Valentiner ... 60 ... Smithe p. 119

Nuson 0.0025

Dessel 0.001

Wien Ber. 106 [Boltzm.-Wind
105 [Jäger U. (4893)
104 I. (2702)
103
102
101
100
99

Proc. R. S. 59 (1896) Roberts-Austen
57 (1895) Fitzgerald, Dunbury
50 Kelvin

Nature 52 (Basset, Culverwell, Bryan, Burbury)
51 (Culv., Burb.,
46 (Watson, Burbury)
45 Korteweg Ray Tait Watson
44 Kelvin

[78057] 53 ff.

T 18 ff. [3511]

Kroker U 12 ff
Kamm. U. 36 ff [41.S.1]

Pr. Lond. math. soc. 27 (1896) Burbury
26 (1895) (Burbury) ~~Procee~~ (9-15) (78157)

Zittingsversl. K.A.W.A. 1896/7 (Lorentz)

Gött. Nachr. 1896 (Voigt) 10792 L.U. ~~XII~~ K3

Rep. B. A. 1894 (Bryan) 1891 (Culverwell)

Pr. Cambr. Ph. S. 8 (1895) (Bryan), 1894, 7 (1891) Lady

Rev. gén. sciences (Poincaré) 5 (1894)

L.U. 4 37-46 (25006)
L.T 5, 13 ff. [3512]
Kamm 5, 25 ~~...~~-28 [20, P. 2]
Wien T 3044
Phil. Mag.
T

Phil. M. 30 Burbury Culv.
33 Thomson Ray
32 Sutherland, Rayleigh
35 Boltzmann)

Maxwell: Illustrations of the Dynamical Theory of Gases

Prop. XIII: $\mu = \frac{1}{3}\rho l v = \frac{1}{3\sqrt{2}} \frac{Mv}{\pi s^2}$

"A remarkable result: ...... friction independent of density. Such a consequence ... very startling, and the only experiment I have met with on the subject does not seem to confirm it."

Prop. XIV. "In a system of moving particles whose density, velocity etc. are functions of x, to find the quantity of matter transferred across the plane of yz, due to the motion of agitation alone."

Quantity of matter, projected from stratum dx, crossing yz in + direction & striking other particles at distances between nl & $(n+dn)l$ is:

$$\frac{MNv(x \mp nl)}{2nl^2} dx\, e^{-n} dn \qquad N, v, l \text{ are functions of } x$$

If U and r are functions (x) not vanishing with x whose variations are very small between the limits $x = +r$ and $x = -r$:

$$\int_{-r}^{+r} \pm U x^m dx = \frac{2}{m+2} \frac{d}{dx}\left(U r^{m+2}\right)$$

$$\therefore \int_{-nl}^{+nl} \frac{MNvx}{2nl^2} dx = \frac{1}{3} \frac{d}{dx}\left(MNvn^2 l\right) \qquad \int_{-nl}^{+nl} \mp \frac{MNv}{2l} dx = -\frac{1}{2} \frac{d}{dx}\left(MNvn^2 l\right)$$

Now we have $\int_{+0}^{\infty} -\frac{1}{6} \frac{d}{dx}(MNvl)\, n^2 e^{-n} dn = \cdot \qquad \cdot\, q = -\frac{1}{3} \frac{d}{dx}(\rho v l)$

Prop. XXI "To find the amount of energy which crosses unit of area in unit of time when the velocity of agitation is greater on one side of the area than on the other"

Substituting $E = \frac{1}{2}\beta M v^2$ for M in Eq. XIV: $E = \rho \frac{\theta}{N} = \beta v \frac{MNv^2}{2N}$

$$J_q = -\frac{1}{3}\frac{d}{dx}\left(\frac{1}{2}\beta M v^2 N v l\right)$$

$\beta = \beta v . M$

Now: $MN = \rho$ $\quad l = \frac{1}{A\rho}$ $\quad$ & $MNl = \frac{1}{A}$

$$J_q = -\frac{1}{2}\frac{\beta v^2}{A}\frac{dv}{dx} \qquad \frac{2}{v}\frac{dv}{dx} = \frac{1}{T}\frac{dT}{dx}$$

$$\therefore \quad J_q = -\frac{3}{4}\beta \rho l v \frac{1}{T}\frac{dT}{dx}$$

[calculation: conductivity of air 10^6 times less than ~~lead~~ copper faulty, see Clausius]

~~k is variable too! dependent on v~~

$$J_q = -\frac{\beta}{6}\frac{\partial}{\partial x}(\rho v^3 \lambda) = -\frac{\beta}{2}\rho\lambda v^2\frac{\partial v}{\partial x} = -\frac{\beta}{2}\rho\lambda v \; v\frac{\partial v}{\partial x}$$

$$= \cancel{\frac{3}{4}\ldots\frac{\partial T}{\partial x}}$$

$$v\frac{\partial v}{\partial x} = \frac{v^2}{2}\frac{1}{T}\frac{\partial T}{\partial x}$$

$\beta = \frac{5}{3}$ $\qquad$ sp. $T = m\frac{v^2}{2}$ $\qquad$ $\frac{1}{m}\frac{\partial T}{\partial x}$

$$q = \frac{3\beta\mu}{2}\frac{1}{m}\frac{\partial T}{\partial x} = \frac{3}{2}\beta\mu\frac{\partial T}{\partial x}$$

$k = \frac{5}{4}\mu$

$\beta\rho$ $\qquad K = \frac{\beta}{2}\frac{3\mu}{m}\frac{\partial T}{\partial x}$

$$J_q = \frac{3}{4}\beta\frac{p\lambda}{T}\frac{\partial T}{\partial x}\cdot\frac{3\mu}{\rho\lambda}$$

$\frac{p}{\rho} = T . n$

$\frac{\frac{nmc^2}{3}}{nm} = \frac{mc^2}{2}\frac{2}{3m}$

$$\lambda = \frac{1}{\pi n \sigma^2} \qquad p v = RT \qquad J_q = -\frac{3}{4}\beta\lambda R\frac{dT}{dx}$$

$$= \frac{9}{4}\beta\mu n\frac{\partial T}{\partial x}$$

$$q = \frac{3}{2}\beta\frac{\mu}{m}\frac{\partial T}{\partial x}$$

für einatomige Moleküle: $\beta = \frac{5}{3}$ somit $K = \frac{5}{2}$ ..

Boltzmann: $E_T = (1+\beta) E_{pr}$

Maxwell II

$k = \gamma_v \mu =$ ‖ $k = \frac{5}{2} \gamma_v \mu$

Maxwell I: berücksichtigt wo ϑ gross [illegible] v grösser daher Factor $\frac{3}{2}$, aber [illegible]

Beibl. 21 p. 212 De Heen [illegible] CO_2 :

p. 625 " [illegible]

findet dass die Dichte der Flüssigkeit doppelt so gross wie die des Dampfes

Dagegen p. 579 Villard [illegible]

Amagat p. 209 [illegible] V.d.W. [illegible] const. [illegible] photogr. [illegible]

	CO_2	C_2H_4	Äther	Luft
kr. temp.	31.35	8.8	195	−140.7
druck	72.9	48.5	36.5	35.9
Dichte	0.464	0.212	0.253	0.344

dennoch erklärt [illegible] das [illegible] Sydney Young [illegible]

---

p. 957 Leduc & Sacerdote [illegible] const [illegible]:

	ϑ	π	
HCl	51.5	96	Vincent Chappuis
	52.3	86	Dewar
	52	83	Led. S.S.
PH_3	52.8	64	"
H_2S	100	88.7	Olszewski
	100.2	92	Dewar
	100	90	Led. S.S.
$(CH_3)_2O$	129.6	—	Nadejdine
	129.6	57	L. S.S.

---

p. 955 Kuenen [illegible] Condens. [illegible]

(Phil. Mag. 44 p. 174)

[illegible]

	ϑ	π	a	b
N_2O	309·	71·9	0·00753	0·00197
C_2H_2	308·25	61·02	881	231
CO_2	304·1	73·26	714	190
		48·8	1078	286
C_2H_6	305			

p. 404 Lorenz [illegible] temp. [illegible]

Theorie der Linde' [illegible]

p. 21 Kamerlingh Onnes [illegible] temp.,

Wasserstoff therm. & Thermoelement, genaue [illegible] Neu [illegible]

p. 718 $\frac{C}{c}$ = 1·26 für Acetylen Maneuvrier & Fournier

c_p = 0·1233 Argon Dittenberger

p. 856 Moissan & Dewar [illegible] Fluors CR 124 p. 1202

[illegible] −185°

p. 851 S. Young Pentan ϑ = 197·2, π = 25100 mm, $\frac{w}{\text{gr}}$ = 4·303 cm^2

p. 329 Dewar [illegible] Roy Inst. 1896 27/3

Beschreibg. d. Apparates

O [illegible]; NO [illegible]

O spec. [illegible] = 1·1375 (bei 766·5 mm)

Wroblewski 1·168

Olszewski 1·124

[illegible]

Luft [illegible] 10 [illegible] ??

Beibl. 22

p. 73 Leduc [illegible] 20

[illegible] :

CO_2	1·5287
N_2O	1·5301
HCl	1·2692
H_2S	1·1895
Cl_2	2·491
NH_3	0·5971
SO_2	2·2639

p. 94 Moissan & Dewar:

Neue Erfahrg. [illegible] Fluor CR 125 p. 505

Siedep. d. Fluor : −187° bei −210° [illegible]

Dichte = 1·14 kleinere Cap. cond. [illegible] 0

keine Absorption, nicht magnetisch

---

p. 265 Lange Spec. W. des flüssigen Ammoniaks)

Sehr ausführl. Beobachtgn zwischen −50 und + 100°

---

p. 392 Kuenen Aethan

Kuenen	Observation		Haenlen	
	−93	1		
$\tau = 32·0$, $\pi = 48·8$	34	50·2	34·5	50

~~Fluor - Dewar~~ Fortsetz. 52 ~~p. 510~~

Rdschau p. 522

Fleming & Dewar magn. Suszept. d. fl. O_2

bei $-182°$: $\mu = 1.0041$ wenig abhängig von Feldstärke ([illegible])

~~[illegible]~~ Mangansulfat = 3·32 mal so groß als bei 25°

~~[illegible]~~ Wismuth Zunahme 16%

Es scheint dass für paramagnet. [illegible] $\frac{\text{Dichte}}{\text{abs. Temp.}}$

Siedetemp. d. flüss. Ozon CR 1898, p. 1751 (−119°) Troost

Rdschau p. 375

A. Schuster (Nature LVIII p. 199 30/6):

Auffallende Ähnlichkeit des Spectrums des [illegible] mit gewöhnl.

C Spectrum 8 Streifen [illegible] ; [illegible] Cyan [illegible]

Observ[illegible] Wied. Ann. 69 184 Helium!

Fortsch. 52 p. 323 The new process for the liquef. of air etc Nature

53 p. 515

Schon vor Linde hat Hampson [Brin's Oxygen Works 21/3 1895]

[illegible] App. constr. [illegible]

Linde Wied. Ann 57 p. 328

p. 510 Fleming Dewar Electr. [illegible] tief. Temp.

---

p. 33 Gladstone Specifische Refraction & period. [illegible]

Nach Rayleigh: spec. [illegible] Argon = 0·159

[illegible] Atomgew. = 20 [illegible]

Lockyer the story of helium Nature 53, 319, 342 (1896)
245
526

Eder & Valenta Wien Denkschr. 64 p. 39 Spectrum des Argons

Wszechświat p. 669:

Brit. Ass. Bristol: Crookes Monium (118)

Ramsay & Travers: Xenon najniższa temp. wrzenia pozostaje gdy inne wyparowały gęstość jego większa niż innych. Widmo analogiczne z argonem

Neon: 19.2, jednoatomowy, w powietrzu 1:40.000

Fortschr. 97 p. 157 Berthelot (CR 124 p. 113,) Recherches sur l'hélium.

Fand chemische Verbindungen mit Benzol, u CS_2 [illegible]

Ersteres [illegible] Spectrum [illegible]

Letzteres Kohleartig; [illegible] He [illegible] Spectr. [illegible]

Analysen nicht ausgeführt.

Wilde glaubt nicht an Berthelots Hyp. dass Argon allotr. N

[illegible] N [illegible].

W. Ramsay Ein unentdecktes Gas

Glaubt dass He und Arg in eine Gruppe [illegible]; [illegible] 3tes

Element [illegible] Atomgew. 20 liegen [illegible] Suchen ohne Erfolg gemacht

Brit. Ass. Toronto 19/VIII 97

CR 126 p. 1610

W. Ramsay, M. Travers & Miss ~~[illegible]~~ Emily Aston

Dr. Hampson lieferte 750 cm³ flüssige Luft

Alles wurde [illegible] 10 cm³

[illegible] O_2 [illegible] Cu [illegible] $\underline{N_2}$ [illegible] MgO & CaO [Selectiv. Funken [illegible] O_2 & N_2O

[illegible] 26·2 cm³ [illegible] [illegible] [2 [illegible] D_3 He [illegible]]

Dichte: 22·47 (O_2 = 16) $\frac{C}{c} = 1·666$ [illegible] Kundt —
22·51

also weniger flüchtig als O_2 N_2 Ar, einatomig = Kr

Rutherford [illegible] (5567) [illegible] Nordlichtes [illegible] Eosinum [illegible]

CR ~~[illegible]~~ 126 p. ~~[illegible]~~ 1762 Dieselben

18 Lit. Argon 216 [illegible]

Dichte 51? ; [illegible] ; [illegible] Ar [illegible]

[illegible] 2 Fractionen à 75 cm³ 160

[illegible] sehr brillantes orange Spectrum [illegible] (5849·6)

Also folgende Linien:

Na	D_1	5895·0
Na	D_2	89·0
He	D_3	75·9
Kr	D_4	66·5
Ne	D_5	49·6

Neon

Dichte 14·67 [illegible] 10-11 ?

Es wird von Alum. Electr. [illegible] absorb. [~~[illegible]~~ [illegible] Ar, He, Kr] [illegible] orange

$\frac{C}{c} = 1·66$

v 6 5 [illegible] Ni – Co

of Fraction ~ 6 Compl. Spectrum

Dichte : 1.987 $\frac{c}{c} = 1.66$ Metargon

Es ist noch fest bei Siedetemp. während Kr [illegible]

daher kann man es sehr leicht rein erhalten

$D_5 = 5849.6$

Beibl. 22 86e

J. Dewar Platinthermometer hat 0 Punkt von – 265.2 Pt Graden

[illegible] H_2 [illegible] – 256.8 [illegible] Scala

Dichte von H_2 bei Siedep. = 0.07

Dichte des gasf. H_2 = 0.55

klar, farblos, kein Absorpt. Spectrum, also kein Metall

Beibl. 22 p. 267 Ramsay & Travers

Helium homogen; [illegible] Ar 2

Beibl. 22 p. 217 Ramsay & Travers [Proc R. S. 62 p. 225 1898]

Brechungs Exp. d. Luft, O_2, N_2, Ar, H_2, He

mit d. Apparat von Rayleigh (siehe weiter)

n Luft = 1

H_2	0.4733
O_2	0.9243
N_2	1.0163
A_2	0.9596
CO_2	1.5316

Beibl. 20 p. 312

Rayleigh [Pr. 59, p. 198, Zeitschr. f. ph. Ch. 19 p. 364, 1896]

1). Außergewöhnlich reines Arg. Dichte $= 19·940$ $(O=16)$

2). $\frac{n_{Ar}}{n_{Luft}} = 0·961$ $\frac{n_{He}}{n_{Luft}} = 0·146$ also die kleinste bekannte

[illegible] $\sqrt{N_2}$ [illegible]

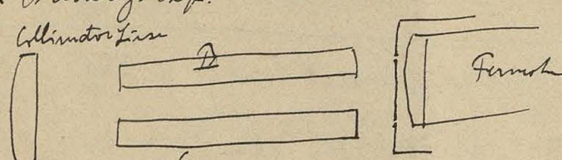
3). Capillare Methode Reibungscoeff. $\eta_{Ar} = 1·21$ $(O_2 = 1)$

$\eta_{He} = 0·96$

Siehe besonders obiges Referat in Zeitschr.

Reinigung von Ar ausführlich beschrieben

Methode für Brechungsexp.



