugi rodzaj: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{r} = 0$

$$\begin{cases} \frac{\partial p}{\partial x} = \mu \nabla^2 u \\ \quad\quad\quad\quad\quad\quad p = \underline{A} \quad \Big\| \; \nabla^2 A = 0 \\ -\,- \end{cases}$$

$$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} = (k-1)\mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{v}{r}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \;-\;\right]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = (k-1)\left[\underbrace{2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2}_{\cancel{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 - 4\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}}} + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right]$$

$$\cancel{u\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + u\frac{\partial^2 v}{\partial x \partial y} = 0}$$

$$\cancel{u\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + u\frac{\partial^2 v}{\partial y^2} = 0}$$

$$\cancel{v\frac{\partial^2 v}{\partial y^2} + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + v\frac{\partial^2 u}{\partial x \partial y} = 0}$$

~~[illegible]~~ Dla każdej powierzchni jest $u = v = 0$

otrzymuje się z III: składowe u w kierunku stycznej

v normalnej

$\frac{\partial u}{\partial x} = 0$ [co dozwolone pozwoli tylko pierwsze pochodne]

$$k\mu\frac{\partial v}{\partial y} = (k-1)\mu\left[\underbrace{-\frac{2}{3}\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2}_{\frac{4}{3}\left(\frac{\partial v}{\partial y}\right)^2} + \left(\frac{\partial u}{\partial y}\right)^2\right] + (k-1)\kappa\,\nabla^2\theta$$

z II wszystkiego wynika : $\frac{\partial v}{\partial y}=0$ z czego wynikałoby $\frac{\partial u}{\partial y}=0$ ‖ ? ? !

To prawda : ciepło zostanie wytwarzane dopóki $\frac{\partial u}{\partial y} \gtrless 0$, podczas gdy ruch $=0$ przy powierzchni, zatem musiałoby nastąpić grzanie – chyba że $\frac{\partial u}{\partial y}=0$

lub też : uwzględnienie przewodzenia ciepła!

ważnem n.p. dla przepływu gazu przez rurki | ważnem tylko dla prędkich ruchów

uwzględnienie przewodnictwa niezbędne

Zatem adiabatyczna teorya będzie niemożliwą n.p. dla rurek włoskowatych

Porówna się wogóle okazać wpływ przewodnictwa cieplnego materyału rurki na prąd gazu! Co prawda, że znaczniejszym może być tylko n.p. dla oporu waty w dośw. Kelvina

Zły przewodnik będzie oporą wiele znaczniejszą niż dobry

Obliczyć zatem różnicę przepływu izotermicznego i adiabat.!

~~$u = u_0 + a\rho + b\rho^2$~~

~~$\frac{\partial u}{\partial \rho} = a + 2b\rho$~~ $= 0$

~~[illegible]~~ $u_0 + a\delta + b\delta^2 = 0$

$u_0 + \underline{\delta(a+2b\delta)} - b\delta^2 = 0$

$b = \frac{u_0}{\delta^2}$

$a = -\frac{2u_0}{\delta}$

$$\rho\left(\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y}\right) + \cancel{u\frac{\partial \rho}{\partial x}} + \cancel{v\frac{\partial \rho}{\partial y}} = 0$$

$$\left(\frac{\partial u}{\partial y}\right)^2 = -\frac{k}{\mu}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right)$$

Robię założenie że $\frac{\partial \theta}{\partial x} = \frac{\partial \theta}{\partial z} = 0$

$$\left(\frac{\partial u}{\partial y}\right)^2 = -\frac{k}{\mu}\frac{\partial^2 \theta}{\partial y^2}$$

Rurki włoskowate

Wprowadzając zwykłe przybliżenia z uwzględnieniem równania termicznego:

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \qquad \frac{dp}{dx} = \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial(\rho u)}{\partial x} = 0$$

$$\text{III).} \quad u\frac{dp}{dx} + kp\frac{\partial u}{\partial x} = (k-1)\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]\mu + \kappa\,\Delta^2\theta$$

$$u = f(x, r) \qquad \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = (f')^2$$

$$\frac{\partial u}{\partial y} = f'\frac{y}{r}$$

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= f''\frac{y^2}{r^2} + f'\left(\frac{1}{r} - \frac{y^2}{r^3}\right) \\ \frac{\partial^2 u}{\partial z^2} &= f''\frac{z^2}{r^2} + f'\left(\frac{1}{r} - \frac{z^2}{r^3}\right) \end{aligned}\right\} \quad \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f'' + f'\frac{1}{r} = \frac{1}{r}\,d(rf')$$

$$\frac{dp}{dx} = \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \mu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] \qquad \rho u = k.(r).$$

$$u\frac{dp}{dx} + kp\frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial r}\right)^2 + \underbrace{\frac{\kappa}{R}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{p}{\rho}\right)\right]}$$

$$= -\frac{\kappa p}{R}\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{r}{\rho^2}\frac{\partial \rho}{\partial r}\right]$$

~~I). [illegible] k = 0:~~

$$u = \frac{r^2}{4}\varphi(x) + \psi(x)\log(r) + \chi(x)$$

$$\frac{\partial u}{\partial r} = \frac{r}{2}\varphi(x) + \frac{1}{r}\psi(x)$$

$$\frac{\partial}{\partial r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = 0$$

$$\mu\cdot\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \mu\cdot\varphi(x) = \frac{dp}{dx} \qquad p = \mu\int\varphi(x)\,dx$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = r\varphi(x)$$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\varphi(x) + \psi(x)$$

Stawiając $x=0$:

$$\frac{r^2}{4}[\varphi(x)]^2 + \lg r\ \psi(x)\varphi(x) + \chi(x)\varphi(x) + k\left[\frac{r^2}{4}\varphi'(x)\int\varphi(x)dx + \lg r\cdot\psi'(x)\int\varphi(x)dx + \chi'(x)\int\varphi(x)dx\right] =$$

$$= (k-1)\left[\frac{r^2}{4}[\varphi(x)]^2 + \psi(x)\varphi(x) + \frac{1}{r^2}[\psi(x)]^2\right]$$

Musi być spełnione dla dowolnych r, x

$$[\varphi(x)]^2 + k\,\varphi'(x)\int\varphi(x)dx = (k-1)[\varphi(x)]^2$$

$$\psi(x)\varphi(x) + k\psi'(x)\int\varphi(x)dx = 0$$

$$\chi(x)\varphi(x) + k\chi'(x)\int\varphi(x)dx = (k-1)\varphi(x)\psi(x)$$

$$0 = \frac{k-1}{r^2}[\psi(x)]^2 \qquad \psi(x) = 0$$

$$\chi\varphi = -k\chi'\int\varphi$$

$$\frac{\chi}{\chi'}\varphi = -k\int\varphi$$

$$\varphi - \frac{\chi\chi''}{\chi'^2}\varphi + \frac{\chi}{\chi'}\varphi' = -k\varphi$$

$$\varphi\left[1+k - \frac{\chi\chi''}{\chi'^2}\right] = -\frac{\chi}{\chi'}\varphi'$$

$$\int\varphi = \frac{(k-2)}{k}\frac{\varphi^2}{\varphi'}$$

$$k\varphi = \frac{k-2}{k}\left[2\varphi - \frac{\varphi^2\varphi''}{\varphi'^2}\right]$$

$$k-4 = (k-2)\frac{\varphi\varphi''}{\varphi'^2}$$

$$\frac{k-4}{k-2}\frac{\varphi'}{\varphi} = \frac{\varphi''}{\varphi'}$$

$$a\frac{k-4}{k-2}\varphi = \varphi'$$

$$\frac{\varphi'}{\varphi} = a\frac{k-4}{k-2}$$

$$\varphi = b\,e^{a\frac{k-4}{k-2}x}$$

$$(1+k)\frac{\chi'}{\chi} - \frac{\chi''}{\chi'} = a\frac{4-k}{2-k}$$

$$(1+k)\lg\chi - \lg\chi' = a\frac{4-k}{2-k}x +$$

$$\frac{\chi'}{\chi^{1+k}} = c\,e^{a\frac{k-4}{k-2}x}$$

$$\chi^{-k} = -ck\int e^{a\frac{k-4}{k-2}x}dx$$

Żeby warunek $u=0$ na pow. był spełniony musiałoby być:
$u=$ skończone dla $r=0$

$$\frac{\delta^2}{4}\varphi(x)+\chi(x)=0$$

$$\chi(x)=-\frac{\delta^2}{4}\varphi(x) \qquad \psi=0$$

To wymagałoby:

$$\varphi^2+k\varphi'\int\varphi=0$$

Co nie zgadza się z równaniem dla φ zatem w ogóle (stały stan) niemożliwy do spełnienia jeśli $\kappa=0$ i jeśli zachowujemy takie przybliżenia

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Spróbować przy $\kappa \geq 0$: $\qquad u=\frac{r^2-\delta^2}{4}\varphi(x)$

$\rho u=f(r)$: $\qquad \frac{\partial u}{\partial r}=\frac{r}{2}\varphi(x)$

$$\frac{r^2}{4}\rho\varphi(x)-\frac{\delta^2}{4}\rho\varphi(x)=\rho u=f.(r)$$

$\frac{\partial}{\partial x}=0$:

$$\frac{r^2-\delta^2}{4}\frac{\partial}{\partial x}(\rho\varphi)=0 \qquad \frac{\partial\rho}{\partial x}\varphi+\rho\frac{\partial\varphi}{\partial x}=0$$

$$\rho\varphi(x)=f(r)$$

$$\frac{1}{\rho}=\frac{\varphi(x)}{f(r)} \quad \Big\| \quad \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\frac{1}{f(r)}\right]=F(r)$$

$$\frac{dp}{dx}=\mu\varphi \qquad p=\mu\int\varphi\,dx$$

$$\frac{r^2-\delta^2}{4}\mu\varphi^2(x)+k\mu\frac{r^2-\delta^2}{4}\varphi'\int\varphi=\mu(k-1)\frac{r^2}{4}\varphi^2(x)+\frac{\kappa\varphi(x)}{R}F(r)$$

Musi być spełnione ~~niezależnie od~~ dla każdego r i x

$$(r^2-\delta^2)\left[\varphi^2_{(x)}+k\,\varphi'_{(x)}\int\varphi\,dx\right]=(k-1)\,r^2\varphi^2_{(x)}+\frac{4K}{\mu R}\varphi_{(x)}\,F_{(r)}$$

$$r^2\left[(2-k)\,\varphi^2_{(x)}+k\,\varphi'_{(x)}\int\varphi\,dx\right]-\delta^2\left[\varphi^2_{(x)}+k\,\varphi'_{(x)}\int\varphi\,dx\right]=\frac{4K}{R\mu}\varphi_{(x)}\,F_{(r)}$$

$$2r\left[\qquad\qquad\qquad\qquad\right]=\frac{4K}{R\mu}\varphi_{(x)}\,F'_{(r)}$$

$$2\left[\qquad\qquad\qquad\qquad\right]=\frac{4K}{R\mu}\varphi_{(x)}\,F''_{(r)}$$

$$\frac{F''}{F'}=\frac{1}{r}$$

$$\log F'=\log r+\ldots$$

$$F'=ar=\frac{dF}{dr}$$

$$F=ar^2+b$$

$$(2-k)\varphi^2+k\varphi'\int\varphi=\frac{2aK}{R\mu}\varphi\quad\Big\|\;-\delta^2\left[\varphi^2+k\varphi'\int\varphi\right]=\frac{4bK}{R\mu}\varphi$$

$$2(2-k)\varphi\varphi'+k\varphi''\int+k\varphi'\varphi=\frac{2aK}{R\mu}\varphi'$$

$$[2-k][\varphi^2\varphi''-2\varphi\varphi'^2]\;k\varphi'^2\varphi=\frac{2aK}{R\mu}[\varphi\varphi''-\varphi'^2]$$

$$[2-k]\left[\frac{\varphi''}{\varphi'}-2\frac{\varphi'}{\varphi}\right]-k\frac{\varphi'}{\varphi}=\frac{2aK}{R\mu}\left[\frac{\varphi''}{\varphi\varphi'}-\frac{\varphi'}{\varphi^2}\right]$$

$$(1-k^2)\varphi=\left[\frac{2aK}{R\mu}+\frac{4bK}{\delta^2R\mu}\right]$$

$\varphi=\text{const}$

Wtedy jednak: $\theta=\frac{p}{R\rho}=\frac{pu}{R\rho u}=\frac{\mu_{(x)}\frac{r^2-\delta^2}{4}\varphi_{(x)}}{R}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \frac{p\varphi}{R}F(r) = \frac{p\varphi}{R}\left(\frac{ar^2}{2}+b\right)$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \frac{p\varphi}{R}\left[\frac{ar^3}{2}+br\right]$$

$$r\frac{\partial\theta}{\partial r} = \frac{p\varphi}{R}\left[\frac{ar^4}{8}+\frac{br^2}{2}\right] + \cancel{f(x)}$$

$$\theta = \frac{p\varphi}{R}\left[\frac{ar^4}{32}+\frac{br^2}{4}\right] + \cancel{f(x)\log r} + f_2(x)$$

$$p = p_1 + (p_2-p_1)\frac{x}{l} \quad \Big\| \quad \frac{dp}{dx} = \frac{p_2-p_1}{l} = \mu\varphi$$

$$\varphi = \frac{p_2-p_1}{l\mu} = \frac{2ak}{(2-k)R\mu} = \frac{-4bk}{\delta^2 R\mu} \qquad a = -\frac{2b}{(2-k)\delta^2}$$

$$a = \frac{2-k}{2}\,\frac{(p_2-p_1)R}{lk} \qquad b = -\frac{(2-k)}{2}\delta^2 \cdot a$$

$$\overset{\theta_0 + \cancel{\frac{\varphi}{R}}}{\theta} = \frac{p_2-p_1}{Rl\mu}\left[p_1+(p_2-p_1)\frac{x}{l}\right]\left[\frac{r^4}{32}-\frac{(2-k)\delta^2 r^2}{8}\right]\frac{2-k}{2}\,\frac{p_2-p_1}{lk}R$$

$$\left[-\frac{\delta^4}{32}+\frac{(2-k)\delta^4}{8}\right]$$

$$\theta-\theta_0 = \left(\frac{p_2-p_1}{l}\right)^2\frac{2-k}{16}\,\frac{1}{k\mu}\left[p_1+(p_2-p_1)\frac{x}{l}\right]\underbrace{\left[\frac{r^4-\delta^4}{4}-(2-k)\delta^2(r^2-\delta^2)\right]}_{\underbrace{(r^2-\delta^2)\left[\frac{r^2}{4}+\frac{\delta^2}{4}-(2-k)\delta^2\right]}_{\frac{1}{4}(r^2-\delta^2)\left[r^2+(4k-7)\delta^2\right]}}$$

$$\rho = \frac{p}{R\theta}$$

$$\rho u = \frac{p\,\frac{r^2-\delta^2}{4}\varphi}{R\left[\theta_0 + \frac{p\varphi}{R}\cdot\frac{2-k}{64}\,\frac{p_2-p_1}{l\,u}R\left(r^4-\delta^4-4(2-k)\delta^2 r^2+4(2-k)\delta^4\right)\right]}$$

$$M = 2\pi\int_0^\delta r\rho u\,dr = \frac{2\pi p\varphi}{2R}\int\frac{(r^3-r\delta^2\quad)\,dr}{\theta_0+\dots}$$

$\downarrow$

Nie można spełnić warunku

$\rho u = f_u(r)$

To samo dwuwymiarowe:

$\frac{\partial p}{\partial y} =$ [illegible] $=0$ $\qquad v =$ [illegible]

$$\begin{cases} \frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} \\ \frac{\partial(\rho u)}{\partial x} = 0 \\ u\frac{\partial p}{\partial x} + kp\frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial y}\right)^2 + \kappa\frac{\partial^2\theta}{\partial y^2}\left(+\frac{\partial^2\theta}{\partial x^2}\right) \end{cases} \longrightarrow$$

$$\frac{\partial^3 u}{\partial y^3} = 0$$

$$u = \frac{y^2}{2}\varphi(x) + y\,\psi(x) + \chi(x)$$

$$\rho u = f(y)$$

$$\frac{\partial p}{\partial x} = \mu\varphi(x)$$

$$p = \mu\int\varphi(x)\,dx + p_0$$

$$u = \frac{y^2}{2}\frac{1}{\mu}\frac{dp}{dx} + \psi(x)\,y + \chi(x) \nearrow$$

$$\frac{\partial u}{\partial y} = \frac{y}{\mu}\frac{dp}{dx} + \psi(x) \nearrow$$

$$\theta = \frac{p}{R\rho} \qquad \frac{\partial^2\theta}{\partial y^2} = \frac{p}{R}\frac{\partial^2}{\partial y^2}\left(\frac{1}{\rho}\right)$$

$$\left.\begin{matrix} y=0 \\ y=\delta\end{matrix}\right\}\ u=0 \qquad \chi = 0$$

$$\psi = -\frac{\delta}{2}\ \text{[illegible]}\ \varphi(x)$$

$$u = \frac{y(y-\delta)}{2}\frac{1}{\mu}\frac{dp}{dx}$$

$$\frac{\partial u}{\partial y} = \frac{2y-\delta}{2\mu}\frac{dp}{dx}$$

$$\rho u = \frac{y(y-\delta)}{2\mu}\frac{dp}{dx}\rho = f(y)$$

$$\rho = \frac{F_1(y)}{\frac{dp}{dx}} \quad\Big|\quad \frac{1}{\rho} = \frac{\frac{dp}{dx}}{F_1(y)}$$

$$\frac{\partial^2\theta}{\partial y^2} = \frac{p\frac{dp}{dx}}{R}\frac{\partial^2}{\partial y^2}\left[\frac{1}{F_1(y)}\right]$$

$$\frac{y^2-y\delta}{2}\left(\frac{dp}{dx}\right)^2 + kp\frac{d^2p}{dx^2}\,\frac{y^2-y\delta}{2} = (k-1)\left(\frac{2y-\delta}{2}\right)^2\left(\frac{dp}{dx}\right)^2 + p\frac{dp}{dx}\,\frac{k}{R}\,F_{(y)}$$

~~$(y-\delta)\left(\frac{dp}{dx}\right)^2$~~

$$\left(\frac{dp}{dx}\right)^2\left[\frac{y^2-y\delta}{2} - (k-1)\left(\frac{2y-\delta}{2}\right)^2\right] + kp\frac{d^2p}{dx^2}\,\frac{y^2-y\delta}{2} = p\frac{dp}{dx}\,\frac{k}{R}\,F_{(y)}$$

~~$\left(\frac{dp}{dx}\right)^2\left[y-\frac{\delta}{2} - (k-1)(2y-\delta)\right] + kp\frac{d^2p}{dx^2}\left(y-\frac{\delta}{2}\right) = p\frac{dp}{dx}\,\frac{k}{R}\,F'_{(y)}$~~

$$y^2\left[\left(\frac{3}{2}-k\right)\left(\frac{dp}{dx}\right)^2 + \frac{k}{2}p\frac{d^2p}{dx^2}\right] + y\delta\left[\left(k-\frac{3}{2}\right)\left(\frac{dp}{dx}\right)^2 - \frac{k}{2}p\frac{d^2p}{dx^2}\right]$$

$$-\frac{(k-1)\delta^2}{4}\left(\frac{dp}{dx}\right)^2 = p\frac{dp}{dx}\,\frac{k}{R}\,F_{(y)}$$

$$(y^2-y\delta)\left[\left(\frac{3}{2}-k\right)\left(\frac{dp}{dx}\right)^2 + \frac{k}{2}p\frac{d^2p}{dx^2}\right]$$

$$(2y-\delta)\left[\qquad\qquad\qquad\right] = p\frac{dp}{dx}\,\frac{k}{R}\,F'_{(y)}$$

$$2\left[\qquad\qquad\qquad\right] = \cdot - F''_{(y)}$$

$$y-\frac{\delta}{2} = \frac{F'}{F''}$$

$$\frac{F''}{F'} = \frac{1}{y-\frac{\delta}{2}}$$

$$\lg F' = \lg\left(y-\frac{\delta}{2}\right) + \cdots$$

$$F' = a\left(y-\frac{\delta}{2}\right)$$

$$F = a\left(\frac{y^2}{2} - y\frac{\delta}{2}\right) + b = \frac{a}{2}y(y-\delta) + b$$

$$(y^2-y\delta)\left[\qquad\qquad\right]-\frac{(k-1)\delta^2}{4}\left(\frac{dp}{dx}\right)^2=p\frac{dp}{dx}\,\frac{\kappa}{R}\left[\frac{a}{2}(y^2-y\delta)+b\right]$$

$$\left.\begin{aligned}\left(\frac{3}{2}-k\right)\left(\frac{dp}{dx}\right)^2+\frac{k}{2}p\,\frac{d^2p}{dx^2}&=\frac{a}{2}\,\frac{\kappa}{R}\,p\frac{dp}{dx}\\ \frac{(k-1)\delta^2}{4}\left(\frac{dp}{dx}\right)^2&=-b\,\frac{\kappa}{R}\,p\,\frac{dp}{dx}\end{aligned}\right\}$$

$$\frac{1}{p}\frac{dp}{dx}=\frac{-4b\kappa}{(k-1)\delta^2R}$$

$$p=A\,e^{-\frac{4b\kappa}{(k-1)\delta^2R}x}$$

$$\left[\frac{4b\kappa}{(k-1)\delta^2R\mu}\right]\left[\underbrace{\frac{3}{2}-k+\frac{k}{2}}_{\frac{3-k}{2}}\right]=-\frac{a}{2}\,\frac{\kappa}{\mu R}$$

$$a=-\frac{4(3-k)b}{(k-1)\delta^2}$$

$$\frac{\partial^2\theta}{\partial y^2}=\frac{p\frac{dp}{dx}}{R}F=\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}(y^2-y\delta)+b\right]$$

$$\frac{\partial\theta}{\partial y}=\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}\left(\frac{y^3}{3}-\frac{y^2\delta}{2}\right)+by\right]+\vartheta_{(x)}$$

$$\theta=\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}\left(\frac{y^4}{12}-\frac{y^3\delta}{6}\right)+b\frac{y^2}{2}\right]+y\,\vartheta_{(x)}+\eta_{(x)}$$

$y=0$: $\theta_0=\eta_{(x)}$

$y=\delta$: $\theta_\delta=\frac{p\frac{dp}{dx}}{R}\left[-\frac{a}{2}\,\frac{\delta^3}{12}y+b\frac{\delta}{2}y\right]+y\,\vartheta_{(x)}+\theta_0$

$$\theta=\theta_0+\frac{p\frac{dp}{dx}}{R}\left[\frac{a}{2}\left(\frac{y^4+\delta^3y}{12}-\frac{y^3\delta}{6}\right)+b\frac{y^2-y\delta}{2}\right]=\frac{pu}{R\rho u}$$

$$R\theta = \frac{p}{\rho} \qquad R\theta = \frac{p(x)}{f(y)} \cdot \frac{y(y-\delta)}{2\mu}\frac{dp}{dx} = F(y)\, p(x)\frac{dp}{dx} = \frac{c}{2} F(y)$$

Niemożliwe do spełnienia dla $y=0$ oprócz jeśli

$$p\frac{dp}{dx} = \text{const} \qquad p^2 = cx + a \quad \Bigg\| \quad p\frac{dp}{dx} = \frac{c}{2}$$

$$p\frac{dp}{dx} = \frac{c}{2} \quad \Bigg\| \quad p\frac{d^2p}{dx^2} + \left(\frac{dp}{dx}\right)^2 = 0$$

$$u\rho = f = \frac{y(y-\delta)}{2\mu}\frac{1}{F(y)}$$

$$\frac{\partial^2\theta}{\partial y^2} = \frac{c}{2R} F''$$

$$u = \frac{y(y-\delta)}{2}\frac{1}{\mu}\frac{dp}{dx}$$

$$\frac{y^2-y\delta}{2\mu}\left(\frac{dp}{dx}\right)^2 + k\,\frac{y^2-y\delta}{2\mu}\,p\frac{d^2p}{dx^2} = (k-1)\mu\left(\frac{2y-\delta}{2\mu}\right)^2\left(\frac{dp}{dx}\right)^2 + \frac{kc}{2R}F''(y)$$

$$p = \sqrt{cx+a} \qquad p\frac{d^2p}{dx^2} = -\frac{c^2}{4(cx+a)}$$

$$\frac{dp}{dx} = \frac{c}{2\sqrt{cx+a}} \qquad \left(\frac{dp}{dx}\right)^2 = \frac{c^2}{4(cx+a)}$$

$$\frac{d^2p}{dx^2} = \frac{-c^2}{4\sqrt{(cx+a)^3}}$$

$$\left[(1-k)(y^2-y\delta) - (k-1)\frac{(2y-\delta)^2}{2}\right] \frac{c^2}{4(cx+a)}$$

$$(1-k)\quad y^2-y\delta - 2y^2 + 2y\delta$$

Więc nie spełnia równania II

($p = f(x)$ etc)

Założenia muszą być sprzeczne ze sobą!

Adiabatyczne

$$\frac{dp}{dx} = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) \qquad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \rho = f_1(x)$$

$$\frac{\partial(\rho u)}{\partial x} = 0 \qquad p^{\frac{1}{k}} \sim \rho$$

$$\rho u = f_c(r)$$

---

p niezależne od r zatem : $\frac{\partial}{\partial r}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] = 0$

$$u = \frac{r^2}{4}\varphi(x) + ~~\psi(x)\log r + f(x)~~$$

$$\frac{dp}{dx} = \mu\,\varphi(x)$$

$$u = \frac{r^2 - \delta^2}{4}\varphi(x)$$

$$u = \frac{r^2-\delta^2}{4\mu}\frac{dp}{dx}$$

$$\frac{1}{p_0}\frac{dp}{dx} = \frac{k}{\rho_0^k}\rho^{k-1}\frac{d\rho}{dx}$$

$$\frac{\partial}{\partial x}\left[\frac{r^2-\delta^2}{4\mu}\frac{k p_0}{\rho_0^k}\rho^k\frac{d\rho}{dx}\right] = 0$$

$$\frac{r^2-\delta^2}{4\mu}\frac{k p_0}{\rho_0^k}\cdot\frac{\rho^{k+1}}{k+1} = x f(r) + \psi(r)$$

$$= (r^2-\delta^2)(Ax + B)$$

$$~~\rho^{k+1} = ax + b~~$$

$$p^{\frac{k+1}{k}} = ax + b$$

$$\frac{p_2^{\frac{k+1}{k}} - p_1^{\frac{k+1}{k}}}{l} = a$$

$$p^{\frac{k+1}{k}} - p_1^{\frac{k+1}{k}} = \frac{x}{l}\left(p_2^{\frac{k+1}{k}} - p_1^{\frac{k+1}{k}}\right)$$

itd.

Podobnie jak przy izotermicznym wychodzi tylko że $\frac{1+k}{k}$ na miejscu 2

Izotermiczne z uwzględnieniem $\frac{\partial^2 u}{\partial x^2}$

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2} + \frac{4}{3}\mu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial(\rho u)}{\partial x} = 0 \quad = \quad \frac{\partial(pu)}{\partial x} = 0$$

$$pu = \varphi(y)$$

$$u = \frac{\varphi(y)}{f(x)} \qquad p = f(x)$$

$$f'(x) = \mu \frac{\varphi''(y)}{f(x)} + \frac{4}{3}\mu\, \varphi(y) \frac{d^2}{dx^2}\left(\frac{1}{f(x)}\right)$$

$$f'(x)\, f(x) = \mu\, \varphi''(y) + \frac{4}{3}\mu\, \varphi(y)\, f(x) \frac{d^2}{dx^2}\left[\frac{1}{f(x)}\right]$$

$\frac{\partial}{\partial y}$:

$$\cancel{0 = \mu\, \varphi'''(y) + \frac{4}{3}\mu\, \varphi'(y)} \uparrow$$

$$\cancel{0 = \mu\, \varphi''''(y) + \frac{4}{3}\mu\, \varphi''(y)} \uparrow$$

$$\cancel{\frac{\varphi^{IV}}{\varphi'''} = \frac{\varphi''}{\varphi'}}$$

Możliwe tylko jeśli:

I/.

$$f(x)\, f'(x) = a$$

$$\underbrace{f(x)\, f'(x)}_{\frac{1}{2}\frac{d}{dx} f^2}$$

$$f^2 = 2ax + b$$

$$f = \sqrt{2ax+b}$$

$$f' = \frac{a}{\sqrt{}}$$

$$f'' = -\frac{a^2}{\sqrt{()^3}}$$

$$f(x) \frac{d^2}{dx^2}\left[\frac{1}{f(x)}\right] = c \qquad \cancel{\varphi'' = c}$$

$$d - \frac{f'}{f^2}$$

$$\underbrace{-\frac{f''}{f} + 2\frac{f'^2}{f^2}}_{} =$$

$$+\frac{a^2}{(2ax+b)^2} + 2\frac{a^2}{()^2} = c$$

Niemożliwe oprócz jeśli $a=0$

t.zn $p = \text{const}$

Druga możliwość: $\varphi''(y) = a \qquad \varphi(y) = b$ niemożliwe bo wtedy $u = \ldots$

Zatem to wogóle nie może być przystosowane do warunków początkowych

Jedyne możliw: $p = \text{const}$ lub $pu = \text{const}$

Izotermiczne z uwzględnieniem zmienności p, jeśli $v, u = 0$

$$\cancel{\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} + \frac{4}{3}\mu \frac{\partial^2 u}{\partial x^2}}$$

$$\cancel{\frac{\partial p}{\partial y} =}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad \rho u = \frac{\partial \psi}{\partial y} \qquad \rho v = -\frac{\partial \psi}{\partial x} \qquad \frac{p}{\rho} = R\theta$$

$$\frac{v}{u} = -\frac{\frac{\partial \psi}{\partial x}}{\frac{\partial \psi}{\partial y}}$$

$$u = \frac{1}{\rho}\frac{\partial \psi}{\partial y} = \frac{R\theta}{p}\frac{\partial \psi}{\partial y}$$

$$v = -\frac{1}{\rho}\frac{\partial \psi}{\partial x} = -\frac{R\theta}{p}\frac{\partial \psi}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{-1}{\rho}\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)$$

$$= -\frac{1}{\rho^2}\left(\frac{\partial \psi}{\partial y}\frac{\partial \rho}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

$$\frac{\partial u}{\partial x} = \frac{1}{\rho}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{1}{\rho^2}\frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial y}$$

$$\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial^2 \psi}{\partial x \partial y} + \frac{1}{\rho^2}\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial y} - \frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial x}\right)$$

$$+ \frac{1}{\rho}\left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}\right)$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = +\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial y} + \frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial x}\right)$$

$$-\frac{1}{\rho}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)$$

$$\Phi = \frac{4}{3}\mu\left[\underbrace{\left(\frac{\partial u}{\partial x}\right)^2 - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2}\right] + \mu\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]^2$$

$$= \frac{1}{\rho^4}\left[\left(\frac{\partial \psi}{\partial y}\frac{\partial \rho}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial x}\right)^2\left(\frac{\partial \rho}{\partial y}\right)^2\right] + \frac{1}{\rho^2}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2 - \frac{1}{\rho^3}\frac{\partial^2 \psi}{\partial x \partial y}\left[\frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial y} + \frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}\right]$$

$$-\frac{1}{\rho^4}\frac{\partial \rho}{\partial x}\frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial y}$$

$$\cancel{-\frac{1}{\rho^2}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2 + \frac{1}{\rho^3}\frac{\partial^2 \psi}{\partial x \partial y}\left[\frac{\partial \rho}{\partial x}\frac{\partial \psi}{\partial y} + \frac{\partial \rho}{\partial y}\frac{\partial \psi}{\partial x}\right]}$$

II).

$$k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\frac{4\mu}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+\left(\frac{\partial v}{\partial y}\right)^2\right]+\mu\left[\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right]^2$$

$$-\frac{\mu}{3}\left[u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]-\mu\left[u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\right]$$

$$=\frac{4}{3}\mu\left[\left(\frac{\partial u}{\partial x}\right)^2-u\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial v}{\partial y}\right)^2-v\frac{\partial^2 v}{\partial y^2}\right]-\frac{1}{3}\mu\left[4\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+u\frac{\partial^2 v}{\partial x\partial y}+v\frac{\partial^2 u}{\partial x\partial y}\right]$$

$$u^2\left[\frac{1}{u^2}\left(\frac{\partial u}{\partial x}\right)^2-\frac{1}{u}\frac{\partial^2 u}{\partial x^2}\right]+v^2\left[\;-\,-\,\ldots\;\right]$$

$$+\mu\left[\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2-u\frac{\partial^2 u}{\partial y^2}-v\frac{\partial^2 v}{\partial x^2}\right]$$

$$=-\,u^2\frac{\partial}{\partial x}\left(\frac{1}{u}\frac{\partial u}{\partial x}\right)$$

$$=-\left[u^2\frac{\partial^2}{\partial x^2}(\log u)+v^2\frac{\partial^2}{\partial y^2}(\log v)\right]\frac{4}{3}-\left[u^2\frac{\partial^2}{\partial y^2}(\log u)+v^2\frac{\partial^2}{\partial x^2}(\log v)\right]$$

$$=\ \mu$$

$$\left[a_1 x^3+a_2 xy^2)+a_3\right]\left[(8a_1+2a_2)x+\frac{2\mu}{3}b_2 y\right]+\left[-\frac{b_2}{3}x^3+b_2 y^2 x+b_3\right]\left[\left(\frac{8}{3}b_2+6b_1\right)x+\frac{2\mu}{3}a_2 y\right]$$

$$+k\left[\frac{p_0}{\mu}+\mu(4a_1+a_2)x^2+\frac{a_2}{3}y^2+\frac{2}{3}b_2 xy\right]\left[3a_1x^2+a_2y^2+2b_2xy\right]=$$

Jeśli u, v, p rzędu		stałych		równań $\frac{\partial^2}{\partial x\partial y}$	równań
	1	3 + 3	} + p_0	$\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 v}{\partial x\partial y^2}=\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 u}{\partial y^3}$:	0
	2	6 + 6			0
	3	10 + 10		x^{n-3} y^{n-3}	1
	4	15 + 15			3
	5	21 + 21			6
	6	28			

Podstawiając do równania dla Φ:

Stopień: $2(n-1)$	równań dla spójności	pozostaje zatem dowolnych:
0	0	6
2	6	p_0 + 6
4	15	5
6	28	2
8	45	−3

$u = e^{a+\varphi}$ $v = e^{b+\psi}$ $p = e^{c+\chi}$

$\frac{\partial u}{\partial x} = \frac{\partial \varphi}{\partial x} e^{\varphi} =$ [illegible] $\frac{\partial v}{\partial y} = \frac{\partial \psi}{\partial y} e^{\psi}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 \varphi}{\partial x^2} e^{\varphi} + \left(\frac{\partial \varphi}{\partial x}\right)^2 e^{\varphi} = \left[\frac{\partial^2 \varphi}{\partial x^2} + \left(\frac{\partial \varphi}{\partial x}\right)^2\right] e^{\varphi}$ $\frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 \psi}{\partial x \partial y} e^{\psi} + \frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial y} e^{\psi}$

$$e^{\chi}\frac{\partial \chi}{\partial x} = \left\{\frac{4\mu}{3}\left[\frac{\partial^2 \varphi}{\partial x^2} + \left(\frac{\partial \varphi}{\partial x}\right)^2\right] + \mu\left[\frac{\partial^2 \varphi}{\partial y^2} + \left(\frac{\partial \varphi}{\partial y}\right)^2\right]\right\} e^{\varphi} + \frac{\mu}{3}\left[\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial \psi}{\partial x}\frac{\partial \psi}{\partial y}\right] e^{\psi}$$

$u = x(2ax - y^2)\varphi$ $v = x(2ax - y^2)\psi$

$\frac{\partial u}{\partial x} =$

$u = x(a_1 x^2 +$ [illegible] $+ a_2 y^2) + a_3$ $v = x(b_1 x^2 +$ [illegible] $+ b_2 y^2) + b_3$

$\frac{\partial u}{\partial x} = 3a_1 x^2 + a_2 y^2$ $\frac{\partial u}{\partial y} = 2a_2 xy$ $\frac{\partial v}{\partial y} = 2b_2 xy$ $\frac{\partial v}{\partial x} = 3b_1 x^2 + b_2 y^2$

$\frac{\partial^2 u}{\partial x^2} = 6a_1 x$ $\frac{\partial^2 u}{\partial y^2} = 2a_2 x$ $\frac{\partial^2 v}{\partial y^2} = 2b_2 x$ $\frac{\partial^2 v}{\partial x^2} = 6b_1 x$

$\frac{\partial^2 u}{\partial x \partial y} = 2a_2 y$ $\frac{\partial^2 v}{\partial x \partial y} = 2b_2 y$

$$\frac{\partial p}{\partial x} = \frac{4\mu}{3}\,6a_1 x + \frac{\mu}{3}\,2b_2 y + \mu\,2a_2 x = \mu(8a_1 + 2a_2)x + \frac{2\mu}{3}b_2 y$$

$$\frac{\partial p}{\partial y} = \frac{4\mu}{3}\,2b_2 x + \frac{\mu}{3}\,2a_2 y + \mu\,6b_1 x = \mu\left(\frac{8}{3}b_2 + 6b_1\right)x + \frac{2\mu}{3}a_2 y$$

$p = p_0 +$ [illegible] ~~$8a_1 + 2a_2$~~ $\mu(4a_1 + a_2)x^2 + \frac{\mu}{3}a_2 y^2 + \frac{2\mu}{3}b_2 xy$

$\frac{2}{3}b_2 = \frac{8}{3}b_2 + 6b_1$

$\underline{b_2 = -3b_1}$

$$\frac{\partial^2}{\partial x \partial y}\log u = \frac{\partial}{\partial y}\left(\frac{1}{u}\frac{\partial u}{\partial x}\right) = -\frac{1}{u^2}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{1}{u}\frac{\partial^2 u}{\partial x \partial y}$$

$$u = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2$$

$$v = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2$$

$$\frac{\partial u}{\partial x} = a_1 + 2a_3 x + a_4 y$$

$$\frac{\partial^2 u}{\partial x^2} = 2a_3 \qquad \frac{\partial^2 u}{\partial x \partial y} = a_4 \qquad \frac{\partial^2 u}{\partial y^2} = 2a_5$$

$$\frac{\partial^2 v}{\partial x^2} = 2b_3 \qquad \frac{\partial^2 v}{\partial x \partial y} = b_4 \qquad \frac{\partial^2 v}{\partial y^2} = 2b_5$$

~~$\frac{\partial p}{\partial x} = \frac{\mu}{3}$~~

$$\frac{\partial p}{\partial x} = \frac{1}{3}\mu\,(2a_3 + b_4) + \mu\,(2a_3 + 2a_5)$$

$$\frac{\partial p}{\partial x} = \mu\left(\frac{8}{3} a_3 + \frac{b_4}{3} + 2a_5\right)$$

$$\frac{\partial p}{\partial y} = \mu\left(\frac{8}{3} b_3 + \frac{a_4}{3} + 2b_5\right)$$

$$\frac{\partial v}{\partial y} = b_2 + b_4 x + 2b_5 y$$

$$\frac{\partial v}{\partial x} = b_1 + 2b_3 x + b_4 y$$

$$\frac{\partial u}{\partial y} = a_2 + a_4 x + 2a_5 y$$

$$p = p_0 + \mu\Big(\overbrace{\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5}^{A}\Big)x + \mu\Big(\overbrace{\frac{8}{3} b_3 + \frac{1}{3} a_4 + 2b_5}^{B}\Big)y$$

$$\mu\left(\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5\right)\left(a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2\right) +$$

$$\left(\frac{8}{3} b_3 + \frac{1}{3} a_4 + 2b_5\right)\left(b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2\right) +$$

$$k\left[p_0 + \mu\left(\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5\right)x + \mu\left(\qquad\qquad\right)y\right]\left[a_1 + b_1 + (2a_3 + b_4)x + (2b_5 + a_4)y\right]$$

$$= (k-1)\left\{\left[\frac{4}{3}\mu\left\{(a_1 + 2a_3 x + a_4 y)^2 + (b_2 + b_4 x + 2b_5 y)^2 - (a_1 + 2a_3 x + a_4 y)(b_2 + b_4 x + 2b_5 y)\right\}\right.\right.$$

$$\left.\left. + \mu\left[a_2 + b_1 + (a_4 + 2b_3)x + (2a_5 + b_4)y\right]^2\right\}$$

$$\left(\frac{8}{3} a_3 + \frac{1}{3} b_4 + 2a_5\right)a_0 + \left(\frac{8}{3} b_3 + \frac{1}{3} a_4 + 2b_5\right)b_0 + k(a_1 + b_1)\frac{p_0}{\mu} =$$

$$= (k-1)\left[\frac{4}{3}\left(a_1^2 + b_2^2 - a_1 b_2\right) + (a_2 + b_1)^2\right]$$

$$A a_5 + B b_5 + k B (2 b_5 + a_4) = (k-1)\left[\frac{4}{3}\left(a_4^2 + 4 b_5^2 - 2 a_4 b_5\right) + (2 a_5 + b_4)^2\right]$$

Spróbować $a_0 = b_0 = a_2 = b_2 = a_5 = b_5 = 0$

tzn. że $u = v = 0$ na osi X

$$\left(\frac{8}{3} a_3 + \frac{1}{3} b_4\right)\left(a_1 x + a_3 x^2 + a_4 x y\right) + \left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right)\left(b_1 x + b_3 x^2 + b_4 x y\right) +$$

$$+ k\left[\frac{p_0}{\mu} + \left(\frac{8}{3} a_3 + \frac{1}{3} b_4\right) x + \left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right) y\right]\left[a_1 + b_1 + (2 a_3 + b_4) x + a_4 y\right]$$

$$= (k-1)\left[\frac{4}{3}\left\{(a_1 + 2 a_3 x + a_4 y)^2 + (b_4 x)^2 - (a_1 + 2 a_3 x + a_4 y) b_4 x\right\} + \left[b_1 + (a_4 + 2 b_3) x + b_4 y\right]^2\right]$$

6 równań na współczynniki $y^2 \quad x^2 \quad xy \quad x \quad y \quad 0$

$u = x(a_1 + a_3 x + a_4 y)$

$v = x(b_1 + b_3 x + b_4 y)$

Jeśli równocześnie mają u, v być $= 0$ na drugiej prostej to chyba tak że

$a_1 : b_1 = a_3 : b_3 = a_4 : b_4$

~~$\left(\frac{8}{3} a_3 + \frac{1}{3} b_4\right) a_4 x + \left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right) b_4 x + k\left\{\left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right)\left[a_1 + b_1 + (2 a_3 + b_4) x + a_4 y\right] +\right.$~~

~~$\left. + a_4\left[\frac{p_0}{\mu} + (\quad) x + \left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right) y\right]\right\} =$~~

~~$= (k-1)\left[\frac{8}{3} a_4 (a_1 + 2 a_3 x + a_4 y) - a_4 b_4 x + 2\left[b_1 + (a_4 + 2 b_3) x + b_4^2 y\right]\right]$~~

~~$k\left\{\left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right) a_4 + a_4\left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right)\right\} = (k-1)\left(\frac{8}{3} a_4^2 + 2 b_4^2\right)$~~

~~$k\, a_4\left(\frac{8}{3} b_3 + \frac{1}{3} a_4\right) =$~~

~~Czy możliwe są rozwiązania formy~~

~~$u = a + e^{\alpha_1 x + \alpha_2 y}$ $\quad \frac{\partial u}{\partial x} = \alpha_1 e^{\alpha_1 x + \alpha_2 y}$ $\quad \frac{\partial u}{\partial y} = \alpha_2 e^{\alpha_1 x + \alpha_2 y}$~~

~~$v = b + e^{\beta_1 x + \beta_2 y}$ $\quad \frac{\partial v}{\partial x} = \beta_1 e^{\beta_1 x + \beta_2 y}$ $\quad \frac{\partial v}{\partial y} = \beta_2 e^{\beta_1 x + \beta_2 y}$~~

~~Znaleźć takie u v któreby jako równania $u=0$ $v=0$ dały np. $x=0$ $y=\cdots$~~

~~$p = R\rho\theta$~~

~~$\rho\frac{\partial\theta}{\partial x} + \theta\frac{\partial\rho}{\partial x} = \frac{\mu}{3R}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\mu}{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$~~

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] = -\frac{\partial p}{\partial x} + \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \qquad \Big| \, u \qquad \frac{\partial^2\rho}{\partial t^2} = -\frac{\partial^2(\rho u)}{\partial x \partial t}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} = -\frac{\partial p}{\partial x} + \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2\rho}{\partial t^2} = \frac{\partial^2(\rho u^2)}{\partial x^2} + \frac{\partial^2 p}{\partial x^2} - \frac{4\mu}{3}\frac{\partial^3 u}{\partial x^3}$$

$$\frac{\partial^2\rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[p + \rho u^2 - \frac{4\mu}{3}\frac{\partial u}{\partial x}\right]$$

$$= \frac{\partial^2}{\partial x^2}\left[\rho(R\theta + u^2) - \frac{4\mu}{3}\frac{\partial u}{\partial x}\right]$$

Zaniedbując tarcie i u^2

$\frac{c}{A}\rho\left(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x}\right)$ + [illegible] $+ p\frac{\partial u}{\partial x} = \Phi + \kappa\nabla^2\theta$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + k p\frac{\partial u}{\partial x} = (k-1)\Phi + (k-1)\kappa\frac{\partial^2\theta}{\partial x^2}$$

$$\cancel{\frac{\partial^2 p}{\partial t^2} = R\frac{\partial^2}{\partial t^2}(\rho\theta) = R\left[\rho\frac{\partial^2\theta}{\partial t^2} + 2\frac{\partial\rho}{\partial t}\frac{\partial\theta}{\partial t} + \theta\frac{\partial^2\rho}{\partial t^2}\right]}$$

Zaniedbując tarcie i wyższe potęgi:

$$\frac{\partial^2\rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[R\rho\theta\right]$$

$$\frac{\partial\rho}{\partial t} + \rho_0\frac{\partial u}{\partial x} + \cancel{u\frac{\partial\rho}{\partial x}} = 0$$

$$\frac{R}{k-1}\,\rho\left(\frac{\partial\theta}{\partial t} + \cancel{u\frac{\partial\theta}{\partial x}}\right) + \overset{R\rho\theta}{\downarrow}\frac{\partial u}{\partial x} = \kappa\frac{\partial^2\theta}{\partial x^2}$$

$$\frac{R}{k-1}\,\rho_0\frac{\partial\theta}{\partial t} - R\theta_0\frac{\partial\rho}{\partial t} = \kappa\frac{\partial^2\theta}{\partial x^2}$$

Przybliżone obliczenie:

supponując $u = a\sin(\alpha x - \beta t)$

$$\rho = -\rho_0\frac{\alpha}{\beta}\sin(\alpha x - \beta t)$$

W jednowymiarowym zadaniu: wpływ κ

$$\begin{cases} \dfrac{dp}{dx} = \dfrac{4}{3}\mu\dfrac{d^2u}{dx^2} \qquad \rho u = \rho_0 u_0 \\ u\dfrac{dp}{dx} + k\,p\dfrac{du}{dx} = (k-1)\dfrac{4}{3}\mu\left(\dfrac{du}{dx}\right)^2 + (k-1)\kappa\dfrac{d^2\theta}{dx^2} \end{cases}$$

$$p = p_0 + \frac{4}{3}\mu\frac{du}{dx}$$

$$\theta = \frac{pu}{R\rho_0u_0}$$

$$\frac{d\theta}{dx} = \frac{1}{R\rho_0u_0}\left[p_0\frac{du}{dx} + \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 + \frac{4}{3}\mu u\frac{d^2u}{dx^2}\right]$$

$$\frac{4}{3}\mu u\frac{d^2u}{dx^2} + k p_0\frac{du}{dx} + \cancel{\frac{4}{3}\mu k\left(\frac{du}{dx}\right)^2} = \cancel{\frac{4}{3}\mu k\left(\frac{du}{dx}\right)^2} - \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 + (k-1)\kappa\frac{d^2\theta}{dx^2}$$

$$\frac{4}{3}\mu\left[u\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2\right] + k p_0\frac{du}{dx} = (k-1)\kappa\frac{d^2\theta}{dx^2}$$

$$\frac{4}{3}\mu u\frac{du}{dx} + k p_0 u = (k-1)\kappa\frac{d\theta}{dx} + (k-1)a$$

$$= (k-1)a + \frac{(k-1)\kappa}{R\rho_0u_0}\left[p_0\frac{du}{dx} + \frac{4}{3}\mu\frac{d}{dx}\left(u\frac{du}{dx}\right)\right]$$

~~$\frac{(k-1)\kappa}{R\rho_0 u_0}\left[\frac{4}{3}\mu\frac{d}{dx}\left(u\frac{du}{dx}\right)+p_0\frac{du}{dx}\right]-\frac{4}{3}\mu u\frac{du}{dx}=k p_0 u-a(k-1)$~~

Z przybliżeniem podstawiając jako pierwsze przybliżenie:

$$R\frac{d\theta}{dx}=-\frac{(k-1)p_0}{\rho_0 u_0}\frac{du}{dx}$$

$$\frac{\frac{4}{3}\mu u-\frac{(k-1)\kappa}{R\rho_0 u_0}p_0}{k p_0 u-a(k-1)}\,du=dx$$

$$\int\frac{\frac{4}{3}\mu u-\frac{(k-1)\kappa}{u_0\theta_0}}{u-\alpha}\,du=k p_0 x-b$$

$$=\frac{4}{3}\mu+\frac{\frac{4}{3}\mu\alpha-\frac{(k-1)\kappa}{u_0\theta_0}}{u-\alpha}$$

$$\frac{4}{3}\mu(u-\alpha)+\left[\frac{4}{3}\mu\alpha-\frac{(k-1)\kappa}{u_0\theta_0}\right]\log(u-\alpha)=k p_0 x-b$$

$$\frac{\partial^2 \rho}{\partial t^2} = R\left[\rho_0 \frac{\partial^2 \theta}{\partial x^2} + \cancel{2\frac{\partial \rho}{\partial x}\frac{\partial \theta}{\partial x}} + \theta_0 \frac{\partial^2 \rho}{\partial x^2}\right]$$

$$\rho_0 \frac{\partial \theta}{\partial t} - (k-1)\theta_0 \frac{\partial \rho}{\partial t} = \frac{(k-1)\kappa}{R}\frac{\partial^2 \theta}{\partial x^2}$$

$$\rho_0 \frac{\partial^3 \theta}{\partial t \partial x^2} = (k-1)\theta_0 \frac{\partial^3 \rho}{\partial t \partial x^2} + (k-1)\frac{\kappa}{R}\frac{\partial^3 \theta}{\partial x^3}$$

$$\frac{\partial^3 \rho}{\partial t^3} = R\theta_0 \frac{\partial^3 \rho}{\partial t \partial x^2} + \kappa(k-1)\frac{\partial^3 \theta}{\partial x^3}$$

↓

w przybliżeniu

$$p \sim \rho^{k} \sim \rho\,\theta$$

$$\theta \sim \rho^{k-1}$$

$$\lg \theta = c + (k-1)\lg \rho$$

$$\frac{1}{\theta_0}\frac{\partial^3 \theta}{\partial x^3} = (k-1)\frac{\partial^3 \rho}{\partial x^3}\frac{1}{\rho_0}$$

$$\frac{\partial^3 \rho}{\partial t^3} = R\theta_0 k \frac{\partial^3 \rho}{\partial t \partial x^2} + \frac{\kappa(k-1)^2\theta_0}{\rho_0}\frac{\partial^3 \rho}{\partial x^3}$$

$$\rho = a \sin(\alpha x - \beta t)$$

$$+\beta^3 = R\theta_0 k\, \alpha^2 \beta - \frac{\kappa(k-1)^2\theta_0}{\rho_0}\alpha^3$$

$$\frac{\beta}{\alpha} = a$$

$$a^3 = R\theta_0 k\, a - \frac{\kappa(k-1)^2\theta_0}{\rho_0}$$

$$a = \sqrt{kR\theta_0} \text{ jako I przybliżenie}$$

$$\left[a^2 - R\theta_0 k\right] = -\frac{\kappa(k-1)^2\theta_0}{\rho_0\sqrt{kR\theta_0}}$$

$$a = \sqrt{kR\theta_0 - \frac{\kappa(k-1)^2\theta_0}{\rho_0 a_0}}$$

$$a = \sqrt{a_0^2 - \frac{\kappa(k-1)^2\theta_0}{\rho_0 a_0}}$$

$$= a_0\left[1 - \frac{1}{2}\frac{\kappa(k-1)^2\theta_0}{\rho_0 a_0^3}\right]$$

$$\frac{1}{2}\,\frac{(0{\cdot}4)^2 \;\; 300 \,.\, 0{\cdot}000\ldots}{0{\cdot}0013 \,.\, (33000)^3} \quad !$$

$$-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2$$

$$-u\frac{\partial}{\partial x}\left[-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial u}{\partial x}\right]-v\frac{\partial}{\partial y}\left[-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial v}{\partial y}\right]$$

$$-u\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]\qquad -v\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]$$

$$k\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\frac{2}{3}\left[u\frac{\partial}{\partial x}\mu\,\mathrm{div}+v\frac{\partial}{\partial y}\mu\,\mathrm{div}-\mu[\mathrm{div}]^2\right]$$

$$-2\left[u\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+v\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)-\mu\left(\frac{\partial u}{\partial x}\right)^2-\mu\left(\frac{\partial v}{\partial y}\right)^2\right]$$

$$-\left[u\frac{\partial}{\partial y}\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+v\frac{\partial}{\partial x}\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)-\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$$

$$=\frac{2}{3}\left[u^2\left\{\frac{1}{u}\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]-\frac{1}{u^2}\frac{\partial u}{\partial x}\cdot\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right\}+v^2\left\{\frac{1}{v}\frac{\partial}{\partial y}\left[\mu(\quad)\right]-\frac{1}{v^2}\frac{\partial v}{\partial y}(\quad)\right\}\right.$$

$+ - .$

$$=\frac{2}{3}\left[u^2\frac{\partial}{\partial x}\frac{\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}{u}+v^2\frac{\partial}{\partial y}\frac{\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}{v}\right]$$

$$-2\left[u^2\frac{\partial}{\partial x}\left(\frac{\mu\frac{\partial u}{\partial x}}{u}\right)+v^2\frac{\partial}{\partial y}\left(\frac{\mu\frac{\partial v}{\partial y}}{v}\right)\right]-\left[u^2\frac{\partial}{\partial y}\frac{\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)}{u}+v^2\frac{\partial}{\partial x}\frac{\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)}{v}\right]$$

$$(k-1)\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

Przybliżone rozwiązanie podstawiając: $p = \sqrt{cx+a} + \varphi$

$$u = \frac{y(y-\delta)}{2\mu}\frac{dp}{dx} + \psi$$

$$\begin{cases} \dfrac{\partial p}{\partial x} = \mu \dfrac{\partial^2 u}{\partial y^2} \\[2ex] \dfrac{\partial p}{\partial y} = \dfrac{\mu}{3} \dfrac{\partial^2 u}{\partial x \partial y} \end{cases}$$

$$\frac{\partial(\rho w)}{\partial x} = 0$$

$$u\frac{\partial p}{\partial x} + kp\frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial y}\right)^2 + \kappa(k-1)\frac{\partial^2\theta}{\partial y^2}$$

$$u = \frac{y(y-\delta)}{4\mu}\cdot\frac{-c}{\sqrt{cx+a}} + \psi$$

~~$\dfrac{\partial u}{\partial y} = \dfrac{2y-\delta}{4\mu}\dfrac{c}{\sqrt{(cx+a)}} + \dfrac{\partial \psi}{\partial y}$~~

$$\frac{\partial^2 u}{\partial y^2} = \frac{c}{2\mu\sqrt{\ }} + \frac{\partial^2\psi}{\partial y^2}$$

$$\frac{\partial u}{\partial x} = \frac{y(y-\delta)\ c^2}{8\mu\sqrt{(cx+a)}^3} + \frac{\partial\psi}{\partial x}$$

~~$\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{-(2y-\delta)\,c^2}{8\mu\sqrt{(\ \ )}^3} + \dfrac{\partial^2\psi}{\partial x\partial y}$~~

~~$\dfrac{c}{2\sqrt{cx+a}} + \dfrac{\partial\varphi}{\partial x} = \mu\dfrac{c}{2\mu\sqrt{\ }} + \mu\dfrac{\partial^2\psi}{\partial y^2}$~~

~~$\dfrac{\partial\varphi}{\partial y} = \dfrac{\mu}{3}\left[-\dfrac{(2y-\delta)\,c^2}{8\mu\sqrt{(cx+a)}^3} + \dfrac{\partial^2\psi}{\partial x\partial y}\right]$~~

} Niemożliwe!

$$\kappa(k-1)\frac{\partial^2\theta}{\partial y^2} = \frac{y(y-\delta)}{4\mu}\frac{-c}{\sqrt{\ }}\ \frac{-c}{2\sqrt{\ }} + k\sqrt{\ }\ \frac{y(y-\delta)\ c^2}{8\mu\sqrt{(\ \ )}^3}$$

$$-(k-1)\mu\ \frac{(2y-\delta)^2}{16\mu^2}\ \frac{c^2}{(cx+a)}$$

$$\kappa\frac{\partial^2\theta}{\partial y^2} = \frac{-2y(y-\delta) - (2y-\delta)^2}{16\mu}\ \frac{c^2}{(a-cx)} = \frac{-\delta^2 + 4y\delta - 4y^2 + 2y\delta - 2y^2}{-\ \ -}$$

$$= -\frac{(\delta^2 - 6y\delta + 6y^2)}{16\mu}\ \frac{c^2}{a-cx}$$

$y=0$ $\quad \delta^2$

N.p. $y = \frac{\delta}{2}$ $\quad \delta^2 - 3\delta^2 + \frac{6}{4} = -\frac{\delta^2}{2}$

$$y^2 - y\delta + \frac{\delta^2}{6} = 0$$

$$y = \frac{\delta}{2} \pm \sqrt{\frac{\delta^2}{4} - \frac{\delta^2}{6}} = \frac{\delta}{2} \pm \frac{\delta}{\sqrt{12}}$$

$y = \frac{\delta}{2}$ $\frac{d\theta}{dy} = 0$ $\frac{1}{2} - \frac{3}{4} + \frac{1}{4}$

$$k\frac{d\theta}{dy} = -\frac{(\delta^2 y - 3\delta y^2 + 2y^3)}{16\mu}\frac{c^2}{a-cx}$$

$$k\theta = -\frac{\left(\frac{\delta^2 y^2}{2} - 2\delta y^3 + \frac{y^4}{2}\right)}{32\mu}\frac{c^2}{a-cx} + b$$

$$\theta = \theta_0 - \frac{y^2(y-\delta)^2}{32\mu k}\frac{c^2}{a-cx}$$

Maximum $y = \frac{\delta}{2}$ $\theta = \theta_0 - \frac{\delta^4}{128\mu k}\frac{c^2}{a-cx}$

$$= \theta_0 - \frac{\delta^4}{128\mu k}\frac{c^2}{p^2}$$

$\frac{1}{3}$

10^2

$$\frac{\cancel{10}^8\cdot\cancel{9}\cdot\frac{1}{3}\cdot\cancel{10}^{12}}{16\cdot 128\cdot\cancel{10}^8\cdot\cancel{10}^2\cdot 42\cdot\cancel{10}^6}$$

$\frac{1}{8}$

Podobnie przy symetryi walcowatej:

$u = \frac{r^2-\delta^2}{8\mu}\frac{-c}{\sqrt{a-cx}}$ $\Big\|$ $p = \sqrt{a-cx}$

$$\frac{dp}{dx} = \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

$$u\frac{dp}{dx} + kp\frac{\partial u}{\partial x} = (k-1)\mu\left(\frac{\partial u}{\partial r}\right)^2 + (k-1)\kappa\cdot\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

$\frac{dp}{dx} = \frac{-c}{2\sqrt{\ }}$ $\frac{\partial u}{\partial x} = \frac{r^2-\delta^2}{8\mu}\frac{-\frac{c^2}{2}}{\sqrt{(\ )^3}}$ $\frac{\partial u}{\partial r} = \frac{r}{4\mu}\frac{-c}{\sqrt{\ }}$

$$\frac{c^2}{2}\frac{r^2-\delta^2}{8\mu}\frac{1}{(\ )} - k\frac{r^2-\delta^2}{8\mu}\frac{\frac{c^2}{2}}{(\ )} - (k-1)\mu\frac{r^2}{16\mu^2}\frac{c^2}{(\ )} = (k-1)\cdot\kappa\,\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

~~$r\frac{\partial\theta}{\partial r} = A - \frac{r^3}{\ } + \frac{c}{r}$~~

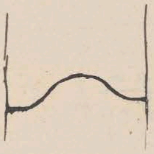
~~$\theta = -\frac{r^4}{4} + m = \frac{\delta^4 - r^4}{4}$~~

~~$0 = -\frac{\delta^4}{4} + -$~~

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \frac{1}{k\mu}\frac{-(r^2-\delta^2)c^2 - r^2c^2}{16(a-cx)} = \frac{c^2}{16k\mu}\;\frac{\delta^2-2r^2}{a-cx}$$

$$\frac{\partial}{\partial r}(\;) = \frac{c^2}{16k\mu}\;\frac{\delta^2 r - 2r^3}{a-cx}$$

$$r\frac{\partial\theta}{\partial r} = \frac{c^2}{16k\mu}\;\frac{\frac{\delta^2 r^2}{2} - \frac{r^4}{2}}{a-cx} + b$$



$$\frac{\partial\theta}{\partial r} = \frac{c^2}{32k\mu}\;\frac{r\delta^2 - r^3}{a-cx} + \frac{b}{r}$$

$$\begin{cases}\theta = \dfrac{c^2}{32k\mu}\;\dfrac{\frac{r^2\delta^2}{2} - \frac{r^4}{4}}{a-cx} + \cancel{b\lg r} + m \\[2ex] \theta_0 = \qquad\qquad \dfrac{\frac{\delta^2}{4}}{a-cx} \qquad + n\end{cases}$$

$$\theta = \theta_0 - \frac{c^2}{128\,k\mu}\;\frac{(\delta^2-r^2)^2}{a-cx} = \theta_0 - \frac{(\delta^2-r^2)^2}{128.\,k\mu}\left[\frac{p_1^2-p_2^2}{l\,p}\right]^2$$

$$\boxed{\theta_0 - \frac{u^2}{2k}}$$

$$\left.\begin{array}{l} p_1^2 = a \\ p_2^2 = a - cl \end{array}\right\}\quad \frac{p_1^2-p_2^2}{l} = c \qquad\qquad r = \frac{\delta}{2}$$

Minimum: $\theta = \theta_0 - \frac{\delta^4}{k\mu}\frac{9}{16.128}\left[\frac{p_1^2-p_2^2}{l\,p}\right]^2$

$$\frac{1}{\cancel{200}}\;\frac{\overset{100}{\cancel{10}}.\,10^{-2}\,\frac{1}{9}.\,10^{12}}{10^{8}.\,42.\,10^{6}} \qquad \frac{5}{16}\,\frac{9}{9}\,.10^{6} \qquad 2.1$$

N. p.
$\delta = 0{\cdot}01$
$p_1 = 10^6 \;\|\; p_2 =< \frac{3}{4}.10^6$
$l = 10$
$k = 0{\cdot}00006$
$\mu = 0{\cdot}00017$

$$\frac{1}{240}\left[\frac{1}{2}\,\frac{10^6}{10}\right]^2.\;\frac{10^{-8}}{1{\cdot}7.\;6.\;10^{-9}.\,42.\,10^6} \qquad \frac{100}{9.\,2.\,42}$$

$$= \frac{10^5}{240.\,4.\,10.\,42} = \frac{10^3}{96.\,42}$$

$$= \frac{10}{40} = \frac{1}{4}\,{}^\circ C$$

Tymczasem robiąc n.p. $\left.\begin{array}{l}\delta = 0{\cdot}02\\ l = 5\,cm\\ p_2 = \frac{1}{2}p_1\end{array}\right\}$ będziemy mieli

$$\Delta\theta = \frac{1}{4}.\,16.\,4.\,3 = 48^\circ\;!!$$

W ogólnym przypadku gdy zboczenia od stanu izotermicznego tylko małe:

$$\theta = \theta_0 + \varphi$$

$$p = R\rho(\theta_0+\varphi) \qquad \rho = \frac{p}{R(\theta_0+\varphi)}$$

$$\frac{\partial}{\partial x}\left(\frac{pu}{\theta_0+\varphi}\right) + \frac{\partial}{\partial y}\left(\frac{pv}{\theta_0+\varphi}\right) = 0 \qquad pu\cdot\left(1-\frac{\varphi}{\theta_0}\right)$$

~~$$\frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) + \frac{1}{\theta_0}\left[\frac{\partial(\varphi u)}{\partial x} + \frac{\partial(\varphi v)}{\partial y}\right] = 0$$~~

$$\frac{\partial}{\partial x}\left[pu\cdot\left(1-\frac{\varphi}{\theta_0}\right)\right]$$

~~II). $\frac{\partial}{\partial x}(pu) + \frac{\partial}{\partial y}(pv) - \frac{p\left[u\frac{\partial\varphi}{\partial x} + v\frac{\partial\varphi}{\partial y}\right]}{\theta_0-\varphi} = 0$~~

~~III). $u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + k\,p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = (k-1)\,\Phi + (k-1)k\,\Delta^2\varphi$~~

Błąd w owym rozwiązaniu jest zatem tylko rzędu $p\,u\,\frac{\frac{\partial\varphi}{\partial x}}{\theta_0}$

$$\theta = \theta_0 + \varphi$$

$$p = R\rho(\theta_0+\varphi) = \pi + \psi$$

Przy tem ~~[illegible]~~ $\frac{\partial\theta}{\partial z} = 0$ dla powierzchni zatem ~~ścianki~~ ściany nie udzielają ciepła [więc to jest faktycznie przybliżone rozwiązanie zadania -- $= \Phi + k\Delta^2\theta$ ale nie $= \Phi$ (bo wtedy musiałoby $\frac{\partial u}{\partial z} = 0$ być na powierzchni)!]

Należy zatem też ~~[illegible]~~ spodziewać się że temp. przeciętna nie zostanie zmniejszona, po wypływie, mimo że ściany nie są adiabatyczne.

byłoby rozwiązaniem ścisłem gdyby $k = \infty$.

II przybliżenie:

$$\frac{\partial}{\partial x}(pu)+\frac{\partial}{\partial y}(pv)-\underbrace{R\rho\left(u\frac{\partial\varphi}{\partial x}+v\frac{\partial\varphi}{\partial y}\right)}_{=R\rho\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)}=0$$

~~Jeżeli przez~~ Czy możliwe $\theta=\text{const}$ ‖ jeżeli $k=0$

$$\left.\begin{aligned}\frac{\partial}{\partial x}(pu)+\frac{\partial}{\partial y}(pv)=0 &= u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\\ p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) &= -\frac{\mu}{3}\left[u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\quad\right)\right]\\ &\quad-\mu\left[u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\right]\end{aligned}\right\}\ I+II$$

Z drugiej strony

$$p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\Phi$$

zatem chyba musiałoby być:

$$\star\ \frac{1}{3}\left[u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]+u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\ \star$$

$$-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2$$

Najprostszy dowód że ogólnie niemożliwe przykłady jednowymiarowe!

Podobieństwo dynamiczne

$$\frac{\partial p}{\partial x}=\frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=(k-1)\mu\left[\left(\frac{\partial u}{\partial x}\right)^2+\dots\right]+k\left[\frac{\partial^2\theta}{\partial x^2}+\dots\right]$$

Mając rozwiązanie $p = f(xyz)$

$\theta = \varphi(xyz)$ w danym naczyniu (o ścianach nieruchomych)

$\left.\begin{matrix} u \\ v \\ w \end{matrix}\right\} = \psi(xyz)$

I).

Otrzymam inne rozwiązanie dla naczynia o rozmiarach n razy większych podstawiając wszędzie nx, ny, nz .. na miejsce xyz

oraz $p = \frac{1}{n} f$

II). $\left.\begin{matrix} u \\ v \\ w \\ p \end{matrix}\right\}$ powiększyć n razy k n^2 razy   rozmiary niezmienione

---

III). rozmiary naczynia n razy większe

podstawić $nx\ y\ z$

k, μ ... n razy

IV). rozmiary naczynia n razy $\left\{\begin{matrix} x\ y\ z \\ u\ v\ w \end{matrix}\right.$ n . p niezmien.

k (n^2) razy

Ogólnie: robiąc podstawienia: $\begin{matrix} n\,x \\ n\,y \\ n\,z \end{matrix}$ zamiast $\begin{matrix} x \\ y \\ z \end{matrix}$

$\begin{matrix} m\,u \\ m\,v \\ m\,w \end{matrix}$ — — .. $\begin{matrix} u \\ v \\ w \end{matrix}$

$\alpha\mu$ .. — μ

$\beta . k$ — k

$b\,p$ .. — p

$n\,\theta$ .. θ

$$\frac{p}{R\theta}\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]+\frac{\partial p}{\partial x}=\frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$ I $= X\frac{p}{R\theta}$

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$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

III

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+k p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=(k-1)\mu\,\dot{\Phi}+\kappa\nabla^2\theta$$

I). $\frac{b}{r}\frac{m^2}{n}\equiv\frac{b}{n}\equiv\alpha\frac{m}{n^2}\quad=\frac{b}{r}$

III). $m\frac{b}{n}\equiv\alpha\frac{m^2}{n^2}\equiv\beta\frac{r}{n^2}$

~~I). $b\left(\frac{m^2}{r}+1\right)\equiv\alpha\frac{m}{n}$~~

~~III). $bm\equiv\alpha\frac{m^2}{n}+\beta\frac{r}{n}$~~

I). Niezmieniające $\alpha=1\quad\beta=1$

$$b\left(\frac{m^2}{r}+1\right)\equiv\frac{m}{n}$$

$$bm\equiv\frac{m^2}{n}+\frac{r}{n}$$

1). niezmieniające $r=1$: $\left.\begin{aligned}b(m^2+1)&\equiv\frac{m}{n}\\ b&\equiv\frac{m}{n}+\frac{1}{nm}\end{aligned}\right\}$ ~~$bm^2\neq\frac{1}{nm}$~~

$$(m^2+1)\left(m+\frac{1}{m}\right)\equiv m\qquad(m^2+1)^2\equiv m$$

Niemożliwe

$$\frac{m}{n}+\frac{r}{nm}\left(\frac{m^2}{r}+1\right)\equiv\frac{m}{n}$$

$$\cancel{m^2}(m^2+r)\equiv m^2$$

Należy zatem $r=-m^2$

Niemożliwe!

Zaniedbując przedtem energię kinet:

~~I.~~ $b = \alpha \frac{m}{n}$

III). $b \frac{m}{n} = \alpha \frac{m^2}{n^2} + \beta \frac{r}{n^2}$

$\alpha \frac{m^2}{n^2} = \cancel{\alpha \frac{m^2}{n^2}} + \beta \frac{r}{n^2}$ ~~Niemożliwe!~~

II. $\frac{b}{r} m^2 = b = \alpha \frac{m}{n}$

III). $mb = \alpha \frac{\cancel{m^2}}{n} = \beta \frac{r}{n}$

3 równania między 6 wielkościami

3 dowolne

I).

$m^2 = r$

$\alpha \frac{m^2}{n} = \beta \frac{r}{n}$

$\alpha m^2 = \beta r$

$\alpha \cancel{r} = \beta \cancel{r}$

$\underline{\underline{\alpha = \beta}}$

zatem pozostają tylko 2 dowolne

$$\left.\begin{aligned} b &= \alpha \frac{m}{n} = b \frac{m^2}{r} \\ mb &= \alpha \frac{r}{n} \end{aligned}\right\} \begin{aligned} m^2 &= r \\ b &= \alpha \frac{m}{n} \end{aligned}$$

przy takiem założeniu

pozostają zatem 2 równania:

$\alpha = \beta$

$$b = \alpha \frac{m}{n} = b \frac{m^2}{r} \Big\} \underline{\underline{\left\{\begin{aligned} b &= \alpha \frac{m}{n} \\ r &= m^2 \end{aligned}\right\}}}$$

$b = \alpha \frac{m}{n}$ $r = m^2$ 5 wielkości z których 3 dowolne

Przykład: ochłodzenie kuli termometr. obustronne

Uzupełnić co do ilości ciepła przenoszonego

1). Niezmieniające $\left\{ \begin{array}{ll} \kappa\ \mu : & \alpha = 1 \\ \theta : & r = 1 \end{array} \right.$

$m = 1$ $b = \frac{1}{n}$

Tj. Rozmiary n razy powiększyć, ochłodzenie $\frac{1}{n}$ zmniejszyć zostaje niezmieniona

Wystyga prędzej [illegible]; masa przestyg. niezmien.

2). Niezmieniające $\kappa\ \mu$: $\alpha = 1$

rozmiarów : $n = 1$

$b = m$ $r = m^2$

to samo także dla zadania z czasem t zmiennym!

Powiększając prędkości m razy: należy ciśnienie n razy, temp. m^2 razy powiększ.

Meteor lecący przez atmosferę: jeśli z prędkością m razy większą leci przez atm. n razy gęstszą ogrz. będzie n razy [tylko, nie m^2!] większy, a temp. ustalona m^2 wyższa.

Uwzględniając zależności $\kappa\ \mu$ od temp. i kładąc je prop. θ:

I). $\frac{b}{r} m^2 = b = r \frac{m}{n}$

II). $m b = \frac{r^2}{n}$

$r = m^2$ $b = \frac{m^3}{n}$

$\frac{m^4}{n} = \frac{m^4}{n}$ same przez się spełnione

Pozostaje warunek: $r = m^2$ $b = \frac{m^3}{n}$

1). Pozostają takie same

2). $n = 1$ $r = m^2$ $b = m^3$

III). $\alpha = 1$ ~~[illegible]~~ $m = \frac{1}{n}$

$b = \frac{1}{n^2}$

$r = \frac{1}{n^2}$

Więc ~~[illegible]~~ powiększając rozmiary n razy i zmniejszając prędkości $\frac{1}{n}$ należy ciśnienie i temp. zmniejszyć $\frac{1}{n^2}$ (porówn. Helmholtz)

IV. $\alpha = 1$ $m = n$

$b = 1$ $r = n^2$

[O jeden stopień większa swoboda przy tych analogiach niż przy innych]

~~$r^2\varphi''\varphi + 2r\varphi\varphi' - r^2\varphi'^2 + k(2\varphi + r\varphi')(p_0 + 2\varphi + r\varphi') = 0$~~

$$r^2\varphi''\varphi + 3r\varphi\varphi' + \varphi^2 + r\varphi\varphi' + r^2\varphi'^2 + k\left[(2\varphi + r\varphi')(p_0 + 2\varphi + r\varphi') - \varphi^2 - r\varphi\varphi' - r^2\varphi'^2\right] = 0$$

$$r^2(\varphi''\varphi + \varphi'^2) + \varphi^2 + 4r\varphi\varphi'$$

$$\left.\underbrace{r^2 d(\varphi\varphi') + 2r\varphi\varphi'}_{d(r^2\varphi\varphi')} + d(r\varphi^2)\right\} = \cancel{d[r\varphi(\varphi + r\varphi')]} = \frac{1}{2} d[2r\varphi^2 + 2r^2\varphi\varphi'] = \frac{1}{2} d^2(r^2\varphi^2)$$

$$r^3\varphi''\varphi + 4r^2\varphi\varphi' + r\varphi^2 + r^3\varphi'^2$$

~~$3r\varphi(r\varphi' + \varphi)$~~

$$\underbrace{r^3 d(\varphi\varphi') + 3r^2\varphi\varphi'}_{d(r^3\varphi\varphi')} + \frac{1}{2} d(r^2\varphi^2)$$

$$+ \frac{1}{2} d(r^2\varphi^2) + k\left[p_0\, d(r^2\varphi) + \underbrace{3\varphi^2 r + 3\varphi\varphi' r^2}_{\frac{3}{2} d(r^2\varphi^2)}\right] = 0$$

$$r^3\varphi\varphi' + \frac{r^2\varphi^2}{2} + k p_0 r^2\varphi + \frac{3}{2} k r^2\varphi^2 = a$$

$$r^3\varphi\varphi' + \frac{r^2\varphi^2}{2}(1 + 3k) + k p_0 r^2\varphi = a$$

Wypływ kołowy dwuwymiarowy:

$$u = z\frac{x}{r} \;\Big\|\; v = z\frac{y}{r}$$
$$= \varphi x \qquad\quad = \varphi y$$

$$\frac{\partial u}{\partial x} = \varphi + \varphi'\frac{x^2}{r} \qquad\qquad \frac{\partial u}{\partial y} = \varphi'\frac{xy}{r}$$

$$\frac{\partial^2 u}{\partial x^2} = \varphi'\frac{3x}{r} - \varphi'\frac{x^3}{r^3} + \varphi''\frac{x^3}{r^2} \qquad\qquad \frac{\partial^2 u}{\partial y^2} = \varphi'\frac{x}{r} - \varphi'\frac{xy^2}{r^3} + \varphi''\frac{xy^2}{r^2}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \varphi'\frac{3x}{r} + \varphi''\cdot x$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 2\varphi + \varphi'\cdot r$$

$$\frac{\partial}{\partial x} = 3\varphi'\cdot\frac{x}{r} + \varphi''\cdot x =$$

$$\frac{dp}{dr}\cdot\frac{x}{r} = \frac{4\mu}{3}\left(\varphi'' x + 3\varphi'\cdot\frac{x}{r}\right)$$

$$\text{I).}\quad \frac{dp}{dr} = \frac{4\mu}{3}\left(\varphi''\cdot r + 3\varphi'\right)$$

$$\Phi = -\frac{2}{3}(2\varphi + r\varphi')^2 + 4\varphi^2 + 2\varphi'^2\frac{x^4+y^4}{r^2}$$
$$+ 4\varphi\varphi' r + 4\varphi'^2\frac{x^2y^2}{r^2}$$
$$= -\frac{2}{3}(2\varphi + r\varphi')^2 + 4\varphi^2 + 4\varphi\varphi' r + 2r^2\varphi'^2$$

$$\text{III).}\quad \cancel{\varphi x\cdot\frac{x}{r}}\ \varphi r\,\frac{4\mu}{3}(\varphi'' r + 3\varphi') + k\,p\,(2\varphi + r\varphi') =$$
$$= \overset{(k-1)}{\frac{4}{3}}\varphi^2 + \frac{4}{3}\,r\varphi\varphi' + \frac{4}{3}\,r^2\varphi'^2 = \overset{(k-1)}{\frac{4}{3}}\left[\varphi^2 + r\varphi\varphi' + r^2\varphi'^2\right]$$

$$\text{I).}\quad \frac{dp}{dr} = \frac{4\mu}{3}\,\frac{d}{dr}\left[\varphi' r + 2\varphi\right]$$

$$p = \frac{4\mu}{3}\left[\varphi' r + 2\varphi\right] + p_0\,\frac{4\mu}{3}$$

$$r^3\varphi\varphi' + \frac{r^2\varphi^2}{2}(1+3k) + k\rho_0 r^2\varphi = a$$

$r^2\varphi = z$

$$z\left(r\frac{dz}{dr} - z\right) + \frac{z^2}{2}(1+3k) + k\rho_0 rz = a$$

$r\,d\varphi + \varphi\,dr = dz$

$r\varphi' = \frac{dz}{dr} - \frac{z}{r}$

$$\cancel{z\frac{dz}{dr} + \frac{z^2}{2}\left(\frac{3k-1}{2}\right)}$$

$$z\frac{dz}{dr} + \frac{z^2}{r}\,\frac{3k-1}{2} + k\rho_0 z = \frac{a}{r}$$

$$\left.\begin{aligned} r^2\varphi &= z \\ \varphi &= \frac{z}{r^2} \\ \varphi' &= \frac{1}{r^2}\frac{dz}{dr} - \frac{2z}{r^3} \end{aligned}\right\}$$

$$rz\left(\frac{1}{r^2}\frac{dz}{dr} - \frac{2z}{r^3}\right) + \frac{z^2}{r^2}\,\frac{1+3k}{2} + k\rho_0 z = a$$

$$z\frac{dz}{dr} - \frac{2z^2}{r^2} + \frac{z^2}{r^2}\left(\frac{1+3k}{2}\right) + k\rho_0 z = \alpha$$

$$z\frac{dz}{dr} + \frac{z^2}{r^2}\,\frac{3(k-1)}{2} + k\rho_0 z = a$$

$r\varphi^2 = z$

$\varphi\varphi' = \frac{1}{2r}\frac{dz}{dr} - \frac{z}{2r^2}$

$$\frac{1}{2}\left(r^2\frac{dz}{dr} - rz\right) + rz\left(\frac{1+3k}{2}\right) + k\rho_0 r\sqrt{rz} = a$$

$\cancel{rz = y^2}$

$\cancel{z + r\frac{dz}{dr} = 2y\frac{dy}{dr}}$

$$\cancel{yr\frac{dy}{dr} - y^2 + y^2\,\frac{1+3k}{2} + k\rho_0 r\,yr = a}$$

$$\cancel{y\frac{dy}{dr} + \frac{y^2}{r}\,\frac{3k-1}{2} + k\rho_0 y = \frac{a}{r}}$$

$$\cancel{\varphi\frac{d\varphi}{dr}} \quad z\frac{dz}{dr} + \frac{z^2}{r}\,\frac{3k+1}{2} + k\rho_0\frac{z}{r} = \frac{a}{r^3}$$

$$z \frac{dz}{d\varphi} \frac{d\varphi}{dr} + z\varphi \frac{3k-1}{2} + k p_0 z = \frac{a\varphi}{z}$$

~~$\varphi = \frac{r}{z} \quad \varphi = \frac{1}{z}\varphi$~~

$$\frac{d\varphi}{dr} = \frac{\varphi^2}{\varphi \frac{dz}{d\varphi} - z}$$

$$r + \varphi \frac{dr}{d\varphi} = \frac{dz}{d\varphi}$$

$$\varphi^2 \frac{dr}{d\varphi} = \varphi \frac{dz}{d\varphi} - z$$

$$z\varphi^2 \frac{dz}{d\varphi} + \left(z\varphi \frac{3k-1}{2} + k p_0 z - \frac{a\varphi}{z}\right)\left(\varphi \frac{dz}{d\varphi} - z\right) = 0$$

$$r^2 \varphi = z \qquad \frac{dr}{d\varphi} = \frac{\varphi \frac{dz}{d\varphi} - z}{2\varphi\sqrt{\varphi z}}$$

$$\varphi r^2 + 2r \frac{dr}{d\varphi}\varphi^2 = \frac{dz}{d\varphi}\varphi$$

$$z \frac{dz}{d\varphi} 2\varphi\sqrt{\varphi z} + \left(\varphi \frac{dz}{d\varphi} - z\right)\left[z\varphi \frac{3(k-1)}{2} + k p_0 z - a\right] = 0$$

$$\searrow \quad z = u.v$$

$$uv\left(u \frac{dv}{dr} + v \frac{du}{dr}\right) + \frac{u^2 v^2}{r^2} \frac{3(k-1)}{2} + k p_0 u v = \frac{a}{r}$$

$$u^2\left[v \frac{dv}{dr} + \frac{v^2}{r^2} \frac{3(k-1)}{2}\right] + u\left[v^2 \frac{du}{dr} + k p_0 v\right] = \frac{a}{r}$$

$$\frac{du}{dr} = -\frac{k p_0}{v}$$

$$v = -\frac{k p_0}{\frac{du}{dr}}$$

$$+ u^2 k^2 p_0^2 \left[-\frac{1}{\left(\frac{du}{dr}\right)^3} \frac{d^2u}{dr^2}\right] + \frac{u^2\, 3(k-1)}{2r^2} \frac{k^2 p_0^2}{\left(\frac{du}{dr}\right)^2} = \frac{a}{r}$$

$$\frac{1}{v} \frac{dv}{dr} + \frac{3(k-1)}{2} \frac{1}{r} = 0$$

~~$v = A e^{\frac{3(k-1)}{r}}$~~

$$\log v = -\frac{3(k-1)}{2} \log r$$

$$v = A r^{-\frac{3k-1}{2}}$$

~~$u\left(\frac{du}{dr} + k p_0 \frac{1}{v}\right) = \frac{a}{v^2} r$~~

$$u \frac{du}{dr} + k p_0 u A r^{\frac{3k-1}{2}} = \frac{a}{A^2} r^{1-3k-1} = \frac{a}{A^2} r^{-3k}$$

~~$\log \frac{du}{dr} r$~~

$\varphi = z r \qquad \varphi' = z + r \frac{dz}{dr}$ 60 – ~~1000~~ 50

$$z r^4 \left(z + r \frac{dz}{dr}\right) + \frac{z^2 r^4}{} \; \frac{3k+1}{2} + k p_0 \, z r^3 = 0$$

3 k

$$r^5 \, z \frac{dz}{dr} + z^2 r^4 \, \frac{3k+1}{2} + z r^3 k p_0 = 0$$

III).

~~$R\theta = \frac{1}{\rho}$~~

~~$R \frac{\partial\theta}{\partial x} + \frac{\partial\theta}{\partial y} = \frac{1}{\rho}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) \; \left(\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\right)$~~

$$\iiint \Phi \, dv = \mu \iint \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 + \ldots \quad - \quad + \frac{1}{3}(\mathrm{div})^2 +$$

$$\mu \iint \Big\{ u \left[m \frac{\partial v}{\partial x} + n \frac{\partial v}{\partial x} \pm l \frac{\partial u}{\partial x} - l \frac{\partial v}{\partial y} - l \frac{\partial w}{\partial z} \right] + v \left[\qquad \right] + w \left[\qquad \right] \Big\} dS$$

$$\iint u \frac{\partial}{\partial x} v_n + v \frac{\partial}{\partial y} v_n + w \frac{\partial}{\partial z} v_n -$$

$$\iint - v_n \,\mathrm{div}\, v$$

$$\int \Phi = \iiint \left(\frac{\partial u}{\partial x}\right)^2 + \ldots - \frac{1}{3} \mathrm{div}^2 \; + \iint \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}\right) v_n - \iint v_n \,\mathrm{div}\, dS$$

$$\iiint \rho \left(u \frac{\partial\theta}{\partial x} + \ldots \right) dv = \iint \rho \,\theta (u l + v m + w n) \, dS$$

$$\iiint p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dv = \iint p (u l + v m + w n) - \underbrace{\iiint u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}}_{}$$

$$= \iint \frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right) dS + \frac{\mu}{3} \iint v_n \,\mathrm{div}\, dS - \mu \iiint \left(\frac{\partial u}{\partial x}\right)^2 \ldots \frac{\mathrm{div}^2}{3}$$

$M_0 \quad M_1$

$c_0 \quad c_1 = \frac{c_0}{\sqrt{M_1}}$

$$\left(\frac{c}{A}+R\right)\iint \rho\,\theta(ul+vm+wn)\,dS =$$

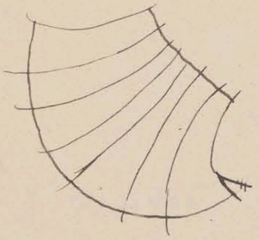
$$=\mu\iint\left[\frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right)+\frac{1}{3}v_n\,\mathrm{div}+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)v_n-v_n\,\mathrm{div}\right]dS+$$

$$=\mu\iint\left[\frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right)+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)v_n-\frac{2}{3}v_n\,\mathrm{div}\right]dS+$$

$$*-\iint\frac{u^2+v^2+w^2}{2}\,\rho\,(v_n)\,dS$$

$$R\left(1+\frac{1}{k-1}\right)\iint\ldots=\frac{Rk}{k-1}\iint\rho\,\theta\,v_n\,dS=\uparrow\qquad=\frac{k}{k-1}\iint p\,v_n\,dS$$

Jeżeli się to zastosuje do rurki prądu



$$\varrho_1\rho_1v_1=\varrho_2\rho_2v_2$$

$$\frac{kR}{k-1}\left(\rho_2\theta_2v_2\varrho_2-\rho_1\theta_1v_1\varrho_1\right)=-\frac{\rho_2v_2^3\varrho_2-\rho_1\varrho_1v_1^3}{2}+\mu\left\{\left[\left(\frac{\partial}{\partial s}\frac{v^2}{2}\right)_2\varrho_2-\left(\frac{\partial}{\partial s}\frac{v^2}{2}\right)_1\varrho_1\right]\right.$$

$$\left.+\left[v_2\frac{\partial v_2}{\partial s}\varrho_2-v_1\frac{\partial v_1}{\partial s}\varrho_1\right]-\frac{2}{3}\left[v_2\frac{\partial v_2}{\partial s}\varrho_2-v_1\frac{\partial v_1}{\partial s}\varrho_1\right]\right\}$$

$$\{\quad\}=\frac{4}{3}\left[v_2\frac{\partial v_2}{\partial s}\varrho_2-v_1\frac{\partial v_1}{\partial s}\varrho_1\right]+\iint\frac{\partial}{\partial n}\left(\frac{u^2+v^2+w^2}{2}\right)df$$

$$+\iiint\nabla^2\frac{u^2+v^2+w^2}{2}\,\underset{\overset{\shortparallel}{\varrho\,ds}}{dv}$$

$$\frac{kR}{k-1}\varrho_1\rho_1v_1\left(\theta_2-\theta_1\right)=\rho_1\varrho_1v_1\frac{v_2^2-v_1^2}{2}+\frac{4\mu}{3}\left[\frac{1}{\rho_2}\frac{\partial v_2}{\partial s}-\frac{1}{\rho_1}\frac{\partial v_1}{\partial s}\right]\rho_1\varrho_1v_1$$

$$\frac{kR}{k-1}\left(\theta_2-\theta_1\right)=-\frac{v_2^2-v_1^2}{2}+\frac{4\mu}{3}\left[\frac{1}{\rho_2}\frac{\partial v_2}{\partial s}-\frac{1}{\rho_1}\frac{\partial v_1}{\partial s}\right]+\int\nabla^2\left(\frac{u^2+v^2+w^2}{2}\right)\frac{ds}{\rho v}$$

$$\iint \frac{\partial}{\partial n}\frac{(u^2+v^2+w^2)}{2}\, dS = \iint S\left(\nabla\frac{(u^2+v^2+w^2)}{2} . N\right) dS =$$

$$\iiint \operatorname{div} \nabla \frac{u^2+v^2+w^2}{2}\, dv = \iiint \nabla^2 \frac{u^2+v^2+w^2}{2}\, dv = \iiint \nabla^2 (v^2)\, q\, ds$$

$u^2+v^2+w^2 = v^2$

$dv = q\, d\bar{s}$

$$= \iint \left[u \frac{\partial u}{\partial n} + v \frac{\partial v}{\partial n} + w \frac{\partial w}{\partial n}\right] dS = \iint S v$$

Dla warstwy przylegającej do ściany wielkości u v w będą rzędu

$\delta . \frac{\partial u}{\partial N}$ $\delta . \frac{\partial v}{\partial N}$ $\delta . \frac{\partial w}{\partial N}$

$\frac{\partial u}{\partial n} =$ ~~$\frac{\partial^2 u}{\partial N^2}$~~ $\dots$

$= \frac{\partial u}{\partial N} + \delta \frac{\partial^2 u}{\partial N^2}$

Zatem:

$\iint u \frac{\partial u}{\partial n}\, dS \dots\dots \delta \iint \left[\left(\frac{\partial u}{\partial N}\right)^2 + \left(\frac{\partial v}{\partial N}\right)^2 + \left(\frac{\partial w}{\partial N}\right)^2\right] ds . dt$

$v \iint \left[\left(\frac{\partial u}{\partial N}\right)^2 + - \right] ds . q$

$q_1 v_1 \rho_1 = q_2 v_2 \rho_2$

$\delta\, dt\, \delta \left(\frac{\partial u}{\partial N} \dots\right) \rho = \text{const}$

$$= \frac{\iint \left[\frac{\partial u}{\partial N} - \right] ds}{v \rho}$$

zatem dla nich sprowadzą $\iint$ nie zniknie

Jeżeli wszędzie $\nabla^2(u^2+v^2+w^2)=0$ wtedy względ. $q_1 \frac{\partial v^2}{\partial s}\Big|_1 = q_2 \frac{\partial v^2}{\partial s}\Big|_2$ zatem

$$\frac{kR}{k-1}(\vartheta_2 - \vartheta_1) = -\frac{v_2^2 - v_1^2}{2}$$

Powolny ruch cieczy: odwracalny

zmieniając znak $p_1 - p_2$ i znak $\begin{matrix}u\\v\\w\end{matrix}$ otrzyma się znów tożsame rozwiązania

Szybki ruch nie odwracalny! Tworzenie się promieni cieczy!