[illegible] Luft [illegible]

[illegible]

[illegible] compens. [illegible]

[illegible] ~~Cylinderlinse~~ [illegible] Cylinder [illegible]

[illegible] vertical; [illegible] horiz. [illegible]

Es genügen Glasstäbe & wassergefüllte Röhren

In Luft ist von He $< \frac{1}{10000}$ [illegible] Ar. & He-Spectr. [illegible] = [illegible]

Fortschr. 96 p. 118

W. Ramsay (Nature 53 p. 598)

He [illegible] Clevit 7·2 cm^3 auf 1 gr.

[illegible]:
- Bröggerit 1 cm^3 "
- Samarskit 0·6 " "
- Fergusonit 1·1 " " } Analyse: 54·7 % H_2, 13·9 % CO_2, 31·2 % He

Dichten: 2·18, 2·12, 2·14

---

p. 120 W. Ramsay The position of ar. and he among the elements

Atomgew. Ar = 39·88

He = 4·28

W. R. & Collie: [illegible] 38

Karl Olszewski Wied. Ann 69, p 184 [illegible]

Helium aus Clevit (v. Ramsay) Dichte (H=1) = 2·133

$$\frac{C_p}{C_v} = 1{\cdot}652$$

100 cm^3 [illegible] 0·7 cm^3 He

[illegible] temp [illegible] −264° [illegible]

Fortschr. 95 Rayleigh Ramsay p. 124

[illegible] atmosph. N : 2·310 [illegible] 0·8 % [illegible]

chem. 2·299

~~100~~ Vol. [illegible] 13° [illegible] 4·0 Vol Ar

$$\frac{C_p}{C_v} = 1{\cdot}645$$

Siedepunkt − 186·9

krit. Punkt − 121°   krit Druck 50·6

Durch Olszewski auch feste Krystalle (wird) [illegible] − 189·5° [illegible]

---

p. 132 Kellas:

100 cm³ Luft enthalten 0·937 cm³ Ar

also auf 100 cm N_2: 1·186 cm³ Ar

Litergewicht von N_2(rein) = 1·2511

atmosph = 1·2572

Ar = 1·782

---

p. 126 Berthelot C.R. 120

Ar + Benzoldampf [illegible]

---

p. 132 Ramsay: Ar:

[illegible] Cleveit (oder Uranpecherz) mit [illegible] H_2SO_4 D_3

[illegible] = 3·89 [illegible]

---

p. 134 Ramsay, Collie, Travers

[illegible] U Y Th

Dichte = 2·13 (O=16)

[illegible] 0·0073 Vol. He $\frac{C}{c}$ = 165 − 163

---

p. 782 Olszewski (1895)

	ϑ	π	Siedep.	Gefrierp.	Gefrierdruck mm	Dichte der Flüssigk. beim Siedep.
H_2	220·0	20·0	?	?	?	?
N_2	146·0	35·0	194·4	−214·0	60	0·885
CO	139·5	35·5	190·0	207·0	100	?
A	121·0	50·6	187·0	189·6	?	1·5
O_2	118·8	50·8	182·7	?	?	1·124
NO	93·5	71·2	153·6	167·0	138	?
CH_4	81·8	54·9	164·0	185·8	80	0·415

250 in Vacuum

Achl. 22 p. 515 Dewar: Solidif. of H_2: -238° ; [illegible] ca -223°, [illegible] ca 15

Nature 58 p. 370 Crookes:

Helium lines distinctly visible in the more volatile portion of liquid air. Helium out from Bath Gas containing Neon.

Nature 58 p. 566

Xenon the heaviest of the three gases (A, Kr, Xe.)

The last fraction of liquefied A, ~~lightest [illegible]~~ remains behind spectr. analogous to A but differing entirely in the position of the lines.

p. 545

Baly [illegible] He [illegible] Ne [illegible] ; Ramsay & Travers [illegible]

~~He~~ Ne [illegible] 9·6 [at.w. = 19·2] [illegible] Fl & Na

	ϑ	π	φ	δ	
CH_4	−81·8	54·9			Olszewski CR 100, p. 940 (1885)
	−95·5	50·0			Dewar } Phil. M. (5) 18 p. 210
C_2H_6	35	45·2			Dewar } (1884)

Dichte: Methan 0·4148 bei $\underbrace{-164·0}_{=\text{Siedep.}}$ Olszewski Bull. Crac. 14 (1886) Beibl. 1886

− 185·8 = δ

t	p
−85·4	49·0
−93·3	40·0
105·8	26·3
110·6	21·4
126·8	11·0
138·5	6·2
153·8	2·24
185·8	0·105 = Gefrierpunkt = 80 mm Hg
201·5	0·066

Obs. CR 100

Äthan 4° : 46 Atm.
Cailletet
CR 85 p. 851
1877

Äthan:

+34° (kr)	50·2 (kr)
29	46·7
23·5	40·4
0	23·8
−93 (Siedep.)	1
−151	Dp.

Propan

+97°	44
49	18
43	15·7
30	11·1
20	8·8
10	7·4
0	5·0
−45 (Sied.)	1·0
−151	Dp.

Olsz. Crac. 1889
Fortschr. 1894 p. 354

Hainlen: Liebigs Ann. 282, p. 229, (1894) Fortschr. 1894 p. 354

Äthan: Siedep.: −89·5°

kr. +34·5 $\pi = 50$

$\delta = 0·446$ bei 0°

0·396 bei +10·5°

Propan: Siedep. −37·0 $\delta = 0·536$ bei 0°

kr. +102° $\pi = 48·5$

δ = 0·536	bei 0°
0·524	6·2
0·520	11·2
0·515	15·9

n-Butan Siedep. +1 $\delta = 0·60$ bei 0°

Propan: t	p
−33	1·8
19	2·7
15	3·1
11	3·6
5	4·1
2	4·8
+1	5·1
+5·5	6·9
+12·5	7·1
22	9
53	17·0
85	35
+102	48·5

S. Young: Journ. Chem. Soc. 71 p. 446 (1897)

Siedep. von normalem Pentan bei 760 mm: 36·3°

sp. G. b. 0°: 0·64536

kr. Temp. 197·2° $\pi = 25100$ mm

Volum eines gramm bei kr. Temp. 4·303 cm³

Dampfdrucke Biot'schen Formel: $\log p = a + b\alpha^t + c\beta^t$

$a = 7·62281$, $b = -4·531970$, $\log b = 0·656866$, $c = -1·213285$, $\log c = 0·0980498$,

$\log \alpha = \bar{1}.\,0·99926637$, $\log \beta = \bar{1}.\,0·99448608$, $t = t^\circ C + 20$.

2510 : 76 = 33·03
230
2

Wied. Ann. 5 p. 405

A. Ritter [illegible] Atm. [illegible] Constit. [illegible]

temp [illegible] $/c_v$ = [illegible]

stabil [illegible] labil [illegible] temp. [illegible]

[illegible]

$dQ = c_p\,dT - Av\,dp = 0$, $dp = -\gamma\,dz$

$$0 = c_p\,dT + A\,dz$$

Für die ganze Höhe der Atm [illegible] $A\int_0^H dz = -c_p\int_{T_0}^{0} dT$

[illegible] für $T_0 = 273$

$H = 27491.1\,m$ [illegible] 3. 705 [illegible]

[Mohn u. Guldberg [illegible] Temp. [illegible] vertic. [illegible] Atm [illegible] d. [illegible] 10 / Met. [illegible] 1878]

[illegible]

Atm [illegible] 0° C. : $H = 348..952\,m$

[illegible] $H = 28\,705\,m$

x kg. [illegible] (1-x) kg [illegible] x = [illegible]

$dQ = c\,dT + T\,d\left(\frac{xr}{T}\right) = 0$

Clapeyron Clausius:

$\frac{r}{T} = A\,u\,\frac{dp}{dT}$ [u = Vol. [illegible]

$\frac{xr}{T} - \frac{x_0 r_0}{T_0} = c\log\left(\frac{T_0}{T}\right)$

$A\,\underbrace{xu}_{=v}\,\frac{dp}{dT} = \frac{x_0 r_0}{T_0} + c\log\frac{T_0}{T}$

$Av\frac{dp}{dT} = \frac{x_0 L_0}{T_0} + \xi\log\left(\frac{T_0}{T}\right)$ $x_0 = 1$

(Somit $AH = \xi T_0 + L_0$

$A\,dz = -\left(\frac{L_0}{T_0} - \xi\log T_0\right)dT + \xi\log T\,dT$

[illegible] 200 km

[illegible] 26 [illegible]

[illegible] R. Pictet [illegible] Cailletet [illegible] adiab. [illegible]

[illegible] p. 416

[illegible] Schneewolken [illegible]

[illegible] Schmelzstumpfe, [illegible] 60 [illegible]

[illegible] H = 349 km

Hergesell Fortschr. 53 p. 192 (II)

18/2 1887 15000 Höhe −66° Luft: 0·033 CO_2 / 20·79 O_2 / 78·27 N_2 / 0·94 Ar } Vol.

10 800 −55°

Strahlungsconstante nach Lehnebach (1874): 0·0152

Christiansen (1883): 0·0167

Kurlbaum (1898): 0·0176

daraus Sonnentemperatur: 6249° − 6737°

(Warburg) je nach Solarconstante $= \frac{3}{60}$ oder $\frac{4}{60}$

V. ph. G. 52 (1898)

Verh. ph. G. (1898) Elster & Geitel [illegible] d. Radium [illegible]

Egon v. Oppolzer Met. Zs. 1896, p. 73

[illegible] $p = R\rho T$

1). $dp = -\rho\, dh$

2). $dp = RT\, d\rho + R\rho\, dT$

3). $RT \frac{d\rho}{dh} = -\rho \left[1 + R \frac{dT}{dh}\right]$ also $\frac{d\rho}{dh} > 0$ wenn $-\frac{dT}{dh} > \frac{1}{R}$

$\frac{1}{R}$ für $H_2 = \frac{1}{422{\cdot}6} = 0{\cdot}0023$ also [illegible] H_2 [illegible]

$\frac{dT}{dh} = \frac{1}{R}$ (cca 6000 [illegible])

Assmann franz. [illegible] Registrierball.

20/X 95 – 22/III 96

↓
0'52° auf 100m | 1° auf 182m

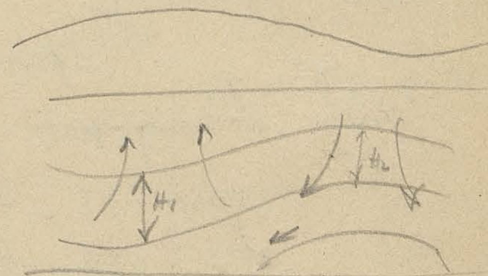
Min. −68 | Min. −63°

in 14000 | in 14000 m

= 0'54/100m

Verh. B. G. [illegible]

W. Bezold



$H_1 : H_2 = T_1 : T_2$

Temp. [illegible] Hann [illegible]

[en relat. [illegible] 2000 – 4000 [illegible] H_2O]

[illegible] 7000 m [illegible]

temp. [illegible]

Ivanof [illegible]

Encke'sche Komet

Newton ↗

Euler [illegible]

Laplace [illegible] 0'2 [illegible]

Bull. Ac. Sc. Pet.

Barth

Edwin Burt [illegible] 19 [illegible] 92

[illegible] $\odot$ [illegible] 28% [illegible] $\odot$ Atm. [illegible]

~~[illegible] 1.7~~

[illegible] 1.7 [illegible] Atm.

[illegible] Photosphäre [illegible] < 0.85 [illegible]

Van Bebber Meteorologie

Br.	temp. Nord	Süd	Luftdruck:	
0°	25.9		758	
10°	26.4	25.0	57.9	59.1
20	25.6	22.7	59.2	60.7
30	20.3	18.5	61.7	63.5
40	9.6	11.9	62.0	60.5
50	2.3	3.2	60.7	53.2
60	−1.3		58.7	48.4
70	−13.3		58.6	38.0
80	−20.0	−9.3	60.5	

Luft: 21.00 Vol. O_2
78.96 N_2
0.04 CO_2

Höhe der Atm.
Alhazen [illegible] $\odot$ [illegible] = 52000[x]

Melanderhjelm 4833 [illegible] = centrif.

G. G. Schmidt [illegible] Aeq. 27.5
Pole 27.1 g. M.

Mendeleef's Formel (Arch. d. sc. ph. Genève Mars 1876)

$$t_h = c + \frac{t_0 - c\,p_h}{p_0}$$

t_0 = temp. [illegible] Station

c = " [illegible] = −36°

(temp. = [illegible]) nach Slaistus [illegible]

Dagegen [illegible] Woerkoff [illegible]: c = −43·6

[illegible] 43·6 [illegible]

Nach Vettin ([illegible]) [illegible] 18·7 16·2

[illegible] 9·5

Nach Hagström
Upsala [illegible]: 6·8

12·9

vertik. Upsala [illegible] 5°20′

[illegible]:

☾ : [illegible]

Nach [illegible] $\frac{1}{900}$ [illegible]

Dagegen Neison [illegible] 2° [illegible] $p_a = \frac{1}{300} p_0$

Ebenso Kästner

⊙ Protuberanzen [illegible] 1–3' [illegible] 4' [illegible] (= 23000 g. M.)
[illegible] 20 g. M. [illegible]

Chromosphäre [illegible] Atmosph. [illegible] 6000 g M [illegible] 1000

Distanz ⊙ ♁ = ~~23.307~~ $r_{♁}$

$\frac{M}{m} = 319{,}500$

$\frac{G}{g} = 27'5$

$\frac{A}{a} = 108'556 \quad A = 187{,}000$ g. M.

Nach Pouillet [illegible] 1/4, [illegible] 0'4 [illegible] Atmosphäre.

Temp [illegible] pro 100 m 0'57 – 0'59 [illegible]

Winter 0'45
Frühjahr 0'67
Sommer 0'70
Herbst 0'53

---

2. Passat Region [illegible] ; [illegible] Continenten [illegible] ⊙ [illegible] Zenith [illegible]
[illegible] 3–5 Monate

2. Calmen: ⊙ [illegible] ; [illegible]

W. Förster Fortschr. 47 III (1891) p. 231 | [illegible]
Himmelsluft [illegible] Polar [illegible] ; [illegible]
⊙ ; [illegible] Encke's Komet ; [illegible] spectrosk. [illegible]
[illegible] Polar [illegible] ; [illegible] 16 [illegible]
Krakatau

Günther Der Weltraum 56 [illegible]

Neubauer Atom [illegible] Äther [illegible]

Die physische [illegible] Berlin 1872 p. 58

Daly Ramsay Fortschr. 5 p. 383

Proc. Phys. S. L. 13 (1895) 187 Nov. 1 [illegible]

Margules Luftbewegungen in einer rotierenden Luftschale

W. Ber. 102 (1893) p. 1369 Wellenbewegung, Ebbe u. Flut d. Atm.

Rudzki [illegible] d. Atm. Mém. Math. Neuruss. Naturf. Ges. Odessa 15 (1893) p. 71 (Russ.

Fortschr. 49 (1893) p. 374

[illegible] Atm. [illegible]

Laplace'sche [illegible] [Gravit. = Centrif. /] [illegible]

[illegible] 10^{-14} mm Hg.

[illegible] Oberbeck Crelle 81 (1876) p. 62 [illegible]

Petrini Fortschr. 48 (1892) p. 224

[illegible] 2 [illegible]

[illegible]: 1). [illegible] 1210 [illegible] temp. [illegible] Nullpunktes ∞

2). [illegible] temp. [illegible] Nullpunkts [illegible] 12020 [illegible]

Meydenbauer Fortschr. 48 [illegible]

Börnstein

Koláček Wied. Ann. 41 (1890) p. 151

[illegible]

adiab. $\frac{\partial U}{\partial V}\,dV + \frac{\partial U}{\partial T}\,dT + p\,dV = 0$ [illegible]

$$k\,\nabla^2 T = \frac{1}{V}\left(\frac{\partial U}{\partial V}\frac{DV}{Dt} + \frac{\partial U}{\partial T}\frac{DT}{Dt} + p\,\frac{DV}{Dt}\right)$$ [illegible]

[illegible] Fortschr. 46 (1890) p. 363

(Fortschr. I bis 1886 incl. [illegible])

Andrée Fortschr. 51 (1895). p. 248 III

bei cyklonalem Wetter 26/2 1894

– 1700	0.72°	} [illegible] pro 100 m	= $\frac{0.76}{100}$
1700 – 2290	1.07°		
2290 – 3130	0.62°		

Anticyklonal 9/VIII 95

0.8	bis 1800	
0.0	– 2300	
1.0	2500	= $\frac{0.63}{100}$
1°	3000	
0.2	3600	

Angot C.R. (1894) 118 p. 218 s. Hermite 282

[illegible] Angot [illegible] Aufstieg [illegible]* [illegible] = –45°

[illegible] Hermite [illegible] Ballon [illegible]

J. Hann Ebbe u. Flut [illegible]

Naturw. Rdsch. 9 (1894) p. 567 Fortschr. 50 (1894) III p. 290

P. Schreiber Zur Thermodynamik der Atm. Met. Zs. 11 p. 464 (1894)

Möller Met. Zs. 10 (1893) p. 169, 290, 327, 302

Zur Dynamik d. Atm. [illegible]

$c(t_2 - t_1) = Ah$

P. Schreiber [illegible] Civiling. 39 (1893) Zs. f. Luftschiffahrt 13 (1894) } Wahrscheinlich Unsinn

Fortschr. 50 p. 454

Egon v. Oppolzer [illegible] Dynamik d. Atm. Met. Zs. 11 (1893) Fortschr. 50 p. 455

[illegible]

Circulation d. Atm. Pernter: Nature 45 p. 593

[illegible]: Science 19 (1892) p. 301

Ph. Tr. 183 (1892) J. Thomson

Ferrel Fortschr. 45 (1889) p. 337 Pop. Treatise on Winds

Fortsch. 49 p. 52.