U gazów: także powolny ruch nie odwracalny [asymetria wypływu etc

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$\xi = \eta = \zeta = 0$

Idealne gazy <u>Niewirowy i dotyczy adiabat, bez tarcia i bez przewodn.</u>

Adiabat

$$-\int \frac{dp}{\rho} = \frac{u^2+v^2+w^2}{2} \qquad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \rho = \rho_0\left(\frac{p}{p_0}\right)^{\frac{1}{k}}$$

$$\int \frac{dp}{\rho_0\left(\frac{p}{p_0}\right)^{\frac{1}{k}}} = \frac{p_0^{\frac{1}{k}}}{\rho_0}\int\frac{dp}{p^{\frac{1}{k}}} = \frac{1}{-\frac{1}{k}+1}\,\frac{p_0^{\frac{1}{k}}}{\rho_0}\,p^{-\frac{1}{k}+1}$$

$$\frac{u^2+v^2+w^2}{2} = -\left(\frac{p_0}{p}\right)^{\frac{1}{k}}\frac{p}{\rho_0}\cdot\frac{k}{k-1}\Bigg| = -\frac{\rho_0}{\rho}\frac{p}{\rho_0}\frac{k}{k-1}\Bigg| = \frac{k}{k-1}R\theta\Bigg|_2^1$$

krytyczna prędkość:
$$k\frac{p_2}{\rho_2} = -2\frac{k}{k-1}\frac{p}{\rho_0}\left(\frac{p_0}{p}\right)^{\frac{1}{k}}\Bigg|_1^2 = +k\frac{p_2}{\rho_0}\left(\frac{p_0}{p_2}\right)^{\frac{1}{k}}$$

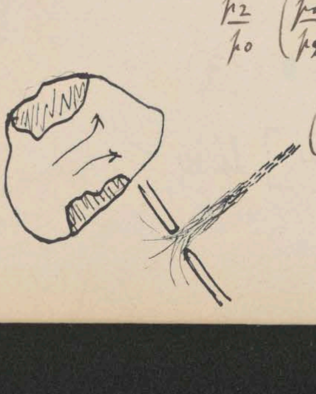
$$= -2\frac{k}{k-1}\left[\frac{p_2}{\rho_0}\left(\frac{p_0}{p_2}\right)^{\frac{1}{k}} - \frac{p_0}{\rho_0}\right]$$

$$\frac{p_2}{\cancel{\rho_0}}\left(\frac{p_0}{p_2}\right)^{\frac{1}{k}}\left(\frac{2}{k-1}+1\right) = \frac{2}{k-1}\frac{p_0}{\cancel{\rho_0}}$$

$$\frac{p_2}{p_0}\left(\frac{p_0}{p_2}\right)^{\frac{1}{k}} = \frac{2}{k+1}$$

$$\left(\frac{p_0}{p_2}\right)^{\frac{1-k}{k}} = \frac{2}{1+k}$$

$$\frac{p_2}{p_0} = \left(\frac{2}{1+k}\right)^{\frac{k}{1-k}}$$



$$\sqrt{k}\sqrt{\frac{p_2}{\rho_2}}\cdot\rho_2 = \sqrt{k}\sqrt{p_2\rho_2}$$

$$= \sqrt{k}\sqrt{p_2\rho_0\left(\frac{p_2}{p_0}\right)^{\frac{1}{k}}}$$

$$= \sqrt{k}\sqrt{\frac{\rho_0}{p_0^{\frac{1}{k}}}}\;p_2^{\frac{k+1}{2k}}$$

$$= \sqrt{k}\sqrt{\frac{\rho_0}{p_0^{\frac{1}{k}}}}\;p_0^{\frac{k+1}{2k}}\left(\frac{2}{1+k}\right)^{\frac{k}{1-k}\frac{k+1}{2k}}$$

$$= \sqrt{k}\sqrt{\rho_0 p_0}\left(\frac{2}{1+k}\right)^{\frac{k+1}{2(1-k)}}$$

Jeśli rotacyjne: $\nabla^2(u^2+v^2+w^2)=0$

N.p. dwuwymiarowe: $\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right)$

$$u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial x^2}+u\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^2 v}{\partial y^2}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2=0$$

$$\frac{4}{3}\left(u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 v}{\partial y^2}\right)+u\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^2 v}{\partial x^2}+\frac{1}{3}\left(u\frac{\partial^2 v}{\partial x\partial y}+v\frac{\partial^2 u}{\partial x\partial y}\right)+$$

$$k\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+k\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)p_0=(k-1)\left[\frac{4}{3}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right\}+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2-\frac{4}{3}\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\right]$$

$$\frac{\partial v}{\partial x}\overset{}{\underset{\overset{\parallel}{\frac{u}{v}}}{\frac{dx}{ds}}}+\frac{\partial v}{\partial y}\frac{dy}{ds}+\cdots$$

$$\frac{kR}{k-1}(\theta_2-\theta_1)=\frac{\mu}{3}\,\frac{1}{\rho}\,\frac{\partial\,[illegible]}{\partial s}$$

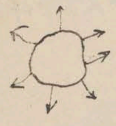
$$=\frac{\mu}{3}\,\frac{1}{\rho}\,\frac{u\frac{\partial\,[illegible]}{\partial x}+v\frac{\partial\,[illegible]}{\partial y}+w\frac{\partial\,[illegible]}{\partial z}\left(\sqrt{u^2+v^2+w^2}\right)}{\sqrt{u^2+v^2+w^2}}$$

$$\frac{kR}{k-1}(\theta-\theta_0)=\frac{\mu}{3}\,\frac{1}{\rho}\,\frac{\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\sqrt{u^2+v^2}}{\sqrt{u^2+v^2}}$$

Wystarczy jeśli całka nad pow. rury $=0$

Aby to w każdym punkcie: tylko jeśli dla każdego elementu powierzchni

$\perp v$ całka nad obiegiem $ds.\frac{\partial}{\partial n}(u^2+v^2+w^2)=0$



$$\int ds.\mathrm{S}\nabla(u^2+v^2+w^2)\,V_{s_0 v_0}=\int ds\,\mathrm{S}.s_0.V_{v_0}\nabla(u^2+v^2+w^2)=0$$

$$=\int\mathrm{S}(ds.V_{v_0}\nabla(u^2+v^2+w^2))$$

$$\parallel$$

$$\int\mathrm{S}(ds.v)=\iint\mathrm{S}.\mathrm{curl}\,v.N\,df\;\Big\|\;\iint\mathrm{S}\left[v_0.\mathrm{curl}\,V_{v_0}\nabla(u^2+v^2+w^2)\right]df=0$$

$$\nabla p = \frac{\mu}{3} \nabla \operatorname{div} v + \mu \nabla^2 v \;\Big|\; \underbrace{-\rho (v \nabla) v}_{= -\rho\left(\nabla \frac{v^2}{2} + \nabla v \operatorname{curl} v\right)}$$

~~Jeżeli wszędzie: curl $v \perp v$~~

~~$\int v \operatorname{curl} v = 0$~~

~~$u\xi + v\eta + w\zeta = 0$~~

$$\nabla^2 v = \nabla \operatorname{div} - \operatorname{curl}^2$$

$$\nabla p = \frac{4}{3}\mu \nabla \operatorname{div} - \operatorname{curl}^2 v$$

$$\operatorname{curl}^2 v = \nabla\left(p - \frac{4}{3}\mu \operatorname{div}\right)$$

$\operatorname{curl}^3 v = 0$

$$\nabla^2\left(p - \frac{4}{3}\mu \operatorname{div} v\right) = 0$$

$$v = \operatorname{curl} \int \frac{\operatorname{curl} v}{r} dv + \nabla \int \frac{\operatorname{div} v}{r} dv$$

$$= ~~\operatorname{curl} \int \frac{\nabla p - \frac{4}{3}\mu \operatorname{div} v}{r}~~ dv + ~~\nabla \int \frac{\operatorname{div} v}{r} dv~~$$

~~Zważywszy ρ stosujemy się $\operatorname{div} v = \frac{1}{\rho}$~~ $\operatorname{div}(\rho v) = 0$

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to znaczy, że curl. $V v_0$ $\nabla(u v w)$ $\perp v_0$

$$\begin{vmatrix} i & j & k \\ \frac{u}{v} & \frac{v}{v} & \frac{w}{v} \\ \frac{\partial v}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

$\frac{u}{V}$

$$\underbrace{\iint S\, n\, \nabla(u v w)\, df}_{\iiint \nabla^2(u v w)\, dv} = (q, v, \frac{dv_1}{ds} \dots)$$

$$\iiint \nabla^2(u \dots)\, q\, ds$$

$$\frac{d}{ds}\left(q \frac{d(u v w)}{ds}\right) = q \nabla^2(u v w)$$

Przy adiabatycznem odkształceniu efuzji

$$\frac{u^2+v^2+w^2}{2} = \frac{k}{k-1}\left[\frac{p_0}{\rho_0} - \frac{p}{\rho_0}\left(\frac{p_0}{p}\right)^{\frac{1}{k}}\right]$$

$$= \frac{k}{k-1}\left[\frac{p_0}{\rho_0} - \frac{p_0^{\frac{1}{k}}}{\rho_0}\underbrace{p^{1-\frac{1}{k}}}_{\left[p_0\left(\frac{\rho}{\rho_0}\right)^{k}\right]^{\frac{k-1}{k}}}\right] \qquad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{k}$$

$$= \frac{k}{k-1}\left[\frac{p_0}{\rho_0} - \frac{p_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{k-1}\right] = \frac{k}{k-1}\,\frac{p_0}{\rho_0}\left[1-\left(\frac{\rho}{\rho_0}\right)^{k-1}\right]$$

oprócz tego: $\xi = \eta = \zeta = 0$

$$u = \frac{\partial\varphi}{\partial x} \qquad \frac{\partial(\rho u)}{\partial x} + \dots = 0$$

$$u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

$$\begin{cases} \dfrac{\partial\varphi}{\partial x}\dfrac{\partial(\log\rho)}{\partial x} + \dfrac{\partial\varphi}{\partial y}\dfrac{\partial(\log\rho)}{\partial y} + \left(\dfrac{\partial^2\varphi}{\partial x^2} + \dfrac{\partial^2\varphi}{\partial y^2}\right) = 0 \\ \left(\dfrac{\partial\varphi}{\partial x}\right)^2 + \left(\dfrac{\partial\varphi}{\partial y}\right)^2 = f(\rho) = \dfrac{k}{k-1}\dfrac{p_0}{\rho_0}\left[1-\left(\dfrac{\rho}{\rho_0}\right)^{k-1}\right] \end{cases}$$

$$\log\left\{1-\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\underbrace{\frac{k-1}{k}\frac{\rho_0}{p_0}}_{a}\right\} = (k-1)\left[\log\rho - \log\rho_0\right]$$

$$(k-1)\frac{\partial\log\rho}{\partial x} = \frac{\partial}{\partial x}\log\left\{1-a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\right\}$$

$$= \frac{-2a\left(\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial\varphi}{\partial y}\frac{\partial^2\varphi}{\partial x\partial y}\right)}{1-a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]}$$

$$2a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2\frac{\partial^2\varphi}{\partial x^2} + 2\frac{\partial\varphi}{\partial y}\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x\partial y} + \left(\frac{\partial\varphi}{\partial y}\right)^2\frac{\partial^2\varphi}{\partial y^2}\right] - (k-1)\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)\left[1-a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\right] =$$

Prościej dla strugi izotermicznej : $\frac{p_0}{\rho_0} = a$

$\frac{u^2+v^2+w^2}{2} = -\frac{p_0}{\rho_0}\lg\frac{\rho}{\rho_0}$ $\qquad \frac{\partial \lg\rho}{\partial x} = \frac{a}{2}\frac{\partial}{\partial x}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]$

$a\left[\left(\frac{\partial\varphi}{\partial x}\right)^2\frac{\partial^2\varphi}{\partial x^2}+2\frac{\partial\varphi}{\partial x}\frac{\partial\varphi}{\partial y}\frac{\partial^2\varphi}{\partial x\,\partial y}+\left(\frac{\partial\varphi}{\partial y}\right)^2\frac{\partial^2\varphi}{\partial y^2}\right]+\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0$ A dalej?

$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \frac{\theta}{\theta_0} = \left(\frac{\rho}{\rho_0}\right)^{k-1} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}$

$$\frac{u^2+v^2+w^2}{2} = \frac{k}{k-1}\left[\frac{p_0}{\rho_0}-\frac{p_0}{\rho_0}\frac{\theta}{\theta_0}\right] = \frac{k}{k-1}R\theta_0\left[1-\frac{\theta}{\theta_0}\right] = \frac{k}{k-1}R(\theta_0-\theta)$$

$u^2+v^2+w^2$ będzie miało maximum na osi symetrii

zatem $\frac{\partial}{\partial n}(u^2+v^2+w^2)$ = nieskończ. małe jak δ (jak

ale pod całką będzie δ^2 tego samego rzędu co ε

→ Jeżeli zadanie jednowymiarowe :

izotermicznie :

~~$a\left(\frac{\partial\varphi}{\partial x}\right)^2\frac{\partial^2\varphi}{\partial x^2}$~~ $\rho u = \text{const}$ $\qquad u\,e^{-\frac{\rho_0}{p_0}\frac{u^2}{2}} = \text{const}$

adiabatycznie:

Równanie to ~~[illegible]~~ nie być spełnione ~~tylko dla pewnej prędkości~~ równocześnie

u – – – –

przy prędkości niezmiennej, chyba :

$$\begin{cases}\frac{\partial\rho}{\partial t}+\frac{\partial\left(\rho\frac{\partial\varphi}{\partial x}\right)}{\partial x}=0\\[2mm] \frac{\partial\varphi}{\partial t}+\frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 = \frac{k}{k-1}\frac{p_0}{\rho_0}\left[1-\left(\frac{\rho}{\rho_0}\right)^{k-1}\right]\end{cases}$$

$$\left.\begin{array}{ll}\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 & \Big|\ u\\[2mm] \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x} = -k\frac{p_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{k-2}\frac{\partial\rho}{\partial x} & \Big|\ \rho\end{array}\right\}\ \frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x} = -k\frac{p_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{k-2}\rho\frac{d\rho}{dx}$$

$\alpha = \frac{1}{\kappa}$

$$\Delta^2\theta = \overset{0\,+}{\cancel{\ \ }}\ \alpha\left[u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + kp\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - (k-1)\Phi\right]$$

~~$\theta = \theta_0 + \alpha\varphi + \frac{\alpha^2}{2}\psi + \cdots$~~

~~$\Delta^2\theta = \alpha\Delta^2\varphi + \frac{\alpha^2}{2}\Delta\psi + \cdots$~~

~~Dla dostatecznie małych α [illegible] wyższe potęgi~~

~~$\Delta^2\varphi = u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \cdots$~~

~~Przy tem zauważmy:~~

~~$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$~~

$$\theta = \theta_0 + \alpha\theta' + \frac{\alpha^2}{2}\theta'' + \cdots$$

$$u = u_0 + \alpha u' + \frac{\alpha^2}{2}u'' + \cdots$$

$\vdots$

$$p = p_0 + \alpha p' + \frac{\alpha^2}{2}p' + \cdots$$

Wtedy w pierwszem przybliżeniu:

$$\frac{\partial p_0}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial x} + \cdots\right) + \cdots$$

$$\frac{\partial(\rho_0 u_0)}{\partial x} + \frac{\partial(\rho_0 v_0)}{\partial y} = 0$$

$$\frac{p_0}{\rho_0} = R\theta_0$$

$$\Delta^2\theta' = \left[u_0\frac{\partial p_0}{\partial x} + \cdots \quad -(k-1)\Phi_0\right]$$

Za pomocą tego drugie przybliżenie:

$$\frac{p_0 + \alpha p'}{\rho_0 + \alpha\rho'} = R(\theta_0 + \alpha\theta')$$

$$p_0 + \alpha p' = R(\theta_0 + \alpha\theta')(\rho_0 + \alpha\rho')$$

$$p' = R[\rho_0\theta' + \rho'\theta_0]$$

$$= p_0\left[\frac{\theta'}{\theta_0} + \frac{\rho'}{\rho_0}\right]$$

$$\boxed{\frac{p'}{p_0} = \cancel{\ \ } = \frac{\theta'}{\theta_0} + \frac{\rho'}{\rho_0}}$$

$$p' = p_0\frac{\theta'}{\theta_0} + R\theta_0\cdot\rho'$$

$$\frac{\partial p'}{\partial x} = \frac{\frac{\partial p_0}{\partial x}\cdot\theta' + p_0\frac{\partial\theta'}{\partial x}}{\theta_0} + R\theta_0\frac{\partial\rho'}{\partial x}$$

$$\rho_0\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) + u_0\frac{\partial\rho'}{\partial x} + v_0\frac{\partial\rho'}{\partial y} = 0$$

$$\frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial^2 u'}{\partial x^2} + \cdots$$

$$R\theta_0\frac{\partial\rho'}{\partial x} = \frac{\partial p'}{\partial x} - \frac{\partial p_0}{\partial x}\frac{\theta'}{\theta_0} - \frac{p_0}{\theta_0}\frac{\partial\theta'}{\partial x}$$

$$\underbrace{R\theta_0\rho_0}_{p_0}\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) + \cancel{u_0\frac{\partial}{\partial x}\ \cdots}$$

$$+ u_0\frac{\partial p'}{\partial x} + v_0\frac{\partial p'}{\partial y} - \frac{\theta'}{\theta_0}\left(u_0\frac{\partial p_0}{\partial x} + v_0\frac{\partial p_0}{\partial y}\right) -$$

$$- \frac{p_0}{\theta_0}\left(u_0\frac{\partial\theta'}{\partial x} + v_0\frac{\partial\theta'}{\partial y}\right) = 0$$

Tj.: $\frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) + \sim$

$$\frac{\partial p'}{\partial y} = --$$

$$\uparrow \qquad \uparrow$$

$$p_0\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) + u_0\frac{\partial p'}{\partial x} + v_0\frac{\partial p'}{\partial y} = \cancel{\frac{u_0}{\theta_0}}\ \cancel{\frac{\partial}{\partial x}}\ \frac{1}{\theta_0}\left(u_0\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}\right)(p_0\theta')$$

Jeżeli chcemy utrzymać $\frac{\partial p'}{\partial y} = 0$ [będzie możliwe tylko jeżeli $\frac{\partial^2 u'}{\partial x \partial y}$ b. mała w porównaniu do $\frac{\partial^2 u'}{\partial y^2}$]

$\therefore v_0 = 0$

$$\begin{cases} \frac{\partial p'}{\partial x} = \mu\frac{\partial^2 u'}{\partial y^2} \\ p_0\frac{\partial u'}{\partial x} + u_0\frac{\partial p'}{\partial x} = \frac{1}{\theta_0}u_0\frac{\partial(p_0\theta')}{\partial x} \end{cases} \qquad \frac{\partial^2 p'}{\partial x^2} = \mu\frac{\partial^3 u'}{\partial x \partial y^2}$$

$$\| : \frac{\partial}{\partial y^2}$$

$$p_0\frac{\partial^3 u'}{\partial x \partial y^2} + \frac{\partial^2 u_0}{\partial y^2}\frac{\partial p'}{\partial x} = \frac{1}{\theta_0}\frac{\partial^2}{\partial y^2}\left(u_0\frac{\partial \text{—}}{\partial x}\right)$$

$$p_0\frac{d^2 p'}{dx^2} + \mu\frac{\partial^2 u_0}{\partial y^2}\frac{dp'}{dx} = \frac{\mu}{\theta_0}\frac{\partial^2}{\partial y^2}\left(u_0\frac{\partial}{\partial x}(p_0\theta')\right)$$

$$\begin{cases} \dfrac{p}{R\theta}\, u \dfrac{\partial u}{\partial x} + \dfrac{\partial p}{\partial x} = \mu \dfrac{\partial^2 u}{\partial x^2} \\ u \dfrac{\partial p}{\partial x} + k\, p \dfrac{\partial u}{\partial x} = (k-1)\mu\, \Phi + (k-1)\kappa\, \nabla^2\theta \end{cases}$$

Podobieństwo dla różnych gazów ?

gęstość zależy od $R = \frac{1}{\rho_0}$ zmieniając zatem R na ρR etc. jak dany;

$$\left.\begin{aligned} \frac{b\, m^2}{\rho r n} &\equiv \frac{b}{n} \equiv \alpha \frac{m}{n^2} \\ \frac{m b}{n} &\equiv \alpha \frac{m^2}{n^2} \equiv \beta \frac{r}{n^2} \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{m^2}{\rho r} &= 1 \\ b = \alpha \frac{m}{n} &= \beta \frac{r}{m n} \end{aligned}\right\}$$ 3 równań na 7 wielkości 4 dowolnych

~~N.p. biorąc dowolnie ρ, α, β~~

$$\frac{m^2}{r} \equiv \rho \equiv \frac{m^2}{r} \equiv \frac{\beta}{\alpha}$$

$$b \equiv \alpha \frac{m}{n}$$

$$\underline{\rho \equiv \frac{\beta}{\alpha}}$$

Podobne mogą być tylko ruchy takich gazów [z tych sk!]

przy których $\frac{K}{\mu R}$ ~~takie~~ same więc przy których

$\frac{\rho_0 K}{\mu}$ takie same

28

H_2	O_2	N_2	CO_2	CH_4	C_2H_4	N_2O
$\frac{7}{0{\cdot}5} = 14$	$\frac{16.1}{1}$	$\frac{14.1}{1}$	$\frac{22.\ 0{\cdot}71}{0{\cdot}82} = 19$ 154	$\frac{8.\ 1{\cdot}66}{0{\cdot}63} = 21$	$\frac{14.\ 0{\cdot}689}{0{\cdot}56} = 22$	$\frac{22.\ 0{\cdot}73}{0{\cdot}82} = 20$

28
16
44
22

więc w ogóle niewielkie różnice umożliwiają przy każdym ze współczynników zmiana

Wtedy redukuje się to na:

$\frac{m^2}{r} = \rho$

$b = \alpha \frac{m}{n}$

obierając dowolnie 4 wielkości

Np. ρ, α, $r = 1$,

$m^2 = \rho$

$bn = \alpha m = \alpha\sqrt{\rho}$

$\left.\begin{matrix} m = \sqrt{\rho} \\ bn = \alpha\sqrt{\rho} \end{matrix}\right\}$

I). robiąc rozmiary niezmienione $n = 1$ pozostają

$$\begin{cases} m = \sqrt{\rho} = \sqrt{\frac{1}{\rho_0}} \\ b = \alpha\sqrt{\rho} = \frac{\alpha}{\sqrt{\rho_0}} \end{cases}$$

Wtedy prędkości wzrastają $\sim \frac{1}{\sqrt{\rho_0}}$

jeśli ciśnienie $\sim \frac{\alpha}{\sqrt{\rho_0}}$

~~ilości~~ ciężar przepływający $\sim$ $\rho u = \frac{p u}{R\theta} \| \equiv \frac{bm}{\rho r} = \frac{\beta}{\rho} = \underline{\underline{\alpha}}$

~~robiąc~~ zatem jeśli

II). ciśnienie niezmienione $b = 1$

$m^2 = \rho$

$n = \alpha m = \alpha\sqrt{\rho}$

$m = \frac{1}{\sqrt{\rho_0}}$

$n = \frac{\alpha}{\sqrt{\rho_0}}$

$\rho = \frac{m^2}{r}$

$b = \alpha\frac{m}{n}$

$m = \cancel{\sqrt{\frac{\rho}{r}}}$

Np. $n = 1$

$r = 1$

$m = \sqrt{\rho}$

$b = \alpha\sqrt{\rho}$

~~Dopędzający~~ $\frac{mb}{\rho r} = \alpha$

czas dla pewnej objętości: $\tau \sim \frac{1}{m}$

$n = 1$

$b = 1$

$m = \frac{1}{\alpha}$

$r = \frac{1}{\alpha^2\rho}$

$\frac{\partial u}{\partial p} = 0$

$\frac{\partial u}{\partial x}$

Ograniczając się na $\rho u \frac{\partial u}{\partial x}$

$\frac{m^2}{\rho r} = 1$

$m = \sqrt{\rho r}$

$(r = 1)$: $m = \sqrt{\rho}$

~~przepływający~~: $\frac{mb}{\rho r} = \cancel{\sqrt{\rho}}$

czas dla pewnej objętości: $\frac{1}{m}$

Emden!

$h = 1$

$n = 1$

$m = \sqrt{\alpha}$

$b = \beta\sqrt{\alpha}$

$h = 1$

$b = 1$

$\alpha = m$

$n = \beta\sqrt{\alpha}$

Bez tarcia, adiabatycznie:

$$\frac{b\,m^2}{\rho\, r\, n} \equiv \frac{b}{n}$$

Tylko takie będą podobne, które mają równe k

$$\frac{m b}{n} \equiv \cancel{\mu}\, k\, \frac{m b}{n} \quad \Big\| \; k=1$$

Izotermicznie, z tarciem :
r=1

$$\frac{b\,m^2}{\rho\,[illegible]\,n} \equiv \frac{b}{n} \equiv \alpha\,\frac{m}{n^2}$$

$$m^2 = \rho$$

$$b n = \alpha m = \alpha\sqrt{\rho}$$

To jest tak samo jak z [illegible] — ~~[illegible]~~ $\Phi \propto \Delta^{5}\theta$
jeśli k = równe !

$$m = \sqrt{\rho} = \frac{1}{\sqrt{\rho_0}}$$

$$b n = \frac{\alpha}{\sqrt{\rho_0}}$$

Zatem jeśli rozmiary zmniejszone:

ciężar przypadający $\sim \alpha$

$$\frac{\text{ciężar przyp.}}{\text{różn. ciśnienia}} \sim \frac{\alpha}{\alpha\,[illegible]}\sqrt{\rho_0} = \sqrt{\rho_0}$$

gęstość $\frac{1}{R\theta} = \frac{\alpha}{\sqrt{\rho_0}}\,\rho_0 = \alpha\sqrt{\rho_0}.$

$$\frac{\text{objętość}}{\text{różn. ciśnień}} \sim \frac{\sqrt{\rho_0}}{\alpha\sqrt{\rho_0}} = \frac{1}{\alpha}$$

↓

To będzie się stosować do dowolnie szerokich rurek jeśli k = k

i jeśli pomiary zrobione przy odpowiednich ciśnieniach.

W ścisłej metodzie transpozycyjnej:

Objętości gazu mierzona pod ciśnieniem $\frac{\frac{1}{2}(p_1+p_2)}{\text{różn. ciśnienia}} \sim \frac{1}{\mu}$

Stawiając $u = u_0 + \mu u'$ etc.

$$\left.\begin{aligned} &u_0\frac{\partial u_0}{\partial x} + \cdots = -\frac{\partial p_0}{\partial x} \\ &\frac{\partial(\rho_0 u_0)}{\partial x} + \cdots = 0 \\ &u_0\frac{\partial p_0}{\partial x} + \cdots + k p_0\left(\frac{\partial u_0}{\partial x} + \cdots\right) = \kappa\,\Delta^2\theta_0 \end{aligned}\right\} \qquad p_0 = R\theta_0\rho_0$$

$$\left\{\begin{aligned} &u_0\frac{\partial u'}{\partial x} + \cdots + u'\frac{\partial u_0}{\partial x} + \cdots = -\frac{\partial p'}{\partial x} + \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y}\right) + \cdots \\ &\rho_0\frac{\partial u'}{\partial x} + u_0\frac{\partial \rho'}{\partial x} + \cdots = 0 \\ &u_0\frac{\partial p'}{\partial x} + \cdots + u'\frac{\partial p_0}{\partial x} + \cdots + k p_0\left(\frac{\partial u'}{\partial x} + \cdots\right) + k p'\left(\frac{\partial u_0}{\partial x} + \cdots\right) = (k-1)\Phi_0 + \kappa(k-1)\,\Delta^2\theta' \end{aligned}\right.$$

$$\cancel{p = R\theta\rho} \qquad p' = R\left[\theta_0\rho' + \theta'\rho_0\right]$$

Bez tarcia i przewodn.

$$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + kp\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad \cancel{\frac{\partial p}{\partial x}}$$

$$u\frac{\partial \lg p}{\partial x} + v\frac{\partial \lg p}{\partial y} + w\frac{\partial \lg p}{\partial z} + k\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$u\frac{\partial \lg \rho}{\partial x} + v\frac{\partial \lg \rho}{\partial y} + \qquad + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cdots\right) = 0$$

$$-\rho\left[u\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}\right) + v\left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} + \cdot\right) + w\left(\quad\right)\right] + kp\left(\frac{\partial u}{\partial x} + \quad\right) = 0$$

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\frac{u^2+v^2+w^2}{2} = \frac{kp}{\rho}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + k\,p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$ III).

IV).

Jeśli się supponuje $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$:

$\therefore \quad k\,u\frac{\partial \rho}{\partial x}\cdot\rho^{k-1} + \dots \quad + k\rho^k\frac{\partial u}{\partial x} \quad = 0$

II). $\frac{\partial(\rho u)}{\partial x} + \frac{\partial \rho v}{\partial y} + \dots = 0$ stąd się łatwo otrzymuje, że zamiast III można także wziąć IV

III otrzyma się wstawiając IV do II

[illegible] Ale z tego nie wynika że z III i II zawsze wypływa IV ?

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\lg p + k\left(\frac{\partial u}{\partial x} + \quad\right) = 0$$

$$\left(u\frac{\partial}{\partial x} + \dots\right)\lg\rho + \left(\frac{\partial u}{\partial x} + \dots\right) = 0 \quad \Big| \; -k$$

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\lg\left(\frac{p}{\rho^k}\right) = 0$$

To znaczy że każda cząsteczka gazu nie zmienia podczas swego biegu $\lg\left(\frac{p}{\rho^k}\right)$ zatem prawo adiabat.

Wynika stąd dla ruchu niewirowego z równania $\int\frac{dp}{\rho} = \frac{k}{k-1}\dots = \frac{u^2+v^2+w^2}{2}$

to samo co dla ruchu ogólnego wzdłuż $\iiint$ -- :

$$\frac{kR(\theta_2 - \theta_1)}{k-1} + \left(\frac{u^2+v^2+w^2}{2}\right)_2 - \left(\frac{u^2+v^2+w^2}{2}\right)_1 = 0$$

~~N.p. przyjmując dla powietrza $u^2+v^2+w^2$~~

Zatem ~~k~~ $\frac{k}{k-1}R\theta + \frac{u^2+v^2+w^2}{2} = \text{const} = \frac{k}{k-1}R\theta_0$ | jeśli w początku w spoczynku

maksymalna wartość dla prędkości gdy $\theta = 0$ $\quad \frac{u^2+v^2+w^2}{2} = \frac{k}{k-1}R\theta_0 = \frac{k}{k-1}\frac{p_0}{\rho_0}$

$\sqrt{u^2+v^2+w^2} = \sqrt{\frac{2}{k-1}}\sqrt{k\frac{p_0}{\rho_0}}$ lub też uwzględniając $a^2 = k\frac{p}{\rho} = kR\theta$

Więc maksymalna możliwa prędkość $= \sqrt{\frac{2}{k-1}}$ razy zwykłej prędkości głosu

$\sqrt{\frac{2}{0{\cdot}4}} = \sqrt{5} = 2{\cdot}4$ wtedy musiałoby już nastąpić oziębienie do (0° abs.)

Tymczasem według St Venant & Wantzel przy efuzji

$$\sqrt{\sim} = a$$

$$\left.\begin{array}{l} \frac{k}{k-1}\cancel{R}\theta + \frac{k\cancel{R}\theta}{2} = \frac{k}{k-1}\cancel{R}\theta_0 \\ (2+k-1)\theta = 2\theta_0 \end{array}\right\} \quad \begin{array}{l} \theta = \frac{2\theta_0}{k+1} \\ \theta_0 - \theta = \theta_0\frac{k-1}{k+1} \end{array}$$

Np. $\theta_0 = 300$ $\theta_0 - \theta = \frac{0{\cdot}4}{2{\cdot}4}\cdot 300 = \underline{\underline{50^\circ}}$ $+27^\circ$ -23°

---

nie zna rozproszenia energii

Odwracalność ruchów cieczy

Idealna ciecz: $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\dots = -\frac{\partial p}{\partial x}$

zmieniając znak u v w: nie zmienia się znak dp

N.p. kula w cieczy poruszającej się

(więc wskutek zmiany prędkości granicznych)

Ten sam kształt linij prądu i rozkładu prędkości

u	$-u$
v	$-v$
w	$-w$
p	$a-p$

Powolne tarcie cieczy: $\frac{\partial p}{\partial x}=\mu\left(\frac{\partial^2 u}{\partial x^2}+\dots\right)$

zmieniając znak u v w: zmienia się znak dp

więc wskutek zmiany ciśnienia

N.p. wypływ przez rurki włoskowate

Szybkie tarcie cieczy: $u\frac{\partial u}{\partial x}+\dots = -\frac{\partial p}{\partial x}+\mu\left(\frac{\partial u}{\partial x^2}+\dots\right)$

Nieodwracalny: $-u, -v, -w$ nie będzie rozwiązaniem jeśli $+u, v, w$ jest rozwiązaniem

---

Idealny gaz: odwracalny tak samo jak ciecz idealna

nie zmienia się znak dp

[bez rozproszenia energii!
~~[illegible]~~ chyba drobne drgania

Powolne tarcie gazu

Nieodwracalne, bo oprócz dp wchodzi także w grę ~~[illegible]~~ bezwzględna wartość p:

$$\frac{\partial p}{\partial x}=\mu\frac{\partial u}{\partial y^2}$$

$$\frac{\partial(pu)}{\partial x}\cdot\ =0$$

$$\downarrow$$

$$p\frac{\partial u}{\partial x}+u\frac{\partial p}{\partial x}$$

jeśli są spełnione przez p, u to będą także spełnione przez −u, −p [ale ujemne ~~[illegible]~~ p niemożliwe: dodatkowy warunek!

Szybkie tarcie oczywiście stałe nieodwracalne

Entropia gazu $S = \cancel{k}c_v \lg\theta + R\lg v + \text{const}$

$$\cancel{R\lg\frac{1}{\rho} - 2k\lg\rho} = c_v\left(\lg\theta + (k-1)\lg v\right)$$

$$= c_v \lg\frac{\theta}{\rho^{k-1}} + \text{const} = c_v \lg\frac{p}{\rho^k} + \text{const}!$$

$$\cancel{\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}}$$

$$\cancel{\ldots}$$

$$\cancel{\frac{\partial \rho u}{\partial x} + \quad = 0}$$

$p = p_1 + p_2$

$u = u_1 + u_2$

$$\cancel{(\rho_1+\rho_2)\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x}\right) + (u_1+u_2)\left(\frac{\partial\rho_1}{\partial x} + \frac{\partial\rho_2}{\partial x}\right) + \ldots = 0}$$

Dodawanie rozwiązań niemożliwe u gazów oprócz dla bardzo małych $\frac{\partial\rho}{\partial x} \neq 0$

$$\rho_1\frac{\partial u_2}{\partial x} + \rho_2\frac{\partial u_1}{\partial x} + u_1\frac{\partial\rho_2}{\partial x} + u_2\frac{\partial\rho_1}{\partial x} + \quad = 0$$

$$\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \quad\right) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \ldots\right)$$

$\vdots$

$p = p_1 + p_2$ $\qquad u = u_1 + u_2$ —

$$\frac{\partial p_2}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_2}{\partial x} + \ldots\right) + \mu\nabla^2 u_2$$

$\vdots$

$$\cancel{\rho\frac{\partial u}{\partial x} + u\frac{\partial\rho}{\partial x} \ldots = 0}$$

$$\cancel{\rho\left(\frac{\partial u_1}{\partial x} + \ldots\right) = 0}$$

$$\cancel{\rho\left(\frac{\partial u_2}{\partial x} + \ldots\right) + u\frac{\partial\rho}{\partial x}} = 0$$

$$(u_1+u_2)\frac{\partial(\rho_1+\rho_2)}{\partial x} \ldots = 0$$

Przybliżone obliczenie dla kuli w gazie

Równania: ~~izotermiczne~~: adiabatyczne

$$\begin{cases}\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div} + \mu\Delta^2 u \\ \cdot \quad - \end{cases} \qquad\Bigg\|\qquad k p \operatorname{div} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = 0$$

Supponując p jako bardzo duże, podczas gdy $\frac{\partial p}{\partial x}$ małe, otrzyma się $\operatorname{div} = 0$

więc ruchy takie jak w cieczy $\qquad p = P + p \qquad P = \text{const}$

$$\operatorname{div} = \cancel{-\frac{1}{kp}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \ldots\right)} = -\frac{1}{P+p}\left(u\frac{\partial}{\partial x} + \ldots\right)$$

$$= -\frac{1}{P}\left(u\frac{\partial p}{\partial x} + \ldots\right) + \frac{p}{P^2}\left(u\frac{\partial p}{\partial x} - -\right)$$

$$p = p_0 + \frac{1}{P}p' + \frac{1}{2}\left(\frac{1}{P}\right)^2 p'' + \ldots$$

$$\begin{cases}\frac{\partial p_0}{\partial x} + \frac{1}{P}\frac{\partial p'}{\partial x} + \ldots = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\overset{=0}{\cancel{\operatorname{div}_0}} + \frac{1}{P}\operatorname{div}' + \right) + \mu\left[\Delta^2 u_0 + \frac{1}{P}\Delta^2 u' + \ldots\right] \\ k\left(P + p_0 + \frac{1}{P}p' + \right)\left(\overset{=0}{\cancel{\operatorname{div}_0}} + \frac{1}{P}\operatorname{div}' + \right) + \left(u_0 + \frac{1}{P}u'\right)\left(\frac{\partial p_0}{\partial x} + \frac{1}{P}\frac{\partial p'}{\partial x}\right) + \ldots = 0\end{cases}$$

~~Równamy rozwijając~~ według $\frac{1}{P}$:

$$\frac{\partial p u}{\partial x} + \ldots = 0 \qquad \cancel{\rho = \rho_0 + \frac{1}{P}\rho'}$$

$$\cancel{\frac{\partial(\rho_0 u_0)}{\partial x} + \ldots = 0} \quad \Big\| \quad \cancel{\frac{\partial}{\partial x}(\rho u + \rho u') + \ldots = 0}$$

$$\cancel{\begin{cases}\frac{\partial p_0}{\partial x} = \mu\Delta u_0 \\ u_0\frac{\partial p_0}{\partial x} + \ldots = 0\end{cases}}$$

$$\frac{p}{\rho} = R\theta \qquad \frac{p_0 + \frac{1}{P}p'}{\rho_0 + \frac{1}{P}\rho'} = R\left(\theta_0 + \frac{1}{P}\theta'\right)$$

$$\cancel{\begin{cases}\frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial \operatorname{div}'}{\partial x} + \mu\Delta^2 u' \\ p_0\operatorname{div}' + u'\frac{\partial p_0}{\partial x} + u_0\frac{\partial p'}{\partial x} + \ldots = 0\end{cases}}$$

$$p' = R(\theta'\rho_0 + \theta_0\rho')$$

$$(\mathrm{div}'+\dots)\left(1+\frac{p_0}{p}+\frac{1}{p^2}p'+\dots\right)+\left(u_0+\frac{1}{p}u'\right)\left(\frac{\partial p_0}{\partial x}+\frac{1}{p}\frac{\partial p'}{\partial x}\right)+\dots=0$$

$$k\,\mathrm{div}'+u_0\frac{\partial p_0}{\partial x}+v_0\frac{\partial p_0}{\partial y}+w_0\frac{\partial p_0}{\partial z}=0$$

$$\uparrow$$

$$\left.\begin{aligned}\frac{\partial p'}{\partial x}&=\frac{\mu}{3}\frac{\partial\,\mathrm{div}'}{\partial x}+\mu\Delta^2u'\\ \frac{\partial p'}{\partial y}&=\frac{\mu}{3}\frac{\partial\,\mathrm{div}'}{\partial y}+\mu\Delta^2v'\\ \frac{\partial p'}{\partial z}&=\frac{\mu}{3}\frac{\partial}{\partial z}+\mu\Delta^2w'\end{aligned}\right\}\qquad\begin{aligned}\Delta^2p'&=\frac{4\mu}{3}\Delta^2\mathrm{div}'\\ p'&=\frac{4\mu}{3}\mathrm{div}'+\varphi\end{aligned}\qquad\Delta^2\varphi=0$$

$$\mu\frac{\partial}{\partial x}\mathrm{div}'+\frac{\partial\varphi}{\partial x}=\mu\Delta^2u'=\frac{\partial}{\partial x}(\mu\,\mathrm{div}'+\varphi)$$

$$\mu\Delta^2v'=\frac{\partial}{\partial y}(\qquad)$$

$$\mu\Delta^2w'=\frac{\partial}{\partial z}(\qquad)$$

---

obliczeni $u_0\frac{\partial p_0}{\partial x}+\dots=k\,\mathrm{div}'$

$$p=p_0-\frac{3}{2}\mu ca\frac{x}{r^3}$$

$\frac{\partial p}{\partial x}=-\frac{3}{2}\mu ca\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right)$	$u=-\frac{3}{4}ca\left(1-\frac{a^2}{r^2}\right)\frac{x^2}{r^3}+c\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$
$\frac{\partial p}{\partial y}=\frac{3}{2}\mu ca\frac{3xy}{r^5}$	$v=-\frac{3}{4}ca\left(1-\frac{a^2}{r^2}\right)\frac{xy}{r^3}$
$\frac{\partial p}{\partial z}=\frac{3}{2}\mu ca\frac{3xz}{r^5}$	$w=-\frac{3}{4}ca\left(1-\frac{a^2}{r^2}\right)\frac{xz}{r^3}$

$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}=$ ~~$-\frac{27}{8}\mu c^2a^2\frac{a^2x^2(x^2+y^2+z^2)}{r^{10}}+\frac{9}{8}\mu c^2a^2$~~

$$=-\frac{27}{8}\mu c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x^2}{r^6}+\frac{9}{8}\mu c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x^2}{r^6}-\frac{3}{2}\mu c^2a\frac{1}{r^3}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$+\frac{9}{2}\mu c^2a\frac{x^2}{r^5}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$-k\,\mathrm{div}' = -\frac{3}{2}\mu a c^2 \frac{1}{r^3}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right) + \frac{9}{2}\mu c^2 a \frac{x^2}{r^5}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$\underbrace{-\frac{18}{8}\mu c^2 a \frac{x^2}{r^5}\left(\frac{a}{r}-\frac{a^3}{r^3}\right)}$$

$$+\frac{9}{2}\mu c^2 a \frac{x^2}{r^5}\left(1-\frac{5}{4}\frac{a}{r}+\frac{1}{4}\frac{a^3}{r^3}\right)$$

symetrycznie dla ± x

Dla $r=a$, $r=\infty$

$\mathrm{div}'=0$

Dla $x=0$: $r>a$ $\quad \mathrm{div}'<0$

Dla $x=r$: (oś X) $\quad \mathrm{div}' = \mu \frac{c^2 a}{r^3}\left[3-\frac{9}{2}\frac{a}{r}+\frac{3}{2}\frac{a^3}{r^3}\right] = 3\mu\frac{c^2 a}{r^3}\left[1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right]$

$\frac{a}{r}=1-\delta \qquad 1-\frac{3}{2}(1-\delta)+\frac{1}{2}(1-\delta)^3 = \cancel{\frac{3}{2}\delta}-\cancel{\frac{3}{2}\delta}+\frac{3}{2}\delta^2-\frac{\delta^3}{2}$

>0

$\mathrm{div}'>0$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z} = \frac{\partial\,\mathrm{div}}{\partial x} = \Delta^2 u + \underbrace{\frac{\partial\zeta}{\partial y}-\frac{\partial\eta}{\partial z}}_{=-\frac{\partial\varphi}{\partial x}}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\partial^2 u}{\partial y^2} \,\Big\|\, \frac{\partial}{\partial z}\left(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\right)+\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial\,\mathrm{div}'}{\partial y} = \Delta^2 v + \underbrace{\frac{\partial\cancel{\xi}}{\partial z}-\frac{\partial\zeta}{\partial x}}_{-\frac{\partial\varphi}{\partial y}}$$

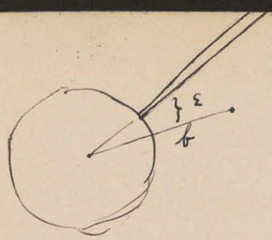
$\xi=0 \qquad \frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=0$

$$\frac{\partial\,\mathrm{div}'}{\partial z} = \Delta^2 w + \underbrace{\frac{\partial\eta}{\partial x}-\frac{\partial\cancel{\xi}}{\partial y}}_{-\frac{\partial\varphi}{\partial z}}$$

~~$\frac{\partial\zeta}{\partial x}=\frac{\partial\varphi}{\partial y} \qquad \frac{\partial\eta}{\partial x}=-\frac{\partial\varphi}{\partial z}$~~

~~$\frac{\partial^2\zeta}{\partial x\partial z}+\frac{\partial^2\eta}{\partial x\partial y}=0$~~

~~$\frac{\partial\zeta}{\partial z}+\frac{\partial\eta}{\partial y}=f(y,z)$~~



$$d\omega \int_a^\infty dr\, \frac{\frac{1}{r^2}\left(1-\frac{5}{4}\frac{a}{r}+\frac{1}{4}\frac{a^3}{r^3}\right)}{\sqrt{b^2+r^2-2br\cos\varepsilon}} \cdot \cos^2\theta$$

$$\frac{\partial}{\partial x}\left(\frac{x^m}{r^n}\right) = m\frac{x^{m-1}}{r^n} - n\frac{x^{m+1}}{r^{n+2}}$$

$$\frac{\partial^2}{\partial x^2}(\quad) = m(m-1)\frac{x^{m-2}}{r^n} - n(2m+1)\frac{x^m}{r^{n+2}} + n(n+2)\frac{x^{m+2}}{r^{n+4}}$$

$$\frac{\partial}{\partial y}\left(\frac{x^m}{r^n}\right) = -n\frac{x^m y}{r^{n+2}}$$

$$\frac{\partial^2}{\partial y^2}(\quad) = -n\frac{x^n}{r^{n+2}} + n(n+2)\frac{x^m y^2}{r^{n+4}}$$

$$\Delta^2\left(\frac{x^m}{r^n}\right) = m(m-1)\frac{x^{m-2}}{r^n} - \frac{x^m}{r^{n+2}}\left[n(2m+1)+2n\right] + \frac{x^m}{r^{n+2}}\, n(n+2)$$

$$= m(m-1)\frac{x^{m-2}}{r^n} + \frac{x^m}{r^{n+2}}\left[\underbrace{n^2+2n-2mn-3n}_{n^2-2mn-n}\right]$$

45
−50

$m=4 \quad n=\begin{cases}10\\8\end{cases}$

100
− 80
− 10
10

64
− 8
−64

$m=2$

$$\Delta^2\left(\frac{x^2}{r^n}\right) = \frac{2}{r^n} + (n^2-5n)\frac{x^2}{r^{n+2}}$$

$n=3$: $\Delta^2\left(\frac{x^2}{r^3}\right) = \frac{2}{r^3} - 6\frac{x^2}{r^5} = 2\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right)$

$n=4$ $\Delta^2\left(\frac{x^2}{r^4}\right) = \frac{2}{r^4} - 4\frac{x^2}{r^6} = 2\left(\frac{1}{r^4}-\frac{2x^2}{r^6}\right)$

$n=5$ $\Delta^2\left(\frac{x^2}{r^5}\right) = \frac{2}{r^5}$

$n=6$ $\Delta^2\left(\frac{x^2}{r^6}\right) = \frac{2}{r^6} + 6\frac{x^2}{r^8} = 2\left(\frac{1}{r^6}+3\frac{x^2}{r^8}\right)$

8

$$\frac{x^2}{r^5} = \frac{1}{3}\frac{1}{r^3} - \frac{1}{6}\Delta^2\left(\frac{x^2}{r^3}\right) \Big\| = \frac{2}{r^8} + 24\frac{x^2}{r^{10}}$$

$$n=10: \| = \frac{2}{r^{10}} + 50\frac{x^2}{r^{12}}$$

$$\frac{x^2}{r^5} = \frac{1}{3}\,\frac{1}{r^3} - \frac{1}{6}\,\Delta^2\left(\frac{x^2}{r^3}\right)$$

$$\frac{x^2}{r^6} = \frac{1}{2}\,\frac{1}{r^4} - \frac{1}{4}\,\Delta^2\left(\frac{x^2}{r^4}\right) \qquad \Bigg|\; -\frac{5}{4}a$$

$$\frac{x^2}{r^8} = -\frac{1}{3}\,\frac{1}{r^6} + \frac{1}{6}\,\Delta^2\left(\frac{x^2}{r^6}\right) \qquad \Bigg|\; \frac{1}{4}a^3$$

$$\frac{x^2}{r^5} - \frac{5}{4}a\,\frac{x^2}{r^6} + \frac{1}{4}a^3\,\frac{x^2}{r^8} = \frac{1}{3}\,\frac{1}{r^3} - \frac{5}{8}\,\frac{a}{r^4} - \frac{1}{12}\,\frac{a^3}{r^6}$$

$$- \Delta^2\left\{\frac{1}{6}\,\frac{x^2}{r^3} - \frac{5}{16}\,\frac{ax^2}{r^4} - \frac{1}{24}\,\frac{a^3x^2}{r^6}\right\}$$

$$\frac{9}{2}\mu c^2 a\left\{\left(\frac{x^2}{r^5} \quad - \quad -\frac{1}{4}a^3\frac{x^2}{r^8}\right) - \frac{1}{3}\left(\frac{1}{r^3} - \frac{3}{4}\,\frac{a}{r^4} - \frac{1}{4}\,\frac{a^3}{r^6}\right)\right\} = \frac{9}{2}\mu c^2 a$$

$$\left(-\frac{5}{8} + \frac{1}{4}\right)\frac{a}{r^4} - \Delta^2\left\{\frac{1}{6}\,\frac{x^2}{r^3} - - \quad\right\}$$

$$= -\frac{9}{2}\mu c^2 a\left[\frac{3}{8}\,\frac{a}{r^4} + \frac{1}{6}\,\Delta^2\left(\frac{x^2}{r^3} - \frac{15}{8}\,\frac{ax^2}{r^4} - \frac{1}{4}\,\frac{a^3x^2}{r^6}\right)\right]$$

$$\Delta^2\left(\frac{1}{r^2}\right) = -\frac{\partial}{\partial x}\left(\frac{2x}{r^4}\right) + \cdots = 2\sum\left(-\frac{1}{r^4} + \frac{4x^2}{r^6}\right) = \frac{2}{r^4}$$

$$= ~~-\mu c^2 a\left[\frac{27}{16}\right.~~ \quad \Delta^2\left\{-\mu c^2 a\left[\frac{27}{32}\,\frac{a}{r^2} + \frac{3}{4}\left(\frac{x^2}{r^3} - \frac{15}{8}\,\frac{ax^2}{r^4} - \frac{1}{4}\,\frac{a^3x^2}{r^6}\right)\right]\right\}$$

Zatem $\operatorname{div}' = \Delta^2\psi$

$$\left.\begin{aligned} \cancel{\mu}\,\Delta^2 u' &= \cancel{\mu}\,\Delta^2\frac{\partial\psi}{\partial x} + \frac{1}{\mu}\frac{\partial\varphi}{\partial x} \\ \cancel{\mu}\,\Delta^2 v' &= \cancel{\mu}\,\Delta^2\frac{\partial\psi}{\partial y} + \frac{1}{\mu}\frac{\partial\varphi}{\partial y} \\ \cancel{\mu}\,\Delta^2 w' &= \cancel{\mu}\,\Delta^2\frac{\partial\psi}{\partial z} + \frac{1}{\mu}\frac{\partial\varphi}{\partial z} \\ \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} &= \Delta^2\psi \end{aligned}\right\}$$

Rozwiązanie tego: $u' = \frac{\partial\psi}{\partial x} + U$

$v' = \frac{\partial\psi}{\partial y} + V$

$w' = \frac{\partial\psi}{\partial z} + W$

$\Delta^2 U = \frac{1}{\mu}\frac{\partial\varphi}{\partial x}$

$\Delta^2 V =$

$\Delta^2 W =$

$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$

$- \Delta^2\varphi = 0$ } Zupełnie to samo jak dla u. v. w.

$$-k\frac{\partial\psi}{\partial x} = -\mu c^2 a\,\frac{1}{2}\left[-\frac{27}{8}\frac{a}{r^3}\frac{x}{r} + 3\left(\frac{x}{r^3} - \frac{15ax}{8r^4} - \frac{a^3x}{4r^6}\right) - \frac{3}{2}\left(\frac{3x^3}{r^5} - \frac{15}{2}\frac{ax^3}{r^6} - \frac{3}{2}\frac{a^3x^3}{r^8}\right)\right]$$ 177

$$= -\mu c^2 a\left[\frac{3}{2}\frac{x}{r^3}\left[1 - 3\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] - \frac{9}{4}\frac{x^3}{r^5}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]\right]$$

$$-k\frac{\partial\psi}{\partial y} = -\mu c^2 a\left[-\frac{27}{16}\frac{ay}{r^4} - \frac{9}{4}\frac{x^2y}{r^5}\left[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right]\right]$$

$$-k\frac{\partial\psi}{\partial z} = -\mu c^2 a\,[\;—\;-\;——\;—\;—\;]$$

Dla $r=a$:

$$-k\frac{\partial\psi}{\partial x}\Big|_{r=a} = -\frac{\mu c^2 a}{2}\left[\frac{3x}{a^3}\left(-\frac{9}{4}\right) - \frac{9}{2}\frac{x^3}{a^5}(-2)\right] = \frac{\mu c^2}{2a}\left[\frac{27}{4}\frac{x}{a} - 9\frac{x^3}{a^3}\right]$$

$$= 9\frac{\mu c^2}{2a}\left[\frac{3}{4}\frac{x}{a} - \frac{x^3}{a^3}\right]$$

$$-k\frac{\partial\psi}{\partial y}\Big|_{r=a} = +\mu c^2 a\left[\frac{27}{8}\frac{y}{a^3} + \frac{9}{2}\frac{x^2y}{a^5}(-2)\right] = 9\frac{\mu c^2}{2a}\left[\frac{3}{8}\frac{y}{a} - \frac{x^2y}{a^3}\right]$$

$$-k\frac{\partial\psi}{\partial z}\Big|_{r=a} = \cancel{\mu c^2 a}\,[\;—\;—\;——\;—\; = \frac{9\mu c^2}{2a}\left[\frac{3}{8}\frac{z}{a} - \frac{x^2z}{a^3}\right]$$

Próba utworzenia $\Delta^2\psi$:

$$3\frac{1}{r^3}\left[1 - 3\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] - 9\frac{x^2}{r^5}\left[1 - 3\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] + 3\frac{x^2}{r^3}\left[3\frac{a}{r^3} + \frac{3}{4}\frac{a^3}{r^5}\right] -$$

$$-\frac{27}{2}\cancel{\frac{x^2}{r^5}\Big[1 - \frac{5}{2}\frac{a}{r}} - \frac{1}{2}\frac{a^3}{r^3}\Big] + \frac{45}{2}\cancel{\frac{x^4}{r^7}\Big[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}}\Big] - \frac{9}{2}\frac{x^4}{r^5}\left[\frac{5}{2}\frac{a}{r^3} + \frac{3}{2}\frac{a^3}{r^5}\right]$$

$$-\frac{27}{8}\frac{a}{r^4} - \frac{9}{2}\cancel{\frac{x^2}{r^5}\Big[1 - \frac{5}{2}\frac{a}{r}} - \frac{1}{2}\frac{a^3}{r^3}\Big] + \frac{27}{2}\frac{ay^2}{r^6} + \frac{45}{2}\cancel{\frac{x^2y^2}{r^7}\Big[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}}\Big] -$$

$$-\frac{9}{2}\frac{x^2y^2}{r^5}\left[\frac{5}{2}\frac{a}{r^3} + \frac{3}{2}\frac{a^3}{r^5}\right]$$

$$-\frac{27}{8}\frac{a}{r^4} - \frac{9}{2}\cancel{\frac{x^2}{r^5}\Big[1 - \frac{5}{2}\frac{a}{r}} - \frac{1}{2}\frac{a^3}{r^3}\Big] + \frac{27}{2}\frac{az^2}{r^6} + \frac{45}{2}\cancel{\frac{x^2z^2}{r^7}\Big[1 - \frac{5}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}}\Big] -$$

$$-\frac{9}{2}\frac{x^2z^2}{r^5}\left[\frac{5}{2}\frac{a}{r^3} + \frac{3}{2}\frac{a^3}{r^5}\right]$$

$$= 3\frac{1}{r^3}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]-\frac{27}{4}\frac{a}{r^4}+\frac{27}{2}\frac{a(y^2+z^2)}{r^6}-\frac{9x^2}{r^5}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]$$

$$+\frac{3x^2}{r^3}\left[\frac{3a}{r^3}+\frac{3}{4}\frac{a^3}{r^5}\right]-\frac{9}{2}\frac{x^2}{r^3}\left[\frac{5}{2}\frac{a}{r^3}+\frac{3}{2}\frac{a^3}{r^5}\right]$$

$$=\frac{3}{r^3}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]-\frac{27}{4}\frac{a}{r^4}+\frac{27}{2}\frac{a}{r^4}\left[1-\frac{x^2}{r^2}\right]-\frac{9x^2}{r^5}\left[1-\frac{3a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]$$

$$+\frac{3x^2}{r^5}\left[3\frac{a}{r}+\frac{3}{4}\frac{a^3}{r^3}\right]-\frac{9}{2}\frac{x^2}{r^5}\left[\frac{5}{2}\frac{a}{r}+\frac{3}{2}\frac{a^3}{r^3}\right] =$$

$$=\frac{3}{r^3}\left[1-\frac{3a}{4r}-\frac{1}{4}\frac{a^3}{r^3}\right]+\frac{x^2}{r^5}\left[\frac{a}{r}\overset{-9+}{\Big(}-\frac{27}{2}+27+9-\frac{45}{4}\Big)+\frac{a^3}{r^3}\left[\cancel{\frac{9}{4}}+\cancel{\frac{9}{4}}-\frac{27}{4}\right]\right]$$

$$=\frac{3}{r^3}\left[1-\frac{3a}{4r}-\frac{1}{4}\frac{a^3}{r^3}\right]+\frac{9}{\text{[illegible]}}\frac{x^2}{r^5}\left[-1+\frac{5}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]$$

Zgadza się

Teraz należy jeszcze U V W tak oznaczyć żeby na powierzchni owe wartości były spełnione:

a w nieskończoności:

$U, V, W = 0$

$$U = -\frac{27\mu c^2}{8a}\frac{x}{a}\left[1-\frac{4}{3}\frac{x^2}{a^2}\right]$$

$$V = -\frac{27\mu c^2}{8a}\frac{y}{a}\left[\frac{1}{2}-\frac{4}{3}\frac{x^2}{a^2}\right]$$

$$W = -\frac{27\mu c^2}{8a}\frac{z}{a}\left[\frac{1}{2}-\frac{4}{3}\frac{x^2}{a^2}\right]$$

Ogólne rozwiązanie:

$$U = \frac{1}{\mu}\sum\left\{\frac{r^2}{2(2n+1)}\frac{dp_n}{dx}+\frac{n r^{2n+3}}{(n+1)(2n+1)(2n+3)}\frac{d}{dx}\left(\frac{p_n}{r^{2n+1}}\right)\right\}+\sum\frac{d\varphi_n}{dx}$$

$$V = \qquad \frac{d}{dy} \qquad\qquad \frac{d}{dy} \qquad\qquad \frac{d\varphi_n}{dy}$$

$$W =$$

~~$p_0 = \text{const}$~~ ~~$p_{-1} =$~~

$\frac{x^2}{r^3}$

~~$p_1 = x$~~

~~$p_{-2} =$~~

$\frac{2x}{r^{n}} \quad - \frac{3x^3}{r^5}$

~~$p_2 =$~~ $\quad p = r^m S_m \qquad \frac{1}{r^{m+1}} S_m$

$p_n =$

$$x u + y v + z w \Big|_{r=\infty} = 0$$

$S_0 = 1$

$S_1 = \cos$ $\qquad x \qquad \frac{x}{r^3}$

$S_2 = \frac{1}{2}(3\cos^2 - 1)$

$S_3 = \frac{1}{2}(5\cos^3 - 3\cos)$

$\frac{1}{r}\ 3 \quad p_0 = 1 \ \| \ p_{-1} = \frac{1}{r}$

$p_1 = x \ \| \ p_{-2} = \frac{x}{r^3}$

$p_2 = \frac{1}{2}(3x^2 - r^2) \ \| \ p_{-3} = \frac{1}{2r^3}\left(3\frac{x^2}{r^2} - 1\right)$

$$x u + y v + z w \Big|_{r\,\infty} = \frac{1}{\mu} \sum \frac{n r}{2(2n+3)} p_n + \frac{1}{r} \sum n \varphi_n \Big|_{\infty} = 0$$

$n < -1 \qquad n < 0$

$$\frac{\partial p_{-3}}{\partial x} = \cancel{\frac{3x}{2r^4}} \ \frac{-3x}{2r^5}\left(\frac{3x^2}{r^2} - 1\right) + \frac{3x}{r^5} - \frac{3x^3}{r^7} = \frac{3x}{r^5}\left[\frac{3}{2} - \frac{5}{2}\frac{x^2}{r^2}\right]$$

$$= \frac{9x}{2r^5}\left[1 - \frac{5}{3}\frac{x^2}{r^2}\right]$$

$$\frac{\partial p_{-3}}{\partial y} = -\frac{3y}{2r^5}\left(\frac{3x^2}{r^2} - 1\right) - \frac{3yx^2}{r^7} = \frac{3y}{r^5}\left[\frac{1}{2} - \frac{5}{2}\frac{x^2}{r^2}\right] = \frac{9}{2}\frac{y}{r^5}\left[\frac{1}{3} - \frac{5}{3}\frac{x^2}{r^2}\right]$$

$$\frac{\partial}{\partial x}\left(\frac{p_{-3}}{r^{-5}}\right) = \frac{\partial}{\partial x}\left[\frac{r^2}{2}\left(\frac{3x^2}{r^2} - 1\right)\right] = \frac{1}{2}\frac{\partial}{\partial x}\left[3x^2 - r^2\right] = 3x - x = 2x$$

$$\frac{\partial}{\partial y}\left(\frac{p_{-3}}{r^{-5}}\right) = \frac{1}{2}\frac{\partial}{\partial y}\left[3x^2 - r^2\right] = -y$$

$n = -3$

$$U = \frac{-r^2}{2.5}\ \frac{3x}{r^5}\left[\frac{3}{2} - \frac{5}{2}\frac{x^2}{r^2}\right] + \frac{\cancel{2}\ 1}{2.5.\cancel{x}}\ \frac{1}{r^3}\ 2x = \frac{1}{10}\frac{x}{r^3}\left[-\frac{5}{2} + \frac{15}{2}\frac{x^2}{r^2}\right]$$

$$= -\frac{1}{4}\frac{x}{r^3}\left[1 - 3\frac{x^2}{r^2}\right]$$

$$V = \frac{-r^2}{2.5}\ \frac{3y}{r^5}\left[\frac{1}{2} - \frac{5}{2}\frac{x^2}{r^2}\right] - \frac{1}{10.}\ \frac{1}{r^3}\ y \qquad = \frac{1}{10}\frac{y}{r^3}\left[-\frac{5}{2} + \frac{15}{2}\frac{x^2}{r^2}\right]$$

$$= -\frac{1}{4}\frac{y}{r^3}\left[1 - 3\frac{x^2}{r^2}\right]$$

$$U = A\frac{x}{r^3}\left[1 - 3\frac{x^2}{r^2}\right] + B\frac{x}{r^3} + C\frac{x}{r^5}\left[\frac{3}{2} - \frac{5}{2}\frac{x^2}{r^2}\right]$$

$$V = A\frac{y}{r^3}\left[1 - 3\frac{x^2}{r^2}\right] + B\frac{y}{r^3} + C\frac{y}{r^5}\left[\frac{1}{2} - \frac{5}{2}\frac{x^2}{r^2}\right]$$

$$\left[A\frac{1}{a^3} + B\frac{1}{a^3} + \frac{3}{2}C\frac{1}{a^5}\right]\cancel{x} = \frac{1}{a}$$

$$\cancel{\left[-3A\frac{1}{a^5} - \frac{5}{2}C\frac{1}{a^7}\right]x^3 = -\frac{4}{3}\frac{x^3}{a^3}}$$

$$\frac{C}{a^5} = \frac{1}{2a}$$

$$C = \frac{a^4}{2}$$

$$A\frac{1}{a^3} + B\frac{1}{a^3} + \frac{1}{2}C\frac{1}{a^5} = \frac{1}{2}\frac{1}{a}$$

$$-3A\frac{1}{a^5} - \frac{5}{2}C\frac{1}{a^7} = -\frac{4}{3}\frac{1}{a^3}$$

$$\frac{3A}{a^5} + \frac{5}{4}\frac{1}{a^3} = \frac{4}{3}\frac{1}{a^3}$$

$$\frac{3A}{a^5} = \cancel{\left(\frac{5}{4} - \frac{4}{3}\right)\frac{1}{a^3}}\left(\frac{4}{3} - \frac{5}{4}\right)\frac{1}{a^3} = \frac{1}{12}\frac{1}{a^3}$$

$$\underline{A} = \frac{a^2}{36}$$

$$A + B + \frac{1}{2} C \frac{1}{a^2} = \frac{1}{2} a^2$$

$$B = \cancel{\frac{1}{2} a^2} - \frac{a^2}{36} - \cancel{\frac{1}{3} \frac{a^2}{2}} = \frac{2}{9} a^2$$

$$U = -\frac{27 \mu c^2}{8a} \left[\frac{a^2}{36} \left\{ \frac{x}{r^3} \left[1 - 3\frac{x^2}{r^2} \right] + 8 \frac{x}{r^3} \right\} + \frac{a^4}{2} \frac{x}{r^5} \left[\frac{3}{2} - \frac{5}{2} \frac{x^2}{r^2} \right] \right]$$

$$U = -\frac{27 \mu c^2}{8a} \left[\overset{\frac{1}{4}\frac{x}{r^3}}{\left(- \frac{a^2}{12} \frac{x^3}{r^5} \right.} + \frac{a^4}{4} \frac{x}{r^5} \left[3 - 5 \frac{x^2}{r^2} \right] \right]$$

$$V = -\frac{27 \mu c^2}{8a} \left[\overset{\frac{y}{4r^3}}{\left(- \frac{a^2}{12} \right.} \frac{y x^2}{r^5} + \frac{a^4}{4} \frac{y}{r^5} \left[1 - 5 \frac{x^2}{r^2} \right] \right]$$

$$U = + \frac{27 \mu c^2 a}{32} \left[\overset{-\frac{x}{r^3} +}{\left(\frac{1}{3} \frac{x^3}{r^5} \right.} - \frac{a^2 x}{r^5} \left(3 - 5 \frac{x^2}{r^2} \right) \right]$$

$$V = \frac{27 \mu c^2 a}{32} \left[\overset{-\frac{y}{r^3} +}{\left(\frac{1}{3} \frac{y x^2}{r^5} \right.} - \frac{a^2 y}{r^5} \left(1 - 5 \frac{x^2}{r^2} \right) \right]$$

Próba: $\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \sim = \frac{x^2}{r^5} - \frac{5}{3} \frac{x^4}{r^7} \Big(- \frac{3a^2}{r^5} + \frac{15 a^2 x^2}{r^7} + \frac{15 a^2 x^2}{r^7} - \frac{35 x^4 a^2}{r^9}$

$$\left.\begin{matrix} +\frac{1}{r^3} - \frac{3x^2}{r^5} \\ +\frac{1}{2} \end{matrix}\right\} = 0$$

$+\frac{1}{3} \frac{y^2}{r^5} - \frac{5}{3} \frac{x^2 y^2}{r^7} \Big| - \frac{a^2}{r^5} + \frac{5 a^2 y^2}{r^7} + \frac{5 a^2 x^2}{r^7} - \frac{35 x^2 y^2 a^2}{r^9}$

$+\frac{1}{3} \frac{z^2}{r^5} - \frac{5}{3} \frac{z^2 x^2}{r^7} \Big) - \frac{a^2}{r^5} + \frac{5 a^2 z^2}{r^7} + \frac{5 a^2 x^2}{r^7} - \frac{35 x^2 z^2 a^2}{r^9}$

$\underbrace{\qquad\qquad\qquad}_{0}$

$\underbrace{\qquad}_{-\frac{5}{3} \frac{x^2}{r^5}}$

$- \frac{5 a^2}{r^5}$

$\underbrace{- \frac{35 x^2 a^2}{r^7}}_{+ \frac{5 a^2 x^2}{r^7}}$

$$= \cancel{- \frac{5 a^2}{r^7} (r^2 - x^2)} + \frac{5 a^2 x^2}{r^7}$$

$$= \frac{1}{r^7} \left\{ r^4 - \frac{5}{3} (x^4 + y^4 + z^4) \right.$$

$= 0$

$$\Delta^2 U \sim \frac{1}{3} \left(6 \frac{x}{r^5} - 10 \frac{x^3}{r^7} \right) \cancel{- 3a^2 \left(10 \frac{x}{r^7} \right)} \cancel{+ 5 a^2 \left(6 \frac{x}{r^7} \right)} = \frac{2x}{r^5} - \frac{10}{3} \frac{x^3}{r^7}$$

$$\Delta^2 V \sim \frac{2}{3} \frac{y}{r^5} - \frac{10}{3} \frac{x^2 y}{r^7}$$

$$\Delta W \sim \frac{2}{3} \frac{z}{r^5} - \frac{10}{3} \frac{x^2 z}{r^7}$$

$$\frac{\partial}{\partial x} (\nabla^2 U) + \frac{\partial}{\partial y} (\nabla^2 V) + \frac{\partial}{\partial z} (\nabla^2 W) = 0$$

zgadza się

Przewodzenie prądu cieplnego

termostatyka nie było elektrostatyki:

temp. stała U stałe

$\Delta^2\theta$ $\Delta^2 U$

$k\frac{\partial\theta}{\partial x}$ prąd el. $\frac{\partial U}{\partial x}$

$\theta = \frac{1}{4\pi}\int\frac{\Delta^2\theta}{r}\,dv$ $U = \frac{1}{4\pi}\int\frac{\Delta^2 U}{r}\,dv$

$\Delta^2\theta\ \frac{\partial\theta}{\partial x}$? siła mech. $\Delta^2 U.\ \frac{\partial U}{\partial x}$

dyf. przew. galw.

$\underbrace{\hspace{3em}}$

w elektrolitach

ścisły związek

grawit. d grawitacje

$$\frac{p_1-p_2}{l}\,\frac{a^4\pi}{8\mu} = F$$

$$p_1-p_2 = \frac{8\mu}{a^4\pi}F\,l \qquad \Bigg\| \qquad \frac{40000.\cancel{8}.\,0{\cdot}0015.\,\cancel{1}000}{\cancel{3}.\,\cancel{16}} = 0{\cdot}06\text{ atm}$$

$$-\frac{5z}{r^7}$$

$$-\frac{5}{r^7}+\frac{35z^2}{r^9}$$

$$-\frac{7z}{r^9} \qquad -\frac{7}{r^9}+\frac{63z^2}{r^{11}}$$

$$\frac{\partial}{\partial x} \qquad \frac{1}{3}\frac{yx^2}{r^5} - \frac{a^2y}{r^5} + \frac{5a^2x^2y}{r^7}$$

$$\cancel{\frac{1}{3}\cdot\frac{2xy}{r^5} - \frac{5x^3y}{3r^7} + \frac{5a^2xy}{r^7} + \frac{15a^2xy}{r^7} - \frac{35a^2x^3y}{r^9}}$$

$$\frac{2}{3}\frac{y}{r^5} - \cancel{\frac{10}{3}\frac{x^2y}{r^7}} - \frac{25x^2y}{3r^7} + \frac{35x^4y}{3r^9} + \frac{15a^2y}{r^7} - 2\cdot3\cdot5\cdot7\frac{a^2x^2y}{r^9} - \cancel{3\cdot5\cdot7\frac{a^2x^2y}{r^9}}$$

$$+9\cdot35\frac{a^2x^4y}{r^{11}}$$

$$\cancel{\frac{1}{3}\frac{x^2}{r^5} - \frac{5x^2y^2}{3r^7} - \frac{a^2}{r^5} + \frac{5a^2y^2}{r^7} + \frac{5a^2x^2}{r^7} - \frac{35a^2x^2y^2}{r^9}}$$

$$-\frac{5}{3}\frac{x^2y}{r^7} - \frac{10x^2y}{3r^7} + \frac{5\cdot7}{3}\frac{x^2y^3}{r^9} + \frac{15a^2y}{r^7} - 35\frac{a^2y^3}{r^9} - \frac{105a^2x^2y}{r^9} + 35\cdot9\frac{a^2x^2y^3}{r^{11}}$$

$$\cancel{\frac{1}{3}yx^2\frac{\cdot 20}{r^7} - a^2y\frac{\cdot 20}{r^7} + a^2x^2y\frac{42}{r^9}}$$

$$-\frac{5}{3}\frac{yx^2}{r^7} + \frac{35}{3}\frac{yx^2z^2}{r^9} + \frac{5a^2y}{r^7} - \frac{35a^2yz^2}{r^9} - 35\frac{a^2x^2y}{r^9} + \overset{9\cdot35}{\cancel{175}}\cdot\frac{a^2x^2yz^2}{r^{11}}$$

$$+\frac{35}{3}\frac{x^2y}{r^7} \qquad -\frac{35a^2yx^2}{r^9} \qquad +9\cdot35\frac{a^2x^2y}{r^9}$$

$$-\frac{35a^2y}{r^7}$$

$$\frac{2}{3}\frac{y}{r^5} - \frac{10}{3}\frac{x^2y}{r^7} \qquad =0$$

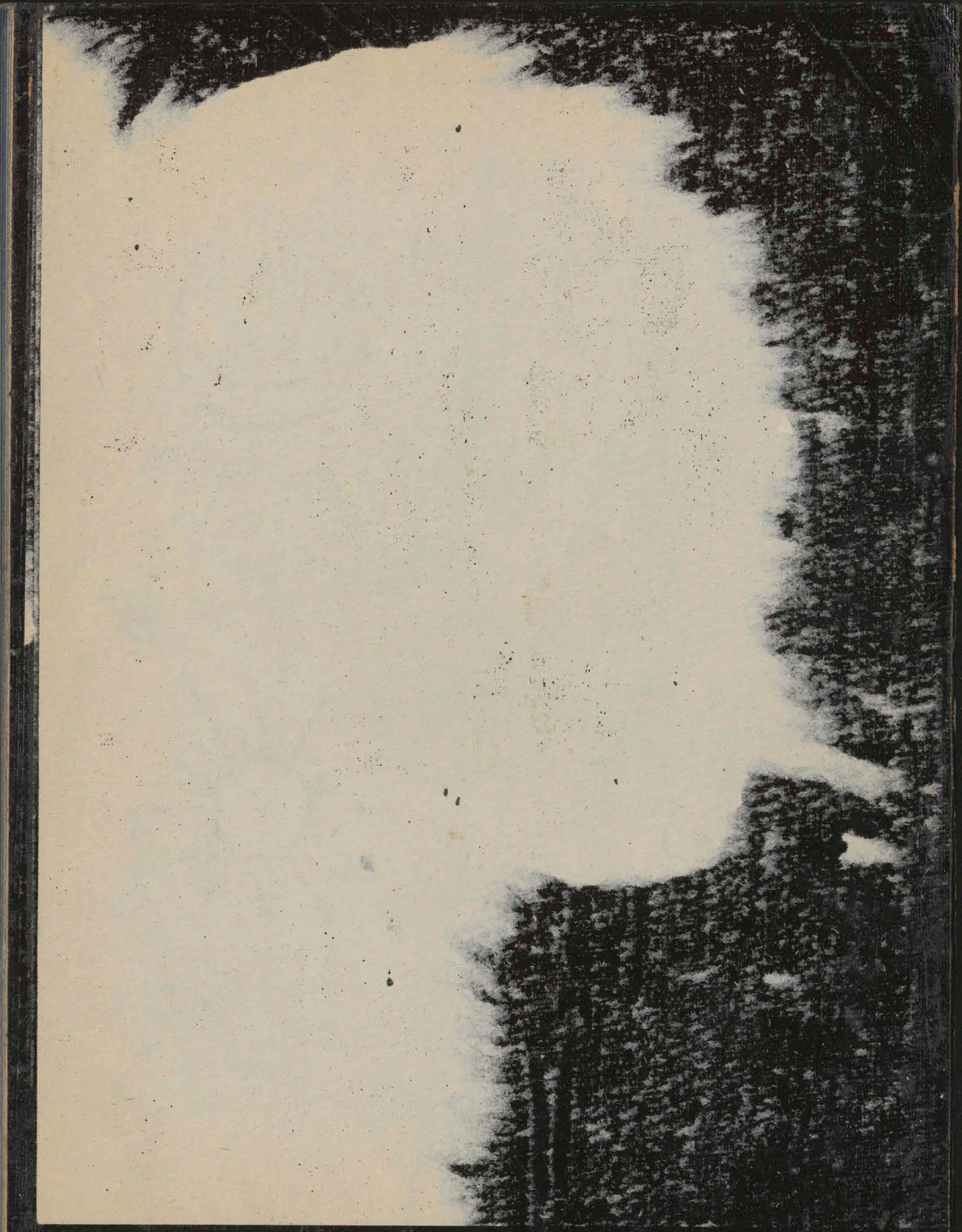
$$\begin{array}{r} -105 \\ -210 \\ -35 \\ \hline 350 \end{array} \quad +315 \qquad = -35\frac{a^2x^2y}{r^9}$$

$$\frac{2}{r^5} - \frac{10x^2}{r^7} - \frac{10x^2}{r^7} + \frac{70x^4}{3r^9} \qquad \cancel{\frac{10}{r^5}}\ \frac{10}{3}\frac{1}{r^5} - \frac{10}{3}$$

$$\frac{2}{3r^5} - \frac{10}{3}\frac{y^2}{r^7} - \frac{10x^2}{3r^7} + \frac{70x^2y^2}{3r^9}$$

$$\frac{2}{3r^5} - \frac{10}{3}\frac{z^2}{r^7} - \frac{10x^2}{3r^7} + \frac{70x^2z^2}{3r^9} \qquad = ?$$

$$\frac{70x^2}{3r^7}$$



[illegible]
[illegible]

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Rudzki O kształcie fali sprężystej w pokładach ziemskich

Rozpr. 33 (13) p. 377 (1898)

Phil Mag. 50 (1900) p. 210 Rayleigh

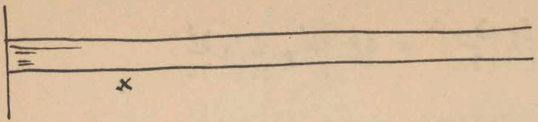
On a theorem analogous to the Virial Theorem

$$2\sum m \frac{dx}{dt}\frac{dy}{dt} = \sum(xY+yX)$$

Thomson J. J. Elements de math. th. de El. & Magn.

BN. 80329 CLXVIII J. 26

C.R. BN 5006 XXXVIII A1 - D1



$$\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] = -r\frac{\partial p}{\partial x}.$$

$$\frac{\partial^2}{\partial r^2}\left(r\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \cancel{r\frac{\partial^2 p}{\partial x\partial r}}$$

$$\cancel{r\frac{\partial u}{\partial r} = f(x) + c}$$

$$r\frac{\partial^2}{\partial r^2}\left(r\frac{\partial u}{\partial r}\right) = +\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

$$\cancel{-\frac{f_1(x)}{r^2} = -r\frac{\partial p}{\partial x}}$$

$$\log\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = +\log(r) + \cancel{\text{const}}\, f_1(x)$$

$r=R \quad u=0$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = r f_1(x)$$

$r=0 \quad u =$ endlich

$$\cancel{u = -\frac{r}{\partial r} f(x) + \varphi(x)}$$

$\psi(x)=0$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\varphi(x) + \psi(x)$$

$$\frac{\partial u}{\partial r} = r\varphi(x) + \frac{1}{r}\psi(x)$$

$$u = \frac{r^2}{2}\varphi(x) + \psi(x)\log r + \chi(x)$$

$$0 = \frac{R^2}{2}\varphi(x) + \chi(x)$$

$$\cancel{\varphi(x) = -\frac{2}{R^2}\chi}$$

$$\chi(x) = -\frac{R^2}{2}\varphi(x)$$

$$u = \frac{r^2-R^2}{2}\varphi(x)$$

$$\frac{\partial}{\partial x}(\rho u) = \overset{0}{\cancel{\text{const}}}$$

$$p u = \cancel{A}\, x F(r) + \Phi(r) = p\,\frac{r^2-R^2}{2}\varphi(x)$$

$$\cancel{\Phi(r)=0 \quad \Phi(x) \quad \varphi(x) = x}$$

$$\cancel{\frac{\partial p}{\partial r}=0 \quad \therefore \quad xF(r) + \Phi(r)=0}$$

$$\cancel{p = \ldots} \qquad p\varphi(x) = \cancel{x}\ \text{const}$$

$$\frac{\partial u}{\partial r} = r\varphi(x) \qquad \frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 2r\varphi(x)$$

$$\left.\begin{array}{l} 2\varphi(x) = -\frac{dp}{dx} \\ p\varphi(x) = c \end{array}\right\} \qquad \begin{array}{l} 2c = -p\frac{dp}{dx} \\ p^2 = Ax+B \end{array}$$

$$u = \frac{r^2-R^2}{2}\,\frac{\cancel{x}\ \text{const}}{p}$$

$u = (a^2 - r^2)\frac{k}{\mu}$

$\frac{\partial u}{\partial r} = -2r\frac{k}{\mu}$

$-p\frac{\partial u}{\partial x}\Big\rfloor + \mu\Big[2\left(\frac{\partial u}{\partial r}\right)^2 = \kappa\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$

$-8\mu\left(\frac{k}{\mu}\right)^2 r^2 = \frac{\partial}{\partial r}\Big($

$\frac{8}{3}\mu\left(\frac{k}{\mu}\right)^2 r^2 = \frac{\partial\theta}{\partial r} + \frac{c}{r}$ $\Big\|$ $\theta = \frac{8}{9}\mu\left(\frac{k}{\mu}\right)^2 r^3 + c$ ~~$c\log r$~~

$\theta_0 = \frac{8}{9}\mu \quad a^3$

$\theta - \theta_0 = \frac{8}{9}\frac{\mu}{\kappa}\left(\frac{k}{\mu}\right)^2(r^3 - a^3)$

$J_\lambda = \frac{A}{\lambda^5}e^{-\frac{B}{\lambda\theta}}$

$\frac{\partial J}{\partial\lambda} = -\frac{5}{\lambda} + \frac{B}{\lambda^2\theta} = 0$

$\lambda_m\theta = \frac{B}{5}$

$B = 5\lambda_m\theta$

$8{\cdot}863 \,.\, \cancel{288} \,.\, 5$

$3\,5452 \quad 144$

3545

$B = 1276{\cdot}3$

$\frac{B}{\theta = 288} = 4{\cdot}4315$

${}^{10}\log J_\lambda = {}^{10}\log A - 5\,{}^{10}\log\lambda - \frac{B}{\lambda\theta}\,{}^{10}\log e$

$\theta = 1440$
273
1167

$\frac{B}{\theta} = 8863 \,.\, 0{\cdot}4343$
3434
35452
2659
3545
205
$3{\cdot}8486$

$\lambda = 1: \quad \log J = -3{\cdot}8486$

0·3514

2·246

58531
23930
0·34601 5

$\lambda = \frac{1}{2}$

$+\;0{\cdot}30103 \,.\, 5$
$1{\cdot}50515$
$-\;7{\cdot}6972$
$38080 \;-10$

0102

~~0·3920~~

~~2·466~~

$\lambda_{max} = 1{\cdot}735$

0080

$0{\cdot}23930$
$-\;1{\cdot}1965$
$-\;2{\cdot}2182\,5$
$-\;3{\cdot}4147\,5$

1·735 . 12
347
2082
4164

6·098

0·7852

$\lambda = 2:$

$-\;1{\cdot}50515$
$-\;1{\cdot}9243$
$-\;3{\cdot}4295$

0·7705

5·895

$\lambda = 3$

−1·28829
~~[illegible]~~
−2·3856
−3·6685

477,2

0·5315 3·400

0·157 24
31·4
628
377

$\lambda = 4$

0·96215
3·0103
−3·97245
0·2275

1·688

θ = 1440
720
2160
273
1887

$\frac{B}{\theta}$ = 1·28287
2·5657

$\lambda = 1$: $\lg J$ = −2·5657
0·6343
−5·1314
+1·5052
−3·6262

$\lambda = 0{\cdot}5$

0·5738

50°

$\lambda = 2$ 4·308

$\lg J$ = −1·28285
1·50515
−2·7880
1·4120

2·582

$\lambda_{\max}$ = 1·157 | 0·40921
0·06333
0·34588
11

0·06333.5
+0·31665

−2·2176
−0·3166
− ~~[illegible]~~
−2·5342
~~[illegible]~~

1·6658

4·632

$\lambda = 3$

0·8552
2·3856
3·2408
0·9592
9·103

$\lambda = 4$ 0·6414
3·0103
3·6517
0·5483

3·748

3·534

$3^4 = 81$

θ = 720
273

$\frac{B}{\theta}$ = ·77572

$\lambda_{\max}$ = 3·47

$\lambda = 3$: −2·3856
−2·5857
−4·9713

0·0287 −1

0·1068

$\theta = 1440 \cdot \frac{5}{4}$

~~7~~200

1800

273

1527

~~25657~~

$38486 \cdot \frac{4}{5}$

~~57~~

153944

$\cancel{\lambda = 1}$ $\cancel{\tfrac{1}{2}}$ 3 = 3'07888

0'1211

1'322

$\lambda = 2$: 1'53944

1'50515

−3'0446

0'1554

1'430

$\lambda = \frac{1}{2}$ −6'15776

+1'50515

−4'63261

0'5474

0'353

1'735. $\frac{4}{5}$

0'347

$\lambda_{\max}$ = 1'388

~~48870~~

0'14239

0'34601

388. 24

776

1552

931

2'21825

0'71195

2'9302

0'2698

$\lambda = 3$ −1'02629

2'3856

−3'4119

0'7881

6'139

1'861

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\eta\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)u+\frac{1}{3}\eta\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)-\frac{\partial p}{\partial x}$$

~~$u = f.(r, x) =$~~

~~$v = \int f.(r,x) \sin\varphi =$~~

~~$w = \int f.(r x) \cos\varphi = q\frac{\sin\varphi}{q\cos\varphi}$~~

$x = r\cos\varphi$ | $u = q\cos\varphi$ $\qquad q, w = f.(r,z)$

$y = r\sin\varphi$ | $v = q\sin\varphi$

z | $w = w$

~~$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$~~

$$\frac{\partial s}{\partial x}=\frac{\partial s}{\partial r}\cos\varphi-\frac{\partial s}{\partial\varphi}\frac{\sin\varphi}{r}$$

$$\frac{\partial s}{\partial y}=\frac{\partial s}{\partial r}\sin\varphi+\frac{\partial s}{\partial\varphi}\frac{\cos\varphi}{r}$$

$$[\theta]=\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}$$

$$\frac{\partial\theta}{\partial x}=\left[\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial r}-\frac{q}{r^2}+\frac{\partial^2 w}{\partial r\,\partial z}\right]\cos\varphi$$

$$\begin{cases}\dfrac{\partial u}{\partial x}=\dfrac{\partial q}{\partial r}\cos^2\varphi+\dfrac{q}{r}\sin^2\varphi\\[2mm]\dfrac{\partial u}{\partial y}=\left(\dfrac{\partial q}{\partial r}\ \cancel{\sin\varphi}-\dfrac{q}{r}\right)\sin\varphi\cos\varphi\\[2mm]\dfrac{\partial u}{\partial z}=\dfrac{\partial q}{\partial z}\cos\varphi\end{cases}\qquad\begin{cases}\dfrac{\partial v}{\partial x}=\left(\dfrac{\partial q}{\partial r}-\dfrac{q}{r}\right)\sin\varphi\cos\varphi\\[2mm]\dfrac{\partial v}{\partial y}=\dfrac{\partial q}{\partial r}\sin^2\varphi+\dfrac{q}{r}\cos^2\varphi\\[2mm]\dfrac{\partial v}{\partial z}=\dfrac{\partial q}{\partial z}\sin\varphi\end{cases}$$

$$\begin{cases}\dfrac{\partial w}{\partial x}=\dfrac{\partial w}{\partial r}\cos\varphi\\[2mm]\dfrac{\partial w}{\partial y}=\dfrac{\partial w}{\partial r}\sin\varphi\\[2mm]\dfrac{\partial w}{\partial z}=\dfrac{\partial w}{\partial z}\end{cases}\qquad\begin{array}{l}\dfrac{\partial p}{\partial x}=\dfrac{\partial p}{\partial r}\cos\varphi\\[2mm]\dfrac{\partial p}{\partial y}=\dfrac{\partial p}{\partial r}\sin\varphi\\[2mm]\dfrac{\partial p}{\partial z}=\dfrac{\partial p}{\partial z}\end{array}$$

$$\frac{\partial^2}{\partial x^2}+ \quad = \frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2}+\frac{1}{r}\frac{\partial u}{\partial r}$$

$$\left[\frac{\partial^2 q}{\partial z^2}+\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial r}-\frac{q}{r^2}\right]\cos\varphi=\nabla^2 u$$

$$\nabla^2 w=\frac{\partial^2 w}{\partial r^2}+\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}$$

$$\frac{\partial(\theta)}{\partial z}=\frac{\partial^2 q}{\partial r\,\partial z}+\frac{1}{r}\frac{\partial q}{\partial z}+\frac{\partial^2 w}{\partial z^2}$$

$$\rho\left[\frac{\partial q}{\partial t}\cancel{\cos\psi}+q\cancel{\cos\psi}\left(\frac{\partial q}{\partial r}\cos^2\psi+\cancel{\frac{q}{r}\sin^2\psi}\right)+q\sin\psi\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)\sin\psi\cancel{\cos\psi}+w\frac{\partial q}{\partial z}\cancel{\cos\psi}\right]$$

$$=\eta\,\cancel{\cos\psi}\left[\frac{\partial^2 q}{\partial z^2}+\frac{4}{3}\left(\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial r}-\frac{q}{r^2}\right)+\frac{1}{3}\frac{\partial^2 w}{\partial r\,\partial z}\right]-\frac{\partial p}{\partial r}$$

$$\left[\frac{\partial w}{\partial t}+q\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right]=\frac{\eta}{\rho}\left[\frac{4}{3}\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{3}\left(\frac{\partial^2 q}{\partial r\,\partial z}+\frac{1}{r}\frac{\partial q}{\partial z}\right)\right]-\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\left[\frac{\partial q}{\partial t}+q\frac{\partial q}{\partial r}+w\frac{\partial q}{\partial z}\right]=\frac{\eta}{\rho}\left[\frac{\partial^2 q}{\partial z^2}+\frac{4}{3}\left(\frac{\partial^2 q}{\partial r^2}+\frac{1}{r}\frac{\partial q}{\partial z}-\frac{q}{r^2}\right)+\frac{1}{3}\frac{\partial^2 w}{\partial r\,\partial z}\right]-\frac{1}{\rho}\frac{\partial p}{\partial r}$$

Equation of motion with ~~[illegible]~~ radial symmetry (independent of ψ)

$$\cancel{c\rho\frac{\partial\theta}{\partial t}}=-\frac{2}{3}\mu\,\boxed{\theta}^2+\mu\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$$

$$-p\,\boxed{\theta}\ \text{[illegible]}$$

$$=$$

$$c\rho\left[\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right]$$

$$c\rho\left[\frac{\partial\theta}{\partial t}+q\frac{\partial\theta}{\partial r}+w\frac{\partial\theta}{\partial z}\right]=-p\left[\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right]-\frac{2}{3}\mu\left[\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right]^2$$

$$+\mu\left[2\left\{\left(\frac{\partial q}{\partial r}\right)^2\cos^4\psi+2\frac{q}{r}\frac{\partial q}{\partial r}\cos^2\psi\sin^2\psi+\left(\frac{q}{r}\right)^2\sin^4\psi \atop \sin^4\psi+2\qquad+\qquad\cos^4\psi\right.\right.$$

$$\left.+\left(\frac{\partial w}{\partial z}\right)^2\right\}$$

$$\underbrace{\sin^4\psi+\cos^4\psi}_{=1-2\sin^2\psi\cos^2\psi}+\cancel{\quad}$$

$$=\left(\frac{\partial q}{\partial r}\right)^2+\left(\frac{q}{r}\right)^2-2\sin^2\psi\cos^2\psi\underbrace{\left[\left(\frac{\partial q}{\partial r}\right)^2+\left(\frac{q}{r}\right)^2-2\frac{\partial q}{\partial r}\frac{q}{r}\right]}_{\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2}$$

$$\left.+\left(\frac{\partial w}{\partial r}\cancel{\quad}+\frac{\partial q}{\partial z}\right)^2\sin^2\psi+\left(\frac{\partial q}{\partial z}+\frac{\partial w}{\partial r}\right)^2\cos^2\psi+4\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2\sin^2\psi\cos^2\psi\right]$$

$$2\left(\frac{\partial w}{\partial z}\right)^2+2\left(\frac{q}{r}\right)^2+2\left(\frac{\partial q}{\partial r}\right)^2\cancel{-4\sin^2\psi\cos^2\psi\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2}+\left(\frac{\partial w}{\partial r}\right)^2+\left(\frac{\partial q}{\partial z}\right)^2+2\frac{\partial q}{\partial z}\frac{\partial w}{\partial r}\cancel{\quad}$$

$$\cancel{+4\sin^2\psi\cos^2\psi\left(\frac{\partial q}{\partial r}-\frac{q}{r}\right)^2}$$

1

$$\frac{c}{A}\rho\left[\frac{\partial\theta}{\partial\tau}+q\frac{\partial\theta}{\partial r}+w\frac{\partial\theta}{\partial z}\right]=-p\left[\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right]+\mu\left[-\frac{2}{3}\left(\frac{\partial q}{\partial r}+\frac{q}{r}+\frac{\partial w}{\partial z}\right)^2+2\left[\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{q}{r}\right)^2+\left(\frac{\partial q}{\partial r}\right)^2\right]+\underbrace{\left(\frac{\partial w}{\partial r}\right)^2+\left(\frac{\partial q}{\partial z}\right)^2+2\frac{\partial q}{\partial z}\frac{\partial w}{\partial r}}_{=\left(\frac{\partial w}{\partial r}+\frac{\partial q}{\partial z}\right)^2}\right]$$

Thermal equation in respect to motion with radial symmetry

F. ex. Stationary flow in tube:

$$q=0 \qquad w=\frac{p_1^2-p_2^2}{8\mu p l}\left(R^2-r^2\right) \qquad p^2=p_1^2-\underbrace{\boxed{\frac{p_1^2-p_2^2}{l}}}_{=\alpha}z$$

$$w=\frac{\alpha(R^2-r^2)}{8\mu p}$$

$$\frac{\partial w}{\partial r}=-\frac{\alpha r}{4\mu p}$$

With neglection of $\mu\frac{\partial w}{\partial z}$

$$\frac{c\rho}{A}w\frac{\partial\theta}{\partial z}=-p\frac{\partial w}{\partial z}+\mu\left(\frac{\partial w}{\partial r}\right)^2$$

$$\frac{c\rho}{A}\frac{\partial\theta}{\partial z}\frac{\alpha(R^2-r^2)}{8\mu p}=+p\frac{\alpha(R^2-r^2)}{8\mu p^2}\frac{dp}{dz}+\mu\frac{\alpha^2r^2}{16\mu^2p^2}$$

$$=\frac{1}{8\mu p^2}\left[-\frac{\alpha^2(R^2-r^2)}{2}+\mu\frac{\alpha^2r^2}{2\mu}\right]$$

$$=\frac{1}{8\mu p^2}\cdot\frac{\alpha^2(2r^2-R^2)}{2}$$

$$\frac{c}{A}\frac{\partial\theta}{\partial z}=\frac{\alpha(2r^2-R^2)}{2p\rho(R^2-r^2)}=\cancel{\frac{\alpha(2r^2-R^2)}{2(R^2-r^2)}}$$

$$\cancel{=\frac{\alpha}{2}\frac{1}{\sqrt{p_1^2-\alpha z}}\left[\frac{R^2}{R^2-r^2}-2\right]}$$

$$\frac{p}{\rho}=\alpha\theta$$

$$\rho=\frac{p}{\alpha\theta}$$

$$p\rho=\frac{p^2}{\alpha\theta}$$

$$\frac{c}{A}\frac{1}{\alpha\theta}\frac{\partial\theta}{\partial z}=\frac{a(2r^2-R^2)}{2p^2(R^2-r^2)}=\frac{a}{2}\frac{1}{p_1^2-az}\left[\frac{R^2}{R^2-r^2}-2\right]$$

$$\frac{c}{\pi R^2}\int 2\pi r\, w\rho\theta\, dr = \frac{2}{R^2}\int w\,\rho\theta\, r\,dr = \frac{2}{R^2\alpha}\int_0^R \frac{p_1^2-p_2^2}{8\mu l}(R^2-r^2)\,r\,dr = \frac{R^2(p_1^2-p_2^2)}{16\alpha\mu l} = \rho\theta\,\frac{R^2}{16\mu}\frac{p_1^2-p_2^2}{l}$$

$\frac{R^2}{2}-\frac{R^2}{4}$

unabhängig von z ↑ !

dagegen wäre $\int w\rho r\,dr = f(z)$

was unmöglich; [illegible] e

[illegible] $w =$ [illegible] $p/p^r\, c\,\theta$

constant

$$\frac{c}{A\alpha}\log\theta = -\frac{1}{2}\log(p_1^2-az)\left[\frac{R^2}{R^2-r^2}-2\right] + \text{const}$$

$$\frac{c\alpha}{A}\log\theta_0 = -\frac{1}{2}\log p_1^2 \quad [\quad\quad] + -$$

~~$\frac{c\alpha}{A}\left[\theta^2-\theta_0^2\right] = \log\frac{p_1^2}{p_1^2-az}\left[\frac{R^2}{R^2-r^2}-2\right]$~~

$$\frac{c}{\alpha A}\log\frac{\theta}{\theta_0} = \log\sqrt{\frac{p_1^2}{p_1^2-az}}\left[\frac{R^2}{R^2-r^2}-2\right]$$

$$\theta = \theta_0\left[\frac{p_1^2}{p_1^2-az}\right]^{\frac{1}{2}\frac{\alpha A}{c}\frac{2r^2-R^2}{R^2-r^2}}$$

$$= \theta_0\left[1-\frac{az}{p_1^2}\right]^{-\frac{1}{2}\frac{\alpha A}{c}\frac{2r^2-R^2}{R^2-r^2}}$$

$z = l$:

$$\theta = \theta_0\left[\frac{p_1}{p_2}\right]^{\frac{2r^2-R^2}{R^2-r^2}\cdot\frac{\alpha A}{c}}$$

$r = 0$:

$$\theta = \theta_0\left(\frac{p_1}{p_2}\right)^{-\frac{\alpha A}{c}}$$

~~$\int\left(\frac{R^2}{R^2-r^2}-2\right)r\,dr = R^2\log(R^2-r^2)$~~

$r = R$:

$$\theta = \infty$$

With taking into account the conduction of heat:

$$\frac{c\rho}{A}\frac{\partial\theta}{\partial z}\frac{a(R^2-r^2)}{8\mu p} = \frac{1}{8\mu p^2}\frac{a^2(2r^2-R^2)}{2} + \kappa\left[\overbrace{\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}}^{=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)}\right]$$

$$\frac{c}{\alpha A}\frac{1}{\theta}\frac{\partial\theta}{\partial z}\frac{a(R^2-r^2)}{8\mu} = \frac{1}{8\mu(p_1^2-az)}\frac{a^2(2r^2-R^2)}{2} + \kappa \nearrow$$

$$\theta = b_0 + b_1 z + (\cancel{b_2} + b_3 z) r^2$$

$$\frac{\partial\theta}{\partial z} = b_1 + b_3 r^2 \qquad r\frac{\partial\theta}{\partial r} = 2r^2(b_2 + b_3 z)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(\ ) = 4(b_2 + b_3 z)$$

$$\frac{c}{\alpha A}\,\alpha(R^2-r^2)(b_1+b_3 r^2)(p_1^2 - az) = a^2\left(r^2 - \frac{R^2}{2}\right)(b_0 + b_1 z + b_2 r^2 + b_3 z r^2) +$$

$$+ 4\kappa(b_2 + b_3 z)(b_0 + b_1 z + b_2 r^2 + b_3 r^2 z)(p_1^2 - az)$$

$$\left.\begin{aligned}
\frac{c}{\alpha A}\,a R^2 b_1 p_1^2 &= -a^2\frac{R^2}{2} b_0 + 4\kappa p_1^2 b_0 b_2 \\
\frac{c}{\alpha A}\,\alpha(R^2 b_3 - b_1) p_1^2 &= a^2\left(b_0 - \frac{R^2}{2} b_2\right) + 4\kappa b_2^2 p_1^2 \\
-\frac{c}{\alpha A}\,a^2 R^2 b_1 &= a^2\left(-\frac{R^2}{2}\right) b_1 + 4\kappa\left[(b_0 b_3 + b_1 b_2) p_1^2 - a b_0 b_2\right] \\
+\frac{c}{\alpha A}\,a^2(b_1 - R^2 b_3) &= a^2\left(b_1 - \frac{R^2}{2} b_3\right) + 4\kappa\left[2 b_2 b_3 p_1^2 - a b_2^2\right]
\end{aligned}\right\}$$

$$0 = a^2\left[- \frac{R^2}{2}(b_1 p_1^2 + a b_0)\right] + 4\kappa p_1^2 (b_0 b_3 + b_1 b_2)$$

$$0 = a^2\left[b_1 p_1^2 + b_0 - \frac{R^2}{2}(b_3 p_1^2 + b_2)\right] + 8\kappa b_2 b_3 p_1^4$$

$$1 + \frac{1}{2}\frac{\alpha A}{c}\frac{a}{p_1^2}\underbrace{\frac{2r^2 - R^2}{R^2 - r^2}}\, z -$$

$$= \frac{R^2}{R^2 - r^2} - 2 = \frac{1}{1 - \frac{r^2}{R^2}} - 2 = 1 + \frac{r^2}{R^2} + \frac{r^4}{R^4} \ldots - 2 = -1 + \frac{r^2}{R^2} + \frac{r^4}{R^4} - \frac{1}{2}\frac{\alpha A}{c}\frac{2r^2 - R^2}{R^2 - r^2} - 1$$

~~$\frac{\partial\theta}{\partial z} = +\theta_0 \frac{a}{p_1^2}\frac{\alpha A}{2c}\frac{2r^2 - R^2}{R^2 - r^2}\left[1 - \frac{az}{p_1^2}\right]$~~

$$= 1 - \frac{1}{\beta}\frac{a^2}{p_1^2} z + \frac{1}{\beta}\frac{a^2}{p_1^2 R^2} z r^2 +$$

~~$\ln\theta = \ln\theta_0 + \ln\left[1 - \frac{az}{p_1^2}\right] - \frac{2\alpha A}{c}$~~ [illegible]

$\frac{1}{\theta}\frac{\partial\theta}{\partial z} =$ ~~$-\frac{a}{p_1^2}\frac{1}{1 - \frac{az}{p_1^2}}$~~ $= \frac{-a}{p_1^2 - az}$

~~$\frac{-a}{p_1^2 - az}\,\frac{c}{\alpha A}\,\frac{\alpha(R^2 - r^2)}{p_1^2}$~~ $= x$

$$\boxed{\begin{gathered}\eta\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)=\frac{\partial p}{\partial z}\\ \frac{\partial}{\partial z}(\rho w)=0\end{gathered}}$$

$$\theta = b_0 + b_1 z \overset{+b_{11}z^2}{} + b_2 z r^2 + b_3 z^2 r^2 + b_4 z r^4 + b_5 z^2 r^4 + \dots$$

$$\frac{2c}{\alpha A}\, a(R^2-r^2)(p_1^2-az)\frac{\partial\theta}{\partial z} = a^2(2r^2-R^2)\,\theta + 16\mu\kappa\,(p_1^2-az)\frac{\theta}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

$$r\frac{\partial\theta}{\partial r} = 2b_2 z r^2 + 2b_3 z^2 r^2 + 4b_4 z r^4 + 4b_5 z^2 r^4$$

$$\frac{\partial}{\partial r}(\ ) = 4b_2 z r + 4b_3 z^2 r + 16 b_4 z r^3 + 16 b_5 z^2 r^3$$

$$\overset{=\beta}{\boxed{\frac{2c\,a}{\alpha A}}}(R^2-r^2)(p_1^2-az)(b_1 \overset{+2b_{11}z}{} + b_2 r^2 + 2b_3 z r^2 + b_4 r^4 + 2b_5 z r^4) =$$

$$= a^2(2r^2-R^2)(b_0 + b_1 z \overset{+b_{11}z^2}{} + b_2 z r^2 + b_3 z^2 r^2 + b_4 z r^4 + b_5 z^2 r^4) +$$

$$+ \overset{64}{}\mu\kappa\,(p_1^2-az)(b_0 + b_1 z \overset{+b_{11}z^2}{} + b_2 z r^2 + b_3 z^2 r^2 + b_4 z r^4 + b_5 z^2 r^4)(b_2 z + b_3 z^2 + 4b_4 z r^2 + 4b_5 z^2 r^2)$$

$$\beta \cancel{R^2} p_1^2 b_1 = -a^2 \cancel{R^2} b_0$$

z)

$$-\beta R^2 a\, b_1 \overset{+2\beta R^2 p_1^2 b_{11}}{} = -a^2 R^2 b_1 + 64\mu\kappa\, p_1^2 b_0 b_2$$

r²)

$$-\beta p_1^2 b_1 + \beta R^2 p_1^2 b_2 = 2a^2 b_0$$

zr²)

$$2\beta R^2 p_1^2 b_3 - \beta R^2 a\, b_2 + \beta\, a\, b_1 = -a^2 R^2 b_2 + 2a^2 b_1$$

I III: $\beta R^2 p_1^2 b_2 = a^2 b_0 = -p_1^2 \rho b_1$

$$b_1 = -\frac{a^2 b_0}{\rho p_1^2}$$

$$b_2 = \frac{a^2 b_0}{\rho p_1^2 R^2}$$

II). $2\beta p_1^2 b_{11} = \rho a b_1 - a^2 b_1 + \underline{64\mu\kappa} \frac{p_1^2 b_0 b_2}{R^2}$

$$b_{11} = b_1 \left[\frac{a}{2p_1^2} - \frac{a^2}{2\rho p_1^2}\right]$$

$$= \frac{-a^3}{2p_1^2} \frac{b_0}{\rho p_1^2} \left(1 - \frac{a}{\rho}\right)$$

$$\left[1-2\alpha+\frac{17}{16}\alpha^2\right]^{-1} = 1+2\alpha-\frac{17}{16}\alpha^2+4\alpha^2 \ldots\ldots$$

$$\frac{1}{1-x} = 1+x+x^2+\cdot \qquad = 1+2\alpha+\frac{47}{16}\alpha^2$$

Dieterici-[illegible]:

$$\frac{v_a}{v_i} = e^{h^2} = e^{\frac{\frac{ms^2}{2}}{\frac{mc^2}{3}}} = e^{\frac{\frac{ms^2}{2}}{RT}}$$

$$\frac{ms^2}{2} = [illegible]$$

$= LK$ [illegible]

$\vee B$

$$\psi_f = \frac{2ma}{v_f}$$

$$\cancel{\psi} = \frac{RT}{2a} \lg \frac{v_a}{v_i} = \frac{m}{v_f}$$

$B = RT \lg \frac{v_a}{v_i}$

$L = \frac{RT}{2}$

$$\lg \frac{v_a - b}{v_i - b} = \lg \frac{v_a}{v_i} \cdot \frac{1-\frac{b}{v_a}}{1-\frac{b}{v_i}}$$

$$= \lg \frac{v_a}{v_i} + \lg\left(1-\frac{b}{v_a}\right) - \lg\left(1-\frac{b}{v_i}\right)$$

$$= \lg \frac{v_a}{v_i} - \frac{b}{v_a} + \frac{b}{v_i} - \frac{b^2}{v_a^2} + \frac{b^2}{v_i^2}$$

$$= \lg \frac{v_a}{v_i} - b\left(\frac{1}{v_a}-\frac{1}{v_i}\right) - b^2\left(\frac{1}{v_a^2}-\frac{1}{v_i^2}\right)$$

(17f)

$$a\left(\frac{1}{v_f}-\frac{1}{v_g}\right) = rT\left[\lg\frac{v_g-b}{v_f-b} - \left(\frac{v_g}{v_g-b} - \frac{v_f}{v_f-b}\right)\right]$$

$$1+\frac{b}{v_g}+\frac{b^2}{v_g^2}$$

$$= rT\left[\lg\frac{v_a}{v_i} - b\left(\frac{1}{v_g}-\frac{1}{v_f}\right) - b^2\left(\frac{1}{v_g^2}-\frac{1}{v_f^2}\right)\right.$$

$$\left. + b\left(\frac{1}{v_g}-\frac{1}{v_f}\right) - b^2\left(\quad\right)\right]$$

$$= p(v_g - v_f)$$

Boltzmanns (178)

$\frac{v_f}{v_f-b} = 1+\frac{b}{v_f-b}$

$$p(v_g - v_f) = rT\left[\lg\frac{v_g-b}{v_f-b} + a\left(\frac{1}{v_g}-\frac{1}{v_f}\right)\right]$$

$$p = \frac{rT}{v-b} - \frac{a}{v^2} \quad = \frac{b}{v_f-b} - \frac{b}{v_g-b}$$

$$rT\underbrace{\left[\frac{-v_g}{v_g-b}+\frac{v_f}{v_f-b}+\lg\frac{v_g-b}{v_f-b}\right]}_{\neq \lg\frac{v_g}{v_f} - 2b\left(\frac{1}{v_g}-\frac{1}{v_f}\right) - 2b^2\left(\frac{1}{v_g^2}-\frac{1}{v_f^2}\right)} = 2a\underbrace{\left(\frac{1}{v_g}-\frac{1}{v_f}\right)}_{=2L_i}$$

= not identical with Molnars Equation

$\lg\frac{v_g}{v_f}\,\frac{1-\frac{b}{v_g}}{1-\frac{b}{v_f}}$

$\lg\frac{v_g}{v_f}$

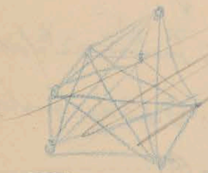
$-\frac{b}{v_g}-\frac{b^2}{v_g^2}$

$+\frac{b}{v_f}+\frac{b^2}{v_f^2}$

$+\cancel{1}+\frac{b}{v_f}+\frac{b^2}{v_f^2}$

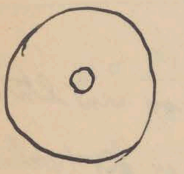
$-\cancel{1}-\frac{b}{v_g}-\frac{b^2}{v_g^2}$

$= \lg\frac{v_g}{v_f} + 2b\left(\frac{1}{v_f}-\frac{1}{v_g}\right) - 2b^2\left(\frac{1}{v_g^2}-\right.$



$$W = \frac{1}{2}\sum \mu u$$

$$\delta W = \mu u$$

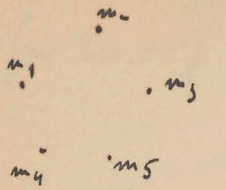


$$4\pi\left[R^2 - \left(\sqrt[3]{R^3-r^3}\right)^2\right]$$

$$= 4\pi R^2\left[1-\left(1-\left(\frac{r}{R}\right)^3\right)^{\frac{2}{3}}\right]$$

$$= 4\pi R^2\ \cancel{1}-\cancel{1}+\frac{2}{3}\left(\frac{r}{R}\right)^3$$

$$= \frac{8\pi}{3}\,\frac{r^3}{R} = \frac{2v}{R}$$



$$W = m_1 m_2 \chi_{12} + m_1 m_3 \chi_{13} + m_1 m_4 \chi_{14} + m_1 \dots$$

$$+ m_2 m_3 \chi_{23} + m_2 m_4 \chi_{24} + \dots$$

$$+ m_3 m_4 \chi_{34} + \dots$$

$$+$$

$$= \frac{1}{2} \sum m_i m_k \chi_{ik}$$

$$= \frac{1}{2} \sum m_i \sum m_k \chi_{ik}$$

$$= \frac{1}{2} \int_0 \underset{M}{\cancel{4\pi\rho r^2 dr}} \int_0^\infty 4\pi\rho r^2 dr\, \chi(r) \qquad = 2\pi M \rho \int_0^\infty r^2 \chi\, dr$$

Es könnte [illegible] $= \frac{nB}{2}$ [illegible] $B =$ [illegible] ∞ [illegible]

[illegible] B [illegible] ; [illegible] B [illegible]

[illegible] $\overset{W=}{\frac{1}{2} \sum m U}$ [illegible] 1). ein Summand fort

2). U wird bei den betrachteten Teilchen kleiner und diese beiden Wirkungen sind gleich groß $= B$

also im Ganzen Arbeit $\delta W = B$

Wenn nun aber der entstandene Hohlraum wieder verschwindet so wird letztere Arbeit $= \frac{1}{2} B$ wieder gewonnen; also wirkliche innere Verd[illegible] für 1 Teilchen $= \frac{B}{2}$!

$$dN = A e^{-h(U+L)}\, du\, dv\, dw \ldots dx\, dy\, dz$$

$$= A e^{-h(U + L_a + L_i)}\, d\omega_a\, d\omega_i\, dv$$

$$N_u = A e^{-hU} dv \int e^{L_a + L_i}\, d\omega_a\, d\omega_i = B e^{-hU} dv$$

$$= N_{ges}\, e^{-hU} dv$$

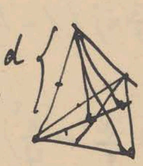
$$\frac{\rho_f}{\rho_g} = e^{-hU} = e^{-h\int}$$

$$p = \frac{mc^2}{3\pi(d-2r)^3}$$

$$\cancel{b = \frac{3}{4}(d-2r)^3}$$

$$4\,\frac{4}{3}\pi r^3 = b \qquad 2r = \sqrt[3]{\frac{3b}{2\pi}} \qquad p = \frac{mc^2}{3\pi\left(\sqrt[3]{v} - \sqrt[3]{\frac{3b}{2\pi}}\right)^3}$$

$$\cancel{d^3 = v} \qquad \cancel{d = \sqrt[3]{v}}$$



$$h = \sqrt{d^2 - \left(\frac{2}{3}\,\frac{d}{2}\sqrt{3}\right)^2}$$

$$= d\sqrt{1 - \frac{1}{3}} = d\sqrt{\frac{2}{3}}$$

$$d\,\frac{1}{2}\sqrt{3}$$

$$p = \frac{mc^2}{3\pi v\left(1 - \sqrt[3]{\frac{3b}{2\pi v}}\right)^3}$$

$$\frac{4}{3}\pi r^3 = b_1 \qquad r = \sqrt[3]{\frac{3b_1}{4\pi}} \qquad p = \frac{mc^2}{3\pi\left(\cancel{\sqrt[3]{v}} - \sqrt[3]{\frac{6b_1}{\pi}}\right)^3}$$

$$\cancel{2}\frac{d}{\cancel{2}} \cdot \frac{d}{2}\sqrt{3} \cdot d\sqrt{\frac{2}{3}} = \frac{d^3}{\sqrt{2}} = v \qquad d = \sqrt[3]{v\sqrt{2}} \qquad \sqrt{2} = \frac{3b}{2\pi v}$$

$$\frac{b}{v} = \frac{\pi\sqrt{8}}{3}$$

$$\boxed{p = \frac{mc^2}{3\pi\left(\sqrt[3]{v\sqrt{2}} - \sqrt[3]{\frac{6b_1}{\pi}}\right)^3}} = \frac{mc^2}{\left(\sqrt[3]{3\sqrt{2}\pi v} - \sqrt[3]{18 b_1}\right)^3} = \frac{mc^2}{3\pi\sqrt{2}\cdot v\left[1 - \sqrt[3]{\frac{6}{\pi\sqrt{2}}\,\frac{b_1}{v}}\right]^3}$$

$$\frac{6\, b_1}{\pi\sqrt{2}} = \mathcal{B} \qquad p = \frac{n\, m\, c^2}{3\pi\sqrt{2}\,.\,v\left(1-\sqrt[3]{\frac{\mathcal{B}}{v}}\right)^3}$$

$$\left[\frac{\mathcal{B}}{v}\right]^{\frac{1}{3}} = \left[1 \cancel{+} - \frac{v-\mathcal{B}}{v}\right]^{\frac{1}{3}} = 1 - \frac{v-\mathcal{B}}{3v}$$

$$p = \frac{n\, m\, c^2}{3\pi\sqrt{2}\,.\,v\left(\frac{v-\mathcal{B}}{3v}\right)^3} = \frac{n\, m\, c^2}{\cancel{3}\pi\sqrt{2}}\,\frac{9\, v^2}{(v-\mathcal{B})^3} = n\, m\, c^2\, \frac{3}{2}\,\frac{\mathcal{B}}{b_1}\,\frac{v^2}{(v-\mathcal{B})^3}$$

$$p v = n\, m\, c^2 .\, \frac{9}{\pi\sqrt{2}}\;\frac{1}{\left(1-\frac{\mathcal{B}}{v}\right)^3} = \frac{9}{\pi\sqrt{2}}\,\frac{[illegible]}{\underline{n m^2}}\left[1 + 3\,\frac{\mathcal{B}}{v}\right.\,.$$

$$\frac{1+\frac{2}{3}x}{1-\frac{x}{3}} = \left(1+\frac{2}{3}x\right)\left(1-\frac{x}{3}\right)^{-1} = .$$

$$= \left(1+\frac{2x}{3}\right)\left(1+\underset{\frac{2x}{3}}{\frac{x}{3}} + \underset{\frac{2x^2}{9}}{\frac{x^2}{9}} + \underset{\frac{2x^3}{27}}{\frac{x^3}{27}}\right)$$

$$= 1 + x + \frac{x^2}{3} + \frac{x^3}{9}$$

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$$W_i' = \sum\sum \overline{r_{hk} f(r)_{hk}} = \frac{1}{t}\int_0^t \sum\sum r\, f(r)\, dt$$

el. Rad.

$$= m\sigma \sum \gamma$$ ~~[illegible]~~

$$= m\sigma N \cdot \overline{\gamma}$$ relat. [illegible]

$$\Sigma_x \gamma = N.\overline{\gamma}$$

~~$N = \frac{\bar{c}}{\lambda} n$~~

$$\gamma = \bar{g} \int_0^{\frac{\pi}{2}} \sin\theta\, d\theta \cos^2\theta = \frac{\bar{g}}{3} = \frac{\bar{c}\sqrt{2}}{3}$$

$$= m\sigma \frac{\bar{c}\,\bar{\gamma}}{\lambda} n \qquad \bar{g} \overset{?}{=} \bar{c}\sqrt{2}$$

$$\lambda = \frac{V}{\sqrt{2}\,\pi\sigma^2 n\left(1+\frac{5b}{8v}\right)}$$

$$= \frac{\sigma}{\lambda}\frac{nm\bar{c}^2}{2}\frac{2\sqrt{2}}{3}$$

$$2L = 3pV - \frac{2\sqrt{2}}{3}\frac{\sigma}{\lambda} L - W_i''$$

$$L\left[1+\frac{\sqrt{2}}{3}\frac{\sigma}{\lambda}\right] = \frac{3}{2}pV - \frac{1}{2}W_i''$$

$$\frac{V}{nm} = v \qquad \frac{2\pi N\sigma^3}{3pm} = b$$

$$L\left[1+\left(1+\frac{5b}{8v}\right)\frac{2\pi\sigma^3 n}{3V}\right]$$

$$\frac{b}{v} = \frac{2\pi\sigma^3 n}{3V}$$

$\parallel$

$$\frac{nm\bar{c}^2}{2} \qquad \bar{c}^2 = 3rT$$

$$W_i'' = 3\rho^2 a V$$

$$rT\left[1+\frac{\sqrt{2}}{3}\frac{\sigma}{\lambda}\right] = pv + \underbrace{\frac{1}{3}\frac{W_i''}{nm}}_{\frac{a}{v}}$$

$$p + \frac{a}{v^2} = \frac{rT}{v}\left[1+\frac{\sqrt{2}}{3}\frac{\sigma}{\lambda}\right]$$

Nach Jäger für Flüssigkeiten und nach Sutherland für feste Körper

$\lambda = d - \sigma$

Sonst

$$p + \frac{a}{v^2} = \frac{rT}{v}\left[1 + \frac{\sqrt{2}}{3}\frac{\sigma}{d-\sigma}\right] \qquad \frac{d}{\sigma} = \sqrt[3]{\frac{v}{v_0}}$$

$$= \frac{rT}{v}\left[1 + \frac{\sqrt{2}}{3}\frac{1}{\sqrt[3]{\frac{v}{v_0}} - 1}\right] \qquad \sqrt[3]{\frac{v}{v_0}} = \left[1 + \frac{v-v_0}{v_0}\right]^{\frac{1}{3}} = 1 + \frac{1}{3}\frac{v-v_0}{v_0}$$

$$= \frac{rT}{v}\left[1 + \frac{\sqrt{2}}{\cancel{3}}\frac{1}{\cancel{\frac{1}{3}}\frac{v-v_0}{v_0}}\right] = \frac{rT}{v}\left[\frac{v - v_0 + v_0\sqrt{2}}{v-v_0}\right]$$

$$= \frac{rT}{v}\frac{v_0\sqrt{2}}{v-v_0} = \frac{rT\sqrt{2}}{v-v_0}$$

~~$pv - \ldots v_0 + \frac{a}{v} - \frac{a v_0}{v^2} = rT\sqrt{2}$~~

~~$\frac{a}{v} - \frac{a v_0}{v^2} = rT\sqrt{2}$~~

~~$\frac{dv}{dT}\left[\frac{a}{v^2} - \frac{2 a v_0}{v^3}\right] = -r\sqrt{2}$~~ ~~$\alpha = \frac{1}{3}\frac{1}{v}\frac{dv}{dT}$~~

$v - v_0 = rT\sqrt{2}\,\frac{v_0^2}{a}$

prakt.: $\frac{a}{v_0^2}(v - v_0) = rT\sqrt{2}$

$\frac{a}{v_0^2}\frac{dv}{dT} = r\sqrt{2}$

$< \frac{a}{v_0^2}(v-v_0)$

$$\alpha = \frac{1}{3}\frac{1}{v}\frac{dv}{dT} = \frac{v_0}{3}\frac{r\sqrt{2}\,T}{a\,T}$$

$$= \frac{v-v_0}{3v_0T} = \frac{r\sqrt{2}}{3}\frac{v_0}{a} = \frac{r}{p}\frac{\sqrt{2}}{3}\frac{1}{a}$$

$$dp \cancel{+} - \frac{2a}{v^3}\,dv = -\frac{rT\sqrt{2}}{(v-v_0)^2}\,dv \qquad \cancel{k = \frac{1}{v}\frac{dv}{dp}} \quad k = v\frac{dp}{dv}$$

$$v\frac{dp}{dv} = \frac{2a}{v^2} - \frac{rT\sqrt{2}\,v}{(v-v_0)^2} = \frac{2a}{v^2} - \frac{a\,v}{v_0^2(v-v_0)} = a\frac{2vv_0^2 - 2v_0^3 - v^3}{v_0^2v^2(v-v_0)}$$

$$= -\frac{a}{v_0(v-v_0)} = \frac{a^2}{r\sqrt{2}\,v_0^3\,T}$$

$$k = \frac{v_0(v-v_0)}{a} = rT\sqrt{2}\,\frac{v_0^3}{a^2}$$

$$\frac{k}{\alpha} = \frac{v-v_0}{r}\,\frac{3}{\sqrt{2}} \neq \frac{2(v-v_0)}{r} = \frac{3}{\sqrt{2}}\,T\sqrt{2}\,\frac{v_0^2}{a} = \frac{3}{\sqrt{2}}\,\frac{T\sqrt{2}}{\rho_0^2}\,\frac{1}{a}$$

$$a = \frac{r}{\rho}\,\frac{\sqrt{2}}{3}\,\frac{1}{\alpha}$$

$$k = \frac{rT\sqrt{2}}{\rho^3}\,\frac{\rho^2}{r^2}\,\frac{9}{2}\alpha^2 = \frac{9}{2}\,\frac{T\sqrt{2}}{\rho r}\,\alpha^2$$

$$k = \frac{2}{9}\,\frac{\rho r}{\sqrt{2}\,T\alpha^2} = \frac{\sqrt{2}}{9}\,\frac{r}{T}\cdot\frac{\rho}{\alpha^2} = \frac{\sqrt{2}}{9}\,\frac{R}{T}\,\frac{\rho}{\mu\alpha^2}$$

$$r = \frac{R}{\mu}$$

$$= \frac{1}{\mu}\,\frac{pv}{T} = \frac{1}{\mu}\,\frac{p}{\rho T} = \frac{1}{\mu}\,\frac{\overset{1000}{\cancel{76\cdot13.6}}\cdot 980}{0.00129\cdot273} \qquad \mu = \frac{\text{Atomgew.}}{14.5}$$

$$\frac{\sqrt{2}}{9}\,\frac{R}{T} = \frac{\sqrt{2}}{9}\,\frac{980.000}{(273)^2\cdot0.00129}$$

2.4362

4.8724

0.1106 − 3

0.9542

2.9372

0.30103

0.1505

5.9912

6.1417

2.9372

3.2045

$$J_c = \frac{6.}{\mu}\,42.10^6\ .\ 252$$

$$\frac{R}{\mu} = \frac{10^6\,.\,14.5}{(273)\cdot0.0013} \quad \frac{14.5}{273.0.0013} =$$

$Cu: \frac{\varrho}{\mu\alpha^2}$ $\quad \frac{8{\cdot}8 \;.\; 14{\cdot}5}{63{\cdot}3 \;.\; [168 \,.\, 10^9]^2} = \frac{14{\cdot}5}{\alpha \,.\, \frac{\alpha A}{\vartheta}} = \frac{14{\cdot}5 \;.\; 10^{15}}{168 \,.\, \cancel{} \,.\, 121 \,.\, \cancel{}}$

$$\begin{array}{r} 3{\cdot}2045 \\ 1{\cdot}1614 \\ \hline 4{\cdot}3659 \end{array} \qquad \begin{array}{r} 19{\cdot}3659 \\ -\;4{\cdot}3081 \\ \hline 15{\cdot}0578 \end{array} \qquad \begin{array}{r} 2{\cdot}2253 \\ 2{\cdot}0828 \\ \hline 4{\cdot}3081 \end{array}$$

$1150 \;.\; 10^{12}$

~~Die~~ Orgel Flötenpfeifen haben nun sehr verschiedenen Druck, während dessen ~~[illegible]~~
Rohr gemacht überall verengt. Warum?

a ist Maß für das Schwingungs Volumen

$m\frac{d^2x}{dt^2} = -kx$

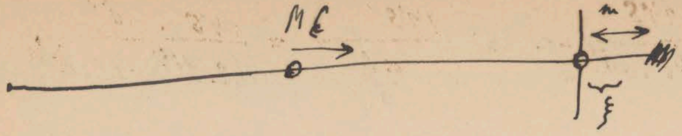
$x = a \sin\sqrt{\frac{k}{m}}\,t$

Bedingung L k dieselbe

$\frac{m a^2 \frac{k}{m}}{2} = C \qquad a = \sqrt{\frac{2c}{k}} \longrightarrow$ also dann auch Schwing. Vol. dasselbe

wenn Kraft, ~~die~~ welche auf d. Rohrwand wirkt, dieselbe ist

~~$m\frac{a^2\alpha^2}{2} = C \quad a = \sqrt{\frac{\frac{2c}{m}}{\alpha}}$~~



$\xi = a \sin \alpha t$

$\xi'|_0 = a\alpha = v$

$\overline{MC^2} = \overline{mc}^2$

$\frac{\overline{MC^2} + \overline{mc^2}}{2} + \overline{V} = \frac{MC_0^2}{2}$

$\overline{V} = \frac{\overline{mc^2}}{2}$

$C^2 = \frac{1}{3} C_0^2$

$M = 2m$

~~$M \neq m$~~

$$\left.\begin{aligned} MV^2 + mv^2 &= MV_0^2 + mv_0^2 \\ MV + mv &= MV_0 + mv_0 \end{aligned}\right\}$$

$M(V^2 - V_0^2) = m(v_0^2 - v^2)$

$M(V - V_0) = m(v_0 - v)$

$V + V_0 = v + v_0$

$V - v = -V_0 + v_0 \quad | m | -M$

$V = \frac{(M-m)V_0 + 2mv_0}{(M+m)} = \frac{V_0 + 2v_0}{3}$

$v = \frac{~~(m-M)v_0 + 2MV_0~~}{M+m} = \frac{-v_0 + 4V_0}{3}$

$\frac{v_0(m-M) + 2MV_0}{m+M}$

$$p + \frac{a}{v(v-b)} = \frac{R\theta}{v-b}$$

$$p_x = \frac{a(2v-b)}{(v-b)v^2} - \frac{a}{v(v-b)}$$

$$= ~~\frac{a\,2v^2 - bv - 2bv + b}{\quad}~~$$

$$= a\,\frac{2v-b-v}{(v-b)^2 v^2} = \frac{a}{v_x^2(v_x-b)}$$

$$p = \frac{R\theta}{v-b}\left[\cancel{1} - \frac{a}{v(v-b)}\right.$$

$$\frac{\partial p}{\partial v} = -\frac{R\theta}{(v-b)^2} + \frac{a\,(2v-b)}{v^2(v-b)^2} = 0 \quad \Big\| \quad \frac{R\theta}{\cancel{v-b}} = \frac{a(2v-b)}{v^2}$$

$$\frac{\partial^2 p}{\partial v^2} = \Big\|$$

$$\frac{d}{dv}\left(-R\theta + \frac{a(2v-b)}{v^2}\right) = 0$$

$$2v^2 - 2v(2v-b) = 0$$

$$2vb = 2v^2$$

$$v_k = b$$

$$p = \frac{R\vartheta}{v-b}\, e^{-\frac{c}{R\vartheta v}} = \frac{R\vartheta}{v-b}\left[1 - \frac{c}{R\vartheta v} + \frac{1}{2}\frac{c^2}{R^2\vartheta^2 v^2} - \right]$$

$$p = \frac{R\vartheta}{v-b} - \frac{c}{v(v-b)} + \cdots$$

$$p = -\frac{a}{v^2} + \frac{rT}{v}\,\frac{v+\frac{2}{3}b}{v-\frac{1}{3}b} = \frac{rT}{v}\,\frac{1+\frac{2}{3}\frac{b}{v}}{1-\frac{1}{3}\frac{b}{v}} = \frac{rT}{v}\left(1+\frac{2b}{3v}\right)\left(1+\frac{1}{3}\frac{b}{v}\right.$$

$$\frac{\partial p}{\partial v} = -\frac{2a}{v^3} \pm \frac{rT}{v^2}\,\frac{(v-\frac{1}{3}b)-(v+\frac{2}{3}b)}{(v-\frac{1}{3}b)^2} = -\frac{2a}{v^3} + \frac{rTb}{v^2(v-\frac{1}{3}b)^2} = 0$$

$$-\frac{2a}{v} + \frac{rTb}{(v-\frac{1}{3}b)^2} = 0 \;\Big|\; \frac{2a}{v} = \frac{rTb}{(v-\frac{b}{3})^2}$$

$$\frac{\partial p}{\partial v^2} = \frac{2a}{v^2} - \frac{2rTb}{(v-\frac{1}{3}b)^3} = 0$$

$$\frac{a}{v^2} = \frac{rTb}{(v-\frac{1}{3}b)^3} = \frac{rTb}{2v(v-\frac{b}{3})^2}$$

$$2v = v - \frac{b}{3}$$

$$\left(1-\frac{b}{3v}\right)^{-1} = \left[1+\frac{1}{3}\frac{b}{v}+\frac{1}{9}\left(\frac{b}{v}\right)^2+\cdots\right]\left[1+\frac{2}{3}\frac{b}{v}\right]$$

$$\left.\begin{array}{l} 1+\frac{1}{3}\frac{b}{v}+\frac{1}{9}\left(\frac{b}{v}\right)^2+\cdots \\ +\frac{2}{3}\frac{b}{v}+\frac{2}{9}\left(\frac{b}{v}\right)^2+\cdots \end{array}\right\} = 1+\frac{b}{v}+\frac{1}{3}\left(\frac{b}{v}\right)^2+\cdots$$

~~$\alpha = \frac{\delta}{3\theta v}$~~ ~~$\beta = \frac{2\alpha}{9}\frac{\delta^3}{b^2R}$~~

~~$p+\frac{a}{v^2} = \frac{R\theta v^2}{\delta}$~~ ~~$\frac{9}{2}R\theta\frac{v^2}{\delta^3}$~~

Nach Jäger:

$$\left(p+\frac{a}{v^2}\right) = 9\frac{b^2 R\theta}{2(v-b)^3}$$

$$1 - \frac{2a}{v^3}\frac{dv}{dp} = -9\frac{b^2R\theta}{2}\frac{3}{(v-b)^4}\frac{dv}{dp}$$

$$1 = \frac{dv}{dp}\left[\frac{2a}{v^3} - \underbrace{\frac{9\cdot 3\, b^2R\theta}{2\,(v-b)^4}}\right]$$

$$\left(p+\frac{a}{v^2}\right)\frac{3}{v-b} = \frac{dv}{dp}\frac{a}{v^2}\left[\frac{2}{v}-\frac{3}{v-b}\right]$$

$$= \frac{dv}{dp}\frac{a\,[2v-2b-3v]}{v^3(v-b)}$$

$$\frac{v^2}{a} = \frac{2(v-b)^3}{9b^2R\theta}$$

$$\frac{dp}{dv} = -\frac{a(v+2b)}{v^3(v-b)}$$

$$\frac{a}{v^2}(v-b)^3 = \frac{9}{2}b^2R\theta$$

$$\left[\underbrace{-\frac{2a}{v^3}(v-b)^3+3\frac{a}{v^2}(v-b)^2}\right]\frac{dv}{d\theta} = \frac{9}{2}Rb^2 = \frac{dv}{d\theta}\,a\left(\frac{v-b}{v}\right)^2\left[\underbrace{-\frac{2(v-b)}{v}+3}_{\frac{v+2b}{v}}\right]$$

~~$\left[-2a(v-b)^3+3av(v-b)^2\right]\frac{dv}{d\theta}\frac{1}{v^3}$~~

~~$\left[-2av^3+6av^2b-6avb^2+2ab^3+3av^3-6av^2b+3avb^2\right]\frac{1}{v^3}\frac{dv}{d\theta}$~~

$$= \frac{a}{v^2}\frac{(v-b)^3}{\theta}$$

$$\frac{1}{v}\frac{dv}{d\theta} = \frac{v-b}{v+2b}\frac{1}{\theta} = \alpha$$

$$\frac{1}{v}\frac{dv}{dp} = \beta = \frac{v^2(v-b)}{a(v+2b)} = \frac{\theta\alpha v^2}{a} = \frac{2\alpha}{9}\frac{(v-b)^3}{b^2R}$$

$\alpha_{Hg} = \frac{1}{5500}$

$v - b = (v + 2b)\,\theta\alpha$

$1 - \frac{b}{v} = \left(1 + \frac{2b}{v}\right)\theta\alpha \mp 3\theta\alpha$

~~280. 3~~

$\frac{3\cancel{00}\;3}{55\cancel{00}} = \frac{9}{55} = \frac{1}{6}$

$b = \frac{5}{6}v$ [wenn $\alpha \to 0$

$b \partial \gamma'$

$R\cancel{A} = \frac{pv}{\theta} = \frac{nm\sigma}{\theta 3} = \rho\frac{c^2}{3\theta}$

$\cancel{(1855)^2} = \frac{(485)^2 \cdot 0{\cdot}00129 \cdot 10^4}{3 \cdot 273}$

2·6857
5·3714
0·1106 − 3
5·4820 − 3
−2·9133
2·5687 = 370 . 10^4

2·4362
0·4771
2·9133

$\rho = \frac{2}{9}\alpha\left(\frac{1}{6}\right)^3 \frac{1}{370 \cdot 10^4} = \frac{2}{9}\,\frac{1}{216}\,\frac{1}{5500}\,\frac{1}{370 \cdot 10^4}$

0·9542
2·3345
3·7404
6·5682
13·5973
3010
13·2963

$= \cancel{4}\; 2.\ 10^{-13}$!

$\rho_{Hg} = 0{\cdot}000004 \cdot \frac{1}{13{\cdot}6 \cdot 76 \cdot 980}$

$= 4.\ 10^{-12}$

also $20 \partial \gamma'$

$(C_2H_5)OH$: $\alpha = 0.0018$.

$$1 - \frac{b}{v} \neq \cancel{2\theta\alpha = 3.000 \cdot 0.0018}$$

$$= 1 \quad \cancel{\text{[illegible]}}\ \text{~~unmöglich!~~}$$

~~[illegible]~~

$$\theta\alpha = \frac{0.54}{1}$$

$$\frac{b}{v} = \frac{1 - \theta\alpha}{1 + 2\theta\alpha}$$

$$\frac{b}{v} = \frac{0.46}{2.08} = 0.23$$

$$\rho = \frac{2}{9}\,\alpha\,(0.23)^{3}\,\frac{1}{370.10^{4}} = \frac{\cancel{4}}{9}\,\frac{\overset{0.0004}{\cancel{0.0018}}\,.\,(0.23)^{3}}{370.\ 10^{4}}$$

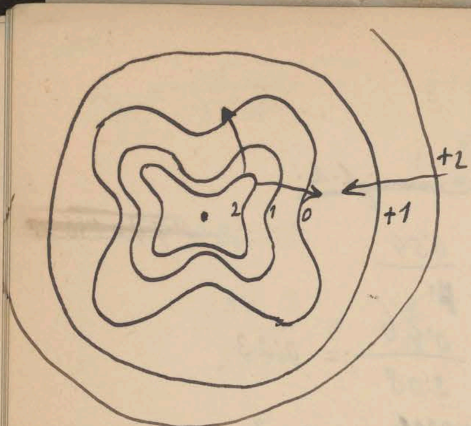
$$\begin{array}{r}
\underline{0.3617 - 1\ .\ 3} \\
0.0851 - 2 \\
\underline{0.6021 - 4} \\
0.6872 - 6 \\
\underline{6.5682} \\
0.1190 - 12
\end{array}$$

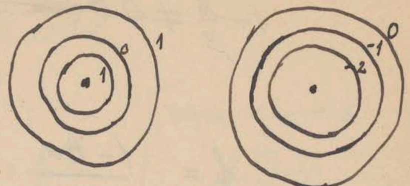
$$= 1.3.\ 10^{-12}$$

In Wirklichkeit:

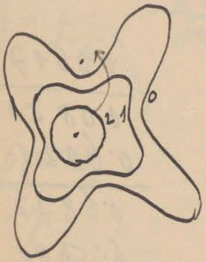
$$0.00009.\ 10^{-6} = 0.9.\ 10^{-10}$$ fast 100 mal so gross!

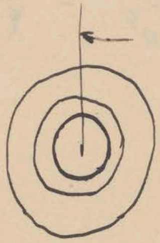
α sollte mit θ kleiner werden, in Wirklichkeit umgekehrt





~~[illegible]~~





$\varphi = \cancel{F_1} + F_2 + F_3 + \cdots$

$= \cancel{(x, y, z)} + \overset{a}{(}x^2 + y^2, z^2) + \cdots$

$X = -\frac{\partial\varphi}{\partial x} = -\Big[\cancel{\frac{\partial F_1}{\partial x}} + \frac{\partial F_2}{\partial x} + \cdots$ $\quad F_1 = 0$

$= -(A_2 x + D y + \cdots) - (A_3 x^2 + \cdots) -$

$\delta\int$ dm ~~[illegible]~~ V_{xyz}
in stabil

$\frac{\partial(F_3 + F_5 + \cdots)}{\partial x} < \frac{\partial(F_2 + F_4 + \cdots)}{\partial x}$

$-\frac{1}{2}(X_x + Y_y + Z_z) = F_2 + \frac{3}{2}F_3 + 2F_4 + \cdots = L$

φ soll < 0 sein $\quad F_2 + F_3 + F_4 + \cdots < 0$

$F_2 + \frac{3}{2}F_3 + 2F_4 + \cdots > 0 > -\varphi$

$\underbrace{3F_3 + 5F_5 + \cdots} < \underbrace{2F_2 + 4F_4 + \cdots}_{>0}$ ~~wenn F_2~~ [illegible]

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Dawniejsza t. g. bez uwzględnienia czasu ~~[illegible]~~ trwania spotkania
miejsca które zajmują [illegible]

stosuje się więc tylko do rzadkich gazów

tłumaczy Boyle Charles zupełnie

tarcie wew., przew. ciepła, dyfuzję qualitatywnie

bo do quantitatywn. zbadania potrzeba prawa siły.

Oprócz tego szczególne zjawiska 2 G, [illegible] etc.

Nowsza t. g. przy uwzględnieniu sił etc.

Fundamenta:

Maxw. ~~[illegible]~~ rozdział en. kin.

Maxw.-Boltzm. rozdział prędkości $h = \frac{1}{\theta}$

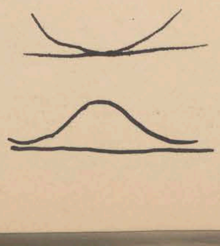
<u>Clausius-Virial</u>

$$\bar{\varphi} = \int \varphi e^{-h\varphi} d\varphi \qquad \nabla\bar{\varphi} = \int \nabla\varphi . e^{-h\varphi}\varphi - h\int \varphi\nabla\varphi . e^{-h\varphi} d\varphi \underbrace{- \nabla h \int \varphi^2 e^{-h\varphi} d\varphi}_{>0}$$

$$\nabla(h\varphi) > 0$$

$$\nabla\left(\frac{\varphi}{\theta}\right) > 0$$

$$\int \nabla(\varphi e^{-h\varphi}) d\varphi < 0$$



$$\frac{2L}{3d^2(d-b)} - p - \frac{\cancel{p}\,2}{\cancel{p}\,d^3}\underbrace{A m \pi \rho \lg \frac{R}{a}}_{\rho} = 0$$

$$p + \frac{a}{v^2} = \frac{R\theta}{d^2(d-b)}$$

$$\cancel{pv+a} = \frac{R\theta\,\cancel{p_4}}{v^{m}\underbrace{\left[1-\frac{b}{d}\right]}_{\frac{1}{3}\frac{v-v_0}{v_0}}}$$

$$= \frac{R\theta.3}{v-v_0}$$

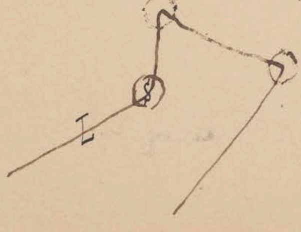
Zanim przejdę do właściwego przedmiotu odczytu z góry ~~zrobić~~ uczynić dwa zastrzeżenia 1). że w obec ogromnej ilości materyału, który się nagromadził w ostatnich latach, muszę zrezygnować z jednakowego traktowania całego przedmiotu i ograniczę się na pewne punkta, które mi się wydają ważniemi, mianowicie zupełnie pominąć muszę teoryę roztworów, bo ta sama przez się już wyrosła do olbrzymich rozmiarów i tworzy całość dziś odrębną która zasługuje na ~~osobne~~ szczególne omówienie, ~~a do tego nie jest w bezpośrednim związku z tak innemi~~ naszemi ~~zagadnieniami~~

2) że nie myślę mówić jako adwokat teoryi kinet., bo to mi się wydaje niepotrzebnem. Wprawdzie walka przeciwko tej teoryi – rozpoczęta przez niektórych fizyków – czasami przyjmuje formy bardzo stanowcze, czego dowód następujący krótki referat o książce Boltzmanna „Über Gastheorie" (w ogólnie znanem czasopiśmie niemieckiem) który tutaj jako Curiosum odczytu przytoczę:

2

Ale przeważna część fizyków przecież przyznaje że zasadniczych błędów (w podstawie tej teoryi) nie wykazano, że ~~ta teorya~~ ona tłomaczy mnóstwo zjawisk inną drogą jeszcze niewyjaśnionych ~~jeszcze~~ i że doprowadziła do nowych wyników i najściślej sprawdzonych, zatem użyteczności jej nie podlega kwestyi. ~~Stąd~~ Ogół więc bez wyjątków [illegible] krytykował, co nie ~~[illegible]~~ wydaje mi się niewątpliwie uzasadnionem, bo sądzę że to się więcej przyda niż wyliczanie wszystkich formułek które na podstawie podobnych rozumowań w ostatnich latach zostały wyśledzone, chociaż że może Panowie odniosą wrażenie iż tytuł mojego referatu mógłby raczej brzmieć: „Braki postępów na polu teoryi kin. materyi."

Dawniejsza teorya kin. t.j. teorya skonstruowana przez Clausiusa i, w znacznie ~~poprawionej~~ doskonalszej formie, przez Maxwella była oparta na pojęciu (t.zw. drogi swobodnej t.j. drogi, ~~podczas~~ na której cząsteczka porusza się z jednostajną prędkością, w tym samym kierunku, aż następuje (dwustronne) spotkanie albo bardzo krótki czas trwające) z inną cząsteczką.



~~[illegible]~~ Średnią energię cząsteczek na tych kawałkach dróg przebytych uważano za ~~[illegible]~~ średnią ~~energię~~ wartość kinet. wogóle ; ~~[illegible]~~

nie uwzględniano 1). ~~zmiany~~ więc zmienności prędkości w bliskości drugiej cząstki (t.j. w obrębie zetknięcia

2). czasu trwania spotkania

3). drogi przebytej podczas tego czasu ~~[illegible]~~ w porównaniu z długością l drogi swobodnej

jednym słowem nie uwzględniano rozciągłości sfery sił wywieranych przez cząsteczki.

To postępowanie oczywiście ~~tu więcej będzie~~ jest usprawiedliwione ~~[illegible]~~ jeżeli gaz jest stosunkowo rozrzedzony; ~~[illegible]~~ jak wszystkie gazy przy ciśnieniu atm.

Tym sposobem wyjaśniono prawo Boyle-Charles-Avogadra: $pv = R\theta$, a jakościowo wyjaśniono zjawiska tarcia wewnętrznego, przewodzenia ciepła i dyffuzyi. ~~Wywód owego prawa jest niezależny od hypotezy co do istoty sił działających przy spotkaniu, podczas gdy ilościowe obliczenie tamtych zjawisk wymagałoby~~ ~~wiadomości~~ ~~takiej hipotezy.~~

A oprócz tego zdołano ~~dało się~~ ~~[illegible]~~ ~~[illegible]~~ ~~można było~~ ~~przewidzieć~~ przepowiedzieć istnienie kilku specyalnych zjawisk, ~~wiadomości odnoszące się do postrzeżeń at~~ których prawa dopiero a posteriori stwierdzono doświadczalnie ~~dając tym sposobem wyraźny dowód~~ co w każdym razie korzystnie świadczy o użyteczności tych teoryi.

Do tego należy przedewszystkiem nagła zmiana (Discontinuität) w stanie gazu ~~w bliskości ściany~~ otaczającego ciała stałe, która się okazuje ~~jako ślizganie się gazu~~ n.p. przy tarciu wewnętrznem i przy przewodzeniu ciepła (także elektryczności ?!)

Według zwykłej ~~[illegible]~~ teoryi hydromechaniki cieczy lub gazów lepkich rozkład prędkości ~~musi~~ miałby być ciągły t.j. nigdzie ~~[illegible]~~ nie mogą graniczyć ~~między któremi istniałaby skończona~~ ze sobą dwie warstwy ~~oddzielone~~ ~~różnicą prędkości~~ ~~[illegible]~~. ~~[illegible]~~ oddzielone prędkościami ~~nigdy~~ różniącemi się o skończenie wielkie

Tak samo przy przewodzeniu ciepła: ~~niemożliwe iżby przy dotykaniu się~~ temperatury dwu ciał ~~[illegible]~~ dotykających się ~~było~~ miałaby być takasama z obu stron ~~[illegible]~~ powierzchni ~~granicznej~~ styczności. Tymczasem przeciwnie ~~takie ślizganie się gazów wzdłuż powierzchni ciał stał~~ teorya gazów wskazuje że ~~podobne zjawisko powinno musi zachodzić~~ ~~nagłe przejście (prędkości albo temp.)~~ (w gazach rozrzedzonych) nagła zmiana, skok, zachodzić musi (na powierzchni oddzielającej gaz od ciała stałego, co tak dla tarcia wewnętrznego (Warburg) jak i dla przewodzenia ~~temperatury~~ ciepła (Sm) zostało doświadczalnie dowiedzionem. Różnica prędkości ~~stycznej~~ ciała stałego i warstwy stycznej gazu oznaczona jest przez $\Delta u = -\beta \frac{\partial u}{\partial n}$, a tak samo skok temperatury $\Delta \theta = -\gamma \frac{\partial \theta}{\partial n}$, przy czem β i γ są ~~[illegible]~~ spółczynnikami (rozmiaru długości) proporcyonalnemi do ~~[illegible]~~ drogi swobodnej λ i nie wiele się od niej różniącemi.

	Cl.-M.	M.	endliche ~~[illegible]~~
η	$\sqrt{\theta}$	θ	
κ	$\sqrt{\theta}$	θ	
D	$\theta^{3/2}$	θ^2	

~~Obliczenie~~

~~Wywód wszystkich tych praw jest [illegible]~~

~~Do wszystkich tych rezultatów jakościowych~~

Do jakościowego tłomaczenia wszystkich tych zjawisk wystarcza przyjęcie ogólnych zasad teoryi kinetycznej, bez specyalnej hipotezy co do ~~na~~ istoty sił działających między cząsteczkami.

Wiadomość tychże jest niezbędna do ilościowo dokładnych obliczeń, (tylko prawo Boyle-Charles-Avogadro tego nie wymaga).

Odnosi się to nietylko do bezwzględnej wartości spółczynników tarcia wewnętrznego η, przewodnictwa ciepła κ, dyfuzyi D, ale zmienności także do ich ~~zmienności~~ zależności od temperatury (a ~~[illegible]~~ ciśnienia) ~~samą~~ aż dotąd tylko pod założeniem że siły są odwrotnie proporcyonalne do 5ej potęgi odległości zdołano wyrachować (Maxwell .. Boltzmann

~~podczas gdy~~ Wszystkie (dotychczasowe) obliczenia tych wielkości ~~oparte na~~ (Clausiusa, Meyera etc.) oparte na hipotezie że cząsteczki wywierają siły (także ~~[illegible]~~ jakby były ~~[illegible]~~ kulami sztywnemi są fałszywe, a właściwa droga (do obliczeń tego rodzaju) wskazana przez Boltzmanna jest niesłychanie mozolna.

~~Co do zmienności tych spółczynników w zależności od temperatury~~ [illegible] że

~~Łatwo jednak wykazać że według drugiej teoryi~~

Spółczynniki ~~k~~, jak ~~[illegible]~~ łatwo [illegible] ~~[illegible]~~ proste rozumowanie wykazuje, musiałyby być proporcyonalne do θ według pierwszego założenia, a do √θ według drugiego, podczas gdy ~~one w rzeczywistości~~ wartości zmierzone między temi

granicami się wahają. Zatem w rzeczywistości inne musi zachodzić prawo siły. (Sutherland ~~przyjął co do tego~~ skombinował obie teorie wytworzył pod tym względem hipotezę, że owe cząstki
Pierwsze są za twarde, drugie za miękkie)
są kulami à la Clausius-Maxwell, które ~~oprócz tego jeszcze wywierają~~ jednak przyciągają się siłą $\frac{a}{r^4}$, ~~więc spopularyzował zwykłą hipotezę~~ t.zn. że przyjął cząsteczki typu Van d. Waalsa.

Pewnie ~~że~~ że dodanie takich sił musiałoby mieć ten skutek że ~~wyglądałyby~~ cząsteczki wydawałyby się mniej „twardemi" t.z. że sfera działalności przy niższej temperaturze, ~~że wskutek mniejszej prędkości cząsteczki już w takiej odległości by się~~ wydawałaby się większą aniżeli przy wyższej temperaturze, i że wskutek tego zmienność owych spółczynników byłaby ~~mniejszą~~ większą aniżeli $\sqrt{\theta}$. ~~Teraz~~ (ale czy to jest wystarczającem do uzasadnienia owego prawa?) Sutherland ma dwie stałe do dyspozycji: ~~wielkość promienia~~ średnicę cząsteczek (i stałą (C) (zawartego) określającą wielkość siły, więc nic dziwnego ~~iż~~ może je tak dobrać że ~~zgadzają się z~~ dla każdego gazu formułka $\eta = \eta_0 \frac{1+\alpha C}{1+\frac{C}{\theta}} \sqrt{1+\alpha\theta}$ zgadza się wystarczająco z doświadczeniami. Ale praw tego rodzaju mnóstwo byłoby można znaleźć.

W prawie Sutherlanda w każdym razie jest zawarta słuszna zasada że siły w większych odległościach mogą być przyciągające w mniejszych silnie odpychające, ale już samo pojęcie kul sztywnych jest niejako naiwną abstrakcyą którą używamy tylko do ułatwienia rachunków i łatwiejszego uzmysłowienia, a żeby prawo Sutherlanda łączące to ~~[illegible]~~ pojęcie z ~~drugiem~~ pojęciem sił przyciągających – miało odpowiadać rzeczywistości, jest to ~~rzeczą~~ hipotezą bardzo nieprawdopodobną przeciwko której w dalszym ciągu jeszcze ważne objekcye uczynić będziemy.

(jest w ogóle przeciwko siłom V. d. W.

Boltzmanna siły $-\frac{1}{r^5}$; Richarz o wartości $\frac{1}{r^n}$ $n > 3$ $n < 8$ (dissoc.)

~~Nowsza teorya kinetyczna~~ Aby módz zastosować teoryę kinetyczną do gazów zgęszczonych, do cieczy i ciał stałych niezbędnem jest uwzględnienie wielkości sfery działania sił odpychających, ponieważ ~~[illegible]~~ przy zbliżaniu się do p. kr. droga swobodna już do tego samego rzędu wielkości się zbliży. ~~gdyż ta już w bliskości punktu krytycznego podobnej wielkości jest jak~~ – To jest celem nowszej teoryi kinetycznej.

Dawniejsze teorye ki. ~~tam~~ używane metody okazują się ~~tu tam~~ niewystarczającemi do pokonania ogromnych trudności ~~z tąd~~ tutaj powstających; ~~a~~ tylko dwa ~~drogi znany~~ ~~aż dotąd, kto~~ instrumenta matematyczne znane aż dotąd, które nam tutaj usługi oddawać mogą i ~~którem~~ na których racyonalna teorya się opierać musi t.j. Clausius'a twierdzenie o wiriale i Maxwella i Boltzmanna prawo o rozdziale energii kinetycznej i prędkości w systemach mechanicznych

~~Zajmiemy się najpierw drugiem twierdzeniem, ponieważ pierwsze nie przedstawia żadnych [illegible]~~

odnosi się do ~~[illegible]~~ ~~konserwatywnego~~ skończonego systemu złożonego z punktów materyalnych między którymi działają siły XYZ (konserwatywne) ~~a na które [illegible]~~ ~~[illegible]~~, ~~pod [illegible]~~

Clausiusa twierdzenie o wiriale ~~[illegible]~~

Oznaczając całk. en. kin. przez L ~~[illegible]~~ wyrażamy zasadę:

$$L\ \cancel{\quad}+\frac{1}{2}\sum(Xx+Yy+Zz)=0$$

gdzie L oznacza całkowitą energię kinetyczną systemu punktów materyalnych, ~~X, YZ składowe całkowitej siły działającej na punkt xyz~~, a suma ~~[illegible]~~ (nazwane wiriałem) odnosi ~~się~~ do wszystkich cząsteczek w-

Dowodzi ~~się~~ tego w bardzo prosty sposób ~~zapomocą~~ zważając że ~~jeśli~~ pod założeniem iż ~~stan~~ systemu jest stały, jakaś bądź suma funkcyj utworzona ~~przez sumowanie~~ prędkości i spółrzędnych musi być ~~niezmienna z czasem~~ niezależna od czasu, zatem także

$$\frac{d}{dt}\sum m\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right)=0$$

co przy wykonaniu różniczkowania daje powyższy wzór, ~~jeśli się~~

Zastosowując ~~[illegible]~~ to twierdzenie do gazu (lub cieczy etc.) poddanego jednakowemu ciśnieniu p w naczyniu o objętości V otrzymuje się, oddzielając p ~~to ciśnienie~~ od reszty sił XYZ, (które mają teraz oznaczać wewnętrzne siły) otrzymuje się

$$L=\frac{3}{2}pv-\sum(Xx+Yy+Zz)$$

~~które~~ co służy jako podstawa do obrachowania równania zasadniczego izotermicznego (Zustandsgleichung) materyi.

Dla porównania ~~[illegible]~~ ~~trudniejsze jest krytyka co do~~ większe trudności nastręcza zrozumienie i dyskusya ~~praw~~ twierdzeń M. i B. ~~Trzymając~~ Zachowując nomenklaturę zaprowadzoną przez Bryana w swem cennym British Ass. Rep. 1894 nazywamy ~~[illegible]~~

prawem Maxwella co do rozdziału energii kin. następujące prawo:

Jeżeli w systemie dynamicznym konserwatywnym obierzemy ~~prędkości~~ spółrzędne p_1, p_2, p_3 tak, że energia będzie ~~wyrażała się~~ wyrażona ~~[illegible]~~ jako suma kwadratów ich (momentów) $\dot{q}_1 = \frac{dp_1}{dt}$ ~~dt~~ prędkości, to w stanie stałym

$$T = \frac{1}{2}\left[\left(\frac{dp_1}{dt}\right)^2 + \left(\frac{dp_2}{dt}\right)^2 + \dots\right]$$

Średnia wartość każdego z tych kwadratów (t.j. części energii przypadającej na ruch p_n) będzie równa: $\overline{\left(\frac{dp_1}{dt}\right)^2} = \overline{\left(\frac{dp_2}{dt}\right)^2} = \dots$

Tych części będzie tyle ile system ma stopni swobody ruchu.

~~Maxwella~~ Maxwell-Boltzmanna twierdzenie zaś opiewa:

Prawdopodobieństwo że w takim systemie mechanicznym spółrzędne ~~[illegible]~~ mają wartości w obrębie $p_1 + dp_1$, $p_2 + dp_2$ .. a prędkości $\dot{q}_1 + d\dot{q}_1$, ..

jest proporcyonalne do $e^{-hU} dp_1\, dp_2 \dots dq_1\, dq_2 \dots$

gdzie U = energia potencyalna w owym układzie spółrzędnych $p_1 \dots p_n$

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Od czasu co Boltzman i Maxwell ogłosił ~~swe~~ te twierdzenie (1879), zadziwiające prostotą i ogólnością zastosowania, aż do dziś dnia ciągle jeszcze trwa dyskusye co do ścisłości tych twierdzeń, Spór Boltzmanna mianowicie Anglicy ogromnie fizycy czynili aby tę ~~xxxx~~ kwestyę wyjaśnić ale mimo to jeszcze ~~kwestyę~~ nie można jej uważać za całkiem rozstrzygniętą.

Zarzuty robiono pewnym niektórym rozumowaniom użytym przez Maxwella,*) ale te udało się poprawić nie zmieniając wniosków ostatecznych. Głównie jednak przeciwnicy owych twierdzeń opierali się na tem, że można ~~xx~~ wynaleźć systemy mechaniczne, które nie czynią zadość owym twierdzeniom, że zatem ~~xx~~ nie mogą one być ogólnie ważne

Tak n. p. Lorda Kelvina test case 1).

2).

Burnside Watson i Burbury tymczasem ~~jednak~~ starali się wydoskonalić metodę dowodu użytą przez Boltzmanna i później jednak także Burbury przeszedł na stronę skeptyków ~~starał się udowodnić~~ wykazał innymy prawo rządzącem [illegible] w kilku rozprawach i w tym roku wreszcie ogłosił książkę która otwartą wojnę wypowiada tamtym twierdzeniom.

*) Np. E ... S

Burbury przedewszystkiem zwraca się przeciwko zasadzie użytej przez Boltzmanna do udowodnienia trwałości rozkładu prędkości.

~~Uważa on~~ Boltzmann przyjmuje że przed spotkaniem dwóch cząstek prawdopodobieństwa ich prędkości ~~[illegible]~~ są zupełnie niezależne. Więc wogóle jeżeli $f(u_1 v_1 w_1)\ du_1 dv_1 dw_1\ dx_1 dy_1 dz_1$ jest prawdopodobieństwo że cząstka ma prędkości $u_1 v_1 w_1$, leżąca w obrębie $dx_1 dy_1 dz_1$, również $f(u_2 v_2 \dots)$ że jakaś cząstka ma prędkości ~~f~~ $u_2 v_2 w_2$; to prawdopodobieństwo równoczesnego istnienia tych zjawisk jest oznaczone przez iloczyn $f(u_1 v_1 w_1)\, f(u_2 v_2 w_2)\ du_1 dv_1 dw_1\ du_2 dv_2 dw_2$ ale właśnie tylko wtedy jeżeli one są od siebie niezależne.

[Dalsze rozumowanie bardzo proste; ilość spotkań jest proporcyonalna do tego wyrażenia przy spotkaniach zmienia się:

$$\frac{m}{2}(u_1^2+v_1^2+w_1^2)+U_1+\frac{m}{2}(u_2+\dots)+U_2=\frac{m}{2}(u_1'^2+v_1'^2+w_1'^2)+U_1'+\dots$$

~~a [illegible]~~ więc jeżeli f ma ową postać eksponencyalną, to $f_1 f_2$ pozostanie $= f_1' f_2'$

z drugiej strony według znanego twierdzenia Liouvilla $\int du_1 dv_1 \dots d\dots = \int \dots$

$f_1' f_2' (d\dots)$ ~~To~~ oznacza jednak równocześnie ilość spotkań przeciwnych t.j. takich które ~~[illegible]~~ zamieniają prędkości cząstek z $u_1' v_1' \dots$ na $u_1 v_1 \dots$ więc, ponieważ jeden proces będzie równoważony przez drugi to wogóle nie nastąpi zmiana w rozkładzie.]

Burbury jednak powiada że jeżeli jakaś cząstka ma pewną prędkość, to ~~[illegible]~~ ona przez to już wpływa na prawdopodobieństwo prędkości u innych (mianowicie sąsiednich) cząstkach, że tych zjawisk nie można zatem uważać jako od siebie niezależnych.

Zarzut ten zdaje mi się całkiem usprawiedliwionym. Dopóty gaz jest bardzo niesk. rozrzedzony, więc ~~długa~~ wielkość sfery sił niesk. mała w porównaniu z odległościami cząsteczek, oczywiście ruchy ich można przyjąć jako niezależne, ale to założenie nie będzie usprawiedliwionem w gęstych gazach (lub cieczach).

Burbury jednak jeszcze dalej idzie. Usiłuje dowieść, że przyjęcie owego ~~[illegible]~~ rozkładu B. M. doprowadza do ~~owego~~ ~~[illegible]~~ sprzeczności, które można tylko usunąć, przyjmując zamiast – nadzwyczajnie skomplikowaną funkcyę która zawiera w liczniku ogółem kwadratów prędkości także

Rachunków, bardzo skomplikowanych Burbury'ego, przeciwko którym niejeden można zarzut podnieść, nie mogę tutaj ~~sz~~ szczegółowo roztrząsać, wspomnę tylko że w jednym z ostatnich zeszytów Wiedemanna An- pan Zemplen Győző kwestye podniesione przez Burb. objaśnia w zadawalający sposób, tak że nie domniemane sprzeczności znikają ~~potrzebujemy~~ ~~jakiejkolwiek zmiany do~~ ~~przemiany prawa B. M.~~ słuszność ~~zarzutów~~ zarzutu przeciwko metodzie dowodu użytej przez B.), nie wynika z tego jednak żeby rezultat był fałszywy.

~~Ale~~ – (Możemy ~~[illegible]~~ przyznać ~~Zarzut przeciwko~~ metodzie dowodu użytej przez Boltzmana ~~jest słuszny~~, ~~ale na szczęście~~ (mamy jeszcze zupełnie odmienny sposób wywodu Utwierdza nas w tym zapatrywaniu to że) oparty na pracy Maxwella i wydoskonalony przez ~~[illegible]~~ Lorda Rayleigha, który zawsze ~~[illegible]~~ stawał po stronie obrońców owego prawa.

Wystawmy sobie system mechaniczny oznaczony przez n współrzędnych $p_1 - p_n$. Jeżeli te współrzędne mają wartości dane nazwiemy to „konfiguracyą" systemu; ~~jk~~ ~~jeżeli~~ konfiguracya będzie się wciąż ciągle zmieniała, ale może znów stanąć się tą samą co pierwotnie. W danej konfiguracyi współrzędne ~~mają pewne~~ mogą mieć ~~[illegible]~~ prędkości, co oznaczamy przez wyrażeniem ~~[illegible]~~ „faza". Wyobr. sobie podwójnie niesk. wiele systemów we wszystkich możl. konf. i prędk.

Wyobraźmy sobie teraz równocześnie ~~ten sam system~~ nieskończenie wielką ilość takich samych systemów, które wszystkie są (w tej samej konfiguracyi [albo właściwie w konfiguracyi wahającej się między granicami $p_1 - p_1+dp_1$ itd.] i ~~[illegible]~~ mają tą samą energię całkowitą [albo właściwie z energią ~~[illegible]~~ zawartą między E i ~~[illegible]~~ $E+dE$]. Wtedy jeszcze sposób w jaki tą energia całkowita składa się z energii kinetycznych pojedyńczych ruchów itd. ~~[illegible]~~, ~~więc~~ jest dowolny więc faza jest do pewnego stopnia dowolna. Ułożymy prędkości w ten sposób, że wszystkie fazy będą zawarte, które ~~zgadzają się~~ odpowiadają danym granicom energii, ~~i do tego~~ ~~to w~~ a ilość systemów pewnej fazy oznaczamy przez założenie rozkłada w pewnym sensie równomiernego t.j. ilość systemów fazy zawartej między p_1 i p_1+dp_1 ma być proporcyonalna do e^{-hL^2} $dp_1\, dq_2 \ldots dq_n\, dp_1 \ldots dp_n$

~~a takich gromad systemów wyobraźmy sobie jeszcze niesk. wiele, tj. że wszystkich możliwych~~

~~[illegible]~~ ~~a te systemy konfigura~~

Wtedy twierdzimy że cały ten układ będzie stały t.j. że ilość systemów pewnej fazy i konfiguracyi ~~pozostanie~~ ~~[illegible]~~ niezmieni się z czasem. jeżeli pozostawimy ~~[illegible]~~ wszystkie systemy same sobie, tak że nastąpi w nich ruch [illegible]

Łatwo tego dowieść. W skutek ruchu nie zmieni się całkowita energia tylko systemy obrębu p_1 p_1+dp_1 / KF q_n q_n+dq_n ~~zamienią się~~ przyjmą inną konfiguracyę i fazę: p_1' $p_1'+dp_1'$ / K'F' q_n' $q_n'+dq_n'$

~~te systemy ilo~~ ilość systemów, które dawniej miały ową konf i fazę K'F' była według założenia $A\,dp_1' \dots dq_n'$, ~~te~~ one wskutek ruchów ~~ina~~ zamieniły się w K''F'', ale za to przybyły owe ~~KF~~ systemy K F których ilość jest $A\,dp_1 \dots dq_n$. Znane twierdzenie Liouvilla jednak opiewa że przy wszystkich ruchach mechanicznych $dp_1 \dots dq_n = dp_1' \dots dq_n'$ ~~[illegible]~~ nie zmienią się w skutek ruchu. Zatem następuje tylko cykliczna przemiana konf. i faz, tak że ilość każdego rodzaju pozostaje niezmienna.

Następuje pytanie ~~ile~~ między temi wszystkiemi systemami jest takich jakie np. q_1 ma pewną wartość $q_1 - q_1+dq_1$ a inne spółrzędne wszystkie mniejsze wartości ~~[illegible]~~ t.j. jakie będzie prawdopodobieństwo—

Całkowitą ilość systemów znajdujemy: ~~[illegible] całą przestrzeń~~

$$\frac{\int dq_1 \dots dq_n}{}$$

$\int\int\int dq_n$

$$T+V=E=V+\tfrac{1}{2}(q_1^2+\dots q_n^2)$$

* $q_1\,dq_1 = dE$ $\qquad dq_1=\frac{dE}{q_1}=\frac{dT}{p_1}$

$$dE\int\frac{dp_2\dots dp_n}{q_1} = dE\int\frac{dq_2 \dots dq_n}{\sqrt{E-V-q_2^2-\dots q_n^2}}$$

przyczem ~~[illegible]~~ uważać należy że $q_2^2 + \dots - q_n^2 < T$

Otrzymuje się $N = \frac{\left[\Gamma(\frac{1}{2})\right]^n}{\Gamma(\frac{n}{2})} \left[2E - 2V\right]^{\frac{n}{2}-1}$

a jako ~~stosunek~~ ilość systemów gdzie $p_1 \dots p_1 + dp_1$ do całkowitej ilości:

$$n_{p_1} = \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{1}{2})\,\Gamma(\frac{n}{2}-\frac{1}{2})} \left[1 - \frac{p_1^2}{2T}\right]^{\frac{n}{2}-\frac{3}{2}} \frac{dp_1}{\sqrt{2T}}$$

[Naturalnie $\int_{\pm\sqrt{2T}}^{+\sqrt{2T}} n_{p_1} = 1$]

jeśli w tym wzorze zrobimy ~~[illegible]~~ $n = \infty$ to

$$n_{p_1} = \frac{e^{-\frac{p^2 n}{4T}}}{\sqrt{2\pi}} \frac{dp}{\sqrt{\frac{2T}{n}}}$$

a jeśli stawimy $\frac{T}{n}$ = średniej energii kin. jednej współrzędnej =

$$\cancel{\Gamma(\tfrac{n}{2}) = e^{-\frac{n}{2}} \left(\tfrac{n}{2}\right)^{\frac{n}{2}} \sqrt{n\pi}} \qquad \cancel{\Gamma(\tfrac{1}{2}) = \sqrt{\pi}}$$

$$\left(\frac{n}{2e\pi}\right)^{\frac{n}{2}} \sqrt{n\pi}$$

$$\cancel{\int \frac{\Gamma(\frac{1}{2})\,\Gamma(\frac{n}{2}-\frac{1}{2})}{\Gamma(\frac{n}{2})}}$$

$$\int \frac{\left[\Gamma(\frac{1}{2})\right]^{n-1}}{\Gamma(\frac{n}{2}-\frac{1}{2})} \underbrace{\left(1 - \frac{p_1^2}{2T}\right)^{\frac{n}{2}-\frac{3}{2}} (2T)^{\frac{n}{2}-\frac{3}{2}}}_{(2T - q_1^2)^{\frac{n-3}{2}}} dq_1\, dp_1 \dots dp_n$$

~~[illegible]~~

Trudność przy tym dowodzie polega na tem żeby ~~wykazać~~ ~~pod~~ czy i pod jakimi warunkiem ten rozkład jest jedynie możliwym rozkładem.

Warunek konieczny do tego jest w każdym razie założenie Maxwella że system musi być tego rodzaju iż prędzej czy później przejdzie przez każdą konfiguracyę z każdą możliwą fazą. Zdaje się że to jest także warunek wystarczający; chociaż nie zdołano tego jeszcze dowieść ~~możnaby~~ wprowadzić ~~pokazać że~~ taki rozkład $f_n =$ $f(p_1 \dots p_n)\,dp_1 \dots dp_n$ jest stałym, gdzie $f(p_1 \dots p_n) = c$ oznaczałoby jedną z całek ruchu

Nie jest zawsze rzeczą łatwą orzec czy dany system jest tego rodzaju, ~~tego czasem nie łatwo~~ iż spełnia ów warunek, ~~to czasem nie tak łatwo orzec~~. Owe ~~[illegible]~~ przedtem wspomniane przykłady które przeciwnicy przytaczali jako contra-argumenty mają to wspólne, że nie czynią ~~[illegible]~~ zadość temu warunkowi więc nie można je uważać jako trafne. ~~Prosty przykład to jeszcze~~ Są to ~~jednak~~ w pewnym sensie przypadki osobliwe.

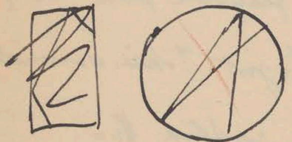
Prosty przykład posłuży do objaśnienia.

ale niezwykły tylko koło

Wyobraźmy sobie ~~zwykły~~ bilard, idealnie gładki tworzący idealny

ścian prawie żadnej siły.. ale tylko pod ogromnym warunkiem prowadzenia

~~czworokąt~~. Jeśli rzucimy kulę w kierunku (~~idealnie~~) prostopadłym ~~całej do~~ ~~jednej ściany~~, to ruch pozostanie periodyczny, prostodrożny, ona będzie się ~~stale~~ wciąż w ten sam punkt odbijała. Jeśli rzucimy pod jakimś kątem, ~~nastąpi~~



odbijanie nastąpi w ten sposób, że kula z czasem po ~~[illegible]~~ całem polu koła przejdzie — z wyjątkiem ~~koła~~ obszaru pewnego w środku — jeśli przypadkiem ten kąt nie był tak obrany

że φ = w którym to wypadku punkt

(Ruch byłby „geordnet" Boltzmanna)

powracałby się tylko po przekątnych w boku. Prawdopodobieństwo żebyśmy właśnie utrafili taki kąt (matematycznie ściśle) jest nieskończenie małe, ponieważ jest nieskończenie więcej liczb niewymiernych niż wymiernych. Zresztą jednak przez każdy punkt płaszczyzny kula przejdzie tylko w dwu kierunkach.

foremnem

Jeśli jednak koło nie jest idealnie regularnym tylko w jakimś miejscu ma małą nieregularność, ~~to z czasem~~ np. ~~[illegible] eliptyczne~~ to ~~z czasem~~ po każdym obiegu kuli kierunek jej trochę się zmieni i z czasem powierzchnia zostanie pokrytą siecią dróg we wszystkich kierunkach. Wtedy więc

~~zamieszanie~~ $\overline{u^2} = \overline{v^2}$ we wszystkich punktach [illegible].

będzie ~~mniej~~

~~Ale wzór M. B. co do rozkładu~~ prędkości ~~więc jeszcze~~ używać ~~nie można~~

~~bo nie można wyłączyć przyjmując wszystkie możliwe konf.~~, ale

~~jeśli ustalić [illegible] [illegible] [illegible] zderzających się~~

~~Inny~~ Dajmy t na to że istnieje teraz wielka ilość takich punktów czyli kul

które jednak na siebie nie działają (przechodzą przez siebie jak duchy).

Czy tu można użyć tw. O M? Oczywiście nie, bo każda kula nie zmieni

swojej dowolności początkowej więc system nie przyjmie wszystkich faz możliwych.

Dopiero jeśli przyjmujemy że one się zderzają jak kule sprężyste albo w

jakiś inny sposób na wzaj. działają, wtedy nastąpi wymiana prędkości

i wtedy ~~można~~ będzie się stosowało nasze twierdzenie.

Inny przykład: Kule w kierunku prostym, odbijające się w A i B
prędkości początkowe $c_1 c_2$ ~

Jeśli masy równe, to zawsze ~~jedna~~ jakaś kula będzie miała

prędkość c_1 jakaś inna c_2 etc. . Dopiero gdy jakaś przyczyna mącąca

regularność n.p. mała różnica w masie, albo uderzenie ekscentryczne etc.

nastąpi rozkład M.D.

Szczególnie ostrożnym należy być ze zastosowaniem M.D. do systemów

drgających.

Można powiedzieć ~~warunek~~ warunkiem jest niestałość (Unstabilität) ruchów składowych,

t.j. jeśli warunki początkowe trochę odmienne to ruch pojedynczych składowych ~~coraz więcej się będzie~~ nie będzie się wahał koło

od pierwotnego typu tylko będzie się coraz więcej od niego oddalał.