J. Stoney [illegible]

[illegible]

1). [illegible]

2). [illegible]

3). [illegible]

R. Ball [illegible] = 2000 [illegible]

[illegible] !

Howard [illegible]

Hutchins [illegible] p. 293 [illegible]

[illegible]

~~Fortsch.~~ 48 p. 281

Trabert Hutchins [illegible] Fortsch. 9 (1893)

Maurer (1887) : $dQ = 6(T - T_0)dt$

0'036 cal [illegible]

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p. 227 Mascart : Sur la masse de l'atmosphère C.R. 114

J. d. Ph. 1 (3)

Laplace [illegible] : $\frac{p}{p_0} = \frac{\rho}{\rho_0} e^{-\frac{R}{H}s}$ $\qquad s = \frac{h}{R+h}$

(const. temp.)

$H = 7994m \times \frac{(273+t)}{273}$ $\qquad \frac{H}{R} = \frac{1}{798} \times \frac{273+t}{273}$ [illegible]

[illegible] : $\frac{\rho}{\rho_0} = f(s)$

[illegible] : $M = 4\pi R^3 \rho_0 \underbrace{\frac{K}{R}}_{= \int_0^1 \frac{f(s)\,ds}{(1-s)^4}}$

[illegible] $(1-s)^4$ [illegible] $f(s) = (1-s)^4 e^{-\alpha s}$

$\frac{K}{R} = \frac{1}{\alpha}$

[illegible] Mont Blanc $\alpha = 637$

$\alpha = 660$ [illegible] Station

[illegible] : $\frac{H}{R} = \frac{1}{769}$ $\frac{K}{H} = 1 + \frac{4}{6}$

[illegible] 64 km [illegible] $< \frac{1}{700}$ [illegible]

Very Distr. of Moon's Heat — Fortschr. 47 p. 62

Nature 44 p. 101

Naturw. Rdsch. 7 p. 115

Boys On the Heat of Moon and Stars ~~[illegible]~~ Fortschr. 46 p. 53

[illegible] Microradiom. [illegible] $\frac{1}{150000}$ [illegible]

[illegible] (Huggins [illegible])

[illegible]

[illegible]

Hutchins p. 75 [illegible] $= \frac{1}{184560}$ [illegible]

Langley [illegible] 15 p. [illegible] $-10°$ bis $+50°$ [illegible]

[illegible]

Maurer Fortschr. 46 (1890) p. 299 [illegible]

M. Zs. 7 , Naturw. Rdsch. 5 (1890)

[illegible] ? [illegible]

~~Hazen Fortschr. 46 p. 458~~

~~[illegible]~~

bis incl. 46 [illegible] 50 p. 442 [illegible]

Helmholtz [illegible] Berl. ~~[illegible]~~ 1888 p. 647 Fortsch. 44 (1888) p. 217

M. Zs. 5, 329 Naturw. Rdsch. 3 p. 465

[illegible] :

[illegible] :

$$-\frac{\partial P}{\partial x} - \frac{1}{\varepsilon}\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \dots - \frac{k^2}{\varepsilon}\left[\frac{\partial^2 u}{\partial x^2} + \dots\right]$$

$$-\frac{\partial \varepsilon}{\partial t} = \frac{\partial(\varepsilon u)}{\partial x} + \dots$$

Statt $x\ y\ z$	$nx\ \ ny\ \ nz$	Sonst gleich $P\ p\ \varepsilon\ u\ v\ w\ \theta$
t	nt	
k^2	nk^2	

[illegible]

[illegible]

[illegible]

[illegible] 8026 m ; [illegible]

[illegible] 10·8 m. red. [illegible]

$\frac{65.5}{8026}$ [illegible]

[illegible]

$$\frac{\partial u}{\partial t} - \frac{k^2}{2}\frac{\partial^2 u}{\partial z^2} = 0 \qquad \left| \begin{array}{l} u=0 \\ z=0 \end{array} \right. \qquad \left| \begin{array}{l} \frac{\partial u}{\partial z}=0 \\ z=h \end{array} \right.$$

$$u = A e^{-nt} \sin(\varphi x) \qquad \frac{du}{dt} = -n u \qquad e^{-nt} = \frac{1}{2}$$

$$n = \frac{k^2}{2}\varphi^2 \qquad \varphi = \frac{\pi}{2h} \qquad + nt \log e = + \log 2$$

$$= \frac{k^2}{2}\frac{\pi^2}{4h^2} \qquad n = \frac{\log 2 \quad {}^e\log 10}{42747 \cdot 365 \cdot 86400}$$

[illegible] $e^{-nt} = \frac{1}{2}$ [illegible] $t = 42747$ [illegible]

[illegible] 10 [illegible] (2h) [illegible] k^2 [illegible]

[illegible]

[illegible]

[illegible]

[illegible] $\frac{1}{2}$ red. [illegible] 36164 [illegible]

„[illegible] Convection [illegible]

[illegible]"

[illegible] Circul. [illegible] 30° [illegible]

[[illegible] 30° [illegible] 133 m [illegible]]

[illegible]

Guldberg u. Mohn in Christiania Z. Met. 1878 VIII p. 113

Fortschr. 34 (1878) p. 1014

[illegible] temp. [illegible] in vertik. [illegible] Atm.

[illegible] temp [illegible]: $\frac{p}{p_0} = \left(1 - \frac{g z}{\alpha m T_0}\right)^m$

$p = \alpha \rho T$

$m = \frac{g}{\alpha_1 \alpha}$

[illegible] temp [illegible]

[illegible] Börnstein 1888 Met. Z. p. 466

[illegible]:

Bezold [illegible] Thermodynamik d. Atm. Met. Z. 6 p. 201, 287 Fortschr. p. 220

[illegible]

potentielle Temp = Temp. [illegible] adiab. [illegible] Normaldr. [illegible]

Trockenstadium

Regen- "

Hagel Schnee "

Oberbeck [illegible] Atm. Met. Z. 5 p. 205 Fortschr. 225 (1888)

Licht [illegible] incompr. [illegible]

[[illegible] Circulation [illegible]!]

[illegible] Max. 45°

[illegible] 35°16' [illegible] Äq. [illegible]

[illegible] Vertical [illegible] Rad.

[illegible]

2). [illegible] Rot. [illegible] Parall. [illegible] NW [illegible] Äq. [illegible] 35°, [illegible] 10

[illegible] NO [illegible] Äq. [illegible] Pol., Max. 54°

André Fatol. 44 p. 238 Sur les mouvements verticaux de l'air

[illegible] Metz [illegible] Lyon [illegible]

<u>Linss</u> [illegible] M.Z. 5 p. 77 (1888) Fatol. p. 239

[illegible] : $v = 0.04$ [illegible] Meter, [illegible] $\frac{mm}{Stunde}$

[illegible] Cyklonen [illegible] $r = 2$ bis 3 mm also [illegible] $v \geq \frac{10\ cm}{sec}$ [illegible]

Very Naturw. Rdsch. 1899 XIV p. 169 Astrophys. J. VIII p. 199

☾ temp [illegible] ca 100°

[illegible] ca −170°

Naturw. Rdsch. 1899 p. 370

Kammer [illegible] Ballonfahrt am 3/X 98

(Hann [illegible] temp [illegible] Anticyklone [illegible]

[illegible] Cyklone

In [illegible] 1500 [illegible] 7500 0.56° pro 100m

[illegible] 0.62°

von 5000 an 0.80

p. 363 Breitenbach [illegible]!!

Rdsch. 29/VII und 27/VII Durchsicht [illegible]

Ucke – Die quantitat. Verhältnisse des O_2 der Luft in den verschiedenen Klimaten Z. ö. g. G. M. X S. 33

Hempel [illegible] B. d. D. Ch. G. XX (1886)

([illegible] Günther) 20.89 (Para) – 20.933 (Cleveland Amer.)

Rudzki

[illegible]: Abbe Atmosph. Radiation of Heat Amer. J. of Science 1892 (Mai)

[illegible]: 1 kg. [illegible] 0.032 – 0.036 Cal.

Fröhlich Rep. f. Meteor. VI [illegible] temp. [illegible] ca −127° – 131°

Mendelejeff [illegible] C. Pet. 1884 (p. 269)

[illegible]?

Oberbeck [illegible] Crelle LXXXI 1876 p. 62

Lamb Motion of Fluids p. 225 [illegible]

[illegible]

Ritter's [illegible]: [illegible] ladrel. [illegible]

[illegible] Centr. [illegible] (?)

Mascart [illegible] Journ. d. Physique ~~1892~~

Gromeka [illegible] temp. [illegible] = const. ([illegible])

Гроикека Kasan 1886 Niekotoryje sluchaj ravnovesija svizmaemago gaza.

Thomson: Equilibrium of a gas under its own gravitation

Phil. Mag. 5 ser. 23 p. 287

Ritter Wied. Ann. 1882

Olszewski −220° 4 mm [illegible] Handb. d. exp. Ch. ~~Dammer~~ Stuttg. 1892

[illegible] (?)

Kin. Earth. [illegible] $> \sqrt{2ga}$ [illegible]

$= 11200$ m

[illegible] Atm. [illegible] (!)

[illegible] Centrif. [illegible]

[illegible] Atm. [illegible]

[illegible]

[O_2 [illegible] : $\frac{1}{2\sqrt{\pi}\alpha}\int_{11200}^{\infty} e\, dv\, e^{-\frac{v^2}{\alpha^2}}$ [illegible] Kranzow [illegible]].

$\alpha = 397$

~~[illegible]~~

[[illegible] 80 km [illegible] $\frac{219}{100000}$ [illegible] O_2]

[illegible] etc. [illegible]

[illegible] Atm. [illegible]

[illegible]

1). station[illegible]

2). [illegible] rot. [illegible]

3). [illegible]

*) $\frac{\partial u}{\partial t} - 2(v\zeta - w\eta) = -\frac{\partial P}{\partial r} - \frac{2\mu}{r\rho}\left(\frac{1}{r}\frac{\partial \zeta}{\partial \theta} + \frac{\zeta \cot\theta}{r} - \frac{1}{r\sin\theta}\frac{d\eta}{d\varphi}\right) + \frac{\mu}{3\rho}\frac{\partial \delta}{\partial r}$

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + v\frac{\partial\rho}{r\partial\theta} + \frac{w}{r\sin\theta}\frac{\partial\rho}{\partial\varphi} + \rho\delta = 0$$

---

*) [illegible] Θ_2 Whitehead Sec. appr. to viscous fluid motion Quart. J. XXIII 1889 p. 145

([illegible] $u = v = 0$, [illegible] Resulta[illegible])

[[illegible] Stokes Math. phys. papers Camb. 1880 I p. 103]

(Edwardes: Steady motion of a viscous fluid in which .. Quart. J. 1892 p. 75

[illegible])

Hicks Recent progress in Hydrodyn. Rep. B. Ass. 1881 p. 80: [illegible] Stokes [illegible]

[illegible]

$P = \frac{1}{2}(u^2 + \dots) + \int\frac{dp}{\rho} - V$ [illegible] $u = v = 0$, $\delta = 0$ (incompr.)

$\xi = 0 \quad 2\zeta = \frac{1}{r}\frac{\partial w}{\partial\theta} + w\frac{\cot\theta}{r} \quad 2\eta = -\frac{\partial w}{\partial r} - \frac{w}{r}$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(wr) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(w\sin\theta)\right] = 0$$

$$w = \frac{\omega_0 a^3}{r^2} \sin\theta \qquad \frac{a^3}{r^3} \sin\theta = 289$$

$$\frac{\partial V}{\partial r} + \frac{a^6 \omega^2}{r^5} \sin\theta = \frac{1}{\rho}\frac{\partial p}{\partial r} \qquad \frac{\partial V}{\partial r} = -\frac{g a^2}{r^2}$$

$$\frac{\partial V}{\partial \theta} + \frac{a^6 \omega^2}{r^4} \sin\theta \cos\theta = \frac{1}{\rho}\frac{\partial p}{\partial \theta}$$

$\frac{\partial p}{\partial r}$ [illegible] < 0 [illegible] Laplace

ρ centrif. [illegible]

~~[illegible]~~ [illegible] $6\frac{1}{2}$ km [illegible] $2 \frac{1m}{m}$ [illegible]

---

transl. [illegible] Crelle LXXXI 1876 p. 62

[illegible] [comp. Lamb motion of fluids p 225]

[illegible] $2+1$ [illegible]

2 Aequator [illegible] $p = \text{const} + \frac{3}{2}\mu \frac{V}{a} \cos\varphi$

$V = $ [illegible] (transl.)

a = Radius

$\mu = 0{\cdot}134\,\rho$

$$\rho = \frac{760}{9{\cdot}81_{m} \cdot 6370000_{m}} \qquad 10^{-14} \text{ mm.}$$ [illegible]

[illegible] incomp. [illegible] φ^2

[illegible] Iden [illegible]

G.S. Schmidt (Gotha) [Gilb. Ann. 62 (1819)]

[illegible]

[illegible] Ner. 1, [illegible] temp. [illegible]

[illegible] sp. Kl. [illegible] temp.

I). temp. Abnahme linear nach Humboldt 121.1 Toisen für 1° Reaumur

ergibt $h = 6.6$ g M [illegible]

II [illegible] prop. [illegible] temp.

Formel: $z = a \log\left(\frac{c+t}{c+x}\right)$ $\quad z = ??$ $\quad x =$ temp.

ergibt 27.5 g M.

Kelvin: Phil. Mag. (5) 23 p. 287

F Neumann theor. Physik

Hicks B.A.R. 1881

Gustav Mie Entwurf einer allg. Th. d. Energieübertragg. !
Wien. Sitzgsber. 107 p. 1113 (1898)

Liénard Die Theorie von Lorenz (L'éclairage électrique 14 p. 417, 1898) 456

Poincaré [illegible] magn. Energie [illegible] Maxwell [illegible] Hertz (L'écl. électr. 18 p. 361 1899)
[illegible] Energie $(c v_0 - b w_0 + \alpha u_0)\, d\tau = \int \frac{[illegible]}{8\pi} d\tau$
[illegible] Energie [illegible] Hertz'sche [illegible] $\int \mu\, d\tau \frac{\alpha^2+\beta^2+\gamma^2}{8\pi}$

$u_0\ v_0\ w_0$ [illegible] Compon. [illegible]

Fabry & Perot C.R. 128 p. 1156 (1899) — intensive monochrom [illegible]
~~[illegible]~~ [illegible] Bog. bogen Lampe im Vacuum 4 [illegible]; [illegible] 22 cm

Planck Max Th. Ann. 23 p. 702 (1899)

Petrinelli [illegible] Nuovo Cim. X (4) p. 117 (99) XIV 9
Wilson Rdsch. XIV 283

Curie [illegible] Radioaktivität C.R. CXXIX p. 714 (1899)

Demarçay Radiumlinien " p. 716

Eisenparaffin Dielektrizität bis 50% Fe Nuovo Cim. IX (4) p. 254 (1899)
Boccara – Gandolfi

N. Cim. VIII (4) (1898) p. 191

	μ	ε	$\sqrt{\varepsilon\mu}$	n
5% Fe	1·040016	1·83333	1·584	1·615
10	1·10018	3·08333	1·857	1·903
15	1·15607	3·58333	2·035	2·096
20	1·22143	4·66666	2·387	2·294
30	1·38631	6·25000	2·954	2·794
40	1·62228	8·33333	3·678	3·564

1826 [illegible] Petersbg. Akad.: [illegible]

Fresnel geb. 1788 gest 1827

1815 Erste Abh. ; vollst. 1818 Mém. sur la diffr. de la lumière

1821 polarisation double réfr.