[Podczas gdy pewne wartości jako funkcje od począt. warunków coraz mniej zależne]

Zdaje się że w gazach, cieczach etc. w każdym razie te warunki są spełnione

z bardzo wielkim przybliżeniem

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Że rozkład B.M. jest jedynym możliwym rozkładem tego B. dowiódł — ale tylko dla gazów rozrzedzonych za pomocą sławnego swego „Minimumtheorem". ~~fi~~ ~~funkcji H.~~ Do gazów zgęszczonych etc. dowodu tego z przyczyn dawniej wspomnianych nie można zastosować, ponieważ używa obliczenia ilości spotkań -- Polega on na ~~stosowaniu~~ ~~sławnej~~ wprowadzeniu funkcji $H = \Sigma f \lg f$ udt, którą Boltzmann identyfikuje z entropią gazu (z ujemnym znakiem).

Pokazuje on, że $\frac{dH}{dt}$ może mieć tylko znak ujemny, zatem H dąży do Minimum, gdzie $\frac{dH}{dt}=0$, i ~~stąd pokazuje się że z wyglądu że~~ a z tego równania otrzymuje się jako ów rozkład stały, do którego gazy dążą : rozkł. B.M.

Przeciwnicy teorii kinetycznej ~~[illegible]~~ zaprzeczali przedewszystkiem ~~[illegible]~~ słuszności tej interpretacji prawa entropii.

Mówiono tak: ~~jeżeli czasowi dt dam znak ujemny, to $\frac{dH}{dt}$ stanie się dodatnim, a mogę to uczynić, bo znaczy to tylko że wszystkim prędkościom~~ Jeśli wszystkie dążności ($\frac{dx}{dt}$ etc.) obiorę się ze znakiem ujemnym (więc tak samo wielkie ale w przeciwnym kierunku) to muszę ~~dt~~ ~~uczynić~~ uwzględnić to w owym wzorze dając dt znak ujemny, z czego by wynikła konsekwencja $\frac{dH}{dt} > 0$.

~~I z drugiej wszystkie~~ To jest prawda, gdyby się wszystkim prędkościom dało znak ujemny, świat by się cofał w tył, ~~całe życie~~ wszystkie procesa następowałyby ~~dawnego~~ wstecz

w przeciwnym kierunku i także prawo entropii trzeba by odwrócić.
Maxwella armia demonów mogłaby to uskutecznić my jednak nie możemy.
Teraz oni jednak mówią: prędkości dodatnie i ujemne są równie prawdopodobne więc i równie prawdopodobnem jest żeby się entropia zwiększała jak zmniejszała.

Otąd to powstaje z tego że nie określamy bliżej co rozumiemy pod prawdopo[dobieństwem] w danym razie — jakie są te możliwe przypadki których ilość jest mianownikiem we wyrażeniu dla prawdopodobieństwa

Jeżeli my obieramy n.p. cząsteczki mające równe prędkości i (Wiedząc jedno naczynie zam) dajemy im różne kierunki — nie wiedząc o wielkości ~~prędkości~~ ich które posiadają — to pewnie będzie rzeczą równie prawdopodobną że $\frac{dH}{dt}$ dodatnie jak ujemne, bo $\frac{dH}{dt}$ będzie właśnie $=0$

Tzn. najprawdopodobniejszy jest wtedy stan zupełnie „nieporządny", stan stały gdzie panuje rozkład B.M.

~~Byłoby n.p. rzeczą nadzwyczajnie nieprawdopodobną, żebyśmy przypadkowo do górnej części naczynia włożyli same cząsteczki~~ ~~szyb~~ ~~obdarzone ruchem w kierunku +X a do dolnej w kierunku −X. Jeżeli się to jednak stało, to będzie rzeczą nadzwyczajnie prawdopodobną że $\frac{dH}{dt}$ będzie ujemnem~~

Jeżeli jednak system rzeczywisty fizyczny nam jest faktycznie dany, to
~~musi~~ „prawdopodobieństwo" pewnego układu jego
możemy uważać układ pewnych warunków początkowych jako dany, co do których
nie możemy mówić o żadnym prawdopodobieństwie, bo nie wiemy ~~wskutek jakich~~
jakie czynniki ~~jako~~ rozkład spowodowały, innemi słowami nie wiemy ~~[illegible]~~
historyi przeszłej systemu, więc wtedy rachunek może tylko znaczyć
prawdopod. że w pewnym momencie czasu syst. dany przyjmie owe ruchy
a co do tego nie mamy żadnego prawa twierdzić że one będzie równe
dla rozkładu prędkości ~~dodatnych i ujemnych.~~ i analogicznego przeciwnego.
Rzecz wyjaśni się jeszcze lepiej ~~wskutek dyskusyi~~ przez uwzględnienie
twierdzenia ogólnego mechanicznego Poincarégo że ~~wszystkie ruchy~~
ruch jakiegoś bądź systemu ~~dowolnego~~ konserwatywnego (skończonego) zawsze po upływie jakiegoś czasu
się powtórzy więc że będzie okresowy. Zermelo twierdził że ~~[illegible]~~ wskutek tego
~~więc~~ twierdzenie o entropii nie mogłyby zastosowane do takich systemów, więc
że ~~[illegible]~~ ono nie da się wytłomaczyć mechanicznie. Boltzmann ~~przeciwnie~~
pokazał odpowiedział jednak że nie zachodzi żadna sprzeczność jeżeli się owe
prawo entropii uważa jako ~~prawo~~ nie jako twierdzenie ~~nie bezwzględne~~
ściśle, tylko jako twierdzenie o prawdopodobieństwie nadzwyczaj wielkiem, tak jak wszystkie
nasze prawa fizyczne empirycznie poznane.

~~[illegible]~~ Jeśli 1 cm³ powietrza i 1 cm³ wodoru pozostawimy wzajemnemu działaniu, to one będą się przenikały wskutek dyfuzji i będą dążyły do równomiernego rozkładu — ~~[illegible]~~ H będzie się zmieszało. Prawo rozkładu Maxw. jest jednak tylko prawem prawdopodobieństwa, więc także ten układ przy którym połowa naczynia jest H_2, druga połowa powietrze, ma pewne pr. choć nadzw. mało prawdop., zatem po upływie jakiegoś ogromnie długiego czasu musi się zdarzyć (raz) znowu całkowite odmieszanie — co również wynika z owego twierdzenia Liouville'a — ale czas ten jest tak ogromny, prawdop. owe tak małe, że nie mamy żadnego widoku, abyśmy mieli raz sposobność ~~Stan~~ sprawdzić doświadczalnie takie zjawisko odmieszania choć jednego mm³.

Inny przykład: ~~tak~~ ~~kula z motocyklu [illegible] [illegible] [illegible] o opór sztywny zostaje tam przytrzymana a en. kin. zamienia się w energię cieplną. Wtedy~~ patrzmy na większe jakieś ciało stałe zwykłej temperatury. Z czasem, ponieważ wszystkie kombinacje prędkości zgadzające się z prawem energii mają pewną prawdop. — nawet w dowodzie owego twierdzenia R. B. ~~[illegible]~~ wyraźnie robimy zastrzeżenie, że system przejdzie przez wszystkie możliwe konfiguracje, — zdarzy się także że ~~wszystkie~~ prędkości wszystkich atomów będą miały przybl. równy kierunek, wtedy ciało samo przez się ruszy się z miejsca, mając temperaturę ~~0~~ bezwzgl. zero — ~~[illegible]~~ prawo entr. zostanie przełamane, ale to tylko na chwilkę, bo potem znów nastąpi niezmiernie długi czas, ~~[illegible] [illegible]~~ nieregularnego ruchu cieplnego cząsteczek.

Wyż. stan ~~porząd~~ początkowy gazów ~~[illegible]~~ przed dyfuzją, przy nierównomiernościach temperatury, ciśnień etc ~~Owe procesy (przy których doświadczamy powstawania entropii (zmniejszenia H)~~ ~~polegają na~~ jest stosunkowo tak bardzo nieprawdopodobny [ze molekular geordnet] że w praktyce zawsze będziemy spostrzegali zwiększanie entropii (zmniejszanie H), ale nie wolno ekstrapolować tego prawa na nieskończenie długi przeciąg czasu; ~~[illegible] jak to czasem [illegible] [illegible]~~ takie twierdzenia jak n.p. ~~o ostatecznym~~ że wszechświat ostatecznie osiągnie stan nieruchomości martwoty wskutek dyssypacji energii, według teorji mechanicznej nie są uzasadnione.

Prawo Maxwella co do rozdziału energii kinetycznej wcześnie zastosowano do interpretacji stosunku ciepła właściwego $\frac{C}{c} = k$ ~~przy gazach~~ gazów.

Jeśli cząsteczka gazu ma n stopni swobody (n ruchów możliwych) ~~to znaczy n~~ to przypadnie na nie energia $E = n\alpha$, ~~gdzie~~ ponieważ każdej współrzędnej odpowiada równa ilość energii kin; z tego 3α na ruch postępowy, co się równa temperaturze θ, zatem $\alpha = \frac{\theta}{3}$, $E = \frac{n}{3}\theta$ z.

Oprócz tego uwzględnić należy (energię potencjalną) (która się może powiększać) przy podwyższeniu temp., n.p. wskutek zmiany odległości atomów składających cz., $U = \nu\alpha = \frac{\nu\theta}{3}$ gdzie ν oznacza pewien stosunek stałą wielkość, i pracę wykonaną ~~przy~~ przeciwko ciśnieniu zewnętrznemu $\int p\,dv = \frac{m\bar{c}^2}{3} = \frac{2}{3}\theta$

$$\text{Zatem } \frac{C}{c} = \frac{n+\nu+2}{n+\nu} = 1 + \frac{2}{n+\nu}$$

Jak wiadomo Boltzmann stosował za pomocą tego wzoru wartości $k = 1{\cdot}66,\ 1{\cdot}4,\ 1{\cdot}33$ które napotykamy ~~p~~ u gazów jedno, dwu- i 3atomowych

1) Przyjmując jako kształt cząsteczek ~~kl~~ kule całkiem gładkie ~~Wtedy~~ wprawdzie mamy ~~6 stopni swobody ruchu~~ 3 współrzędne ruchu postępowego i 3 ruchu obrotowego — ale tych ostatnich nie należy uwzględnić ponieważ, ~~jeśli kule są gładkie~~ ruchy obrotowe w ogóle nie będą żadnego miały wpływu, zatem $n = 3$, $\nu = 0$; $k = 1 + \frac{2}{3}$

2). elipsoidy albo wąskie ciała obrotowe $n=5, v=0$; $\kappa = 1·4$

3). elipsoidy trójosiowe albo wogóle ciało sztywne dowolnego kształtu

$$n=6, v=0; \quad \kappa = 1·33$$

Mniejsze wartości stosunku κ łatwo wytłomaczyć przyjmując większą ilość ruchów wewnętrznych, trudności tylko nastręczają te wielkie wartości κ. Trudności polegają na tem:

1). Stosunek osi elipsoidy trójosiowej nie zmienia stosunku κ, ale on ma wpływ na czas relaksacyi t, t.j. na przeciąg czasu potrzebny do osiągnięcia stanu stałego, czem mniejsza różnica między osiami, tem dłuższego czasu będzie potrzeba aż wszystkie ruchy się wyrównają.

Aby więc n.p. gaz miał $\kappa = 1·4$, cząsteczki musiałyby być matematycznie idealnemi ciałami obrotowemi, gładkiemi, najmniejszy brak symetryi zrobi $\kappa = 1·33$. Taka symetrya kulista nie jest jednak rzeczą ~~nie~~ ~~bardzo~~ prawdopodobną przynajmniej dla niektórych gazów n.p. N_2, który jest trój- albo pięcio wartościowy.

Więc Boltzmann sądzi, że owe wartości κ mogą być ewentualnie za wielkie, że gazy mają w rzeczywistości mniejsze κ, tylko że to z powodu wielkiego czasu relaksacyi uszło dotychczasowym badaniom.

Trudno sądzić o prawdopodobieństwie tego przypuszczenia dla tamtych gazów, w każdym razie otwiera się tu nowe pole ~~dla badań doświadczalnych~~ do doświadczeń: badania zmienności ciepła właściwego z czasem wpływ czasu (Nachwirkung d. sp. W.), która z pewnością odgrywa znaczną rolę u gazów wieloatomowych i może przyczyni się do wytłomaczenia licznych sprzeczności dotychczasowych resultatów różnych badaczy.

2). Gdyby cząsteczki były ciałami sprężystemi, toby ~~jest~~ z czasem cała energia ruchu postępowego musiała się zamienić w drgania sprężyste tych cząsteczek, ponieważ ~~Ale~~ ~~jak~~ (Kelvin powiada) ciało sprężyste ma n=∞ (nieskończ. wiele drgań fundamentalnych). Wprawdzie zdaje mi się iż zachodzi pewna wątpliwość czy założeni Maxwella tutaj jest ściśle spełnione, ale trzeba przyznać że ostatecznie energia ruchów wewnętrznych musiałaby być bez wszelkiego porównania wyższą aniżeli w rzeczywistości.

Oczywiście jednak owe pojęcia kul materyalnych były tylko wprowadzone do uzmysłowienia sfery działania sił i nie można myśleć o tem aby jawnie przypisać objektywną egzystencyę. ~~Zastępując więc owe kule sferami~~ Wracając więc do racyonalniejszego pojęcia sfery sił omijamy tę trudność. Ale z drugiej strony musimy przyznać że wśród cząsteczek muszą się odbywać jakieś ruchy drgające innego rodzaju, które powodują promieniowanie, ~~zatem musi pozatem być więcej stopni swobody ruchu~~ Więc ilość [illegible] n przecież ~~istnieć~~ musiałaby być większą, n mniejsze.

Tu jednak w grę wchodzi ośrodek, eter, ~~w~~ na który owe ruchy się przenoszą. Jego wpływu nie można subsumować w siły konserwatywne supponowane przy wywodzie prawa M., zatem do tych ruchów nie możemy stosować prawa rozdziału równego energii. Wkracza to w pole nieznanego związku promieniowania z teoryą kinetyczną.

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A propos [illegible] zrobić uwagę na interesującą

Pod tym względem Stoney podaje myśl że fosforescencya może ~~być~~ polegać na takich ruchach wewnętrznych które (są tylko w małym stopniu związane z innemi ruchami więc) mają stosunkowo długi czas relaksacyi.

3). Jeśli więc sobie wyobrażamy atomy jako punkty materyalne otoczone sferą sił, cząsteczka jednoatomowa będzie miała $n = 3$, tak samo jak kula; cząsteczka dwuatomowa $n = 5$, ale tylko jeśli odstęp dwuch atomów jest niezmienny; w ~~innym przypadku~~ przeciwnym razie $n = 6$; trzyatomowe w ogólnym razie $n = 9$ etc.

Więc jeśli połączenie między oba atomami jest idealnie sztywne, matematycznie niezmienne, to będzie $n = 5$, $\kappa = 1.4$. Nie mogłoby to być n.p. urzeczywistnione przez połączenie fizycznie praktycznie sztywne, za pomocą ciała sprężystego, bo wtedy byłoby $n = 6$ a oprócz tego jeszcze $v = 1$ (zob później)

Co to znaczy jednak: poł. matemat. sztywne, w mechanice wyobrażamy sobie połączenie urzeczywistnione tylko przez siły, a więc nie mamy

Warto ~~jednak~~ dla tego uwagę zwrócić na to, że możemy otrzymać $\kappa = 1.4$, (T.j. rezygnując z wprowadzenia połączenia idealnie sztywnego) a wtedy zmienności energii pot. ujemnej) wtedy $n = 6$, $v = -1$. Wielkość v może mieć także wartości ujemne, bo niekoniecznie musi się powiększać wewnętrzna energia potencyalna przy podwyższeniu temperatury; ~~Ona przy~~ v będzie dodatnie jeśli siły są tego samego rodzaju jak sprężyste, będzie jednak ujemnem jeśli n.p. przyciąganie według prawa $\frac{1}{r^2}$ bo wtedy musi się zmniejszyć ~~się~~ odstęp ~~[illegible]~~ (więc wielkość cząsteczek! zob. przegląd, Sutherland) (przy podwyższeniu temperatury. jeśli układ ~~będzie~~ na przestrzeni stałym

Supponując jako przykład ruch po obwodzie koła

$\alpha = \frac{m}{2} r^2 \omega^2$; siła odśrodkowa $m r \omega^2 = \frac{A}{r^\mu}$

$$U = \frac{-A}{(\mu-1) r^{\mu-1}} = -\frac{m r^2 \omega^2}{(\mu-1)} = \frac{-2\alpha}{\mu-1} \qquad \text{więc } v = -\frac{2}{\mu-1}$$

N.p. $v=-1$

dla $\mu=3$

Nie sądzę naturalnie żeby to było wyrazem prawa istniejącego bo przykład ten polega na bardzo uproszczonych podstawieniach. Przy prawie Dulong i Petit o tem jeszcze wspomniemy; ale może nie byłoby zbytkiem – wspomnieć o częściowo Staigmüller ~~[illegible]~~ wprowadza takie połączenia absolutnie sztywne, (n.p. tam gdzie κ = 1.4) ~~[illegible]~~ przy innych zaś inne znów połączenia sprężyste [gdzie U=], przez co otrzymuje wielkość n+r zgadzającą się z faktycznem κ. ~~Nie mamy jednak żadnej przyczyny supponować siły [illegible]~~ Nie wyjaśnia to jednak wcale kwestyi tu poruszonej; do tego znaczenie ~~sprężystego~~ niektórych ~~jakichś~~ połączeń jako ~~się~~ sztywnych ~~[illegible]~~ innych jako sprężystych ~~[illegible]~~ jest bardzo dowolnem.

Powiedzieliśmy że zastosowanie twierdzenia o viriale i twierdzenia M.B. tworzy jedyną drogę racyonalną do teoryi kinetycznej gazów zgęszczonych i cieczy i wart. ciał stałych.

Jak wiadomo Van d. Waals wyprowadził swą znaną formułkę

$p + \frac{a}{v^2} =$ ~~[illegible]~~ $\frac{R\theta}{v-b}$ ze założenia że cząsteczki uważać ~~należy~~ można jako kule sztywne między którymi działają jeszcze siły przyciągające których sfera jest ~~[illegible]~~ wielka w stosunku do ~~[illegible]~~ cząsteczek samych. On posługiwał się przy tem twierdzeniem

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Albowiem pierwotny wzór wprawdzie oddaje ~~[illegible]~~ w świetny sposób jakościowo zjawiska zachodzące przy zgęszczaniu i skraplaniu gazów ale ~~[illegible]~~ dalekim jest od ilościowej dokładności.

Pokazuje się to n.p. mianowicie w objętości krytycznej, w odpowiadającym punktowi krytycznemu ~~krytycznej~~ ~~który określony~~ ~~[illegible]~~ ~~jest przez~~ ~~t.j. punktem~~ zostaje $\frac{\partial p}{\partial v}=0$, $\frac{\partial^2 p}{\partial v^2}=0$]
[t.j. objętości ~~[illegible]~~ punktu

Z równania otrzymałoby się $v_k =$ t.j. $\frac{1}{2\cdot 8}$ części objętości, która by odpowiadała zwykłemu prawu Boyle Charlesa, podczas gdy liczne doświadczenia mian. S. Younga podały jako wartość owego stosunku $\frac{1}{3\cdot 7}$ – ..

Dieterici pokazuje że także z formy ( ) otrzymałoby się $\frac{1}{3}$, ~~[illegible]~~ ~~[illegible]~~ a uwzględniając dalsze dodatnie wyrażenia szeregu ( ) jeszcze wyższe wartości. Z tego on wnioskuje że w ogóle równania formy nie może odpowiadać rzeczywistości.

Ten wniosek zdaje mi się przedwczesnym, bo nie mamy żadnej przyczyny do przypuszczenia, że dalsze potęgi muszą być dodatnie; przeciwnie ~~[illegible]~~ wyrażenie / będzie miało znak – a ostatnie dalsze wyrażenia przy p. kr. mamy będą miały wpływ [illegible], więc możeby być żebyśmy z ~~zupełnego~~ równania uzupełnionego otrzymali wartość przybliżoną do owego $\frac{1}{3\cdot 7}$.

Dieterici podaje dwa inne wzory:

$pv + \frac{a}{v^{3/2}} = \frac{R\theta}{v-b}$ i $pv =$

z których pierwszy dałby stosunek $\frac{v_k}{v} =$ a drugi

Pierwszy jednak nie jest wcale teoretycznie uzasadniony, a drugi jest oparty na formule z teoryi cieczy Dieterici'ego, którą dalej będziemy krytykowali. Ani jeden ani drugi nie czyni zadość warunkowi ( ) $p = \infty$ $v = \frac{b}{3}$. Więc ~~teoretycznie~~ nie można ich uznać ze stanowiska teoretycznego a pod względem doświadczalnym wiemy tylko że zgadzają się w tym jednym punkcie z doświadczeniem, ~~dalej~~ pod innym względem ~~inne konsekwencye~~ nie zostały one jeszcze bliżej sprawdzone.

~~[illegible]~~ Równanie w każdym razie musi mieć kształt $p + \frac{a}{v^2}) = T\varphi(v)$, jeżeli uważamy cząsteczki za ciała sztywne jakiegokolwiek kształtu; wynika to z tego że można wszystkie prędkości zmienić w pewnym stosunku, przez co nie zmieni się ~~kształt~~ formy ruchu, tylko ciśnienie. (Rayleigh). Z sił atrakcyjnych ~~się powodu~~ powstające owe wyrażenie $\frac{a}{v^2}$, niezależnie od prawa działania, jeżeli sfera sił wielka w porównaniu do ~~odstępów~~ odległości cząsteczek. Gdyby była mała, to ~~[illegible]~~ trzebaby je zastąpić jakąś funkcyą v i T.

Jeżeli jednak cząsteczki nie są idealnie sztywne, ~~tylko ciśnienie~~ t. j. jeżeli siły odpychające nie stają się ∞ -- to proporcyonalność ciśnienia całkowitego do temperatury ustaje. Tak n.p. pod założeniem Maxwellowskich sił $\frac{A}{r^5}$ otrzymałoby się równanie (Boltzm.

$$pv + \frac{a}{v} = r\theta\left[1 + \frac{b}{v}\right]$$

gdzie jednak b będzie proporcyonalne (do gęstości i) do potęgi $\theta^{-\frac{3}{4}}$.

Tego rodzaju jest także równanie kształtu .. $\left(p + \frac{a}{v^2}\right)\cancel{(v-b)^4} = \frac{RT\,v^3}{(v-b)^4}$,

~~ponieważ tutaj a i b są zmiennemi~~ które Reinganum wywiódł jako rezultat empiryczny ze ~~licznych~~ spostrzeżeń Younga ~~[illegible]~~ co do izopentanu, gdzie a i b są wielkościami zmiennemi:

$$a = A\,e^{\frac{0{\cdot}0345\,A\left[(v-2\beta)^3 + 12{\cdot}2\frac{\beta^4}{v}\right]}{\beta v^3 R\theta}}$$

$$b = \beta\,e^{\frac{0{\cdot}0726\,A\left[(v-2\beta)^3 + 3{\cdot}34\frac{\beta^4}{v}\right]}{\beta v^3 R\theta}}$$

Równanie to ~~jest~~ mimo swej skomplikowanej formy zawiera tylko trzy ~~wielkości~~ spółczynniki stałe, i czyni ~~zadość~~ ~~spełnia~~ twierdzeniu ~~do~~ o ~~do~~ stanach coresp. ; ~~jest~~ stosuje się ono z wielką dokładnością do doświadczeń, ale ~~teoretycznie~~ nie ma żadnych teoretycznych podstaw, ~~i~~ i ~~nie~~ więc może być uważane tylko jako empiryczna formułka.

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* Rozumowania teoretyczne p. Reinganum są błędne, mianowicie ~~[illegible]~~ uważa temperaturą jest średnia energią kinet. cząstek wzdłuż całej drogi, a nie tylko w częściach drogi gdzie siła = 0.

Jeśli nie udało nam się

Z góry można przewidzieć że zastosowując pojęcie V. d. W. do cieczy nie możemy ~~nam oczekiwać~~ się spodziewać pomyślnych rezultatów, dopóki nie znamy sposobu ~~do obliczania~~ do obliczenia wartości ~~więcej~~ owego szeregu ( ) z większą dokładnością, ponieważ tam $\frac{b}{v}$ już znaczną może mieć wartość (zazwyczaj >1). ~~Wystarcza to do [illegible]~~ ~~ścisłości Voigta,~~ Z tej przyczyny też ~~obliczenia~~ teorya kinetyczna cieczy w tej formie jak została na podstawie zasad V. d. V. rozwinięta przez Voigta, Dieterici'ego i Molnera, jest bez praktycznej wartości. Voigta pierwotna teorya była zupełnie teoryą jakichś „idealnych" albo raczej urojonych cieczy ponieważ nie uwzględniała wcale wielkości sfery ich odpychających t. j. uważała cząsteczki jako punkty nie jako kule.

W drugiej rozprawie wprowadził Voigt ~~[illegible]~~ zwykłe pojęcie V. de W. (co do niedostatecznie szeregu $(1+\frac{b}{v}\ldots)$ ale przeprowadzenie teoryi — pominąwszy nawet ów ważny zarzut — jest nieścisłe, oparte na przybliżeniach wrodzonych obliczeń tarcia Clausiusa i innych ~~[illegible]~~ których ogólnie trudno ocenić. To samo stosuje się do teoryi Dieterici'ego i Milnera.

Jako słuszną uważamy ~~podstawę~~ zasadę że także w cieczy średnia energia kinet. ~~jest~~ miarą temperatury, ale Voigt i Diet. nie przyjmują ~~[illegible]~~ jej ~~tego~~

jako rezultat prac Rosz.-P. tylko starając się je dowieść — w sposób co prawda stanowczo nieścisły nieprawidłowy.

Celem tych prac jest przedewszystkiem obliczenie ~~jedna wielkość~~ ciepła utajonego parowania. Voigt (i Drud.) otrzymuje wzór:

$$L_i = \frac{1}{2}\frac{R\theta}{\mu}\left[\lg\frac{v_f - b}{v-b} + \frac{b}{v-b} - \frac{b}{v_f-b}\right]$$

gdzie L_i oznacza ciepło parowania wewnętrzne [t.j. po odciągnięciu uwzględnieniu pracy zewnętrznej, $v =$

Okazuje się, że w rzeczywistości L_i jest u wielu cieczy prawie dwa razy tak wielkie, Milner także rzeczywiście otrzymuje ze swoich obliczeń podwójną wartość owego wyrażenia, ale rezultat ten polega na ~~błędzie~~ omyłce [rachunkowej

Można sobie zresztą oszczędzić wszystkie rozumowania kinetyczne co do tej wielkości L_i ~~zważając że~~ robiąc użytek ~~używając~~ z ~~wprost~~ warunku ~~wzoru~~ termodynamicznego dla stanu równowagi parowania ciecz-para: $v_l - v_g = \int_{v_g}^{v_l} v\,dp$ i zważając że

$\frac{a}{v} =$

co daje nam ~~[illegible]~~ pod założeniem równania pierwotnego V.d.Wal

Niezgodność z rzeczywistością zaś nie może nas wcale dziwić ponieważ nie można myśli o tu siły równanie samo zastosować do stanów gdzie $\frac{b}{v}$ ma wartości

Wychodząc z formy która jest już o jeden stopień dokładniejsza, otrzymuje się według Boltzmanna:

ale zapewne i to nie będzie dokładnie spełnione.

~~Jäger, zdaje się, jest Ściśnienie zgęszczenie nadzwyczaj wielkie cząsteczek w cieczy, w skutek którego $\frac{b}{v}$ staje się tak wielkie, zostało uwzględnione, o ile wiem, jedynie przez Jägera; o równaniu jego wspomnimy przy innej sposobności.~~

W skutek owych wzorów obliczenie L (ciepła parowania w ogóle jest rzeczą ~~[illegible]~~ zawiłą jeszcze) ~~będzie zależnem od~~ dane jest przez równanie zasadnicze~~go~~.

Całkiem nowego obliczenia jednak wymagałyby zjawiska tarcia wewnętrznego, przewodzenia ciepła i dyfuzyi dla cieczy i gazów zgęszczonych. Łatwo jednak zrozumiemy że trudności tu zachodzące nie zostały jeszcze pokonane, gdyż nawet dla gazów doskonałych nie zdołano jeszcze tych rachunków wykonać ~~pod~~ ~~przyjmując~~ teoryę kul elastycznych (a znów metody Maxwella nie można zastosować jeśli ~~[illegible]~~ zgęszczenie jest znaczne). Ale możemy się spodziewać ~~[illegible]~~ osiągnięcia jakichś rezultatów przynajmniej co do zmienności tych wielkości z temperaturą – którą tam w ~~całkiem~~ dla gazów dosk. prosty sposób można obliczyć.

I tutaj jeśli ciecz zachowuje stałą objętość, wszystkie ruchy pozostaną geometrycznie podobne jeżeli ~~je~~ prędkości ich w pewnym stosunku α powiększymy t. zn. (ruchy cząsteczkowe i ~~ruchy~~ prądów ~~wypadkowe~~ cieczy samej). Przez to jednak wszystkie ~~ciśnienia~~ natężenia w elementach cieczy się powiększą w stosunku α^2 a temperatura w stosunku ~~α~~ α^2.

Więc również natężenie styczne tarcia wewnętrznego,

N. p. $p_{xy} = \alpha^2 (p_{xy})_0 = \cancel{\eta}$ z czego wynika tak samo jak przy gazach idealnych:

$$(p_{xy})_0 = \eta_0\left(\frac{\partial u_0}{\partial y} - \frac{\partial v_0}{\partial x}\right)$$

$$p_{xy} = \eta\left(\frac{\partial u_0\alpha}{\partial y} - \frac{\partial v_0\alpha}{\partial x}\right) = \alpha^2\,\eta_0\left(\frac{\partial u_0}{\partial y} \cdots\right)$$

więc $\eta = \alpha\eta_0$, $= \eta_0\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{2}}$

Tutaj jednak tarcie nie jest niezależnem od ~~cie~~ gęstości jak przy gazach, więc ~~następuje dla~~ przy stałym ciśnieniu (trzeba jeszcze uwzględnić zmienność) wskutek powiększenia średnich odległości cząsteczek ~~z rosnącą~~ z podnoszeniem temperatury.

$$\frac{d\eta}{d\theta} = \left(\frac{\partial\eta}{\partial\theta}\right) + \left(\frac{\partial\eta}{\partial v}\right)\left(\frac{\partial v}{\partial\theta}\right).$$

$\frac{\partial v}{\partial\theta}$ jest spółczynnik rozszerzalności termicznej

a $\frac{\partial\eta}{\partial v}$ można obliczyć ~~dla kilku cieczy~~ jeżeli znamy ~~zmienność~~ zależność tarcia wewnętrznego od ~~[illegible]~~ ciśnienia $\frac{\partial\eta}{\partial p}$ i ściśliwość cieczy $\frac{\partial v}{\partial p}$: $\frac{\partial\eta}{\partial v} = \frac{\frac{\partial\eta}{\partial p}}{\frac{\partial v}{\partial p}}$.

Znamy te wartości ~~[illegible]~~ n. p. dla Benzolu i Eteru dla których otrzymujemy n. p. dla θ=20° : $\frac{d\eta}{d\theta} =$ resp.

podczas gdy w rzeczywistości :

Więc pokazuje się kompletne fiasko naszej formułki, zatem i zasady V. d. Waals – Clausiusa, t. zn. że nie można cząsteczek uważać jako kul sprężystych. ~~co się tu~~ ~~je~~ Wiemy to już z odpowiednich wzorów dla gazów doskonałych, ale tutaj sprzeczności założenia i rzeczywistości występuje wiele jeszcze jaskrawiej.

W dalszym ciągu wrócimy jeszcze do ~~[illegible]~~ omówienia wniosków z tej przyczyny się nasuwających.

Należałoby tu jeszcze obszerniej pomówić o teorji kinet. Jägera który w ~~licznych~~ ~~rozprawach~~ pracach starał się wybudować teorję kin. cieczy na podstawie pojęć V. d. W. Jakkolwiek tam w specyalnych problemach dużo jest myśli zawartych ~~vat~~ ciekawych ~~obiecujących~~ [n. p. ~~[illegible]~~ ciągłe skorzystanie używanie korzystanie ze związku między stałymi włoskowatości a wielkością $\frac{a}{v^2}$ (wskazanego przez Stefana)] nie ~~[illegible]~~ myślę jednak wchodzić w ~~ich~~ jej specyalne rozstrzyganie, bo ~~on~~ jest w sprzeczności ~~[illegible]~~ ~~[illegible]~~ z ~~zasadniczą~~ fundamentalną zasadą (B. R.) iż energia kinet. w cieczy i w gazie jest równą. Jäger sądzi że ona w stanie ciekłym jest znacznie mniejszą i stara się wyprowadzić różnicę na podstawie swej teorji.

W kinetycznej teoryi cieczy więc nie postąpiliśmy jeszcze daleko,
Kinetyczna teorya ciał stałych jednak jeszcze mniej jest wykończona,
~~[illegible]~~ ~~dodać nam jeszcze o zdaniu~~ nie zgodzono się nawet jeszcze co do
głównych zarysów które tej teoryi trzeba nadać.

Pod
~~W~~ jednym ~~punktem~~ względem jednak tutaj w korzystniejszej jesteśmy sytuacyi
niż wobec cieczy t. j. że mamy ~~[illegible]~~ prawo tak zadziwiającej
ogólności i prostoty – choć nie dokładnie spełnione – jak prawo Dulong'a
i Petit'a co do ciepła właściwego.
ciepła atom.

A te regularności zdołano wytłumaczyć w sposób zupełnie zadawalający
jedynie z przyjęcia że cząsteczki albo raczej atomy poruszają się koło jakiejś średniej pozycyi równowagi
wychodząc ~~z ogólnych zasad~~ ~~[illegible]~~
zasad ~~teoryi kinetycznej~~ –

Można użyć w tym celu albo (Riecke)
~~Riecke w tym celu~~ użyć twierdzenie co do wiriału, z którego wynika, ~~stosując~~ pomijając pv jako małe:

$$\overline{L} = -\tfrac{1}{2}\Sigma(Xx+Yy+Zz)$$

~~[illegible]~~

$$= +\tfrac{1}{2}\Sigma\left(x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}+z\frac{\partial U}{\partial z}\right)$$

albo też ~~łapiąc~~ wprost powołać się na prawo Maxwella-B., według którego:

$$\overline{U} = \frac{\int U e^{-hU}\,dx\,dy\,dz\dots}{\int e^{-hU}\,dx\,dy\,d\dots}$$

gdzie h = odwrotn. temperat.

Jeśli cząsteczki poruszają się koło pewnej średniej pozycji równowagi, to siły w pewnym dostatecznie małym obrębie będzie można przyjąć jako prop. do wychylenia, więc U jako funkcję drugiego stopnia współrzędnych, ~~[illegible]~~ co według jednego F_2 i drugiego wzoru daje $\overline{U}=\overline{L}$ (prawo zachodzące przy wszystkich drganiach sprężystych (Huyghens)), więc też zmiana energii pot. będzie równa zmianie energii kinet., zatem całkowita zmiana energii będzie dwa razy tak wielka jak gdyby $U=0$; t.j. (dwa razy tak wielkie jak w gazach jednoatomowych t.j. ciepło właściwe jednego atomu)

Uwzględniając wzór funkcyjny jednorodnej wyższego stopnia otrzymałoby się:

$c = 6.$

Czem większe wychylenia cząsteczek poza obręb sfery gdzie siły można uważać za proporcjonalne do odstępów, tem mniej dokładnie będzie się ~~zgadzać to prawo~~ sprawdzało to prawo, więc różnicy największej można oczekiwać u tych pierwiastków

1). które mają najmniejszy ciężar ~~[illegible]~~ atomowy, bo te przy równej temperaturze muszą mieć odpowiednio większe prędkości, więc i wychylenia ~~[illegible]~~ większe.

2). które mają małą wartość objętości atomowej, więc małe odstępy atomowe, ponieważ tam siły w małym obrębie wolniej będą zmienne.

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Ta reguła ~~sprawdza~~ zgadza się z ~~[illegible]~~ najzupełniej z doświadczeniem jak to wskazuje następująca tabliczka

Wpływ ciężaru atomowego jeszcze lepiej uwydatnia się w ~~[illegible]~~ niższej temperatur

Dziwne więc jest nadzwyczajnie mała wart. dla C, tam trzebaby chyba $\frac{\Delta \Pi}{dt}$ przyjąć jako ujemne, co jednak przy ciałach stałych trudniej usprawiedliwić ~~jak~~ niż przy gazach. Boltzmann tłumaczy to zjawisko znów ~~opóźnieniem~~ ~~[illegible]~~ ~~[illegible]~~ zbytnią wielkością czasu relaksacji niektórych ruchów i z tego powstającą „Nachwirkg Wartoby było ~~wyjaśnić~~ rozstrzygnąć tę kwestyę doświadczalnie.

Że ciepło właściwe się zmniejsza (ze zniżeniem temp. (mimo zmniejszenia prędkości)) wskazuje na przewagę wyższych potęg wpływu F_3 etc... więc na bardzo szybkie wzmaganie się sił odpychających przy zbliżeniu się cząsteczek.

Staigmüller ~~podaje~~ wywodzi także prawo Koppa-Neumanna co do ciepła właściwego związków chemicznych w ten sam sposób jak prawo Dul. Pet. nie tłumaczy jednak wcale czemu tutaj trzeba obierać dla wielu pierwiastków zupełnie inne wartości ciepła właściwego np.

aniżeli wtedy gdy one są ~~[illegible]~~ niepotężone.

Według mojego zdania ~~[illegible]~~ zmusza nas to ~~bezpośrednio znowu do na potrzebę~~ do przyjęcia asymetryi sił działających, wskutek której siła (i potencyał) zależy nie tylko od odległości cząsteczek ale też od orientacyi ich względem pewnych kierunków. O tem jeszcze więcej będziemy mówili.

~~Oprócz tego pr.~~ Wyjąwszy ~~[illegible]~~ Pominąwszy te zjawiska dyte. Ter- rezultaty dotychczasowe ~~na mase~~ teoryi kinet. na innych polach fizyki ciał stałych są prawie równe zeru.

~~Sutherland próbował opisać równania zasadnicze ciał stałych~~

Zdaje się że jedyna próba zastosowania ~~teoryi~~ pojęć kinet. do ciał stałych została podjęta zresztą przez Sutherlanda, ~~który~~ oczywiście znów ~~pod~~ założeniem cząsteczek à la Van der Waals.

Poprawiając ~~[illegible]~~ kilka błędów w jego rozumowaniu możemy tak wnioskować:

Z twierdzenia o wirialu wynika:

$$p+\frac{a}{v^2} = R\theta + \overline{\sum\sum r f_{(r)}}$$

przyczem $\overline{\sum\sum r f_r} = \frac{1}{t}\int_0^t \sum\sum r f_{(r)}\,dt = ~~6\sum m\rho~~\ 6\sum m\rho$

6 = ~~pr~~ średnica

gdzie $\sum m\rho$ oznacza sumę momentów (ilości ruchu) wymienionych (prędkości ~~w ta~~ względnych składowych w kierunku osi zderzenia) między cząsteczkami ~~podczas~~ przez zderzenia podczas jedności czasu.

~~$\Sigma\rho$ możemy zastąpić przez $N\bar{\rho}$ gdzie N= ilość zderzeń~~

Wprowadzając średnią wartość $\bar{\rho}$, ilość zderzeń N, i drogę swobodną λ będziemy jednak mieli:

$$\Sigma\rho = N\bar{\rho} = \frac{\bar{c}}{\lambda}\bar{\rho} \quad \text{a wstawiając} \quad \bar{\rho} = \frac{\bar{q}}{\sigma_3} = \frac{\bar{c}\sqrt{2}}{3}$$ (Boltzm..

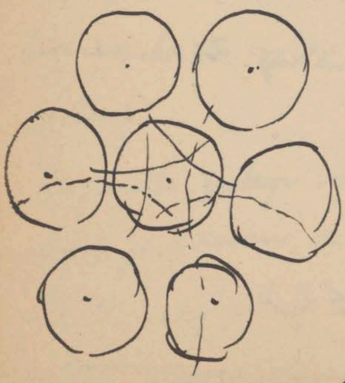
otrzymujemy:

$$pv + \frac{a}{vp} = R\vartheta\left[1 + \frac{\sqrt{2}}{3}\,\frac{\sigma}{\lambda}\right]$$

Chodzi teraz o wyrażenie ~~[illegible]~~ ~~[illegible]~~ wartości średniej drogi λ;

co do tego Sutherland to samo założenie robi jak Jäger w odpowiednim rachunku dla cieczy t.j. że $\lambda = d - \sigma$.

Jest to uzasadnione, jeżeli wystawiamy sobie wszystkie cząsteczki w przypadkach normalnych (z wyjątkiem jednej ruchomej); udzielając im podwójne promienie a uważając ruchomą cząsteczkę jako punkt otrzymujemy rysunek



² którego pokazuje się że ~~miejsca wolne dla ru~~ miejsce wolne dla ruchu cząsteczki (ruchomej) przybliżenie będzie kulą o owym promieniu. (Jäger Winkelm

W rzeczywistości trzeba sobie jednak cząsteczki są rozłożone wyobrazić we wszystkich możliwych, nie tylko tych normalnych przypadkach; λ w skutek tego ~~[illegible]~~

otrzyma inną wartość, ale zawsze jeszcze przybliżenie podobnej wielkości.

Wstawiając $\frac{d}{\sigma} = \sqrt[3]{\frac{v}{v_0}} = 1 + \frac{1}{3}\frac{v-v_0}{v_0}$ otrzymujemy wreszcie uważając $v-v_0$ jako małe:

$$p + \frac{a}{v^2} = \frac{r\theta\sqrt{2}}{v-v_0}$$

Sutherlanda ~~sprowadza się~~ równanie jest (p. 544) ~~przytacza explicite swoje równania, dla symbole~~ ~~jego~~ równoważne ~~z~~ do

$$p + \frac{a}{v^2} = \frac{3r\theta}{v-v_0}$$

, co polega na przeoczeniu czynnika $\frac{\sqrt{2}}{3}$

Z tego równania otrzymuje się związek między współczynnikiem rozszerzalności α i ściśliwością $\frac{1}{\kappa}$: $\kappa = \frac{\varrho}{\sqrt{2}}\frac{\theta}{R\rho}\alpha^2$

który jednak zupełnie nie sprawdza się w rzeczywistości, obliczone wartości ~~[illegible]~~ ściśliwości są kilkadziesiąt −100 razy za małe!

Sutherland ~~[illegible]~~ myśli wtedy poprawić założenie wtedy przyjmując promień cząsteczek zależny od ~~zmienny z~~ temperatury, ale te obliczenia i hipotezy jego są już bardzo problematycznej wartości.

Uderzającym może się wydać fakt, że Jäger ~~w celu [illegible]~~ równanie dla cieczy otrzymuje wychodząc z zupełnie tych samych założeń; ~~różnica~~ w rzeczywistości Jäger ~~postępując~~ powinien był otrzymać zupełnie to samo równanie i różnica jego równania od naszego polega tylko na innym, zdaje mi się mniej ścisłym, sposobie ~~[illegible] przybl.~~ pomijania wielkości mniejszego rzędu w ~~wprowadzaniu warunków przybliżenia w~~ jego pracy. Jeśli jednak to równanie jest ważne równocześnie dla cieczy, jak wtedy Sutherland wytłomaczy sprężystość postaciową?

Istotnie, jest wprawdzie u Sutherlanda dotyczące obliczenie ale zupełnie ~~[illegible]~~ błędne. Jest to w ogóle już ~~z góry~~ samo przez się zrozumiałe, że ~~wprowadzając~~ ~~[illegible]~~ ~~sfery~~ siły przyciągające ~~[illegible]~~ (à la V. d. W. których sfera bardzo) wielka w porównaniu z odległościami cząsteczek, spowodują tylko ~~pr~~ ciśnienie wewnętrzne, a supponując cząsteczki jako gładkie sztywne kule nie otrzymamy żadnych sił które by ~~[illegible]~~ mogły służyć do przytrzymania cząsteczek ~~[illegible]~~w pewnych stałych średnich pozycjach równowagi. One będą ciągle zmieniały swe położenie, może z wielkim tarciem wewnętrznym — ale zawsze będziemy mieli tylko ciecz.

Możnaby sądzić że ~~otrzymaliśmy~~ doszliśmy tak przynajmniej do równania zasadniczego cieczy, ale obliczenie ~~ściśliwości i zależności od~~ związku między ściśliwością i ~~[illegible]~~ rozszerzalnością termiczną według (tak samo także według) daje znowu rezultaty zupełnie niezgodne z rzeczywistością

Otrzym

~~[illegible]~~ Zatem wynik ten sam jak dawniej: porzucenie ~~[illegible]~~ założenia ~~co do cząsteczek jako kul sztywnych~~ V. d. Waalsa.

Do wytłomaczenia sprężystości ciał stałych jest niezbędnie potrzebne założenie że siły są znacznie zmienne w obrębie średniej odległości dwóch cząsteczek. Zdaje się więc najracyonalniej że porzucimy hipotezę kul sztywnych i zastąpimy je sferą sił odpychających. ~~obie sfery i druga możliwość~~: ~~zachowanie kul sztywnych i przyjęcie sił atrakcyjnych bardzo rosnących ze zbliżeniem nie jest z góry wykluczone.~~

Tym sposobem zbliżyliśmy się znów do dawnych historycznych teoryi molekularnych sprężystości Naviera i Poissona etc.. Wiadomo, że porzucono tamte teorye ponieważ z nich otrzymuje się dla ciał ~~jedn~~ bezpostaciowych tylko jedną stałą sprężystości, podczas gdy w rzeczywistości mamy dwa współczynniki niezależne. Podobnież dla ~~z~~ kryształów systemu regularnego (trójosiowego dają one tylko dwie stałe niezależne, podczas gdy ~~bardzo dokł~~ liczne doświadczenia Voigta ~~wyk~~ dowiodły że musimy przyjąć 3 niezależne współczynniki, które bynajmniej nie spełniają ~~związek~~ warunku podanego przez Poissona.

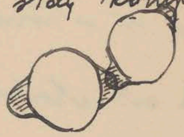
Voigt z tej przyczyny porzucił założenie, że siły są tylko funkcyami promienia r i rozwinął teoryę ogólną sprężystości przyjmując ~~siły~~ pewną kierunkowość (Polarität) sił t.j. ~~pewne kierunki w cząsteczkach~~ ~~zal~~ zależność siły nie tylko od oddalenia r ale także od orientacyi promienia wodzącego r względem stałych kierunków w cząsteczce.

Nasuwają się jednak pewne wątpliwości, czy jesteśmy do tego także zmuszeni w teoryi kinetycznej. Albowiem ~~Voigta~~ Naviera Poissona i Voigta teorye są teoryami statycznemi; byłoby rzeczą nadzwyczaj ważną rozstrzygnąć czy ~~wzorek~~ formułka Poissona, ~~którą teoryą~~ musiałaby także zachodzić w kinetycznej teoryi cząsteczek niekierunkowych.

Należy bowiem podnieść, że sam fakt krystalizacyi nie zmusza nas jeszcze do przyjęcia kierunkowości sił, ~~cząsteczek~~ które cząsteczki wywierają na siebie. ~~[illegible]~~ Średnie pozycye cząsteczek, które one zajmą pod działaniem silnego ciśnienia zewnętrznego i sił odpychających, wzmagających się bardzo silnie ~~przy ro~~ ~~dla r < b~~ przy zbliżeniu w danej przestrzeni ~~będzie po~~ będą oznaczone przez układ, najgęstszy jaki jest możliwy tych cząsteczek ~~kul b~~ „the closest possible package."

Kelvin w ~~swoich~~ kilku nadzwyczajnie interesujących dużo cennego materyału zawierających rozprawach dz zajmuje się takiemi układami. Jeśli cząsteczki są kulami, to powstaje t. zw. equilateral homogeneous assemblage, składające się z równoległych warstw  w których każda cząsteczka jest w styczności z 12oma sąsiadami i ten system ~~ma~~ posiada trzy kierunki prostop. do osi równouprawnione, przedstawia nam to więc kryształ systemu regularnego, bez przyjęcia asymetryi w cząsteczkach składających samych.

Z drugiej strony przyznać należy że ~~[illegible]~~ pominąwszy krystalizacyę dużo innych zjawisk nas i tak już skłania do przyjęcia sił kierunkowych, przedewszystkiem zjawiska chemiczne. Chemicy, mówiąc o różnej wartościowości pierwiastków, którą to czasem widać ~~schem~~ schematycznie-naiwnie reprezentowaną przez haczyki  , supponują w rzeczywistości takie atrakcyjne siły kierunkowe.

Boltzmann te pojęcia ujął we formę matematyczną w swej teorii dyssocyacyi gazów; wystawia on sobie wykreślone na powierzchni kuli, która ma przedstawiać cząsteczkę, pewne obręby „empfindliche Bezirke", które na siebie wywierają siły zwykle nieznaczne ale przy zupełnym zbliżeniu nadzwyczaj wielkie, tak że dwa atomy ~~utworzą~~ połączą się i utworzą stały kompleks, jeżeli ich „empfindliche Bezirke" się przecinają : 

Boltzm. pokazuje też na przykładach, że teorya jego zgadza się z doświadczeniem a z drugiej strony tłomaczy że nie możnaby otrzymać teoryi dysoc. na podstawie sił które by były tylko f.(r), bo ~~wtedy~~ w takim razie jeżeli n.p. ilość cząsteczek dwuatomowych ~~jest znacz~~ przeważa nad ilością jednoatomowych, to również musiałaby być także wielka ilość cząsteczek trzy, cztery etc. atomowych, — ~~[illegible]~~ ~~to~~ ~~doświadczenie~~ ~~[illegible]~~ co się sprzeciwia z doświadczeniem.

Jeżeli się nie przyjmuje sił kierunkowych w cząstkach, trudno także sobie wytłumaczyć, co właściwie spowodowuje nagłe przejście ze stanu ciekłego do stanu krystalicznego ~~ps~~ w punkcie ~~topliwości.~~ topn. krzepnięcia.

Przyjmując asymetrię sił łatwo zaś pojąć że przy ~~tt~~ w tej samej temperaturze, więc przy tej samej energii kinetycznej, dwa mogą istnieć układy ~~stałe~~ o różnych wartości funkcji U, jeden gdzie osie są nieregularnie rozdzielone (ciecz) a drugi gdzie mają pewne kierunki (kryształ) i gdzie te nieregularności zostają utrzymane przez ruchy nieregularne cząsteczek) przez co ~~siły~~ powstają siły orientacyjne które ograniczają swobodę ruchu cząsteczek.

Ciepło utajone krzepnięcia odpowiada wtedy zmianie U wskutek orientacji osi – również jak ciepło utajone skroplenia odpowiada zmianie U wskutek sił molekularnych $\left(\frac{a}{v}\right)$.

Za tą hipotezą przemawia też fakt że istnieją t. zw. kryształy ciekłe

n. p.

t. j. ~~substancje~~ kryształy których ~~przy pewnej~~ wysokiej ~~temperaturze mają~~ tarcie wewnętrzne ~~tak małe jak lepkie ciecze~~ (z podniesieniem temperatury tak się zmniejsza że przedstawiają ~~nakoni~~ wreszcie masę na pozór jak lepkie ciecze) ale które zachowują jeszcze wszelkie optyczne właściwości struktury krystalicznej. Dopiero przy pewnej oznaczonej temperaturze naraz (podobnie zatem mamy) ustaje, ~~[illegible]~~ z czem jest połączone pochłonięcie pewnej ilości ciepła utajonego

Także Tamman, któremu zawdzięczamy wiele nadzwyczaj ciekawego materyału co do ~~stanu~~ zjawisk przejścia ze stanu ciekłego do stałego, ~~[illegible]~~ podtrzymuje hipotezę kierunkowości cząst. i wnioskuje z niej, że nie istnieje żaden punkt krytyczny dla tej przemiany, ponieważ w takim punkcie ~~nie tylko~~ ~~ciepło~~ równocześnie ciepło utajone i zmiana objętości ~~ale~~ musiałoby być = 0, co ~~[illegible]~~ jest zjawisko niemożliwe, jeżeli się przyjmuje że to ostatnie zależy do pewnego stopnia także od orientacyi cząsteczek. Rzeczywiście doszedł on przy

już bardzo blisko do punktu gdzie ~~[illegible]~~ $v_s - v_l = 0$

a $r - r$ wcale nie dążyło do wartości 0.

Trzeba się tu starać jak najusilniej o dalszy materyał doświadczalny.

Nie mówiliśmy wcale jeszcze o ciałach stałych t.zw. ~~[illegible]~~ bezpostaciowych. To co zwykle ~~tą~~ nazwą określamy są tylko agregaty krystaliczne, ciała mikrokrystaliczne jak n.p. metale w zwykłej formie. Dużo innych substancyj jak n.p. wosk, szkło, drzewo itd. ~~[illegible]~~ ~~[illegible]~~ są mieszaninami różnych ciał. Ale istnieją także kilka ciał ~~elementarnych~~ czystych, jednolitych, bezpostaciowych które można uważać jako ścisłe n.p.

Według trafnego wyrażenia Tammana należy te sobie wystawić jako ciecze przechłodzone (unterkühlte Flüssigkeit). Nie różnią się zasadniczo od cieczy, nie mają żadnej struktury ani kierunkowości, ale tarcie wewnętrzne stało się tak wielkie (swoboda ruchu jest tak mała) że (cząsteczki) nie mogą ~~więcej nabrać~~ już nabrać orientacji regularnej. Takie ciała istotnie nie mają oznaczonego punktu topnienia. One mięknią stopniowo przy ogrzaniu i nie ma żadnej temperatury gdzie by nastąpiło nagłe pochłonięcie ciepła utajonego.

Voigt - Jankiewicz

Mówiąc o teoryi kinet. ciał stałych nie chciałbym pominąć też wspomnieć aby wskazać na doświadczenia nadzwyczajnie ważne i interesujące Sir R. Austena i Springa co do dyfuzyi metalów stałych.

Nie ma zjawiska, któreby wyraźniej przemawiało za kinetyczną naturą ~~wszystkich~~ ciał stałych jak fakt że one dyfundują w siebie ze szybkością którą można nawet dokładnie mierzyć.

Jak n. p. Roberts Austen ~~postawił~~ przycisnął słup ołowiu do płyty złotej jako podstawy i ~~trzymając~~ zachowując cały system w pewnej temperaturze aż do 30 dni, mógł się przekonać potem za pomocą analizy chemicznej że złoto przeniknęło, czasami w znacznych ilościach aż do samego końca (

Przytaczam kilka wartości dla współczynnika dyf.

Jest to zupełnie naturalne według teoryi kinet. ponieważ cząstki mają wszystkie możliwe drgania czasem także tak wielkie, że opuszczają swe pozycye średniej równowagi i wędrują dalej jak cząstki cieczy.

Spring pokazał znów że proces ten zwykle bardzo bywa przyspieszony przez wielkie ciśnienie, n. p. Pb – Sn przyłożone do siebie i poddane ciśnieniu 10000 Atm przy- utworzyły (aliaż) warstwę grubą 6 mm w przeciągu kilku godzin.

Z proszków takich metalów które ze węglem skłonne do tworzenia ~~węgli~~ aliaży, można otrzymać diament przez samą kompresyę bez wysokiego ogrzania.

Można więc powtórzyć słowa filozofa w dosłownym sensie:

Πάντα ῥεῖ.

Jeżeli ~~tu~~ na koniec rzucimy raz jeszcze okiem na rezultaty pozytywne tu-taj omawianej nowszej teoryi kinetycznej materyi — pominąwszy właściwe jej dokonania — to łatwo się nam wydawało może iż plon nie był ~~z~~ zbyt obfitym. Ale myślę, że i negatywne rezultaty mają swoją wartość i również też liczne znaki zapytania, które ~~[illegible]~~ przy omówieniu różnych ~~kwestyj~~ teoryi musieliśmy umieszczać, i które nam wskazują, jakie kwestye należy uważać za najważniejsze ~~najpilniejsze~~ i w którym kierunku należy głównie pracować.

Pod tym względem chciałem mianowicie wskazać na problematy teoryi kinetycznej ciał stałych, ponieważ sądzę że racyonalna teorya kon. materyi od tego musi rozpocząć, i ~~[illegible]~~ myślę że tu (prędzej) dojdziemy do ważniejszych rezultatów aniżeli w teoryi cieczy. ~~Nble~~ ~~Najważ~~ Nadzwyczaj ważnem byłoby co do tego przedewszystkiem nagromadzenie dalszego materyału doświadczalnego mianowicie co do ~~stałych~~ sprężystości kryształów i ciał właściwych bezpostaciowych (nie kryptokrystalicznych, te są bez wartości) przy bardzo niskich i wysokich temperaturach, dalsze ~~badania~~. krytyczna badań Tammana etc. bo ~~[illegible]~~ teraz najmniej nam jest znaną: fizyka ciał stałych.

gazy 2[illegible] 5
[illegible] 4
ciecze 7½
stałe 9 [illegible] + 20

$-\frac{1}{v}\frac{\partial v}{\partial p} = 0{\cdot}0001\cancel{7}0$ $0{\cdot}000\,650$

ith.   Am.

$$\begin{array}{r} 424 \\ 315 \\ \hline 11{\cdot}0 \end{array} : 37 = 3$$

Am. $1{\cdot}5\,\%$

$0{\cdot}015$

$\alpha = 0{\cdot}0011{\cdot}8$

$\frac{\partial \eta}{\partial p} = 0{\cdot}00093$

$- \frac{\cancel{0{\cdot}000}93}{\cancel{0{\cdot}00065}\,5} \cdot \cancel{0{\cdot}00118}$ $0{\cdot}00009$

$837 : 5$

~~0·00118~~

$$\begin{array}{r} -\;0{\cdot}00167 \\ +\;0{\cdot}00183 \\ \hline +\;0{\cdot}00015 \end{array}$$

$$\begin{array}{r} 19{\cdot}3 \\ 18{\cdot}9 \\ \hline 0{\cdot}4 \end{array} \qquad \begin{array}{r} 14{\cdot}5 \\ 11{\cdot}7 \\ \hline 2{\cdot}8 \end{array}$$

Am.

$28 : 20$

$14 : \cancel{1{\cdot}1}\,131 = 11$

$0{\cdot}00148$

$0{\cdot}00073$

$- \frac{\cancel{0{\cdot}000}73}{17} \cdot 0{\cdot}00148$

$148 \cdot 73$

$$\begin{array}{r} 1036 \\ 444 \\ \hline 1080 \end{array} : 17 = 63$$

60

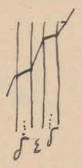
$$\begin{array}{r} -0{\cdot}0063 \\ +0{\cdot}0018 \\ \hline -0{\cdot}0045 \end{array}$$

Przewodnictwo cieplne proszków musi zależeć od ~~ciała~~, jakości gazu

→  otaczającego i od temp. spręż. [illegible]. zatem od ciśnienia.

Można sobie to wystawić w przybliżeniu pod obrazem szeregu blaszek o grubości podobnej jak ziarnka proszku:

→ 



k przewodn. ciała stałego

κ " gazu

χ " proszku

$$k \frac{d\theta_s}{dx} = \kappa \frac{d\theta_g}{dx} = \chi \frac{d\theta_p}{dx}$$

$$\delta \frac{d\theta_s}{dx} + \varepsilon \frac{d\theta_g}{dx} = \frac{d\theta_p}{dx} (\delta+\varepsilon)$$

$$\delta + \varepsilon \frac{k}{\kappa} = \frac{k}{\chi}(\delta+\varepsilon) \qquad \frac{\delta}{k} + \frac{\varepsilon}{\kappa} = \frac{\delta+\varepsilon}{\chi}$$

$$\cancel{\frac{1}{k} + \frac{1}{\kappa} = \frac{2}{\chi}}$$

albo nazywając μ procentową zawartość (względem objętości) ciała stałego, zatem $\frac{\varepsilon}{\delta+\varepsilon} = 1-\mu$:

$$\frac{\mu}{k} + \frac{1-\mu}{\kappa} = \frac{1}{\chi}$$

Wskutek skoku temp. zmieni się to na:

$$\delta \frac{d\theta_s}{dx} + (\varepsilon + 2\gamma) \frac{d\theta_g}{dx} = (\delta+\varepsilon) \frac{d\theta_p}{dx}$$

$$\frac{\delta}{k} + \frac{\varepsilon+2\gamma}{\kappa} = \frac{\delta+\varepsilon}{\chi} \qquad \frac{\mu}{k} + \frac{1-\mu}{\kappa} + \frac{\frac{2\gamma}{\delta+\varepsilon}}{\kappa} = \frac{1}{\chi}$$

N. p. dla powietrza przy 760 mm: $\lambda = 10^{-5}$ cm

$\rho = 1{\cdot}7 . 10^{-5}$ cm

Jeśli kule to przybliżenie: $\mu = \frac{3}{4}$

$$\frac{1}{\lambda} \neq \frac{1}{K}\left[\frac{1}{4} - \frac{3{\cdot}4 . 10^{-5}\,\text{cm}}{\delta+\varepsilon}\right]$$

N. p. $\delta+\varepsilon =$ ~~[illegible]~~ $0{\cdot}1$ mm

$$\frac{1}{\lambda} = \frac{1}{K}\left[\frac{1}{4} - 3{\cdot}4 . 10^{-3}\right] = \frac{1}{4K}\left[1 - 1{\cdot}3 . 10^{-2}\right] \qquad 1{\cdot}3\,\%$$

$\delta+\varepsilon = 0{\cdot}01$ mm $\qquad \frac{1}{\lambda} = \frac{1}{4K}\left[1 - 1{\cdot}3 . 10^{-1}\right] \qquad 13\,\%$

~~$\delta+\varepsilon = 0{\cdot}001$ mm~~ ~~$\frac{1}{\lambda}$~~

$\delta+\varepsilon = 0{\cdot}001$ mm tutaj λ byłoby 10 razy większe, zatem już nie można zastosować takich obliczeń, ale ~~odtąd zaczyna się~~ tu jest ~~[illegible]~~ granica ~~gdzie~~ poniżej której przewodnictwo będzie się zmniejszało proporcyonalnie do ciśnienia gazu, a nie będzie zależnem od grubości ziarn (jeśli ~~[illegible]~~ $\delta\varepsilon < 0{\cdot}001$) i ściśnienia proszku

Jaka jest średnia prędkość ~~[illegible]~~ środka ciężkości dwóch cząstek?

$$f(u_1,v_1,w_1) = A e^{-h(u_1^2+v_1^2+w_1^2)} du_1\, dv_1\, dw_1$$

$$\iiint\!\!\iiint A^2 e^{-h[(u_1^2+v_1^2+w_1^2)+(u_2^2+v_2^2+w_2^2)]} du_1\, dv_1\, dw_1\, du_2\, dv_2\, dw_2 \frac{1}{2}\sqrt{(u_1+u_2)^2+(v_1+v_2)^2+(w_1+w_2)^2}$$

$$e^{-h(V_1^2+V_2^2)} \quad V_1^2 V_2^2 \sin\theta_1 \sin\theta_2\, d\theta_1\, d\theta_2\, dV_1\, dV_2\, d\varphi_1\, d\varphi_2 \sqrt{V_1^2+V_2^2-2V_1V_2\cos}$$

Jeśli przybliżenie: średnia prędkość c

$$\iiint \sin\theta_1 \sin\theta_2\, d\theta_1\, d\theta_2\, d\varphi_1\, d\varphi_2 \frac{1}{\sqrt{2}}\sqrt{1-\cos{\theta_1\varphi_1 \atop \theta_2\varphi_2}}$$

$\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos(\varphi-\varphi)$

$$\iiint \uparrow$$

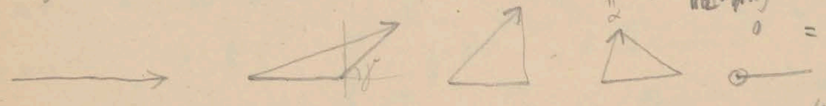
$$= \iint 2\pi \sin\gamma\, d\gamma \frac{\sqrt{1+1+2\cos\gamma}}{4\pi} = \frac{\sqrt{2}}{2}\int_0^\pi \underbrace{\sqrt{1+\cos\gamma}}_{\sqrt{2}\sin\frac{\gamma}{2}} \sin\gamma\, d\gamma = \frac{1}{\sqrt{2}} \cancel{\frac{(1+\cos\gamma)^{3/2}}{3/2}\Big|_0^\pi}$$

$$\frac{\gamma}{2} = \alpha$$

$$= [illegible] \int_0^{\frac{\pi}{2}} \sin\alpha \sin 2\alpha\, d2\alpha$$

$$= 4 [illegible] \int_0^{\frac{\pi}{2}} \sin^2\alpha \cos\alpha\, d\alpha$$

$$= \frac{4}{3} [illegible] \sin^3\alpha \Big|_0^{\frac{\pi}{2}} = \frac{4 [illegible]}{3}$$



ca $\frac{\sqrt{2}}{2} c = \frac{1}{\sqrt{2}} c$

Średnia prędkość 4 cząsteczek: można wyobrazić sobie złożoną z

2 grup à 2 cząsteczek $= \left(\frac{1}{\sqrt{2}}\right)^2 c$

8 cząsteczek $= \left(\frac{1}{\sqrt{2}}\right)^3 c$

16 " $\left(\frac{1}{\sqrt{2}}\right)^4 c$

2^n " $\left(\frac{1}{\sqrt{2}}\right)^n c = \frac{1}{2^{\frac{n}{2}}} c = \frac{c}{\sqrt{2^n}}$

n.p. ~~[illegible]~~ prędkość środka masy 1000 cząstek ca $\left(\frac{1}{2}\right)^5 c =$

$= \frac{c}{32}$

Jeśli $\delta = 10^{-8}$ cm

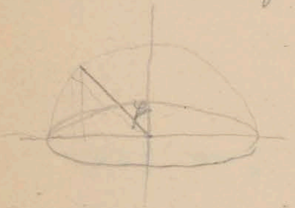
to przypadnie na $[10^{-3}\ \mathrm{mm}]^3 = 10^{12}$ cząsteczek

średnia ich prędkość: $\frac{c}{\sqrt{10^{12}}} = \frac{c}{10^6}$

$c = \frac{400\ \mathrm{m}}{\mathrm{sec}}$ $\quad \Vert\ \frac{400 \cdot 100}{10^6} = \frac{4}{100}\ \frac{\mathrm{cm}}{\mathrm{sec}}$

$= 0{,}4\ \frac{\mathrm{mm}}{\mathrm{sec}}$

F Exner mierzył jednak tylko średnią prędkość <u>rzutu</u> drogi



$$\int_0^{\frac{\pi}{2}} \frac{2\sin\varphi\, d\varphi}{2\pi} = \frac{1}{\pi}$$

jego wartość trzeba pomnożyć jeszcze przez $\underline{\pi}$

Jaka będzie średnia wartość ~~średniej~~ drogi wykonanej w czasie 2τ jeśli τ odpowiada długości średniej drogi λ?

Podobnie ~~Tak samo~~ jak w tamtym wypadku ca $\frac{\lambda}{\cancel{2}}\sqrt{2}$ / ale nie dzieli się przez 2!

Średnia wartość drogi składającej się z $\overset{m=}{2^n}$ kawałków λ (więc odbytej w czasie $2^n.\tau$) będzie $\frac{\lambda(\sqrt{2})^n}{\cancel{2}}$ przybliż.: $\cancel{\frac{\lambda}{\sqrt{2}} = \frac{\lambda}{\sqrt{m}}}$ $\lambda\sqrt{2}^n$

$= \lambda\sqrt{m}$

po upływie

$\tau c\sqrt{2}$ 2τ

$\tau c(\sqrt{2})^2$ 4τ

$\tau c(\sqrt{2})^3$ 8τ

$\tau c\sqrt{m}$ $m\tau$

$\underbrace{\qquad}_{t}$

$= \sqrt{m\tau}\cdot$ ~~$\sqrt{m}\,c\,\sqrt{c}$~~

$\sqrt{\tau c}\cdot\sqrt{c}$

$l = \sqrt{t\lambda c}$

n.p. jeżeli $\lambda = 10^{-8}$ cm

$t = 1$

$c = 400.100$

$l = \sqrt{4.10^{-4}} = 2.10^{-2}$

$= 0{\cdot}02$ cm $= 0{\cdot}2$ mm

$= 200\mu$

$$\left.\begin{aligned}\lambda &= 10^{-8}\text{ cm}\\ t &= 1\\ c &= 0{\cdot}04\text{ cm}\end{aligned}\right\}\quad l = \sqrt{4.10^{-10}} = 2.10^{-5}\text{ cm} = 0{\cdot}0002\text{ mm}$$

$= 0{\cdot}2\mu$

więc „scheinbares" $c = c' = 0{\cdot}0002\ \frac{\text{mm}}{\text{sec}}$

Emulsye można uważać jako roztwory, kropelki drobne w nich zaś [illegible] jako molekuły. Dyfuzya.

A zatem kropelki same jako gas.

$\frac{1}{\rho}dp = -g\,dz \qquad p = \frac{p_0}{\rho_0}\rho$

$\rho = \rho_0\, e^{-g\frac{\rho_0}{p_0}z}$

$\frac{980.\ 0{\cdot}0012}{980.\ 76.\ 13{\cdot}6} = 10^{-6}.z_{cm} = 10^{-4}z_m$

$\frac{p}{\rho} = R\theta$ θ to samo

R w stosunku ciężarów molek.

jeżeli n.p. $\delta = 0{\cdot}1\mu$ (kropelki) to będzie w nich cca 10^9 molekułów

zatem stała gazowa

$\rho = \rho_0\, e^{-10^3.z_{cm}}$

więc już w 1mm wysokości ρ prawie zero

jeżeli jednak $\delta = 0{\cdot}01\mu$ to będzie $\rho = \rho_0\, e^{-2\,cm}$ zatem cząsteczki tej wielkości wcale nie osadzą się, jeszcze w odległości kilku cm $\rho > 0$

„Fällen" przez dodawanie sól etc. polega na tem że się zmienia [illegible] i że one się zlewają w większe kropelki.

~~Rudzki w ostatnich latach ogłosił szereg prac, w których rozwija matematyczną teoryę [illegible] ziemi z uwzględnieniem~~

~~Praca powyższa~~ Jest to czwarta z szeregu prac, w których Rudzki rozwija matematyczną teoryę wnętrza ziemi, tworząca dalszy ciąg rozprawy

~~XXXII~~ O kształcie fali sprężystej w pokładach ziemskich. Rozpr. XXXIII t.

Rudzki ~~zwrócił uwagę~~ zauważył ~~zatem~~, że pokłady ziemskie nie można uważać ~~jako~~ ~~ciała~~ za ośrodki izotropowe. ~~Uwzględnia~~ ~~Uwzględnia~~ ~~ziarnistość~~ Nie polega to na ziarnistości struktury ~~które~~ ~~ciała~~ skał, która przy tego rodzaju zjawiskach, gdzie chodzi o stosunkowo długie fale, nie wchodzi w rachubę, ale na uwarstwowaniu ich i na wpływie ciśnienia górnych warstw na dolne, wskutek którego sprężystość w kierunku pionowym będzie różna od sprężystości w kierunku poziomym, (a zatem równanie ~~sp~~ ~~[illegible]~~ sprężystości, ~~które~~ ~~[illegible]~~ 5 spółczynników musiały będą zawierać ~~5 zł spółczynników spręż.~~ (zamiast 2 jak w ośrodkach izotr.)

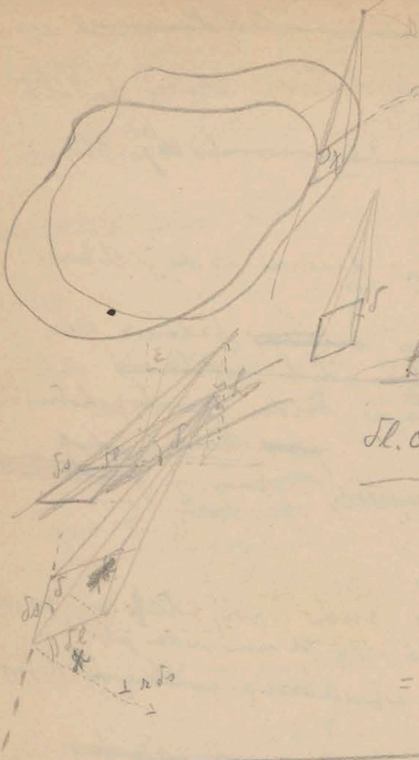
~~wynika, że fale powstające wskutek dystrakcyi w jednym punkcie nie będą miały kształtu kul [illegible] jak w ośrodk. izotr. lecz będą się składały z~~

Ruch drgający który powstanie wskutek wstrząśnienia w jednym punkcie nie będzie się rozchodził równomiernie we wszystkich ~~stro~~ kierunkach, ~~jako~~ tworząc fale kuliste jak w ośrodkach izotropowych, tylko z prędkością różną w różnych kierunkach.

Obliczenia ~~kształt~~ wykazują że powierzchnia fali ~~będzie się kształtu~~ [illegible] rozpada się na dwie części
I. elipsoidę obrotową i ~~III~~ II. inną powierzchnię obrotową, której kształt
określony jest przez równanie ~~bardzo skomplikowane, bo III~~ 6go stopnia.

~~Nie znając jeszcze z doświadczenia~~
Nie rozporządzając jeszcze doświadczalnemi danemi co do wielkości
owych 5 spółczynników, ~~Radziki~~ nie można ~~ścisłego~~ badania dalej
~~Radziki oblicza jednak kształt owej~~
przeprowadzić w ścisły sposób; na podstawie bardzo prawdopodobnych
~~jednak~~ Radziki) jednak
założeń co do owych wielkości ↑ ~~jako przykład~~ (wykonywuje ~~jednak Radziki~~
dyskusyę owego równania
~~tu rachunek~~ (na podstawie

wynika z niego, że owa druga powierzchnia znów rozpada się na dwie
mniej więcej tak samo prędko jak
części, wewnętrzną ~~III~~ która się rozchodzi ~~z prędkością podłużną do~~ elipsoidalną
falą i zewnętrzną rozchodzącą się z prędkością ~~prawie podwójną~~ [illegible]
znacznie większą. (mniej więcej podwójną) Jednorazowe wstrząśnienie środka
sprowadzi zatem we wszystkich (...) punktach
~~Wskutek wstrząśnienia jednorazowego punkt~~ ziemi trzy (...) wstrzą-
~~Każdy punkt ziemi (z wyjątkiem punktów osobliwych) doznaje więc trzech~~ ~~punktu~~
~~jednorazowego wstrząśnienia środka,~~
(~~centralnie i wyżej~~) wstrząśnień ~~wskutek~~
zresztą
~~z których każdy się rozchodzi~~ które nie będą (ani wyłącznie dylatacyjne
(jak fale głosu) ani torsyonalne (jak ~~para~~ fale światła) tylko charakteru mieszanego.



$$\sum i\,d\omega = \sum \frac{i}{r^2}\, r\,\delta\,.\,ds\,.\,dl$$
$\cos\chi$

~~$f = r \sim \delta\cos\chi$~~

$$\frac{\delta l.\cos\chi\,.\,\delta s \sim \delta\,.\,\frac{r}{3}}{\frac{4}{3}\pi r^3}\,.\,4\pi$$

$$= \sum \frac{\delta l\,\delta s \sim \delta\cos\chi}{\cancel{4\pi}\, r^2}$$

Lit. Exner Brown 1827

Exner Wiener Pogg. Ann. CXVIII | Renard J. d. pharm. 3. XXXIV

[illegible] S Mastix [illegible] C [illegible]

[illegible]

[illegible] Vol. [illegible] 0.004 mm [illegible]

| [illegible] ², Vol < l³

[illegible]

[illegible] Archi[illegible] [illegible]

[illegible]

[illegible] | [illegible]

Gouy : Note préliminaire : Journal de Physique 31/12 1888

Attention : quelques substances se déposent à la paroi, gomme gutte jamais

d'autant plus vif que moindre viscosité

phénomène général

Le mouvement brown. est donc un phénomène général d'autant plus sensible que la viscosité du liquide est plus petite. Le point le plus important est la régularité du phénomène ; des milliers de part. ont été examinées et dans aucun cas on n'a vu une part. en suspension qui n'offrit pas le mouvement habituel, avec son intensité ordinaire, eu égard à la grosseur de la particule.

Lumière n'a pas d'influence

des particules de même grosseur, mais de nature diverse solides, liquides, gazeuses, sont animées de mouvements peu différents.

Très rapide à la limite de visib., beaucoup plus que pour $\delta = 1\mu$

vitesses ca quelques $\frac{\mu}{\text{sec}}$, ce qui répond peut être à l'objection qu'on pourrait tirer de la loi des grands nombres, en considérant l'extrême petitesse des molécules.

[illegible] Wied. Ann. 36 p. 334-346 1889

Galitzine { [illegible] Z. ph. Ch. 4 p. 417 (1889)
Wied. Ann. 55 p. 200 (1888)

Schwedoff ~~8 J. [illegible]~~ Rech. exp. sur la cohésion des liquides J. d. ph. 8 p. 341 (1889)

Barus Subsidence of fine solid particles in liquids

Sill. J. 37 p 122 Beibl. 12 p 563 (1888) Rdschau 4, p 451

Fortschr. 1889 p 513

[illegible] [[illegible]] [illegible]

[illegible]

[illegible] permanent aufgeschlämmt, [illegible] HCl

L'Eclairage électrique

La polarisation rotatoire magnétique et l'axiome de Clausius

Wien contre Brillouin (L'Ecl. El. XV p. 265 1898)

↓ Wied. Ann. 52 p. 132 (1894)

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p_{H_2O} =	0.39	-30°
	0.91	-20°
	2.08	-10°
	4.60	+0°
	9.16	10°

$$p = 4.525 \cdot 10^{\frac{7.4475\,t}{234.69+t}} = a \cdot 10^{\frac{bt}{c+t}}$$

n.p. $t = -20$

$$\frac{-148.95}{214.69} = \quad \begin{array}{r} 1731 \\ -\,3318 \\ \hline 0.8413 - 1 \end{array}$$

$$\downarrow$$

$$10^{0.6938} = 4.941$$

$\lg p = 0.6556 + 0.6938$

$= 0.9618 - 10$

$= 0.916$

$$\frac{dp}{dt} = a \cdot 10^{\frac{bt}{c+t}} \cdot {}^{e}\log 10 \cdot \frac{b}{(c+t)^2}\left[(c+t)-t\right]$$

$$= p \cdot {}^{e}\log 10 \cdot \frac{bc}{(c+t)^2}$$

$$s = \frac{1}{p} \qquad \log s = -\log p$$

$$\frac{d\log s}{dt} = -\frac{d\log p}{dt} = -\frac{1}{p}\frac{dp}{dt} = -\,{}^{e}\log 10 \cdot \frac{bc}{(c+t)^2}$$

$$J\lambda = T\frac{dp}{dt} \qquad\qquad V = \frac{v_0}{273} \cdot T$$

$$\frac{dv}{dt} = -\frac{JC - \lambda J\, V\frac{d\log s}{dt}}{p + \lambda J} = -\frac{JC + T\left(\frac{dp}{dt}\right)^2 \cdot \frac{V}{p}}{p + T\frac{dp}{dt}}$$

$$\frac{dx}{dt} = p\frac{dv}{dt} - \frac{pv}{T}$$

$$= -P\left\{\frac{JC + T\left(\frac{dp}{dt}\right)^2\frac{V}{p}}{p + T\frac{dp}{dt}} + \frac{V}{T}\right\}$$

$$= -P\left\{\frac{JC + \left(T\frac{dp}{dt}\right)^2\frac{1}{p}\frac{v_0}{273}}{p + T\frac{dp}{dt}} + \frac{v_0}{273}\right\}$$

$$\frac{771}{273}\Big\}\log = \frac{43616}{45089}$$

70716
$-$97676
73040

53752
2824
56·576 . 1034
1697
226
$\boxed{58.500} = \frac{585\,m}{8100}$
88705

$JC = 9·897$

$C = 0·2377$

$$J = \frac{42200 \cdot 10^3}{980 \cdot 76 \cdot 13·6} = 41·64$$

$$\left(\frac{dp}{dt}\right)_0 = p_0\, {}^{e}\log 10 \cdot \frac{b}{c} = 4·525 \cdot 2·3026 \cdot \frac{7·4475}{234·69}$$

88081
99149
13354
00584

62531
00584
61947
37603
99550

65562
87201
~~27046~~
36222
~~[illegible]~~
$-$88985
51936

51936
43616
95552

$T\frac{dp}{dt} =$ 90·265
4·525
94790

91104
45089
36193
$-$65562
70631

50853
9·9
5095·2

$$X_x = \overset{=2M}{a}\frac{\partial\xi}{\partial x} + \overset{\parallel L}{b}\,\theta \qquad X_y = Y_x = \frac{a}{2}\left(\frac{\partial\xi}{\partial y} + \frac{\partial\eta}{\partial x}\right)$$

$$Y_y = a\frac{\partial\eta}{\partial y} + b\theta \qquad Y_z = Z_y = \frac{a}{2}\left(\frac{\partial\eta}{\partial z} + \frac{\partial\zeta}{\partial y}\right)$$

$$Z_z == a\frac{\partial\zeta}{\partial z} + b\theta \qquad Z_x = X_z = \frac{a}{2}\left(\frac{\partial\zeta}{\partial x} + \frac{\partial\xi}{\partial z}\right)$$

$$\cancel{a}\,\rho\frac{\partial^2\xi}{\partial t^2} = \frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} = a\frac{\partial^2\xi}{\partial x^2} + \frac{a}{2}\left(\frac{\partial^2\xi}{\partial y^2} + \frac{\partial^2\eta}{\partial x\partial y}\cancel{\xi} + \frac{\partial^2\zeta}{\partial x\partial z} + \frac{\partial^2\xi}{\partial z^2}\right) + b\frac{\partial\theta}{\partial x}$$

$$= \frac{a}{2}\nabla^2\xi + \frac{a}{2}\frac{\partial\theta}{\partial x} + b\frac{\partial\theta}{\partial x} = \frac{a}{2}\nabla^2\xi + \left(b + \frac{a}{2}\right)\frac{\partial\theta}{\partial x}$$

$$\left.\begin{array}{l|l}
\rho\dfrac{\partial^2\xi}{\partial t^2} = \dfrac{a}{2}\nabla^2\xi + \left(\dfrac{a}{2} + b\right)\dfrac{\partial\theta}{\partial x} & \dfrac{\partial}{\partial x} \\
\rho\dfrac{\partial^2\eta}{\partial t^2} = \dfrac{a}{2}\nabla^2\eta + \left(\dfrac{a}{2} + b\right)\dfrac{\partial\theta}{\partial y} & \dfrac{\partial}{\partial y} \\
\rho\dfrac{\partial^2\zeta}{\partial t^2} = \dfrac{a}{2}\nabla^2\zeta + \left(\dfrac{a}{2} + b\right)\dfrac{\partial\theta}{\partial z} & \dfrac{\partial}{\partial z}
\end{array}\right\} \quad \rho\frac{\partial^2\theta}{\partial t^2} = (a+b)\nabla^2\theta$$

~~[illegible]~~ Dilotacyjne prędkości $\sqrt{\frac{a+b}{\rho}}$

$$\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial\xi}{\partial y} - \frac{\partial\eta}{\partial x}\right) = \frac{a}{2}\nabla^2\left(\frac{\partial\xi}{\partial y} - \frac{\partial\eta}{\partial x}\right)$$

} Torsyonalne prędkości $\sqrt{\frac{a}{2\rho}} = \sqrt{\frac{T}{\rho}}$

$$\frac{a}{2} = T$$

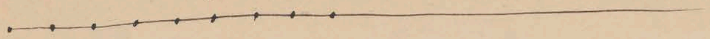
$$x_{n-m}\frac{d^2x_{n-m}}{dt^2} + x_{n-m+1}\frac{d^2x_{n-m+1}}{dt^2} + \dots + x_{n+m}\frac{d^2x_{n+m}}{dt^2} =$$

$$= E\left[x_{n-m}(x_{n-m-1} - 2x_{n-m} + x_{n-m+1}) + x_{n-m+1}(x_{n-m} - 2x_{n-m+1} + x_{n-m+2}) + \dots\right]$$

$$= -E\left[(x_{n-m} - x_{n-m+1})^2 + (x_{n-m+2} - x_{n-m+1})^2 + \dots\dots\right] +$$

$$+E\left[+x_{n-m}x_{n-m-1} - x_{n-m}^2 + x_{n+m}x_{n+m+1} - x_{n+m}^2\right]$$

$$= \frac{d}{dt}\left[x_{n-m}\frac{dx_{n-m}}{dt} + \dots\right] - \left[\left(\frac{dx_{n-m}}{dt}\right)^2 + \dots \left(\frac{dx_{n+m}}{dt}\right)^2\right]$$



~~$d^2\xi$~~ $$\frac{d^2x_n}{dt^2} = E\left[(x_{n+1} - x_n - l) + (l - (x_n - x_{n-1}))\right]$$

$$= E\left[x_{n+1} - 2x_n + x_{n-1}\right]$$

$$\frac{d^2x_1}{dt^2} = E(x_2 - x_1 - l)$$

$$\frac{d^2x_2}{dt^2} = E(x_3 - 2x_2 + x_1)$$

$$\frac{d^2x_3}{dt^2} = E(x_4 - 2x_3 + x_2)$$

$$\frac{d^2x_4}{dt^2} = E(x_5 - 2x_4 + x_3)$$

$$\frac{d^2(x_2+x_1)}{dt^2} = E(x_3 - x_2 - l)$$

$$\frac{d^2(x_3+x_2)}{dt^2} = E(x_4 - x_3 - x_2 + x_1)$$

- - - - - - - - -

$$\frac{d^2x_n}{dt^2} = E(x_{n+1} - 2x_n + x_{n-1})$$

$$\frac{d^2x_{n+1}}{dt^2} = E(x_{n+2} - 2x_{n+1} + x_n)$$

$$\overbrace{+\frac{dx_{n-1}}{dt^2}\quad \frac{d^2x_n}{dt^2}\frac{dx_n}{dt} + \frac{d^2x_{n+1}}{dt^2}\frac{dx_{n+1}}{dt}} = E\Big\{\overbrace{\underbrace{x_n\frac{dx_{n-1}}{dt} + x_{n-1}\frac{dx_n}{dt}}}^{\frac{d}{dt}(x_n x_{n-1})} - 2x_{n-1}\frac{dx_{n-1}}{dt} + x_{n-2}\frac{dx_{n-1}}{dt}$$

$$-2x_n\frac{dx_n}{dt} + \underbrace{x_{n+1}\frac{dx_n}{dt} + x_n\frac{dx_{n+1}}{dt}} - 2x_{n+1}\frac{dx_{n+1}}{dt} + x_{n+2}\frac{dx_{n+1}}{dt}$$

$$\frac{1}{2}\frac{d}{dt}\left[\left(\frac{dx_{n-m}}{dt}\right)^2 + \left(\frac{dx_{n-m+1}}{dt}\right)^2 + \cdots \left(\frac{dx_1}{dt}\right)^2 + \cdots \left(\frac{dx_{n+m}}{dt}\right)^2\right] = E\left[\frac{d}{dt}\left(x_{n-m}\cdot x_{n-m+1} + x_{n-m+1}x_{n-m+2} + \cdots\right)\right.$$

$$\left. - \frac{d}{dt}\left[x_{n-m}^2 + x_{n-m+1}^2 + \cdots\right] + x_{n-m-1}\frac{dx_{n-m}}{dt} + x_{n+m}\frac{dx_{n+m+1}}{dt}\right]$$

$$= -E\left[\frac{d}{dt}\frac{1}{2}\left[(x_{n-m+1} - x_{n-m})^2 + (x_{n-m+2} - x_{n-m+1})^2 + \cdots\right] + x_{n-m-1}\frac{dx_{n-m}}{dt} + x_{n+m}\frac{dx_{n+m}}{dt}\right]$$

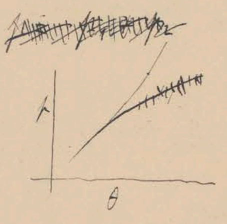
$dQ = A\,p\,dv + dU$ pro 1 gr.

$0 = A\,p\,dv + c\,d\theta$ ~~+~~ $+\ r\,dx$

$$dx = \frac{\partial x}{\partial p}\,dp + \frac{\partial x}{\partial \theta}\,d\theta$$

$$\frac{1}{\varrho}\frac{dp}{dz} = g$$

$$\frac{p}{\rho} = R\theta$$



x = [illegible]

Wenn 1 kg. [illegible] x kg [illegible] temp. θ p, wie viel wird sich ausscheiden wenn $p+dp$ $\theta+d\theta$? $= dx$

[illegible] θ [illegible] P

[illegible] cond. [illegible] ~~$\frac{p\,x}{\varrho d}$ = [illegible] Vol. = $R_{22}\theta$~~ $\quad x = \rho d.$

[illegible] Vol. $\frac{1}{\rho}$ [illegible] P [illegible] ρw $\quad x = \frac{\rho d}{\rho d}\cdots = \frac{1}{s\rho}$

~~[illegible]~~ $\quad s = \frac{RT}{P} \qquad s = s_0\,\frac{T}{T_0}\cdot\frac{P_0}{P}$

~~750~~ ~~$\frac{18.5}{750}\;\frac{99.6}{2105}\cdot 55746$~~ $\quad \frac{pM}{\rho} = R\theta \qquad R = \frac{760.}{0.001293\;.\;273}$

2553 $\qquad s = \frac{1}{0.627}\,\frac{R_{Lp}\,T}{P}$

~~[illegible]~~ $\quad x = \frac{P}{\rho\,R_w\,\theta} \qquad r = A\,T\,\frac{dP}{dT}\,s = A\,\frac{\theta\,R_w}{p}\,\frac{dP}{d\theta}$

$=$

$v\,dp = g\,dz$

$pv = R\theta$

$p\,dv = R\,d\theta - g\,dz$

$x = \frac{P}{R_v\theta}\,v$

$dx =$

$p = \frac{R\theta}{v}$

$\cancel{v\,dp = -\frac{R\theta}{v}\,dv}$

$-x$ $\quad \underbrace{\;}_{}^{+r}$

$A p\,dv + c\,d\theta = \underset{A}{+}\, d\left(\frac{Pv}{R\theta}\right) \cdot \frac{\theta^2 R}{P}\frac{dP}{d\theta}$

$pv = R\theta$

$\frac{v}{R\theta} = \frac{1}{p}$

!

$v\,dp = g\,dz$

$pv = R\theta$

$\frac{AR\theta}{v}\,dv + c\,d\theta = A\frac{\theta^2 R}{P}\frac{dP}{d\theta}$ $\quad d\left(\frac{Pv}{R\theta}\right) \underset{A}{=} -\frac{\theta^2 R}{P}\frac{dP}{d\theta}\left[P\,d\left(\frac{v}{R\theta}\right) + \frac{v}{R\theta}\,dP\right]$

$v\,d\left(\frac{R\theta}{v}\right) = -g\,dz$

$AR\,d\theta + c\,d\theta + Ag\,dz = -A\frac{\theta^2 R}{P}\frac{dP}{d\theta}\left[d\left(\frac{1}{p}\right) + \frac{1}{p}\frac{dP}{P\,d\theta}\,d\theta\right]$

$\frac{R\theta}{p}\,dp = -g\,dz$

$d\theta\left[\underbrace{AR + c}_{AR\frac{k}{k-1}} + A\frac{\theta^2 R}{pP}\left(\frac{dP}{d\theta}\right)^2\right] = -Ag\,dz - A\,\theta\,\frac{dP}{d\theta}\,\frac{1}{p}\,g\,dz$

$= -\left(A + \frac{\theta}{p}\frac{dP}{d\theta}\right)g\,dz$

$\frac{d\theta}{dz} = \frac{g}{R}\,\frac{1 + \frac{\theta}{p}\frac{dP}{d\theta}}{\frac{k}{k-1} + \frac{\theta^2}{pP}\left(\frac{dP}{d\theta}\right)^2} = -\underbrace{\frac{k-1}{k}\frac{g}{R}}_{=1}\,\frac{1 + \frac{\theta}{p}\frac{dP}{d\theta}}{1 + \frac{k-1}{k}\frac{\theta^2}{pP}\left(\frac{dP}{d\theta}\right)^2}$

$= 1\,\frac{1 + \left(\frac{1}{A}\frac{\theta}{p}\frac{dP}{d\theta}\right)}{1 + \left(\frac{1}{A}\frac{\theta}{p}\frac{dP}{d\theta}\right)\cdot\left(\frac{k-1}{k}\frac{\theta}{P}\frac{dP}{d\theta}\right)}$

-25° -15° -5° 0° +5° +10° + 15°

0·~~309₅~~ 0·340 0·451 0·609₅

$\frac{dP}{d\theta}$	$\frac{dP}{d\theta}$	θ	P	p	
-30°					
-25°					
-20°	0·071	.253	0·91	519	35
-15°	0·1185	.258	1·44	554	
-10	0·1695	.263	2·15	591	
-5	0·238	.268	3·16	629	
0	0·340	.273	4·52	671	
5	0·451	.278	6·51	715	
10	0·609₅	.283	9·14	760	

9·767
8·548
1·219 : 2

6·971
069
0·902 : 2

4·909
569
340

~~4·569~~
~~3·950~~
~~0·619~~

.3406
2930
0·476

1·562
325
237

2·327
1·988
0·339

$$\theta = 273 \,\|\, \frac{1 + 273\,\frac{0\cdot34}{760}}{1 + 273\cdot\frac{0\cdot34}{760}} \cdot \frac{0\cdot4}{1\cdot4}\;\frac{2}{7}\;\frac{\overset{39}{\cancel{273}}\cdot 0\cdot34}{\cancel{4\cdot52}\;\;2\cdot26}$$

$$\frac{p}{p_0} = \cancel{\left[1 - \frac{k-1}{k}\,\frac{gx}{R\theta_0}\right]^{\frac{k}{k-1}}} \left(\frac{\theta}{\theta_0}\right)^{\frac{k}{k-1}}$$

$$p = 760 \cdot \frac{283}{27} \left(\frac{\theta_0 - \Delta}{\theta_0}\right)^{\frac{k}{k-1}}$$

$$= 760\left[1 - \frac{k}{k-1}\,\frac{\Delta}{\theta_0} + \frac{\alpha(\alpha-1)}{1\cdot2}\left(\frac{\Delta}{\theta_0}\right)^2 - \frac{\alpha(\alpha-1)(\alpha-2)}{1\cdot2\cdot3}\left(\frac{\Delta}{\theta_0}\right)^3 \cdots\right]$$

$\frac{7}{2}\cdot\frac{5}{283} = 3500 : 566 = 0\cdot0618$
104
47

$\frac{7}{2}\cdot\frac{30}{283} =$ ~~210~~ $105 : 283 =$

$\frac{2}{7}$

30103
84510
45593

~~$\frac{1+\alpha}{1-\alpha} = \sqrt{1+\frac{\alpha}{1-\alpha}}$~~

$\frac{7}{2}\cdot\frac{5}{2}\,\frac{1}{2} = \frac{35}{8}$

47712
45179
0·02533 ^{2}
0·05066
0·07599

75
37

1·01354
−0·18551
0·82803 . 760
579621
49682
629303

~~1~~0·106005
0·742035
0·371017
1855085
309181

519

629

760

110

131

0·01124
0·00281 . 35
843
2405
0·10835
~~0~~1·05417
−0·37102
0·68315 . 760
478205
40989
519194

~~44~~
44
45

0·001191

0·05417 . ~~[illegible]~~ $\frac{\cancel{10}}{36.4}$
135425
33856
~~56~~
1·03762
−0·30918
0·72844 : 760
509908
437064
553614

$$\frac{k-1}{k}\,\frac{\int}{u_0 g} = \frac{2}{7}\cdot\frac{0·0033\ .\ \cancel{0·05}\ \big|\ \frac{\cancel{60}}{\cancel{425}}\ .\ \cancel{980}\ .\ \cancel{10}^2}{0·02\ \ \cancel{10}\ .\ \cancel{980}}$$

70

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k \qquad \frac{\frac{p}{\rho}}{\frac{p_0}{\rho_0}} = \left(\frac{\rho}{\rho_0}\right)^{k-1} = \frac{\theta}{\theta_0}$$

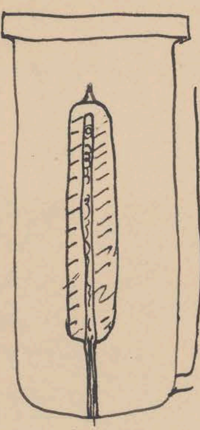
$$= \left(\frac{\theta}{\theta_0}\right)^{\frac{k}{k-1}}$$

$$\left(\frac{4}{760}\right)^{\frac{k-1}{k}}\theta_0 = \theta \qquad = \left(\frac{1}{190}\right)^{\frac{2}{7}} . 283$$

2·4362
−0·6511
1·7851

2·2788 . 2
~~[illegible]~~
4·5576 :7
−0·6511
+2·4518
1·8007

273
63·2
210°



$100\,cm^2 \,.\, \frac{50^\circ}{1\,cm}\ 0{\cdot}0001 =$

$= 0{\cdot}5\ \frac{cal.}{sec.} = 2$ Volt Am

50 Ohm 10 Volt

$i^2 w = \frac{e^2}{w}$

$\frac{4}{3}\,(1{\cdot}4)^2\,(\cancel{980})^2\ \frac{\cancel{0{\cdot}00}1293}{(76.\ 13{\cdot}6)^2\,\cancel{980}^2}\left(\frac{\cancel{273}}{3}\right)^{\cancel{91}\,\cancel{44}\,6} .\ 1{\cdot}67\ .\ \cancel{10}\ .\ \cancel{10}$

$(9{\cdot}1)^6$

$\theta = 3$
$u_0 = 10$

$$\left(\frac{1{\cdot}4}{76.\ 13{\cdot}6}\right)^2 .\ \frac{4.\ 1293\ .\ 167}{3}\ .(9{\cdot}1)^6$$

$$\left(\frac{1}{74}\right)^2 \qquad \left(\frac{9{\cdot}1^3}{740}\right)^2 .\ 4 . 1{\cdot}67 .\ 0{\cdot}431$$

$$\left(\frac{754}{740}\right)^2 = 1{\cdot}04\ .\ \frac{\cancel{1{\cdot}67 . 1{\cdot}72}}{(1{\cdot}7)^2}$$

$\cancel{91}$
819
91
8281
74529
8281
75357

$$\overbrace{V \rho c}^{10}\, \frac{\partial \theta}{\partial t} = -0{\cdot}01\ \theta .$$

$\theta = \theta_1 e^{-\alpha t}$

$\alpha = 0{\cdot}001$

$e^{-\alpha t} = 1\%$

$\cancel{\frac{1}{\alpha t}}$

$100 = e^{\alpha t}$

$lg\,100 = \alpha\, t$

$t = \frac{lg\,100}{\alpha}$

$= \frac{4{\cdot}6}{10^{-3}}$

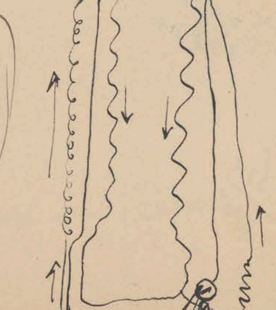
$= 4{\cdot}6 . 10^3$

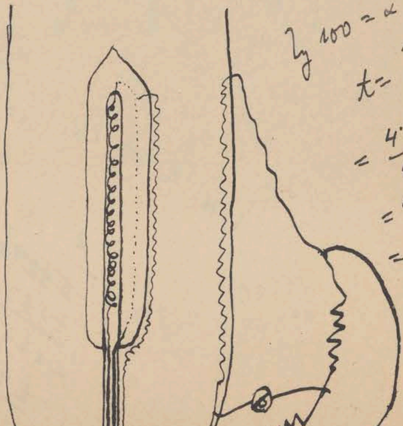
$= 0{\cdot}7 . 10^2$ min

$= 70$ min

= 1 godzi

$0.034 .\ 13{\cdot}6$
408
544
$0{\cdot}46$





$$\theta = a \int_0^{\frac{x}{\sqrt{t}}} e^{-\frac{c\rho}{4k} y^2} dy + b$$

$x = 0 \ : \ \theta = \theta_2$

$t = 0 \ : \ \theta = \theta_1$

$$\theta_1 = a \underbrace{\int_0^{\infty} e^{-\frac{c\rho}{4k} y^2} dy}_{} + b$$

$$\left.\begin{aligned} \theta_1 &= a\, \cancel{2}\sqrt{\frac{k}{c\rho}}\, \frac{\sqrt{\pi}}{\cancel{2}} + b \\ \theta_2 &= b \end{aligned}\right\} \quad a = (\theta_1 - \theta_2) \sqrt{\frac{c\rho}{k\pi}}$$

$$\theta = (\theta_1 - \theta_2) \sqrt{\frac{c\rho}{k\pi}} \int_0^{\frac{x}{\sqrt{t}}} e^{-\frac{c\rho}{4k} y^2} dy + \theta_2$$

$$= (\theta_1 - \theta_2) \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{t}}\sqrt{\frac{4k}{c\rho}}} e^{-y^2} dy + \theta_2 = \frac{2}{\sqrt{\pi}} (\theta_1 - \theta_2) \int_0^{\frac{2x}{\sqrt{t}}\sqrt{\frac{k}{c\rho}}} e^{-y^2} dy + \theta_2$$

więc zależnem będzie od

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = \frac{2}{\sqrt{\pi}} (\theta_1 - \theta_2)\, e^{-\frac{4x^2}{t}\frac{k}{c\rho}} \cdot \left.\frac{2}{\sqrt{t}} \sqrt{\frac{k}{c\rho}}\right|_{x=0} = \frac{4(\theta_1 - \theta_2)}{\sqrt{t}} \sqrt{\frac{k}{\pi c\rho}}$$

$$k \frac{\partial\theta}{\partial x} = \frac{4}{\sqrt{t}} (\theta_1 - \theta_2) \frac{k}{\sqrt{\pi}} \sqrt{\frac{k}{c\rho}} =$$ na powierzchni absorbowane ciepło

Ściana zimna w kontakcie z parą wodną, jeśli n.p. parę wpuszczamy do cylindra.

$$\frac{\partial\theta}{\partial t} = \frac{k}{c\rho}\frac{\partial^2\theta}{\partial x^2}$$

Warunek początkowy $t=0$ $\theta=0 \begin{cases} x=0 \\ x=l \end{cases}$

$t \ldots$ $\theta=\theta_1 \mid x=0$

$\sqrt{t} = x z$

$$\theta = f.\left(\overbrace{\frac{\sqrt{t}}{x}}^{z}\right) \qquad \frac{\partial\theta}{\partial t} = f'(z)\frac{1}{2\sqrt{t}\cdot x} = f'(z)\cdot\frac{z}{2t} = f'(z)\frac{1}{2z x^2}$$

$$\cancel{\frac{\partial\theta}{\partial t} = \frac{f'(z)}{x^2}\frac{\partial\theta}{\partial x}} \cdot \quad \frac{\partial\theta}{\partial x} = -f'(z)\frac{\sqrt{t}}{x^2}$$

$$\theta = \qquad \frac{\partial^2\theta}{\partial x^2} = +f''(z)\frac{t}{x^4} + 2f'(z)\frac{\sqrt{t}}{x^3} = \frac{f''(z)\,z^2 + 2f'(z)\,z}{x^2}$$

$$\frac{f'(z)}{2z\cancel{x^2}} = \frac{k}{c\rho}\,\frac{f''(z)\,z^2 + 2f'(z)\,z}{\cancel{x^2}}$$

$$f'(z) = e^{\varphi(z)}$$

$$f''(z) = \varphi' e^{\varphi(z)}$$

$$\cancel{F'/F}$$

$$\varphi'\cdot z^2 + 2z - \frac{c\rho}{2kz} = 0$$

$$\varphi' + \frac{2}{z} - \frac{c\rho}{2kz^3} = 0$$

$$\varphi = \frac{c\rho}{4kz^2} - 2\log z \quad \Big\| \quad f'(z) = \frac{e^{\frac{c\rho}{4kz^2}}}{z^2}$$

$$\theta = A\int\frac{e^{-\frac{c\rho}{4kz^2}}}{z^2}\,dz \qquad \frac{1}{z} = y \qquad = -A\int e^{-\frac{c\rho}{4k}y^2}\,dy = A\int^{\frac{x}{\sqrt{t}}} e^{-\frac{c\rho}{4k}y^2}\,dy + \overset{\theta_1}{\text{const}}$$

$$\frac{\partial\theta}{\partial t} = -e^{-\frac{c\rho}{4k}\frac{x^2}{t}}\frac{x}{2\sqrt{t^3}} \qquad \frac{\partial\theta}{\partial x} = e^{-\frac{c\rho}{4k}\frac{x^2}{t}}\frac{2}{\sqrt{t}} \qquad \cancel{\frac{\partial^2\theta}{\partial x^2} = e^{\cdots}\left[\frac{2}{t} + \frac{4x^2}{t^2}\right]}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{c\rho}{4k}\frac{2x}{\sqrt{t^3}}\,e^{\frac{c\rho}{\cdots}}$$

Ilość ciepła przewodzonego

przez gaz

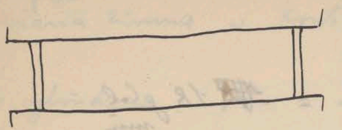
$$\frac{0{\cdot}000066 \cdot 100}{0{\cdot}8} \cdot 35 = \frac{0{\cdot}231}{0{\cdot}8} = 0{\cdot}3\ \frac{g\,Cal}{sec} = 18\ \frac{g\,Cal}{min}$$

przez szkło:

$$\frac{0{\cdot}001 \cdot 100}{0{\cdot}8} \cdot 2 = 0{\cdot}25\ \frac{g\,Cal}{sec}$$

przez sztabę żelazną $q = 10\,mm^2$

$$\frac{0{\cdot}15 \cdot 100}{0{\cdot}8} \cdot 0{\cdot}1 = 2\ \frac{g\,Cal}{sec}$$



$$\gamma_0 (1+\varepsilon\theta)\frac{d\theta}{dz} = \text{const} = a$$

$$\gamma_0\theta + \gamma_0\varepsilon\frac{\theta^2}{2} = az + \text{const}$$

$$\cancel{\gamma_0\theta_0 + \gamma_0\varepsilon}\,\theta \qquad z=0 : \quad \theta=0 \quad \text{const}=0$$

$$\cancel{\gamma_0\theta + \gamma}\ \gamma_0\theta\left[1+\varepsilon\frac{\theta}{2}\right] = az$$

$$\gamma_0\theta_1\left[1+\varepsilon\frac{\theta_1}{2}\right] = ah$$

$$\theta\left(1+\varepsilon\frac{\theta}{2}\right) = \frac{z}{h}\,\theta_1\left(1+\varepsilon\frac{\theta_1}{2}\right)$$

N.p. $z=\frac{h}{2}$ $\qquad \theta\left(1+\varepsilon\frac{\theta}{2}\right) = \frac{1}{2}\theta_1\left(1+\varepsilon\frac{\theta_1}{2}\right)$ $\qquad \theta.$

$$\cancel{\left(\frac{\theta}{2} - \theta_1\right) + \varepsilon\left(\frac{\theta^2}{2} - \theta_1^2\right) = 0}$$

$$\underbrace{\theta - \frac{\theta_1}{2}}_{\delta} + \frac{\varepsilon}{2}\left(\frac{\theta^2}{1} - \frac{\theta_1^2}{2}\right) = 0$$

$$\int\theta\,dz = \int_0^{\theta_1}\frac{\theta\,\gamma_0(1+\varepsilon\theta)\,d\theta}{a} = \frac{\cancel{\gamma_0}}{a}\left(\frac{\theta_1^2}{2} + \varepsilon\frac{\theta_1^3}{3}\right) = \frac{\cancel{\gamma_0}}{a}\,\frac{\theta_1^2}{2}\left[1+\frac{2\varepsilon}{3}\theta_1\right]$$

$$= h\,\frac{\frac{\theta_1^2}{2}\left[1+\frac{2\varepsilon\theta_1}{3}\right]}{\theta_1\left[1+\frac{\varepsilon\theta_1}{2}\right]} \mp \frac{h\theta_1}{2}\left[1+\frac{1}{6}\varepsilon\theta_1\right]$$ więc różnica przeciętnej = $\left(\frac{1}{6}\varepsilon\theta_1\right)$ od średniej temp.

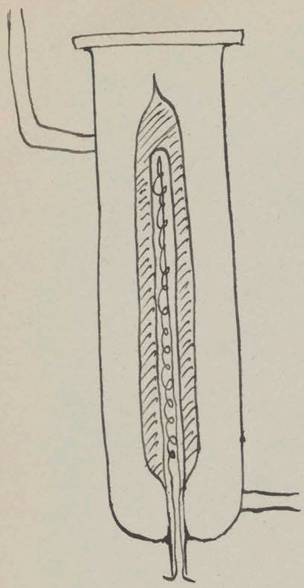
N.p. $\varepsilon = \frac{1}{273} = 0{\cdot}0036$

$\theta_1 = 100$ $\qquad \frac{\varepsilon\theta_1}{6} = 0{\cdot}06 = 6\%$ więc przeciętna temperatura o 3°C = 53°C

wyższa od średniej 50°

Temu odpowiada różnica w ciśnieniu lub objętości $\frac{3}{273} = \frac{1}{90}$

t.j. 8 mm Hg = ~~80~~ 87 mm gliceryny



gr cal

$K \cdot \frac{30\,cm^2}{1\,cm}\,\theta^{=50}$ K H_2O = 0·0015 | 0·045 . 50 = 2·5

Glas

K Luft = 0·00006 | 0·0018 . 50 = 0·09

1 Volt Ampere = 0·24 $\frac{g\,cal}{sec}$ 10 ‒ …. 0·3 Voltamp

$i^2 w = \frac{e^2}{w}$ $w = 2$ Ohm

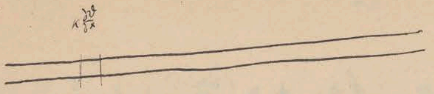
$i^2 = 5$ …. 0·15

$i = 2$ ‒ ‒ ‒ ‒ 0·4 Amp

239

[illegible]

[illegible]



$i =$ [illegible] pro cm²

$$\varphi = i^2 w = \frac{i^2}{\lambda_\theta} = -\frac{\partial}{\partial x}\left(\kappa\frac{\partial\vartheta}{\partial x}\right)$$

$$= -\frac{d\kappa}{dx}\frac{d\theta}{dx} - \kappa\frac{d^2\theta}{dx^2}$$

$$= -\frac{d\kappa}{d\theta}\left(\frac{d\theta}{dx}\right)^2 - \kappa\frac{d^2\theta}{dx^2}$$

$$\kappa = \kappa_0(1+\alpha\theta)$$

$$\lambda = \lambda_0(1+\beta\theta)$$

Jeśli: Przybliżeni $\kappa = \kappa_0$

$$\frac{i^2}{\cancel{\Xi}} = -\lambda_0(1+\beta\theta)\,\kappa_0\frac{d^2\theta}{dx^2}$$

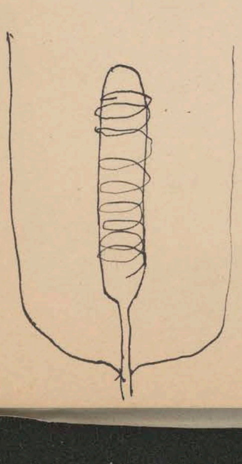
W pierwszym przybliżeniu: $i^2 = -\lambda_0\kappa_0\frac{d^2\theta}{dx^2}$

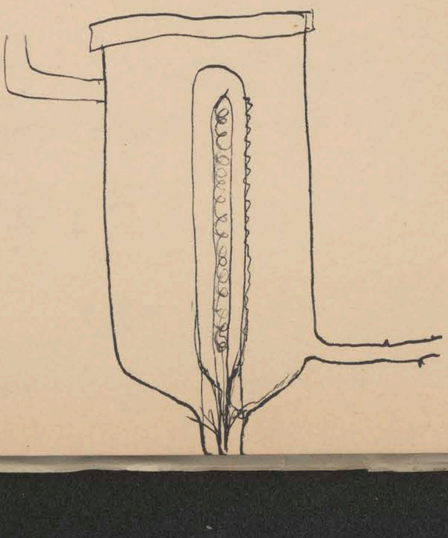
$$\theta = \theta_0 \cancel{\lambda_0\kappa_0} - \frac{i^2}{\lambda_0\kappa_0}x^2 + \frac{i^2}{\lambda_0\kappa_0}x\,l$$

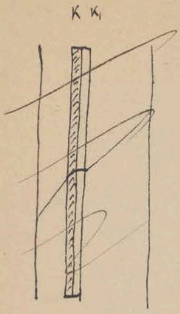
Jeśli $\theta_0 = 0$

$$W = \int\frac{dx}{\lambda_\theta} = \int\frac{dx}{\lambda_0\left(1+\beta\theta_0+\beta\frac{i^2}{\lambda_0\kappa_0}lx - \beta\frac{i^2}{\lambda_0\kappa_0}x^2\right)} = \frac{1}{\lambda_0}\int\frac{dx}{1+\frac{\beta i^2 l}{\lambda_0\kappa_0}x - \frac{\beta i^2}{\lambda_0\kappa_0}x^2}$$

$$= \frac{1}{\lambda_0}\,\frac{}{1-\sqrt{\frac{\beta}{\lambda\kappa}}\,i x}$$







$$\frac{c}{A} f(r) \frac{\partial\theta}{\partial x} = R f(r) \frac{\partial\theta}{\partial x} - w \frac{\partial p}{\partial x} + \mu\left(\frac{\partial u}{\partial r}\right)^2 + K \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

$$\left(\frac{c}{A} - R\right) f(r) \frac{\partial\theta}{\partial x} = -\frac{w}{\rho}\frac{2\mu}{r^2-R^2} f(r) + \mu\left(\frac{\partial u}{\partial r}\right)^2 + K \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)$$

$$-(x+y+z)^2 + 3\left[x^2+y^2+z^2\right]$$

~~$-2(x+y+z) + 6x$~~

~~$4x - 2y - 2z$~~

~~$2x^2 - 2xy + 2y^2 + 2z^2 - 2xz - 2xy$~~

$$-(1+\alpha+\beta)^2 + 3\left[1+\alpha^2+\beta^2\right]$$

$$= 2\left[1+\alpha^2+\beta^2-\alpha-\beta-\alpha\beta\right]$$

$$= \left[(\alpha-\beta)^2 + (1-\alpha)^2 + (1-\beta)^2\right]$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + \cdots = \left(\frac{\partial y}{\partial x}\right)^2 +$$

$$\cancel{\frac{\partial w}{\partial t}} + w\cancel{\frac{\partial w}{\partial z}} = \frac{\eta}{\rho}\left[\cancel{\frac{4}{3}\frac{\partial^2 w}{\partial z^2}} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{c}{A}\rho\, w\frac{\partial \theta}{\partial z} = -p\frac{\partial w}{\partial z} + \mu\left[\cancel{-\frac{2}{3}\left(\frac{\partial w}{\partial z}\right)^2} + \cancel{2\left(\frac{\partial w}{\partial z}\right)^2} + \left(\frac{\partial w}{\partial r}\right)^2\right] + \kappa\, \underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)}_{\overset{\|}{\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}}}$$

$\theta, \rho, v = fc.(r, z)$

$$\frac{p}{\rho} = R\theta$$

$(\rho . w) = fc.(r)$

$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

$\rho w = fc.(r)$ $\qquad \frac{\partial}{\partial x}(\rho w) = 0$ $\qquad \frac{p}{\rho} = R\theta$

$$\frac{c}{A}\rho w\frac{\partial\theta}{\partial x} = -p\frac{\partial w}{\partial x} + \mu\left(\frac{\partial w}{\partial r}\right)^2 + \kappa\left(\underbrace{\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}}_{\frac{1}{r}\frac{\partial\theta}{\partial r}}\right)$$

$p = fc.(x)$

$$\frac{\partial}{\partial r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = 0$$

$$\underbrace{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}}_{\cancel{\frac{1}{r}}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}r\right)} = \varphi(x)$$

$$\cancel{\frac{1}{r}}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}r\right) = r\,\varphi(x)$$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\varphi(x) + \psi(x)$$

$$u = \frac{r^2}{2}\varphi(x) + \cancel{\psi(x)\log(r)} + \chi(x)$$

$$u = \frac{r^2 - R^2}{2}\varphi(x) = \frac{r^2 - R^2}{2}\frac{1}{\mu}\frac{\partial p}{\partial x}$$

$$\rho\frac{r^2 - R^2}{2\mu}\frac{\partial p}{\partial x} = f(r) \qquad \rho\frac{\partial p}{\partial x} = \frac{2\mu}{r^2 - R^2}f(r)$$

$$R\frac{\partial\theta}{\partial x} = \frac{w}{f(r)}\frac{\partial p}{\partial x} - p\frac{\partial}{\partial x}\left(\frac{w}{f(r)}\right)$$

$$= \frac{w}{f(r)}\frac{\partial p}{\partial x} - \frac{p}{f(r)}\frac{\partial w}{\partial x}$$

$$R f(r)\frac{\partial\theta}{\partial x} = w\frac{\partial p}{\partial x} - p\frac{\partial w}{\partial x}$$

C, c ciał stałych, Dulong Petit, związki, ciecze! temper.!

Przy przekroczeniu wytrzymałości ciało staje się płynnem:

magnetyz.
elektrolisa, opór zmniejszony?
podchłodzenie przy ~~poruszaniu~~ mieszaniu cieczy!

Unterkühlung durch Bewegung: Depretz CR II Pogg. Ann. XLI

Leitungsvermögen von Salzen: Grätz Wied. Ann. 40 p. 18 (1890)
KCl St. Andrews Edinb. Proc. 1884/5 13 p. 275
(auch f. fest/fl.) NaCl geschm. 850°

Salzkrystalle in verschied. Richtg. [illegible]!, dann bei Druck bis zur Zerdrückung

---

Kann die Capillaritätsconstante ρ Eis-Wasser theoretisch berechnet werden?
Capillaritätsconstante α von Eis?

Emulsionen: Quincke Pflüger's Archiv 1879 Bd. 19
Ctbl. 1880 p. 110
Brücke " " p. 109

Wied. Ann 67 p. 702 (1899) Richarz [illegible]

1). [illegible] = 20 [illegible]

Molec. = [illegible]

2) [illegible]:

$c_p - c_v = \frac{0\cdot0691}{d}$ $c_{v\,H_2} = \frac{3\cdot 0\cdot0691}{2d}$ $d = \frac{A}{2\cdot 14\cdot5}$

[illegible]

$A\,,\; c_v = 3\cdot0\cdot0691\cdot14\cdot5 = 3\cdot006$

C_v [illegible] $= 2c_v = 6\cdot012$

$A.\; C_p = 6\cdot072 - 6\cdot252$

6 [illegible]

p. 670 [illegible] Theorie [illegible] Const. [illegible] Dulong Petit

$C_v = \frac{6\, r\, c_v}{\vartheta}$ [illegible] $r =$ ~~[illegible] 1 Atom [illegible]~~ = [illegible] Atom

$\left[c_v = 0{\cdot}98835 \frac{\vartheta}{\mu}\right] = \frac{5{\cdot}93}{\frac{\mu}{\vartheta} = \alpha = \text{At.}\,1}$

[illegible]: [illegible] a + b + c Atom $\mu = \text{Mol.}\,1 = \alpha a + \beta b + \gamma c$

$\mu C_v = (a+b+c)\, 5{\cdot}93$

$\frac{a \mu C_v}{a+b+c} = a \alpha A$ [illegible]

$\frac{b \mu C_v}{a+b+c} = b \beta B$

$\frac{c \mu C_v}{a+b+c} = c \gamma C$

$\mu C_v = a \alpha A + b \beta B + c \gamma C$

[[illegible] A B C [illegible] normalen [illegible]!]

Eisen Wied. Ann 67 p. 695 [illegible]

8°	4·58
-70°	0·284
90°	0·029
600	0·008
700	0·002
800	0·0004

$x = \frac{k}{k-1} \cdot \frac{RA}{g}$

$= \frac{7}{2}$ ~~780 · 760 · 13·6 · 1000²~~ / ~~0·0012 · 780~~

=

38 / 266

$\frac{7}{2}$ ~~780~~ 76[illegible] · 13·6 · ~~[illegible]~~ · ~~[illegible]~~ / 1·293 · ~~780~~ ~~[illegible]~~

4249
1335
~~[illegible]~~
~~1584~~
1116
~~8830~~
4468
0156
4624

$\frac{283}{273} =$ 4518
4362
0156

Wied. Ann. 65 p. 655 (1898)

Staigmüller [illegible]

[illegible] Gasmol. [illegible] 3n Vorrat [illegible] n Atome:

$x\,y\,z$ [illegible] Atoms^{A} Mol. ; [illegible] B [illegible] C [illegible] (φ, ψ) [illegible]

$\int . z\, \partial x\, \chi$ $\qquad$ $\int_8 = 6$ äussere Coord.

[illegible] $(3n-6)$ [illegible] Coord.

[illegible] ε [illegible]:

$$\left.\begin{array}{lll} 3\varepsilon & = \text{[illegible]} & \\ 3\varepsilon & \text{rot.} & " \\ 3n-6 & \text{[illegible]}\ \varepsilon\ \text{[illegible]} & " \\ 3n-6 & \text{pot.} & "\quad \text{[illegible]} \end{array}\right\} = E = \underbrace{6(n-1)}_{\vartheta = \text{Wärmedimension}}\varepsilon$$

[illegible] Coord. [illegible] $= 0$ [illegible] Relat[illegible] Vor. [illegible]

[illegible] Coord. [illegible] ; [illegible] $\vartheta = a + 2i$ [illegible]

$$k = \frac{\vartheta + 2}{\vartheta}$$

$\vartheta = 5$: H_2; O_2, N_2, CO, NO, HCl, HBr, HJ $\left(\begin{array}{l} a = 5 \\ i = 0 \end{array}\right.$

$= 7$: $\underbrace{Cl_2,\ Br_2,\ J_2,\ ClJ}_{a=5,\ i=1}$, $\underbrace{H_2O,\ N_2O,\ CO_2,\ H_2S,\ SO_2}_{\text{[illegible]}}$

$\underbrace{NH_3,\ CH_4}_{6}$ $\begin{array}{l} a = 6 \\ i = 0 \end{array}$ (?) [illegible]

$$\frac{1}{\eta}\frac{d\eta}{d\theta} = \frac{1}{\eta}\frac{\partial\eta}{\partial\theta} + \left(\frac{v}{\eta}\frac{\partial\eta}{\partial v}\right)\cdot\left(\frac{\partial v}{\partial\theta}\frac{1}{v}\right)$$

$$\downarrow$$

$$= \frac{1}{\eta}\frac{\partial\eta}{\partial p} : \frac{1}{v}\frac{\partial v}{\partial p}$$

$$= \frac{1}{\eta}\frac{\partial\eta}{\partial p}\cdot\frac{\alpha}{\beta}$$

Äther $\theta = 20^\circ$: $\frac{1}{\eta}\frac{\partial\eta}{\partial p} = 7{\cdot}30 \cdot 10^{-4}$ | Benzol $9{\cdot}3 \cdot 10^{-4}$

p. Atmosph.

1·1139
2·4116
−3·5255
+1·5185
0·9930 −3

0·00739
561
178 : 18·645

2·8096
1·2553
−4·0649
+2·2504
0·1855 −2

0·0153

$\frac{1}{\eta}\frac{d\eta}{d\theta} =$ 0·00278
245
33 : [illegible] 0·0098
: 13·258
für 18.5°

für 21°

$\beta =$ 254 190 $\cdot 10^{-6}$
13·5 168
178 $\cdot 10^{-6}$ (Atm)

Amagat

6 : 83·0
15·4 : 87·1
17·9 : 91·7
50 : 111

94
90 | $92 \cdot 10^{-6}$ (Atm)

$\alpha =$ 0·00148
13
0·00161

0·00116
0·00008
0·00124

350 · 34 185
105
140
119

35 · 37
105
245
1295

0·0018
− 0·0066
− 0·0048
zu. 0·0098

$$\frac{7{\cdot}3 \cdot \cancel{10^{-4}} \cdot 0{\cdot}0016}{1{\cdot}78 \cdot \cancel{10^{-4}}}$$

73
438
1168 : 1·78 = 656
100
11

$$\frac{9{\cdot}3 \cdot \cancel{10^{-4}} \cdot 0{\cdot}00124}{0{\cdot}92 \cdot \cancel{10^{-4}}}$$

− 0·0125
+ 0·0018
− 0·0107

− 0·0153 zu.

Benzol $\frac{286 \cdot (0{\cdot}00116)^2 \cdot 39}{R \quad 0{\cdot}892}$

$0{\cdot}645 \cdot 2$
0·1290 −6
2·4564
1·5911
0·1765 −2
− 6·5443
0·6322 −9

0·9504 −1
6·5939
6·5443

ob. ~~4~~ $4{\cdot}288 \cdot 10^{-9}$

zn. $8{\cdot}3 \cdot 10^{-11}$

Alk. $\frac{293 \cdot (0{\cdot}00111)^2 \cdot 23}{R \cdot 0{\cdot}794}$

0·0453 −3 ^{2}
0·0906 −6
2·4669
1·3617
0·4061 −7
0·3253 −~~11~~9
− 0·8998 −1
0·4255 −~~11~~9

− 6·5939
= 0·4061 −7

$2{\cdot}66 \cdot 10^{-9}$

$1 \cdot 10^{-10}$

Hg: $\frac{273 \cdot \left(\frac{1}{550}\right)^2 \cdot \cancel{100}}{R \cdot 13{\cdot}6}$

2·7404
5·4808
1·1335
6·4434
13·0577

2·4362
−13·0577
0·3785 −11

$2{\cdot}39 \cdot 10^{-11}$

zn.: $3{\cdot}9 \cdot 10^{-12}$

Pentan: $\frac{288 \cdot (0{\cdot}00159)^2 \cdot 36}{R \cdot 0{\cdot}626}$

0·2014 −3 ^{2}
0·4028 −6
1·5563
2·4594
0·4185 −2
− 6·2400
0·1785 −8

0·7966 −1
6·4434
6·2400

$1{\cdot}51 \cdot 10^{-8}$

zn. $2{\cdot}9\cancel{1} \cdot 10^{-10}$

$$\beta = -\frac{\alpha^2\,\theta}{\sqrt{2}\,R\rho} = -\frac{\alpha^2\theta}{\sqrt{2}.\,r}\left(\frac{\mu}{\rho}\right)$$

$\alpha = 0{\cdot}00116$

Benzol ca 13°

$\mu = \frac{78}{m}$, $\rho = 0{\cdot}892$

$\beta = 0{\cdot}000085$

0·00170

Äther 0–50°

~~$\mu = 46$~~

$\mu = 74$ $\rho = 0{\cdot}736$

0·000189

~~0·00127~~ 0·00111

~~$\alpha\,S_z$~~ ~~50°~~ Alk 0–40°

$\mu = \frac{46}{2}$

$\rho = 0{\cdot}794$

0·000100

Petroleum 0·00099

0·000069

$\rho = 0{\cdot}76$

Pentan 0–30° 0·00159

0·000292

$\mu = \frac{72}{2}$

$\rho = 0{\cdot}6263$

98% H_2SO_4 18° : $0{\cdot}00055 = \alpha$

$\beta = 0{\cdot}0000462$. 0·730

3234
1386

$\rho = 1{\cdot}841$ $3{\cdot}3726.10^{-11}$

$$\mu = \begin{array}{r} 2 \\ 32 \\ 64 \\ \hline 98 \end{array} = \frac{98}{2}$$

$R = \cancel{\mu}\,\frac{r}{\mu}$

Hg : $\frac{1}{5500}$

(Amagat) 0·0000039

$\rho = 13{\cdot}6$

$\mu = \frac{200}{2}$

~~$\frac{r}{\rho}\mu = R\mu$~~

$$R = \frac{\overset{1000}{\cancel{76.13{\cdot}6}}.\,980}{0{\cdot}001293\,.\,273} = \begin{array}{r} 9912 \\ 5478 \\ \hline 4434 \end{array}$$

~~110~~
1116
4362
5478

~~25~~ ~~10^{-6}~~ 105

$= 2{\cdot}776\,.\,10^6$

$\log R = 6{\cdot}4434$

$+\log\sqrt{2} = 0{\cdot}1505$

6·5939

$$\frac{0{\cdot}00055^2\,.\,29\,.\,49}{2{\cdot}78\,.\,10^6\,.\,1{\cdot}84\,.\,1{\cdot}42}$$

121 . 25

3025 . 29
6050
27225
87725

87725 . 49
350900
789525
42985

2·78 . 1·84
2224
111
5115 . 142
20460
102
7263

$42985 : 7263 = 5{\cdot}918\,.\,10^{-11}$
6670
133
60

CO_2: 0·0001 9~~6~~ 6
1 88
~~8~~ 6

$\frac{40·3}{32·6}$
~~30~~ $-\frac{7·7}{2.\ 305·6}$ =

~~8~~ 6 : 1~~8~~0 =
88
2 : 47 = 0·042
~~3 : 12 94~~ = 0·033
28

7·7 : 611 = 0·012~~0~~ 6
1·6
~~154 : 30~~

5

$$\cancel{p}+\frac{a}{v^2}=\frac{R\theta\sqrt{2}}{v-v_0}$$

$$1-\frac{2a}{v^3}\frac{\partial v}{\partial p}=-\frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial p}.$$

$$\frac{\partial v}{\partial p}=\frac{1}{\frac{2a}{v^3}-\frac{R\theta\sqrt{2}}{(v-v_0)^2}}$$

$$=\frac{1}{2\frac{R\theta\sqrt{2}}{v(v-v_0)}-\frac{R\theta\sqrt{2}}{(v-v_0)^2}}=\frac{(v-v_0)^2}{\sqrt{2}.R\theta}\left[\frac{v}{2(v-v_0)-v}\right]$$

$$=\frac{(v-v_0)^2 v}{\sqrt{2}.R\theta\,(v-2v_0)}=\qquad v-v_0=\delta$$

$$\beta=\frac{1}{v}\frac{\partial v}{\partial p}$$

$$v\beta=-\frac{\delta^2.(\cancel{v_0}+\cancel{\delta})}{\sqrt{2}R\theta.\cancel{v_0}}$$

$$-\frac{2a}{v^3}\frac{\partial v}{\partial\theta}=\frac{R\sqrt{2}}{v-v_0}-\frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial\theta}$$

$$\left[-\frac{2a}{v^3}+\frac{R\theta\sqrt{2}}{(v-v_0)^2}\right]\frac{\partial v}{\partial\theta}=\frac{R\sqrt{2}}{v-v_0}$$

$$\left[-\frac{2R\theta\sqrt{2}}{(v-v_0)v}+\frac{R\theta\sqrt{2}}{(v-v_0)^2}\right]\frac{\partial v}{\partial\theta}=\nearrow$$

$$\cancel{R}\theta\cancel{\sqrt{2}}\left[-2(v-v_0)+v\right]\frac{\partial v}{\partial\theta}=\cancel{R}\cancel{\sqrt{2}}.\cancel{v}(v-v_0)$$

$$2v_0-v$$

$$\frac{\partial v}{\partial\theta}=\frac{\delta}{\theta}=\frac{v-v_0}{\theta}$$

$$v.\beta=-\frac{\delta^2}{\sqrt{2}R\theta}$$

cub. stg

$$v.\alpha=\frac{\delta}{\theta}$$

$$\beta=-\frac{\theta\alpha^2 v}{\sqrt{2}R}$$

$$\boxed{\beta=-\frac{\alpha^2\theta}{\sqrt{2}R\rho}}$$

$$\int_{-\infty}^{+\infty}\left(\overset{a}{x^2}+\overset{b}{y^2}+\cdot\right)e^{-h(x^2+y^2)}\,dx\,dy\cdots$$

$$\int_{-\infty}^{+\infty}e^{-hx^2}dx = \sqrt{\frac{\pi}{ah}} \qquad a\int x^2e^{-ahx^2}dx = ax\cancel{\frac{e^{-hx^2}}{-2ah}} + \int\frac{e^{-ahx^2}}{2h}dx$$

$$= \frac{1}{2h}\sqrt{\frac{\pi}{ah}}$$

$$\frac{1}{2h}\left(\sqrt{\frac{\pi}{h}}\right)^n \qquad \int_{-\infty}^{+\infty}e^{-h(x^2+y^2\cdots)}\cdot dx\cdot dy = \left(\sqrt{\frac{\pi}{h}}\right)^n$$

$$\frac{c}{A}\frac{d\theta}{dx} + \frac{p}{b}\frac{du}{dx} = \frac{J\rho}{b}$$

$$\frac{1}{\rho}\frac{dp}{dx} = -g$$

$$\rho u = b$$

$$\frac{p}{\rho} = R\theta$$

$$d\rho\,\sigma = \rho g u_0 = \rho g\frac{dx}{dt} = dp$$

$$\sigma\rho = R\rho\frac{d\theta}{dx}\frac{dx}{dt}$$

$$\delta\varphi = \alpha\cdot\rho\cdot\delta t$$

$$= c\rho\cdot\delta x\cdot\frac{g}{R}\quad\frac{k-1}{k}$$

$$\frac{\delta x}{\delta t} = \frac{\alpha}{c}\frac{R}{g}$$

$$\cancel{\beta = \frac{\alpha^2\rho\cdot q\cdot\theta}{\sqrt{2}\cdot\rho\cdot R}}$$

$$\frac{p}{\rho u}\frac{du}{dx} = R\theta\cdot\frac{1}{u}\frac{du}{dx}$$

$$= -R\theta\frac{1}{\rho}\frac{d\rho}{dx}$$

$$\boxed{\rho = \frac{p}{R\theta}}$$

$$= -\frac{\theta}{\rho}\left(\frac{1}{\theta}\frac{dp}{dx} - \frac{p}{\theta^2}\frac{d\theta}{dx}\right)$$

$$= -\frac{1}{\rho}\frac{dp}{dx} + \frac{p}{\rho\theta}\frac{d\theta}{dx}$$

$$= g + R\frac{d\theta}{dx}$$

$$\frac{k-1}{k} = \frac{2}{7}$$

$$\frac{42\cdot\cancel{10}\cdot 0{\cdot}00033}{\cancel{10^7}\ 20}\cdot\frac{10^6}{10^3}\,\frac{}{b}$$

$$= \frac{0{\cdot}6\cdot 10^3}{b}\,\mathrm{cm} =$$

$$= \frac{0{\cdot}6\cdot 10^3}{0{\cdot}001\cdot u}\,\mathrm{cm} = \frac{6\cdot 10^5}{u}\,\mathrm{cm}$$

$$= \frac{6\cdot}{u_0}10^3\,\mathrm{m} = \frac{6}{u_0}\,\mathrm{km}$$

$\eta = \eta_0(1+\alpha t)$

C_6H_6: 0·00093

$(C_2H_5)_2O$: 0·00073

Compr.

~~0·00018~~

0·00019

0·00067

β

0·00149

0·00118

$$\frac{0{\cdot}00148 \cdot \cancel{0{\cdot}00093}}{\cancel{0{\cdot}00019}}$$

$$\frac{0{\cdot}00072 \cdot \cancel{0{\cdot}00159}}{\cancel{0{\cdot}00064}}$$

8

1359 : 8 =
55

1062
359

10974 : 19 =
147
144

$- \lfloor 0{\cdot}00577$

C_6H_6

$+ 0{\cdot}00167$

$\frac{1}{300}$

$- 0{\cdot}0041$

$- \lfloor 0{\cdot}0017$

$(C_2H_5)_2O$

$+ 0{\cdot}0017$

0·000

42·4
31·5

10·9 : (37 · 20)
350
170
= 295

0·0147

Müller p. 298 [illegible]

Met. Ztschr. 10 (1893)

255

[illegible] 27km [illegible]

[illegible] 200 [illegible] (!)

[illegible] 200km [illegible]

[illegible] [~~[illegible]~~]

Durchl.:	Σ - Δt	[illegible]	Nach den [illegible]
0-1000m	0.50	0.61	49
2000	0.50	0.54	50
3	0.54	0.54	54
4	0.53	0.54	53
5	0.64	0.64	64
6	0.69	0.69	69
7	0.68	0.68	68
8	0.~~72~~ 67	0.72	72
000-9000	0.90	0.90	80
			80
			639

B. N. .... Sygnatura: ....

Dzieło: (Autor) ....

(Tytuł) ....

....

(Stan dzieła) ....

tom wol.

pożyczyłem z c. k. biblioteki uniwersyteckiej i zwrócę je w ciągu tygodni .... przyjmując odpowiedzialność za wszelkie uszkodzenia.

We Lwowie dnia

Mieszkanie: Podpis:

G. Cantoni: Su alcune condizioni fisiche dell' affinità e sul moto browniano. Rendic. Lomb. (2) I p. 56, Cimento XXVII 156.

[illegible] Affinität [illegible]

p. Mol. [illegible] B. M. [illegible]

[illegible] H_2 reducirte [illegible] Ag Cu Fe | S | C

[illegible]

[illegible] $\frac{1}{1600}$ mm [illegible] Uranoxyd [illegible]

[illegible] Alkohol [illegible] Äther [illegible]

[illegible] H_2O

[illegible] Atom [illegible]

B. N. .... Sygnatura : ....

Dzieło : (Autor) ....

(Tytuł) ....

....

(Stan dzieła) ....

tom wol.

pożyczyłem z c. k. biblioteki uniwersyteckiej i zwrócę je w ciągu tygodni .... przyjmując odpowiedzialność za wszelkie uszkodzenia.

We Lwowie dnia

Mieszkanie : Podpis :

• •

Tytuł dzieła:

Sygnatura:

Data:	Nazwisko pożyczającego:

$u=0 \quad w=0 \quad v = f_1(x)$ $\qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$0 = -\frac{\partial p}{\partial x} +$ ~~[illegible] $\mu\nabla^2 u$~~

$p = f_2(y)$

$\rho\frac{\partial v}{\partial t} +$ [illegible] $= -\frac{\partial p}{\partial y} + \mu\nabla^2 v$

$0 = -\frac{\partial p}{\partial z}$

$\rho\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu\frac{\partial^2 v}{\partial x^2}$ $\qquad \frac{\partial \rho}{\partial t} = 0$

$\rho\frac{\partial v}{\partial t} = \mu\frac{\partial^2 v}{\partial x^2}$

$v = e^{-\gamma t}\cos\alpha x$

$-\gamma\rho = -\alpha^2\mu$

$\gamma = \frac{\alpha^2\mu}{\rho}$

$v = e^{-\frac{\alpha^2\mu}{\rho}t}\sin\alpha x$

$v = e^{-\gamma x}\sin\alpha(x-\beta t)$

$\frac{\partial v}{\partial x} = -\gamma e^{-\gamma x}\sin + \alpha e^{-\gamma x}\cos$

$\frac{\partial^2 v}{\partial x^2} = \gamma^2 e^{-\gamma x}\sin - 2\alpha\gamma e^{-\gamma x}\cos - \alpha^2 e^{-\gamma x}\sin$

$\frac{\partial v}{\partial t} = -\alpha\beta e^{-\gamma x}\cos$

$\frac{2\mu\alpha^2}{\rho} = \varepsilon$

$\alpha = \sqrt{\frac{\varepsilon\rho}{2\mu}}$

$$\left.\begin{aligned} -\alpha\beta\rho &= -\mu 2\alpha\gamma \\ 0 &= \gamma^2-\alpha^2 \end{aligned}\right\} \quad \begin{aligned} \gamma &= \alpha \\ \beta &= \frac{2\mu\alpha}{\rho} \end{aligned}$$

$$v = e^{-\alpha x}\sin\alpha\left[x - \frac{2\mu\alpha}{\rho}t\right]$$

$$= e^{-x\sqrt{\frac{\varepsilon\rho}{2\mu}}}\sin\left[x\sqrt{\frac{\varepsilon\rho}{2\mu}} - \varepsilon t\right]$$

$\rho = 0{'}0012$

$\mu = 0{'}00019$

$\varepsilon = \frac{2\pi}{T} = 2n\pi$

W próżni 10^{-6} atm: $\sqrt{\frac{2n\pi\cdot 1{'}2}{2\cdot 0{'}18}\cdot 10^{-6}}$

$n = 500$

$\parallel$ ~~$\sqrt{\frac{1}{2}\,\frac{2\cdot 1{'}2}{0{'}18}\cdot 1000^{-2}} =$~~ $\sqrt{100} = 10^{-1}$

$\boxed{e^{-\frac{x}{10}}}$

Próba czy $\Delta^2 \cdots = \mathrm{div}'$

$$\Delta^2 \left[\frac{27}{32}\frac{a}{r^2} + \frac{3}{4}\left(\frac{x^2}{r^3} - \frac{15}{8}\frac{ax^2}{r^4} - \frac{1}{4}\frac{a^3x^2}{r^6}\right)\right] =$$

$$= \frac{27}{32}\frac{2a}{r^4} + \frac{3}{4}\left[\overset{-6\frac{x^2}{r^5}}{2\frac{1}{r^3}} - \frac{15a}{8}\left(\frac{2}{r^4} - \frac{4x^2}{r^6}\right) - \frac{1}{4}a^3\left(\frac{2}{r^6} + 6\frac{x^2}{r^8}\right)\right]$$

$$= \frac{27}{16}\frac{a}{r^4} + \frac{3}{2}\frac{1}{r^3} - \frac{45a}{16r^4} + \frac{45}{8}\frac{ax^2}{r^6} - \frac{3a^3}{8r^6} - \frac{9}{8}\frac{a^3x^2}{r^8} - \frac{9}{2}\frac{x^2}{r^5}$$

$$- \frac{\cancel{18}}{\cancel{16}}\frac{9}{8}\frac{a}{r^4}$$

$$= \frac{3}{2}\frac{1}{r^3}\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right] - \frac{9}{2}\frac{x^2}{r^5}\left[1 - \frac{5}{4}\frac{a}{r} + \frac{1}{4}\frac{a^3}{r^3}\right]$$

zatem $k\,\mathrm{div}' = \Delta^2\left\{+\mu c^2 a\left[\frac{27}{32}\frac{a}{r^2} + \frac{3}{4}\left(\;-\;-\;\text{---}\;\right)\right]\right\}$

$$\mathrm{div}' = \Delta^2\psi$$

Rozwiązanie systemu równań

$$\begin{cases} \frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial\,\mathrm{div}'}{\partial x} + \mu\Delta^2 u' \\ \frac{\partial p'}{\partial y} = \\ \frac{\partial p'}{\partial z} = \end{cases}$$

$$\Delta^2 p' = \frac{4\mu}{3}\Delta^2\mathrm{div}'$$

$$\underline{\underline{p' = \frac{4\mu}{3}\mathrm{div}' + \varphi}}$$

$$\Delta^2\varphi = 0$$

$$\mu\frac{\partial\,\mathrm{div}'}{\partial x} + \frac{\partial\varphi}{\partial x} = \mu\Delta^2 u'$$

$$u' = \frac{\partial\psi}{\partial x} + U$$

$$\begin{cases} \frac{\partial\varphi}{\partial x} = \mu\Delta^2 U \\ \frac{\partial\varphi}{\partial y} = \mu\Delta^2 V \\ \frac{\partial\varphi}{\partial z} = \mu\Delta^2 W \end{cases}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} = 0$$

~~$\Delta^2 u$~~ Z warunków powierzchniowych

$$U = -\frac{27\mu c^2 a}{32\,k}\left[-\frac{x}{r^3} + \frac{1}{3}\frac{x^3}{r^5} - \frac{a^2 x}{r^5}\left(3 - 5\frac{x^2}{r^2}\right)\right]$$

$$V = -\frac{27\mu c^2 a}{32\,k}\left[-\frac{y}{r^3} + \frac{1}{3}\frac{x^2 y}{r^5} - \frac{a^2 y}{r^5}\left(1 - 5\frac{x^2}{r^2}\right)\right]$$

$$W =$$

$$\mu\Delta^2 U = -\frac{27\mu^2 c^2 a}{32\,k}\left(\frac{2x}{r^5} - \frac{10}{3}\frac{x^3}{r^7}\right) = \frac{\partial}{\partial x}\left[-\frac{27\mu^2c^2a}{32k}\,\frac{2}{3}\left(-\frac{1}{3r^3} + \frac{x^2}{r^5}\right) = \frac{9}{16}\frac{\mu^2c^2a}{k}\left[\frac{1}{3r^3} - \frac{x^2}{r^5}\right]\right.$$

$$\mu\Delta^2 V = -\frac{27\mu^2c^2a}{32\,k}\left(\frac{2}{3}\frac{y}{r^5} - \frac{10}{3}\frac{x^2y}{r^7}\right) = \frac{\partial}{\partial y}$$

$$\left.\begin{aligned}\frac{2}{3}\frac{\partial}{\partial x}\left(\frac{x^2}{r^5}\right) &= \frac{4x}{3r^5} - \frac{10x^3}{3r^7}\\ \cancel{+}-\frac{2}{9}\frac{\partial}{\partial x}\left(\frac{1}{r^3}\right) &= -\frac{3x}{r^5}\cdot\frac{2}{9}\end{aligned}\right\}\qquad\begin{aligned}&\frac{\partial}{\partial x}\left|\,\frac{2}{3}\left(-\frac{1}{3r^3} + \frac{x^2}{r^5}\right)\right.\\ &\frac{\partial}{\partial y} = \frac{2}{3}\left(+\frac{y}{r^5} - \frac{5x^2y}{r^7}\right)\end{aligned}$$

$$\varphi = \frac{9}{16}\frac{\mu^2c^2a}{k}\left[\frac{1}{3r^3} - \frac{x^2}{r^5}\right]$$

$$p = P + (p_0) - \frac{3}{2}\mu c a\frac{x}{r^3} + \frac{1}{P}\left\{\frac{4\mu}{3k}\left[\frac{3}{2}\mu a c^2\frac{1}{r^3}\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right) - \right.\right.$$

$$\left.\left.-\frac{9}{2}\mu c^2 a\,\frac{x^2}{r^5}\left(1 - \frac{5}{4}\frac{a}{r} + \frac{1}{4}\frac{a^3}{r^3}\right)\right] + \frac{9}{16}\frac{\mu^2c^2a}{k}\left[\frac{1}{3r^3} - \frac{x^2}{r^5}\right]\right\}$$

$$\{\ \} = \frac{\mu^2c^2a}{k}\left|\begin{aligned}&\frac{2}{r^3}\left(1 - \frac{\ddot{2}}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right) + \frac{9}{16}\frac{1}{3r^3}\\ &-\frac{6x^2}{r^5}\left(1 - \frac{5}{4}\frac{a}{r} + \frac{1}{4}\frac{a^3}{r^3}\right) - \frac{9}{16}\frac{x^2}{r^5}\end{aligned}\right|$$

$$u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial y}+w_0\frac{\partial u_0}{\partial z}=\frac{9}{16}c^2a^2\frac{x}{r^3}\left(1-\frac{a^2}{r^2}\right)\left[x^2\left(M+\frac{2}{r^3}-\frac{2a^2}{r^5}\right)+y^2M+z^2M\right]$$

$$-\frac{3}{4}c^2a\frac{x}{r^3}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)\left(1-\frac{3x^2}{r^2}-\frac{3a^2}{r^2}+\frac{5a^2x^2}{r^4}\right)$$

$$=\frac{9}{16}c^2a^2\frac{x}{r^3}\left(1-\frac{a^2}{r^2}\right)\Big[\underbrace{-\frac{1}{r}-\frac{3x^2}{r^3}-\frac{a^2}{r^3}+\frac{5a^2x^2}{r^5}+\frac{2x^2}{r^3}-\frac{2a^2x^2}{r^5}}_{-\frac{1}{r}-\frac{x^2}{r^3}-\frac{a^2}{r^3}+\frac{3a^2x^2}{r^5}}\Big]-\uparrow$$

$$\cancel{=\frac{3}{4}c^2\Big\{\frac{3}{4}\frac{a^2x}{r^3}}$$

$$=\frac{3}{4}\frac{c^2ax}{r^3}\Big\{\frac{3}{4}\Big[-\cancel{\frac{a}{r}}-\frac{ax^2}{r^3}-\cancel{\frac{a^3}{r^3}}+\frac{3a^3x^2}{r^5}+\cancel{\frac{a^3}{r^3}}+\cancel{\frac{a^3x^2}{r^5}}+\cancel{\frac{a^5}{r^5}}-\frac{3a^5x^2}{r^7}\Big]$$

$$-1+\frac{3x^2}{r^2}+\frac{3a^2}{r^2}-\frac{5a^2x^2}{r^4}+\frac{3}{4}\cancel{\frac{a}{r}}-\frac{9}{4}\frac{x^2a}{r^3}-\frac{9a^3}{4r^3}+\frac{15a^3x^2}{4r^5}+\frac{a^3}{4r^3}-$$

$$-\cancel{\frac{3a^3x^2}{4r^5}}-\cancel{\frac{3a^5}{4r^5}}+\frac{5a^5x^2}{4r^7}\Big\}$$

$$=\frac{3}{4}\frac{c^2ax}{r^3}\Big\{-1+\frac{3x^2}{r^2}+\frac{3a^2}{r^2}-\frac{5a^2x^2}{r^4}-3\frac{ax^2}{r^3}-2\frac{a^3}{r^3}+6\frac{a^3x^2}{r^5}-\frac{a^5x^2}{r^7}\Big\}$$

$$\left.\begin{aligned}&\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)\\&+\frac{\partial}{\partial y}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)\\&+\frac{\partial}{\partial z}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)\end{aligned}\right\}=\begin{aligned}&\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}\quad\underbrace{\cancel{+u\frac{\partial}{\partial x}\operatorname{div}+v\frac{\partial}{\partial y}\operatorname{div}+w\frac{\partial}{\partial z}\operatorname{div}}}_{=0}\\&+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+\left(\frac{\partial v}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial v}{\partial z}\\&+\ -\end{aligned}$$

$$=\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2+2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+2\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}+2\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}$$

$$\pm\frac{1}{2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)^2+\ -\ -$$

$$=\frac{1}{2}\left[\Phi-\xi^2-\eta^2-\zeta^2\right]$$

Czy $f_1 = \frac{\partial U}{\partial x}$, $f_2 = \frac{\partial U}{\partial y}$?

W takim razie musiałoby być $\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$

$$\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) =$$

$$= u\left(\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial^2 v}{\partial x^2}\right) + v\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x\partial y}\right) + w\left(\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial^2 v}{\partial x\partial z}\right)$$

$$+ \underbrace{\frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\partial v}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)}_{} + \frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial z}$$

$$= -\frac{\partial w}{\partial z}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

t.zn. czy $\mathrm{curl}\,\underbrace{(\sigma\nabla)\sigma}_{\frac{1}{2}\nabla(\sigma^2) + V\sigma\,\mathrm{curl}\,\sigma} = 0 \Big\} = \mathrm{curl}\,V\sigma\,\mathrm{curl}\,\sigma = (\sigma\nabla)\,\mathrm{curl}\,\sigma - (\mathrm{curl}\,\sigma\,\nabla)\sigma$

To będzie oczywiście w każdym razie tam spełnione, gdzie ruch taki sam jaki by mógł istnieć bez tarcia ~~tak to będzie zawsze w tych rozwiązaniach~~

$$u\frac{\partial \xi_1}{\partial x} + v\frac{\partial \xi_1}{\partial y} + w\frac{\partial \xi_1}{\partial z} - \left(\xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z}\right)$$

$$\eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z} = 0$$

$$u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + w\frac{\partial \eta}{\partial z} - \left(\cancel{\xi\frac{\partial v}{\partial x}} + \eta\frac{\partial v}{\partial y} + \zeta\frac{\partial v}{\partial z}\right) =$$

$\frac{\partial \eta}{\partial x} = +\frac{3}{2}\frac{c a x z}{r^5}$ $\qquad \frac{\partial \eta}{\partial y} = +\frac{3}{2}\frac{c a y z}{r^5}$ $\qquad \frac{\partial \zeta}{\partial z} = -\frac{1}{2}\frac{c a}{r^3} + \frac{3}{2}\frac{c a z^2}{r^5}$

$$= \frac{\mu^2 c^2 a}{k}\left[\frac{1}{r^3}\left(\frac{35}{16} - \frac{3}{2}\frac{a}{r} - \frac{1}{2}\frac{a^3}{r^3}\right) - 3\frac{x^2}{r^5}\left(\frac{35}{16} - \frac{5}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)\right]$$

$$-p = P - \frac{3}{2}\mu a c \frac{x}{r^3} + \frac{\mu^2 c^2 a}{kP}\frac{1}{2r^3}\left[\left(\frac{35}{8} - \frac{3a}{r} - \frac{a^3}{r^3}\right) - \frac{3x^2}{r^2}\left(\frac{35}{8} - \frac{5a}{r} + \frac{a^3}{r^3}\right)\right]$$

Rozmiary P --- $\frac{\mu c}{a}$ $\qquad \mu c \propto aP$

Tak samo: $\frac{\partial p}{\partial x}$ - $\mu \frac{\partial^2 u}{\partial x^2}$ zgadza się

Równanie termiczne ściśliwe:

$$k\, \mathrm{div}' = -\left[u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z}\right] + (k-1)\mu\left[\underbrace{\left(\frac{\partial u_0}{\partial x}\right)^2 + \cdots}_{\Phi} + k\Delta^2\theta\right]$$

Jeżeli się nie robi $k=\infty$ izotermicznie to Φ tego samego rzędu wielkości co ... zatem adiabatyczne założenie nieuzasadnione!

$$\frac{\partial u_0}{\partial x} = -\frac{3}{4}ca\left[\frac{2x}{r^3} - \frac{3x^3}{r^5} - \frac{2a^2x}{r^5} + \frac{5a^2x^3}{r^7}\right] + c\left[\frac{3}{4}\frac{ax}{r^3} + \frac{3}{4}\frac{a^3x}{r^5}\right]$$

$$\frac{\partial u_0}{\partial x} = -\frac{3}{4}ca\left[\frac{x}{r^3} - \frac{3x^3}{r^5} - \frac{3a^2x}{r^5} + \frac{5a^2x^3}{r^7}\right]$$

$$\frac{\partial v_0}{\partial y} = -\frac{3}{4}ca\left[\frac{x}{r^3} - \frac{3xy^2}{r^5} - \frac{a^2x}{r^5} + \frac{5a^2xy^2}{r^7}\right]$$

$$\frac{\partial w_0}{\partial z} = -\frac{3}{4}ca\left[\frac{x}{r^3} - \frac{3xz^2}{r^5} - \frac{a^2x}{r^5} + \frac{5a^2xz^2}{r^7}\right]$$

$$\frac{\partial u_0}{\partial y} = -\frac{3}{4}ca\left[-\frac{3x^2y}{r^5} + \frac{5a^2x^2y}{r^7}\right] + c\left[\frac{3ay}{4r^3} + \frac{3a^3y}{4r^5}\right]$$

$$\frac{\partial u_0}{\partial y} = -\frac{3}{4}ca\left[-\frac{3x^2y}{r^5} + \frac{5a^2x^2y}{r^7} - \frac{y}{r^3} - \frac{a^2y}{r^5}\right]$$

$$\frac{\partial u_0}{\partial z} = -\frac{3}{4}ca\left[-\frac{3x^2z}{r^5} + \frac{5a^2x^2z}{r^7} - \frac{z}{r^3} - \frac{a^2z}{r^5}\right].$$

$$\frac{\partial v_0}{\partial x} = -\frac{3}{4} ca \left[\left(1-\frac{a^2}{r^2}\right)\frac{y}{r^3} - \frac{3x^2y}{r^5}\left(1-\frac{a^2}{r^2}\right) + \frac{2a^2x^2y}{r^7}\right]$$

$$\frac{\partial v_0}{\partial x} = -\frac{3}{4} ca \left[\left(1-\frac{a^2}{r^2}\right)\frac{y}{r^3} - \frac{3x^2y}{r^5} + \frac{5a^2x^2y}{r^7}\right]$$

$$\frac{\partial v_0}{\partial z} = -\frac{3}{4} ca \left[\frac{2a^2zxy}{r^7} - \frac{3xyz}{r^5}\left(1-\frac{a^2}{r^2}\right)\right]$$

$$\left.\begin{matrix}\dfrac{\partial v_0}{\partial z} = \\ \\ \dfrac{\partial w_0}{\partial y} = \end{matrix}\right\} \begin{matrix} -\dfrac{3}{4} ca \left[-\dfrac{3xyz}{r^5} + \dfrac{5a^2xyz}{r^7}\right] \quad = \\ = \nearrow \end{matrix}$$

$$\frac{\partial w_0}{\partial x} = -\frac{3}{4} ca \left[\left(1-\frac{a^2}{r^2}\right)\frac{z}{r^3} - \frac{3x^2z}{r^5} + \frac{5a^2x^2z}{r^7}\right]$$

$$\frac{\partial u_0}{\partial x} = -\frac{3}{4} ca \; x\left[\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{3a^2}{r^5} + \frac{5a^2x^2}{r^7}\right] -$$

$$\frac{\partial u_0}{\partial y} = -\frac{3}{4} ca \; y\left[-\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2x^2}{r^7}\right]$$

$$\frac{\partial u_0}{\partial z} = -\frac{3}{4} ca \; z\left[\qquad\qquad\qquad\right]$$

$$\frac{\partial v_0}{\partial x} = -\frac{3}{4} ca \; y\left[\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2x^2}{r^7}\right]$$

$$\frac{\partial v_0}{\partial y} = -\frac{3}{4} ca \; x\left[\frac{1}{r^3} - \frac{3y^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2y^2}{r^7}\right] -$$

$$\frac{\partial v_0}{\partial z} = -\frac{3}{4} ca \cdot \frac{xyz}{r^5}\left[-3 + \frac{5a^2}{r^2}\right]$$

$$\frac{\partial w_0}{\partial y} = \nearrow$$

$$\frac{\partial w_0}{\partial x} = -\frac{3}{4} ca \; z\left[\frac{1}{r^3} - \frac{3x^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2x^2}{r^7}\right]$$

$$\frac{\partial w_0}{\partial z} = -\frac{3}{4} ca \; \cancel{x}\left[\frac{1}{r^3} - \frac{3z^2}{r^5} - \frac{a^2}{r^5} + \frac{5a^2z^2}{r^7}\right] -$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{3}{2} ca \frac{y}{r^3} = -\zeta$$

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = +\frac{3}{2} ca \frac{z}{r^3} = -\eta$$

$$\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} = 0$$

Przybliżone uwzględnienie inercyi:

$\frac{p}{\rho} = R\theta$ czem większe R, tem mniejsze ρ zatem tem mniejszy wpływ inercyi. ~~ściśle R~~ Zatem rozwiniemy według

$\frac{p}{\rho} = \frac{p_0}{\rho_0}$ potęg $r \sim \frac{1}{R} \sim \rho_0$

$$\begin{matrix} u \\ v \\ w \end{matrix} = u_0 + \rho_0 u' + \frac{\rho_0^2}{2} u'' + \dots$$

$$\rho_0 \frac{p}{p_0}\left\{ u_0 \frac{\partial u_0}{\partial x} + \dots + \rho_0\left[u_0 \frac{\partial u'}{\partial x} + u' \frac{\partial u_0}{\partial x} + \dots\right] \right\} + \frac{\partial p_0}{\partial x} + \rho_0 \frac{\partial p'}{\partial x} =$$
$$= \mu\left[\Delta^2 u_0 + \rho_0 \Delta^2 u' + \dots\right]$$

$$p_0 \operatorname{div}_0 + \rho_0(p' \operatorname{div}_0 + p_0 \operatorname{div}') + k\left(u_0 \frac{\partial p_0}{\partial x} + \dots\right) + k\rho_0\left[u' \frac{\partial p_0}{\partial x} + u_0 \frac{\partial p'}{\partial x} + \dots\right] =$$
$$= (k-1)\Phi_0 + \rho_0 \Psi$$

$$\begin{cases} \frac{\partial p_0}{\partial x} = \frac{\mu}{3} \frac{\partial}{\partial x} \operatorname{div}_0 + \mu \Delta^2 u_0 \\ p_0 \operatorname{div}_0 + k\left(u_0 \frac{\partial p_0}{\partial x} + \dots\right) = (k-1)\Phi_0 \end{cases}$$

Poprawki:

$$\left.\begin{aligned} \underbrace{\left(u_0 \frac{\partial u_0}{\partial x} + \dots\right)}_{f_1} + \frac{\partial p'}{\partial x} &= \frac{\mu}{3} \frac{\partial}{\partial x} \operatorname{div}' + \mu \Delta^2 u' \\ f_2 + \frac{\partial p'}{\partial y} &= \frac{\mu}{3} \frac{\partial}{\partial y} \operatorname{div}' + \mu \Delta^2 v' \\ f_3 + \frac{\partial p'}{\partial z} &= \frac{\mu}{3} \frac{\partial}{\partial z} \operatorname{div}' + \mu \Delta^2 w' \end{aligned}\right\}$$

Izotermicznie:

$$p' \operatorname{div}_0 + p_0 \operatorname{div}' + \left(u' \frac{\partial p_0}{\partial x} + \dots\right) + \left(u_0 \frac{\partial p'}{\partial x} + \dots\right) = 0$$

$$\Delta^2 p' + \underbrace{\left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}\right)} = \frac{4\mu}{3} \Delta^2 \operatorname{div}'$$

$$\underbrace{\operatorname{div}\left[(\sigma\nabla)\sigma\right]}_{= F} = \operatorname{div}\left[\nabla \frac{\sigma^2}{2} + \bar{V} \sigma \operatorname{curl} \sigma\right] = \nabla^2 \frac{\sigma^2}{2} + \sigma \operatorname{curl}^2 \sigma - (\operatorname{curl} \sigma)^2$$

$$p' = \frac{4\mu}{3} \operatorname{div}' - \int \frac{F\, dv}{4\pi r} + \varphi$$

Dla cieczy, albo wogóle jeśli P bardzo duże w porównaniu do $\frac{\partial p}{\partial x}$ etc.

$$\left.\begin{aligned} f_1 + \frac{\partial p'}{\partial x} &= \mu \Delta^2 u' \\ f_2 + \frac{\partial p'}{\partial y} &= \mu \Delta^2 v' \\ f_3 + \frac{\partial p'}{\partial z} &= \mu \Delta^2 w' \end{aligned}\right\}$$

$$\underbrace{\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}}_{F} + \Delta^2 p' = \cancel{\mu \Delta^2 \operatorname{div}'} = 0$$

$$p' = \underbrace{\int \frac{F\, dv}{4\pi r}}_{\Omega} + \varphi$$

$$u' = u + U$$

$$f_1 = \mu \Delta^2 u$$

$$\vdots \quad u = \frac{1}{\mu} \int \frac{f_1\, dv}{4\pi r}$$

etc.

$$\frac{\partial p'}{\partial x} = \mu \Delta^2 U$$

$$\frac{\partial p'}{\partial y} = \mu \Delta^2 V$$

$$\frac{\partial p'}{\partial z} = \mu \Delta^2 W$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = -\left(\frac{\partial u}{\partial x} + \dots\right)$$

$$\mu \Delta^2 \operatorname{div} U = \Delta^2 p' = -\mu \Delta^2 \operatorname{div} u$$

$$p' = -\mu \operatorname{div} u + \varphi \quad \Big\| \quad \Delta^2 \varphi = 0$$

$$-\mu \frac{\partial}{\partial x} \operatorname{div} u + \frac{\partial \varphi}{\partial x} = \mu \Delta^2 U$$

$$-\mu \frac{\partial}{\partial y} \operatorname{div} u + \frac{\partial \varphi}{\partial y} = \mu \Delta^2 V \qquad \Delta^2 \varphi = 0$$

$$-\mu \frac{\partial}{\partial z} \operatorname{div} + \frac{\partial \varphi}{\partial z} = \mu \Delta^2 W$$

$$-\mu \nabla \operatorname{div} u + \nabla \varphi = \mu \nabla^2 U$$

$$\cancel{u' = \nabla \int \frac{\operatorname{div} u'\, dv}{r} + \operatorname{curl} \int \frac{\operatorname{curl} u'}{r} dv}$$

$$\cancel{u' = \frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_3}{\partial y} - \frac{\partial \varphi_2}{\partial z}}$$

$$\left\{\begin{aligned} f_1 + \frac{\partial \Omega}{\partial x} + \frac{\partial \varphi}{\partial x} &= \mu \Delta^2 u' \\ f_2 + \frac{\partial \Omega}{\partial y} + \frac{\partial \varphi}{\partial y} &= \mu \Delta^2 v' \\ f_3 + \frac{\partial \Omega}{\partial z} + \frac{\partial \varphi}{\partial z} &= \mu \Delta^2 w' \end{aligned}\right.$$

$$\cancel{F_1 + p' = \Delta^2 \varphi_1}$$

$$\cancel{F_2 + p' = \Delta^2 \varphi_1}$$

$$\frac{\partial f_1}{\partial y} - \frac{\partial f_2}{\partial x} = \mu \Delta^2 \zeta$$

$$\xi = \int \frac{\frac{\partial f_1}{\partial y} - \frac{\partial f_2}{\partial x}}{} +$$

$$\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z} = 2\frac{yz}{r^5}x\left[-3+\frac{5a^2}{r^2}\right]$$

$$\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} = 2z\left[-\frac{3x^2}{r^5}-\frac{a^2}{r^5}+\frac{5a^2x^2}{r^7}\right] = 2\frac{z}{r^3}\left[-\frac{a^2}{r^2}+\frac{x^2}{r^2}\overbrace{\left[-3+\frac{5a^2}{r^2}\right]}^{N}\right]$$

$$\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} = 2y\left[\underbrace{\qquad\nearrow\qquad}_{M}\right]$$

~~$\frac{\partial u}{\partial x} = x\left(M+\frac{1}{r^3}-\frac{2a^2}{r^5}\right)$~~

~~$\frac{\partial v}{\partial y} = x\left(M+\frac{1}{r^3}\right.$~~

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2 =$$

$$\frac{x^2}{r^6}\left\{\left[\left(1-\frac{3a^2}{r^2}\right)-x^2\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right)\right]^2+\left[\left(1-\frac{a^2}{r^2}\right)-y^2\left[\frac{3}{r^2}-\frac{5a^2}{r^4}\right]\right]^2+\left[\left(1-\frac{a^2}{r^2}\right)-z^2\left(\qquad\right)\right]^2\right\}$$

$$=\frac{x^2}{r^6}\left\{\begin{array}{ll} 1-\frac{6a^2}{r^2}+\frac{9a^4}{r^4} & -2r^2\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right) \\ +1-\frac{2a^2}{r^2}+\frac{a^4}{r^4} & +2\left(\frac{3a^2}{r^2}x^2+\frac{a^2y^2}{r^2}+\frac{a^2z^2}{r^2}\right)\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right) \\ +1-\frac{2a^2}{r^2}+\frac{a^4}{r^4} & +(x^4+y^4+z^4)\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right)^2 \end{array}\right.$$

$$=\frac{x^2}{r^6}\left\{\cancel{3}-\frac{10a^2}{r^2}+\frac{11a^4}{r^4}-6+\frac{10a^2}{r^2}+\frac{4a^2x^2}{r^2}\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right)+\frac{6a^2}{r^2}-\frac{10a^4}{r^4}\right.$$
$$\left.+(x^4+y^4+z^4)\left(\frac{3}{r^2}-\frac{5a^2}{r^4}\right)^2\right.$$

$$=\frac{x^2}{r^{10}}\left\{-3+\frac{6a^2r^2}{\cancel{r^2}}+\frac{a^4}{\cancel{r^4}}+\frac{4a^2x^2}{\cancel{r^2}}\left(\frac{3}{\cancel{r^2}}-\frac{5a^2}{r^2}\right)+(x^4+y^4+z^4)\left(\frac{3}{\cancel{r^2}}-\frac{5a^2}{r^2}\right)^2\right\}$$

$$\left(\frac{\partial w}{\partial y}+\dots\right)^2+(\ )^2+(\ )^2=\frac{4}{r^6}\left\{\frac{N^2x^2y^2z^2}{r^4}+(z^2+y^2)\left[-\frac{a^2}{r^2}+\frac{x^2N}{r^2}\right]^2\right\}$$

$$=\frac{4}{r^{10}}\left\{\frac{N^2x^2y^2z^2}{\cancel{r^4}}+a^4(z^2+y^2)-2a^2x^2(z^2+y^2)N+x^4(z^2+y^2)N^2\right\}$$

$$\Phi = \frac{2}{r^{10}} \Big\{ \overset{-3x^2r^4}{4a^4x^2} + 6a^2r^2x^2 \underbrace{- 4a^2x^4N}\ + \underbrace{x^2(x^4+y^4+z^4)N^2} +$$

$$2a^4(z^2+y^2) \underbrace{- 4a^2x^2(z^2+y^2)N} + \underbrace{2x^4(z^2+y^2)N^2 + 2N^2x^2y^2z^2} \Big\}$$

$$-4a^2Nx^2\underbrace{[x^2+z^2+y^2]}_{~~[illegible]~~\ r^2} + \underbrace{N^2x^2\{x^4+y^4+z^4+2x^2z^2+2x^2y^2+2y^2z^2\}}_{= N^2x^2r^4}$$

$$= \frac{2}{r^{10}}\Big\{ \overset{-3x^2r^4}{a^4(x^2+2r^2-2x^2)} + 6a^2r^2x^2 - 4a^2Nx^2r^2 \ [illegible] + N^2x^2r^4 =$$

$$\overset{-3x^2r^4}{=} 2r^2a^4 - a^4x^2 + 6a^2r^2x^2 - 4a^2r^2x^2N \ [illegible] + x^2r^4N^2$$

$$N = -3 + 5\frac{a^2}{r^2}$$

$$\overset{-3x^2r^4}{=} +2a^4r^2 - a^4x^2 + 6a^2r^2x^2 + 12a^2r^2x^2 - 20a^4x^2 - [illegible] +$$

$$+ 9x^2r^4 - 30a^2x^2r^2 + 25a^4x^2 =$$

$$= +2a^4r^2 + 4a^4x^2 - 12a^2x^2r^2 - [illegible] + 6x^2r^4$$

$$\Phi = 2\cdot\frac{9}{16}\frac{c^2a^2}{r^{10}}\Big\{ \qquad \Big\}$$

$$= \frac{9}{8}\frac{c^2a^2}{r^4}\Big\{\Big(\overset{6\frac{x^2}{r^2}+}{2\frac{a^4}{r^4}} - 12\frac{a^2x^2}{r^4} + 4\frac{a^4x^2}{r^6}\Big\} ~~[illegible]~~ \Big\}$$

$$k\,\mathrm{div}' + \left(u_0\frac{\partial p_0}{\partial x} + \ldots\right) = (k-1)\Phi\mu$$

$$\mathrm{div}' = -\frac{1}{k}\left(u_0\frac{\partial p_0}{\partial x} - \ldots\right) + \frac{k-1}{k}\mu\Phi + \nu\Delta\vartheta$$

zatem chodzi o wyrażenie Φ jako $\Delta^2\chi$

Do dyspozycji:

$\alpha \quad \Delta^2\left(\frac{x^4}{r^{10}}\right) = 12\frac{x^2}{r^{10}} + 10\cdot\frac{x^4}{r^{12}}$

$\beta \quad \Delta^2\left(\frac{x^4}{r^8}\right) = 12\frac{x^2}{r^8} - 8\frac{x^4}{r^{10}}$

$\gamma \quad \Delta^2\left(\frac{x^2}{r^8}\right) = \frac{2}{r^8} + 24\frac{x^2}{r^{10}}$

$\delta \quad \Delta^2\left(\frac{x^2}{r^6}\right) = \frac{2}{r^6} + 6\frac{x^2}{r^8}$

$\varepsilon \quad \Delta^2\left(\frac{x^2}{r^4}\right) = \frac{2}{r^4} - 4\frac{x^2}{r^6}$

$\zeta \quad \Delta^2\left(\frac{1}{r^4}\right) = \frac{12}{r^6}$

$\eta \quad \Delta^2\left(\frac{1}{r^2}\right) = \frac{2}{r^4}$

$\vartheta \quad \Delta^2\left(\frac{1}{r^6}\right) = \frac{30}{r^8}$

$\cancel{10\,a\,\alpha}$

$10\alpha = 10a^6$

$+8\beta = 6a^4$

$12\alpha + 24\gamma = 9a^6$

$12\beta + 6\delta = -20a^4$

$2\gamma + 30\vartheta = 2a^6$

$-4\varepsilon = 9a^2$

$2\delta + 12\zeta = 0$

$2\varepsilon + 2\eta = 0$

$4\alpha + 8\gamma = 3a^6$

$\gamma = \frac{3a^6 - 4a^6}{8}$

$3\delta = -10a^4 - 6\beta = -10a^4 - \frac{9}{2}a^4 = -\frac{29}{2}a^4$

$30\vartheta = 2a^6 + \frac{1}{4}a^6 = \frac{9}{4}a^6$

$$\begin{cases} \alpha = a^6 \\ \beta = \frac{3}{4}a^4 \\ \gamma = -\frac{1}{8}a^6 \\ \delta = -\frac{29}{6}a^4 \\ \vartheta = \frac{3}{40}a^6 \\ \varepsilon = -\frac{9}{4}a^2 \\ \eta = +\frac{9}{4}a^2 \\ \zeta = \cancel{-\frac{1}{6}} = \frac{29}{36}a^4 \end{cases}$$

$\Phi = \Delta^2\chi$

$$\chi = \frac{9c^2}{8}\left[\frac{a^6x^4}{r^{10}} + \frac{3}{4}\frac{a^4x^4}{r^8} - \frac{1}{8}\frac{a^6x^2}{r^8} - \frac{29}{6}\frac{a^4x^2}{r^6} - \frac{9}{4}\frac{a^2x^2}{r^4} + \frac{29}{36}\frac{a^4}{r^4} + \frac{9}{4}\frac{a^2}{r^2} + \frac{3}{40}\frac{a^6}{r^6}\right]$$

$$\frac{k-1}{k}\Gamma\chi = \frac{k-1}{k}\mu\frac{9c^2}{8}\Big[\frac{a^6x^4}{r^{10}} + \frac{3}{4}\frac{a^4x^4}{r^8} \cancel{+} - \frac{1}{8}\frac{a^6x^2}{r^8} - \frac{29}{6}\frac{a^4x^2}{r^6} - \frac{9}{4}\frac{a^2x^2}{r^4}$$

$$+\frac{3}{40}\frac{a^6}{r^6} + \frac{29}{36}\frac{a^4}{r^4} + \frac{9}{4}\frac{a^2}{r^2}\Big]$$

$$\frac{\partial}{\partial x} \quad = \frac{k-1}{k}\mu\frac{9c^2}{8}\Big[\frac{4a^6x^3}{r^{10}} + \frac{3a^4x^3}{r^8} - \frac{1}{4}\frac{a^6x}{r^8} - \frac{29}{3}\frac{a^4x}{r^6} - \frac{9}{2}\frac{a^2x}{r^4} +$$

$$-10\frac{a^6x^5}{r^{12}} - 6\frac{a^4x^5}{r^{10}} + \frac{a^6x^3}{r^{10}} + 29\frac{a^4x^3}{r^8} + 9\frac{a^2x^3}{r^6}$$

$$-\frac{9}{20}\frac{a^6x}{r^8} - \frac{29}{9}\frac{a^4x}{r^6} - \frac{9}{2}\frac{a^2x}{r^4}\Big]$$

$r=a$: $\big[\ \big] = -16\frac{x^5}{a^6} + 46\frac{x^3}{a^4} - \frac{2033}{90}\frac{x}{a}$

$-$

$$\begin{array}{r} 3 \\ +54 \\ 116 \\ \hline 173 \end{array}$$

$$\frac{173}{12} + \frac{99}{20} + \frac{29}{9} =$$

$$= \frac{\begin{array}{r} 173 \\ 865 \\ 2595+ \end{array}}{186}$$

$$\begin{array}{r} 2595 \\ 891 \\ 580 \\ \hline 4066 \end{array}$$

$$\frac{4066}{180} = \frac{2033}{90}$$

$(a\nabla)y =$

$$-\frac{9}{8}\frac{c^2a^2x^3z}{r^8}\left(1-\frac{a^2}{r^2}\right)+\frac{3}{2}\frac{c^2axz}{r^5}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)+$$

$$-\frac{9}{8}\frac{c^2a^2xy^2z}{r^8}\left(\quad\right)-\frac{9}{8}\frac{c^2a^2xz^3}{r^8}\left(\quad\right)+\frac{3}{8}\frac{c^2a^2xz}{r^6}\left(1-\frac{a^2}{r^2}\right)$$

$$=-\frac{9}{8}\frac{c^2a^2xz}{r^6}\left(\quad\right)+\frac{3}{8}\cdots$$

$$=-\frac{3}{4}\frac{c^2a^2xz}{r^6}\left(1-\frac{a^2}{r^2}\right)+\frac{3}{2}\frac{c^2axz}{r^5}\left(\qquad\right)$$

$$-\left\{\frac{3}{8}\frac{c^2a^2xz}{r^3}\left[\frac{1}{r^3}-\cancel{\frac{3y^2}{r^5}}-\frac{a^2}{r^5}+\cancel{\frac{5a^2y^2}{r^7}}\right]-\frac{3}{8}\cancel{\frac{c^2a^2xy^2z}{r^8}\left[-3+\frac{5a^2}{r^2}\right]}\right\}$$

$$=\frac{3}{4}\frac{c^2a^2xz}{r^6}\left\{-1+\frac{a^2}{r^2}+\frac{2r}{a}-\frac{3}{2}-\frac{1}{2}\cancel{\frac{a^2}{r^2}}-\frac{1}{2}+\cancel{\frac{a^2}{2r^2}}\right.$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{9}{16}c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x}{r^3}\Big\{xy\left[\frac{1}{r^3}-\frac{3x^2}{r^5}-\frac{a^2}{r^5}+\frac{5a^2x^2}{r^7}\right]+$$

$$xy\left[\frac{1}{r^3}-\frac{3y^2}{r^5}-\frac{a^2}{r^5}+\frac{5a^2y^2}{r^7}\right]+\frac{xyz^2}{r^5}\left[-3+\frac{5a^2}{r^2}\right]\Big\}-\frac{3c^2}{4}a\,y\uparrow\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$=\frac{9}{16}c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x}{r^3}\Big\{\underbrace{2xy\left(\frac{1}{r^3}-\frac{a^2}{r^5}\right)-3\frac{xy}{r^3}}_{-\frac{xy}{r^3}+\frac{3a^2xy}{r^5}}+\frac{5a^2xy}{r^5}\Big\}-\nearrow$$

$$=\frac{9}{16}c^2a^2\left(1-\frac{a^2}{r^2}\right)\frac{x^2y}{r^6}\left[-1+\frac{3a^2}{r^2}\right]-\frac{3}{4}c^2a\frac{y}{r^3}\left[1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right]\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]$$

$$=-\frac{3}{4}c^2a\frac{y}{r^3}\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]+\frac{3}{4}\frac{c^2ay}{r^6}\Big\{\frac{3}{4}ax^2\left(-1+\frac{2a^2}{r^2}-\frac{6a^4}{r^4}\right)+$$

$$+\frac{3}{4}a\left(\cancel{r^2}-\cancel{3x^2-a^2}+\frac{5a^2x^2}{r^2}\right)+\frac{1}{4}a^3\left(1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right)\Big\}$$

$$\{\,\}=+\frac{3}{4}ar^2-3ax^2-\frac{1}{2}a^3+\frac{a^3x^2}{r^2}-\frac{1}{4}\frac{a^5}{r^2}-\frac{13}{4}\frac{a^5x^2}{r^4}$$

$$=-\frac{3}{4}\frac{c^2ay}{r^3}\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]+\frac{3}{4}\frac{c^2ay}{r^6}\{\,\}$$

$$u\frac{\partial w}{\partial x}+\cdots=-\frac{3}{4}\frac{c^2az}{r^3}\left[1-\frac{3x^2}{r^2}-\frac{a^2}{r^2}+\frac{5a^2x^2}{r^4}\right]+\frac{3}{4}\frac{c^2az}{r^6}\{\,\}$$

$$\Phi=c^2a^2\frac{1}{4}\left(\frac{z^2}{r^6}+\frac{y^2}{r^6}\right)+\frac{3}{2}c^2a\Big[\cancel{-\frac{1}{r^3}}+\cancel{\frac{3x^2}{r^5}}+\frac{a^2}{r^5}-\cancel{\frac{5a^2x^2}{r^7}}-\cancel{\frac{3ax^2}{r^6}}-\cancel{\frac{2a^3}{r^6}}+\cancel{\frac{6a^3x^2}{r^8}}-$$

$$-\cancel{\frac{a^5x^2}{r^{10}}}-\frac{3x^2}{r^5}\Big[\cancel{-1}+\frac{a^2}{r^2}+\cancel{\frac{3a^2}{r^2}}-\cancel{\frac{5a^2x^2}{r^4}}-\frac{3a^2}{r^7}-\cancel{\frac{2a^3}{r^3}}+\cancel{\frac{6a^3}{r^5}}-\cancel{\frac{a^5x^2}{r^7}}\Big]+\frac{x}{r^3}\Big[\cancel{\frac{6x}{r^2}}-$$

$$-\frac{6x^3}{r^4}-\cancel{\frac{6a^2x}{r^4}}-\cancel{\frac{10a^2x}{r}}+\frac{20a^2x^3}{r^6}-\cancel{\frac{6ax}{r^3}}+\frac{9ax^3}{r^5}+\cancel{\frac{6a^3x}{r^5}}+\cancel{\frac{12a^3x}{r^5}}-\cancel{\frac{30a^3x^3}{r^7}}-\cancel{\frac{2a^5x}{r^7}}+\cancel{\frac{7a^5x^3}{r^9}}\Big]$$

$$-\frac{2}{r^3}\Big[\cancel{1}-\cancel{\frac{3x^2}{r^2}}-\frac{a^2}{r^2}+\cancel{\frac{5a^2x^2}{r^4}}\Big]+\frac{2}{r^6}\Big[\cancel{\frac{3}{4}ar^2}-\cancel{3ax^2}-\cancel{\frac{1}{2}a^3}+\cancel{\frac{a^3x^2}{r^2}}-\frac{1}{4}\frac{a^5}{r^2}-\frac{13}{4}\cancel{\frac{a^5x^2}{r^4}}\Big]$$

$$-\underset{=(x^2-r^2)}{(y^2+z^2)}\Big[\frac{15x^2}{r^7}+\frac{5a^2}{r^7}-\frac{35a^2x^2}{r^9}+3\frac{a}{r^6}-18\frac{ax^2}{r^8}+3\frac{a^3}{r^8}-8\frac{a^3x^2}{r^{10}}+2\frac{a^5}{r^{10}}+\frac{65}{\cancel{2}}\frac{a^5x^2}{r^{12}}\Big]\Big]$$

$$\Phi = \frac{c^2a^2}{4}\left(\frac{r^2-x^2}{r^6}\right) + \frac{3}{2}c^2a$$

$$\frac{85}{2}\frac{a^5x^4}{r^{12}} - 40\frac{a^5x^2}{r^{10}} - 56\frac{a^3x^4}{r^{10}} + 43\frac{a^3x^2}{r^8} - \cancel{a} - \frac{5}{2}\frac{a^5}{r^8} - 30\frac{ax^2}{r^6}$$

$$-6\frac{a^3}{r^6} - \frac{3}{2}\frac{a^4}{r^4} + \cancel{\frac{a^2x^3}{r^9}}\ \cancel{\frac{a^2x^2}{r^7}} + \frac{3x^3}{r^5} - \frac{3}{r^3}$$

Przy powierzchni $u\,v\,w = 0$ wyjdzie warunek:

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \cancel{u\frac{\partial \rho}{\partial x}} + \cancel{v\frac{\partial \rho}{\partial y}} + \cancel{w\frac{\partial \rho}{\partial z}} = 0 \qquad \underline{\underline{\operatorname{div} = 0}}$$

istotnie $\operatorname{div} = 0$ dla $x = a$

to znaczy: układając x, y (w kierunek prądu i normalnej jako styczne do powierzchni)

$$\frac{\partial w}{\partial z} = 0 \quad | \text{ ponieważ } \quad v = 0, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial w}{\partial x} = 0, \quad \frac{\partial w}{\partial y} = 0$$

$$u = 0 \qquad \frac{\partial u}{\partial x} = 0 \qquad \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial u}{\partial z} \gtrless 0$$

$$w = 0 \qquad \frac{\partial v}{\partial x} = 0 \qquad \frac{\partial v}{\partial y} = 0 \qquad \frac{\partial v}{\partial z} = 0$$

Jeśli jednak zastosujemy $\boxed{III}$:

$$\cancel{k\rho \operatorname{div}} + \cancel{\left(u\frac{\partial u}{\partial x} + \right)} = (k-1)\Big[\underset{\hat{u}}{\Phi} + \kappa\,\Delta^2\theta\Big]$$

$$0 = \cancel{\mu}\left(\frac{\partial u}{\partial z}\right)^2 + \kappa\,\Delta^2\theta$$

t. zn. że przy powierzchni będzie $\kappa\frac{\partial^2\theta}{\partial n^2} = \cancel{\mu}\left(\frac{\partial u}{\partial z}\right)^2$

więc gaz cieplejszy się stanie? A jeśli κ się może $= 0$ wtedy musi być $\frac{\partial u}{\partial n} = 0$

κ w mierze mechanicznej: $K = 0{\cdot}000058 \cdot 42.10^6 = 42.58 = 243$

Φ

Nie zaniedbując κ otrzyma się jako równanie III:

$$\theta = \theta_0 + \frac{1}{p}\theta'$$

$$k\, \mathrm{div}' = -\left(u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z}\right) + (k-1)\left[\Phi_0 + \kappa \Delta^2 \theta_0\right]$$

$$p = \overset{P+}{p_0} + \frac{1}{p} p' + \frac{1}{2}\frac{1}{p^2} p'' + \cdots$$

$$\begin{cases} \dfrac{\partial p_0}{\partial x} = \cancel{\dfrac{\mu}{3}\dfrac{\partial}{\partial x}\mathrm{div}_0} + \mu \Delta^2 u_0 \\ - - - - \end{cases}$$

$$\mathrm{div}_0 = 0$$

wystarczy do oznaczenia $p, u, v, w,$

$$\frac{\partial}{\partial x}(\rho_0 u_0) + \frac{\partial}{\partial y}(\rho_0 v_0) + \frac{\partial}{\partial z}(\rho_0 w_0) = 0$$

$$\cancel{\rho_0\, \mathrm{div}_0} + u_0 \frac{\partial \rho_0}{\partial x} + v_0 \frac{\partial \rho_0}{\partial y} + w_0 \frac{\partial \rho_0}{\partial z} = 0$$

będzie spełnione przez $\rho_0 = \text{const}$

~~zatem θ_0 = const~~

(ale czy koniecznie?)