Laplace, Poisson etc. [illegible]

Wollaston 1802, Brewster, Fraunhofer 1821 (–30) [illegible]

Hamilton – Lloyd 1832

1858 Balfour Stewart The absorption of a plate equals the radiation and that for every description of heat.

(1855 Angström [illegible])

Wied. Ann. 67 p. 452

Jaumann [illegible] – Voigt [illegible]

[illegible] Voigt W. Ann. 53 p. 43 (1894)

[illegible] – Constante [illegible]

[illegible] 145 [illegible] 10 – 413 [illegible]

[illegible]

Ketteler [illegible] Wied Ann. 49 p. 507 !

Drude 48. p. 542 ! Dielectr [illegible]

[illegible] Van der Waals' [illegible] Boltzmann – Mech. [illegible] 68 p. 350

$$\left(pT + \frac{a}{v}\right)\left(v - \frac{1}{3}b\right) = rT\left(v + \frac{2}{3}b\right)$$

[illegible] $\left(v - \frac{b}{3}\right)^{-1} \infty$ [illegible] $v = \frac{1}{3}b$

Amagat CO_2

Humphreys Boltz. p 219

[illegible]

1). [illegible]

2). prop. [illegible] 3). [illegible] 4). [illegible] temp.

II 5). [illegible] 6). [illegible] 7). [illegible]

8). [illegible] [reduc. [illegible]] [illegible]

9). [illegible] prop. [illegible] prop. [illegible] (? 1)

11). [illegible] $\frac{1}{2}$ [illegible] $\sqrt[3]{}$at. vol.

12). [illegible] (Atomgew.)

pag. 558 Ann. 22

Wirkung [illegible]

[illegible] Lommel's Absorpt. theorie: [illegible] harm. [illegible] period. [illegible]

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + p^2x + b\varepsilon x^2 + \cdots + f\sin qt = 0$$

Absorpt. Max. wenn $q = p$ [illegible]

[illegible] Period [illegible]

[illegible]

[illegible]

$$\frac{d^2x}{dt^2} + 2k_1\frac{dx}{dt} + p_1 x^2 + \cdots + f\sin qt$$

$$k_1 = \frac{k}{1+g} \qquad f_1 = \frac{f}{1+g} \qquad p_1^2 = \frac{p^2}{1+g}$$

Hauptabs. für $q^2 = p_1^2$ [[illegible] $g = 8.10^{-6}$]

$\frac{k}{p^2}$ < [illegible] ~~[illegible]~~

[[illegible] Spectra [illegible]]

Evershed Phil Mag (5) 39. p. 460 1895 [illegible] !!

[illegible] Phil [illegible] 1895

[illegible] Ann. 20 (1896) p. 975 Phil [illegible]

Humphreys & Mohler Astroph. (20) 1896 p. 503

[illegible] Rowland [illegible] bis 0.2 A.

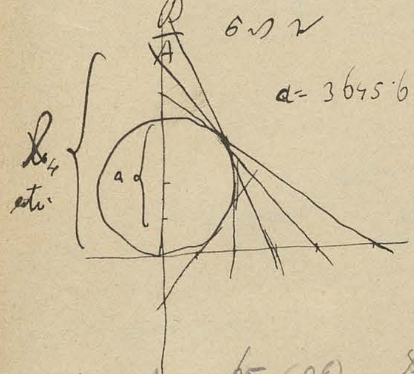
[illegible] ca 2. 10^{-3} Angstr. [illegible]

Balmer W. Ann. 60 (1897) p. 380 [illegible] Spectralen

$\tau_n = A - \frac{B}{n^2} - \frac{C}{n^4}$ nach KR. ([illegible] ca $10^{-5}\lambda$)

$\frac{1}{\lambda} = \tau_n = A - \frac{B}{(n+c)^2}$ [illegible] Balmer

B [illegible] [illegible] = [illegible] Rydberg



Wied. Ann. 65 (98). Spectrum des [illegible] p. 257

[illegible] p. 440

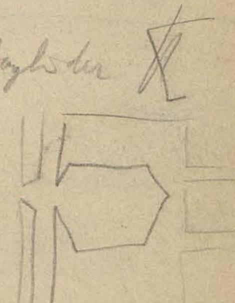
69 (1898) Heydweiller [illegible] p. 531

65 (98) Kurlbaum [illegible] 100°

	$\frac{g\ cal}{cm^2 sec}$	$h_{100} - h_0$	6.10^{12}
Schneebeli	1874	0.0152	1.10
K & Warb.	75	0.014	1.01
Grätz	80	0.0150	1.08
Christiansen	83	0.0167	1.21
Kurlbaum	98	0.0176	1.28

(~~[illegible]~~

Arbeitsleiter



Innere Reibg. Schumann Wied. Ann 23 p. 353 (1884)

O.E. Meyer Pogg. Ann. 127 p. 253 (1866); 148 p. 1, 203 (1873)

Notiz [illegible] Wanner Wied. Ann 68 p. 143 (1899)

[illegible]

[illegible] Siehe Voigt Wied. Ann 68 p. 604!!

Ibid. p. 129 Mie [illegible] Ätherbewegung event. [illegible]

p. 125 Ketteler Magnetooptik [illegible] Theorie [illegible]

[illegible]

Wied. Ann 40 p. 424 (1890)

[illegible]

[illegible]

---

41 ~~[illegible]~~ 565

Christiansen Die [illegible] !!!

[illegible]

41 ~~Ibid~~ 626, 770 ← Theorie

Schütze? 8 [illegible] /

42. p. 348 [illegible] Bemerkung dazu

p. 465 Riecke 8 Elast. [illegible]

---

p. 674 Michaelis 8 Kol. Th. [illegible] Elast. [illegible]

Sutherland Phil. Mag. 22 p. 81 (1886)
24 113, 168 (1887)
27 305 (1889) } $n = 4$

De Heen (1883) Wied. Ann. 46 p. 775 Anm. $n = 7$

43 p. 158 [illegible]

1). [illegible] 0.01 < [illegible]

2). [illegible] $= 17 . 10^{-6}$ mm

3). [illegible] $< 8.5 . 10^{-6}$ mm

43 p. 629 Raps Selbsttätige Hg-Pumpe! Siehe auch 48 p. 377 Raps!

45 [illegible] ; Lebedew p. 292 [illegible] $R = r = 2$ mm [illegible]

[illegible] ~~1 [illegible] = $\frac{2}{3}$ mg~~

$$1 \text{cm}^2 = \frac{2}{3} 10^{-7} g = 0.6 . 10^{-4} \text{Dyn}$$

[illegible] Newton'sche Attraction pro 1g 10 : 0.6 Dyn

p. 685 Wien [illegible] Localisierung der Energie

47 p. 208 Snow [illegible] ultrarot. Emissionsspectrum der Alkalien
dazu 48 p. 150 Kayser & Runge

47 p. 671 Voigt [illegible]

48 p. 467 ~~Zur kinetischen Theorie mehratomiger Gase Richarz~~ !!!

~~708 Über das Gesetz Dulong & Petit~~

49. 347
45. p. 428 } Pringsheim Kirchhoffs Gesetz [illegible]

49 p. 459 Riecke Molecular Th. d. pyroel. u. piezoel. Erg.

49. p. 651 Electr. Erg. molecularer Vg. Ebert

50. p. 377 Voigt } Bey. zur Lichttheorie (— Ketteler)
p. 381 Drude

50. ~~Emission erhitzter Gase~~ Paschen { 50 p. 409
51 p. 1
p. 40
Pringsheim 51 p. 441 Voigt
Bing 52 p. 205 } 52 p. 209

50. p. 625 Ueber die Serien der Spectra Rydberg

51 p. 638 Ueber molec. Th. d. Piezoelectr. Voigt

52 p. 93 Spectra d. Pb As Sb Bi Kayser & Runge

~ Serien d. Sn u. Pb Kayser & Runge

p. 114 Kayser & Runge
Ueber die Serien der Spectra

56 p. 78 Galitzin (Th. d. Spectralen
p. 741 Lommel) ...

57 p. 447 Fluorescenz d. Na u. K Wiedemann – Schmidt

57 ~~Lommel 485 59 p. 793~~
~~Galitzin 773 60 p. 392~~

58 Schmidt Polarisirte Fluorescenz ... Ueber die ... p. 417

Nature 57 p. 320 (1898) A Schuster Recall 22
Runge's u. Paschen's Erg. d. Spectra d. O S Se

Wied. Ann.
61 Runge u Paschen üb Serien ... p. 641

63 Lummer-Pringsheim üb ... 100° u 1200° p. 395

~~Kamerlingh-Onnes Allg. Th. ... Versl. 21 p. 207~~ !!
~~Arch. Neerland. 30 p. 101 (1896)~~

~~H. A. Lorentz üb Entropie ... Versl. 21 p. 400~~
~~Zittingsverslag Kon. Akad. Wet. Amsterd. 1896/7 p. 262~~

Cantor üb ... regen Gasmol. Wied. 62 p. 482

Duhem. Avogadro's ... 62 p. 204
Traube 61 p. 380, 391, 396

Tamman ... 62 p. 280

Wied. Ann. 62 p. 490 üb osmot. ... electrolyt. Diss. Traube!!!
62 p. 528 üb Lambert'sche ... Polar. ...
[illegible]!!

~~Burbury üb Boltzmann's ... Energie ...~~
~~Lond. math. soc. 27 p. 214 (1896)~~

$c_1 e^{-\frac{c_2}{...}}$ $J_m = 0.03124$
$\lambda_m = 8.863$
$\theta = 288°$ abs.

Traube's Theorie:

H. Jahn [illegible] — Chem. Ber. 30 p. 2982 (1898)
Traube [illegible] 31 p. 154
[illegible] Jahn 31 p. 160
A. Fock [illegible] 31. p. 506
Traube " [illegible] Fock 31. p. 1081

Beibl. 22 p. 540

Vasini [illegible] Rendic. R. Acc. d. Lincei (5) 6 p. 175, 199 (1897)
Gaz. Chim. It. 27 p. 533 (1897)
Beibl. 22 p. 541

O. E. Meyer Pogg. Ann. 127 p. 253

$$\rho \frac{du}{dt} = \eta D^2 u + \frac{1}{3}\eta \frac{\partial \Delta}{\partial x} - \frac{\partial p}{\partial x}$$

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$D^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$\rho u \frac{\partial u}{\partial x} = \eta \left[\frac{4}{3} \frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right)\right] - \frac{\partial p}{\partial x}$$

$$0 = \frac{1}{3}\eta \frac{\partial^2 u}{\partial x \partial r}$$

$$0 = \frac{\partial(\rho u)}{\partial x}$$

Wenn [illegible]:
p, ρ [illegible]
$v = w = 0$

[illegible] $u, \rho, p = f(x)$ [illegible]

[illegible] $u = 0$ [illegible]

[illegible]

$u \frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 u}{\partial x \partial z}$ [illegible]

[illegible]

$$0 = \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{a}\frac{\partial u}{\partial x}$$

$$\frac{dp}{dx} = \eta \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

u: p [illegible] x

$$0 = \frac{\partial(\rho u)}{\partial x}$$

$\frac{\partial u}{\partial r} = 0$ für $r = 0$

$$p = k\rho(1+\alpha\vartheta)$$

Rowland: $D_1 = 5896{\cdot}16$

$D_2 = 5890{\cdot}19$

Wied. Ann. 61!
63!

~~Wied. Ann. 64~~

Lummer & Pringsheim [illegible] 10: [illegible] 1887

Luft 1·4025

O_2 1·3977

CO_2 1·2995

H_2 1·4084

p. 555

p. 549 Anderson [illegible] osmot. [illegible]

p. 752 Meyer [illegible] ~~[illegible]~~ const. [illegible] Hg.

Wied. Ann. 65 p. 655 ~~[illegible]~~ [illegible]

p. 670 ~~[illegible] const. [illegible] Dulong & Petit~~

66 p. 473 Tammann [illegible] II → Heydweiller p. 1194

~~p. 826 Dieterici Kinetische Theorie d. Flüssigkeiten~~

67 p. 291 Schiller [illegible] osmot. [illegible] Thermod. [illegible]

Wied. Ann. 67 p. 702 ~~Richarz~~ [illegible]

p. 803 ~~[illegible]~~ Mertenbach [illegible]

p. 894 G. Jäger [illegible]

899 Müller Erzbach [illegible]

Naturw. Rundschau 9 p. 224 (1894)

Richarz Verh. phys. G. Berlin 10 p. 73 (1891) [illegible]

Wiedem. Beiblätter

22 Rayleigh Phil. Mag. 44 p. 356 (1897)
[illegible] (! [illegible]!)

$\left\{\begin{array}{l}123 \text{ p. } 626, 871 \text{ (1896)} \\ 125 \text{ p. } 1078 \text{ (1897)}\end{array}\right.$

Painlevé ~~[illegible]~~ CR 125 p. 1078 (1897) [illegible] 3 [illegible] 2 [illegible]

Poincaré 123 p. 1031

Natanson [illegible] thermokin. [illegible] thermodyn. [illegible]

C. H. Wind [illegible] Boltzmann'schen [illegible] Gastheorie

Wien. Ber. 106 p. 21 (1897) Kritik Boltzmanns [illegible] !!

G. Jäger Wien. Ber. 105 p. 791 (1896) [illegible]

[illegible] ~~[illegible]~~ ([illegible] 3 [illegible])

V. d. Waals [illegible]

Mewald [illegible] p. 23

Guglielmo [illegible] p. 546

Lincei 6 p. 254 (1897)

~~Milner Phil. Mag. 43 p. 291 (1897)~~

20 Boltzmann Planck Helm Energetik Wiedemann 57

Ostwald Boltzmann 58

G. Jäger [illegible] , [illegible] Wiener Ber. 103 p. 251 (1894)
104 p. 408 (1895)

Wyatt [illegible] Am. Chem. J. 17 p. 461 (1895)
Chem. Centralblatt 2 p. 278 (1895) p. 13

Roberts Austen Diff. [illegible] Metalle Pr. R. S. 59 p. 281 (1896) !!
p. 846

Töpler Zur Gasdiffusion 58 599

C. del Lungo [illegible]
Zur Polemik Ostwald–Boltzmann p. 855
Lincei 5 p. 467 (1896)

Slotte [illegible] p. 257 !
Verh. 21 p. 119

Basevi Clausius' Virial theorem Nat. 52 p. 413 (1895) [illegible]

~~G. J. Stoney [illegible] Ph. Mag. 40 p. 362 (1895)~~

G. Jäger Z. Theorie d. Diss. d. Gase Wien Ber. 100 (1891)
104 p. 671 (1895)

Bakker Zur Th. [illegible] Ztschr. f. ph. Ch. 20 p. 461, 21 p. 556
17 p. 678 (1895)
18 p. 335 (1895)
Baynes

Culverwell Boltzmann's Minimum theorem Nat. 52 p. 149 (1895)
wenn $\frac{dH}{dt} \geq$ [illegible]

~~Burbury [illegible] Pr. Lond. math. S. 26 p. 431 (1895)~~ p. 350

Sutherland Visc. [illegible] Ph. Mag. 40 p. 421 (1895) schwund!

G. J. Stoney Pr. R. S. 58 p. 147 (1895) p. 351

S. Jäger } ... Wien. Ber. 105 p. 97 (1896)
" p. 15 / p. 755

$pv = RT\left(1 + \frac{ab}{v} + \frac{10b^2}{v^2}\right)$

Bertrand ... CR. 122 p. 963 (1896) Journal physique 5 p. 285

~~1083~~
1173, 1174, 1314

21 ~~Boltzmann ...~~

Levi Civita ... Lincei 5 p. 164 (1896) p. 173

Heydweiller ... 63 56

Noyes, Goodwin ... p. 110 Ph. Rev. 4 p. 207 (1896)

V. d. Waals ... Potentials p. 206

Kamerlingh Onnes Allg. Th. ... p. 207

H. A. Lorentz & Entropie einer Gasmasse p. 400

Zittingsversl. Kon. Ak. Wet. Amst. 1896/7 p. 252

Burbury & Boltzmann ... Ph. mag. 27 p. 2.. (1896)

Ausnahme ...

V. d. Waals ... p. 210

Bakker ... Vorwale Mol. p. 211 (Z. ph. Ch. 21 p. 497)

22 p. 277

~~Voigt p. 854 Gött. Nachr. 1896 ...~~

19 S. Arlen ... Energetik 55

Energetiker ... ! Vectoren ...! p. 736 [Rend. ... 9 p. 119 (1895)]

Petrini & ... Hydrodyn. ...