Rozwiązanie:

$$\frac{dx}{u_0} = \frac{dy}{v_0} = \frac{dz}{w_0}$$

określa położenie linij prądu

Zatem ono równanie wypowiada, że wzdłuż linij prądu ρ niezmienne, ale od jednej do drugiej może być zmienne, tutaj jednakże, bo w nieskończoności ρ będzie jednakowe wszędzie

zatem wogóle $\rho_0 = \text{const}$

$$\left(P + p_0 + \frac{1}{p} p' ..\right) = R\left(\theta_0 + \frac{1}{p}\theta' ..\right)\left(\text{const} + \frac{1}{p}\rho' + .\right)$$

zatem θ_0 = $\underline{\text{const}}$

$$\frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\mathrm{div}' + \mu \Delta^2 u'$$

$$k\, \mathrm{div}' = -\left(u_0 \frac{\partial p_0}{\partial x} + \cdots\right) + (k-1)\Phi_0$$

$$\frac{\partial}{\partial x}(\rho_0 u' + \rho' u_0) + \frac{\partial}{\partial y} \cdots + \frac{\partial}{\partial z} \cdots = 0$$

to będzie oznaczenie dla ρ' i z tego wynikanie θ'

$$\rho_0\, \mathrm{div}' + \underline{\rho'\, \mathrm{div}_0} + u' \frac{\partial \rho_0}{\partial x} + \cdots + u_0 \frac{\partial \rho'}{\partial x} + \cdots = 0$$

$$u_0 \frac{\partial \rho'}{\partial x} + v_0 \frac{\partial \rho'}{\partial y} + w_0 \frac{\partial \rho'}{\partial z} = -\underbrace{\left[\frac{\partial}{\partial x}(\rho_0 u') + \frac{\partial}{\partial y}(\rho_0 v') + \frac{\partial}{\partial z}(\rho_0 w')\right]}_{R}$$

Rozwiązanie:

$$\frac{dx}{u_0} = \frac{dy}{v_0} = \frac{dz}{w_0} = \frac{d\rho'}{R}$$

to nie prawda, bo jeśli $\frac{dp}{p}$ małe

to także $\frac{d\rho}{\rho}$ małe

$$\rho = \rho_0 + \frac{1}{p}$$

$$\text{I}).\ \underbrace{u_0 \frac{\partial \rho_0}{\partial x} + v_0 \frac{\partial \rho_0}{\partial y} + \cdots}_{\text{małe wobec } \rho_0} + \rho_0\, \mathrm{div}_0 = 0$$

zatem $\mathrm{div}_0 = 0$

Inna metoda: rozwiązać wszystko izotermicznie i potem wprowadzić (I przybliżenie temperatury

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Jeszcze raz gruntowniej:

Założenie fundamentalne: ruch taki że Δp mało wobec p

~~[illegible]~~

$p = R\theta\rho$ [N.p. zamknięte naczynie

$$\frac{dp}{p} = \frac{d\theta}{\theta} + \frac{d\rho}{\rho}$$

I). $\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \right) + \mu\nabla^2 u$

IV). $p = R\theta\rho$

II). $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$

III). $k\,p\,\mathrm{div} + \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) = (k-1)\left[\mu\,\Phi + \kappa\Delta^2\theta\right]$

II). $\rho\,\mathrm{div} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0$

II). $\mathrm{div} + \frac{u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}}{p} - \frac{u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}}{\theta} = 0$

III). $(1-k)\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) + k\frac{p}{\theta}\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z}\right) = (k-1)\left[\mu\,\Phi + \kappa\Delta^2\theta\right]$

$$\frac{kR}{k-1}\rho\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial y}\right) = \mu\,\Phi + \kappa\Delta^2\theta + \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right)$$

jeżeli $\Delta\rho$ mało w porównaniu do ρ, to także div bardzo mało $\qquad \rho = \underset{\overset{\|}{\text{const}}}{\rho_0} + \rho'$

$u\ v\ w\ p \ldots$ itd. $= u_0 + \frac{1}{\rho_0} u' + \ldots$

$$\text{II}).\ (\rho_0 + \rho')\left(\operatorname{div}_0 + \frac{1}{\rho_0}\operatorname{div}' + \ldots\right) + \left(u_0 + \frac{1}{\rho_0}u' + \ldots\right)\frac{\partial\rho'}{\partial x} + \left(v_0 + \frac{1}{\rho_0}v' \ldots\right)\frac{\partial\rho'}{\partial y} + \ldots = 0$$

1). $\operatorname{div}_0 = 0$

2). $\cancel{\rho' \operatorname{div}_0} + \operatorname{div}' + u_0 \frac{\partial\rho'}{\partial x} + v_0 \frac{\partial\rho'}{\partial y} + \ldots = 0$

$$\operatorname{div}' = -\left(u_0 \frac{\partial\rho'}{\partial x} + v_0 \frac{\partial\rho'}{\partial y} + w_0 \frac{\partial\rho'}{\partial z}\right)$$

$$\text{IV}).\ p_0 + \frac{1}{\rho_0} p' + \ldots = R\left(\theta_0 + \frac{1}{\rho_0}\theta' + \ldots\right)(\rho_0 + \rho')$$

Jeżeli założymy że $\frac{p_0}{R\theta_0}$ tego samego rzędu co ρ_0 itd.

1). $p_0 = R\theta_0\rho_0$

2). $p' = R\theta' + R\theta_0\rho'$

$$\text{I}).\ 1).\ \frac{\partial p_0}{\partial x} = \cancel{\frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div}_0} + \mu\nabla^2 u_0$$

$$2).\ \frac{\partial p'}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div}' + \mu\nabla^2 u'$$

$$\text{III}).\ k\left(p_0 + \frac{1}{\rho_0}p' + \ldots\right)\left(\cancel{\operatorname{div}_0} + \frac{1}{\rho_0}\operatorname{div}' + \ldots\right) + \left(u_0 + \frac{1}{\rho_0}u' + \ldots\right)\left(\frac{\partial p_0}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial x} + \ldots\right) =$$
$$= (k-1)\left[\mu\left(\Phi_0 + \frac{1}{\rho_0}\Phi' + \ldots\right) + \kappa\left(\Delta^2\theta_0 + \frac{1}{\rho_0}\Delta^2\theta'\right)\right]$$

~~k~~ Albo:

$$\frac{kR}{k-1}(\rho_0 + \rho')\left[\left(u_0 + \frac{1}{\rho_0}u' + \ldots\right)\left(\frac{\partial\theta_0}{\partial x} + \frac{1}{\rho_0}\frac{\partial\theta'}{\partial x} + \ldots\right) + \ldots\right] = \mu\Phi_0 + \frac{1}{\rho_0}\Phi' + \ldots +$$
$$\mu\left(u_0 + \frac{1}{\rho_0}u' + \ldots\right)\left(\nabla^2 u_0 + \frac{1}{\rho_0}\ldots\right)$$
$$+ \kappa\Delta^2\theta_0 + \frac{1}{\rho_0}\Delta^2\theta' + \ldots$$

$$\underbrace{k\frac{p_0}{\rho_0}}\operatorname{div}' + u_0\frac{\partial p_0}{\partial x} = (k-1)\left[\mu\Phi_0 + \kappa\Delta^2\theta_0\right]$$

$$kR\theta_0\operatorname{div}' + u_0\frac{\partial p_0}{\partial x} = \nearrow$$

więc dla θ równanie formy:

$$\Delta^2\theta + \theta\,\varphi(x,y,z) = \psi(x,y,z)$$

$p =$

$= f_c(\underbrace{\frac{1}{P}, \Theta, x, y, z})$

$\left.\begin{array}{l} P = \text{mierzone w wielkiej długości} \\ \quad = \text{stała całkow.} \\ \Theta \end{array}\right\}$

$=$ Zmieniając $\frac{1}{P}$: każdym razem inny rozkład p w

$\frac{1}{P}$ = parametr

Czy będzie można stawić:

$$p = \text{const} + p_0 + \frac{1}{P}p' + \frac{1}{P^2}p'' + \ldots \quad ?$$

$$= f(0, \Theta_0, x_0, y_0, z) + \frac{1}{P}\frac{\partial f}{\partial(\frac{1}{P})} + \Theta\frac{\partial f}{\partial}$$

---

uwzględniając najpierw III dir $= 0$:

otrzymuje się równanie dla θ a z tego za pomocą II dir.

Eliminując dir z II III:

w mierze mechanicznej $= 0{\cdot}000057 \cdot 42{\cdot}10^6$

III. $(1-\kappa)(u\frac{\partial f}{\partial x} + \ldots) + \kappa\frac{p}{\theta}(u\frac{\partial\theta}{\partial x} + \ldots \quad) = (\kappa-1)[\mu\Phi + \kappa\Delta^2\theta]$

Obierając jako pierwsze przybliżenie rozkład adiabat.:

$$\mu\Phi + \kappa\Delta^2\theta = 0$$

z tego θ' a z tego za pomocą II dir

Obierając jako pierwsze przybliżenie rozkład izotermiczny $\rightarrow$ $\mu\Phi + \kappa\Delta^2\theta' = -(u_0\frac{\partial p_0}{\partial x} + \ldots)$

Która z wszystkich tych metod najracjonalniejsza?

Metoda:

Pierwsze przybliżenie div = 0

$$\text{I).}\quad \begin{cases} \frac{\partial p_0}{\partial x} = \mu D^2 u_0 \\ \mathrm{div}_0 = 0 \end{cases}$$

~~Drugie.~~ ~~III)~~ III). $u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z} = (k-1)[\mu \Phi_0 + \kappa \Delta^2 \theta_0]$

z tego θ_0

Drugie przybliżenie:

$$\boxed{\text{IV).}\quad \frac{d\rho}{\rho} = \frac{dp}{p} - \frac{d\theta}{\theta}}$$

II). $\mathrm{div}' = -\frac{u_0 \frac{\partial p_0}{\partial x} + v_0 \frac{\partial p_0}{\partial y} + w_0 \frac{\partial p_0}{\partial z}}{p_0} + \frac{u_0 \frac{\partial \theta_0}{\partial x} + \cdots}{\theta_0}$

I). $\frac{\partial p'}{\partial x} = \frac{\mu}{3} \frac{\partial\, \mathrm{div}'}{\partial x} + \mu \nabla^2 u'$

$\nabla^2 p' = \frac{4\mu}{3} \nabla^2 \mathrm{div}'$

$p' = \frac{4\mu}{3} \mathrm{div}' + U$

Pytanie jak pogodzić równania

II). $\mathrm{div} + \frac{u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} +}{p} - \frac{u \frac{\partial \theta}{\partial x} + v -}{\theta} = 0$

III). $k p\, \mathrm{div} + (u \frac{\partial p}{\partial x} + \cdots \quad) = (k-1)[\mu \Phi + \kappa \Delta^2 \theta]$

w drugiem przybliżeniu

uwzględniając najpierw II). z div = 0 otrzymałoby się $\frac{dp}{p} = + \frac{d\theta}{\theta}$

$p \sim \theta$

z tego za pomocą II: div = ...

~~$(k-1)[\mu \Phi + \kappa \Delta^2 \theta] = \frac{p}{\theta} u \partial$~~

Odejmujemy II od III:

$$(k-1)\,p\,\mathrm{div} = -\frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)+(k-1)\left[\mu\Phi+\kappa\Delta^2\theta\right]$$

Dwa stany które charakteryzować będę [przy danem κ itd.] szczególny ruch:

~~P~~ P i Θ

Najprostszy jeśli P bardzo duże i Θ bardzo duże | w porównaniu do $\frac{\Delta P}{\Delta\Theta}$

Wtedy II): div = 0

zatem ruch przez I i II zupełnie określony

θ jest wtedy obojętne ~~i które nie podlega [illegible] szczególny~~

musi być obliczone z III z uwzględnieniem szczególnych wartości P Θ:

~~[illegible]~~ $(1-k)\left(u\frac{\partial\theta}{\partial x}+\dots\right) = -k\frac{P}{\Theta}\left(u\frac{\partial\theta}{\partial x}+\dots\right)+(k-1)\left(\mu\Phi+\kappa\Delta^2\theta\right)$

Szczególne założenie jeszcze że

1. $\frac{1}{p}\frac{\partial p}{\partial x} \ll \frac{1}{\theta}\frac{\partial\theta}{\partial x}$

Wtedy: III przyjmuje kształt: $u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=0$

$\theta = \mathrm{const}$ izoterm.

---

2. $\frac{1}{p}\frac{\partial p}{\partial x} \gg \frac{1}{\theta}\frac{\partial\theta}{\partial x}$

$$(1-k)\left(u\frac{\partial p}{\partial x}+\dots\right) = (k-1)\left[\mu\Phi+\kappa\Delta^2\theta\right]$$

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+\mu\Phi = -\kappa\Delta^2\theta$$

|| to samo byłoby drugiem przybliżeniem z (1)

zatem ostrożniej lepiej używać tego ~~[illegible]~~ sposobu

Pytanie czy zbliżenie równania III słuszne?

2 θ i p w tr. spół. obliczeniu można użyć w II). [illegible] dziś!

i z pomocą I rozkład poprawiony

Zawsze dodatnie bo

$$= \ldots\ldots \left\{ 2\frac{a^4}{r^4} + \left(6 - \frac{12a^2}{r^2} + \frac{4a^4}{r^4}\right)\cos^2\theta \right\}$$

minimum dla $\theta = 0$

$$= 6\left[1 - \frac{a^2}{r^2}\right]^2 \geqq 0$$

$$\Phi = \frac{q}{g}\,\frac{c^2a^2}{r^4}\left\{6\frac{x^2}{r^2} + \frac{2a^4}{r^4} - \frac{12a^2x^2}{r^4} + \frac{4a^4x^2}{r^6}\right\}$$

$$\Delta^2\left(\frac{x^2}{r^8}\right) = \frac{2}{r^8} + 24\frac{x^2}{r^{10}}$$

$$\Delta^2\left(\frac{x^2}{r^6}\right) = \frac{2}{r^6} + 6\frac{x^2}{r^8}$$

$$\Delta^2\left(\frac{x^2}{r^4}\right) = \frac{2}{r^4} - 4\frac{x^2}{r^6}$$

$$\Delta^2\left(\frac{1}{r^4}\right) = \frac{12}{r^6}$$

$$\Delta^2\left(\frac{1}{r^2}\right) = \frac{2}{r^4}$$

$$\Delta^2\left(\frac{1}{r^6}\right) = \frac{30}{r^8}$$

α

β

γ

δ

ε

ζ

$$24\alpha = 4a^4$$

$$30\zeta + 2\alpha = 2a^4$$

$$6\beta = -12a^2$$

$$2\beta + 12\delta = 0$$

$$-4\gamma = 6$$

$$2\gamma + 2\varepsilon = 0$$

$$\alpha = \frac{a^4}{6}$$

$$15\zeta = a^4 - \alpha = \frac{5}{6}a^4 \qquad \zeta = \frac{a^4}{18}$$

$$\beta = -\frac{a^2}{2}$$

$$\delta = \frac{a^2}{12}$$

$$\gamma = -\frac{3}{2}$$

$$\varepsilon = \frac{3}{2}$$

$$\Phi_0 = \Delta^2\left\{\frac{q}{8}c^2a^2\left[\frac{1}{6}\frac{a^4x^2}{r^8} - \frac{1}{2}\frac{a^2x^2}{r^6} + \frac{1}{18}\frac{a^4}{r^6} - \frac{3}{2}\frac{x^2}{r^4} + \frac{1}{12}\frac{a^2}{r^4} + \frac{3}{2}\frac{1}{r^2}\right]\right\}$$

$$u_0\frac{\partial p_0}{\partial x} + v_0\frac{\partial p_0}{\partial y} + w_0\frac{\partial p_0}{\partial z} = \Delta^2\left\{-\mu c^2a^2\left[\frac{27}{32}\frac{1}{r^2} + [illegible] - \frac{45}{32}\frac{x^2}{r^4} - \frac{3}{16}\frac{a^2x^2}{r^6}\right] - \mu c^2a\cdot\frac{3}{4}\frac{x^2}{r^3}\right\}$$

$$\Delta^2\theta = \Delta^2\Psi$$

$$\Delta^2 U = 0$$

$$\theta = \Psi + U$$

$$r = a \Big| \quad \theta_0 = \Psi_0 + U_0$$

$$r = \infty \quad U = 0$$

$$\Psi = \frac{\mu}{k}c^2a^2\left[-\frac{27}{32}\frac{1}{r^2} + \frac{9}{32}\frac{x^2}{r^4} - \frac{3}{32}\frac{a^2}{r^4} - \frac{1}{16}\frac{a^4}{r^6} + \frac{3}{8}\frac{a^2x^2}{r^6} - \frac{3}{16}\frac{a^4x^2}{r^8}\right] + \frac{\mu}{k}c^2a\frac{3}{4}\frac{x^2}{r^3}$$

$$U_0\Big|_{r=a}^{\theta_0} = -\frac{\mu}{k}\, c^2 \cancel{\phi}\left[\underbrace{-\frac{27}{32}-\frac{3}{32}-\frac{3}{48}}_{-\frac{3}{8}\left(\underbrace{\frac{9}{4}+\frac{1}{4}+\frac{1}{6}}\right)} + \underbrace{\left(\frac{9}{32}+\frac{3}{8}-\frac{3}{16}\right)\frac{x^2}{a^2}+\frac{3}{4}\frac{x^2}{a^2}}_{\frac{9+12-6+24}{32}=\frac{39}{32}}\right]$$

$$-\frac{3}{8}\,\frac{64}{24} = \cancel{-\frac{8}{8}} = -1$$

$$U_0\Big|_{r=a}^{\theta_0} = +\frac{\mu}{k}\, c^2\left[1-\frac{39}{32}\frac{x^2}{a^2}\right]$$

$$U = \theta_0 + \frac{A}{r} + B\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right)$$

$$\left.\begin{aligned} \frac{A}{a}+\frac{B}{a^3} &= \frac{\mu c^2}{k} \\ \cancel{B\left(\frac{1}{a^3}-\frac{3x^2}{a^5}\right)}\ -\frac{3B}{a^3} &= -\frac{39}{32}\,\frac{\mu c^2}{k} \end{aligned}\right| 3 \quad \left.\right\} \quad \begin{aligned} 3\frac{A}{a} &= \left(2-\frac{7}{32}\right)\frac{\mu c^2}{k} \\ &= \frac{57}{32}\,\frac{\mu c^2}{k} \\ \frac{A}{a} &= \frac{19}{32}\,\frac{\mu c^2}{k} \\ \frac{B}{a^3} &= \frac{13}{32}\,\frac{\mu c^2}{k} \end{aligned}$$

$$U = \theta_0 + \frac{\mu c^2}{32k}\left[19\frac{a}{r} + 13\left(\frac{a^3}{r^3}-\frac{3x^2a^3}{r^5}\right)\right]$$

$$\theta = \theta_0 + \frac{\mu c^2}{k}\left\{\frac{3a^2}{32}\left[-\frac{9}{r^2}+3\frac{x^2}{r^4}-\frac{a^2}{r^4}-\frac{2}{3}\frac{a^4}{r^6}+4\frac{a^2x^2}{r^6}-2\frac{a^4x^2}{r^8}\right]+\right.$$

$$\left.\frac{a}{r}\left[\frac{19}{32}+\frac{13}{32}\frac{a^2}{r^2}+\frac{3}{4}\frac{x^2}{r^2}-\frac{39}{32}\frac{x^2a^2}{r^4}\right]\right\}$$

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Gdyby się to wstawiło znowu do III

~~po przyjęciu poprawki byłoby rzędu $\frac{c\mu}{A\theta k} = \frac{c}{42.273}$~~

$$\frac{k}{k-1}\frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x}+\dots\right)=\frac{k}{k-1}\frac{p}{\theta}\frac{\mu c^3}{K}\left[\;\dots\;\right]$$

podczas gdy $\left(u\frac{\partial p}{\partial x}+\dots\right)=\mu c^2\left[\;\dots\;\right]$

zatem stosunek poprawki : $\frac{c\,p}{k\,\theta}=\frac{c.\cancel{10^6}}{0{\cdot}000057\;42.\cancel{10^6}.273}=\frac{c}{0{\cdot}6}$

zatem już większa niż tamta jeśli $c>0{\cdot}6\,\frac{cm}{sec}$!

$$\frac{\mu c^2}{k}=\frac{0{\cdot}000{\cdot}19.\,c^2}{42.10^6.\;0{\cdot}000{\cdot}057}=\frac{c^2}{15.10^6}$$

zatem to osiągnęłoby dopiero 1°

$=\frac{c^2.}{15.10^6}$ ~~12~~ $\frac{2}{3}$

gdy $c=\frac{4.10^3\,cm}{sec}$!

Zatem ten sposób rozwinięcia zupełnie nieusprawiedliwiony

W równaniu III :

$$\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)-\frac{k}{k-1}\frac{P}{\Theta}\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)+\mu\Phi+\kappa\Delta^2\theta=0$$

(1) (2) (3) (4)

1 i 3 są rzędu wielkości $\mu\frac{c^2}{a^2}=0{\cdot}00019\cdot\frac{c^2}{a^2}$

$(4)=2400.\,\Delta^2\theta$

ażeby więc 4 stało się wielkością tego samego rzędu, musiałoby $\Delta^2\theta\neq\frac{\mu c^2}{\kappa a^2}$

$$\Delta^2\theta\neq\frac{0{\cdot}00019}{42.10^6.0{\cdot}000057}\frac{c^2}{a^2}=\frac{1}{15.10^6}\frac{c^2}{a^2}$$

co łatwo osiągnąć

podczas gdy (2) : $\frac{10^6}{273}\,c\,\frac{\partial\theta}{\partial x}$

$\mu\frac{c^2}{c}$

Przy wszystkiem ze zaniedbaniem wyrazów drugiego stopnia

wynika że ρu^2 mało wobec $\mu\frac{\partial u}{\partial x}$

$\rho u \ll \mu/x$ $\qquad$ $a\rho c\ll\mu$

$$(A).\ \Delta^2\theta + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = \Psi$$

Jeśli 2 rozwiązania θ_1 θ_2, to suma:

$$\Delta^2(\theta_1+\theta_2) + u\frac{\partial(\theta_1+\theta_2)}{\partial x} + v\frac{\partial(\theta_1+\theta_2)}{\partial y} + w\frac{\partial(\theta_1+\theta_2)}{\partial z} = 2\Psi$$ } zatem i średnia arytmetyczna także zadość czyni

różnica $\Delta^2(\theta_1-\theta_2) + u\frac{\partial(\theta_1-\theta_2)}{\partial x} + \cdots = 0$

Jeśli mamy rozwiązanie ~~zadania~~ równania

$$(B).\ \Delta^2\theta + u\frac{\partial\theta}{\partial x} + \cdots = 0$$

‖ Np. $\theta = e^{\varphi(x,y,z)}$

$$\frac{\partial\theta}{\partial x} = \frac{\partial\varphi}{\partial x}e^{\varphi}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\varphi}{\partial x^2}e^{\varphi} + \left(\frac{\partial\varphi}{\partial x}\right)^2 e^{\varphi}$$

$$\Delta^2\varphi + \left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right] + u\frac{\partial\varphi}{\partial x} + v\frac{\partial\varphi}{\partial y} + w\frac{\partial\varphi}{\partial z} = 0$$

Jeśli warunek dla θ na powierzchni $\theta = \text{const}$, to zgadza się, że $\theta = \text{const}$ jest jedynem rozwiązaniem dla skróconego równania $\Delta^2\theta + u\frac{\partial\theta}{\partial x} + \cdots = 0$.

Z tego wynika, że $\theta_1 - \theta_2 = \text{const}$, zatem tylko jedno rozwiązanie możliwe

---

Stawiając $\Psi = \Delta^2\chi$ w równaniu (A).

i podstawiając $\theta = \vartheta + \chi$ otrzymuje się:

$$\Delta^2\vartheta + u\frac{\partial\vartheta}{\partial x} + \cdots = -\left(u\frac{\partial\chi}{\partial x} + v\frac{\partial\chi}{\partial y} + w\frac{\partial\chi}{\partial z}\right)$$

$\boxed{\Delta^2 p = 0}$

Podstawiając w B: $\theta = \vartheta + p$:

$$\Delta^2\vartheta + u\frac{\partial\vartheta}{\partial x} + \cdots + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \cdots = 0$$

$\boxed{\vartheta = \theta_0 - p}$

$$\longrightarrow \Psi = \frac{Ax^2}{r^6} + \frac{B}{r^8} + \frac{Cx^2}{r^8} + \frac{Dx^2}{r^{10}}$$

$\cancel{\theta = \sum m_k \frac{x}{r^k}}$ Ponieważ Ψ parzyste potęgi , u v

$\cancel{\Delta^2\theta = \sum}$ to θ musi zawierać mieszane parzyste i nieparzyste

---

Gdyby $\Phi = 0$ to mielibyśmy jako jedno możliwe rozwiązanie

$$\theta = \theta_0 + \alpha p$$

$$\left(u\frac{\partial p}{\partial x} + \cdot \quad\right)\left[1 - \frac{k}{k-1}\frac{P}{\Theta}\alpha\right] + \cancel{k\Delta^2 p} = 0$$

t. znaczy $\alpha = \frac{k-1}{k}\frac{\Theta}{P}$

$\theta = \theta_0 + \frac{k-1}{k}\frac{\Theta}{P}p$ ‖ to nie byłoby jednak w zgodności z warunkiem dla $r=a$

zatem tu nie możemy zastosować

Gdyby $\kappa = 0$ to musiałoby być $\left.\frac{\partial v}{\partial r}\right|_{r=a} = 0$

Czyby to już wystarczyło do oznaczenia ruchu?

Znając rozwiązanie

$\Delta^2\theta + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + .. = \Phi$ otrzyma się III przez podstawienie $\theta = \vartheta + \alpha p$

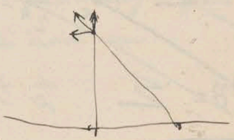
$$u^2+v^2+w^2 = c^2\Big\{\underbrace{\frac{9}{16}\frac{a^2}{r^4}\left(1-\frac{a^2}{r^2}\right)^2 x^2 - \frac{3}{2}\frac{a}{r^3}\left(1-\frac{a^2}{r^2}\right)\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)x^2}_{} + \left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)^2\Big]$$

$$x^2\left(1-\frac{a^2}{r^2}\right)\left[-\frac{3}{2}\frac{a}{r^3} + \underbrace{\frac{3}{2}\frac{a^2}{r^4}\left(\frac{3}{4}+\frac{1}{4}\frac{a^2}{r^2}\right) + \frac{9}{16}\frac{a^2}{r^4}\left(1-\frac{a^2}{r^2}\right)}_{\frac{a^2}{r^4}\left(\frac{27}{16}-\frac{3}{16}\frac{a^2}{r^2}\right)}\right] + \nearrow$$

Wskutek osiowości zadanie dwuwymiarowe ; przy czym

$u = -\frac{1}{\omega}\frac{\partial\psi}{\partial\omega} \qquad v = \frac{1}{\omega}\frac{\partial\psi}{\partial x}$

$$2\frac{\partial^2(\theta r)}{\partial r^2} + \frac{1}{\sin\varphi}\frac{\partial}{\partial\varphi}\left(\sin\varphi\frac{\partial\theta}{\partial\varphi}\right) +$$



$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y}{}^2 + \frac{\partial^2 V}{\partial z^2}$$

$\cancel{y = \omega\sin\varepsilon}$

$\cancel{z = \omega\cos\varepsilon}$

$\cancel{\frac{\partial V}{\partial y} = \frac{\partial V}{\partial\omega}\frac{\partial}{\partial z}} \quad \cancel{\frac{\partial^2 V}{\partial y^2}\,\partial^2}$

$$\frac{\partial V}{\partial y} = \frac{dV}{d\omega}\frac{\partial\omega}{\partial y} = \frac{\partial V}{\partial\omega}\frac{y}{\omega}$$

$$\frac{\partial^2 V}{\partial y^2} = \cancel{\frac{\partial V}{\partial\omega}}\frac{\partial}{\partial\omega}\cdot\frac{y}{\omega} = \frac{\partial^2 V}{\partial\omega^2}\frac{y^2}{\omega^2} - \frac{\partial V}{\partial\omega}\frac{y^2}{\omega^3}$$

$$\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2 V}{\partial\omega^2} - \frac{1}{\omega}\frac{\partial V}{\partial\omega}$$

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial\omega^2} - \frac{1}{\omega}\frac{\partial\theta}{\partial\omega} - \frac{1}{\omega}\frac{\partial\psi}{\partial\omega}\frac{\partial\theta}{\partial x} + \frac{1}{\omega}\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial\omega} = \Phi$$

Gdyby się wprowadziło współrzędne

$$\cancel{\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial r^2}} = \frac{\partial\theta}{\partial x} = \frac{\partial\theta}{\partial r}\frac{\partial r}{\partial x} + \cancel{\frac{\partial\theta}{\partial\omega}\frac{\partial\omega}{\partial x}} = \frac{x}{r}\frac{\partial\theta}{\partial r} = \frac{\sqrt{r^2\omega^2}}{r}\frac{\partial\theta}{\partial r}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{r}\frac{\partial\theta}{\partial r} - \frac{x^2}{r^3}\frac{\partial\theta}{\partial r} + \frac{x^2}{r^2}\frac{\partial^2\theta}{\partial r^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial\xi^2}\left(\frac{\partial\xi}{\partial x}\right)^2 + \frac{\partial^2 z}{\partial\eta^2}\left(\frac{\partial\eta}{\partial x}\right)^2 + 2\frac{\partial^2 z}{\partial\xi\partial\eta}\frac{\partial\xi}{\partial x}\frac{\partial\eta}{\partial x} + \frac{\partial z}{\partial\xi}\frac{\partial^2\xi}{\partial x^2} + \frac{\partial z}{\partial\eta}\frac{\partial^2\eta}{\partial x^2}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial\xi}\frac{\partial\xi}{\partial x} + \frac{\partial z}{\partial\eta}\frac{\partial\eta}{\partial x}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\theta}{\partial r^2}\left(\frac{\partial r}{\partial x}\right)^2 + \frac{\partial^2\theta}{\partial}$$

~~$\frac{\partial\theta}{\partial r} = \frac{\partial\theta}{\partial x}\frac{x}{r} + \frac{\partial\theta}{\partial\omega}\frac{\omega}{r}$~~

~~$\frac{\partial\theta}{\partial\omega} = \frac{\partial\theta}{\partial r}\frac{\omega}{r}$~~

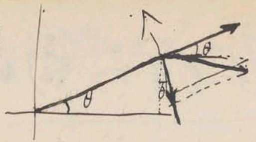
Gdyby było $x=0$ to

$$\frac{dx}{u\Phi} = \frac{dy}{v\Phi} = \frac{dz}{w\Phi} = d\theta$$

$r\frac{\partial\theta}{\partial r} + \theta$

$\cos\varphi\frac{\partial\theta}{\partial\varphi} + \sin\varphi\frac{\partial^2\theta}{\partial\varphi^2}$

$r^2\frac{\partial^2\theta}{\partial r^2} + 2\frac{\partial\theta}{\partial r}\cdot r$



$-u\sin\theta + q\cos\theta =$

$= -u\frac{\omega}{r} + q\frac{x}{r}$

$= +\frac{3}{2}\frac{ca}{r^3}\left(1-\frac{a^2}{r^2}\right)\frac{x^2\omega}{r} -$

$- c\frac{\omega}{r}\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$

$r^2\frac{\partial^2\theta}{\partial r^2} + 2r\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial\varphi^2} + \frac{\cos\varphi}{\sin\varphi}\frac{\partial\theta}{\partial\varphi} +$ ~~$f(r)\cos^2\varphi$~~ itd.

$$\rightarrow \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial\omega^2}\underbrace{-\frac{1}{\omega}\frac{\partial\psi}{\partial\omega}}\frac{\partial\theta}{\partial x} + \underbrace{\frac{1}{\omega}\left(\frac{\partial\psi}{\partial x}-1\right)}_{\varphi}\frac{\partial\theta}{\partial\omega} = \underset{\substack{\downarrow\\ u}}{\Phi}$$

$R = 1$ $\qquad P = -\frac{1}{\omega}\frac{\partial\psi}{\partial\omega}$

$S = 0$

$T = 1$

$k^2 + 1 = 0$

$k = \pm i$

$\frac{\partial\xi}{\partial x} = i\frac{\partial\xi}{\partial\omega} \qquad \frac{\partial\eta}{\partial x} = -i\frac{\partial\eta}{\partial\omega}$

$\xi = x + i\omega \qquad 4\frac{\partial\xi}{\partial y}\frac{\partial\eta}{\partial\omega} = +4$

$\eta = x - i\omega$

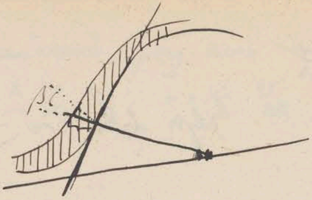
$\| r^2 = \xi\eta \;\|\; x = \frac{\xi+\eta}{2} \|$

$\frac{\partial^2\eta}{\partial x^2}$

$$\frac{\partial^2\theta}{\partial\xi\partial\eta} \pm \underbrace{\frac{1}{4\omega}\frac{\partial\psi}{\partial\omega}}_{L}\frac{\partial\theta}{\partial\xi} \pm \underbrace{\frac{1}{4\omega}\left(1-\frac{\partial\psi}{\partial x}\right)}_{M}\frac{\partial\theta}{\partial\eta} = +\frac{\Phi}{4}$$

Czy spełnione: $LM + \frac{\partial L}{\partial\xi} = 0$?

$$\psi = -\frac{c}{2}\left(1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right) r^2 \sin^2\vartheta$$



$\frac{dr}{r\,d\varphi} = \operatorname{ctg}\alpha$

$$\delta\psi = \frac{\partial\psi}{\partial r}\,\delta r + \frac{\partial\psi}{\partial\vartheta}\,\delta\vartheta$$

$$0 = \frac{\partial\psi}{\partial r}\,dr + \frac{\partial\psi}{\partial\vartheta}\,d\vartheta$$

$$\frac{1}{r}\frac{dr}{d\vartheta} = \frac{-\frac{\partial\psi}{\partial\vartheta}}{r\frac{\partial\psi}{\partial r}} = \operatorname{tg}\beta$$

$$\delta = \frac{\partial\psi}{\partial r}\,\delta r \,.\, \cos\beta$$

$$\delta\psi\int_0^\infty \omega\,\frac{\partial\omega}{\partial\psi}\,\Phi(x,\omega)\,dx = \delta\psi\int \omega\,\frac{\Phi}{\frac{\partial\psi}{\partial\omega}}\,dx$$

$$= \delta\psi\int \frac{1}{2}\,\frac{\Phi(x,\omega)}{\frac{\partial\psi}{\partial(\omega^2)}}\,dx$$

$$\psi = -\frac{c}{2}\left(1 - \frac{3}{2}\frac{a}{\sqrt{x^2+\omega^2}} + \frac{1}{2}\frac{a^3}{(x^2+\omega^2)^{3/2}}\right)\omega^2$$

$$\frac{\partial\psi}{\partial\omega^2} = -\frac{c}{2}\left[1 - \frac{3}{2}\frac{a}{\sqrt{x^2+\omega^2}} + \frac{1}{2}\frac{a^3}{(x^2+\omega^2)^{3/2}}\right]$$

$$-\frac{c}{2}\,\omega^2\left[\frac{3}{4}\frac{a}{\sqrt{(x^2+\omega^2)}^3} - \frac{3}{4}\frac{a^3}{(x^2+\omega^2)^{5/2}}\right]$$

$$= -\frac{c}{2}\left[1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3} + \frac{3}{4}\frac{a\,\omega^2}{r^3} - \frac{3}{4}\frac{a^3\omega^2}{r^5}\right]$$

$\omega^2 = r^2 - x^2$

$$= -\frac{c}{2}\Big[\underbrace{1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3} + \frac{3}{4}\frac{a}{r} - \frac{3}{4}\frac{a x^2}{r^3} - \frac{3}{4}\frac{a^3}{r^3} + \frac{3}{4}\frac{a^3x^2}{r^5}}_{1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3} - \frac{3}{4}\frac{a x^2}{r^3} + \frac{3}{4}\frac{a^3x^2}{r^5} = -\frac{u}{2}}\Big]$$

$$\Phi = \frac{q}{8}\frac{c^2a^2}{r^4}\left\{\underbrace{\frac{2a^4}{r^4} + 6 - \frac{6\omega^2}{r^2} - \frac{12a^2}{r^2} + \frac{12a^2\omega^2}{r^4} + 4\frac{a^4}{r^4} - \frac{4a^4\omega^2}{r^6}}_{\left\{6 - 12\frac{a^2}{r^2} + 6\frac{a^4}{r^4} - \left(6 - \frac{12a^2}{r^2} + \frac{4a^4}{r^4}\right)\frac{\omega^2}{r^2}\right\}}\right\}$$

$$dx = \frac{x}{r}dr = \frac{dr}{r}\sqrt{r^2-\omega^2}$$

$$\frac{q}{8}c^2a^2\frac{1}{2}\int_\rho^\infty dr \frac{\frac{1}{r^5}\left[6\left(1-\frac{2a^2}{r^2}+\frac{a^4}{r^4}\right) + \left(6-\frac{12a^2}{r^2}+\frac{4a^4}{r^4}\right)\frac{2\psi}{cr^2\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right)}\right]}{-\frac{c}{2}\left[1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}+\frac{3}{4}\frac{a}{r^3}\left(1-\frac{a^2}{r^2}\right)\frac{2\psi}{c\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right)}\right]}$$

$$i\sqrt{r^2 - \frac{2\psi}{c\left(1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right)}}$$

$\psi = \text{const}$

Jeśli się uda ~~[illegible]~~ znaleźć rozwiązanie

~~(B). $\Delta^2\theta + \left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) = 0$~~

wtedy potrzeba tylko znaleźć taką funkcję φ : $\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2 = \psi$

starczy ~~[illegible]~~ $\varphi = \log\vartheta$ ~~i stawić $\vartheta = e^\varphi$~~

$\frac{\partial\varphi}{\partial x} = \frac{1}{\vartheta}\frac{\partial\vartheta}{\partial x}$ $\vartheta = e^\varphi$

$$\Delta^2\theta + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = 0$$

~~$\theta = e^{\int\frac{\varphi}{2}dx}$~~ ~~jeśli to możliwe.~~

~~Jako~~ Partykularne rozwiązanie $\Delta^2\theta = 0$ $u\frac{\partial\theta}{\partial x} + \dots = 0$

$$\theta = \frac{a}{r} + b\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + c\frac{\partial}{\partial x^2}\left(\frac{1}{r}\right) + \dots$$

$$\theta = \frac{a}{r} + b\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + c\frac{\partial}{\partial x^2}\left(\frac{1}{r}\right) + \text{---}$$

$$\frac{\partial\theta}{\partial x} = a\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + b\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) + \cdots = \frac{x}{r}\overbrace{\left[a + b(=) + \cdots\right]}^{F_1} + F_2 \text{ --}$$

$$\frac{\partial\theta}{\partial y} = a\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + b\frac{\partial}{\partial x}\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + \cdots = \frac{y}{r}\left[\cdots\cdots = \right]$$

$$\frac{\partial\theta}{\partial z} = a\frac{\partial}{\partial z}\left(\frac{1}{r}\right) + \text{---} \qquad = \frac{z}{r}\left[\cdots\cdots = \right]$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = -\,\cancel{\frac{3}{4}\frac{ca}{r^2}\left(1-\frac{a^2}{r^2}\right)xF_1 + G_1 = f_1(x,z) = \theta}$$

$$= \frac{cx}{r}\left(1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)F_1 + F_2\, u$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = -\frac{x}{r^3} \qquad \frac{\partial}{\partial y}\left(\frac{1}{r}\right) = -\frac{y}{r^3}$$

$$\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) = +\frac{3x^2}{r^5} - \frac{1}{r^3} \qquad \frac{\partial^2}{\partial y\,\partial x}\left(\frac{1}{r}\right) = +\frac{3xy}{r^5}$$

$$\frac{\partial^3}{\partial x^3} = -\frac{15x^3}{r^7} + \frac{3x}{r^5} + \frac{6x}{r^5} \quad\Big\|\quad \frac{\partial^3}{\partial x^2\,\partial y} = -\frac{15x^2y}{r^7} + \frac{3y}{r^5}$$

$$F_1 = \left[-\frac{ax}{r^3} + b\left[\frac{3x^2}{r^5} - \frac{1}{r^3}\right] + c\left[-\frac{15x^3}{r^7} + \frac{3x}{r^5}\right] + \cdots\right]$$

$$F_2 = -\frac{b}{r^3} + \frac{6cx}{r^5}$$

$$\theta = \varphi\psi$$

$$\frac{\partial\theta}{\partial x} = \frac{\partial\varphi}{\partial x}\psi + \varphi\frac{\partial\psi}{\partial x}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\varphi}{\partial x^2}\psi + 2\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial x} + \varphi\frac{\partial^2\psi}{\partial x^2}$$

$$\Delta^2\varphi\cdot\psi + 2\left[\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial\varphi}{\partial z}\frac{\partial\psi}{\partial z}\right] + \varphi\Delta^2\psi + \psi\left[u\frac{\partial\varphi}{\partial x} + \cdots\right] + \varphi\left[\cancel{u\frac{\partial\psi}{\partial x} + \text{--}}\right] = 0$$

Będzie spełniać A jeśli ψ spełnia B podczas gdy φ:

$$\Delta^2\varphi + 2\left[\frac{\partial\varphi}{\partial x}\frac{\partial \lg\psi}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial \lg\psi}{\partial y} + \frac{\partial\varphi}{\partial z}\frac{\partial \lg\psi}{\partial z}\right] + u\frac{\partial\varphi}{\partial x} + \dots = 0$$

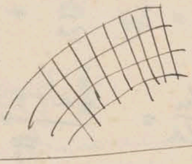
zatem jeżeli się da znaleźć taki φ ażeby

$$2\frac{\partial\varphi}{\partial x}\frac{\partial \lg\psi}{\partial x} + \dots = \Phi$$

$$\iint \quad \Delta^2\theta + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = 0 \qquad dv$$

$$\iint \frac{\partial\theta}{\partial n}\,dS + \iint \underbrace{\left[u\theta\,dS\cos nx + v\theta\,dS\cos ny + \dots\right]} \cancel{- \iiint \theta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)dv}$$

$$\iint \frac{\partial\theta}{\partial n}\,dS + \iint \theta\, v_n\, dS = 0$$



Zastosujmy to do rurek prądu ciepła

$$\left.\begin{aligned} &\Delta^2\theta + u\frac{\partial\theta}{\partial x} + \dots = 0 \\ &\Delta^2\theta' + u\frac{\partial\theta'}{\partial x} + \dots = 0 \end{aligned}\right\} \quad \cancel{\Delta^2(\theta+\theta')=0}$$

$$\Delta^2(\theta-\theta') + u\frac{\partial(\theta+\theta')}{\partial x} + v\frac{\partial(\theta+\theta')}{\partial y} + \dots = 0$$

$$\Delta^2(\theta+\theta') + u\frac{\partial(\theta-\theta')}{\partial x} + v\frac{\partial(\theta-\theta')}{\partial y} + \dots = 0$$

Jednowymiarowe zadanie zupełne

$$\cancel{u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div} + \mu\Delta^2}$$

$$\text{I). } \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$$

$$\text{II). } \frac{\partial(\rho u)}{\partial x} = 0 \qquad \rho u = \text{const} = b$$

$$\text{III). } kp\frac{\partial u}{\partial x} + u\frac{\partial p}{\partial x} = (k-1)\,\mu\,\frac{4}{3}\left(\frac{\partial u}{\partial x}\right)^2 \;\Big\|\; + \kappa\frac{\partial^2\theta}{\partial x^2}$$

$$\text{I). } b\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$$

$$bu + p = \frac{4\mu}{3}\frac{\partial u}{\partial x} + a$$

$$\frac{du}{dx} = \frac{bu + p - a}{\frac{4\mu}{3}}$$

$$\text{II). } kp\,\cancel{\frac{\partial u}{\partial x}} + u\frac{dp}{du}\cancel{\frac{\partial u}{\partial x}} = (k-1)\frac{4\mu}{3}\left(\frac{\partial u}{\partial x}\right)^{\cancel{2}}$$

$$\cancel{kp\frac{(bu + p - a)}{\frac{4\mu}{3}} + u\frac{dp}{dx}}$$

$$kp + u\frac{\partial p}{\partial u} = (k-1)(bu + p - a)$$

$$u\frac{dp}{du} = \underbrace{(k-1)bu - p - (k-1)a}_{= z}$$

$$p = (k-1)bu - (k-1)a - z$$

$$\frac{dp}{du} = (k-1)b\frac{du}{du} - \frac{dz}{du}$$

$$u(k-1)b - u\frac{dz}{du} = z$$

$$\frac{dz}{du} + \frac{z}{u} = (k-1)b$$

$$\underbrace{u\,dz + z\,du}_{} = (k-1)\,b\,du\,u$$

$$zu = (k-1)b\frac{u^2}{2} + c$$

$$z = (k-1)\frac{bu}{2} + \frac{c}{u}$$

$$\frac{dp}{du} < 0 \qquad z < 0$$

z dla $c < 0$

~~$\frac{u(k-1)b}{2} \pm \frac{c}{u} = \frac{(k-1)bu}{2} \pm \frac{c}{u} + (k-1)a - ka + a$~~

$$p = (k-1)\,bu - (k-1)a - (k-1)\frac{bu}{2} - \frac{c}{u}$$

$$p = \frac{(k-1)b}{2}\,u - \frac{c}{u} - (k-1)a$$

$$\frac{4\mu}{3}\,\frac{du}{dx} = bu - a + \frac{(k-1)b}{2}\,u - \frac{c}{u} - (k-1)a$$

$$= (k+1)\frac{b}{2}\,u - \frac{c}{u} - ka$$

$$\frac{\frac{4\mu}{3}\,u\,du}{(k+1)\frac{b}{2}u^2 - kau - c} = dx$$

---

wprowadzajmy zamiast a i c inne stałe:

$$A = \frac{ak}{(k+1)\frac{b}{2}} \qquad C = \frac{-c}{(k+1)\frac{b}{2}} \qquad a = \frac{(k+1)}{k}\frac{b}{2}A \qquad c = -(k+1)\frac{b}{2}C$$

$$\frac{u\,du}{u^2 - Au + C} = \frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\,dx = \underbrace{\frac{(u - \frac{A}{2})\,du}{u^2 - Au + C}}_{\frac{1}{2}\log(u^2 - Au + C)} + \frac{\frac{A}{2}\,du}{u^2 - Au + C}$$

prawdopod. $A > 0$

$C > 0$

$$\int \frac{du}{u^2 - Au + C} = \frac{2}{\sqrt{4C - A^2}}\,\operatorname{arctg}\frac{2u - A}{\sqrt{4C - A^2}} \qquad 4C > A^2$$

$$= \frac{1}{\sqrt{A^2 - 4C}}\,\log\left\{\frac{2u - A - \sqrt{}}{2u - A + \sqrt{}}\right\} \qquad A^2 > 4C$$

$$= \frac{1}{\sqrt{}}\,\log\frac{\left(u - \frac{A}{2} - \sqrt{\left(\frac{A}{2}\right)^2 - C}\right)\left(u - \frac{A}{2} + \sqrt{}\right)}{\left(u - \frac{A}{2} + \sqrt{}\right)^2}$$

$$= \frac{1}{\sqrt{}}\left[\log\left(u^2 - uA + \cancel{\frac{A^2}{4}} - \cancel{\frac{A^2}{4}} + C\right) - 2\log\left(u - \frac{A}{2} + \sqrt{}\right)\right]$$

W przypadku logarytmicznym: $u^2 - Au + C > 0 \qquad \frac{A^2}{4} > C$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\, x + \text{const} = \left[\frac{1}{2} + \frac{\frac{A}{2}}{\sqrt{A^2-4C}}\right] \log\left(u - \frac{A}{2} - \sqrt{\frac{A^2}{4} - C}\right)$$

$$\boxed{(1a)} \qquad + \left[\frac{1}{2} - \frac{\frac{A}{2}}{\sqrt{A^2-4C}}\right] \log\left(u - \frac{A}{2} + \sqrt{\frac{A^2}{4} - C}\right)$$

arctg : $u^2 - Au + C > 0 \qquad \frac{A^2}{4} < C$

$$\boxed{(1b)} \quad \frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\, x + \text{const} = \frac{1}{2} \log(u^2 - Au + C) + \frac{A}{\sqrt{4C-A^2}} \operatorname{arctg} \frac{2u-A}{\sqrt{4C-A^2}}$$

$$\boxed{(2)} \quad p = \frac{b}{2}\left\{(k-1)u + (k+1)\frac{C}{u} - \frac{(k-1)(k+1)}{k} A\right\} \qquad \Big\|$$ zupełnie nie zawiera μ!

$$\boxed{(3)} \quad \theta = \frac{pu}{R\rho u} = \frac{pu}{Rb} = \frac{1}{2R}\left\{(k-1)u^2 - \frac{(k^2-1)}{k}\underline{A}u + (k+1)C\right\} \Big\|$$ niezależnie od b!

$$\frac{(1)}{(2)} : \quad \frac{(k+1)}{\frac{4\mu}{3}} \frac{b}{2}\, x\, p = \left[f_k(u, A, C) - \text{const}\right] \frac{b}{2}\, \varphi_k(u, A, C)$$

$$\left(\frac{1}{2}\right) \qquad p x = \frac{4\mu}{3} \frac{1}{k+1} \left[f(u\, A C) - \text{const}\right] \varphi(u\, AC)$$

Jeśli np. θ dane przy dwóch prędkościach to można obliczyć $\underline{A}$, $\underline{C}$ oprócz tego wartość px przy pewnym u to wynika wartość const

~~N.p. dla $x=0$ $u = u_0$ $\theta = \theta_0$~~
~~dla $u = u_1$ $\theta = \theta_1$~~ ~~$\Big\}$ z tych zapomocą (3) otrzymuje się A i C~~

~~zapomocą (1): const~~

~~$=$~~

~~dalej: równanie $(\frac{1}{2})$: px = znana funkcja z~~ ~~$R\Theta\rho x$~~

~~$$R\Theta\rho x = \frac{4}{3}\mu\frac{1}{k+1}(f-\mathrm{const})\varphi$$~~

~~zatem $\rho = \frac{\frac{4}{3}\mu\frac{1}{k+1}(f-\mathrm{const})\varphi}{R\Theta x} = \frac{\frac{4}{3}\mu\frac{1}{k+1}(f-\mathrm{const})}{\frac{1}{2}u x}$~~

~~zatem $b = \rho u = \frac{\frac{4}{3}\mu\frac{1}{k+1}}{\frac{1}{2}x}(f-\mathrm{const})$~~

$\frac{dp}{dx}$ musi w każdym razie być wszędzie ujemne, inaczej by ruch nie nastąpił

$$\frac{dp}{dx} = \frac{dp}{du}\frac{du}{dx} = \frac{b}{2}\left\{(k-1) - \frac{(k+1)C}{u^2}\right\}\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\frac{u^2-Au+C}{u}$$

$$= \frac{b^2}{4}\frac{k+1}{\frac{4\mu}{3}}\frac{\left[k-1-\frac{(k+1)C}{u^2}\right]\left[u^2-Au+C\right]}{u} \qquad <0$$

więc albo $k-1 < \frac{(k+1)C}{u^2}$
albo $u^2 - Au + C < 0$ $\Big\}$ stanowi w każdym razie górną granicę dla u

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Próba: wstawić oba wyrażenia do I II III

~~$$b\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\frac{u^2-Au+C}{u} + \frac{b^2}{4}\frac{(k+1)}{\frac{4\mu}{3}}\frac{\left[(k-1)-\frac{(k+1)C}{u^2}\right]\left[u^2-Au+C\right]}{u} =$$~~

~~$$\frac{4\mu}{3}\underbrace{\left\{\frac{2u-A}{u} - \frac{u^2-Au+C}{u^2}\right\}}_{=}\frac{u^2-Au+C}{u} \longrightarrow \frac{\cancel{2}u^2-\cancel{Au}-\cancel{u^2}+\cancel{Au}-C}{u^2}$$~~

$$\frac{\frac{b^2}{2}(k+1)}{4\mu/3} + \frac{\frac{b^2}{4}(k^2-1)}{4\mu/3} - \frac{\frac{b^2}{4}(k+1)^2}{4\mu/3}\,\frac{C}{u^2} = 4\mu/3\left(1-\frac{C}{u^2}\right)$$

Spełnione tylko jeśli:

$$\frac{b^2}{2}(k+1) + \frac{b^2}{4}(k^2-1) = \left(4\mu/3\right)^2 = \frac{b^2}{4}\left[k^2-1 + 2(k+1)\right] = \frac{b^2}{4}\left[k^2+2k+1\right] = \frac{b^2}{4}(k+1)^2$$

$$\frac{b^2}{4}(k+1)^2 = \left(4\mu/3\right)^2$$

Identyczne !!

$$\frac{b(k+1)}{2} = 4\mu/3$$

$$b = \frac{8\mu}{3(k+1)}$$

Trzy równania przyjmują kształt:

(1)a $\quad x + \mathrm{const} = \left[\frac{1}{2} + \frac{1}{\sqrt{}}\right] \dots$

$=$

(2) $\quad p = \frac{4\mu}{3(k+1)}\left\{(k-1)u + (k+1)\frac{C}{u} - \frac{(k-1)(k+1)}{k}A\right\}$

(3) $\quad \theta = \frac{1}{2R}\left\{(k-1)u^2 + (k+1)C - \frac{(k^2-1)}{k}Au\right\}$

$$\rho = \frac{b}{u} = \frac{8\mu}{3(k+1)}\frac{1}{u}$$

$u = \frac{8\mu}{3(k+1)}\frac{1}{\rho}$ Np. $\mu = 0{\cdot}00019$ $\quad u = \frac{1}{10}$?

$\rho = 0{\cdot}0012$

Wstawiając 1a — 3 w III:

$$k\frac{b}{2}\left\{(k-1)u+(k+1)\frac{C}{u}-\frac{(k-1)(k+1)}{k}A\right\}+\cancel{u}\frac{b^2}{4}\frac{k+1}{\frac{4\mu}{3}}\frac{\left[k-1-\frac{(k+1)C}{u^2}\right]\left[u^2-Au+C\right]}{\cancel{u}}=$$

$$=(k-1)\frac{4\mu}{3}\left[\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\frac{u^2-Au+C}{u}\right]^2$$

$$k\frac{4\mu}{3}\left\{(k-1)u+(k+1)\frac{C}{u}-\frac{(k-1)(k+1)}{k}A\right\}+\frac{b}{2}(k+1)\left\{k-1-\frac{(k+1)C}{u^2}\right\}\left\{u^2-Au+C\right\}=$$

$$=(k-1)(k+1)^2\frac{b}{2}\left\{u-A+\frac{C}{u}\right\}^2$$

$$\frac{4\mu}{3}k\left[(k-1)u\cancel{+u}-\frac{(k-1)(k+1)}{k}A+(k+1)\frac{C}{u}\right]=\frac{b}{2}(k+1)\left[u-A+\frac{C}{u}\right]\Big\{(k+1)\left(u-A+\frac{C}{u}\right)-$$

$$\underbrace{-u(k-1)+\frac{(k+1)C}{u}\Big\}}$$

$$=\frac{b}{2}(k+1)\left[u-A+\frac{C}{u}\right]\left[2u-(k+1)A+\frac{2(k+1)C}{u}\right]$$

Nie może być spełnione!

I). ~~$\varrho u\frac{du}{dp}+1=\frac{4\mu}{3}\frac{d^2u}{dx\,dp}$~~ ‖ II). $\underbrace{kp\frac{du}{dp}+u=(k-1)\frac{4\mu}{3}\frac{du}{dx}\frac{du}{dp}}$

Spełnione

~~II). [illegible]~~

I). $b\frac{du}{dp}+1=\frac{4\mu}{3}\frac{d}{dp}\left(\frac{du}{dx}\right)$

$$bu+p=\frac{4\mu}{3}\frac{du}{dx}+a$$

Spełnione

$$2\cancel{bu}+\frac{b}{2}\left\{\cancel{(k+1)}p+\cancel{(k+1)}\frac{C}{u}-\frac{(k-1)\cancel{(k+1)}}{k}A\right\}=\cancel{\frac{4\mu}{3}}\frac{\cancel{(k+1)}\frac{b}{2}}{\cancel{\frac{4\mu}{3}}}\frac{\cancel{u^2}-Au+C}{u}+\frac{\cancel{k+1}}{k}\frac{b}{2}A)$$

II).

II).

$$\left\{k\left\{\cancel{(k-1)u+(k+1)\frac{C}{u}-\frac{(k-1)(k+1)}{k}A}\right\}-(k-1)\,(k+1)\left[\cancel{u-A+\frac{C}{u}}\right]\right\}$$

$$=\cancel{u\,\frac{k}{2}\left\{(k-1)-(k+1)\frac{C}{u^2}\right\}}$$

Wstawiając w III:

$$k\left\{(k-1)u+(k+1)\frac{C}{u}-\frac{(k-1)(k+1)}{k}A\right\}\cancel{(k+1)\frac{u^2-Au+C}{u}}+u\,\cancel{(k+1)}\left[k-1-\frac{(k+1)C}{u^2}\right]\cancel{\frac{u^2-Au+C}{u}}$$

$$=(k-1)\,(k+1)\left(\frac{u^2-Au+C}{u}\right)$$

$$\cancel{(k^2-k)u}+\cancel{(k-1)u}-\cancel{(k^2-1)u}+\cancel{k(k+1)\frac{C}{u}}-\cancel{(k+1)\frac{C}{u}}-\cancel{(k^2-1)\frac{C}{u}}+$$

$$\cancel{-(k-1)(k+1)A}+\cancel{(k-1)(k+1)A}=0$$

~~k^2-~~

I). $$\cancel{k}\,\frac{\cancel{(k+1)}}{\cancel{k}}\,2\,\frac{u^2-Au+C}{\cancel{u}}+\cancel{k+1}\,\frac{\left[k-1-\frac{(k+1)C}{u^2}\right]\left[u^2-Au+C\right]}{\cancel{u}}=$$

$$=(k+1)\,\frac{u^2-C}{u^2}\,\frac{u^2-Au+C}{\cancel{u}}$$

$$\underbrace{2+k-1}_{k+1}-\frac{(k+1)C}{u^2}=(k+1)\left(1-\frac{C}{u^2}\right)$$

Spełniony

~~Jeżeli możliwe bardzo małe u~~ to tam będzie

i zaczynając ~~[illegible]~~ z tamtąd rachować x:

$\lim u$ ..

~~$p = \frac{b}{2}(k+1)\frac{C}{u}$~~

~~(1a) nie możliwe~~

~~(1b): $\cancel{(k+1)\frac{b}{2}\frac{4}{3}} x +$ const $= \frac{1}{2}\lg C + \frac{A}{\sqrt{4C-A^2}} \operatorname{arctg}\frac{-A}{\sqrt{4C-A^2}}$~~

~~$\theta = \frac{(k+1)C}{2R}$~~

~~$\rho = \frac{p}{R\theta} = \frac{b}{u}$~~

Przy (1a) najmniejsza możliwa prędkość:

$$u > \frac{A}{2} + \sqrt{\frac{A^2}{4} - C}$$

zatem:

$$u - \frac{A}{2} > \sqrt{\frac{A^2}{4} - C}$$

$$u^2 - Au + \frac{A^2}{4} > \frac{A^2}{4} - C$$

$$u^2 - Au + C > 0$$

największa możliwa z powodu że $\frac{dy}{dx}$ ~~[illegible]~~ < 0:

$$u^2 < \frac{(k+1)C}{k-1}$$

$$Au - C < u^2 < \frac{k+1}{k-1}C$$

$$< \left(1 + \frac{2}{k-1}\right)C$$

$$Au < \underbrace{\left(\frac{k+1}{k-1} + 1\right)}C$$

$$Au < \frac{2k}{k-1}C$$

$$u < \left(\frac{2k}{k-1}\right)\frac{C}{A}$$

~~[illegible]~~

$$\frac{C}{A} < \frac{A}{4}$$

$$u < \frac{k}{2(k-1)}A$$

[illegible]

$$u^2 < \frac{k+1}{k-1}\frac{A^2}{4}$$

$$u < \frac{A}{2}\sqrt{\frac{k+1}{k-1}}$$

~~$\frac{k}{k-1} = \frac{1{\cdot}4}{0{\cdot}4} = \frac{7}{2} = 3{\cdot}5$~~

~~$\sqrt{\frac{k+1}{k-1}} = \sqrt{\frac{2{\cdot}4}{0{\cdot}4}} = \sqrt{6} = 2{\cdot}5$~~

Przy (1a) warunek: $\frac{A^2}{4} > C$

$$\theta < \frac{1}{2R}\left\{(k-1)u^2 - \frac{(k^2-1)}{k}Au + (k+1)\frac{A^2}{4}\right\}$$

$$p < \frac{b}{2}\left\{(k-1)u + \frac{(k+1)}{u}\frac{A^2}{4} - \frac{(k-1)(k+1)}{k}A\right\}$$

$$p < \frac{b}{2}\left\{\frac{kA}{2} - \frac{k^2-1}{k}A + \frac{(k+1)A^2}{4(Au-C)}\right\}$$

$$\theta < \frac{1}{2R}\left\{\cancel{(k-1)}\frac{A^2}{4}\frac{k+1}{\cancel{k-1}} + (k+1)\frac{A^2}{4} - \frac{(k^2-1)}{k}Au\right\}$$

~~$$< \frac{1}{2R}(k+1)\left\{\frac{A^2}{2} - \frac{k-1}{k}Au\right\}$$~~

~~$u^2 - Au > C$~~

~~$\frac{A^2}{4} > C > Au - u^2$~~

~~$\frac{A^2}{4} > Au - u^2$~~

~~$u^2 - Au + \frac{A^2}{4} > 0$~~ ~~$\left(u - \frac{A}{2}\right)^2 > 0$~~

W (1a) i (1b) warunek konieczny $u^2 - Au + C > 0$

zatem $\frac{du}{dx} > 0$ prędkość wzmaga się z długością x

zatem aby były spełnione $\frac{dp}{dx} < 0$

musi być $\frac{dp}{du} < 0$ więc $u^2 < \frac{k+1}{k-1} C$

$$\text{const} = [\quad] \log\left(u_1 - \frac{A}{2} - \sqrt{}\right) + [\quad] -$$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}} x = \left[\frac{1}{2} + \frac{1}{\sqrt{A^2 - 4C}}\right] \log\left(\frac{u - \frac{A}{2} - \sqrt{\frac{A^2}{4} - C}}{u_1 - \frac{A}{2} - \sqrt{\frac{A^2}{4} - C}}\right) + [\quad - \quad]$$

$$\frac{b}{2}(k+1) C = k p_0 \alpha \qquad C = \frac{k p_0 \alpha}{(k+1)\frac{b}{2}}$$

$$\frac{b}{2}\frac{(k+1)}{k} A = p_0 \qquad A = \frac{k p_0}{(k+1)\frac{b}{2}}$$

$$\frac{1}{2}\log\frac{u-A}{u_1-A} = \frac{1}{2}\log\frac{1-\frac{u}{A}}{1-\frac{u_1}{A}} = \frac{1}{2}\frac{u_1-u}{A} = \frac{1}{2}\frac{u_1-u}{k p_0}(k+1)\frac{b}{2}$$

$$\frac{1}{2}\log \frac{\frac{u}{A} + \frac{A}{2}\left[-1 + \left(1 - \frac{4C}{A^2}\right)^{\frac{1}{2}}\right]}{\frac{u_1}{A} + \frac{A}{2}\left[-1 + \left(1 - \frac{4C}{A^2}\right)^{\frac{1}{2}}\right]} = \frac{1}{2}\log\frac{\frac{u}{A} + \frac{A}{2}\left[-\cancel{1} + \cancel{1} - \frac{2C}{A^2}\right]}{\frac{u_1}{A} + \frac{A}{2}\left[\quad - \frac{2C}{A^2}\right]} =$$

$$= \frac{1}{2}\log\frac{\frac{u}{A} - \frac{C}{A^2}}{\frac{u_1}{A} - \frac{C}{A^2}} = \frac{1}{2}\log\frac{u - \frac{C}{A}}{u_1 - \frac{C}{A}} = \frac{1}{2}\log\frac{u-\alpha}{u_1-\alpha}$$

Zatem $$\frac{(k+1)\frac{b}{2}x}{\frac{4\mu}{3}} = \frac{1}{2}\left[\frac{(u_1-u)(k+1)}{k p_0}\frac{b}{2} + \log\frac{u-\alpha}{u_1-\alpha}\right]$$

Co jeśli $u^2 - Au + C < 0$ Jeśli $\frac{C > 0}{A > u + \frac{c}{4}}$

$$\int \frac{u\,du}{u^2 - Au + C} = \cancel{\frac{(u - \frac{A}{2})\,du}{-(u^2 - Au + C)} + \frac{\frac{A}{2}\,du}{\cdots}} = -\int \frac{u\,du}{Au - u^2 - C}$$

$$= \cancel{-\frac{1}{2}\lg\left[-(u^2 - Au + C)\right]}$$

$$= \int \frac{(\frac{A}{2} - u)\,du}{Au - u^2 - C} - \underbrace{\frac{A}{2}\frac{du}{Au - u^2 - C}}$$

$$= \lg(Au - u^2 - C) - \underbrace{\frac{A}{2}\int \frac{du}{\frac{A^2}{4} - C - (u - \frac{A}{2})^2}}$$

$$\frac{A}{2} \lg \frac{\sqrt{\frac{A^2}{4} - C} - u - \frac{A}{2}}{\sqrt{\quad} + \cdots}$$

W granicznym przypadku, gdzie $C = \frac{A^2}{4}$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}} x + \text{const} = \lg\left(u - \frac{A}{2}\right) - \frac{A}{2(u - \frac{A}{2})}$$

dla $x=0$: u bardzo małe $= u_0$

$\theta = \theta_0$

$p = p_0$

$$\theta_0 = \frac{(k+1)C}{2R}$$ ~~$p_0 = \frac{(k+1)C\,b}{2u_0}$~~

$$C = \frac{2R\theta_0}{k+1} \qquad b = \frac{p_0 u_0}{R\theta_0}$$

~~$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}$~~ $$\text{const} = \frac{1}{2}\log C + \frac{A}{\sqrt{4C-A^2}}\operatorname{arctg}\frac{-A}{\sqrt{\ }}$$

Przy wzrastającem u :

arctg może tylko dojść do $\frac{\pi}{2}$ gdy u od 0 do ∞ ; podczas gdy log. wzrasta stale z u

zatem dla wielkich u będzie już tylko:

~~$x = \frac{\frac{4\mu}{3}}{\frac{b}{2}(k+1)}\log u$~~

~~$u = e^{\frac{\frac{b}{2}(k+1)x}{\frac{4\mu}{3}}}$~~

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}x = \frac{1}{2}\log\frac{u^2-Au+C}{C} + \frac{A}{\sqrt{4C-A^2}}\left[\underbrace{\operatorname{arctg}\frac{2u-A}{\sqrt{\ }} + \operatorname{arctg}\frac{A}{\sqrt{\ }}}\right]$$

$$\operatorname{tg}(\varphi+\psi) = \frac{\operatorname{tg}\varphi+\operatorname{tg}\psi}{1-\operatorname{tg}\varphi\operatorname{tg}\psi}$$

$$\varphi+\psi = \operatorname{arctg}\frac{\operatorname{tg}\varphi+\operatorname{tg}\psi}{1-\operatorname{tg}\varphi\operatorname{tg}\psi}$$

$$= \operatorname{arctg}\frac{\frac{2u}{\sqrt{\ }}}{1-\frac{(2u-A)A}{4C-A^2}} = \operatorname{arctg}\frac{\frac{2u}{\sqrt{\ }}}{\frac{4C-2uA}{4C-A^2}}$$

$$= \operatorname{arctg}\frac{u\sqrt{4C-A^2}}{2C-uA}$$

Dla u bardzo dużego:

$$\frac{u}{\sqrt{C}} = e^{\cdots -}\cdot e^{\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}x}$$

dla małych u potem przybliżenie:

~~$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}x = \frac{A}{\sqrt{4C-A^2}}\cdot\frac{\sqrt{4C-A^2}\cdot u}{2C} = \frac{Au}{2C}$~~

~~$\frac{(k+1)\,p_0u_0}{\frac{4\mu}{3}\,2R\theta_0}\,x = \frac{Au\,(k+1)}{2R\theta_0}$~~

~~x~~ ~~$u = \frac{p_0u_0}{\frac{4\mu}{3}A}x$~~

$$\frac{du}{dx} = \frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\,\frac{u^2-Au+C}{u}\Big|_{u=u_0} = \frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\,\cancel{u}\cdot\frac{C}{u_0} = \frac{(k+1)\frac{p_0u_0}{R\theta_0}\,\frac{R\theta_0}{k+1}}{\frac{4\mu}{3}} = \frac{p_0u_0}{\frac{4\mu}{3}}$$

więc tam będzie: $u = u_0 + \frac{p_0u_0}{\frac{4\mu}{3}}x$

$$\frac{dp}{dx}\Big|_{u=u_0} = \frac{\cancel{b}}{\cancel{2}}\,\frac{\cancel{k+1}}{\frac{4\mu}{3}}\,\frac{\left[\cancel{kx}+\cancel{\frac{2R\theta_0}{k+1}}\right]\left[\cancel{u_0^2}-\cancel{Au_0}+\cancel{\frac{2R\theta_0}{k+1}}\right]}{u_0}\left(\frac{p_0\cancel{u_0}}{\cancel{R\theta_0}}\right)^2$$

$$= [illegible] = \frac{p_0^2}{\frac{4\mu}{3}u_0}$$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}x + \text{const} = \frac{1}{2}\log\left(u^2-Au+C\right) + \frac{A}{2}\,\frac{1}{\sqrt{A^2-4C}}\,\log\left[\frac{A+\sqrt{A^2-4C}-2u}{A-\sqrt{A^2-4C}-2u}\right]$$

Gdy u od małych wartości począwszy | dla których: $\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}x + \text{const} = \frac{1}{2}\log C + \frac{A}{2}\frac{1}{\sqrt{\ }}\log\frac{A+\sqrt{\ }}{A-\sqrt{\ }}$

zbliży się do $u = \frac{A}{2} - \sqrt{\frac{A^2}{4}-C}$ to równanie owe przyjmie kształt:

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}x + \text{const} = -\infty + \Delta$$

wtedy będzie $\frac{du}{dx}=0$ rozw— dla u

gdy A, C wielkie:

$$u = \frac{A}{2}\left[1-\left(1-\frac{4C}{A^2}\right)^{\frac{1}{2}}\right] = \frac{A}{2}\left[1-1+\frac{2C}{A^2}\right]$$

$$= \frac{C}{A} \quad \| = \alpha$$

$\frac{2u}{\sqrt{\ }}$

$\frac{4C-2uA}{4c-…}$

Obliczenie owej wartości $-\infty + \infty$:

~~$\log \sqrt{u^2 - Au + C}$~~

$$\lim_{u = \frac{A}{2} - \sqrt{\ }} \frac{1}{2} \log \left\{ (u^2 - Au + C) \left[\frac{\frac{A}{2} + \sqrt{\frac{A^2}{4} - C} - u}{\frac{A}{2} - \sqrt{\frac{A^2}{4} - C} - u} \right]^{\frac{A}{\sqrt{A^2 - 4C}}} \right\}$$

$$\downarrow$$

$$\left(\frac{A}{2} - \sqrt{\ } - u\right)\left(\frac{A}{2} + \sqrt{\ } - u\right)$$

$$\frac{A}{2} + \sqrt{\frac{A^2}{4} - C} - u = \delta + \sqrt{A^2 - 4C}$$

$$\underbrace{\cancel{\frac{A^2}{4} - \frac{A^2}{4} + C + u^2 - Au}}_{\delta} \qquad = \frac{1}{2} \log \frac{\left[\delta + \sqrt{A^2 - 4C}\right]^{1 + \frac{A}{\sqrt{A^2-4C}}}}{\delta\left(\frac{A}{\sqrt{A^2-4C}} - 1\right)}$$

więc będzie $= +\infty$ ~~jest~~ ponieważ $A > \sqrt{A^2 - 4C}$

zatem dla $x = \infty$ u osiąga wartość maksymalną $u = \frac{A}{2} - \sqrt{\frac{A^2}{4} - C} = \alpha$

$$u^2 = \frac{A^2}{2} - C - A\sqrt{\ }$$

~~Oznaczenie stałych:~~

~~Dla $x = \infty$ $u = \alpha$~~

~~$x = 0$ $u = u_0$~~

Tam będzie:

$$\theta = \frac{1}{2R} \left\{ (k-1)\left(\frac{A^2}{2} - C - A\sqrt{\ }\right) - \frac{(k^2-1)}{k}\left(\frac{A^2}{2} - A\sqrt{\ }\right) + (k+1)C \right\}$$

$$\theta_\infty = \frac{1}{2R} \left\{ \frac{1-k}{k}\frac{A^2}{2} + 2C + \frac{k-1}{k} A \sqrt{\frac{A^2}{4} - C} \right\}$$

$$\alpha^2 - \alpha A + C = 0$$

$$A = \frac{\alpha^2 + C}{\alpha}$$

$$\theta_\infty = \frac{1}{2R}\left[-\frac{k-1}{k} A \left(\frac{A}{2} - \sqrt{\frac{A^2}{4} - C}\right) + 2C \right] = \frac{1}{2R}\left[2C - \frac{(k-1)}{k}(\alpha^2 + C) \right]$$

$$\theta_\infty = \frac{1}{2R}\left[C\left(2 - \frac{k-1}{k}\right) - \frac{k-1}{k}\alpha^2 \right] = \frac{1}{2kR}\left[(k+1)C - (k-1)\alpha^2 \right]$$

$$p_\infty = \frac{R b \theta_\infty}{\alpha} = \frac{b}{2k\alpha}\left[(k+1)C - (k-1)\alpha^2 \right]$$

$$\sqrt{A^2 - 4C} = \sqrt{\left(\frac{\alpha^2 + C}{\alpha}\right)^2 - 4C}$$

$$= \sqrt{\frac{\alpha^4 + 2\alpha^2 C + C^2 - 4\alpha^2 C}{\alpha^2}}$$

$$= \frac{\alpha^2 - C}{\alpha}$$

$$\frac{p}{p_\infty} = k\alpha\,\frac{\left[(k-1)u + (k+1)\frac{C}{u} - \frac{(k^2-1)}{k}\,\frac{\alpha^2+C}{\alpha}\right]}{(k+1)C - (k-1)\alpha^2}$$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\,x + \text{const} = \frac{1}{2}\log\left[u^2 - \frac{\alpha^2+C}{\alpha}u + C\right]$$

$$+ \frac{\alpha^2+C}{2\alpha}\,\frac{\alpha}{\alpha^2-C}\,\log\frac{\alpha + \frac{\alpha^2-C}{\alpha} - u}{\alpha - u}$$

$$= \frac{1}{2}\log\left[u^2 - \alpha u - \frac{C}{\alpha}u + C\right] + \frac{\alpha^2+C}{2(\alpha^2-C)}\log\left[\frac{2 - \frac{C}{\alpha^2} - \frac{u}{\alpha}}{1 - \frac{u}{\alpha}}\right]$$

---

punkt gdzie $u = 0$: $x = 0$

$$\text{const} = \frac{1}{2}\log C + \frac{\alpha^2+C}{2(\alpha^2-C)}\log\left(2 - \frac{C}{\alpha^2}\right)$$

$$\frac{(k+1)\frac{b}{2}}{\frac{4\mu}{3}}\,x = \frac{1}{2}\log\left[\frac{u^2-\alpha u}{C} - \frac{u}{\alpha} + 1\right] + \frac{\alpha^2+C}{2(\alpha^2-C)}\log\left[\frac{1 - \frac{u\alpha}{2\alpha^2-C}}{1 - \frac{u}{\alpha}}\right]$$

$$p_0 = \frac{b}{2}(k+1)\frac{C}{u_0} = \frac{(k+1)C}{2}\rho_0$$

tam także $\frac{du}{dx} = \infty$ punkt osobliwy

Pierwsze przybliżenie:

$$\theta = \theta_0 + \frac{\mu c^2}{32K}\left\{ 3a^2\left[-\frac{9}{r^2} + \frac{3x^2}{r^4} - \frac{a^2}{r^4} - \frac{2}{3}\frac{a^4}{r^6} + \frac{4a^2x^2}{r^6} - 2\frac{a^4x^2}{r^8}\right] + \right.$$

$$\left. + \frac{a}{r}\left[19 + 13\frac{a^2}{r^2} + 24\frac{x^2}{r^2} - 39\frac{x^2a^2}{r^4}\right]\right\}$$

dla $x=a$:

$$\{\ \} = 3\Big[\underbrace{-9-1-\tfrac{2}{3}}_{-32} + 3\frac{x^2}{a^2} + 4\frac{x^2}{a^2} - 2\frac{x^2}{a^2}\Big] + \Big[19+13-15\frac{x^2}{a^2}\Big]$$

$$-32 \quad + 15\frac{x^2}{a^2} \quad + \quad 32 - 15\frac{x^2}{a^2} \qquad = 0$$

dla $x=0$:

$$\theta = \theta_0 + \frac{\mu c^2}{32K}\left\{ -3a^2\left[\frac{9}{r^2} + \frac{a^2}{r^4} + \frac{2}{3}\frac{a^4}{r^6}\right] + \frac{a}{r}\left[19 + 13\frac{a^2}{r^2}\right]\right\}$$

$$\frac{\partial\theta}{\partial r} = \frac{\mu c^2}{32K}\left\{ 3a^2\left[\frac{18}{r^3} + \frac{4a^2}{r^5} + \frac{4a^4}{r^7}\right] - \frac{a}{r^2}\left[19 + 39\frac{a^2}{r^2}\right]\right\}\Bigg|_{r=a} =$$

$$= \frac{\mu c^2}{32ka}\left\{ \overset{\cancel{26a}}{78} - 58\right\} = \frac{5}{8}\frac{\mu c^2}{Ka}$$