... p. 385

A. Heydweiller [illegible] 55 561

G. Jäger [illegible] 141 Wien Ber. 103 p. 251 (1894)
53 955

Planck } [illegible] 55 220
Boltzmann } 55 223

~~Boltzmann~~ [illegible]:

Culverwell x Burbury Nature 51 p. 246, 320

Bryan } Nature 52 p. 29, 104 (1895)
Burbury }

Maxwell's [illegible] Energie [illegible]:

Bryan [illegible] Proc. Camb. Phil. Soc. 8 p. 250 (1895)

Fitzgerald [illegible] Proc. R. S. London 57 p. 312 (1895) p. 768

Schütz [illegible] (p. 766 [illegible] p. 768) Gött. N. 1895 p. 30

Burbury [illegible] 769 Pr. R. S. 57 p. 302 (1895)

Riecke: [illegible] Clausius 54 739.

Weinstein [illegible] 53 544

Poincaré [illegible] Rev. gén. d. sc. 5 p. 513 (1894) Kelvin's test case!

Bryan Rep. Br. As. 1894 (39 p.)

Petrini [illegible] eindimensionales Gas p. 148

Sutherland Phil Mag. 38 p. 150, 188 (1894) p. 507
39 p. 1 (1895) 664

Bakker Zs. ph. Ch. 14 p. 671, 446, 456, (1894

J. v. d. Waals [illegible] thermod. Pot. p. 552

Amagat [illegible] CR 120 p. 489 (1895

Kool [illegible] $\Sigma \frac{mv^2}{2} = \frac{3}{2} PV$ [illegible] p. 764

18

Burkhardt × Vectorfunctionen ! p. 283 Gött. Nachr. 4 p. 155 1893

! Heaviside × Operatoren [illegible] Pr. R. S. 52 p. 504
p. 410 ! 54 p. 105 1893

Molenbroek [illegible] !! p. 496
Verh. K. A. Amst. 2 p. 1 (1893)

Culverwell [illegible] Rep. Br. Ass. p. 704 (1891)

Boltzmann [illegible] 53 959

Sutherland [illegible] Phil. Mag. 36 p. 507 1893

Gosiewski [illegible] Krakau Anz. p. 311 (1893)

! Stotto [illegible] ! p. 47

! ~~Sutherland [illegible] (?!) Ph. M. 32 p. 31 215, 524 (1891)~~

~~Watson Burbury Maxw. [illegible] Nature 46 p. 100 (1892)~~

Burbury Ph. M. 37 p. 143 (1894)
"on the II Law of Th." Ph. M. 37 p. 574

Natanson Dynam. [illegible] v. d. Waals [illegible] p. 440
Arch. sc. ph. 28 p. 112 (1892)
[illegible]

V. d. Waals [illegible] Mol. Attraction Versl. Amst. 1893 p. 20 ‖ $U = \frac{e^{-\frac{r}{\lambda}}}{r}$

Ztschr. ph. Ch. 13 p. 657

Riecke [illegible] N. [illegible] 11 p. 221 237 (1894)

Amagat [illegible] C. R. 118 p. 326, 566 (1894)

28

Bryan Proc. Camb. Phil. Soc. 1894 $\frac{24}{11}$

Boltzmann Phil. Mag. ~~XXXV~~ 35 p. 156 (1893) motion of ball upon billiard table

Maxwell Coll. Sc. Pap. $\frac{II}{p}$ 713 (1879)

Lord Kelvin Proc. R. S. 50 p. 85 (1891)

Rayleigh ~~Pr. R. S.~~ Ph. Mag. April (1892) p. 356

Kelvin Phil. Mag. May (1892) p. 466

~~Burbury Nature 46 p. 100 (1892)~~

Stoney Phil. Mag. 40 p. 362

On the Kinetic Th. of Gas regarded as ill. Nature

Not necessary reality but ideal mechanism

A = 3 motions of centre of gravity

B = intramolec. { Ba, Bb (much / little) slowly } influenced by A; Bc not influenced by A

~~aftereffect of~~ γ must depend on time

Effect of aether may be to link the B motions in a molecule with themselves

(See Fitzgerald) so that there will be much less degrees of freedom than without aether.

electrons associated with Ba events will produce absorption spectrum

Bc - - - - gas transparent } p. 377

Bb = [phosphorescence] intermediate } { Bb_1 fast, Bb_2 slower vanishing

[It is possible that Bb motions influence very much the A or Ba motions, but in contrary are little influenced by them (?) p. 396

In fact transparent diatomic gases have γ = ca 1.4 smaller [there may be 6 8 degrees more but linked by aether, therefore there appears but a small fraction]

~~not~~ coloured

5 freedom

What may be the effect of ~~stat~~ perturbating forces not obeying the conditions of Maxw.-Boltz.?

Intermolec. bonds within a crystal ~~of those~~ of the same kind as interatomic in a molecule =
= forces between electrons

In chemical actions: if electrons associated with Ba events : colour effect

Bb_2 " lightning "

Bb_1 - both of them

~~the~~ temperature in Plücker tube need not to be high

Lewis Proc. R. Soc. March 1895 Radiant heat = first cause of chemical action [illegible]

Rayleigh Phil. Mag. 45 p. 522 (1898) Pressure of Radiation !!

17 Heaviside p.1000 Analogie Gravitation u. Elektromagn. Electrician 31 p. 281 (1893)

p. 702 Operatoren i. d. math. Physik Pr. R. S. 52 p. 504 "

~~Sir W. Thomson Kinetische Theorie d. Magnetisierung~~ Ph. M. 33 p. 291 (1892)

[illegible]

Jäger Theorie d. inneren [illegible] Wien Ber. p. 253 (1893) p. 615

Jäger [illegible] Cap. Const. [illegible] v. temp. [illegible] 101 p. 954 (1892)

[illegible] p. 87

Pirogov [illegible] Boltzmanns [illegible] Rep. 27 p. 515 (1891)

Kelvin Viriale: [illegible] Math. Ph. 37 p. 257 (1892)

Testcase Phil. Mag. 33 p. 466 1892

Kool 1. [illegible] Virialsatz [illegible] Arch. Genève 27 p. 474 (1892)

2. [illegible] Correction [illegible] 28 p. 72 ("

3. 2^te^ λ 28 p. [illegible]

p. 94 [illegible] Clausius [illegible]

~~Vertoog [illegible] Isothermen [illegible] v. d. Waals Nat. 45 p. 277 (1892)~~

~~[[illegible]]~~

Jäger Zur Theorie d. [illegible] Wien Ber. 101 p. 920 (1892)
Wärmeleitg. in [illegible] 102 p. 483 (1893)
p. 98

Natanson Wahrscheinlichkeit molekularer Zustände Phil Mag. 34 p. 51 (1892)

[illegible] thermod. Pot. p. 423

Studien zur Th. der Lösungen p. 423

Jäger [illegible] temp. [illegible] v. d. W. [illegible]
W. Ber. 101 p. 1675 (1892)

[illegible] 101 p. 1520 (1892) [illegible]

Tait Grundlagen d. [illegible] Gasth. Pr. R. S. Edinb. 19 p. 32 (1892)

~~[illegible]~~ Leahy [illegible] Proc. Camb. 7 p. 322 (1892)

p. 433 [illegible]

Cornelius [illegible] Ostw. Zeitschr. 11 p. 403 (1893) p. 518

Poincaré [illegible] Gasth. CR. 116 p. 1017 (1893)

Sutherland [illegible] Phil. Mag. 35 p. 211 (1893) 1809

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16 Mc Aulay Quaternions [illegible] R. M. 33 p. 477 (1894)

Wien [illegible] Localisirung d. Energie 45 685

Pirogow p. 171 x l Virial [illegible] Journ. russ. Ges. 2 p. 117 (1890)

[illegible]

1). [illegible] 2). [illegible]

3). [illegible] 4). [illegible]

Pirogow p. 173 l N Körper Problem

motion of matter is only transfer of modification of structure or energy or other qualities of ammovable aether to other places

Durton Theorie d. Constitution d ~~Körper~~ Materie Ph. M. 33 p. 191 (1892)

Theorie d. Lösungen pag XI

~~Rayleigh 8 Virial [illegible] Nature 45 p. 80 (1891)~~ } p. 263
~~Korteweg 8 [illegible] V. d W. 152~~
~~Tait 8 Virial [illegible] 199~~

~~Sutherland [illegible] Ph. Mag. 32 p. 31 (1891)~~

Schuster Electr. Nottze Vectorpotential Ph. Mag. 32 p. 9 (1891)

~~Tolver Preston [illegible] Ph. Mag. 31 p. 441 (1891)~~

~~W Thomson p. 191 [illegible] Nature 44 p. 355~~ 1891

Tait p. 192 [illegible] Trans. R. S. Ed. 36 p. 257 (1891)

V. d W. s Virial [illegible]

Leray [illegible] 8 [illegible] Ann. Chim. Phys. 25 p. 89 (1892) p. 339

eindimensionale Problem

~~Rayleigh p. 341 Dyn. [illegible] Ph. M. 32 p. 424 (1891)~~

Jäger [illegible] 99 p. 860, 1028 (1891), 100, p. 245, p. 1122, p. 1233

Waterston Physik d. [illegible] Trans. R. S. 1892, 183, p. 1

historisch [illegible] 1846 publ.

Nasse – Boltz. Gesetz.

~~Rayleigh Phil. Mag. 33 p. 356 (1892)~~

~~Watson Nature 45 p. 512, 46 p. 29 (1892)~~

~~Culverwell " 46 p. 76~~

Jäger Diss. Ber. 100 p. 1182 (1891)

15 Burbury Problème der kin. Gasth. (68 Phil.R.S.) Ph. M. 30 p. 298 (1890)

Culverwell Uhng 8 Phil.R.S. etc " p. 95

Michaelis Sol. Th. d. El. Ann. 42 6 74 Th. d. d. Ltg. d. Wärme; vgl. die pol. Th. von Voigt

Warburg 4 p. 232 (1878)

Wied. 17 p. 726 (1882)

14 Pirogow J. russ. chem. G. 21 p. 44, 76, (1889)

Pol. v. Natanson 33 683 / 34 970

Tait p. 488 Tr. R.S. E. 33 p. 251 (1886-7), 35 p. 1029 (1890)

Pr. R.S. E 16 p. 65 (1889)

Wied. Ann. 68 p. 441 x .. Poincaré, (4,6) & Weinstein]]!

Dieterici Wied. Ann. p. 826 Kin. Th. d. Fl.

[illegible]

Jäger Sbr. 99 p. 681, 861 [illegible]

Diet. W. Ann 50 p. 79 (1893) [illegible]

Voigt Gött. N. p. 341 (1896), 19, 261 (1897) [illegible]

[illegible] Theorie

Milner Phil. Mag 43 p. 291 (1897)

a homog. [illegible]
A ———
inh. D.
O ———
inh. Fl.
J ———
i homog.

Annahmen:

1). [illegible]

2). [illegible]

3). [illegible]

[illegible] $\frac{N_i}{2\sqrt{k_i m\pi}}$ [illegible] $\frac{N_a}{\sqrt{2k_a m\pi}}$ $\left(= N_a\sqrt{-}\int_0^\infty u e^{-k_a m u^2} du\right.$

[illegible] Schwellenwerth ([illegible])

$$n = N_i\sqrt{\frac{k_i m}{\pi}}\int_s^\infty = \frac{N_i}{2\sqrt{k_i m\pi}} e^{-k_i m s^2} = \frac{N_a}{2\sqrt{k_a m\pi}}$$ || wobei $\frac{1}{2} m c_i^2 = \frac{3}{4k_i}$

also $N_i c_i e^{-h^2} = N_a c_a$ (3) [illegible]: $k_i m s^2 = h^2$

Energien:

$$U_{n} = N_i\sqrt{\frac{k_i m}{\pi}}\int_s^\infty \frac{1}{2} m u^2 u e^{-k_i m u^2} du = \frac{N_i}{2\sqrt{k_i m\pi}} e^{-k_i m s^2}\left[\frac{ms^2}{2}+\frac{1}{2k_i}\right]$$

$$= n\left\{\frac{1}{2}ms^2 + \frac{1}{2k_i}\right\}$$

davon ist [illegible] Austrittsarbeit $\frac{ms^2}{2}\cdot n$ am Austritt es bleibt also $n\frac{1}{2k_i}$ was [illegible]

$$= \frac{N_a}{2\sqrt{k_a m\pi}}\frac{1}{2k_a}$$

(4) also $N_i m c_i^3 e^{-h^2} = N_a m c_a^3$

Somit aus (3): $c_i = c_a$

|| Dasselbe schon bei Voigt

Druck: $\mathfrak{p}_{iA} = N_i \sqrt{\frac{k_i m}{\pi}} \int_0^\infty m u^2 e^{-k_i m (u^2+s^2)} du = \frac{N_i}{4 k_i} e^{-k_i m s^2} =$

$\mathfrak{p}_{aA}$ $= \frac{N_a}{4 k_a}$

(5) $N_i m c_i^2 e^{-h^2} = N_a m c_a^2$

Somit wieder $c_i = c_a$

Dies hat Voigt [illegible] Hyp. [illegible]

Ebenso Boltzm. Zeitschr. ph. Ch. 6 p. 474 (1890), 7 p. 88 (1891)

Riecke " 7 p. 36

Lorentz " 6 p. 564

Aus der [illegible] Energie [illegible]

[illegible] $\frac{mc^2}{2}$ [illegible]

Die Gleichungen geben also: $N_i m e^{-h^2} = N_a m$

[illegible] spec. Vol.: $\frac{v_a}{v_i}$ (6) $\frac{v_a}{v_i} = e^{h^2}$ $h^2 = \frac{3}{2} \frac{m s^2}{m c^2}$

Physik. Bedeutung der Grösse h^2 nach Voigt: $\frac{m s^2}{2} =$ J. [illegible]

[illegible] (6) [illegible] $\frac{v_a}{v_i}$ [illegible]; dabei [illegible]!

Man könnte (6) corrigiren durch Einführung der V. d. Waals'schen Correctionen:

$$\frac{v_a - b_a}{v_i - b_i} = e^{+h^2}$$

aber dann folgt für $v_i - b_i$ ausserordentlich kleine Werthe, so dass dann [illegible]

[illegible] (Voigt)

Es muss also $\frac{m s^2}{2} = J \rho$ [illegible]; dies ist schon d. Kilner [illegible], welche Voigt [illegible]

[illegible]:

Jäger Wied. Ann. 67 p. 894 [shorthand]

[shorthand] Diet. [shorthand]

Literatur. ~~99 p. 679, 860, 1028~~, ~~100 p. 245, 693, 1122~~, 101 103, ~~426, 553, 920~~, 954 102 253, 483, 103 251, 105 p. 15

Kritik: 1). [shorthand] Vogt [shorthand] Diet. [shorthand] Mol. [shorthand] prop. [shorthand] comp. [shorthand] Mol. [shorthand] (Siehe 105)

2). [shorthand] Jäger [shorthand] 16 [shorthand] Milner

$\frac{v_2}{v_1} = e^{k^2}$ [shorthand] Jäger [shorthand] Vogt (3), (10) [shorthand]

[shorthand] (6) [shorthand]

Diet. [shorthand] Vogt [[shorthand] Covolumen [shorthand] Vol.]

3). [shorthand] Lokasions druck [shorthand] (3) [shorthand]:

„[shorthand] const. [shorthand] prop. [shorthand] Conc., [shorthand] mol. [shorthand] const., [shorthand] Subst. – const. [shorthand]"

4). Vogt's [shorthand] Mol. [shorthand] Nov. [shorthand] Gastk.

[shorthand]

[shorthand] Mol. [shorthand] $\frac{1}{84}$ [shorthand]

[shorthand]

Phil. Mag. 43 (1897) p 291

Milner The Heats of Vaporization of Liquids

Assumption that no more degrees of freedom for molecules in liquid than in gas so the average kinetic energies the same, and difference in potential energy = this surface force per g. = "internal" heat of vap. || Only difference between l. & g. = mean free path (v. d. Waals)

Similar conceptions as Voigt's but with taking account of encounters:

Layers of thickness dx

incr. of potential ↓

$$N d\varphi = J \rho \, dL_i \quad (1)$$

if no collisions minim. velocity for a mol. going into gas-space: $\frac{m v_1^2}{2} \cos^2\theta = d\varphi$

but in fact: collisions take place

assuming with V. d. W. $\frac{\lambda - s}{\lambda} = \frac{v-b}{v}$ (2)

effect of collision: that the molecule struck takes up a similar motion to that of the striking one but at an average distance of s (in direction θ) in advance. (this without work)

~~thus~~ for every collision the ~~work~~, necessary to do in moving upwards, will be diminished by $s \cos\theta \frac{d\varphi}{dx}$

Therefore minim. kin. energy necessary to pass:

$$\frac{1}{2} m v_1^2 \cos^2\theta = d\varphi - \frac{s}{\lambda} d\varphi = d\varphi \frac{v-b}{v} \quad (3)$$

eliminating $d\varphi$ (1)(3): $v_1 = \sqrt{2J \, dL_i \, \frac{v-b}{v}} \, \frac{1}{\cos\theta}$

Supposition: Maxwell's law: $\frac{2N}{\sqrt{\pi}\alpha^3} \sin\theta \, q^2 e^{-\frac{q^2}{\alpha^2}} dq \, d\theta \cdot q \cos\theta$ | would be number striking # on plane

but on account of collisions is got to be $\times \frac{\lambda}{\lambda - s} = \frac{v}{v-b}$

Thus number striking the division plane from below is:

$$N_1 = \frac{2N}{\sqrt{\pi}\,\alpha^3}\,\frac{v}{v-b}\int_0^{\frac{\pi}{2}}\int_{v_1}^{\infty} \sin\vartheta\cos\vartheta\,\varrho^3\, e^{-\frac{\varrho^2}{\alpha^2}}\,d\varrho\,d\vartheta$$

$$= \frac{N\alpha}{2\sqrt{\pi}}\,\frac{v}{v-b}\, e^{-\frac{2J\,dL_i}{\alpha^2}\frac{v-b}{v}} \qquad (7)$$

In the upper layer number has become $N+dN$ and sp. volume $v+dv$, all mol. striking the plane from above pass in the layer below, that is:

$$N_2 = \frac{2(N-dN)}{\sqrt{\pi}\,\alpha^3}\,\frac{v+dv}{v+dv-b}\int_0^{\frac{\pi}{2}}\int_0^{\infty} \sin\vartheta\cos\vartheta\,\varrho^3\, e^{-\frac{\varrho^2}{\alpha^2}}\,d\varrho\,d\vartheta$$

$$\therefore\quad N_2 = \frac{N-dN}{2\sqrt{\pi}}\,\alpha\,\frac{v+dv}{v+dv-b} \qquad (8)$$

$$\therefore\quad N_1 = N_2 \qquad \underbrace{e^{\frac{2J\,dL_i}{\alpha^2}\frac{v-b}{v}} = \frac{v+dv-b}{v-b}}_{\text{by expansion:}} \qquad \text{since } \frac{N}{N-dN} = \frac{v+dv}{v}$$

$$\boxed{\alpha^2 = \frac{2RT}{M}}$$

$$dL_i = \frac{RT}{JM}\,\frac{v\,dv}{(v-b)^2}$$

dasselbe wie unten: Ist ($\sqrt{}$ $L_i \sim$ [illegible] $\frac{1}{2}$ [illegible])

and by integrating: $L_i = \frac{RT}{JM}\left\{\log\frac{v'-b}{v-b} + \frac{b}{v-b} - \frac{b}{v'-b}\right\}$ (9)

The same ~~[illegible]~~ assumptions in V. d. Waals' formula, therefore don't vary:

(10) $p(v-b) = \frac{RT}{M}$ [p = total pressure, molec. included]

differentiating it at const. temp.:

(11) $dp\,(v-b) + \frac{RT}{M}\frac{dv}{v-b} = 0$

potential of bodily forces

hydrost. equilibrium: $dp = \rho\, dV$

falsch $L_i = \frac{1}{2}(V'-V)$

$$V'-V = \frac{RT}{M}\int_v^{v'} \frac{v\,dv}{(v-b)^2} = J L_i$$ which gives the same result as (9)

He says $v'-v = \int_v^{v'} v\,dp$ and not as usually assumed $= \int_v^{v'} p\,dv$ 34

Taking $p = p' + \frac{a}{v^2}$ from the last formula would follow $L_i = \frac{a}{J}\left(\frac{1}{v} - \frac{1}{v'}\right)$ which

from the first $\frac{2a}{J}\left(\frac{1}{v} - \frac{1}{v'}\right)$ wrong

This equation does not seem to give accurate results.

Trying the formula by calculating from it b and a in diff. temperatures ~~at~~ results (Water, Benzene, etc.) shows that b does not change very much generally diminishing with temp.; a sensibly constant

Trouton's equation $\frac{ML}{T}$ roughly constant for different liquids seems comprehensible now, because v' (which is varying much from one substance to other) is only in logarithm and has little effect, and $\frac{b}{v}$ is not greatly different for liquids at their boiling points $\frac{ML}{T}$ varies about 20–30%

Sitzber. 99 Jäger p. 679 Zur Th. d. Dampfspanny

η v Mol. ~ $1/b$ v0 $/6$ $\frac{mu^2}{2} > a$

$$Z = N\sqrt{\frac{km}{\pi}}\, e^{-kmu^2}\, du = N\sqrt{\frac{3}{2\pi c^2}}\, e^{-\frac{3}{2}\left(\frac{u}{c}\right)^2} du = \frac{N}{\sqrt{\pi}}\, e^{-x^2} dx \qquad x = \frac{u}{c}\sqrt{\frac{3}{2}}$$

$$n = \frac{N}{\sqrt{\pi}} \int_{\sqrt{\frac{3a}{mc^2}}}^{\infty} e^{-x^2} dx$$

$$\left.\begin{array}{l} a = a_0(1 - \varepsilon\,\cancel{t}) \\ c^2 = c_0^2(1 + \alpha t) \end{array}\right\} \sqrt{\frac{3a}{mc^2}} = \sqrt{\frac{3a_0}{mc_0^2}\,\frac{1-\varepsilon t}{1+\alpha t}}$$

$$N = N_0(1 + \gamma t)$$

$\underbrace{n = \text{prop. e}}_{((\alpha ?))}$ Effg d ex: $d = C\,(1+\gamma t) \int_{k\sqrt{\frac{1-\varepsilon t}{1+\alpha t}}}^{\infty} e^{-x^2} dx$

$$\int_x^\infty e^{-x^2}dx = \frac{e^{-x^2}}{2x}\Big[\underbrace{1 - \frac{1}{2}\frac{1}{1+x^2}}_{!} + \frac{1}{4}\frac{1}{(x^2+1)(x^2+2)} - \dots\Big]$$

$\alpha = 0{\cdot}00366$

$\varepsilon = 0{\cdot}00230$

[illegible] α d_0 d_{50} d_{100}:

$C = \cancel{3437000}\ 6876000$

$\gamma = -0{\cdot}00385$

$k = 3{\cdot}499$

[illegible] Exp. [illegible] d_0 bis d_{100} [Abw. $< \frac{1}{2}\%$]

[Kein Wunder mit 3 Konst.!]

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pg 860 Die [illegible] der Flores mol.

[illegible] α 3 [illegible]

$C = 6936000$

$\gamma = -0{\cdot}00390$

$k = 3{\cdot}500$

[illegible]

$$\frac{mu^2}{2} - \frac{mu'^2}{2} = a \qquad u' = \sqrt{u^2 - \frac{2a}{m}}$$

$$\int_0^\infty Z u' = \text{[illegible]} = N\sqrt{\frac{3}{2\pi c^2}}\int_{\sqrt{\frac{2a}{m}}}^\infty \sqrt{u^2 - \frac{2a}{m}}\, e^{-\frac{3}{2}\left(\frac{u}{c}\right)^2} du =$$

$$= N_c\sqrt{\frac{2}{3\pi}}\int_{\sqrt{\frac{3a}{mc^2}}}^\infty \sqrt{x^2 - \frac{3a}{mc^2}}\, e^{-x^2} dx \qquad \left[\frac{3}{2}\frac{u^2}{c^2} = x^2\right]$$

Für di [illegible] $a = 0$ $\therefore$ $N'c'\sqrt{\frac{2}{3\pi}}\int_0^\infty x e^{-x^2} dx = N'c'\sqrt{\frac{1}{6\pi}}$

[illegible]; [illegible] $d = \dfrac{N'mc'^2}{3}$ (2)

also:

$$N'c' = 2N_c\int_{\sqrt{\frac{3a}{mc^2}}}^\infty \sqrt{x^2 - \frac{3a}{mc^2}}\, e^{-x^2} dx \quad (1)$$

$$= \frac{2}{3} N m c c' \int_{\sqrt{\dots}}^\infty \sqrt{\dots}\, dx = \frac{2}{3} p\alpha(1-\beta t)(1+\alpha t) c_0 c_0' \int_{k\sqrt{\frac{1-\dots}{\dots}}} \sqrt{x^2 - \dots}$$

$N_m = \rho_0(1-\beta t)$ $\quad\beta$ = Ausdehnungs Coeff. [illegible]

$c\,c' = c_0\,c_0'(1+\alpha t)$ $\quad$ weil [illegible]

Dies Integral angenähert$^{(?)}$ ausgewerthet:

$$d = 0\cdot523\,\rho_0(1-\beta t)\,c_0 c_0'(1+\alpha t)\left[\frac{1}{\frac{3a}{mc^2}} + \frac{0\cdot726}{\left(\frac{3a}{mc^2}\right)^2}\right]e^{-\frac{3a}{mc^2}}$$

Aus d_0 und d_{100} ergibt sich $\quad 0'523\,\rho_0 c_0 c_0' = 67.10^4$

$$\frac{3a_0}{mc_0^2} = 0\cdot66$$

Daraus berechnen sich d zwischen 0° und 100° mit bis zu 4% Abweichung

Nach Einsetzung von ~~der~~ Dampfdichte $H_2O = 0'623$, $\rho_0 = 1$ ergibt sich aus

$$(2)\;:\quad \left.\begin{aligned} c_0' &= 61400\text{ cm}\\ c_0 &= 27800\text{ cm}\end{aligned}\right\}\;((?!))$$

Auch noch auf andere Weise:

$\dfrac{N}{N'}$ = [illegible] $= \dfrac{\sigma}{\sigma'}$ $\qquad$ Somit [illegible] ((1))

$$c' = 2\frac{\sigma}{\sigma'}\,c\int_{\sqrt{\;}}^{\infty}\sqrt{x^2-\frac{3a}{mc^2}}\;e^{-x^2}dx$$

und bei Einsetzen des Werthes dieses Integrals

$$c_0 = 26700\text{ cm}$$

[illegible] $\dfrac{a}{m}$ ~~[illegible]~~ = [illegible]

[illegible]!

[illegible]!

$\dfrac{a_0}{m} = 25.10^8$ (CGS) $\quad$ [illegible] 10 mal [illegible]!!

[illegible] incompr. [illegible] Stefan (Wied. Ann. 29 p 658) $\quad \rho A = p_2 - p_1$

$$A = \frac{a}{2m}$$

$p_2 - p_1 = 1235$ Atm. [illegible] (Fick 1221) [illegible] Mol. [illegible] : $\frac{a}{2m}$

[illegible] $N\pi r^2$

$$\delta = \frac{3m\alpha}{a} \qquad \alpha = \text{Cap} \;\checkmark$$

" 76.7 (CSS.)

$$= \cancel{\#}\,\cancel{\varphi}\,2\cdot 10^{-9}\ \text{cm}$$

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p. 1028 [illegible] Vol. 16^{tel} [illegible] Vol. [illegible] temp.

$v\,dp = S\,ds$ = [illegible] Hydr.

$\int_{v_1}^{v_2} v\,dp = \underline{A}$ = [illegible] v_1 [illegible] v_2

$= \frac{a}{m}$

$p(v-b) = \frac{Nmc^2}{3}$ (Jüllner I. p. 454)(!)

[illegible] momentum [illegible] c_6 [illegible] (!!!)

[illegible] temp. [illegible] (compressibel + [illegible])

[illegible]:

$c^2 = c'^2\left(1+\frac{\beta}{v}\right)$ [illegible]

$c^2 = c_0^2(1+\alpha t)\left(1+\frac{\beta}{v}\right)$

$p = \frac{c_0^2(1+\alpha t)\left(1+\frac{\beta}{v}\right)}{3(v-b)}$ $\qquad dp = \dots$

$v\,dp = \dots$

$\int_{v_1}^{v_2} v\,dp = \dots$

$\underbrace{\frac{\beta}{v_2-b} + \frac{\beta}{b}\lg\frac{v_2}{v_2-b}}_{z+\xi t} + \lg\frac{v_1}{v_2} = \delta\,\frac{1-\vartheta t}{1+\alpha t}$

$z + \xi t$ [illegible]

[illegible] v_1, v_2 [illegible]

[illegible] (Kein Wunder!!) [illegible]

[illegible]

[illegible]

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100 p. 245 [illegible]

[illegible]

[illegible] $d = C(1+\gamma\vartheta)\int_{k\sqrt{\frac{1-\varepsilon t}{1+\alpha t}}}^{\infty} e^{-x^2}dx$ [illegible]

[illegible]

[illegible] $e^{\int\frac{1-\varepsilon t}{1+\alpha t}}$

[illegible] $d = \frac{2}{3}\rho_0 (1-\rho) \ldots c_0'(1+\alpha t)\int_{k\sqrt{\ldots}}^{\infty}\sqrt{x^2-k^2}\ldots e^{-x^2}dx$

[illegible] c_0 [illegible]

[illegible]

$\delta =$ [illegible] $= 49.10^{-9}\sqrt[3]{\frac{\mu}{18 s}}$ $\mu =$ Mol.G, $s =$ [illegible]

$\delta = \frac{3m\alpha_0}{a_0\rho_0}$ [illegible] $A = \frac{a_0\rho_0}{2m}$ [illegible] c_0 [illegible] $k^2 = \frac{3a_0}{mc_0^2}$

[illegible] c_0 [illegible] c_0' [illegible] (... 3 ...) [illegible]

$(\frac{1}{2}$ bis $\frac{2}{3})$

Nature 45 (1892)

Rayleigh p. 80. On the Virial of a system of hard colliding bodies.

1). Van d. Waals 1873 by a peculiar way of argumenting $p(v-b) = \frac{1}{3}\sum m V^2$

2). Maxwell (Nature X p. 477, 1874) says: v is the volume of the vessel and not subject to correction. But by calculating the Virial he finds:

$$pv = \frac{1}{3}\sum m V^2 \left\{1 - 2\,\mathrm{lg}\left(1 - 8\frac{\rho}{6} + \frac{17\rho^2}{6^2} - \dots\right)\right\} \quad \left| \frac{\rho}{6} = \frac{b}{4v} \right| \quad \begin{array}{l}\rho = \text{mean density}\\ 6 = \text{density of mol.}\end{array}$$

$$= \text{approx. } \frac{1}{3}\sum m V^2 \left(1 + \frac{4b}{v}\right)$$ which certainly is not ~~true~~ erroneous.

3). Lorentz Wied. Ann. 12 p. 127 and
4). Tait Ed. Tr. 33 p. 90 1886
} arrive by calculating the virial according to Maxwell's suggestion to V. d. W. Formula

with the same order of approximation

But Maxw. Eq., though numerically erroneous, very remarkable suggestion:

p must be proportional to $\underline{\sum m V^2}$ because all velocities can be altered in ~~xxx~~ a constant ratio, without changing the manner of motion. Also if the molecules be bodies of any form.

$$\therefore \quad p = T\varphi(v)$$

If there are forces, besides, there must be added to p a term $\frac{a}{v^2}$, if the range of forces large in comparison with molecular distances.

$$p = T\varphi(v) - \frac{a}{v^2}$$

relation between pressure and temperature: linear verified by Ramsay – Young

$$p = T\varphi(v) + \chi(v)$$

~~$\chi(v)$ cannot be~~ Young-capillary
as long as there are only the two sorts of forces: repulsive and (Laplace Forces) 37
$\chi(v)$ must be $= -\frac{a}{v^2}$, other functions require assumption of intermediate
kinds of forces.

If all bodies of similar shape, there can be only 2 constants: V. d. W. law
of corresponding --

Korteweg p. 152 defending V. d. W. way of calculation
~~for~~ example in one dimension | $p = \frac{\Sigma m v^2}{L - n d}$

The same result: that only the $1 - \frac{4 b_1}{v}$ part of distance has to be
travelled over by the centres of molecules [still to be corrected ~~by~~ on account of
change in mean path in $\left(1 - \frac{4 b_1}{v} \frac{v}{v - 4 b_1}\right)$]

but under assumption that the encounters are independent
whilst in reality "for a short time after every collision the possibilities of fresh
collisions are considerably influenced by the proximity of the departing molecule".