$x=r$

$$\theta = \theta_0 + \frac{\mu c^2}{32K}\left\{ 3a^2\left[-\frac{9}{r^2} + \frac{3}{r^2} - \frac{a^2}{r^4} - \frac{2}{3}\frac{a^4}{r^6} + \frac{4a^2}{r^4} - 2\frac{a^4}{r^6}\right] + \right.$$

$$\left. + \frac{a}{r}\left[19 + 13\frac{a^2}{r^2} + 24 - 39\frac{a^2}{r^2}\right]\right\}$$

$$\{..\} = \left\{ 3a^2\left[-\frac{6}{r^2} + \frac{3a^2}{r^4} - \frac{8}{3}\frac{a^4}{r^6}\right] + \frac{a}{r}\left[43 - 26\frac{a^2}{r^2}\right]\right\}$$

$$\frac{\partial\theta}{\partial r} = 3a^2\left[\cancel{\frac{12}{r^3}} - \cancel{12\frac{a^2}{r^5}} + \frac{48}{3}\frac{a^4}{r^7}\right] - \frac{a}{r^2}\left[43 - \underset{\frac{43}{35}}{78}\frac{a^2}{r^2}\right]$$

$$\frac{\mu c^2}{aK}\left[\frac{48}{32} + \frac{35}{32}\right] = \frac{\mu c^2}{aK}\frac{83}{32} = \frac{\mu c^2}{aK}\left(2 + \frac{19}{32}\right)$$

$$\frac{\partial\theta}{\partial x} = \frac{\mu c^2}{32k}\left\{3a^2\left[+\frac{18x}{r^4} - \frac{12x^3}{r^6} + \frac{4a^2x}{r^6} + \cancel{\frac{4a^4x}{r^8}} - \frac{24a^2x^3}{r^8} + \frac{16a^4x^3}{r^{10}} + \right.\right.$$

$$\left.\left.\underbrace{+\frac{6x}{r^4} + \frac{8a^2x}{r^6} - \cancel{\frac{4a^4x}{r^8}}}\right] + \left[-19\frac{ax}{r^3} - 39\frac{a^3x}{r^5} - 72\frac{x^3a}{r^5} + 5.39\frac{x^3a^3}{r^7} + \underbrace{\frac{48xa}{r^3} - 2.39\frac{a^3x}{r^5}}\right]\right\}$$

$$= Mx + Nx$$

$$= \frac{\mu c^2}{32k}\left\{3a^2\left[\frac{24x}{r^4} + \frac{12a^2x}{r^6} - \frac{12x^3}{r^6} - \frac{24a^2x^3}{r^8} + \frac{16a^4x^3}{r^{10}}\right] + \right.$$

$$\left. + \left[29\frac{ax}{r^3} - 9.13\frac{a^3x}{r^5} - 72\frac{ax^3}{r^5} + 15.13\frac{a^3x^3}{r^7}\right]\right\}$$

$$\frac{\partial\theta}{\partial y} = \frac{\mu c^2}{32K}\left\{3a^2\left[\frac{18}{r^4} - \frac{12x^2}{r^6} + \frac{4a^2}{r^6} + \frac{4a^4}{r^8} - \frac{24a^2x^2}{r^8} + \frac{16a^4x^2}{r^{10}}\right]y\right.$$

$$\left. + \underbrace{\left[-\frac{19a}{r^3} - 39\frac{a^3}{r^5} - 72\frac{x^2a}{r^5} + 5.39\frac{a^3x^2}{r^7}\right]y}_{\text{[illegible]}}\right\} \qquad = My$$

$$= Mz$$

$$\frac{\partial\theta}{\partial z} = z \cdots$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = -\frac{3}{4}ca\left(1 - \frac{a^2}{r^2}\right)\frac{x}{r}M + c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)Mx$$

$$+ Nux$$

$$= M cx\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3} - \frac{3}{4}\frac{a}{r} + \frac{3}{4}\frac{a^3}{r^3}\right] + Nux$$

$$= Mcx\left[1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right] + Nux$$

$$N = 3a^2\left[\frac{6}{r^4} + \frac{8a^2}{r^6} - \frac{4a^4}{r^8}\right] + \frac{48a}{r^3} - 2.39\frac{a^3}{r^5}$$

$$\left[\frac{18}{r^4}-\frac{12x^2}{r^6}+\frac{4a^2}{r^6}+\frac{4a^4}{r^8}-\frac{24a^2x^2}{r^8}+\frac{16a^4x^2}{r^{10}}\right]3a^2+\left[-\frac{19a}{r^3}-\frac{39a^3}{r^5}-72\frac{x^2a}{r^5}+5.39\frac{a^3x^2}{r^7}\right]$$

$$+3a^2\left[\frac{19}{2r^4}+36\frac{x^2}{r^6}+\frac{39a^2}{2r^6}\quad-\frac{5.39\,a^2x^2}{2r^8}\right]-\frac{81a^3}{r^5}+\frac{54x^2a^3}{r^7}-\frac{18a^5}{r^7}-\frac{18a^7}{r^9}+\frac{108a^5x^2}{r^9}-\frac{72a^7x^2}{r^{11}}$$

$$-\frac{19}{2}\frac{a^4}{r^6}-\frac{39}{2}\frac{a^6}{r^8}-36\frac{x^2a^4}{r^8}+\frac{5.39}{2}\frac{a^6x^2}{r^{10}}+\frac{27a^5}{r^7}-\frac{18a^5x^2}{r^9}+\frac{6a^7}{r^9}+\frac{6a^9}{r^{11}}-\frac{36a^7x^2}{r^{11}}+\frac{24a^9x^2}{r^{13}}$$

$$-36\frac{a^2}{r^4}-12\frac{a^4}{r^6}-36\frac{a^2x^2}{r^6}+36\frac{a^4x^2}{r^8}\qquad+\frac{48a}{r^3}$$

$$+18\frac{a^2}{r^4}\qquad-\frac{27}{2}\frac{a^3}{r^5}-\frac{9}{2}\frac{a^5}{r^7}-27\frac{a^3x^2}{r^7}+\frac{27}{2}\frac{a^5x^2}{r^9}$$

$$+24\frac{a^4}{r^6}\qquad-18\frac{a^5}{r^7}-6\frac{a^7}{r^9}-18\frac{a^5x^2}{r^9}+18\frac{a^7x^2}{r^{11}}$$

$$-12\frac{a^6}{r^8}\qquad+9\frac{a^7}{r^9}+3\frac{a^9}{r^{11}}+9\frac{a^7x^2}{r^{11}}-9\frac{a^9x^2}{r^{13}}$$

$$-6.13\frac{a^3}{r^5}$$

$$+\frac{9}{2}13\frac{a^4}{r^6}+\frac{3}{2}\cdot 13\frac{a^6}{r^8}+\frac{9}{2}\cdot 13\frac{a^4x^2}{r^8}-\frac{9}{2}\cdot 13\frac{a^6x^2}{r^{10}}$$

$$=\frac{a^2}{r^4}\left[3.18+\frac{3.19}{2}-36+18\right]-\frac{a^2x^2}{r^6}\left[3.12-3.36+36\right]+\frac{a^4}{r^6}\left[\cancel{12}+\frac{3.39}{2}-\frac{19}{2}-\cancel{12}+24+\frac{9}{2}\cdot 13\right]$$

$$+\frac{a^6}{r^8}\left[\cancel{12}-\frac{39}{2}-\cancel{12}+\frac{3}{2}\cdot 13\right]+\frac{a^4x^2}{r^8}\left[-72-\frac{3.5.39}{2}-\cancel{36}+\cancel{36}+\frac{9}{2}\cdot 13\right]+$$

$$+\frac{a^6x^2}{r^{10}}\left[48+\frac{5.39}{2}-9\cdot\frac{13}{2}\right]+\frac{a}{r^3}\left[-19\overset{+48}{}\right]+\frac{a^3}{r^5}\left[-39-81-\frac{27}{2}-6.13\right]+$$

$$+\frac{ax^2}{r^5}\left[-72\right]+\frac{a^3x^2}{r^7}\left[5.39+54-\frac{27}{2}\right]+\frac{a^5}{r^7}\left[-18+27-\frac{9}{2}-18\right]+$$

$$+\frac{a^7}{r^9}\left[-18+\cancel{6}-\cancel{6}+9\right]+\frac{a^5x^2}{r^9}\left[108-18+\frac{27}{2}-18\right]+\frac{a^7x^2}{r^{11}}\left[-72-36+18+9\right]$$

$$+\frac{a^9}{r^{11}}\left[6+3\right]+\frac{a^9x^2}{r^{13}}\left[24-9\right]$$

72
57
129

117
24
141
282
−19

2.9.13
234
72
−306

39
48

$$= \frac{a^2}{r^4}\,\frac{129}{2} + \frac{a^2x^2}{r^6}\cdot 36 + \frac{a^4}{r^6}\,\frac{263}{2} + \text{[illegible]} + \frac{a^4x^2}{r^8}(-306) + \frac{a^6x^2}{r^{10}}\cdot 87$$

120
78
198

396
27

195
54
249 · 2
498
24

$$+\frac{a}{r^3}\cdot 29 + \frac{a^3}{r^5}\left(-\frac{423}{2}\right) - \frac{ax^2}{r^5}\cdot 72 + \frac{a^3x^2}{r^7}\left[\left(+\frac{471}{2}\right)\right.$$

144
27

$$+\frac{a^5}{r^7}\left(-\frac{27}{2}\right) + \frac{a^7}{r^9}(-9) + \frac{a^5x^2}{r^9}\,\boxed{\frac{171}{2}} + \frac{a^7x^2}{r^{11}}\,\boxed{-81} + 9\frac{a^9}{r^{11}} + 15\frac{a^9x^2}{r^{13}}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z} = \frac{\mu c^3 x}{32k}\Big[\frac{129}{2}\frac{a^2}{r^4}+36\frac{a^2x^2}{r^6}+\frac{263}{2}\frac{a^4}{r^6}-306\frac{a^4x^2}{r^8}+87\frac{a^6x^2}{r^{10}}$$
$$+29\frac{a}{r^3}-\frac{423}{2}\frac{a^3}{r^5}-72\frac{ax^2}{r^5}+\frac{471}{2}\frac{a^3x^2}{r^7}-\frac{27}{2}\frac{a^5}{r^7}-9\frac{a^7}{r^9}+\frac{171}{2}\frac{a^5x^2}{r^9}-81\frac{a^7x^2}{r^{11}}$$
$$+9\frac{a^9}{r^{11}}+15\frac{a^9x^2}{r^{13}}\Big]$$

~~352~~ 196 −225

~~$\Big|_{r=a} = \frac{\mu c^3 x}{32a^2}\Big[\frac{129}{2}+\frac{263}{2}-306+29-\frac{423}{2}-\frac{27}{2}-9+9+$~~

6

~~$\frac{x^2}{a^2}\Big(36+87-72-\frac{471}{2}+\frac{171}{2}-81+15\Big)\Big]$~~

−150

196
29
225

306
225
531
225
−306
153

42
15
+57

−222
57
−165

~~$\Big|_{r=a} = \frac{\mu c^3 x}{k a^2}\Big[\frac{153}{16}-\frac{165}{32}x^2\Big]$~~

$\frac{129}{2}$
$\frac{263}{2}$
29
$-\frac{423}{2}$
$-\frac{27}{2}$
−43

$\frac{392}{2} = 196$
29
225

450 = 225

144

48

−60
−178
+144
54
258
−138
120

−712
−166

$\Big|_{r=a} = 0$

do x = r

$$u\frac{\partial\theta}{\partial x} = \frac{\mu c^2}{32k}\Big[\overbrace{(29-72)}\frac{a}{r^2}+\frac{a^3}{r^4}\overbrace{\left(-\frac{423}{2}+\frac{471}{2}\right)}^{24}$$
$$+\frac{a^5}{r^6}\underbrace{\left(-\frac{27}{2}+\frac{171}{2}\right)}_{+72}+\frac{a^7}{r^8}(\underbrace{-9-81}_{-90})+24\frac{a^9}{r^{10}}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{y}\;w\frac{\partial\theta}{z} = v\cdot\nabla\theta\cdot\cos(v.\nabla\theta)$$

$$= \frac{\mu c^2}{32\kappa}\left\{\frac{129}{2}\frac{a^2x}{r^4} + 36\frac{a^2x^3}{r^6} + \frac{263}{2}\frac{a^4x}{r^6} - 306\frac{a^4x^3}{r^8} + 87\frac{a^6x^3}{r^{10}}\right.$$

$$+ 29\frac{ax}{r^3} - \frac{423}{2}\frac{a^3x}{r^5} - 72\frac{ax^3}{r^5} + \frac{471}{2}\frac{a^3x^3}{r^7} - \frac{27}{2}\frac{a^5x}{r^7} - 9\frac{a^7x}{r^9} + \frac{171}{2}\frac{a^5x^3}{r^9} -$$

$$\left. -81\frac{a^7x^3}{r^{11}} + 9\frac{a^9x}{r^{11}} + 15\frac{a^9x^3}{r^{13}}\right\}$$

$\Delta^2\left(\frac{x}{r}\right) = -2\frac{x}{r^3}$

$\Delta^2\left(\frac{x}{r^2}\right) = \frac{x}{r^4}$

$\Delta^2\left(\frac{x}{r^3}\right) = 0$

$\Delta^2\left(\frac{x}{r^4}\right) = 4\frac{x}{r^6}$

$\Delta^2\left(\frac{x}{r^5}\right) = 10\frac{x}{r^7}$

$\Delta^2\left(\frac{x}{r^6}\right) = 18\frac{x}{r^8}$

$\Delta^2\left(\frac{x}{r^7}\right) = 28\frac{x}{r^9}$

$\Delta^2\left(\frac{x}{r^8}\right) = 40\frac{x}{r^{10}}$

$\Delta^2\left(\frac{x}{r^9}\right) = 54\frac{x}{r^{11}}$

$\Delta^2\left(\frac{x^3}{r^3}\right) = \frac{6x}{r^3} - 12\frac{x^3}{r^5}$

$\Delta^2\left(\frac{x^3}{r^4}\right) = \frac{6x}{r^4} - 12\frac{x^3}{r^6}$

$\Delta^2\left(\frac{x^3}{r^5}\right) = \frac{6x}{r^5} - 10\frac{x^3}{r^7}$

$\Delta^2\left(\frac{x^3}{r^6}\right) = \frac{6x}{r^6} - 6\frac{x^3}{r^8}$

$\Delta^2\left(\frac{x^3}{r^7}\right) = \frac{6x}{r^7}\;\; 0$

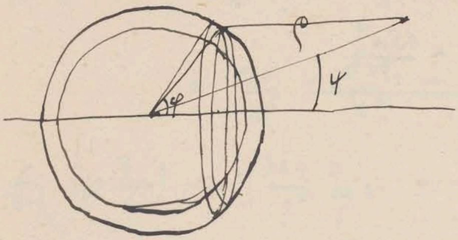
$\Delta^2\left(\frac{x^3}{r^8}\right) = \frac{6x}{r^8} - 8\frac{x^3}{r^{10}}$

$\Delta^2\left(\frac{x^3}{r^9}\right) = \frac{6x}{r^9} - 18\frac{x^3}{r^{11}}$

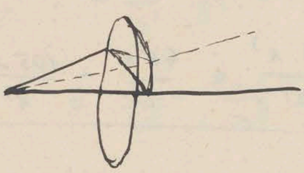
$\Delta^2\left(\frac{x^3}{r^{11}}\right) = \frac{6x}{r^{11}} - 44\frac{x^3}{r^{13}}$

$$\text{pot}\left(\frac{x}{r^5}\right) = \text{pot}\,\frac{\cos\varphi}{r^4} = \text{pot}_{(-\frac{1}{4})}\frac{\partial\left(\frac{1}{r^4}\right)}{\partial x}$$

$$\cos\chi = \cos\varphi\cos\psi + \sin\varphi\sin\psi\cos\varepsilon$$



$$\frac{1}{\rho} = \frac{1}{\sqrt{r^2 + R^2 - 2Rr\cos\chi}}$$



$$\int_{r=a}^{\infty}\int_{\varphi=0}^{\pi}\int_{\varepsilon=0}^{2\pi} \frac{dr\,d\varphi\,\sin\varphi\,d\varepsilon\,.\,\cos\varphi}{r^2\sqrt{r^2+R^2-2Rr\underbrace{(\cos\varphi\cos\psi+\sin\varphi\sin\psi\cos\varepsilon)}_{\cos\chi}}}$$

$a = R^2$

$b = -2R\cos\chi$

$c = 1$

$$\int_a^R \frac{dr}{r^2\sqrt{r^2+R^2-2Rr\cos\chi}} = -\frac{\sqrt{}}{R^2 r} + \frac{2R\cos\chi}{2R^2}\underbrace{\int\frac{dr}{r\sqrt{}}}\ \cancel{-0}$$

$$= \frac{1}{R}\lg\frac{2R\sqrt{} - (2R^2 - 2Rr\cos\chi)}{r}$$

$$\text{pot}\left(\frac{x}{r^n}\right) = -\frac{\text{pot}}{n-2}\frac{\partial}{\partial x}\left(\frac{1}{r^{n-2}}\right) = -\frac{1}{n-2}\,\frac{\partial}{\partial x}\cdot\text{pot}\left(\frac{1}{r^{n-2}}\right)$$

$$\int_a^R \frac{r^2\,dr}{r^{n-2}} = \int r^{4-n}\,dr = \frac{R^{4-n+1}}{(4-n+1)R}\Bigg|_a^R = \frac{R^{4-n}}{4-n+1} - \frac{a}{R}\ \Bigg\|\ \frac{\partial}{\partial x} = \frac{4-n}{4-n+1}\,\frac{x}{R^{n-2}} = \text{pot}\left(\frac{x}{r^n}\right)$$

$$\overset{5-n}{\cancel{+\frac{a\,x}{R^3}}}$$

$$\cancel{-\frac{1}{n-2}\,R^{4-n}\frac{x}{R} = -\frac{1}{n-2}\,\frac{x}{R^{n-2}}}$$

$$\frac{\partial\left(\frac{1}{r}\right)}{\partial x} = -\frac{x}{r^3}$$

$$\frac{\partial^2}{\partial x^2} = -\frac{1}{r^3} + \frac{3x^2}{r^5}$$

$$\frac{\partial^3}{\partial x^3} = -\frac{3x}{r^5} + \frac{6x}{r^5} - \frac{15x^3}{r^7} = \frac{3x}{r^5} - \frac{15x^3}{r^7}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r^3}\right) = \frac{-3x}{r^5}$$

$$\frac{\partial^2}{\partial x^2}\left(\quad\right) = \frac{-3}{r^5} + \frac{15x^2}{r^7}$$

$$\frac{\partial^3}{\partial x^3} = +\frac{15x}{r^7} + \frac{30x}{r^7} - \frac{3.5.7.\,x^3}{r^9} = \frac{45x}{r^7} - \frac{105x^3}{r^9}$$

$$105\,\frac{x^3}{r^9} = -\frac{\partial^3}{\partial x^3}\left(\frac{1}{r^3}\right) \pm \frac{45}{5}\,\frac{\partial}{\partial x}\left(\frac{1}{r^5}\right)$$

$$\frac{x^3}{r^9} = -\frac{1}{105}\,\frac{\partial^3}{\partial x^3}\left(\frac{1}{r^3}\right) - \frac{3}{35}\,\frac{\partial}{\partial x}\left(\frac{1}{r^5}\right)$$

$$\mathrm{pot}\left(\frac{x^3}{r^9}\right) = -\frac{1}{105}\,\frac{\partial^3}{\partial x^3}\,\underbrace{\mathrm{pot}\left(\frac{1}{r^3}\right)}_{\frac{\lg r}{r}} - \frac{3}{35}\,\frac{\partial}{\partial x}\,\underbrace{\mathrm{pot}\left(\frac{1}{r^5}\right)}_{\frac{1}{2r^3}}$$

$$\int^{R} \frac{r^2\,dr}{r^3} = \lg R \qquad \int \frac{r^2\,dr}{r^5} = \int \frac{dr}{r^3} = -\frac{1}{2r^2}$$

$$\frac{\partial}{\partial \cancel{x}}\left(\frac{\lg r}{r}\right) = \left(\frac{1}{r^2} - \frac{\lg r}{r^2}\right)\frac{x}{r} \qquad \frac{x}{r^4}$$

$$\frac{\cancel{x}}{r^3} \qquad \frac{x}{r^4} \qquad \frac{x}{r^3}\lg r$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\lg r\right) = \frac{1}{r^3}\lg r - \frac{3x^2}{r^5}\lg r + \frac{x^2}{r^5}$$

$$\frac{\partial^2}{\partial x^2}(\quad) = -\frac{3x}{r^5}\lg r + \frac{x}{r^5} - \frac{6x}{r^5}\log r + 15\frac{x^3}{r^7}\lg r - \frac{3x^3}{r^7} + \frac{2x}{r^5} - \frac{5x^3}{r^7}$$

$$= \frac{3x}{r^5} - \frac{8x^3}{r^7} - \frac{9x}{r^5}\lg r + \frac{15x^3}{r^7}\lg r$$

$$\frac{\partial}{\partial y}\left(\frac{x}{r^3}\lg r\right) = -\frac{3xy}{r^5}\lg r + \frac{xy}{r^5}$$

$$\frac{\partial^2}{\partial y^2} \qquad = -\frac{3x}{r^5}\lg r + \frac{15xy^2}{r^7}\lg r - \frac{3xy^2}{r^7} + \frac{x}{r^5} - \frac{5xy^2}{r^7}$$

$$= \frac{x}{r^5} - \frac{8xy^2}{r^7} - \frac{3x}{r^5}\lg r + \frac{15xy^2}{r^7}\lg r$$

$$\Delta^2\left(\frac{x}{r^3}\lg r\right) = \frac{5x}{r^5} - \frac{8x}{r^5} - \cancel{\frac{15x}{r^5}\log r} + \cancel{\frac{15x}{r^5}\lg r} = -\frac{3x}{r^5}$$

$$\frac{\partial^3}{\partial x^3}\left(\frac{x}{r^3}\lg r\right) = \cdots\cdots + 45\frac{x^2}{r^7}\lg r - \frac{9}{r^5}\log r + \frac{45x^2}{r^7}\log r - 15.7\frac{x^4}{r^9}\lg r$$

$$\frac{\partial^2}{\partial x^2}\ \frac{x^2}{r^7}\lg r = \frac{2x}{r^7}\lg r - 7\frac{x^3}{r^9}\lg r + \frac{x^3}{r^9} \quad\Big|\quad \frac{\partial}{\partial y} = -7\frac{x^2y}{r^9}\lg r + \frac{x^2y}{r^9}$$

$$\frac{2}{r^7}\lg r - \frac{35x^2}{r^9}\lg r - \cancel{\frac{21x^2}{r^9}\lg r} + \frac{63x^4}{r^{11}}\lg r + \frac{2x^2}{r^9} - \frac{7x^4}{r^{11}} + \frac{3x^2}{r^9} - \frac{9x^4}{r^{11}}$$

$$-\frac{7x^2}{r^9}\lg r + 63\frac{x^2y^2}{r^{11}}\lg r \qquad -\frac{7x^2y^2}{r^{11}} + \frac{x^2}{r^9} - \frac{9x^2y^2}{r^{11}}$$

$$\Delta^2\left(\frac{x^2}{r^7}\lg r\right) = \frac{2}{r^7}\lg r - \frac{49x^2}{r^9}\lg r + \frac{63x^2}{r^9}\lg r + \frac{2x^2}{r^9} - \frac{7x^2}{r^9} + \frac{5x^2}{r^9} - \frac{9x^2}{r^9}$$

$$= \frac{2}{r^7}\lg r \ \text{[illegible]} + \frac{14x^2}{r^9}\lg r - \frac{9x^2}{r^9}$$

$$\frac{\partial}{\partial x}\left(\frac{x}{2^5}\lg 2\right)$$

$$\frac{1}{2^5}\lg 2-\frac{5x^2}{2^7}\lg y+\frac{x^2}{2^7}$$

$$-\frac{5xy}{2^7}\lg y+\frac{xy}{2^7}$$

$$\frac{\partial^2}{\partial x^2}$$

$$-\frac{15x}{2^7}\lg y+\frac{35x^3}{2^9}\lg y+\frac{x}{2^7}-\frac{5x^3}{2^9}+\frac{2x}{2^7}-\frac{7x^3}{2^9}$$

$$-\frac{5x}{2^7}\lg y+\frac{35xy^2}{2^9}\lg y-\frac{5xy^2}{2^9}+\frac{x}{2^7}-\frac{7xy^2}{2^9}$$

$$-\frac{25x}{2^7}\lg 2+\frac{35x}{2^7}\lg 2+\frac{5x}{2^7}-\frac{12x}{2^7}$$

$$=\frac{10x}{2^7}\lg 2-\frac{7x}{2^7}$$

$$\frac{x^3}{2^7}\lg 2$$

$$\frac{3x^2}{2^7}\lg y-\frac{7x^4}{2^9}\lg y+\frac{x^4}{2^9}$$

$$-\frac{7x^3y}{2^9}\lg y+\frac{x^3y}{2^9}$$

$$\frac{\partial^2}{\partial x^2}$$

$$\frac{6x}{2^7}\lg y-\frac{49x^3}{2^9}\lg y+\frac{63x^5}{2^{11}}\lg y+\frac{3x^3}{2^9}-\frac{7x^5}{2^{11}}+\frac{4x^3}{2^9}-\frac{9x^5}{2^{11}}$$

$$-\frac{7x^3}{2^9}\lg y+\frac{63x^3y^2}{2^{11}}\lg y\qquad-\frac{7x^3y^2}{2^{11}}+\frac{x^3}{2^9}-\frac{9x^3y^2}{2^{11}}$$

$$\cdot\qquad\cdot\qquad\cdot\qquad\cdot\qquad\cdot$$

$$\Delta^2=\frac{6x}{2^7}\lg 2+\cancel{\frac{9x^3}{2^9}}-\cancel{\frac{9x^3}{2^9}}-\frac{7x^3}{2^9}$$

$$\frac{x}{2^7}=\Delta^2\left(\frac{x}{10\,2^5}\right)$$

$$\cancel{7}\frac{x^3}{2^9}=\frac{6x}{7\,2^7}\lg 2-\Delta^2\left(\frac{x^3}{7\,2^7}\lg 2\right)\qquad 5$$

$$\cancel{7}\frac{x}{2^7}=\frac{1}{7}\frac{10x}{2^7}\lg 2-\Delta^2\left(\frac{x}{7\,2^5}\lg 2\right)\qquad 3$$

$$\frac{5x^3}{2^9}-\frac{3x}{2^7}=\Delta^2\left(\frac{3x}{7\,2^5}\lg 2-\frac{5x^3}{7\,2^7}\lg 2\right)$$

$$\frac{5x^3}{r^9} = \Delta^2\left(\frac{3}{10}\,\frac{x}{r^5} + \frac{3x}{7r^5}\lg r - \frac{5x^3}{7r^7}\lg r\right)$$

$$\frac{x^3}{r^9} = \Delta^2\left\{\frac{3}{50}\,\frac{x}{r^5} + \frac{3x}{35r^5}\lg r - \frac{x^3}{7r^7}\lg r\right\} = \Delta^2\left\{\frac{3x}{5r^5}\left[\frac{1}{10} + \frac{\lg r}{7}\right] - \frac{x^3}{7r^7}\lg r\right\}$$

$$\frac{x}{r^5} = -\frac{1}{3}\,\Delta^2\left(\frac{x}{r^3}\lg r\right)$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}$$

$$\rho\frac{\partial v}{\partial t} = -\cancel{\frac{\partial p}{\partial x}} + \mu\frac{\partial^2 v}{\partial x^2}$$

$$\cancel{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x}} = 0 \quad \rightarrow \quad \frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial p}{\partial t} + k p\frac{\partial u}{\partial x} + u\frac{\partial p}{\partial x} = (k-1)\left[\mu\Phi + \kappa\frac{\partial^2\theta}{\partial x^2}\right]$$

$$v = A e^{-\alpha x}\cos(\beta x + \gamma t)$$

$$\frac{\partial v}{\partial x} = A e^{-\alpha x}\left[-\alpha\cos - \beta\sin\right] \qquad \frac{\partial v}{\partial t} = -A e^{-\alpha x}\gamma\sin$$

$$\frac{\partial^2 v}{\partial x^2} = A e^{-\alpha x}\left[\alpha^2\cos + \alpha\beta\sin - \beta^2\cos\right]$$

$$dx\sqrt{} = dt\cdot\gamma$$
$$\frac{dx}{dt} = \frac{\gamma}{\sqrt{}}$$

$$-\rho\gamma = \mu\alpha\beta$$
$$\alpha = \beta$$
$$\alpha = \sqrt{-\frac{\rho\gamma}{\mu}}$$

~~[illegible]~~ $v = A e^{-x\sqrt{\frac{\rho\gamma}{\mu}}}\cos\left(x\sqrt{\frac{\rho\gamma}{\mu}} - \gamma t\right)$

współczynnik zanikania $\sqrt{\frac{\mu}{\rho\gamma}}$

prędkość fali: $= \frac{\gamma}{\sqrt{\frac{\rho\gamma}{\mu}}} =$

tem większa im większe γ

N.p. $\gamma = 2\pi n$ N.p. $n = 140$

$\gamma = 1000$

$pr. = \frac{1000}{\sqrt{\frac{\cancel{13}}{6\cdot 00019}}} = 10\ \frac{cm}{sec}$

$\sqrt{\frac{\mu}{\rho\gamma}} = 0\cdot 01$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + k p\frac{\partial u}{\partial x} = (k-1)\left[\mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^2\theta}{\partial x^2}\right]$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}$$

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} = 0$$

Gdyby u było $= 0$, $p = \text{const}$ Niemożl. μv^2 $\frac{\kappa}{\mu}\frac{\partial^2\theta}{\partial x^2} = \cancel{A e^{-2\alpha x}\alpha\left[\cos + \sin(\alpha x - \gamma t)\right]}$

$$= \alpha^2 A^2 e^{-2\alpha x}\left[1 + 2\sin\cos\right]$$

$$= \alpha^2 A^2 e^{-2\alpha x}\left[1 + \sin 2(\alpha x - \gamma t)\right]$$

$$\theta = B e^{-ax}\left[1 + m\cos(bx + ct)\right]$$

$$\frac{\partial^2\theta}{\partial x^2} = B e^{-ax}\left[a^2 + \cancel{(a^2-b^2)m \cos} + ab\, m \sin\right] = \frac{\alpha^2 A^2}{\frac{\kappa}{\mu}} e^{-2\alpha x}\left[1+\cos 2(\alpha x-\gamma t)\right]$$

$a=b$ $bx+ct = 2(\alpha x-\gamma t)$

$a=2\alpha$ $b=2\alpha$ $c=-2\gamma$

$$4\cancel{\alpha^2} B \frac{\kappa}{\mu} = \cancel{\alpha^2} A^2 \qquad m=1 \qquad B = \frac{A^2}{4\frac{\kappa}{\mu}}$$

$$4\alpha^2 m B \frac{\kappa}{\mu} = \alpha^2 A^2$$

$$\theta = \overset{f(t)}{M} + \frac{A^2}{4\frac{\kappa}{\mu}} e^{-2\alpha x}\left[1+\cos 2(\alpha x-\gamma t)\right] + \cancel{c x}$$

$$\theta_0 = F(t) + \frac{A^2}{4\frac{\kappa}{\mu}}\left[1+\cos 2\gamma t\right]$$

$$F(t) = \theta_0 - \frac{A^2}{4\frac{\kappa}{\mu}}\left[1+\cos 2\gamma t\right]$$

$$\alpha = \sqrt{\frac{\varrho}{\mu}\gamma}$$

$$\theta = \theta_0 + \frac{A^2}{4\frac{\kappa}{\mu}}\left\{e^{-2\alpha x}\left[1+\cos(2\alpha x-\gamma t)\right] - 1 - \cos 2\gamma t\right\}$$

$$\frac{1}{p}\frac{\partial p}{\partial t} = \frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{1}{\theta}\frac{\partial\theta}{\partial t} \qquad \frac{\partial p}{\partial t} = R\left(\theta\frac{\partial\rho}{\partial t} + \rho\frac{\partial\theta}{\partial t}\right) = -p\frac{\partial u}{\partial x} + R\rho\frac{\partial\theta}{\partial t}$$

zaniedbując $u\frac{\partial p}{\partial x} = -\rho u \frac{\partial u}{\partial t}$

$$(k-1)p\frac{\partial u}{\partial x} + R\rho\frac{\partial\theta}{\partial t} = (k-1)\left[\mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^2\theta}{\partial x^2}\right]$$

~~Zaniedbując $\frac{\partial u}{\partial x}$~~ Gdyby $\frac{\partial u}{\partial x}=0$ lub gdyby $k=1$ $\alpha' = \sqrt{\frac{R\rho}{\kappa}\gamma}$

$$\theta = \frac{A^2}{4\frac{\kappa}{\mu}}\left\{e^{-2\alpha x}\left[1+\cos 2(\alpha x-\gamma t)\right] - e^{-2\alpha' x}\cos 2(\alpha' x-\gamma t)\right\} + \theta_0 \qquad \alpha = \sqrt{\frac{\rho}{\mu}\gamma}$$

~~$\sqrt{\frac{R}{\kappa}} = \sqrt{\frac{10^6\,\mu}{0.0013\cdot 273\cdot 0.00006\cdot 4.2\cdot 10^6}} = \sqrt{\frac{1\quad 0.00019}{0.00006\cdot 1.5}} = \sqrt{\frac{1}{0.0009}} = \frac{1}{0.03}$~~

~~zatem α' [illegible] większe od α~~

O ile wytwarzanie ciepła następuje periodycznie?

$$\int_0^\infty \Phi\,dx = \alpha^2 A^2 \int_0^\infty e^{-2\alpha x}\left[1+\sin 2(\alpha x-\gamma t)\right]dx =$$

$$= \alpha^2 A^2 \int_0^\infty \left[e^{-2\alpha x} + e^{-2\alpha x}\sin 2\alpha x\cos\gamma t - e^{-2\alpha x}\cos 2\alpha x\sin\gamma t\right]dx$$

$$\int_0^\infty e^{-y}\sin y\,dy = -e^{-y}\cancel{\sin y}\Big|_0^\infty + \int_0^\infty e^{-y}\cos y\,dy = \frac{1}{2}$$

$$\int_0^\infty e^{-2\alpha x}\sin 2\alpha x\,dx = \frac{1}{2\alpha}\cdot\frac{1}{2} = \frac{1}{4\alpha}$$

$$\int_0^\infty e^{-2\alpha x}\,dx = \frac{1}{2\alpha}$$

$$\int\Phi = \alpha^2 A^2\left[\frac{1}{2\alpha} + \frac{1}{4\alpha}\cos\gamma t - \frac{1}{4\alpha}\sin\gamma t\right]$$

$= \frac{\alpha A^2}{4}\left[2 + \cos\gamma t - \sin\gamma t\right] =$ ~~$\frac{\alpha A^2}{4}\Big[2 + \cos\frac{\gamma t}{2} - \sin\frac{\gamma t}{2} - 2\sin\frac{\gamma t}{2}\cos\frac{\gamma t}{2}$~~ $\Big]$

Więc superpozycja period. ważna

Przybliżenie dla małych ruchów cieczy ~~nieściśliwej~~:

$$\left.\begin{aligned}\rho\frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial x} + \frac{\mu}{3}\frac{\partial\,\mathrm{div}}{\partial x} + \mu\Delta^2 u\\ \rho\frac{\partial v}{\partial t} &= -\frac{\partial p}{\partial y} + \frac{\mu}{3}\frac{\partial\,\mathrm{div}}{\partial y} + \mu\Delta^2 v\\ \cdot\ \cdot\ &= \cdot - - -\end{aligned}\right\}$$

$$\frac{\partial\rho}{\partial t} + \rho\,\mathrm{div} + \cancel{u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z}} = 0$$

$$\frac{\partial^2\rho}{\partial t^2} + \underline{\rho\frac{\partial}{\partial t}\mathrm{div}} + \mathrm{div}\frac{\partial\rho}{\partial t} + \cancel{\underline{\frac{\partial u}{\partial t}\frac{\partial\rho}{\partial x}+\dots}} \cancel{+ u\frac{\partial^2\rho}{\partial x\,\partial t}} = 0$$

$$\rho\frac{\partial}{\partial t}\mathrm{div} + \cancel{\left(\frac{\partial\rho}{\partial x}\frac{\partial u}{\partial t} + \frac{\partial\rho}{\partial y}\frac{\partial v}{\partial t} + \dots\right)} = -\nabla^2 p + \frac{\mu}{3}\nabla^2\mathrm{div} + \mu\nabla^2\mathrm{div}$$

$$= -\nabla^2 p + \frac{4}{3}\mu\nabla^2\mathrm{div}$$

$$-\left[\frac{\partial^2\rho}{\partial t^2} + \overset{?}{\overbrace{\frac{\partial\rho}{\partial t}\mathrm{div}}} + \cancel{u\frac{\partial^2\rho}{\partial x\,\partial t} + \dots}\right] = \nearrow$$

$$\rho\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\partial\rho}{\partial y}\frac{\partial u}{\partial t} - \frac{\partial\rho}{\partial x}\frac{\partial v}{\partial t} = \mu\Delta^2\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

Podobieństwo dynam. dla powolnych ruchów:

$\cancel{\frac{b}{n} = \alpha \frac{m}{n^2}}$ z siłą zewn. $\sim$ [illegible] ρX

$$b \equiv \alpha \frac{m}{n} \equiv \frac{bn}{r}$$

$$mb \equiv [illegible] \equiv \beta \frac{r}{n}$$

$$\alpha \frac{m^2}{n} \equiv \beta \frac{r}{n} \qquad \frac{r}{n} \equiv 1$$

$$\beta = \alpha \frac{m^2}{n} \qquad \underline{\underline{r = n}} \qquad \underline{\underline{m}} = \sqrt{\frac{n\beta}{\alpha}}$$

$$b = \frac{\beta}{m} \qquad \underline{\underline{b}} = \frac{\beta}{m}$$

Dla powolnych ruchów różnych gazów

$$\frac{\partial p}{\partial x} = [illegible] X + \mu \frac{\partial^2 u}{\partial x^2}$$

$$k\rho \frac{\partial u}{\partial x} = \mu \left(\frac{\partial u}{\partial x}\right)^2 + k \Delta^2 \vartheta$$

$$\rho \qquad \alpha$$

$$\frac{b}{n} \equiv R \frac{b}{[illegible]} \equiv \alpha \frac{m}{n^2}$$

$$\frac{bm}{n} \equiv \cancel{\alpha \left(\frac{m}{n}\right)^2} \equiv \beta \frac{r}{n^2}$$

$$\frac{\alpha m^2}{n} = \beta \frac{r}{n} \qquad n = \frac{r}{R}$$

$$\frac{\alpha m^2}{n} \doteq \beta R$$

$$\nabla^2_p = -\varepsilon \cancel{\nabla^2 U} - \left[\frac{\partial \varepsilon}{\partial x} \frac{\partial U}{\partial x} + \frac{\partial \varepsilon}{\partial y} \frac{\partial U}{\partial y} + \quad \right] \cancel{+ \mu \nabla^2 \mathrm{div}}$$

$$\downarrow$$

$$\iint \frac{\partial p}{\partial n} do = -\iint \qquad = \cancel{\int \varepsilon . \nabla^2 U} - \int \varepsilon . \frac{\partial U}{\partial n} do$$

$$= -\iint \varepsilon \frac{\partial U}{\partial n} do$$

ε zmienne tylko w kierunku n $\parallel$ $\frac{\partial \varepsilon}{\partial x} = \frac{\partial \varepsilon}{\partial n}$ [illegible] $\cos nx$

[illegible] „ „ „ „ stycznym rot $\frac{\partial \varepsilon}{\partial x} \frac{\partial U}{\partial x} + \ldots = 0$

jeśli ścianka izolująca $\nabla^2_p = 0$

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= X + \mu \nabla u &&= -\varepsilon \frac{\partial U}{\partial x} + \mu \nabla^2 u \\ \frac{\partial p}{\partial y} &= Y + \mu \nabla^2 v &&= -\varepsilon \frac{\partial U}{\partial y} + \mu \nabla^2 v \\ \frac{\partial p}{\partial z} &= Z + \mu \nabla^2 w &&= -\varepsilon \frac{\partial U}{\partial z} + \mu \nabla^2 w \end{aligned}\right\} \nabla p = -\varepsilon \nabla U + \mu \nabla^2 v$$

U nie zmienia się znacznie w obrębie δ ‖

$$\underbrace{\left[\frac{\partial p}{\partial x}\cos nx + \frac{\partial p}{\partial y}\cos ny + \dots\right]}_{\frac{\partial p}{\partial n}} = -\varepsilon \underbrace{\left[\frac{\partial U}{\partial x}\cos nx + \dots\right]}_{\frac{\partial U}{\partial n}} + \mu\left[\nabla^2 u \cdot \cos nx + \dots\right]$$

$$\cancel{u = u_0 + \xi u_1 + \frac{\xi^2}{2} u_2}$$
$$\cancel{v = v_0 + \eta v_1 + \frac{\eta^2}{2} v_2}$$

nie tylko w punkcie $0 = 0$ ale w płaszczyźnie dla niektórych

dla $\xi \cos nx + \eta \cos ny + \zeta \cos nz = 0$

będzie także $u = v = w = 0$

powierzchnia gdzie

u będzie $= 0$

$$u = \overset{=0}{\cancel{u_0}} + \xi\left(\frac{\partial u}{\partial x}\right)_0 + \eta\left(\frac{\partial u}{\partial y}\right)_0 + \zeta\left(\frac{\partial u}{\partial z}\right)_0 + \frac{\xi^2}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_0 + \dots$$

$$\zeta \cos nz = -(\xi \cos nx + \eta \cos ny)$$

$$0 = \xi \underbrace{\left[\frac{\partial u}{\partial x}\cos nz - \frac{\partial u}{\partial z}\cos nx\right]}_{=0} + \eta \underbrace{\left[\frac{\partial u}{\partial y}\cos nz - \frac{\partial u}{\partial z}\cos ny\right]}_{=0}$$

zatem $$\frac{\frac{\partial u}{\partial x}}{\cos nx} = \frac{\frac{\partial u}{\partial y}}{\cos ny} = \frac{\frac{\partial u}{\partial z}}{\cos nz} = \frac{\partial u}{\partial n}$$

zatem : $$\left.\begin{aligned} u &= \left(\frac{\partial u}{\partial n}\right)_0 \left[\xi \cos nx + \eta \cos ny + \zeta \cos nz\right] + \frac{\xi^2}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_0 + \dots \\ v &= \\ w &= \end{aligned}\right\}$$ zatem prop. odległości

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ zatem: $\left(\frac{\partial u}{\partial n}\right)_0 \cos nx + \left(\frac{\partial v}{\partial n}\right)_0 \cos ny + \left(\frac{\partial w}{\partial n}\right)_0 \cos nz = 0$

T. zn. że warstwy równoległe płyną do powierzchni

$\Delta^2 u = \left(\frac{\partial^2 u}{\partial x^2}\right)_0 + \cdots$

$$\underbrace{\int \frac{\partial p}{\partial x}\, d\nu}_{} = -\underbrace{\int \varepsilon \frac{\partial U}{\partial x}\, d\nu}_{} + \mu \int \nabla^2 u\, d\nu$$

$p \cos nx\, d\sigma$

$$\cancel{\varepsilon \frac{\partial U}{\partial x}} = \cancel{\frac{\partial U}{\partial x} \int_0^\delta \varepsilon\, d\nu} = \int \frac{\partial U}{\partial x}\, d\sigma \int \varepsilon\, d\delta +$$

---

s kierunek styczny linij prądu w stałej odległości ν od powierzchni.

$\frac{\partial p}{\partial s} = \varepsilon \frac{\partial U}{\partial s} + \mu \nabla^2 v_s$ ponieważ w pierwszem przybliżeniu v równoległe do powierzchni, tylko różnem prędkości (lamellar)

zatem $\nabla^2 v_s = \frac{\partial^2 v_s}{\partial \nu^2}$ $\frac{\partial U}{\partial s}$ = stałe w głębi ν

$\varepsilon = \frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial \nu^2}$

$$\underbrace{\int_0^{\nu} \frac{\partial p}{\partial s}\, d\nu}_{} = \frac{1}{4\pi} \frac{\partial \varphi}{\partial \nu} \frac{\partial U}{\partial s} + \mu \frac{\partial v_s}{\partial \nu} + \text{const}$$

$\downarrow$ fc. (s)?

$= \delta \left(\frac{\partial p}{\partial s}\right)_{\nu=\delta}$ = przybliżenie = 0

zatem $\mu v_s = -\frac{1}{4\pi} (\varphi_1 - \varphi_2) \frac{\partial U}{\partial s}$

ale oprócz tego istnieje v_n ! ?

ponieważ $\nabla^2 U = 0$

zatem takie prędkości powierzchniowe (proporcjonalnie) jak gdyby U było potencjałem prędkości

w reszcie cieczy będą równania

$$\left.\begin{aligned}\frac{\partial p}{\partial x} &= \mu \nabla^2 u\\ \frac{\partial p}{\partial y} &= \mu \nabla^2 v\\ \frac{\partial p}{\partial z} &= \mu \nabla^2 w\end{aligned}\right\}$$

one będą zatem spełnione przez ~~$u \sim \frac{\partial U}{\partial x}$~~

~~$v \sim \frac{\partial U}{\partial y}$ etc.~~

~~$w \sim \frac{\partial U}{\partial z}$~~

~~$p = \text{const.}$~~

~~warunki powierzchniowe $\overset{?}{\text{spełnione}}$, bo $\nabla^2 u = 0$~~

Nieprawda: bo przecież do tego przykładu nie uwzględniono v_n

~~$u \sim \frac{\partial U}{\partial x}$ na powierzchni etc.~~

~~układ U będzie zatem takie potencjałem prędkości dla $u\ v\ w$~~

~~prędkości prop $\frac{1}{4\pi}\frac{\varphi_i - \varphi_e}{\mu}$~~

Przyjmując $U \cdot \frac{\varphi_i - \varphi_e}{4\pi\mu}$ jako potencjał prędkości istotnie otrzymuje się

prędkości odpowiednie (czysto styczne!) na powierzchni i spełnienie

równań hydr. biorąc $p = \text{const.}$ [ale p_{xx} etc ≥ 0!]

Ażeby otrzymać jednak nieruchomość dla $r = \infty$ trzeba by superponować

rozwiązanie: ruch $-\frac{\varphi_i - \varphi_e}{4\pi\mu}\frac{\partial U}{\partial x}$ dla $r = \infty$ spoczynek dla $r = a$

t.j. (nieruchoma kula w cieczy poruszającej się z prędkością

zatem ciśnienie : $p = \frac{\varphi_i - \varphi_e}{4\pi\mu}\underset{\overset{\shortparallel}{A}}{\left(\frac{\partial U}{\partial x}\right)}\frac{x}{r^3}$

Energia zużyta:

$$\Phi = \mu\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \cdots\right\} + \left\{\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + -\right\}\right]$$

$$u = \alpha \frac{\partial U}{\partial x} \quad \text{etc.}$$

patrz Lamb p. 519 r. 51

$$\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 2\frac{\partial^2 U}{\partial y \partial z}$$

$$\Phi = \mu\alpha^2\left[2\left\{\left(\frac{\partial^2 U}{\partial x^2}\right)^2 + \left(\frac{\partial^2 U}{\partial y^2}\right)^2 + \cdots\right\} + 4\left\{\left(\frac{\partial^2 U}{\partial y \partial z}\right)^2 + \cdots +\right\}\right] = -\mu\iint \frac{\partial(\text{[illegible]}^2)}{\partial n}\, dS$$

$$p_{xx} = -p + 2\mu\frac{\partial u}{\partial x} = -p + 2\mu\alpha\frac{\partial^2 U}{\partial x^2}$$

$$p_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = 2\mu\alpha\frac{\partial^2 U}{\partial z \partial y}$$

$$p_x = -p\cos nx + 2\mu\alpha\left[\frac{\partial^2 U}{\partial x^2}\cos nx + \frac{\partial^2 U}{\partial x \partial y}\cos ny + \frac{\partial^2 U}{\partial x \partial z}\cos nz\right]$$

$$\int p_x = 2\mu\alpha\int\underbrace{\left[\frac{\partial^2 U}{\partial x^2}\cos nx + \cdots\right]}_{}d\sigma$$

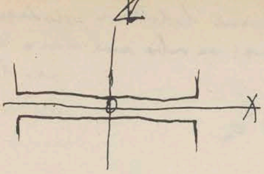
$$\int\frac{\partial}{\partial x}\cos nx + \frac{\partial}{\partial y}\cos ny + \quad = \int\frac{\partial}{\partial n}\frac{\partial U}{\partial x}\, d\sigma$$

wypadkowa ciśnień na kulę w kierunku $X = 2\mu\alpha\int\frac{\partial^2 U}{\partial x \partial n}\, d\sigma$

ponieważ jednak $\frac{\partial U}{\partial n} = 0$ wzdłuż całej powierzchni zatem $P = 0$?

~~$\int U.\nabla^2 U\, d\sigma = \int U\frac{\partial U}{\partial n}\, d\sigma - \int\left[\left(\frac{\partial U}{\partial x}\right)^2 + \cdots\right]$~~ $d\sigma$

Praca powierzchniowa $\int(p_x . u + p_y . v + p_z . w)\, d\sigma$



Dwa krążki ~~pr~~ zbliżające się w cieczy lepkiej

Zadanie dwuwymiarowe :

$$u = u_m \cdot \frac{4z(\delta - z)}{\delta^2}$$

$$\int_0^\delta u\,dz = u_m \cdot \frac{2}{3}\delta$$

Nieściśliwość : $2x\frac{d\delta}{dt} = \int_0^\delta u\,dz = \frac{2}{3}\delta\, u_m$

$$u_m = 3\frac{x}{\delta}\frac{d\delta}{dt} = \frac{3x}{\delta}\cdot c$$

$$u = 12 \cdot \frac{x\, z(\delta - z)}{\delta^3}\cdot c$$

$$\mu\nabla^2 u = \frac{\partial p}{\partial x} = -\frac{24 \cdot x c}{\delta^3}\mu$$

$$p = -12\frac{x^2 c\mu}{\delta^3} + \text{const}$$

$$p = -12(b^2 - x^2)\frac{c\mu}{\delta^3} + p_0$$

$$\int_{-b}^{+b} p\,dx = 12\frac{c\mu}{\delta^3}\cdot 2\left[b^3 - \frac{b^3}{3}\right] = 16 \cdot \frac{b^3 c\mu}{\delta^3}$$

Zaniedbuję oddziaływanie dodatkowe pochodzące z siły wywieranej przez ε na siebie same wobec U mogąc jako stałe w obrębie δ

$$\frac{\partial p}{\partial x} = -\varepsilon\frac{\partial U}{\partial x} + \mu\Delta^2 u$$
$$\frac{\partial p}{\partial y} = -\varepsilon\frac{\partial U}{\partial y} + \mu\Delta^2 v$$
$$\frac{\partial p}{\partial z} = -\varepsilon\frac{\partial U}{\partial z} + \mu\Delta^2 w$$

$$\frac{\partial p}{\partial n} = -\varepsilon\frac{\partial U}{\partial n} + \mu\Delta^2 v_n$$
$$\frac{\partial p}{\partial s} = -\varepsilon\frac{\partial U}{\partial s} + \mu\Delta^2 v_s$$

$$\Delta^2 p = -\underbrace{\left[\frac{\partial\varepsilon}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial\varepsilon}{\partial y}\frac{\partial U}{\partial y} + \dots\right]}\ \cancel{=0}$$

[przyjmując że ε zmienne tylko w kierunku ~~stycznym~~ normalnym U tylko w stycznym]

$$\cancel{= -\iint\varepsilon\frac{\partial U}{\partial n}\,do + \iiint\varepsilon\,\nabla^2 U\,dx\,dy\,dz}$$
$$= -\iint\varepsilon\frac{\partial U'}{\partial n}\,do + \iiint\varepsilon\,\nabla^2 U$$

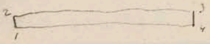
zatem będzie to tylko ważnem, dopóki się pomija V i v w porównaniu do U t.j. dla Endosmozy elektr.

$$= \int\nabla\varepsilon\,\nabla U'$$

$$= 0$$

zatem p ~~wogóle~~ może mieć Maxima i Minima tylko na powierzchni warstwy

A ponieważ $\Delta^2 p = 0$ także też gdzie $\varepsilon = 0$ więc wogóle Maxima Minima tylko w nieskończ. albo na powierzchni (metal) itd.



$$\int_1^2 + \int_2^3 = \int_1^4 + \int_4^3$$

~~[illegible]~~

Przy kuli w cieczy ni–

$$p = p_0 - \frac{3}{2}\mu\frac{ca}{r^3}x = p_0 - \frac{3}{2}\mu\frac{ca}{r^2}\cos\vartheta$$

$$\frac{\partial p}{\partial x} = \mu\Delta^2 v_x = +\frac{3}{}\mu\frac{ca}{r^3}\cos\vartheta \qquad \Delta^2 v_x = 3\frac{ca}{r^3}\cos\theta$$

W płaszczyźnie: $\qquad v = f.(n)$ 296

$$\frac{\partial p}{\partial s} = -\varepsilon \frac{\partial U}{\partial s} + \mu \Delta^2 v = -\varepsilon \frac{\partial U}{\partial s} + \mu \frac{\partial^2 v}{\partial n^2}$$

$$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial U}{\partial n}$$

$$+ \delta n \; \varepsilon_1 \frac{\partial U}{\partial n_1}$$

~~$p_0 + \delta n \; \varepsilon \frac{\partial U}{1\,\partial n_1} + \mu \frac{\partial^2 v}{\partial n^2_{ex}} \delta n = p_0 + \delta n \, \mu \frac{\partial^2 v}{\partial n^2_{int}} + \varepsilon \frac{\partial U}{\partial s_{int}}$~~

$$\frac{\partial^2 p}{\partial s \partial n} = -\varepsilon \frac{\partial^2 U}{\partial n \partial s} = -\varepsilon \frac{\partial^2 U}{\partial n \partial s} + \mu \frac{\partial^3 v}{\partial n^3} \underline{- \frac{\partial \varepsilon}{\partial n} \frac{\partial U}{\partial s}}!$$

$$\frac{\partial^3 v}{\partial n^3} = 0 \qquad \frac{\partial^2 v}{\partial n^2} = \text{const}$$

~~$\frac{dv}{dn} = c\,x + a$~~

~~$v = \frac{c x^2}{2} + a x + b$~~

~~$0 = c \frac{\delta^2}{2} + a \delta$~~

~~$a = -c \frac{\delta}{2}$~~

~~$v = \frac{c}{2}(x^2 - x\delta)$~~

~~$\frac{\partial p}{\partial s} = -\varepsilon \frac{\partial U}{\partial s} + \underline{\underline{c}}\,\mu$~~

~~$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial U}{\partial n}$~~

~~$p = -\int \varepsilon \frac{\partial U}{\partial n} dn + \int \varepsilon \frac{\partial U}{\partial s} ds + c s \mu$~~

~~$p = p_0 + \mu c x - \int \varepsilon \frac{\partial U}{\partial n} dn = p_0 + c x \mu - \frac{1}{4\pi} \frac{\partial \varphi}{\partial n} \frac{\partial U}{\partial n}$~~

I

v wcale nie będzie zmienione, U nie określone

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = 0$$

$$\frac{\partial^2}{\partial x^2}(p_{int}) = 0$$

$$p = \int \varepsilon \Big(\frac{\partial U}{\partial x} dx + \underbrace{\frac{\partial U}{\partial z}}_{=0} dz \Big) + \mu \int \frac{d^2 v}{dz^2} dx$$

$$= p_0 + \int \Big(\varepsilon \frac{\partial U}{\partial x} + \mu \frac{d^2 v}{dz^2} \Big) dx$$

$$\frac{d^2 v}{dz^2} = -\varepsilon \frac{\partial U}{\partial x} = -\frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial z^2} \frac{\partial U}{\partial x}$$

$$v = \frac{1}{4\pi}(\varphi_i - \varphi_a) \frac{\partial U}{\partial x} + \frac{c}{2}(z^2 - z\delta)$$

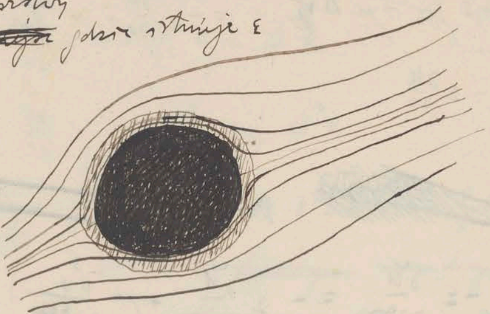
= Elektrische Endosmose

Równanie zasadnicze dla zjawiska odwrotnego: Strömungsströme

$\Delta^2 U = 0$ wszędzie z wyjątkiem ~~miejsc~~ warstwy gdzie istnieje ε

Na tej powierzchni będzie

$$\lambda_1 \frac{\partial U}{\partial n_1} = \lambda_1 \frac{\partial U}{\partial n_2} + \varepsilon \cdot v_n$$



Ogólne równanie wszędzie ważne:

$$\operatorname{div}\left[\lambda \nabla U \overset{+\Delta U' (= \text{„eingeprägte" Kraft})}{} + \varepsilon v\right] = 0$$

$$\lambda \nabla^2 U + \frac{\partial}{\partial x}(\varepsilon u) + \frac{\partial}{\partial y}(\varepsilon v) + \quad = 0$$

$$\underline{\lambda . \nabla^2 U + u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z} = 0} \quad \Bigg\| = -4\pi\lambda\,\varepsilon + \underbrace{u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + \ldots}_{} = 0$$

$$= \frac{D\varepsilon}{Dt} - \frac{\partial \varepsilon}{\partial t}$$

$$= \frac{D\varepsilon}{Dt} \quad (\text{w rozkładzie stałym})$$

$$\left\{ \frac{\partial p}{\partial x} = -\frac{1}{4\pi} \nabla^2 U \frac{\partial U}{\partial x} + \mu \Delta^2 u \right.$$

$$\underbrace{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}_{\text{Strömungslinien}} = \frac{d\varepsilon}{4\pi\lambda\varepsilon}$$

$$4\pi\lambda \cdot \log \varepsilon = \int \frac{dx}{u}$$

~~zatem ε funkcyą linij prądu ?!~~

$$\varepsilon = \varepsilon_0 \, e^{4\pi\lambda \int \frac{dx}{u}}$$

$$x - \frac{x a^3}{2 r^3}$$

$$\left(r - \frac{a^3}{2 r^2}\right) \cos\vartheta$$

$$1 - \frac{a^3}{r^3}$$

Więc jeśli w stanie początkowym ε' i temu odpowiadający U'

wtedy w stanie ruchu równanie:

~~$\lambda \nabla^2 U + u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z} +$~~

$$\operatorname{div} \left[\lambda \nabla (U - U') + \varepsilon \mathfrak{v} \right] = 0$$

$$\lambda \nabla^2 U + u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z} = \quad = -4\pi\lambda \varepsilon'$$

~~jeśli prędkości nie zanadto duże wtedy można stać~~

zatem:

$$4\pi\lambda(\varepsilon' - \varepsilon) + u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z} = 0$$

$$\varepsilon = \varepsilon' + \nu$$

$$4\pi\lambda \nu = u \frac{\partial \varepsilon'}{\partial x} + v \frac{\partial \varepsilon'}{\partial y} + w \frac{\partial \varepsilon'}{\partial z} + \underbrace{\left(u \frac{\partial \nu}{\partial x} + v \frac{\partial \nu}{\partial y} + w \frac{\partial \nu}{\partial z} \right)}_{\neq 0}$$

$$\boxed{\text{pot } \varepsilon = \text{pot } \varepsilon' + \text{pot } \nu + \Phi} \quad \text{zatem: } U = U' + V + \Phi$$

$$\text{pot } \nu = \int \frac{\nu}{r} d\omega = \frac{1}{4\pi\lambda} \int \frac{1}{r} \underbrace{\underbrace{\left[u \frac{\partial \varepsilon'}{\partial x} + \cdots \right]}_{\mathfrak{v} \cdot \nabla \varepsilon' \quad \underbrace{\cos(\mathfrak{v} \cdot \nabla \varepsilon')}_{\cos(n\mathfrak{v})}}}_{\mathfrak{v}_n \cdot \nabla \varepsilon'} d\omega$$

powiedzmy w pierwszem przybliżeniu

$$w = \frac{z^2}{2} \left(\frac{\partial u}{\partial z^2} \right)$$

$$= \frac{1}{4\pi\lambda} \int \frac{\mathfrak{v}_n \frac{\partial \varepsilon'}{\partial n}}{r} d\omega = \quad = \frac{1}{4\pi\lambda} \iint \frac{d\sigma}{r} \underbrace{\int \frac{\partial n^2}{2} \overset{\frac{\partial^2 \mathfrak{v}_n}{\partial n^2}}{\cancel{d\partial n}} \cdot \frac{\partial \varepsilon'}{\partial n}}_{\int \frac{z^2}{2} \frac{\partial \varepsilon'}{\partial z} dz}$$

$$\mathfrak{v}_n = \mathfrak{v}_{n0} + \frac{\partial n^2}{2} \frac{\partial^2 \mathfrak{v}_n}{\partial n^2} = \frac{\partial n^2}{2} \frac{\partial^2 \mathfrak{v}_n}{\partial n^2}$$

$$= \int \frac{\partial^3 U'}{\partial z^3} \frac{z^2}{2} dz$$

$$= \frac{\partial U}{\partial z^2} \frac{z^2}{2} - \int \frac{\partial^2 \overset{U'}{U'}}{\partial z^2} z\, dz =$$

$$U = U' + V + \Phi$$

zaniedbujące co prawda tylko jeśli V małe

$$V = \frac{\varphi_i - \varphi_a}{4\pi\lambda} \iint \frac{\partial^2 v_n}{\partial n^2} \cdot \frac{do}{r}$$

$$\frac{\partial p}{\partial n} = \overbrace{-\varepsilon \frac{\partial U}{\partial n}} + \mu \underbrace{\Delta^2 v_n}_{\frac{\partial^2 v_n}{\partial n^2} + \cancel{\frac{\partial^2 v_n}{\partial s^2} + \frac{\partial^2 v_n}{\partial t^2}}}$$

$$\neq \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} \iint \frac{1}{r} \frac{\partial p}{\partial n}\, do$$

~~zatem V także jak~~ ścisłem rozwiązaniem będzie: $V = \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} p$ ||

otrzymamy transformacyę tylko jeśli $\frac{\partial^2 v_n}{\partial n^2} = \frac{\partial p}{\partial n}$

t. j. jeśli $\frac{\partial U}{\partial n} = 0$

~~Tymczasem~~: Na powierzchni jeśli ciało izolujące, będzie:

zatem $\frac{\partial (V + \Phi)}{\partial n} = 0$

$$\lambda \frac{\partial (U - U')}{\partial n} = 0$$

$$\frac{\partial U}{\partial n} = \frac{\partial U'}{\partial n}$$

przy założeniu że $\Delta^2 v_n = \frac{\partial^2 v_n}{\partial n^2}$ ...!!!

$$\text{zatem: } \frac{\partial^2 v_n}{\partial n^2} = \frac{1}{\mu} \frac{\partial p}{\partial n} + \underbrace{\varepsilon \frac{\partial U'}{\partial n}}$$

zależne tylko od ~~x~~ δn, nie od s, t

$$V = \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} \iint \frac{1}{r} \frac{\partial p}{\partial n}\, do + \underbrace{\frac{\varphi_i - \varphi_a}{4\pi\lambda} \iint \frac{1}{r}\, \varepsilon \frac{\partial U'}{\partial n}\, do}_{f(z)}$$

rozwiązanie ścisłe:

$$V = \frac{\varphi_i - \varphi_a}{4\pi\lambda\mu} p + f(z)$$

~~$\frac{\partial v_s}{\partial s} = -c \cos\theta \left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right]$~~

$$v_s = -u \frac{x}{r} + v \frac{y}{r} = -u \sin\theta + v \cos\theta$$

$$= -c \sin\theta \left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right]$$

$$\frac{\partial v_s}{\partial r} = -c \sin\theta \left[\frac{3}{4}\frac{a}{r^2} + \frac{3}{4}\frac{a^3}{r^4}\right]$$

$$\frac{\partial^2 v_s}{\partial r^2} = +c \sin\theta \left[\frac{3}{2}\frac{a}{r^3} + 3\frac{a^3}{r^5}\right]$$

W tamtym rozkładzie $U = U' + V + \Phi$

oznacza zatem U' = pot. wewnętrzny podr. (w sobie)

Φ = zewnętrzny potencyał

V = zmiana wskutek ruchu mechan.

$$u = -\frac{3}{4}\frac{ca}{r}\left(1-\frac{a^2}{r^2}\right)\cos^2\theta + c\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$q = -\frac{3}{4}\frac{ca}{r}\left(1-\frac{a^2}{r^2}\right)\sin\theta\cos\theta$$

$$v_r = u\frac{z}{r} + v\frac{x}{r} = u\cos\theta + v\sin\theta$$

$$= -\frac{3}{4}\frac{ca}{r}\left(1-\frac{a^2}{r^2}\right)\cos\theta + c\cos\theta\left(1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}\right)$$

$$= c\cos\theta\left[1-\frac{3}{4}\frac{a}{r}-\frac{1}{4}\frac{a^3}{r^3}-\frac{3}{4}\frac{a}{r}+\frac{3}{4}\frac{a^3}{r^3}\right]$$

$$v_r = c\cos\theta\left[1-\frac{3}{2}\frac{a}{r}+\frac{1}{2}\frac{a^3}{r^3}\right]$$

$$\frac{\partial v_r}{\partial r} = c\cos\theta\left[\frac{3}{2}\frac{a}{r^2}-\frac{3}{2}\frac{a^3}{r^4}\right] \qquad \left.\frac{\partial v_r}{\partial r}\right|_{r=a} = 0$$

$$\frac{\partial^2 v_r}{\partial r^2} = c\cos\theta\left[-\frac{3a}{r^3}+\frac{6a^3}{r^5}\right] \qquad \left.\frac{\partial^2 v_r}{\partial r^2}\right|_{r=a} = \frac{3c\cos\theta}{a^2}.$$

$$p = p_0 - \frac{3}{2}\frac{\mu c a}{r^2}\cos\theta$$

$$\frac{\partial p}{\partial r} = 3\mu\frac{ca}{r^3}\cos\theta \qquad \left.\frac{\partial p}{\partial r}\right|_{r=a} = 3\frac{\mu c}{a^2}\cos\theta$$

$$\therefore\ \frac{\partial p}{\partial r} = \mu\frac{\partial^2 v_r}{\partial r^2}$$

$$\cancel{\Delta^2 v_s = \frac{1}{\mu}\frac{\partial p}{\partial s} = +\frac{3}{2}\frac{ca}{r^2}\sin\theta}$$

$$= \frac{1}{\mu}\frac{\partial p}{\partial\theta}\frac{\partial\theta}{\partial s} = \frac{3}{2}\frac{ca}{r^3}\sin\theta$$

$ds = r\,d\theta$

Równanie mechaniczne

Dwojaki ruch : jonowy
molarny

W stanie spoczynku :

~~$\frac{\partial p}{\partial x} = \varepsilon \frac{\partial}{}$~~

$u = v = w = 0$

$$U = U' \left[+ \Phi \right]$$ ← jeśli to się zgadza z warunkiem $u = v = w = 0$!]

$$\varepsilon = \varepsilon'$$

$$\frac{\partial p}{\partial x} = \varepsilon' \frac{\partial U'}{\partial x} = \varepsilon \frac{\partial U}{\partial x}$$

$$\left. \begin{aligned} \frac{\partial p}{\partial x} &= -\varepsilon \frac{\partial U}{\partial x} + \mu \Delta^2 u \\ \frac{\partial p}{\partial y} &= -\varepsilon \frac{\partial U}{\partial y} + \mu \Delta^2 v \\ \frac{\partial p}{\partial z} &= -\varepsilon \frac{\partial U}{\partial z} + \mu \Delta^2 w \end{aligned} \right\}$$

$$\Delta^2 p = -\left[\frac{\partial \varepsilon}{\partial x} \frac{\partial U}{\partial x} + \frac{\partial \varepsilon}{\partial y} \frac{\partial U}{\partial y} + \quad \right] - \varepsilon . \Delta^2 U$$

~~$p = \int$~~

$$\int \Delta^2 p . dv = -\int \underbrace{\quad - - \quad}_{} - \int \varepsilon . \Delta^2 U . dv$$

$$-\int \varepsilon \frac{\partial U}{\partial n} do + \int \varepsilon \Delta^2 U dv \qquad = -\int \varepsilon . \frac{\partial U}{\partial n} . do$$

$$= -\int \varepsilon \frac{\partial U'}{\partial n} do$$

Rozkład : ~~$p = p_1 + p_2$~~

~~$\frac{\partial p_1}{\partial x} = -\varepsilon \frac{\partial U'}{\partial x}$~~

~~$\frac{\partial p_2}{\partial x} = -\varepsilon \frac{\partial V}{\partial x} + \mu \Delta^2 u$~~ ~~$\frac{\partial p_3}{\partial x} = -\varepsilon \frac{\partial \Phi}{\partial x} + \mu \Delta^2 u_3$~~

~~$\frac{\partial p_1}{\partial y} = -\varepsilon \frac{\partial U'}{\partial y}$~~

~~$\frac{\partial p_2}{\partial x} = -\varepsilon \frac{\partial (V + \Phi)}{\partial x} + \mu \Delta^2 u$~~

~~$\frac{\partial p_2}{\partial y} = -\varepsilon \frac{\partial}{\partial y} \quad + \mu \Delta^2 v$~~

~~$\frac{\partial p_1}{\partial z} = -\varepsilon \frac{\partial U'}{\partial z}$~~

~~$\frac{\partial p_2}{\partial z} = -\varepsilon \frac{\partial}{\partial z} \quad + \mu \Delta^2 w$~~

~~$\Delta^2 p_1 = -\varepsilon\,\Delta^2 U' \left[\frac{\partial\varepsilon}{\partial x}\frac{\partial U'}{\partial x} + \dots\right]$~~

~~$\Delta^2 p_2 =$~~

~~pot~~

~~$p = z$~~

$$\varepsilon\,\Delta^2 U + \frac{\partial\varepsilon}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial\varepsilon}{\partial y}\frac{\partial U}{\partial y} + \frac{\partial\varepsilon}{\partial z}\frac{\partial U}{\partial z} = \Delta^2 p$$

z drugiej strony przedtem mieliśmy:

$$\varepsilon\,\lambda\,\Delta^2 U + \frac{\partial\varepsilon}{\partial x}\cdot u\varepsilon + \frac{\partial\varepsilon}{\partial y}\cdot v\varepsilon + \frac{\partial\varepsilon}{\partial z}\,w\varepsilon = -4\pi\lambda\,\varepsilon'\varepsilon$$

Czy nie możnaby stawić:

~~$\frac{\partial U'}{\partial x} = \frac{1}{\lambda}\,u\varepsilon$~~ ~~$\Delta^2 p = -4\pi\lambda\varepsilon\varepsilon$~~

~~$\Delta^2 U = \frac{1}{\lambda}\left(u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}\right)$~~

$$\frac{\partial(V+\Phi)}{\partial n} = 0$$

$$\Delta^2(V+\Phi) = u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

$p = p_1 + p_2$

$U = U' + V + \Phi$

$$\begin{cases} \frac{\partial p_1}{\partial x} = -\varepsilon\,\frac{\partial U'}{\partial x} \\ \frac{\partial p_1}{\partial y} = - \end{cases}$$

$$\frac{\partial p_2}{\partial x} = -\varepsilon\,\frac{\partial(V+\Phi)}{\partial x} + p\,\Delta^2 u$$

$$\frac{\partial p_2}{\partial y} =$$

$$\Delta^2 p_2 = -\left[\frac{\partial\varepsilon}{\partial x}\frac{\partial(V+\Phi)}{\partial x} + \dots\right] - \varepsilon\,\Delta^2(V+\Phi)$$

~~$p_2 = \int \frac{1}{r}\frac{\partial}{\partial n}\,\varepsilon$~~

$$\int_0 \Delta^2 p_2\,do = -\int \varepsilon\,\frac{\partial(V+\Phi)}{\partial n}\,do + \cancel{\int \varepsilon\,\Delta^2(V+\Phi) - \int \varepsilon\,\Delta^2(V+\Phi)}$$

$$= 0$$

Jaka jest wartość $\Delta^2 v_s$ przy powierzchni ? Czy równo $\frac{\partial^2 v_s}{\partial n^2}$?

$$\Delta^2 v_s = \left(\frac{\partial^2}{\partial n^2} + \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial t^2}\, v_s\right)$$

$$p = p_0 - \frac{3\mu c a}{r^2} \cos\theta \qquad \Big| \qquad v_r = c\left[1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right]\cos\theta$$

$$v_{r\theta} = -c\sin\theta\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right]$$

$$\frac{\partial}{\partial r} v_{r\theta} = -c\sin\theta\left[\frac{3}{4}\frac{a}{r^2} + \frac{3}{4}\frac{a^3}{r^4}\right] \qquad \Big|^{r=a} = -\frac{3}{2}\frac{c\sin\theta}{a}$$

$$\frac{\partial^2}{\partial r^2} v_{r\theta} = +c\sin\theta\left[\frac{3}{2}\frac{a}{r^3} + 3\frac{a^3}{r^5}\right] \qquad = \frac{9}{2}\frac{c\sin\theta}{a^2}$$

$$v = \sqrt{v_r^2 + v_{r\theta}^2} = c\sqrt{\left(1 + \frac{9}{4}\frac{a^2}{r^2} + \frac{1}{4}\frac{a^6}{r^6} - \frac{3}{2}\frac{a}{r} - \frac{3}{4}\frac{a^4}{r^4} + \frac{1}{2}\frac{a^3}{r^3}\right)\cos^2\theta \quad \left(1 + \frac{9}{16}\frac{a^2}{r^2} + \frac{1}{16}\frac{a^6}{r^6} - \frac{3}{4}\frac{a}{r} - \frac{3}{16}\frac{a^4}{r^4} - \frac{1}{4}\frac{a^3}{r^3}\right)\sin^2\theta}$$

$$= c\sqrt{1\qquad\qquad}$$

$$\frac{\partial p}{\partial x\,\partial y} \not\parallel \qquad \mu\,\Delta^2\underbrace{\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)} = \frac{\partial \varepsilon}{\partial y}\frac{\partial U}{\partial x} - \frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial y}$$

$$\mu\,\Delta^2\,\zeta \quad = \quad \nearrow$$

$$\mu\,\Delta^2 \operatorname{curl} v = \begin{vmatrix} i & j & k \\ \frac{\partial \varepsilon}{\partial x} & \frac{\partial \varepsilon}{\partial y} & \frac{\partial \varepsilon}{\partial z} \\ \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} & \frac{\partial U}{\partial z} \end{vmatrix}$$

~~[illegible]~~

Uproszczone zadanie:

$$\varepsilon \frac{\partial U}{\partial x} = \mu \Delta^2 u$$

$$\varepsilon \frac{\partial U}{\partial y} = \mu \Delta^2 v$$

$$\varepsilon \frac{\partial U}{\partial z} = \mu \Delta^2 w$$

w przestrzeni zewnętrznej u, v, w tak jak ciecz idealna niewirowa

Toroid ~~[illegible]~~ może się zgadzać

z równaniem $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

to mocy musiałby wszędzie być: ?

$$\varepsilon \Delta^2 U + \left(\frac{\partial \varepsilon}{\partial x}\frac{\partial U}{\partial x} + \dots\right) = 0$$

$$u = \int \frac{\varepsilon \frac{\partial U}{\partial x}}{4\pi r}\, dv + \psi$$

$$= \frac{1}{4\pi}\int_1 \frac{\partial U}{\partial x}\, dv \int \frac{\varepsilon}{r}\, dr + \frac{1}{4\pi}\int_2 \frac{\partial U}{\partial x}\, dv \int \frac{\varepsilon}{r}\, dr$$

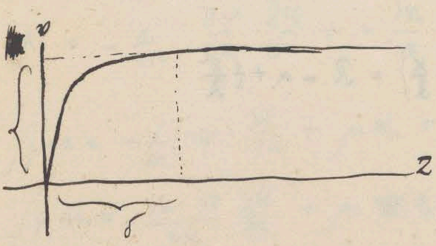
$$\varepsilon \frac{\partial U}{\partial n} = \mu \Delta^2 v_n = \mu\left[\frac{\partial^2 v_n}{\partial n^2} + \frac{\partial^2 v_n}{\partial s^2} + \frac{\partial^2 v_n}{\partial t^2}\right]$$

$$\int dz\, z\, \varepsilon \frac{\partial U}{\partial n} = \frac{\partial U}{\partial n}\frac{1}{4\pi}\int \frac{\partial^2 \varphi}{\partial z^2} z\, dz = \cancel{\frac{1}{4\pi}\frac{\partial U}{\partial n}(\varphi_i - \varphi_a)} = \mu\left[\int z \frac{\partial^2 v_n}{\partial z^2}\, dz + \int z \frac{\partial^2 v_n}{\partial s^2}\, dz + \int\right]$$

$$= \frac{1}{4\pi}\frac{\partial U}{\partial n}\left[\cancel{\frac{\partial \varphi}{\partial z} z} - \int \frac{\partial \varphi}{\partial z}\, dz\right] = \frac{1}{4\pi}\frac{\partial U}{\partial n}(\varphi_i - \varphi_a) =$$

$$= \mu\left\{\left[\frac{\partial v}{\partial z}\cdot z\Big|^{\delta} - \int_0^{\delta} \frac{\partial v}{\partial z}\, dz\right] + \left(\frac{\partial^2 v_n}{\partial s^2}\right)_{\frac{\delta}{2}} \frac{\delta^2}{2} + \left(\frac{\partial^2 v_n}{\partial t^2}\right)_{\frac{\delta}{2}} \frac{\delta^2}{2}\right\}$$

$$\int_0^{\delta} \varphi f\, dx = f_m \cdot \int_0^{\delta} \varphi\, dx$$



$$\int_0^{\delta} z \frac{\partial^2 v}{\partial z^2}\, dz = -\left[v_\delta - v_0\right]$$

$$= \cancel{[illegible]} = \delta \cdot M\left\{z \frac{\partial^2 v}{\partial z^2}\right\}$$

$$= \cancel{[illegible]} = \frac{\delta^2}{2} M\left\{\frac{\partial^2 v}{\partial z^2}\right\}$$

$$= \frac{\partial v}{\partial z}\Big|_0^{\delta} M(z) = -\frac{\partial v}{\partial z}\Big|_0 M(z)$$

w części 1: $\frac{\partial U}{\partial x}$ może być uważane za stałe i powierzchnię za płaską

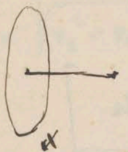
2: różnica w r tak mała w obszarze δ że $r = f.(s,t)$

$$u = \frac{\partial U}{\partial x}\int_0^0 d\rho \int \frac{\Sigma}{r}\,dn + \underbrace{\int_0^\infty \frac{1}{r}\frac{\partial U}{\partial x}\,d\rho \int \Sigma\, dn}_{} + \psi$$

$$\int \Sigma\, dn \downarrow \frac{1}{4\pi}\frac{\partial^2\varphi}{\partial n^2}$$

$$\frac{\frac{\partial\varphi}{\partial n}}{4\pi}\int_0^\infty \frac{1}{r}\frac{\partial U}{\partial x}\,d\rho$$

~~$\int_2 \Sigma \frac{dn}{r} = \frac{x}{r^2}\frac{\partial r}{\partial n}$~~



$$\int_0^R \frac{2\pi\rho\, d\rho}{\sqrt{x^2+\rho^2}} = 2\pi\sqrt{x^2+\rho^2}\Big|_0^R = 2\pi\sqrt{x^2+R^2} - 2\pi x$$

$$2\pi\int_0^x \left(\sqrt{R^2+x^2}-x\right)\frac{dx}{4\pi}\frac{\partial^2\varphi}{\partial x^2}$$

$$= \frac{1}{2}\frac{\partial\varphi}{\partial x}\left[\sqrt{R^2+x^2}-x\right]\Big|_0^x - \frac{1}{2}\int \frac{\partial\varphi}{\partial x}\left[\frac{x}{\sqrt{R^2+x^2}} - 1\right]dx$$

$$= \frac{1}{2}\left[\frac{\partial\varphi}{\partial x}\Big|_0\left(\sqrt{R^2+x^2}-x\right) - R\,\frac{\partial\varphi}{\partial x}\Big|_x\right] - \uparrow$$

$$+ \frac{\partial\varphi}{\partial x}$$

$$\sqrt{R^2+x^2} - x = R\left\{\left[1+\left(\frac{x}{R}\right)^2\right]^{\frac{1}{2}} - \frac{x}{R}\right\} = R\left\{1 + \frac{1}{2}\frac{x^2}{R^2} - \frac{x}{R}\right\} = R - x + \frac{1}{2}\frac{x^2}{R}$$

W każdym razie ważne:

$$\varepsilon \frac{\partial U}{\partial n} = \mu \frac{\partial^2 v_n}{\partial n^2} = \frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial n^2} \frac{\partial U}{\partial n}$$

$$\mu \frac{\partial v_n}{\partial n}\Big|^{\delta} = \frac{1}{4\pi} \frac{\partial \varphi}{\partial n}\Big|_0^{\delta} \frac{\partial U}{\partial n}$$

$$\int\left(\frac{\partial \varphi}{\partial n_\delta} - \frac{\partial \varphi}{\partial n_0}\right) dn = \varphi_a - \varphi_i - \delta \cdot \frac{\partial \varphi}{\partial n_i}$$

$$\int n \, dn \; --$$

$$\mu v_n\Big|^{\delta} = \frac{1}{4\pi} (\varphi_i - \varphi_a) \frac{\partial U}{\partial n}$$

} to wystarczy do oznaczenia ponieważ w reszcie przestrzeni będzie $\Delta^2 \psi = 0$

~~$\Delta^2 \varphi = 0$~~ ~~$\frac{\partial \varphi}{\partial n}$~~

$$\begin{cases} \Delta^2 \psi = 0 \\ \dfrac{\partial \psi}{\partial n}\Big|^{\delta} = \dfrac{\varphi_i - \varphi_a}{4\pi\mu} \dfrac{\partial U}{\partial n} \end{cases}$$

$u = \frac{\partial \psi}{\partial x}$ $v =$ $w =$

Ale to ważne tylko jeśli rzeczywiście takie rozwiązanie jednoznacznie możliwe bo nie wiadomo jeszcze czy styczne składowe odpowiednie.

Czy nie można tego samego rozwiązania użyć w ogólniejszym przykładzie

$$\Delta^2 v_n = \frac{\partial^2 v_n}{\partial z^2} + \left(\frac{1}{R} + \frac{1}{R_1}\right) \frac{\partial v_n}{\partial z}$$

$$\frac{\partial p}{\partial n} = -\varepsilon \frac{\partial U}{\partial n} + \mu \Delta^2 v_n$$

$$\frac{\partial p}{\partial z} = -\frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial z^2} \frac{\partial U}{\partial n} + \mu \frac{\partial^2 v_n}{\partial z^2}$$

$\int dz$

$$p = -\frac{1}{4\pi} \frac{\partial \varphi}{\partial z} \frac{\partial U}{\partial n} + \mu \frac{\partial v_n}{\partial z} + \text{const}$$

$\int dz$

$$\int p \, dz = -\frac{1}{4\pi} \varphi \cdot \frac{\partial U}{\partial n} + \mu v_n + z.\text{const} + \text{const}'$$

$$\underbrace{\int_0^{\delta} p \, dz}_{= p_m \cdot \delta} = \frac{\varphi_a - \varphi_i}{4\pi} \frac{\partial U}{\partial n} + \mu (v_{n\delta} - \underset{0}{\overset{\shortparallel}{v_{n0}}}) + \delta.\text{const}$$

to nie wystarczy jednak do oznaczenia ruchu bo teraz mamy $\mu \Delta^2 v_n = -\frac{\partial p}{\partial x}$ etc.

↓

$$\mu v_{n\delta} = \frac{\varphi_a - \varphi_i}{4\pi} \frac{\partial U}{\partial n}$$

$$v_n = \frac{\varphi_a - \varphi_i}{4\pi\mu} \frac{\partial U}{\partial n}$$

Czy możliwy rozkład

$u = u_1 + u_2$

$v = v_1 + v_2$

$w = w_1 + w_2$

Takie, że $p_1 = 0$ poza warstwą

~~p_1~~

$$\frac{\partial p_1}{\partial x} + \varepsilon \frac{\partial U}{\partial x} = \mu \Delta^2 u_1 \qquad \frac{\partial p_2}{\partial x} = \mu \Delta^2 u_2$$

$$\frac{\partial p_1}{\partial y} + \varepsilon \frac{\partial U}{\partial y} = \mu \Delta^2 v_1 \qquad \frac{\partial p_2}{\partial y} = \mu \Delta^2 v_2$$

$$\underbrace{\frac{\partial p_1}{\partial z} + \varepsilon \frac{\partial U}{\partial z} = \mu \Delta^2 w_1}_{} \qquad \frac{\partial p_2}{\partial z} = \mu \Delta^2 w_2$$

poza warstwą musiałby to być ruch potencyalny; będzie oznaczony i możliwy jeśli prędkości na powierzchni takie jakie

$\Delta^2 \psi = 0 \qquad u = \frac{\partial \psi}{\partial x}$; odpowiadają zadaniu potencyalnemu.

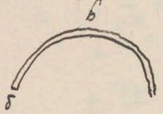
Co do prędkości normalnych mamy

$$v_n = \frac{\partial \psi}{\partial n} = \frac{\varphi_a - \varphi_i}{4\pi\mu} \frac{\partial U}{\partial n}$$

| Gdyby założenie $\lambda \frac{\partial U}{\partial n} = 0$ to by było $v_n = 0$

To prawda bo v_n będzie do zaniedbania w porównaniu z prędkościami stycznemi jeśli δ dostatecznie małe

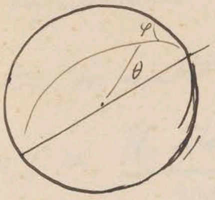
$$v_n = \cancel{v_{n0}} + z \left(\frac{\partial v_n}{\partial z}\right)_0 + \frac{z^2}{2} \left(\frac{\partial^2 v_n}{\partial z^2}\right)_0$$



v_n do v_δ będzie w stosunku $\delta : b$

porównanie ciągłości : $v_n . b = v_\delta . \delta$

$v_n : v_\delta = \delta : b$



$$\Delta^2(u) = \frac{1}{r}\left[\frac{\partial^2(ru)}{\partial r^2} + \frac{1}{r\sin\theta}\,\frac{\partial\left[\sin\theta\,\frac{\partial u}{\partial\theta}\right]}{\partial\theta} + \frac{1}{r\sin^2\theta}\,\frac{\partial^2 u}{\partial\varphi^2}\right]$$

$$r = a + z$$

$$\Delta^2(v) = \frac{1}{a}\left[\frac{\partial^2}{\partial z^2}\left[(a+z)v\right]\right] + \frac{1}{a\sin\theta}\,\underbrace{\frac{\partial}{\partial\theta}\left[\sin\theta\,\frac{\partial v}{\partial\theta}\right]}_{\left[\sin\theta\,\frac{\partial^2 v}{\partial\theta^2} + \cos\theta\,\frac{\partial v}{\partial\theta}\right]} + \underset{0}{\underset{\parallel}{\cancel{\frac{1}{a\sin\theta}\,\frac{\partial^2 v}{\partial\varphi^2}}}}$$

$$\frac{\partial}{\partial z}\left[(a+z)v\right]$$

$$= \frac{\partial v}{\partial z}(a+z) + v$$

$$\frac{\partial^2}{\partial z^2} = \frac{\partial^2 v}{\partial z^2}(a+z) + 2\frac{\partial v}{\partial z}$$

$$\Delta^2(v) = \left(1+\frac{z}{a}\right)\frac{\partial^2 v}{\partial z^2} + \frac{2}{a}\frac{\partial v}{\partial z} + \frac{1}{a}\frac{\partial^2 v}{\partial\theta^2} + \frac{1}{a}\frac{1}{\operatorname{tg}\theta}\frac{\partial v}{\partial\theta}$$

$$v_{r\theta} = -c\sin\theta\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right]$$

$$\left.\begin{aligned} \frac{\partial}{\partial\theta} &= \cos\theta\left[\;-\;\text{---}\;\right] \\ \frac{\partial^2}{\partial\theta^2} &= \sin\theta\left[\qquad\right] \end{aligned}\right\} = 0 \qquad /r=a$$

$$\Delta^2(v)_{r\theta} = \frac{\partial^2 v_{r\theta}}{\partial z^2} + \frac{2}{a}\frac{\partial v_{r\theta}}{\partial z} =$$

istotnie

$$\Delta^2 = \frac{3}{2}\,ca\frac{\sin\theta}{r^3}\Big|^{a} = \frac{3}{2}\frac{c}{a^2}\sin\theta$$

$$\left.\begin{aligned} \frac{\partial}{\partial z} &= -\frac{3}{2}\,\frac{c\sin\theta}{a} \\ \frac{\partial^2}{\partial z^2} &= \frac{9}{2}\,\frac{c\sin\theta}{a^2} \end{aligned}\right\} \quad \frac{\partial^2}{\partial z^2} + \frac{2}{a}\frac{\partial}{\partial z} = \frac{3}{2}\,c\frac{\sin\theta}{a^2}$$

Zatem, przynajmniej dla Kuli: $\Delta^2 v_{r\theta} = \frac{\partial^2 v_{r\theta}}{\partial z^2} + \frac{2}{a}\frac{\partial v_{r\theta}}{\partial z}$

$$\int_0^{\delta} \Delta^2\, z\,dz = \int_0^{\delta}\frac{\partial^2}{\partial z^2}\,z\,dz + \frac{2}{a}\int_0^{\delta}\frac{\partial}{\partial z}\,z\,dz = v_0 - v_\delta + \underbrace{\frac{2}{a}\; v\Big|_0^{\delta} M(z)}_{(v_\delta - v_0)\frac{\delta}{a}}$$

$$\lim (v_\delta - v_0)\frac{\delta}{a} = 0$$

Zatem istotnie:

$$\underbrace{\int_0^\delta \frac{\partial p}{\partial s}\, z\, dz}_{\lim = 0} = -\frac{1}{4\pi}\frac{\partial U}{\partial s}\int_0^\delta \frac{\partial^2 \varphi}{\partial z^2}\, z\, dz + \mu\left(v_0 - v_\delta\right)$$

$$v_\delta = \frac{\varphi_i - \varphi_a}{4\pi\mu}\,\frac{\partial U}{\partial s}$$

ważne pod założeniem: 1). $\lim \frac{\delta}{a} = 0$ 2). $v_0 = 0$ 5). $\varepsilon = \frac{\partial^2\varphi}{\partial z^2}$

3). $\frac{\partial U}{\partial s}$ = const w obrębie δ 4). μ skończone

---

ruchu laminarnego:

$U = U' + V + \Phi$ przy powierzchni: $\frac{\partial}{\partial n}(V+\Phi) = 0$

$$\frac{\partial U}{\partial n} = \frac{\partial U'}{\partial n}$$

$$\frac{\partial U'}{\partial s} = 0$$

$$\frac{\partial U}{\partial s} = \frac{\partial(V+\Phi)}{\partial s}$$

~~Zatem w przestrzeni zewnętrznej:~~ ~~$V = \frac{\varphi_i - \varphi_a}{4\pi\mu}$~~

~~$$\begin{cases} \frac{\partial p}{\partial x} = \mu\Delta^2 u \\ \frac{\partial p}{\partial y} = \mu\Delta^2 v \\ \frac{\partial p}{\partial z} = \mu\Delta^2 w \end{cases}$$~~

właściwie w poprzednim rachunku:

$$\varepsilon = \frac{1}{4\pi}\Delta^2 U = \frac{1}{4\pi}\left[\frac{\partial^2}{\partial z^2} + \frac{2}{\partial\vartheta} \dots\right]$$

Poprzedni wywód wymaga: $\frac{\partial^2 U}{\partial s\,\partial n} = 0$ co spełnione z powodu $\cancel{\frac{\partial U}{\partial n}}$ $\frac{\partial U'}{\partial s} = 0$ $\frac{\partial}{\partial n}(V+\Phi) = 0$

$$= \frac{\partial^2 U'}{\partial s\,\partial n} + \frac{\partial^2(V+\Phi)}{\partial s\,\partial n}$$

a przy całkowaniu $\int \varepsilon z\,dz$ i lim. odpadają inne wyrażenia

zatem ów rezultat będzie ściśle ważny.

$$v_s\delta = \frac{U_i - U_0}{4\pi\mu}\frac{\partial U}{\partial s} = \frac{U_i - U_0}{4\pi\mu}\frac{\partial}{\partial s}(V+\Phi)$$

w warstwie δ różnica V, Φ będzie jednak mała w porównaniu do $\varphi_i - \varphi_0$

$$v_s\delta = \frac{\varphi_i - \varphi_0}{4\pi\mu}\frac{\partial}{\partial s}(V+\Phi) = \frac{\varphi_i - \varphi_0}{4\pi\mu}\left[\frac{\varphi_i - \varphi_0}{4\pi\lambda\mu}\frac{\partial p}{\partial s} + \frac{\partial \Phi}{\partial s}\right]$$

$$V = \frac{\varphi_i - \varphi_0}{4\pi\lambda\mu}\, p + f(x)$$

Zatem zadanie : $\nabla^2\Phi = 0$ w wielkiej odległości $\frac{\partial \Phi}{\partial x} = 0$

$$\begin{cases} \frac{\partial p}{\partial x} = \mu\nabla^2 u \\ \frac{\partial p}{\partial y} = \mu\nabla^2 v \\ \frac{\partial p}{\partial z} = \mu\nabla^2 w \end{cases} \qquad v_{s0} = \frac{\varphi_i - \varphi_0}{4\pi\mu}\left[\frac{\varphi_i - \varphi_0}{4\pi\lambda\mu}\frac{\partial p}{\partial s} + \frac{\partial \Phi}{\partial s}\right]$$

$$v_n\delta = 0$$

$$\therefore \nabla^2 p = 0$$

oznaczając $\frac{\varphi_i - \varphi_0}{4\pi\lambda\mu} p + \Phi$ przez Ψ $\qquad \Delta^2\Psi = 0$

jest warunek graniczny $v_{s0} = \frac{\varphi_i - \varphi_0}{4\pi\mu}\frac{\partial \Psi}{\partial s}$

dotąd $v_{n0} = 0$

---

zatem będzie rozwiązanie możliwe $v = \nabla\Psi \cdot \frac{\varphi_i - \varphi_0}{4\pi\mu}$

ogólniej : $v = \frac{\varphi_i - \varphi_0}{4\pi\mu}\nabla\Psi + v_0$ $\qquad v_{s0} = 0$

$v_{n0} = 0$

$$\begin{cases} \frac{\partial p}{\partial x} = \mu\nabla^2 u_0 \\ \ldots \end{cases} \qquad \Delta^2 p = 0$$

zwykłe tarcie

Zatem znając dla danego kształtu powierzchni etc. rozwiązanie ~~potencyjalne~~ przewodnictwa elektrycznego ψ i tarcia wewnętrznego v_0 można superponować:

$$v = \frac{\varphi_i - \varphi_0}{4\pi\mu} \nabla\psi + v_0$$

dane doświadczalne będą: ciśnienia p_1, p_2

potencyał obserwowany elektrometrycznie A_1 A_2

lub galwanometrycznie D_1 D_2

ponieważ ~~$\frac{\partial \Phi}{\partial n}$~~ ~~$\lambda \nabla \Phi = i$~~ ~~zatem~~

$$i = \lambda \nabla (U - U') + \varepsilon v$$

$$= \lambda \nabla (V + \Phi) + \varepsilon v$$

$$\int i_n \, dw = \lambda \int \frac{\partial}{\partial n}(V + \Phi)\, dw \underset{\tilde{q}}{+} \underbrace{\int \frac{1}{4\pi} \frac{\partial^2 \varphi}{\partial z^2} v \, dw}$$

$$= \int \frac{ds}{4\pi} \int \frac{\partial^2 \varphi}{\partial z^2} \, z \left(\frac{\partial v_1}{\partial z}\right) dz$$

$$= \int \frac{ds}{4\pi} (\varphi_i - \varphi_0)\left(\frac{\partial v_0}{\partial z}\right)$$

(A obserw. np. w drugim połączeniu w którym p bardzo duże)

$$\rho\, w \frac{\partial w}{\partial z} = \rho g - \frac{\partial p}{\partial z} + \frac{\mu}{3}\frac{\partial}{\partial z}\,\mathrm{div} + \mu\,\Delta^2 w$$

$$\frac{\partial(\rho w)}{\partial z} = 0$$

$$w\frac{\partial p}{\partial z} + k p\frac{\partial w}{\partial z} = (k-1)\mu\left[\frac{4}{3}\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] + (k-1)\,\kappa\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial x} = \frac{dw}{dr}\frac{x}{r}$$

$$\left.\begin{aligned}\frac{\partial^2 w}{\partial x^2} &= \frac{d^2 w}{dr^2}\frac{x^2}{r^2} + \frac{dw}{dr}\frac{1}{r} - \frac{dw}{dr}\frac{x^2}{r^3}\\ \frac{\partial^2 w}{\partial y^2} &= \frac{d^2 w}{dr^2}\frac{y^2}{r^2} + \frac{dw}{dr}\frac{1}{r} - \frac{dw}{dr}\frac{y^2}{r^3}\end{aligned}\right\}\quad \Delta^2 = \frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)$$

$$\cancel{\frac{\partial}{\partial r}(r w) = w + r\frac{dw}{dr}}$$

$$\cancel{\frac{\partial^2}{\partial r^2} =}$$

$$\rho\, w\frac{\partial w}{\partial z} = \rho g - \frac{\partial p}{\partial z} + \frac{4\mu}{3}\frac{\partial^2 w}{\partial z^2} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

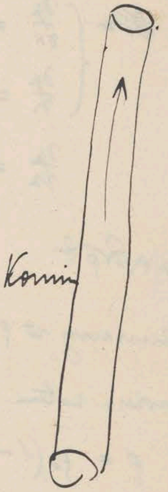
$$\frac{\partial}{\partial z}(\rho w) = 0$$

$$w\frac{\partial p}{\partial z} + k p\frac{\partial w}{\partial z} = (k-1)\mu\left[\frac{4}{3}\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2\right] + (k-1)\,\kappa\left[\frac{\partial^2\theta}{\partial z^2} + \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right]$$

Pytanie, czy to może być rozwiązaniem pod założeniem

$\cancel{\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y}}$ $\frac{\partial p}{\partial r} = 0$ $(u = v = 0)$

1). Pomijamy $\frac{\partial^2 w}{\partial z^2}$ i ~~[illegible]~~ Φ i $\frac{\partial^2\theta}{\partial z^2}$ i $w\frac{\partial w}{\partial z}$:



$$\rho w = \mathrm{fc.}(r)$$

$$\cancel{\left(\rho w\frac{\partial w}{\partial z}\right)} = \rho g - \frac{dp}{dz} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right)$$

$$w\frac{\partial p}{\partial z} + k p\frac{\partial w}{\partial z} = (k-1)\,\kappa\left[\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right]$$

W równaniu termicznem:

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=(k-1)\,\Phi+(k-1)\,\kappa\,\Delta^2\theta$$

co pochodzi z ciepła właściwego?

$$c\rho\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)=c\left[\frac{\partial}{\partial x}(\rho u\theta)+\frac{\partial}{\partial y}(\rho v\theta)+\dots\right]-c\,\theta\,\underbrace{\cancel{\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]}}_{=0}$$

$$=\frac{c}{R}\left[\frac{\partial}{\partial x}(pu)+\frac{\partial}{\partial y}(pv)+\frac{\partial}{\partial z}(pw)\right]$$

Biorąc c_v otrzymałoby się: $\frac{c_v}{R}=\frac{1}{k-1}$

zatem: $u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+p\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=(k-1)\,\kappa.\Delta^2\theta$

Biorąc zaś c_p:

$$k\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\right)+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=(k-1)\,\kappa.\Delta^2\theta$$

Więc tutaj przypadek pośredni, między c_v i c_p.

$$\begin{cases}\frac{\partial p}{\partial x}=\frac{\mu}{3}\frac{\partial}{\partial x}\,\mathrm{div}+\mu\Delta^2 u\\ \frac{\partial p}{\partial y}=\\ \frac{\partial p}{\partial z}=\rho g+\frac{\mu}{3}\frac{\partial}{\partial z}\,\mathrm{div}+\mu\Delta^2 w\end{cases}\qquad \frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

~~$p = R\rho\theta$~~ Jeśli p znaczne to przy wszystkich prądach termokonwekcyjnych zmiany w ρ będą prawie wyłącznie pochodziły z zmiany w θ

można zatem stawić

$$\rho=\rho_0(1-\alpha\theta)$$

$$\cancel{p}\,\mathrm{div}\,\cancel{w} = \alpha\cancel{\rho_0}\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right]$$

$$\frac{\partial}{\partial x}\,\mathrm{div} = \alpha\left[\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}+\cdots\quad + u\frac{\partial^2\theta}{\partial x^2}+\cdots\right]$$

Zapomnijmy tymczasowo, że to małe w obec $\Delta^2 u$ etc.