In 1881 Lorentz by calculating the virial: $p v = \frac{1}{3} \Sigma m v^2 \left(1 + \frac{4 b}{v}\right)$
This has to be still corrected just as above: Mean free path according to
Korteweg: Verslagen en Med. 2, Deel X, Archives Neerland. XII, and V. d. Waals
is shortened in proportion $\frac{v}{v - 4 b_1}$

Polemics: Rayleigh 44: p. 499, 59~~7~~

Tait 45 p. 199; 44 p. 546, 627

Nature 45 p 512

Watson On the B. M. L. of P. of KE

the B.A Report quotes a paper of Prof Burnside where he proves that ~~that~~ it does not hold for a system of colliding spheres, where the centre of mass is at a small distance of the centre of figure.

But Watson shows that Burnside has omitted to take the frequency-factor of collisions ~~&~~ and says that by introducing it the law becomes confirmed.

Calculates ~~after~~ the same example after Boltzmann method.

Burnside p. 533 : Boltzmann published in 1888 a criticism of the same paper in Wi. Sitzb. an objected to ~~the~~ Burnsides frequency factor and corrected his result

Bryan p. 533 : Shows more specially ~~also~~ in that Burnside's error consists.

Nature 46 p. 76 Culverwell against Lord K. Tait case on the Maxwell B. Law (with A B C particles)

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p. 29 ~~A~~ Watson

On a prop in the Kin Th of G. Liouville's theorem

Ray. in Ph M. objects to Maxwell's demonstration and ~~s~~ suggests the use of Hamilton's S function.

The same objection Boltzmann Ph. M. 1882. ~~to Watson's~~ Watson used instead the S function with t independent (the same as Rayleigh)

But now he sees that this does not help for proving the particular case for which he was using it.

Nature 45 p. 277

Korteweg On account of approximation in calculation b cannot be considered independent of v, because when greatest possible compression is reached we must have: $p_1(v - \mu b_1) = \frac{1}{3}\lambda \sum mu^2$ where $\mu b_1 = 3\sqrt{2}/\pi\ b_1 = 1.35 \ldots b_1$,

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Burbury – Watson p. 100

Exceptions of M.D.L. Lord Kelvin's case is a periodic motion and to these that law does not seem to apply.

Kelvin ~~does~~ prescribes a special direction of motions and other condition.

Probably the law holds only for irregular heat motion.

Nature 44 p. 355 Kelvin On some test cases for the M.M. doctrine regarding distr. of E.

History: Maxwell Phil. Mag. 1860 On the collision of elastic spheres

" Phil. Tr. 1866 On the Foundation of K.T. of G.

Boltzmann Wien Ber 1868 [illegible]

a large generalisation of it

Maxwell Camb. Ph. Soc. Trans. 1878, = Sc. papers II p. 713–741

still wider generalisation:

In the ultimate state of the system the average kinetic energy of two given portions of the system must be in the ratio of the number of degrees of freedom of those portions.

Motion of single particle in plane if $V = \frac{1}{2}(\alpha^2 x^2 + \beta^2 y^2 + c x^2 y^2)$

objects to ~~Maxwell's~~ applicability of Maxwell's proof in this case

Ebert Wied. Ann 52 (1894) p. 420

Ann: [illegible] Inv. Th. [illegible] 3 [illegible] 3 Invarianten [illegible]

[illegible]

[illegible] Functionaldeterminante [illegible]

$$\Delta = \begin{pmatrix} p_1 & p_2 & \cdots \\ q_1 & q_2 & \cdots \end{pmatrix}$$

[illegible] Fundamental [illegible]

[illegible] Inv. [illegible] C. Th. [illegible] transf. [illegible] $p_i = f_i(a_1 a_2 a_3 \dots)$

Pol. Coord. [illegible]:

$$q_i = \frac{\partial f_i}{\partial a_1} b_1 + \frac{\partial f_i}{\partial a_2} b_2 + \cdots \qquad \text{wo } b_k = \frac{da_k}{dt}$$ [illegible]

$$\begin{pmatrix} q_1 & q_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \begin{pmatrix} p_1 & p_2 & \cdots \\ a_1 & a_2 & \cdots \end{pmatrix} = D$$ [illegible] Mult. Th. [illegible] F.D.

$$\begin{pmatrix} p_1 & p_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \begin{pmatrix} p_1 & p_2 & \cdots \\ q_1 & q_2 & \cdots \end{pmatrix}\begin{pmatrix} q_1 & q_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \Delta . D$$ [illegible] Δ – Inv.

[illegible] $(-1)^2$ (2 = [illegible]) mult. Det. & Δ [illegible] Transf. [illegible]

[illegible] Fl. Mod. [illegible]

Voigt-Jamurk. 67 p. 452

[illegible]: $570 \frac{gr.}{mm^2}$ [illegible] $= 10 \frac{gr}{mm^2}$ [illegible] 52 [illegible] $= 520 \frac{gr}{mm^2}$

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p. 871 Tammann 87 [illegible]

[illegible]; [illegible] Deydovillu 64 p. 730

Boltzmann J. f. r. u. ang. Math. XCVIII p. 68, 206

Sitzgsb. 63 p. 707, 397

L. Natanson Wied. Ann. 37 (1889) p. 341 Cooling of gas flowing through plug

Meyer-Baynes p. 139

1885 B A, Nature 32 (1885) p. 352, 533 [Phil. Mag. p. 144

Sutherland Phil. Mag. p. 219, 226 p. 425 [illegible]

Effect ([illegible] No. 10?) [illegible]

V. d. Waals correction of b Phil. Mag. p. 423

Dissociation:

Boltzmann W. B. 88 (1883) p. 861, Wied. Ann. 22 (1884) p. 39

Natanson Wied. Ann. 38 (1889) p. 288

Gibbs (Wirkschaften)

Trombini Untersuchg. von Flüssigkeiten 100 p. 1197

Phil. Mag. p. 231 etc. Influence of Dissociation on Viscosity; tries to explain Warburg-Babo CO_2 by Dissociation! No agreement.

100 p. 1122 [illegible]

[illegible]:

1). [illegible] : $= pv = a_0(1+\alpha t)$

2). [illegible] $= b_0(1 \pm \varepsilon t)$

3). [illegible] Vol. [illegible] [illegible]

$$c = c_0(1+\beta t)$$

$$r = r_0 + (\alpha a_0 - \varepsilon b_0 + \beta c_0)\, t$$

$r = A\, T\, u \frac{dp}{dT}$ [illegible]: $\frac{r'}{r} = \frac{u' \frac{dp'}{dT'}}{u \frac{dp}{dT}} \neq \frac{p \frac{dp'}{dT}}{p' \frac{dp}{dT}}$

$$p = (99,\ 679,\ \overset{100}{511}) = C\, e^{-k \frac{1-\varepsilon t}{1+\gamma t}}$$

$$\therefore \quad \frac{r'}{r} = \frac{k'(\varepsilon' + \gamma)}{k(\varepsilon+\gamma)} \qquad k = \frac{3a}{mc^2} \quad k' = \frac{3a'}{mc'^2}$$

$$a' = a(1+\varphi\kappa)$$

$$mc'^2 = mc^2(1+\varphi\lambda)$$

$$\therefore \quad \frac{r'}{r} = 1 + \varphi(\kappa - \lambda)$$

„[illegible] > [illegible] Vol. prop."

[illegible] A [illegible]

100. p. 493 [illegible]

[illegible] Energie [illegible] E, [illegible] $E+e+a'$, [illegible] $E+e+a$

[illegible] $\frac{a'-a}{m}$ prop. Cap. const $= k(\alpha'-\alpha)$

[illegible] Cap. const [illegible] Conc. [illegible]

„[illegible] mol. [illegible] Cap. const. [illegible] const. [illegible]"

[illegible]

osm. Dr.:
$p = p_0(1+\gamma t)$ $\qquad \gamma =$ [illegible] Coeff. [illegible]

$\therefore k(\alpha'-\alpha) = p_0(1+\gamma t)$

$\therefore \alpha' = \alpha + k_0 A i(1+\gamma t) = \alpha_0'(1-\varepsilon' t)$

$$\therefore \varepsilon' = \frac{\alpha_0 \varepsilon - 0{\cdot}56\, A i \gamma}{\alpha_0 + 0{\cdot}56\, A i}$$

$$\neq \varepsilon - 0{\cdot}56\, A i \,\frac{\gamma+\varepsilon}{\alpha_0}$$

„Die mol. [illegible] Temp. coeff. [illegible] Cap. const. [illegible] const. [illegible]"

[illegible] $d =$ [illegible] $= C e^{-\frac{3a}{mc^2}}$ $\qquad [c^2 =$ [illegible] prop. [illegible] mol.

$a = a_0(1-\varepsilon t)$

$c^2 = c_0^2(1+\gamma t)$

[illegible] $a >$ [illegible] $mc^2 <$

[illegible]: „[illegible] Energie [illegible] Conc. [illegible]

[illegible] Mol. [illegible] En. [illegible] const. [illegible]"

$\frac{mc'^2}{2} = \frac{mc^2}{2}(1+\varrho\lambda)$ $\qquad \varrho =$ [illegible] Mol. [illegible]

100 p. 1233 [illegible]

[Thomson's [illegible]]

[illegible] $2 =$ [illegible]

$$4\pi\,(2-\sqrt[3]{4})\,r^2\alpha = m u^2 \qquad \left(m = \frac{4\pi}{3} r^3 \rho\right)$$

$$r = \frac{3(2-\sqrt[3]{4})\alpha}{\rho u^2} = \frac{1{\cdot}24\,\alpha}{\rho u^2}$$

[illegible]?

---

p. 1182 Zur Th. [illegible]

[illegible] Natanson [illegible]!

---

101 p. 158 [illegible]

$r = a + b + c$ [illegible]:

$$\frac{d-d'}{d} = \rho\,(\kappa-1)\,k \qquad b = \frac{\kappa-1}{\kappa}\,r$$

$$A = \frac{b\rho}{2} \qquad \delta = \frac{3\alpha}{b\rho}$$ [illegible]

---

101 p. 400 Zur Stöchiometrie [illegible]

---

101 p. 553 [illegible]

Horrendum! [illegible] $\frac{1}{v}$

---

101 p. 920 Zur Th. [illegible]

[illegible] (wertlos)

$$p v = RT = R_1 (1 + \alpha t) = \frac{N m u^2}{3}$$

$$\left.\begin{array}{ll} 1^{0} & R_1 \\ 1^{\prime} & R_1' \end{array}\right\} \quad \frac{R_1}{R_1'} = \frac{u_0^2}{u_0'^2}$$ [illegible]

$$p + P = \frac{RT}{v-b} = \frac{R_1(1+\alpha t)}{v-b} \qquad P = [illegible] = \frac{a_0 \rho}{2m}(1-\varepsilon t) = P_0(1-\varepsilon t)$$

[illegible]:

$$P_0(1-\varepsilon t) = \frac{R_1(1+\alpha t)}{v-b}$$

$$v = b + \frac{R_1}{P_0}\,\frac{1+\alpha t}{1-\varepsilon t}$$ [illegible] Vol. b [illegible] $\frac{R_1}{P_0}$ [illegible]

[illegible] R_1 [illegible] P_0 [illegible]

[illegible] Vol. [illegible]

berechnet [illegible] k^2 [$d = e^{-k^2 \frac{1-\varepsilon t}{1+\alpha t}}$] [illegible]

[illegible]

$$v = v_0(1-\kappa p) = b + \frac{R_1}{P}(1+\alpha t)\left(1-\frac{p}{P}\right) = v_0\left[1-\frac{R_1(1+\alpha t)}{v_0 P^2}\,p\right]$$

$$\kappa = \frac{R_1(1+\alpha t)}{v_0 P^2}$$ [illegible]

[illegible] Compr[illegible] [illegible] Strom [illegible]

[illegible] ganz [illegible] P_0 [illegible]

[illegible] Verteilung $(1, d)$ $\frac{a}{v^2}$ [illegible]

Tammann [illegible] (II) Wied. Ann. 66 p. 495

Planck (Thermod. p. 18, 152), Ostwald Chemie p 373, 385 (1893),

Poynting Phil Mag 12 p 32 (1887) [illegible]

[illegible] continuirlich [illegible]

Tammann [illegible]

Voigt Gött Nachr. 1897 p. 261 [illegible]

[illegible]

[illegible]

1). [illegible]

2). Correction [illegible]

[illegible]

Milner [illegible]

[illegible] Correction

[illegible]

$$dq = c_v\, dT + \frac{T}{A}\frac{dp}{dT}\, dv \qquad Q = \frac{BT}{A}\int_{v_1}^{v_2}\frac{dv}{v-b} = \frac{BT}{A}\log\left(\frac{v_2-b}{v_1-b}\right)$$

[illegible] [sv + n].

[illegible] : [illegible]

[illegible] [[illegible] Mol. [illegible]] [illegible] momentan

[illegible] $G =$ [illegible], $c =$ [illegible]:

$$c^2 = G^2 \lg \frac{v_a - b}{v_i - b}$$

[illegible] c_p contin. [illegible]

$$\psi = \frac{1}{2} m G^2 \int_{v_i}^{v_a} \frac{dv}{v-b} \qquad c^2 = G^2 \int_{v_i}^{v_a} \frac{dv}{v-b}$$ [illegible] constants [illegible]

[illegible]

[illegible] $\frac{1}{2} c^2 dM$ [illegible] Mol. [illegible]

[[illegible]] [illegible] compensiren [illegible]:

[illegible] $= \frac{1}{2} c^2 dM$

$$\therefore \varphi = \frac{G^2}{2A} \int_{v_i}^{v_a} \frac{dv}{v-b} = \frac{\beta T}{A} \int_{v_i}^{v_a} \frac{dv}{v-b}$$

[illegible] [[illegible] constant [illegible]] [illegible] [ca 2 [illegible] Young für Pentan, Hexan etc.]

[illegible] Energie [illegible] Mol.

[illegible] : V. d. Waals [illegible]

Naturw. Rdsch. XV p. 285 (1900) 9/VI

Spring [illegible] Plastiz. [illegible]

? [illegible] etc. [illegible] Cryst [illegible]?

[illegible] 10000 Atm

1). [illegible]

[illegible]

2). [illegible]

[illegible]

$Cu + Cu$ = [illegible] $Zn + Cu$ = [illegible]

[illegible]

[illegible]

Zn [illegible] Pb [illegible]

[illegible]

Au, Pt, Ag, Cu, Zn, Pb, Bi [illegible]

[illegible] ~~Pt 100~~ 3 – 12 [illegible]

[illegible] $Cu + Zn$ = [illegible]

$Cu - Pb$ -- [illegible]

[illegible] [Zn Pb, Zn Bi] [illegible]

[illegible]: $\underbrace{BaSO_4 \;\; Na_2CO_3}$ $\underbrace{Ba(CO_3)_2,\; Na_2SO_4}$

[illegible] Reaktion

[illegible]

[illegible]

[illegible] Vol. [illegible]

[illegible] Vol. [illegible]

[illegible] 48 S

[illegible] Vol. [illegible]

[illegible] la acetat

[illegible] hydrat

[illegible] alloten [illegible] Vol. [illegible]

[illegible] prism. [illegible] octaed.

[illegible]

[illegible] Fe

[illegible] Vol. [illegible] alloten [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(10000 At.)

[illegible]

Tammann Ztschr. ph. Ch. 21 p. 17 (1896)

Wied. Ann. 62 p. 280

[illegible] Se [illegible] 50° amorph [illegible] [Hittorf P. Ann. 84 p. 217 (1851)

Betol Schm. 95° [illegible] Glas [illegible] −10° [illegible] −200 continuirliche [illegible] Visc.

[illegible] amorph [illegible]

Lithofellinsäure (Wöhler)	205°	105 – 110°
Amygdalin	200°	125 – 130°
Rohrzucker	160°	90 – 100°

[illegible] zu amorph [illegible] Kryst. [illegible] Kerne [illegible]

[illegible] Unterkühlung

Zweiter [illegible] : O. Lehmann: Se [illegible] amorph Se [illegible] 50°.

[illegible] 90° [illegible] grau [illegible]

[illegible], [illegible] 217° [illegible]

[illegible]

[illegible] amorph [illegible]

Chloroform (Pictet) Ztschr. ph. Ch. 16 p. 422 (1895) : [illegible] −68.5, [illegible] −81°

Phil. Mag 33 p. 306

Rayleigh Remarks on K's Investigations respecting B's theorem

Reduction of T on sum of squares ^necessary^ only for given configuration.