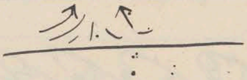
$$\left.\begin{aligned}\frac{\partial p}{\partial x} &= \mu\,\Delta^2 u\\ \frac{\partial p}{\partial y} &= \mu\,\Delta^2 v\\ \frac{\partial p}{\partial z} &= \rho g + \mu\,\Delta^2 w \;=\; f(r) + \mu\,\Delta^2 w\end{aligned}\right\}\quad \Delta^2 p = \frac{\partial f(r)}{\partial z}\;\cancel{+\mu\,\Delta^2\,\mathrm{div}}$$

Zupełne równania wypadkowe:

$$\Delta^2 p = \frac{4}{3}\mu\,\Delta^2\,\mathrm{div} + g\frac{\partial\rho}{\partial z}\qquad\qquad p = R\rho\theta$$

$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}+k\,p\,\mathrm{div} = (k-1)\,\kappa\,\Delta^2\theta$$

Dwurozmiarowe zadanie powolne:



$$u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+k\,p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = (k-1)\,\kappa\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)$$

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2} = g\frac{\partial\rho}{\partial y}$$

$$\frac{\partial p}{\partial x} = \mu\,\Delta^2 u$$

$$\frac{\partial p}{\partial y} = \mu\,\Delta^2 v + \rho g$$

Ogólniej: czy może być rozwiązanie $p = \text{const}$ przy takich prądach?

$$\begin{cases} \cancel{\rho\theta} = \frac{\mu}{3}\frac{\partial}{\partial x}\,\text{div} + \mu\,\Delta^2 u + \rho X \\ 0 = \frac{\mu}{3}\frac{\partial}{\partial y}\,\text{div} + \mu\,\Delta^2 v + \rho Y \\ 0 = \qquad\qquad\qquad\qquad + \rho Z \end{cases} \qquad p_0 = R\rho\theta$$

$$\cancel{k}\, p_0\, \text{div} = (k-1)\,\Phi + (k-1)\kappa\,\Delta^2\theta$$

$$0 = \frac{4\mu}{3}\Delta^2\,\text{div} + \frac{\partial}{\partial x}(\rho X) + \frac{\partial}{\partial y}(\rho Y) + \frac{\partial}{\partial z}(\rho Z)$$

~~Dan~~ Pomijając Φ: $\quad \text{div} = \frac{k-1}{k\,p_0}\cancel{p}\,\kappa \cdot \Delta^2\theta$

N.p. przy ciężkości:

$$0 = \frac{4\mu}{3}\Delta^2\,\text{div} + g\frac{\partial\rho}{\partial z} \qquad \rho = \frac{p_0}{R\theta}$$

$$\cancel{\frac{4\mu}{3}\cdot\frac{k-1}{k\,p_0}\cdot\kappa\cdot\Delta^2\theta = g\frac{p_0}{R}\frac{\partial}{\partial z}\frac{\partial\theta}{\partial z}}$$

$$\left.\begin{aligned} 0 &= \frac{\mu}{3}\frac{\partial}{\partial x}\cdot\text{div} + \mu\,\Delta^2 u \\ 0 &= \frac{\mu}{3}\frac{\partial}{\partial y}\cdot\text{div} + \mu\,\Delta^2 v \\ 0 &= \frac{\mu}{3}\frac{\partial}{\partial z}\cdot\text{div} + \mu\,\Delta^2 w + \rho g \end{aligned}\right\} \quad \left.\begin{aligned} \frac{\partial\rho}{\partial z}\,g &= -\frac{4}{3}\mu\,\Delta^2\,\text{div} \\ \frac{\partial\rho}{\partial y}\,g &= -\mu\,\Delta^2\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ \frac{\partial\rho}{\partial x}\,g &= -\mu\,\Delta^2\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) \end{aligned}\right\}$$

$$\cancel{\rho\,\text{div} + \mu\frac{\partial\rho}{\partial x}} \qquad g\,\Delta^2\rho = -\mu\,\Delta^2\left[\Delta^2 w - \frac{\partial}{\partial z}\,\text{div} + \frac{4}{3}\frac{\partial}{\partial z}\,\text{div}\right]$$

$$= -\mu\,\Delta^2\left[\Delta^2 w + \frac{1}{3}\frac{\partial}{\partial z}\,\text{div}\right]$$

natur. div!

Czy jest jaki ruch możliwy $v = w = 0$

~~$\rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{\mu}{3} \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial^2 u}{\partial z^2}$~~

$$\rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \frac{\mu}{3} \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \mu \Delta^2 u$$

$$\frac{\partial p}{\partial y} = \frac{\mu}{3} \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial p}{\partial z} = \frac{\mu}{3} \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial(\rho u)}{\partial x} = 0$$

$$u \frac{\partial p}{\partial x} + k p \frac{\partial u}{\partial x} = (k-1)\left[+\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] + (k-1)\kappa \, \Delta^2 \vartheta$$

Czy jest możliwe np. $p = p_0 + \frac{\mu}{3} \frac{\partial u}{\partial x}$?

$$\rho u \frac{\partial u}{\partial x} = \mu \Delta^2 u \qquad \rho u \frac{\partial^2 u}{\partial x^2} = \mu \Delta^2 \frac{\partial u}{\partial x}$$

$$u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} = \frac{\mu}{3}\left[u \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \dots\right] + \mu\left[\underbrace{u \Delta^2 u + v \Delta^2 v + w \Delta^2 w}\right]$$

$$= \frac{\partial}{\partial x}\left(u \frac{\partial u}{\partial x}\right) - \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial}{\partial y}\left(u \frac{\partial u}{\partial y}\right) - \left(\frac{\partial u}{\partial y}\right)^2 + \dots$$

$$+$$

$$= \Delta^2 \frac{(u^2 + v^2 + w^2)}{2} - \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \dots + \left(\frac{\partial v}{\partial x}\right)^2 + \dots\right]$$

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ \frac{\partial p}{\partial y} &= \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{aligned}\right\} \qquad \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$$

~~[illegible]~~ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\left.\begin{aligned} &\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\,\mathrm{div} + \mu\Delta^2 u \\ \text{I).}\quad &\frac{\partial p}{\partial y} = \quad -- \\ &\frac{\partial p}{\partial z} = \rho g + \frac{\mu}{3} - \end{aligned}\right\} \quad \Delta^2 p = \frac{4\mu}{3}\,\Delta^2\mathrm{div} + g\frac{\partial\rho}{\partial z}$$

$$p = R\rho\theta \qquad \rho = \frac{p}{R\theta}$$

$$\text{II).}\quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \quad \cdots\cdots$$

$$\text{III).}\quad u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + k\,p\,\mathrm{div} = \cancel{(k-1)\,\Phi} + (k-1)\,\kappa\,\Delta^2\theta$$

$$\frac{\partial}{\partial x}\left(\frac{pu}{\theta}\right) + -- \qquad = 0$$

$$\frac{p}{\theta}\,\mathrm{div} + \frac{1}{\theta}\left(u\frac{\partial p}{\partial x} + --\right) - \frac{p}{\theta^2}\left(u\frac{\partial\theta}{\partial x} + -- \right) = 0$$

$$p\,\mathrm{div} + \left(u\frac{\partial p}{\partial x} + ---\right) = \frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)$$

$$\text{III). a).}\quad p\,\mathrm{div} = \frac{-1}{k-1}\,\frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) + \kappa\,\Delta^2\theta$$

$$\text{b).}\quad u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} = \frac{-k}{k-1}\,\frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) + \kappa\,\Delta^2\theta$$

W każdym razie: $\frac{1}{p}\frac{\partial p}{\partial x}$ bardzo mało w porównaniu do $\frac{1}{\theta}\frac{\partial\theta}{\partial x}$ i tem mniejsze czem wyższe p_0, zatem z wielkim przybliżeniem:

$$p\,\mathrm{div} = \frac{p}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)$$

$$\text{II).}\quad \mathrm{div} = \frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$u\frac{\partial\theta}{\partial x} - \theta\frac{\partial u}{\partial x} + v\frac{\partial\theta}{\partial y} - \theta\frac{\partial v}{\partial y} + w\frac{\partial\theta}{\partial z} - \theta\frac{\partial w}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{u}{\theta}\right) + \frac{\partial}{\partial y}\left(\frac{v}{\theta}\right) + \frac{\partial}{\partial z}\left(\frac{w}{\theta}\right) = 0$$

III b): $\frac{k}{k-1}\frac{p}{\theta}\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right] = \kappa.\Delta^2\theta = \frac{k}{k-1}\,p\,\mathrm{div}$

I). $$\Delta^2 p = \frac{4\mu}{3}\Delta^2\mathrm{div} + \underbrace{\frac{g}{R\theta}\frac{\partial p}{\partial z}} - \underbrace{\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}} \mp \frac{4\mu}{3}\Delta^2\mathrm{div} - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial x}$$

małe w porównaniu do ↑

$p = \frac{4\mu}{3}\mathrm{div} - \mathrm{pot}\,\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z} + A$ $\qquad \nabla^2 A = 0$

~~$\mu\frac{\partial}{\partial x}\mathrm{div} - \frac{\partial}{\partial x}\mathrm{pot} + \frac{\partial A}{\partial x} = \mu\Delta^2 u$~~

I). $\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\mathrm{div} + \mu\Delta^2 u$

$\frac{\partial p}{\partial y} = \frac{\mu}{3}\frac{\partial}{\partial y}\mathrm{div} + \mu\Delta^2 v$

$\frac{\partial p}{\partial z} = \frac{\mu}{3}\frac{\partial}{\partial z}\mathrm{div} + \mu\Delta^2 w - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}$ }

~~$\frac{b}{m} = \alpha\frac{p}{m} = \frac{bm}{Rt}$~~ ~~$bm = \beta\frac{t}{m}$~~

$\mu\Delta^2\xi = -\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial x}$

$\mu\Delta^2\eta = -\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial y}$

$\Delta^2\zeta = 0$

Graniczny przypadek

1).. $k = \infty$

III). $\Delta^2\theta = 0$ $\qquad \theta = f\,xyz$ ---

$\left\{\begin{matrix}\mathrm{II}\\ \mathrm{I}\end{matrix}\right.$ $\qquad$ Przybliżenie:

$\frac{\partial p}{\partial x} = \mu\Delta^2 u$

$\vdots$ - - -

$\frac{\partial p}{\partial z} = \mu\Delta^2 w - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}$ }

div = 0

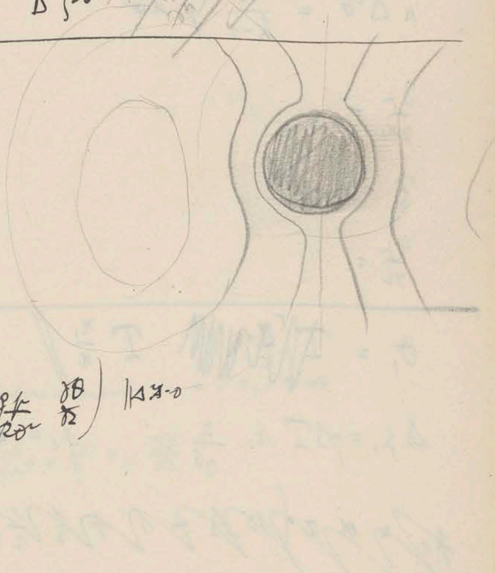
$p = A - \mathrm{pot}\left(\frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}\right)$ $\quad \|\Delta A = 0$

$\mathrm{div} = \frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x} + \ldots\right)$

$\frac{\partial p}{\partial x} = $ ~

$\frac{\partial p}{\partial y} = $

$\vdots$ =



Lepiej tak:

I Przybliżenie: div = 0

$$\Delta^2\theta = 0$$

$$\left.\begin{aligned} \frac{\partial p}{\partial x} &= \mu \Delta^2 u \\ \frac{\partial p}{\partial y} &= \mu \Delta^2 v \\ \frac{\partial p}{\partial z} &= \mu \Delta^2 w - \frac{gp}{R\theta} \end{aligned}\right\} \quad \Delta^2 p = - \frac{gp}{R\theta^2}\frac{\partial\theta}{\partial z}$$

$u, v, w = 0 \dots$

II. Przybliżenie:

$$\mathrm{div} = \frac{1}{\theta}\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right)$$

$$\kappa\Delta^2\theta = \frac{k}{k-1}\, p\, \mathrm{div}$$

$\frac{\partial p}{\partial x} = \dots$

$\frac{\partial p}{\partial y} = \dots$

$\frac{\partial p}{\partial z} = \dots$

$$\Delta^2(x^2+y^2+z^2) = 6$$

$$\Delta^2(r^2) = 6$$

~~$\frac{\partial r}{\partial x} = \frac{x}{r}$~~

~~$\frac{\partial^2 r}{\partial x^2} = \frac{1}{r} - \frac{x^2}{r^3}$~~

$\vartheta_0 = T\frac{r_1}{r}$

$\alpha = -\beta T r_1$

$\Delta p_1 = -\beta T r_1 \frac{z}{r^3}$

$p_1 = \frac{\alpha z}{2r}$

~~$p_1 = \alpha z \{$~~ [illegible] ~~$\frac{B}{r^3}\}$~~

~~$\Delta^2 p_1 =$~~

$$p_1 = \alpha z\left[A + \frac{1}{2r} + \frac{B}{r^3}\right] + \frac{C}{r}$$

$$\Delta^2 u_1 = -\alpha 2x\left[\frac{1}{2r^3} + \frac{3B}{r^5}\right] - \frac{Cx}{r^3}$$

$$\Delta^2 v_1 = -\alpha 2y\left[\qquad\qquad\right] - \frac{Cy}{r^3}$$

$$\Delta^2 w_1 = +\alpha\left[A + \frac{1}{2r} + \frac{B}{r^3}\right] - C\frac{z}{r^3} - \alpha z^2\left[\frac{1}{2r^3} + \frac{3B}{r^5}\right] - \alpha\left[\frac{1}{r}\right]$$

$$= \alpha\left[A - \frac{1}{2r} + \frac{B}{r^3}\right] - C\frac{z}{r^3} - \alpha z^2\left[\frac{1}{2r^3} + \frac{3B}{r^5}\right]$$

$$u_1 = \alpha x z\left[\frac{1}{8r} + \frac{B}{2r^3}\right]$$

$$v_1 = \alpha y z\left[\frac{1}{8r} + \frac{B}{2r^3}\right]$$

$$w_1 = \alpha\left[\frac{A}{10}(2r^2 - z^2) + \frac{Bz^2}{2r^3} + \frac{1}{8}\left(\frac{z^2}{r} - 3r\right)\right]$$

To byłoby dla $r = \infty$:

$$A = 0$$

$$\left.\begin{aligned} u_\infty &= \frac{\alpha}{8}\, r.\cos\alpha\cos\varphi \\ v_\infty &= \frac{\alpha}{8}\, r.\cos\beta\cos\varphi \end{aligned}\right\} \quad q = \frac{\alpha}{8}\, r\cos\varphi$$

$$w_\infty = \frac{\alpha}{8}\, r(\cos^2\varphi - 3)$$

Ten sposób przybliżenia dałby zawsze w nieskończoności: $u = v = w = \infty$

Polega to na opuszczeniu w III wyrazu: $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$

$$[illegible]\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial\theta}{\partial y}+\frac{\partial\theta}{\partial z^2}\right)-a\overbrace{\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)}^{=\theta\,\mathrm{div}}=0$$

$$\frac{\partial}{\partial x}\left[\frac{\partial\theta}{\partial x}-au\theta\right]+\frac{\partial}{\partial y}\left[\quad\right]+\frac{\partial}{\partial z}\left[\quad\right]=-a\,\theta\underbrace{\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]}_{=-\Delta^2\theta}$$

$$\cancel{\frac{\partial}{\partial x}(u\theta)+\frac{\partial}{\partial y}(v\theta)+\frac{\partial}{\partial z}}$$

$$\frac{\partial}{\partial x}\left[2\frac{\partial\theta}{\partial x}-au\theta\right]+\frac{\partial}{\partial y}\left[2\frac{\partial\theta}{\partial y}-av\theta\right]+\frac{\partial}{\partial z}\left[2\frac{\partial\theta}{\partial z}-aw\theta\right]=0$$

Chodzi o równanie kształtu : $\Delta^2\theta = a\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)$

Stawiając $\Delta^2\varphi=0 = \cancel{u\,\Delta^2 v+v\,\Delta^2 u+2\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial}}$

$$\varphi=\psi\cdot\chi$$

$$\frac{\partial\varphi}{\partial x}=u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial x}$$

$$\frac{\partial^2\varphi}{\partial x^2}=u\frac{\partial^2 v}{\partial x^2}+2\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+v\frac{\partial^2 u}{\partial x^2}$$

$$\Delta^2\varphi=\psi\,\Delta^2\chi+\chi\,\Delta^2\psi+2\left[\frac{\partial\psi}{\partial x}\frac{\partial\chi}{\partial x}+\frac{\partial\psi}{\partial y}\frac{\partial\chi}{\partial y}+\frac{\partial\psi}{\partial z}\frac{\partial\chi}{\partial z}\right]$$

Przyjmując zatem :

$$\chi=\theta\qquad u=\frac{1}{\psi}\frac{\partial\psi}{\partial x}\qquad v=\frac{1}{\psi}\frac{\partial\psi}{\partial y}\qquad w=\frac{1}{\psi}\frac{\partial\psi}{\partial z}\qquad \Delta^2\psi=0$$

$$u=\frac{\partial}{\partial x}(\log\psi)\qquad v=\frac{\partial}{\partial y}(\log\psi)\qquad w=\frac{\partial}{\partial z}(\log\psi)$$

$$\mathrm{div}=\cancel{\frac{1}{\psi}\Delta^2\psi}-\frac{1}{\psi^2}\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2\right]$$

Jeśli zatem u, v, w mogą być wyrażone przez taką funkcję ψ że

$$-\frac{a}{2}u = \frac{\partial}{\partial x}(\lg \psi)\,;\quad -\frac{a}{2}v = \frac{\partial}{\partial y}(\lg \psi)\,;\quad -\frac{a}{2}w = \frac{\partial}{\partial z}(\lg \psi) \qquad \Delta^2\psi = 0$$

to stawiając $\theta = \frac{\varphi}{\psi}$, gdzie $\Delta^2\varphi = 0$

będziemy mieć rozwiązanie równania: $\Delta^2\theta - a\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z}\right) = 0$

~~Zatem superpozycja i ruch potencjalny~~

$$\mathrm{div} = \frac{1}{\theta}\left[u\frac{\partial\theta}{\partial x} + \cdots\right]$$

Przy powierzchni $u = v = w = 0$ $\quad \mathrm{div} = 0$

$$\Delta^2\theta = 0 \qquad \frac{\partial\theta}{\partial s} = 0 \qquad \frac{\partial\theta}{\partial n} = \cdot\cdot$$

$$\theta = \theta_0 + n\left(\frac{\partial\theta}{\partial n}\right)_0 + \frac{n^2}{2}\left(\frac{\partial^2\theta}{\partial n^2}\right)_0 + \frac{\xi^2}{2}\frac{\partial^2\theta}{\partial\xi^2} + \frac{\eta^2}{2}\frac{\partial^2\theta}{\partial\eta^2} + \cancel{\eta\xi\frac{\partial^2\theta}{\partial\xi\partial\eta}} + n\xi\frac{\partial^2\theta}{\partial n\partial\xi} + n\eta\frac{\partial^2\theta}{\partial n\partial\eta}$$

$$\Delta^2\theta = a\,\mathcal{S}(v.\nabla\theta)$$

Specjalny przykład: ~~$v = w = 0$~~ $\quad w = u = 0$ $\qquad \mathrm{div} = \frac{1}{\theta}\left[v\frac{\partial\theta}{\partial y}\right] = \frac{\partial v}{\partial y} = \frac{1}{a}\Delta^2\theta$

~~$\frac{\partial p}{\partial y} = \partial p$~~ $\qquad$ ~~$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial z} = 0$~~ $\qquad$ ~~$p = p_0$~~ $\qquad \frac{1}{\theta}\frac{\partial\theta}{\partial y} = \frac{1}{v}\frac{\partial v}{\partial y}$

$$\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial}{\partial x}\mathrm{div} \qquad = \frac{\mu}{3}\frac{\partial^2 v}{\partial x\partial y}$$

$$\frac{\partial p}{\partial z} = \qquad = \frac{\mu}{3}\frac{\partial^2 v}{\partial z\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\mu}{3}\frac{\partial}{\partial y}\mathrm{div} + \mu\nabla^2 v + g\rho \qquad = \frac{\mu}{3}\frac{\partial^2 v}{\partial y^2} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g$$

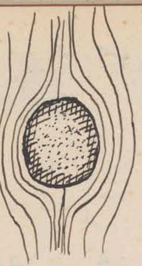
$$\Delta^2 p = \frac{4\mu}{3}\Delta^2\frac{\partial v}{\partial y} + g\frac{\rho_0}{\theta_0}\frac{\partial\theta}{\partial y}$$

Przybliżenie, jeśli $\frac{\partial\theta}{\partial y} = \frac{\partial v}{\partial y} = \underset{0}{\text{małe}}$ $\qquad p = \mathrm{fc.}(y) \qquad \frac{\partial^2\theta}{\partial x^2} = 0 \qquad \frac{\partial\theta}{\partial x} = \frac{\theta_1 - \theta_2}{\delta}$

$$\frac{dp}{dy} = \mu\frac{\partial^2 v}{\partial x^2} + g\rho_0\left[1 + \frac{\theta}{\theta_0}\right] \qquad \frac{\partial}{\partial x}:\quad \frac{\partial^3 v}{\partial x^3} = \rho_0 g\frac{\theta_1 - \theta_2}{\delta}$$

$$v = \left[\rho_0 g\frac{\theta_1 - \theta_2}{\delta}\frac{x^3}{6} + ax^2 + bx + c\right]$$

$$\frac{dp}{dy} = \mu\left(\rho_0\frac{\theta_1 - \theta_2}{\delta}\cancel{x} + 2a\right) + g\rho_0\left[1 - \cdots\cdots\right.,$$



Przypuśćmy ruch „lamellarny"

Można supponować że tylko warstwy w bezpośredniej styczności ścian zmieniają znacznie temperaturę

$\Delta^2\theta = a\int(v.\nabla\theta)$

$\nabla\theta$ przyściennie $\perp s$

$v \parallel s$

$\Big\}\ \Delta^2\theta = 0$

$\mathrm{div} = 0$

$\mathrm{div} = \alpha\left(u\frac{\partial\theta}{\partial x} + \right)$

$\frac{\partial p}{\partial x} = 0$

$\frac{\partial p}{\partial s} = \mu\frac{\partial^2 v}{\partial n^2} + g\rho_0\left[1 - \frac{\theta}{\theta_0}\right]\cdot\frac{\partial y}{\partial s}$

$\mu\frac{\partial^2 v}{\partial n^2} = \frac{\rho g}{\theta_0}\theta$

$\mu\frac{\partial^3 v}{\partial n^3} = \frac{g\rho}{\theta_0}\frac{\partial\theta}{\partial n}$

Zaniedbujemy zupełnie różnicę p w kierunku poziomym:

$\frac{\partial p}{\partial z} + \rho g = \frac{\theta}{\theta_0}\rho_0 g$

$0 = \mu\frac{\partial}{\partial x}\cancel{\mathrm{div}} + \mu\Delta^2 u$

$0 = [illegible]\frac{\partial}{\partial y}\cancel{\mathrm{div}} + \mu\Delta^2 v$

$0 = \mu\frac{\partial}{\partial z}\cancel{\mathrm{div}} + \mu\Delta^2 w + \frac{\theta}{\theta_0}\rho_0 g$

$a\left[u\Delta^2\frac{\partial w}{\partial x} + v\Delta^2\frac{\partial w}{\partial y} + w\Delta^2\frac{\partial w}{\partial z}\right] = \Delta^2\Delta^2 w$

$\vartheta = -\frac{\eta}{\rho}\frac{T}{g}\frac{\partial^2 w}{\partial x^2}$

~~$w\frac{\partial^2 w}{\partial x^2} = \kappa\left(\frac{\partial^4 w}{\partial x^4}\right)$~~

$0 = g\frac{\vartheta}{T} + \frac{\eta}{\rho}\frac{\partial^2 w}{\partial x^2}$

$\vartheta w = \kappa\frac{\partial^2\theta}{\partial x^2}$

$w\frac{\partial^2 w}{\partial x^2} = \kappa\frac{\partial^4 w}{\partial x^4}$

~~$\frac{\partial\vartheta}{\partial x}\ \ \vartheta\frac{\partial w}{\partial x} = \kappa\frac{\partial^2\theta}{\partial x^2} = -\kappa\frac{\partial^2\theta}{\partial x^2}\ \frac{\frac{\partial\theta}{\partial x}}{\theta} + \theta\frac{\partial w}{\partial x}$~~

$\frac{\partial^2 w}{\partial x^2} = \frac{\kappa}{w}\frac{\partial^4 w}{\partial x^4}$

$w = \frac{a}{x^2}$

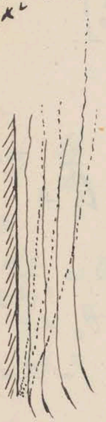
$\frac{a}{x^4} = \frac{\kappa}{a}x^2\,\frac{2.3.4.5}{x^6}$

$a = 20\kappa$

$$w = \frac{\kappa}{\theta}\frac{\partial^2\theta}{\partial x^2}$$

$$\frac{\partial w}{\partial x} = -\frac{\kappa}{\theta^2}\frac{\partial\theta}{\partial x}\frac{\partial^2\theta}{\partial x^2} + \frac{\kappa}{\theta}\frac{\partial^3\theta}{\partial x^3}$$

$$\frac{\partial^2 w}{\partial x^2} = +\frac{2\kappa}{\theta^3}\left(\frac{\partial\theta}{\partial x}\right)^2\frac{\partial^2\theta}{\partial x^2} - \frac{\kappa}{\theta^2}\left(\frac{\partial^2\theta}{\partial x^2}\right)^2 - 2\frac{\kappa}{\theta^2}\frac{\partial\theta}{\partial x}\frac{\partial^3\theta}{\partial x^3} + \frac{\kappa}{\theta}\frac{\partial^4\theta}{\partial x^4} = -\frac{g}{T}\frac{f}{3}\theta$$



$$\frac{dw}{dx} = p \qquad \frac{d^2w}{dx^2} = p\frac{dp}{dw} \qquad \cancel{\frac{d^3w}{dx^3} = \frac{d}{dp}}$$

$$\frac{d^3w}{dx^3} = p\left(\frac{dp}{dw}\right)^2 + p^2\frac{d^2p}{dw^2}$$

$$\frac{d^4w}{dx^4} = p\left[\left(\frac{dp}{dw}\right)^3 + 4p\frac{dp}{dw}\frac{d^2p}{dw^2} + p^2\frac{d^3p}{dw^3}\right]$$

$$w\frac{dp}{dw} = \kappa\left[p^2\frac{d^3p}{dw^3} + 4p\frac{dp}{dw}\frac{d^2p}{dw^2} + \left(\frac{dp}{dw}\right)^3\right]$$

$u = \frac{\partial}{\partial x}(\lg\psi) + u'$

$v = \qquad + v'$

$w = \qquad + w'$

Zatem: $\psi = \frac{c}{2}\frac{a^3 z}{r^3} + cz + b$

~~$\frac{\partial\psi}{\partial z} = \frac{c}{2}\frac{a^3}{r^3} + c - \frac{3}{2}\frac{c a^3 z^2}{r^5} = -\frac{3}{2}\sin^2$~~

~~$\frac{\partial\psi}{\partial x} = -\frac{3}{2}\frac{c a^3 z x}{r^5} = \frac{3}{2}$~~

$$\left.\frac{\partial\psi}{\partial z}\right| = -\frac{3}{2}\sin^2\varphi$$

$$\left.\frac{\partial\psi}{\partial\omega}\right|_{r=a} = -\frac{3}{2}\cos\varphi\sin\varphi$$

$\Delta^2\varphi = 0$ $\quad\varphi = \theta.\psi$

~~$r=a \quad \varphi = \theta_1\left[b + \frac{3c}{2} a\cos\varphi\right]$~~

~~$r=\infty \quad \varphi = \theta_0 c z_\infty$~~

~~$\Psi = l$~~

$\lg\varphi = \Psi$

$\varphi = e^{\Psi}$

$r=a \quad \varphi = \theta_1 e^{b + \frac{3c}{2} a\cos\varphi} = \theta_2 e^{\frac{3ca}{2}\cos\varphi} = \theta_2 e^{\frac{3c}{2}x}$

$r=\infty \quad \varphi = \theta_0 e^{c z_\infty} = \theta_0 e^{c r_\infty \cos\varphi} = \theta_0 e^{cx}$

W każdym razie: rozwiązanie równania $\Delta^2\theta + a\left(u\frac{\partial\theta}{\partial x} + \dots\right) = 0$

możliwe wykonanie jeśli ruch potencjalny

$$\varphi = A_0 + A_1 P_1 r + A_2 P_2 r^2 + \dots$$
$$+ B_0\frac{1}{r} + \frac{B_1 P_1}{r^2} + \frac{B_2 P_2}{r^3} + \dots$$

$$\int_{-1}^{+1} e^{\alpha x} . P_{(x)}\, dx$$

I Przybliżenie : $\frac{\partial\theta}{\partial y} = \frac{\partial v}{\partial y} = 0$ $\quad = u = 0$ $\longrightarrow$ $\frac{\partial p}{\partial x} = 0$, $p = f_1(y)$

$div = 0$

$v = f_2(x)$

$\cancel{u\frac{\partial\theta}{\partial x}} + v\frac{\partial\theta}{\partial y} = 0$

$\theta = f_2(x)$

$$\frac{d^2\theta}{dx^2} = 0$$

$$\theta = \theta_1 + (\theta_2 - \theta_1)\frac{x}{\delta}$$

$$\frac{dp}{dy} = \mu\frac{d^2v}{dx^2} + g\rho_0 \, \frac{\theta}{\theta_1}\Big] = \mu\frac{d^2v}{dx^2} + g\rho_0\frac{\theta_2-\theta_1}{\theta_1}\frac{x}{\delta} + g\rho_0$$

$$\cancel{0 = \mu\frac{d^3v}{dx^3} + g\rho_0\frac{\theta_2-\theta_1}{\theta_1}\frac{1}{\delta}}$$

$$\frac{d^2p}{dy^2} = 0$$

$$p = p_0 + y\frac{p_1-p_0}{h} \qquad \frac{dp}{dy} = \frac{p_1-p_0}{h} = \nearrow$$

$$\left(\frac{p_1-p_0}{h} \overset{-g\rho_0}{}\right)x = \mu\frac{dv}{dx} + g\rho_0\frac{\theta_2-\theta_1}{\theta_1}\frac{x^2}{2\delta} + b$$

$$\frac{p_1-p_0}{h}\frac{x^2}{2} = \mu v + \rho_0 g\frac{\theta_2-\theta_1}{\theta_1}\frac{x^3}{6\delta} + bx + c$$

$$\left.\begin{matrix} x=0 \\ x=\delta \end{matrix}\right\} v=0 \qquad c=0 \qquad \frac{p_1-p_0}{h}\frac{\delta^2}{2} = \rho_0 g\frac{\theta_2-\theta_1}{\theta_1}\frac{\delta^2}{6} + b\delta$$

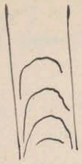
$$\left[g\rho_0 - \frac{p_1-p_0}{h}\right]\frac{x^2-\delta x}{2} = \mu v + \rho_0 g\frac{\theta_2-\theta_1}{\theta_1}\frac{x^3-x\delta^2}{6}$$

II). $u \neq 0$ $\qquad v_0 = \uparrow \; ax + bx^2 + cx^3$

$$\alpha^2\Delta^2\theta = (ax+bx^2+cx^3)\frac{\partial\theta}{\partial y} = \alpha^2\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) = \cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y}$$

$p = f_2(y)$

$$\frac{dp}{dy} = \left(\mu\left(\frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}}\right)\right) + g\rho_0\frac{\theta}{\theta_0}$$



$$\frac{\partial p}{\partial x} = \cancel{\frac{\mu}{3}\frac{\partial}{\partial x}\operatorname{div}} + \cancel{\mu\Delta^2 u} \neq 0$$

$$\frac{\partial p}{\partial y} = \cancel{\frac{\mu}{3}\frac{\partial}{\partial y}\operatorname{div}} + \mu\Delta^2 v + g\rho_0\frac{\theta}{\theta_0}$$

$p = f_1(y)$ $\qquad$ $\theta = f_2(x, y)$

$$\alpha^2\Delta^2\theta = u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} \; \bigg| + \theta \operatorname{div}$$

$u = 0$

$$\frac{dp}{dy} = \mu\frac{\partial^2 v}{\partial x^2} + \rho_0 g\frac{\theta}{\theta_0} \qquad \Bigg|\!\Bigg|\!\Bigg| \qquad 0 = \mu\frac{\partial^3 v}{\partial x^3} + \frac{\rho_0 g}{\theta_0}\frac{\partial\theta}{\partial x}$$

$$\alpha^2\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) = v\frac{\partial\theta}{\partial y} = \theta\frac{\partial v}{\partial y} \qquad \theta = \frac{\mu\theta_0}{g\rho_0}\frac{\partial^2 v}{\partial x^2} + \varphi(y)$$

$$\alpha^2\left[\underbrace{\frac{\mu\theta_0}{\rho_0 g}\left(\frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial x^2\partial y^2}\right)} + \frac{d^2\varphi}{dy^2}\right] = \cancel{\frac{\mu\theta_0}{g\rho_0} v\frac{\partial^3 v}{\partial x^2\partial y} + v\frac{d\varphi}{dy}}$$

$$\frac{\partial}{\partial y}\left(\frac{v}{\theta}\right) = 0 \qquad \frac{v}{\theta} = \psi(x)$$

$$v = \theta\cdot\psi(x)$$

$$\alpha^2\left(\frac{\partial^2\theta}{\partial x^2} + \cancel{\frac{\partial^2\theta}{\partial y^2}}\right) = \theta\frac{\partial\theta}{\partial y}\psi(x)$$

$$\theta = \frac{\mu\theta_0}{g\rho_0}\left[\frac{\partial^2\theta}{\partial x^2}\psi + 2\frac{\partial\theta}{\partial x}\frac{d\psi}{dx} + \theta\frac{d^2\psi}{dx^2}\right] + \varphi(y)$$

Dla wielkiego y w każdym razie $\theta = \text{const}$

zatem $\dfrac{d^2\psi}{dx^2} = a$

$$\psi = a\frac{x^2}{2} + bx + c$$

wzdłuż ściany $\dfrac{\partial\theta}{\partial y} = \dfrac{\partial^2\theta}{\partial x^2} = 0$

$\varphi(y) = \text{const}$

$$\theta = \frac{\mu \theta_0}{g \rho_0} \frac{\partial^2 v}{\partial x^2} + \mathrm{const}$$

$$\theta = \mathrm{const}.$$

W linij środkowej: $\frac{\partial \theta}{\partial x} = 0 \qquad \frac{d^2 v_1}{dx^2} = a$

$$\theta\left[1 - \frac{\mu \theta_0}{g \rho_0} a\right] = \frac{\mu \theta_0}{g \rho_0}\left(a \frac{x^2}{2} + bx + c\right) \frac{\partial^2 \theta}{\partial x^2} + \mathrm{const} \quad \Big| \quad \alpha^2$$

$$\theta \frac{\partial \theta}{\partial y} 4 \qquad = \alpha^2 \frac{\partial^2 \theta}{\partial x^2} \quad \Big| \quad 4 \cdot \frac{\mu \theta_0}{g \rho_0}$$

$$\alpha^2 \theta\left[1 - \frac{\mu \theta_0}{g \rho_0} a\right] - \frac{\mu \theta_0}{g \rho_0} \theta \frac{d\theta}{dy} (4^2)_{\frac{1}{2}} = \alpha^2 \cdot \mathrm{const}$$

$$\theta = m + ny \qquad \theta \frac{d\theta}{dy} + A\theta = B$$

$$\frac{\theta \, d\theta}{B - A\theta} = dy$$

$$y =$$

O stateczności ruchów (dla cieczy)

Ruch który jest stały:

~~$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\mu\nabla^2 u$~~ $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$

⋮ =

⋮ =

~~$\frac{\partial U}{\partial t}+\frac{\partial}{\partial x}(U+u)\frac{\partial(U+u)}{\partial x}+\dots=\frac{1}{\rho}\frac{\partial(p+P)}{\partial x}+\mu\nabla^2(u+U)$~~

~~$\frac{\partial U}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\mu\nabla^2 U-\left[u\frac{\partial U}{\partial x}+U\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial y}+V\frac{\partial u}{\partial y}+w\frac{\partial U}{\partial z}+W\frac{\partial u}{\partial z}\right]$~~

~~$\frac{\partial v}{\partial t}+(v\nabla\cdot v)=\frac{1}{2}\nabla(v^2)+\nabla v\,\mathrm{curl}\,v=\frac{1}{\rho}\nabla P+\mu\nabla^2 v\left(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)+v$~~

~~$\frac{\partial\,\mathrm{curl}\,v}{\partial t}+\mathrm{curl}\,\nabla v\,\mathrm{curl}\,v=\mu\nabla^2\mathrm{curl}\,v$~~

~~$\frac{\partial\,\mathrm{curl}(v+V)}{\partial t}+\mathrm{curl}\,\nabla(v+V)\,\mathrm{curl}(v+V)=\mu\nabla^2\mathrm{curl}(v+V)$~~

~~$\frac{\partial\,\mathrm{curl}\,V}{\partial t}+\mathrm{curl}\,\nabla\left[V\,\mathrm{curl}\,v+v\,\mathrm{curl}\,V\right]=\mu\nabla^2\mathrm{curl}\,V$~~

$$\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{xy}}{\partial y}+\frac{\partial p_{xz}}{\partial z}=-\frac{\partial p}{\partial x}+\lambda\frac{\partial}{\partial x}\mathrm{div}+2\mu\underbrace{\left(\frac{\partial a}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial e}{\partial z}\right)}$$

$$=\mu\left[2\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\partial z}\right)\right]$$

$$=-\frac{\partial p}{\partial x}+(\lambda+\mu)\frac{\partial}{\partial x}\mathrm{div}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\lambda=-\frac{2}{3}\mu$$

$$=-\frac{\partial p}{\partial x}+\frac{1}{3}\frac{\partial}{\partial x}\mathrm{div}+$$

~~$p+2\mu\frac{\partial u}{\partial x}+$~~

$-p+(2\mu+\lambda)\frac{\partial u}{\partial x}$

$p=R\rho\theta$ $\rho\,\mathrm{div}+u\frac{\partial\rho}{\partial x}+\dots=0$

$2\mu+\lambda=3\mu$ Ok. Reyn

$p=p_0+\alpha\,\mathrm{div}$

$\lambda=\mu$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \nu\frac{\partial}{\partial x}\operatorname{div} + \mu\nabla^2 u = -\frac{\partial p}{\partial x} + (\nu+\mu)\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial\rho}{\partial t} = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + kp\frac{\partial u}{\partial x} = (k-1)\Phi$$

$$\frac{\partial\rho}{\partial t} + \rho\frac{\partial u}{\partial x} = 0$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + (\nu+\mu)\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial p}{\partial t} + kp\frac{\partial u}{\partial x} = (k-1)\Phi$$

Który wpływ znaczniejszy czy dynamiczny czy termiczny ?

I). $\cancel{\frac{\partial^2\rho}{\partial t^2} = -\frac{\partial\rho}{\partial t}\frac{\partial u}{\partial x} - \rho\frac{\partial^2 u}{\partial x\partial t}}$

$\cancel{\frac{\partial\rho}{\partial x}\frac{\partial u}{\partial t} + \rho\frac{\partial^2 u}{\partial x\partial t} = -\frac{\partial^2 p}{\partial x^2} + (\nu+\mu)\frac{\partial^3 u}{\partial x^3}}$

[illegible]

$\cancel{p \sim \rho^{k}}$

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$$

E

2). $\cancel{\frac{\partial\rho}{\partial t}\frac{\partial u}{\partial t}} + \rho\frac{\partial^2 u}{\partial t^2} = -\frac{\partial^2 p}{\partial x\partial t} + (\nu+\mu)\frac{\partial^3 u}{\partial x^2\partial t}$

↓

$$= k\cancel{\frac{\partial p}{\partial x}\frac{\partial u}{\partial x}} + kp\frac{\partial^2 u}{\partial x^2} + (\nu+\mu)\frac{\partial^3 u}{\partial x^2\partial t}$$

3).

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + (\nu+\mu)\frac{\partial^3 u}{\partial x^2\partial t}$$

$$u = \sin(\alpha x + \beta t)\,e^{-\gamma t}$$

$$\frac{\partial u}{\partial t} = \beta\cos e^{-\gamma t} - \gamma\sin e^{-\gamma t}$$

$$\frac{\partial^2 u}{\partial t^2} = -\beta^2\sin e^{-\gamma t} - 2\beta\gamma\cos e^{-\gamma t} + \gamma^2\sin e^{-\gamma t} =$$

$$(\gamma^2-\beta^2)\sin - 2\beta\gamma\cos = -\alpha^2 a^2\sin + (\nu+\mu)\left[\alpha^2\gamma\sin - \alpha^2\beta\cos\right]$$

$\cancel{\alpha^2\sin\; e^{-\gamma t}}$

$$\begin{cases}\gamma^2-\beta^2 = -\alpha^2 a^2 + (\nu+\mu)\alpha^2\gamma \\ -2\beta\gamma = -\alpha^2\beta(\nu+\mu) \\ \gamma = \frac{\alpha^2}{2}(\nu+\mu) \\ \beta^2 = \alpha^2 a^2 + \frac{\alpha^4(\nu+\mu)^2}{4} - \frac{\alpha^2 a^2}{2}(\nu+\mu)^2 \\ \beta = a\alpha\sqrt{1 - \frac{(\nu+\mu)^2}{2} + \frac{(\nu+\mu)^4\alpha^2}{4a^2}}\end{cases}$$

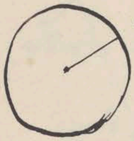
II). $\rho\frac{\partial^2 u}{\partial t^2} = -\frac{\partial^2 p}{\partial x\partial t}$

$$\frac{\partial p}{\partial t} = -kp\frac{\partial u}{\partial x} + (k-1)\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2$$

$$\frac{\partial^2 p}{\partial x\partial t} = -kp\frac{\partial^2 u}{\partial x^2} - k\cancel{\frac{\partial p}{\partial x}\frac{\partial u}{\partial x}} + \underbrace{\frac{8\mu}{3}(k-1)\frac{\partial u}{\partial x}\cdot\frac{\partial^2 u}{\partial x^2}}_{}$$

mało wyższego rzędu

Kula rozszerza się



jeżeli zaniedbujemy inercję to odkształcenie

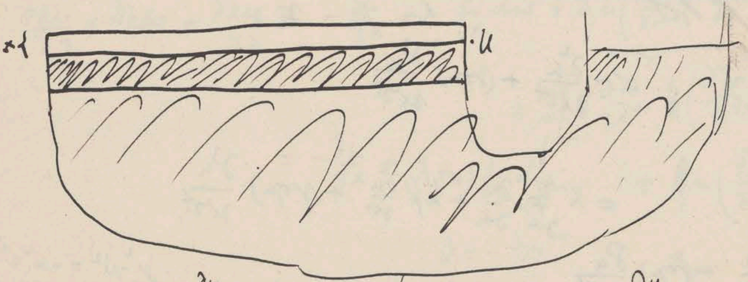
~~przyśpieszenie~~ będzie jednorodne

~~$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=\frac{\partial w}{\partial z}$~~ ~~$x = \frac{x_0}{\dots}$~~ $x : x_0 = a : a_0$

$\frac{dx}{dt} = u = \frac{x_0}{a_0}\frac{da}{dt}$. $u = x\frac{1}{a}\frac{da}{dt}$

$\frac{\partial u}{\partial x} = \frac{1}{a}\frac{da}{dt} = c$ $\frac{\partial u}{\partial y} = 0$

$\Phi = 0$!



$u = U\frac{\xi}{x}$

$\frac{\partial u}{\partial x} = \frac{U}{x}$

$$p_{xx} = -p + 2\mu\frac{\partial u}{\partial x} - \frac{2\mu}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) = -p + \frac{4}{3}\mu\frac{\partial u}{\partial x} = -p + \frac{4}{3}\mu\frac{U}{x}$$

$$M\frac{d^2x}{dt^2} = -2\varphi x s g - p_0 + p - \frac{4}{3}\frac{\mu}{x}\frac{dx}{dt} \qquad p = p_0\frac{a}{x}$$

$$M\frac{d^2x}{dt^2} = -p_0 - 2\varphi s g\,.\,x + \frac{p_0 a}{x} - \frac{4}{3}\frac{\mu}{x}\frac{dx}{dt}$$

$$\frac{dx}{dt} = z \qquad \frac{d^2x}{dt^2} = \frac{dz}{dt} = \frac{dz}{dx}\cdot z$$

$$M z\frac{dz}{dx} + \frac{4}{3}\mu\,\frac{z}{x} - \frac{p_0 a}{x} + 2\varphi s g\, x + p_0 = 0$$

$$dm\,\frac{D}{Dt}\left\{\left[\frac{c}{A}\theta + \frac{1}{2}(u^2+v^2+w^2) + U\right]\right\} = \left[\frac{\partial}{\partial x}(u\,.\,p_{xx}) + \frac{\partial}{\partial y}(v\,.\,p_{yy}) + \frac{\partial}{\partial z}(w\,.\,p_{zz})\right.$$

$$\left. + \underbrace{\frac{\partial}{\partial y}(u\,.\,p_{xy}) + \frac{\partial}{\partial x}(v\,.\,p_{yx})} + \smile + \smile\right] dx\,dy\,dz$$

$$\cancel{\frac{D}{Dt}\Sigma} = \cancel{p_{xx}\frac{\partial u}{\partial x} + p_{yy}\frac{\partial v}{\partial y} + -} + \cancel{p_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)} + \cdots$$

$$\cancel{+ u\frac{\partial p_{xx}}{\partial x} + v\frac{\partial p_{yy}}{\partial y} +} \Big|\, \cancel{u\frac{\partial p_{xy}}{\partial y} + v\frac{\partial p_{xy}}{\partial x}} \cancel{+ - - -} + \smile$$

$$= u\left(\rho\frac{Du}{Dt} - \rho X\right) + v\left(\rho\frac{Dv}{Dt} - \rho Y\right) + w\left(\quad\right) - p\,\mathrm{div} + \Phi$$

$$\cancel{\frac{D(u^2)}{Dt} = 2u\frac{Du}{Dt}}$$

$$\cancel{2u\frac{\partial u}{\partial t} + 2u^2\frac{\partial u}{\partial x} + 2vu\frac{\partial u}{\partial y} + 2wu\frac{\partial u}{\partial z}}$$

$$\cancel{= 2u\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{-}\ \_\!\_\right]}$$

$$\rho\,\frac{c}{A}\frac{D\theta}{Dt} = -p\,\mathrm{div} + \Phi$$

~~$\rho u = \frac{\partial \varphi}{\partial x}$ $\rho v = \frac{\partial \varphi}{\partial y}$ $\rho w = \frac{\partial \varphi}{\partial z}$~~

$u = f(x, t)$ $w = 0$ $\mathrm{div} = \frac{\partial u}{\partial x}$

$v = \varphi(x, t)$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] = -\frac{\partial p}{\partial x} + \frac{4\mu}{3}\frac{\partial^2 u}{\partial x^2}$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x}\right] = \mu\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + (k\, p\frac{\partial u}{\partial x} = \Phi$$

$$= \frac{4\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\Delta^2\theta$$

$$\frac{\partial v}{\partial t} = \frac{\mu}{\rho_0}\frac{\partial^2 v}{\partial x^2}$$

$$\left.\begin{array}{ll} \frac{\partial p}{\partial t} + (k\, p_0\frac{\partial u}{\partial x} = \mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^2\theta}{\partial x^2} & \Big| \; \frac{\partial}{\partial x} \\ \frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} & \Big| \; \frac{\partial}{\partial t} \end{array}\right\} \rho_0\frac{\partial^2 u}{\partial t^2} - k p_0\frac{\partial^2 u}{\partial x^2} = \mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^3\theta}{\partial x^3}$$

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + \frac{\mu}{\rho_0}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)^2 + \kappa\frac{\partial^3\theta}{\partial x^3}$$

$$v = Ae^{-\alpha x}\cos(\beta x + \gamma t)$$

$$\frac{\partial v}{\partial x} = Ae^{-\alpha x}\left[-\alpha\cos + \beta\sin\right]$$

$$\left(\frac{\partial v}{\partial x}\right)^2 = A^2e^{-2\alpha x}\left[\alpha^2\cos^2 + \beta^2\sin^2 - 2\alpha\beta\cos\sin\right]$$

$$\gamma = 2\pi n$$

$$-\sin = \left(\frac{\mu}{\rho_0}\right)\left[(\alpha^2 - \beta^2)\cos + 2\alpha\beta\sin\right]$$

$$\alpha = \beta$$

$$1 = 2\frac{\mu}{\rho_0}\alpha^2$$

$$\alpha = \sqrt{\frac{\rho_0}{2\mu}}$$

$$v = Ae^{-x\sqrt{\frac{\rho_0}{2\mu}}}\cos\left(2\pi n t - x\sqrt{\frac{\rho_0}{2\mu}}\right)$$

$$\left(\frac{\partial v}{\partial x}\right)^2 = A^2e^{-2\alpha x}\cdot\alpha^2\underbrace{\left[1 - 2\cos\sin\right]}_{\left[1 - \sin 2(\gamma t - \beta x)\right]}$$

~~$c = \gamma\sqrt{\frac{\rho_0}{2\mu}}$~~

$$\gamma\sqrt{\frac{2\mu}{\rho_0}}$$

$$2n\pi\sqrt{\frac{2\mu}{\rho_0}}$$

$$\rho \frac{\partial v}{\partial t} = \mu \frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial p}{\partial t} = \mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa \frac{\partial^2 \theta}{\partial x^2}$$

$$\frac{\partial \rho}{\partial t} + \quad = 0 \qquad \rho = \mathrm{const} = \frac{p}{R\theta}$$

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$

$$R\rho_0 \frac{\partial \theta}{\partial t} = \mu\left(\frac{\partial v}{\partial x}\right)^2 + \kappa \frac{\partial^2 \theta}{\partial x^2}$$

$$\rho_0 \frac{\partial v}{\partial t} = \mu \frac{\partial^2 v}{\partial x^2}$$

$$\cancel{\frac{\partial p}{\partial t}} + u \cancel{\frac{\partial p}{\partial x}} + kp \frac{\partial u}{\partial x} = \left[\frac{4\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \kappa \frac{\partial \theta}{\partial x^2}\right](k-1)$$

$$u = f(t)\, x = x f$$

$p_{xx} = p$ [illegible] $\frac{4}{3}\mu \frac{\partial u}{\partial x} = a$

$$p = a + \frac{4}{3}\mu \frac{\partial u}{\partial x}$$

$$\frac{4\mu}{3} \frac{\partial f}{\partial t} + kf\left[a + \frac{4}{3}\mu f\right] = \frac{4\mu}{3} f^2 (k-1)$$

$$\frac{4\mu}{3} \frac{\partial f}{\partial t} + akf + \frac{4\mu}{3} f^2 = 0$$

$$-\frac{1}{f^2} \frac{\partial f}{\partial t} + \frac{3ak}{4\mu} \frac{1}{f} + 1 = 0$$

$$\frac{1}{f} = A e^{\alpha t} + B$$

$$-\frac{\partial}{\partial t}\left(\frac{1}{f}\right) + \frac{3ak}{4\mu}\left(\frac{1}{f}\right) + 1 = 0$$

$$-A\alpha e^{\alpha t} + \frac{3ak}{4\mu}\left[A e^{\alpha t} + B\right] + 1 = 0$$

$$\alpha = + \frac{3ak}{4\mu}$$

$$B = -\frac{4\mu}{3ak}$$

$$u = \frac{x}{\underline{A}\, e^{+\frac{3ak}{4\mu} t} - \frac{4\mu}{3ak}}$$

$$\frac{1}{\underline{A} - \frac{4\mu}{3ak}} = -(a - p_0)\frac{3}{4\mu}$$

$$\underline{A} = \frac{4\mu}{3ak} + \frac{4\mu}{3\,(p_0 - a)}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}$$

$$\frac{\rho}{\rho_0}=\left(\frac{p}{p_0}\right)^{\frac{1}{k}}$$

$$\rho\left(u\frac{\partial v}{\partial x}+\qquad\right)=-\frac{\partial p}{\partial y}$$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial x}=\frac{1}{k}\frac{1}{p}\frac{\partial p}{\partial x}$$

$$\rho\left(u\frac{\partial w}{\partial x}+\qquad\right)=-\frac{\partial p}{\partial z}$$

$$p_1^2=\sqrt{a}$$
$$p_2^2=\sqrt{a-cl}$$
$$p_1^2-p_2^2=cl$$
$$c$$

$$\frac{\partial \rho u}{\partial x}+\dots=0$$

$$p=\sqrt{a-cx}=\sqrt{p_1^2-\frac{x}{l}(p_1^2-p_2^2)}$$

$$u=\frac{(y\delta-y^2)\,c}{4\mu\sqrt{a-cx}}$$

$$\frac{\partial u}{\partial x}=+\frac{(y\delta-y^2)\,c^2}{8\mu\sqrt{(\quad)}^3}$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{3(y\delta-y^2)\,c^3}{16\mu\sqrt{(\quad)}^5}$$

$$\frac{\partial u}{\partial y}=\frac{(\delta-2y)\,c}{4\mu\sqrt{\quad}}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{-2c}{4\mu\sqrt{\quad}}$$

$$\frac{\partial^2 u}{\partial x\,\partial y}=\frac{(\delta-2y)\,c^2}{8\mu\sqrt{(\quad)}^3}$$

$$\frac{\frac{\partial^2 u}{\partial x^2}}{\frac{\partial^2 u}{\partial y^2}}\sim\frac{c^2\delta^2}{(a-cx)^2}\sim\frac{\delta^2}{l^2}\dots\dots$$

$$\left.\begin{aligned}\frac{\partial p}{\partial x}&=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2}\\ \frac{\partial p}{\partial y}&=\frac{\mu}{3}\frac{\partial^2 u}{\partial x\,\partial y}\end{aligned}\right\}\quad 0=\mu\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\Delta^2 p=\mu\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{1}{3}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial v}{\partial y}\right)\right)$$

$$=\frac{4\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=\frac{4\mu}{3}\frac{df}{dx}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f.(x)=f(x)$$

$$\frac{\partial}{\partial x}(\rho u)=0$$

$$\rho u=f.(y)=\varphi(y)$$

$p = \frac{4\mu}{3}\frac{\partial u}{\partial x} + \psi(x,y)$ ‖ $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$

$\frac{\partial p}{\partial y} = \frac{4\mu}{3}\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial\psi}{\partial y}$

$\frac{\partial\psi}{\partial y} = -\mu\frac{\partial^2 u}{\partial x\,\partial y} = -3\frac{\partial p}{\partial y}$

$\psi(x,y) = \chi(x) - 3p$

~~$\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} + \frac{1}{4}\frac{d\chi}{dx} = \frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} + \mu f(x)$~~

$p = \frac{\mu}{3}\frac{\partial u}{\partial x} + \frac{1}{4}\chi(x)$

$\Delta^2 u =$ [illegible] $= \frac{1}{4\mu}\frac{d\chi}{dx}$ ‖ $pu = \varphi(y)$

~~$\frac{\partial p}{\partial x} = \frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} + \frac{1}{4}\frac{d\chi(x)}{dx} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\mu}{3}\frac{\partial^2 u}{\partial x^2}$~~

~~$\Delta^2 p = \frac{4\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \frac{\mu}{3}\frac{d^2\chi(x)}{dx^2} = \frac{1}{12}$~~ [illegible]

$z = \frac{1}{y}$ $\frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$

~~$pu = \frac{\mu}{3}u\frac{\partial u}{\partial x} + \frac{1}{4}u\chi(x) = \varphi(y)$~~

~~$= \frac{\mu}{6}\frac{\partial(u^2)}{\partial x} + \frac{1}{4}u\chi(x)$~~

~~$\frac{\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\mu}{3}u\frac{\partial^2 u}{\partial x^2} + \frac{1}{4}\frac{\partial u}{\partial x}\chi(x) + \frac{u}{4}\frac{d\chi}{dx} = 0$~~

$z\frac{dz}{dx} + zX = a$

$-\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y}X = a$

$\frac{dy}{dx} + ay^3 - Xy^2 = 0$

~~$p\frac{\partial u}{\partial x} + u\frac{\partial p}{\partial x} = 0$~~

~~$\frac{\mu}{3}\left(\frac{\partial u}{\partial x}\right)^2$~~

$\frac{\varphi(y)}{u} = \frac{\mu}{3}\frac{\partial u}{\partial x} + \frac{1}{4}\chi(x)$

~~$\frac{\mu}{3}\frac{u\,du}{\varphi(y) - \frac{u}{4}\chi(x)} = dx$~~

$$\boxed{\frac{\mu}{3}u\frac{\partial u}{\partial x} + \frac{1}{4}u\chi(x) = \varphi(y)}$$

$\frac{1}{4}\frac{d\chi}{dx} = -\frac{\mu}{3}\frac{\partial^2 u}{\partial x^2} - \frac{\varphi(y)}{u^2}\frac{\partial u}{\partial x} = \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$

$\frac{\partial^2 u}{\partial y^2} =$ [illegible] $-\frac{4}{3}\frac{\partial^2 u}{\partial x^2} - \frac{\varphi(y)}{u^2\mu}\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{\varphi(y)}{\mu u}\right]$

$$= \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial u}{\partial r^2}$$

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{dp}{dx} \quad \Big\| \quad \frac{\partial p}{\partial r} = 0$$

~~[illegible]~~

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = r\frac{dp}{dx}$$

$$r\frac{\partial u}{\partial r} = \frac{r^2}{2}\frac{dp}{dx} + \varphi(x)$$

$$\frac{\partial u}{\partial r} = \frac{r}{2}\frac{dp}{dx} + \frac{1}{r}\varphi(x)$$

$$\rho = \frac{p}{R\theta}$$

$$\theta = \theta_0 -$$

$$u = \frac{r^2}{4}\frac{dp}{dx} + \cancel{\varphi(x)\log(r)} + \psi(x)$$

$$\rho u = fc.(r)$$

$$u = \frac{r^2-\delta^2}{4}\frac{dp}{dx}$$

$$\frac{p}{R\left\{\theta_0 - \frac{(\delta^2-r^2)^2}{128\mu k}\left[\frac{p_1^2-p_2^2}{l}\right]^2\frac{1}{p^2}\right\}}\;\frac{r^2-\delta^2}{4}\frac{dp}{dx} = fc.(r)$$

$$\frac{(r^2-\delta^2)\quad p^2\cdot d(p^2)}{8R\left[p^2\theta_0 - \frac{(\delta^2-r^2)^2}{128\mu k}\left[\frac{p_1^2-p_2^2}{l}\right]^2 \cancel{\#}\right]} = fc(r).dx$$

$$\int\frac{z\,dz}{az-b} = \frac{1}{a}\int\left\{dz + \frac{b\,dz}{az-b}\right\} = \frac{1}{a}\left[z + \frac{b}{a}\log(az-b)\right]$$

$$= \frac{1}{a}\left[p^2 + \frac{b}{a}\log(ap^2-b)\right]$$

$$\frac{1}{a}\left[p^2 + \frac{b}{a}\log(ap^2-b)\right] = \frac{8R}{r^2-\delta^2}fc(r).x + \text{const}$$

$$\frac{1}{a}\left[p_1^2 + \frac{b}{a}\log(ap_1^2-b)\right] = \text{const}$$

$$\frac{1}{a}\left[p_1^2 + \frac{b}{a}\log(ap_1^2-b)\right] = \frac{8R}{r^2-\delta^2}fc(r)\,l + \text{const}$$

$$\frac{8R}{r^2-\delta^2}fc(r) = \frac{1}{a}\left[p_2^2 - p_1^2 + \frac{b}{a}\log\left(\frac{ap_2^2-b}{ap_1^2-b}\right)\right]\cancel{\frac{r^2-\delta^2}{8R}}\frac{1}{l}$$

Supozycja: $\frac{\partial v_n}{\partial r} \odot \delta$

$$\frac{\partial v_s}{\partial \theta} + v_s \operatorname{ctg}\theta = 0$$

$$\log v_s = -\int \operatorname{ctg}\theta \, d\theta$$

$$= -\log \sin\theta$$

$$v_s \sin\theta = f_0(r)$$

$$V = -cx\left(1 + \frac{a^3}{2r^3}\right)$$

$$= -c r \cos\theta \left[1 + \frac{a^3}{2r^3}\right]$$

$$\frac{\partial V}{\partial r} = -c\cos\theta\left[1 + \cancel{\frac{a^3}{2r^3}}\right] + \frac{3c\cos\theta\,\cancel{a^3}}{2\cancel{r^3}}$$

$$\Delta^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial}{\partial \theta}$$

$$\lambda \frac{\partial}{\partial r}(V + \Phi) = 0$$

$$\Delta^2 V = \frac{\partial^2 V}{\partial r^2} + \left[\frac{1}{a^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\cos\theta}{a^2 \sin\theta}\frac{\partial V}{\partial \theta}\right] = v_n \frac{\partial \varepsilon'}{\partial n} + v_s \frac{\partial \varepsilon'}{\partial s} \Big[+ v_n \frac{\partial V}{\partial n} + v_s \frac{\partial V}{\partial s}$$

$$\Delta^2 V = \nu \odot \frac{1}{\delta} \qquad \varepsilon' \odot \frac{1}{\delta^2}$$

$$\begin{cases} \dfrac{\partial^2 V}{\partial r^2} = v_n \dfrac{\partial \varepsilon'}{\partial n} + v_s \dfrac{\partial V}{\partial s} \\ \dfrac{\partial p}{\partial n} = \varepsilon' \dfrac{\partial V}{\partial n} + \Big[\mu \dfrac{\partial^2 v_n}{\partial r^2} \\ \dfrac{\partial p}{\partial s} = \varepsilon' \dfrac{\partial V}{\partial s} + \mu\left(\dfrac{\partial^2 v_s}{\partial r^2} + \dfrac{2}{a}\dfrac{\partial v_s}{\partial r}\right) - \cancel{\dfrac{1}{a}\dfrac{\partial^2 v_n}{\partial r \partial \theta}} \end{cases}$$

$$\left.\begin{matrix} \dfrac{\partial}{\partial s} \\ \dfrac{\partial}{\partial r} \end{matrix}\right\} 0 = \frac{\partial \varepsilon'}{\partial r}\frac{\partial V}{\partial s} + \frac{\partial}{\partial r}\mu\left(\frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r}\right)$$

$$\mu v_s = \frac{\varphi_1 - \varphi_0}{4\pi}\frac{\partial V}{\partial s}$$

$$\mu \frac{\partial^2 v_s}{\partial r^2} = -\varepsilon' \frac{\partial V}{\partial s} = \frac{1}{4\pi}\frac{\partial^2 U'}{\partial r^2}\frac{\partial V}{\partial s}$$

$$\frac{\partial v_s}{\partial r} = \frac{1}{4\pi}\frac{\partial U'}{\partial r}\frac{\partial V}{\partial s} + f(s)$$

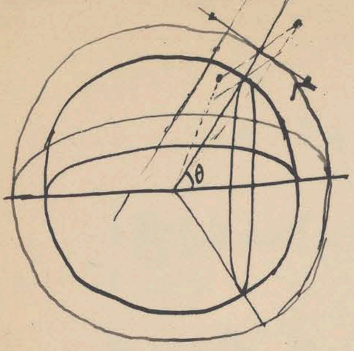
$$v_s = \frac{\varphi_1 - U'}{4\pi}\frac{\partial V}{\partial s} + \xi f(s) = \frac{f_0(r)}{\sin\theta}$$

$$\frac{\partial v_n}{\partial r} + \frac{\partial v_s}{\partial s} + \frac{v_s \operatorname{ctg}\theta}{a} = 0 \int d\xi$$

$v_n +$

$$\frac{\partial V}{\partial s} = \frac{f_0(r)}{\sin\theta} + \delta$$

$$V = f_0(r)\int \frac{d\theta}{\sin\theta} + \delta \ldots$$



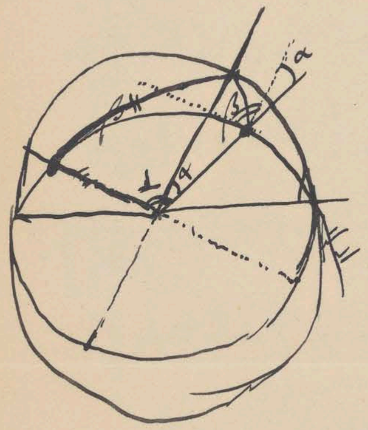
v_n v_s

$\cos\theta\cos\theta_1 - \sin\theta\sin\theta_1 = \cos(\theta+\theta_1)$

~~$v_\xi = v_r$ [illegible]~~

$$v_\xi = v_n\left[\sin\theta_1\cos\theta + \cos\theta_1\sin\theta\cos\varphi\right] + v_s\left[\sin\theta_1\sin\theta + \cos\theta_1\cos\theta\cos\varphi\right]$$

$$v_\xi = v_n\cos\alpha + v_s\cos\beta$$



$$\cos\alpha = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos\varphi$$

~~$\cos\beta = \cos\alpha \ldots + \sin\alpha \ldots \quad \cos\beta\cos\alpha = \frac{\cos\theta_1}{\sin\theta\sin\alpha}$~~

~~$= \frac{\cos^2\theta\cos\theta_1 - \sin\theta\cos\theta\sin\theta_1\cos\varphi - \cos\theta_1}{\sin\theta}$~~

$$\cos\beta = -\sin\theta\cos\theta_1 + \sin\theta_1\cos\theta\cos\varphi$$

$$v_\xi = v_n(\cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos\varphi) + v_s(-\sin\theta\cos\theta_1 + \sin\theta_1\cos\theta\cos\varphi)$$

$$v_\xi = \cos\theta_1(v_n\cos\theta - v_s\sin\theta) + \sin\theta_1(v_n\sin\theta + v_s\cos\theta)\cos\varphi$$

$$\Delta^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\cos\theta}{r^2\sin\theta}\frac{\partial}{\partial\theta}$$

$$\Delta^2 v_\xi = \cos\theta_1\left[\frac{\partial^2 v_n}{\partial r^2}\cos\theta - \frac{\partial^2 v_s}{\partial r^2}\sin\theta\right] + \sin\theta_1\left[\frac{\partial^2 v_n}{\partial r^2}\sin\theta + \frac{\partial^2 v_s}{\partial r^2}\cos\theta\right]\cos\varphi +$$

$$+ \frac{2}{a}\cos\theta_1\left[\frac{\partial v_n}{\partial r}\cos\theta - \frac{\partial v_s}{\partial r}\sin\theta\right] + \frac{2}{a}\sin\theta_1\left[\frac{\partial v_n}{\partial r}\sin\theta + \frac{\partial v_s}{\partial r}\cos\theta\right]\cos\varphi +$$

$$- \frac{1}{a^2\sin^2\theta}\sin\theta_1\left[v_n\sin\theta + v_s\cos\theta\right]\cos\varphi +$$

~~$+ \frac{1}{a^2}\cos\theta_1\left[v_n\cos\theta \right.$ [illegible]~~

$$v_r = -\cos\theta\left[1-\frac{a^3}{r^3}\right]$$

$$\frac{\partial v_r}{\partial r} = -\cos\theta\ \frac{3a^3}{r^4}$$

$$\frac{\partial^2 v_r}{\partial r^2} = +\cos\theta\ \frac{12a^3}{r^5}$$

$$\frac{\partial v_r}{\partial\theta} = +\sin\theta\left[1-\frac{a^3}{r^3}\right]$$

$$\frac{\partial^2 v_r}{\partial\theta^2} = +\cos\theta\left[1-\frac{a^3}{r^3}\right]$$

$$v_s = +\frac{3}{2}\sin\theta$$

$$\frac{\partial v_s}{\partial\theta} = \frac{3}{2}\cos\theta$$

$$\Delta^2 v_\xi = +12\frac{\cos\theta}{a^2} \pm 6\frac{\cos\theta}{a^2} - \frac{3\cos\theta}{a^2} - \frac{3}{a^2}\cos\theta = 0$$

W ogóle ~~[illegible]~~ ~~[illegible]~~ na powierzchni (gdzie $v=0$):

$$\Delta^2 v_\xi = \frac{\partial^2 v_r}{\partial r^2} + \frac{2}{a}\frac{\partial v_r}{\partial r}$$

$$\Delta^2 v_\xi = \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r}$$

$$v_s = \sin\theta\left[\frac{r}{a}+\frac{a^2}{2r^2}\right]$$

$$\frac{\partial v_s}{\partial r} = \sin\theta\left[\frac{1}{a}-\frac{a^2}{r^3}\right]\Big|=0$$

$$\frac{\partial^2 v_s}{\partial r^2} = \sin\theta\left[\frac{3a^2}{r^4}\right]$$

$$\frac{\partial v_s}{\partial\theta} = \cos\theta\left[\quad\right]$$

$$\frac{\partial^2 v_s}{\partial\theta^2} = -\sin\theta\left[\quad\right]$$

$$\Delta^2 v_\xi = \frac{3}{a^2}\sin\theta - \frac{3}{2a^2}\frac{\sin\theta}{\sin^2\theta} - \frac{\sin\theta}{a^2}\frac{3}{2} + \frac{1}{a^2}\frac{3}{2}\frac{\cos^2}{\sin}$$

$$= \frac{\sin\theta}{a^2}\left[\frac{3}{2} - \frac{3}{2\sin^2} + \frac{3}{2}\frac{\cos^2}{\sin^2}\right] = \cancel{\frac{3}{2a^2\sin}}\left[0\right. = 0$$

$$\cos\theta_1\left[\frac{\partial v_n}{\partial\theta}\cos\theta-\frac{\partial v_s}{\partial\theta}\sin\theta - v_n\sin\theta - v_s\cos\theta\right]+\sin\theta_1\left[\frac{\partial v_n}{\partial\theta}\sin\theta+\frac{\partial v_s}{\partial\theta}\cos\theta + v_n\cos\theta \pm v_s\sin\theta\right]\cos\varphi$$

$$\frac{\cos\theta_1}{a^2\sin\theta_1}\left[\frac{\partial v_n}{\partial\theta}(\cos^2\theta_1+\sin^2\theta_1\cos\varphi)-\cancel{\frac{\partial v_s}{\partial\theta}(1-\cos\varphi)\sin\theta_1\cos\theta_1}\pm v_n\cancel{\sin\theta_1\cos\theta_1(1-\cos\varphi)}\pm v_s(\cos^2\theta_1+\sin^2\theta_1\cos\varphi)\right]$$

$$\cos\theta_1\left[\frac{\partial^2 v_n}{\partial\theta^2}\cos\theta-\frac{\partial^2 v_s}{\partial\theta^2}\sin\theta -2\frac{\partial v_n}{\partial\theta}\sin\theta-2\frac{\partial v_s}{\partial\theta}\cos\theta - v_n\cos\theta+v_s\sin\theta\right] \Bigg| +\sin\theta_1\cos\varphi\left[\frac{\partial^2 v_n}{\partial\theta^2}\sin\theta+\frac{\partial^2 v_s}{\partial\theta^2}\cos\theta \Bigg| +2\frac{\partial v_n}{\partial\theta}\cos\theta-2\frac{\partial v_s}{\partial\theta}\sin\theta - v_n\sin\theta - v_s\cos\theta\right]$$

$$\frac{1}{a^2}\left\{\frac{\partial^2 v_n}{\partial\theta^2}(\cos^2\theta_1+\sin^2\theta_1\cos\varphi)-\cancel{\frac{\partial^2 v_s}{\partial\theta^2}(1-\cos\varphi)\sin\theta_1\cos\theta_1}-2\cancel{\frac{\partial v_n}{\partial\theta}\sin\theta_1\cos\theta_1(1-\cos\varphi)}- -2\frac{\partial v_s}{\partial\theta}(\cos^2\theta_1+\sin^2\theta_1\cos\varphi)-v_n(\cos^2\theta_1+\sin^2\theta_1\cos\varphi)+\cancel{v_s\sin\theta_1\cos\theta_1(1-\cos\varphi)}\right\}$$

$$\varphi=0 \,\|\, \cos\varphi=1$$

$\cos^2\theta_1+\sin^2\theta_1\cos\varphi = 1$ ~~[illegible]~~

$$\Delta^2 v_\xi=\frac{\partial^2 v_n}{\partial r^2}+\frac{2}{a}\frac{\partial v_n}{\partial r}-\frac{v_n}{a^2}-\frac{v_s}{a^2}\operatorname{ctg}\theta_1+\frac{1}{a^2}\left(\frac{\partial^2 v_n}{\partial\theta^2}-2\frac{\partial v_s}{\partial\theta}-v_n\right)$$

$$+\frac{1}{a^2}\operatorname{ctg}\theta_1\left[\frac{\partial v_n}{\partial\theta}-v_s\right]$$

$$\Delta^2 v_\xi=\frac{\partial^2 v_n}{\partial r^2}+\frac{2}{a}\frac{\partial v_n}{\partial r}-\frac{2v_n}{a^2}+\frac{1}{a^2}\frac{\partial^2 v_n}{\partial\theta^2}+\frac{1}{a^2}\operatorname{ctg}\theta_1\frac{\partial v_n}{\partial\theta}-\frac{2v_s}{a^2}\operatorname{ctg}\theta_1-\frac{2}{a^2}\frac{\partial v_s}{\partial\theta}$$

Jsouli $v_n=0\quad \frac{\partial v_n}{\partial r}\odot\delta\quad \frac{\partial^2 v_n}{\partial r^2}\odot 1$

$$\Delta^2 v_\xi=\frac{\partial^2 v_n}{\partial r^2}-\cancel{\frac{2v_s}{a^2}\operatorname{ctg}\theta_1}-\cancel{\frac{2}{a}\frac{\partial v_s}{\partial s}}$$

$$=\frac{\partial^2 v_n}{\partial r^2}+\left(\frac{2}{a}+\frac{2r}{a^2}\right)\frac{\partial v_n}{\partial r}+\frac{2v_n}{a^2}+\frac{1}{a^2}\frac{\partial^2 v_n}{\partial\theta^2}+\frac{1}{a^2}\operatorname{ctg}\theta\frac{\partial v_n}{\partial\theta}$$

$$\Delta^2 v_\xi = \frac{\partial^2 v_s}{\partial r^2} + \frac{1}{a}\frac{\partial v_s}{\partial r} + 1$$

$$- \frac{1}{a^2 \sin^2\theta_1}\left[v_r \cos\theta_1 \sin\theta_1 + v_s \sin^2\theta_1\right]$$

$$+ \frac{\cos\theta_1}{a^2\sin\theta_1}\left\{\frac{\partial v_r}{\partial\theta}\left[-\cancel{\sin\theta_1\cos\theta + \cos\theta_1\sin\theta\cos\varphi}\right] + \frac{\partial v_s}{\partial\theta}\left[\sin\theta_1\sin\theta + \cos\theta_1\cos\theta\cos\varphi\right]\right.$$

$$\left. + v_r\left[\sin\theta_1\sin\theta + \cos\theta_1\cos\theta\cos\varphi\right] + v_s\left[\sin\theta_1\cos\theta - \cancel{\cos\theta_1\sin\theta\cos\varphi}\right]\right\}$$

$$+ \frac{1}{a^2}\left\{\cancel{\frac{\partial^2 v_r}{\partial\theta^2}} + \frac{\partial^2 v_s}{\partial\theta^2} + 2\frac{\partial v_r}{\partial\theta}\left[\sin^2\theta_1 + \cos^2\theta_1\cos\varphi\right] + \cancel{[illegible]} + \cancel{v_r \sin\theta_1\cos\theta_1} - v_s\left(\sin^2\theta_1 + \cos^2\theta_1\cos\varphi\right)\right\}$$

$$= \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r} \cancel{- \frac{v_r}{a^2}\operatorname{ctg}\theta_1} - \frac{v_s}{a^2}\operatorname{ctg}^2\theta_1 \cancel{+ v_r \frac{[illegible]}{a^2}\operatorname{ctg}\theta_1} + \frac{\partial v_s}{\partial\theta}\frac{\operatorname{ctg}\theta_1}{a^2}$$

$$+ \frac{1}{a^2}\frac{\partial^2 v_s}{\partial\theta^2} + \frac{2}{a^2}\frac{\partial v_r}{\partial\theta} - \frac{v_s}{a^2}$$

$$1 + \operatorname{ctg}^2 = 1 + \frac{\cos^2}{\sin^2} = \frac{1}{\sin^2}$$

$$= \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r} - \frac{v_s}{a^2\sin^2\theta_1} + \frac{1}{a^2}\frac{\partial^2 v_s}{\partial\theta^2} + \frac{1}{a^2}\frac{\partial v_s}{\partial\theta}\operatorname{ctg}\theta_1 + \frac{2}{a^2}\frac{\partial v_r}{\partial\theta}$$

Ny. ~~[illegible]~~

$$\cancel{\Delta r}\frac{\partial}{\partial r}\left(\cancel{4} r \cancel{\pi}\, \cancel{[illegible]}\, v_r\right) . \sin\theta \cancel{\Delta\theta} + \cancel{4} r \cancel{\pi} \cancel{\Delta r}\frac{\partial}{\partial\theta}\left(\sin\theta . v_s\right)\cancel{\Delta\theta} = 0$$

$$\sin\theta \frac{\partial}{\partial r}\left(v_r . r^2\right) + r\frac{\partial}{\partial\theta}\left(v_s \sin\theta\right) = 0$$

$$2\frac{v_r}{r} + r\frac{\partial v_r}{\partial r} + \frac{\partial v_s}{\partial\theta} + v_s \operatorname{ctg}\theta = 0 \quad = \operatorname{div}$$

$$\frac{\cos}{\sin}' = -\frac{\cos^2}{\sin^2} - 1$$

~~[illegible]~~ $\frac{\partial}{\partial\theta}$:

$$\frac{2}{a^2}\frac{\partial v_r}{\partial\theta} + \frac{r}{a^2}\frac{\partial^2 v_r}{\partial r\,\partial\theta} + \frac{\partial^2 v_s}{a^2\,\partial\theta^2} + \frac{\partial v_s}{a^2\,\partial\theta}\operatorname{ctg}\theta - \frac{v_s}{a^2\sin^2\theta} = 0$$

$$\Delta^2 v_\xi = \frac{\partial^2 v_s}{\partial r^2} + \frac{2}{a}\frac{\partial v_s}{\partial r} - \frac{r}{a^2}\frac{\partial^2 v_r}{\partial r\,\partial\theta}$$

$$\frac{\partial p}{\partial x} = \mu \nabla^2 u + \frac{\rho}{r^2}\frac{x}{r}$$

$$\frac{\partial p}{\partial y} = \mu \nabla^2 v + \frac{\rho}{r^2}\frac{y}{r}$$

$$\frac{\partial p}{\partial z} = \mu \nabla^2 w + \frac{\rho}{r^2}\frac{z}{r}$$

~~$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \Delta^2 p = 0$~~

$$\rho\left(\frac{\partial u}{\partial x} + \quad\right) + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0$$

$$\left(u\frac{\partial}{\partial x} + \quad\right) - \frac{k}{k-1} R \rho \left(u\frac{\partial \theta}{\partial x} + \quad\right) + \mu\Phi + \kappa\Delta^2\theta = 0$$

$$\frac{1}{a}\left[p^2 + \frac{b}{a}\lg(ap^2 - b)\right] = \frac{1}{a}\left[p_2^2 - p_1^2 + \frac{b}{a}\lg\left(\frac{ap_2^2 - b}{ap_1^2 - b}\right)\right]\frac{x}{l} + \frac{1}{a}\left[p_1^2 + \frac{b}{a}\lg(ap_1^2 - b)\right]$$

$$p^2 - p_1^2 + \frac{b}{a}\lg\frac{ap^2 - b}{ap_1^2 - b} = \left[p_2^2 - p_1^2 + \frac{b}{a}\lg\left(\frac{ap_2^2 - b}{ap_1^2 - b}\right)\right]\frac{x}{l}$$

$$2\pi\int \rho u \, r\,dr = \frac{2\pi}{R}\int p\frac{u}{\theta} r\,dr \quad \frac{2\pi}{R}\int \frac{r^2 - \delta^2}{4}\, p\frac{dp}{dx}\, r\,dr$$

$$= 2\pi\int r\,dr\left[\theta_0 - \frac{(\delta^2 - r^2)^2}{128\mu\kappa}\left(\frac{p_2^2 - p_1^2}{l}\right)^2\frac{1}{p^2}\right]\frac{(r^2 - \delta^2)}{8R}\;\frac{1}{\theta_0}\left[p_2^2 - p_1^2 + \frac{b}{a}\lg\left(\frac{ap_2^2 - b}{ap_1^2 - b}\right)\right]$$

$$\lg(ap^2 - b) = \lg ap^2 + \lg\left(1 - \frac{b}{ap^2}\right) = \lg(ap^2) - \frac{b}{ap^2}$$

~~$u^2 = \frac{(y\delta - y^2)^2 c^2}{16\mu^2(a - cx)}$~~

~~$\frac{(y\delta - y^2)^2 c^3}{16\mu^2(a - cx)^2}$~~ $\frac{1}{Ru}$

$$\frac{R\theta\, c^2(r^2 - \delta^2)}{16\mu(a - cx)^2} = \frac{(y\delta - y^2)c^2}{4\mu(a - cx)}$$

$$\frac{1}{\rho}\frac{\partial u}{\partial x} = \frac{R\theta}{p}\frac{\partial u}{\partial x}$$

$$= \frac{R\theta\,(r^2 - \delta^2)\,c^2}{16\mu\sqrt{(\quad)^3}}$$

$$\frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = \int \frac{(r^2 - \delta^2)^2 c^3}{128\mu^2(a - cx)^2}\, r\,dr =$$

$$\int \frac{\partial u}{\partial x}\cdot u\, r\,dr = \frac{r^2 - \delta^2 \cdot c^3}{(a - cx)^2}$$

$$\int p\, u\, r\,dr = \frac{k}{k-1}\frac{\Delta\theta}{\theta}\; c\int_{\delta}^{r} (r^2 - \delta^2)\, r\,dr$$