Emphasizes conditions 1) every possible system of velocities in a given space

2) " " " energy

Easy to find exceptions f.i.: circular table: if projected under angle, central part never be crossed.

Proof of $dq' \ldots = dq \ldots$ changed by introduction of N with $t-t' = \text{const}$ (as ~~xxx~~ usually done, now)

Phil. Mag. 31 p. 441

Preston Some remarks about the kin. th. of gases

He ~~gets~~ gets crazy about velocities of molecules possibly attaining ∞!

Friction of ether would prevent it!!

Phil Mag 33 p. 209

Rayleigh On the theory of surface Forces.

Calculation under assumption of ~~x~~ varying density in the layer of transition $(p = w + K\rho^2)$ ——

33 p. 291

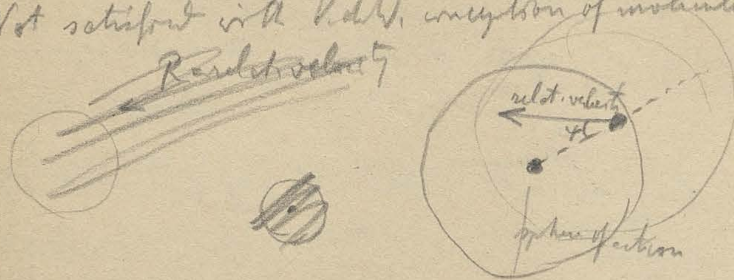
J.W. Thomson The Kin. Th. of the Dissip. of Energy

usual explanation, only probabilities but enormous ones (calculation of an experiment of diffusion)

~~XX~~ Phil Mag. 33 p 301 $= \int \frac{c}{2}$

<u>Natanson</u> Dynamical Illustr. of the Isothermal Formula

Not satisfied with V.d.W. conception of molecules.



(for a molecule

Number of encounters of w, ψ description:

$\frac{2\pi N R^2}{V} w F(v,w) \sin\psi\cos\psi\, d\psi\, dw$

$N F(v,w)\, dw$ Mol. in volume V which have absolute velocity v relative to a — w

time of encounter corresponding to $w\psi$: $= \tau$

Number of systems: $c = \frac{2\pi N R^2}{V}\iiint \tau w f(v) F(v,w) \sin\psi\cos\psi\, dv\, dw\, d\psi$

free molecules: $N(1-c)$

$\bar{L} - \bar{S} = \frac{m w R\cos\psi}{2\tau}$ (Wied. Ann. XXXIII p 698)

putting in Virial equation he gets: finds $\frac{3}{2} pV = N E \left(1+\frac{b}{V}\right)$

$b = 2\pi N R^2 \iiint \left\{\frac{2R\cos\psi}{w} - \tau\right\} w f(v) F(v,w) \sin\psi\cos\psi\, dv\, dw\, d\psi$

if double and triple etc. encounters and systems

General form $\frac{p}{\rho} = Rt\left(1 + T_1\rho + T_2\rho^2 + \dots\right)$

E_0 k.E. free molec.

E_c centre of inertia of binaries

$\frac{3}{2} pV = N(1-c)E + \frac{1}{2} N c E_c + \frac{\pi N^2 R^2}{V^2}\iiint (L-S)\tau w f(v) F(v,w) \sin\psi\cos\psi\, dv\, dw\, d\psi =$

$$= NE - \tfrac{1}{2}Ne\left[2E - E_c\right] + \frac{N\omega}{2V} \iint \tfrac{1}{2} n u^2 \text{ [illegible]}$$

$$\cancel{\#}\ \omega = \tfrac{4}{3}\pi N R^3$$

$$\underbrace{\cancel{\text{must be}}}_{= 2E - E_c}$$

$$= NE + \tfrac{1}{2}N(2E - E_c)\left(\frac{\omega}{V} - 1\right)$$

$$1 - \frac{E_c}{2E} = \varepsilon \qquad \varepsilon\left(\frac{\omega}{V} - 1\right) = \frac{b}{V}$$

For passive molecules $\tau = \dfrac{2R\,\text{avg}}{w}$ | $b = 0$

elastic spheres $\tau = 0.$ | $b = \tfrac{1}{2}\omega$ because $\varepsilon = \tfrac{1}{2}$

in accordance with VdW.

Naturw. Rdsch. XV p. 221

1. Dulong s Petit R. Lasse

Wied. Ann. 48 p. 467

$$\sum \frac{m}{2} u^2 = -\frac{1}{2} \sum (Xx + Yy + Zz)$$

$$\Lambda = \frac{3}{2} pv + \frac{1}{2} \sum r f(r) - \frac{1}{2} \sum (\mathfrak{X}x + \mathfrak{Y}y + \mathfrak{Z}z) = \Lambda_i + \Lambda_a$$

[illegible] [illegible]

[illegible] Mol.

[illegible] Mol.

$\mathfrak{X}\,\mathfrak{Y}\,\mathfrak{Z}$ = [illegible] Mol. [illegible]

$\Lambda_i = \frac{1}{2} \sum r f(r)$ [[illegible] $\frac{1}{2}$ [illegible] + A] [illegible]

$\Lambda_a = -\frac{1}{2} \sum (\mathfrak{X}x + \mathfrak{Y}y + \mathfrak{Z}z) + \frac{3}{2} p$ [= A

[illegible] Mol. [illegible] [$\frac{1}{2}$ [illegible]]

III. [illegible] (endoth.) [illegible] CHN, C_2H_2 (Acetylen), SeH, TeH, O_3, Si_2Cl_3,

Si_2Cl_4?, SiH, PtCl, Ag_2O

Si nicht [illegible]

[illegible]

[illegible] , [illegible] Newton [illegible] W.

$\overline{\sum r f(r)}$ [illegible]

>0

[illegible]

$L_i = \frac{1}{2}\rho f(\rho)$

$$\frac{dL_i}{dT} = \frac{1}{2}\left[f(\rho)+\rho f'(\rho)\right]\frac{d\rho}{dT}$$

IV. [illegible]

$f(r) = \frac{c^2}{r^n}$ $\qquad \overline{L_i} = \frac{c^2}{2}\,\overline{\frac{1}{r^{n-1}}}$

[illegible] pot. Energie:

$$\varphi(r) = -\int_r^\infty f(r)\,dr = -\frac{c^2}{n-1}\frac{1}{r^{n-1}}$$

$$\overline{\varphi(r)} = -\frac{c^2}{n-1}\,\overline{\frac{1}{r^{n-1}}} = -\frac{2}{n-1}\overline{L_i}$$

[illegible] Energie: $\overline{E_i} = \overline{L_i} + \overline{\varphi(r)} = \frac{n-3}{n-1}\overline{L_i}$

$$\frac{d(\overline{E_i})}{dT} = \frac{n-3}{n-1}\,\frac{d(\overline{L})}{dT}$$

[illegible]

[illegible] $n-3>0$

[illegible] $\frac{1}{r^n}$ [illegible] $n=3$ [illegible] stabile [illegible] [Lipschitz] Crelle 66 p. 363 (1866)

$$\frac{d(\overline{E_i})}{dT} = \frac{d(\overline{L})}{dT} - \frac{2}{n-1}\frac{d(\overline{L})}{dT}$$

[[illegible] $n=7$ $\quad \frac{dL}{dT} - \frac{1}{3}\frac{dL}{dT}$]

[illegible] Potenz [illegible] [[illegible] O.E.M. Seeth. p. 99]

V. [illegible] mehratom. Mol. $\quad$ W.B. 63 p. 417 (1871)

[illegible] Atoms [illegible] $= \frac{3}{2h}$ [illegible] Mol.

[illegible] LkA [illegible]

[illegible]

[illegible] Atome [illegible]

[illegible]

VI. [illegible] Stabilität

[illegible] Boltzm. [illegible] $\rho \ldots \rho + d\rho$

$$dW_\rho = \frac{e^{-h\chi}\rho^2 d\rho}{\int_0^R e^{-h\chi}\rho^2 d\rho}$$

[illegible]

[illegible]

[illegible]

$$e^{-h\chi}R^3 \log R = 0$$

$$e^{-h\chi} r^3 \log r \Big|_0 = 0$$

VII [illegible] Boltzm. [illegible]

$$f(r) = \frac{d\chi}{dr}$$

$$\overline{rf} = \frac{\int_0^R r^3 \frac{d\chi}{dr} e^{-h\chi} dr}{\int_0^R e^{-h\chi} r^2 dr} = \frac{-\frac{1}{h} e^{-h\chi} r^3\Big|_0^R + \frac{3}{h}\int^R e^{-h\chi} r^2 dr}{\int \ldots} = \frac{3}{h} = 2L_i$$

$$L_i = \frac{1}{2}\,\overline{rf}$$

[illegible] Atome [illegible] Mol. [illegible]

[illegible]

$\mathfrak{S}$ eltz e Stabil.

0°76 e h.U $\frac{N_3 U}{N_2 L_i}$ = 25 2 positiv

= p

u = Dm / ... = $25 \cdot 10^6 \frac{erg}{ccm}$ (NO_2)

$54 \cdot 10^6$ " (HJ) $p = 1.01 \cdot 10^6 \frac{Dyn}{cm^2}$

$483 \cdot 10^6$ – (H_2)

Voigt p 674 Elastizität isotroper Metalle

s = Elastizitätsmodul s_2 = Torsionsmodul

$$c = \frac{4s - s_2}{s_2(3s - s_2)}, \quad c_1 = \frac{s_2 - 2s}{s_2(3s - s_2)}, \quad c_2 = \frac{1}{s_2}$$

	$c \cdot 10^{-12}$	$c_1 \cdot 10^{-12}$	$c_2 \cdot 10^{-12}$
Al	0.811	0.307	0.252
Bro (Bronze)	1.41	0.61	0.40
Cd	2.33	1.85	0.24
Fe	1.46	0.43	0.51
Au	1.11	0.546	0.280
Cu	1.11	0.173	0.468
Mg	0.498	0.163	0.167
Msg	1.08	0.355	0.36
Ni	2.69	1.16	0.76
Ag	1.08	0.499	0.29
St	2.47	0.89	0.79
Bi	0.41	0.17	0.12
Zn	1.49	0.73	0.38

[illegible] p. 708

Boltzmann [illegible] 63 p. 731 (1871) [illegible]

[illegible]

Boltzmann. 53, p. 195 (1866), 63 p. 712 (1871), 90 p. 231 (1884)

Clausius Poggend. 142 p. 433 (1871)

Helmholtz Crelle 97 p. 111, 317 (1884), Sitzgsb. Berl. Ak. 1884 p. 159
p. 311

[illegible]

$$\varphi = \text{Pot.} = F_1 + F_2 + F_3 + \cdots \quad (\text{homogen})$$

$$X = -\frac{\partial \varphi}{\partial x} = -\frac{\partial F_1}{\partial x} - \frac{\partial F_2}{\partial x} - \cdots$$

$\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\} = 0$ [illegible] $X = Y = Z = 0$ [illegible] $\frac{\partial F_1}{\partial x} = \cdot \frac{\partial F_1}{\partial y} = 0 \Big\} F_1 = 0$

$$\varphi = F_2 + F_3 + \cdots$$

$$-\tfrac{1}{2}(Xx + Yy + Zz) = \frac{x}{2}\frac{\partial F_2}{\partial x} + \frac{y}{2}\frac{\partial F_2}{\partial y} + \frac{z}{2}\frac{\partial F_2}{\partial z} + \frac{x}{2}\frac{\partial F_3}{\partial x} + \cdots = F_2 + \tfrac{3}{2}F_3 + 2F_4 + \tfrac{5}{2}F_5 + \cdots$$

Nach Virial: $\overline{\varphi} = \overline{F_2} + \overline{F_3} + \cdots$

$$\overline{L} = \overline{F_2} + \tfrac{3}{2}\overline{F_3} + 2\overline{F_4} + \cdots$$

[illegible] $\overline{\varphi}$ = [illegible] Multiplum [illegible] $\overline{L}$ [illegible]

[illegible] $\overline{F_2} = \overline{L} = \overline{\varphi}$

Distribution of velocities of particles which at a given moment have a velocity $(p_1, p_2 \dots)$

$$N_{(p_1 p_2 \dots)} = A e^{-h\left[p_1^2 + \dots p_n^2 + V\right]} dp_1 dp_2 \dots$$

$$n\left(\overset{p_1'}{p_1 + \delta p_1}, \overset{p_2'}{p_2 + \delta p_2} \dots\right) = f_{\dots}(p_1 p_2 \dots, \overset{p_1'}{\delta p_1} \overset{p_2'}{\delta p_2} \dots, t) \quad dp_1 dp_2 \dots dp_1' dp_2' \dots$$

Stability requires:

$$\int_{-\infty}^{+\infty} n_{p_1 p_2} = N_{p_1' p_2' \dots} = A e^{-h\left[p_1'^2 + p_2'^2 + \dots p_n'^2 + V'\right]} dp_1' dp_2' \dots dp_n'$$

$$\int_{-\infty}^{+\infty} dp_1 dp_2 \dots dp_n \; A e^{-h\left[p_1^2 + p_2^2 + \dots + p_n^2 + V\right]} f_{\dots}(p_1 p_2, \dots p_n, p_1' p_2' \dots p_n') dp_1' dp_2' \dots dp_n' =$$

$$f_{\dots}(p_1 p_2 \dots p_1' p_2' \dots) = \frac{\alpha}{t} e^{-\frac{\beta}{t}\left[(p_1' - p_1)^2 + (p_2' - p_2)^2 + \dots\right]}$$

[illegible] Vol?

$k = \frac{C_p}{C_v} =$

Ag	1'02
Fe	1'01
Cu	1'015
Pt	1'007
Au	1'016
Al	1'02
Pb	1'039
Zn	1'040

[illegible]

[illegible] $\varphi + L$ [illegible] prop. T [illegible] $= T\,\psi(T)$

[illegible]

[illegible]

1). [illegible] Atom [illegible] Atomvol. $\left(\frac{C_p}{C_v}\right)$ [illegible]

2). [illegible]

[illegible] Atomgew.

	Atomgew.	Atomvol.	Atomwärme
Li	7.0	11.9	6.6
Be	9.1	5.6	3.6-5.3
B	10.9	4.0	2.5-5.5
C	12.0	3.6	1.4-5.5
Na	23.0	23.7	6.7
Mg	23.9	13.8	5.9-6.0
Al	27.0	10.6	5.5-5.8
Si	28.0	11.2	4.6-5.7
P	31.0	13.5	5.3-5.9
S	32.0	15.7	5.2-5.7
K	39.0	45.4	6.5
Ca	39.9	25.4	6.8

[illegible]

Report of the meetings of the british association for the advancement of science London

Wien U [I 37571]

Wien T (5018 I)

Prag I (St. 516)

Brünn I [4308]

X

Proceedings of the London mathematical society London X

Kraków — 22 ff.

Wien U. (I 8182) Graz U. 27 ff (I 91800)

Revue générale des sciences pures et appliquées X

Krakau U (7 ff) —

Proceedings of the Cambridge Phil. Soc.

Wien U (5 ff) (I 47845)

Proceedings of the royal soc. of Edinburgh

Wien U. (1–12) (I 154888)

~~Abstracts of the papers printed~~ Proceedings of the royal soc. of London

Lemberg U (17–19) (25167) Krakau U (57 ff) Czernowitz (57 ff) [82 E 4]

Kammerlingh Onnes: arch. neerl. T 30 S 7 p. 128 (1881)

Phil. Mag. 3512 I

Wiedemann XI (1891)

Markovsky Electr. f. of gas battery.

Ad Maxwell-Boltzmann. 1

1). mathem. [illegible] physik. [illegible] ; [illegible] 2 [illegible]

2). Punkt Atome [illegible] ; 2 Punkt Atome [illegible]

3). [illegible] !

4). [illegible] Confix. [illegible] Phase [illegible]

5). Maxw. [illegible]

(Discret)

6). [illegible] ∞ Coordin. [illegible] continu. [illegible] ? [illegible] ?

[illegible]

[illegible] Confix. [illegible] Energie [illegible]

[illegible] elast. Kugeln [illegible] ∞ [illegible] = statt. Maxw. [illegible]

Ist es richtig [illegible] Confix. [illegible]

oder ist es richtig [illegible] — — " " [illegible] Confix. [illegible] ?

[illegible] Coord. $-\infty$ $+\infty$?

Was wenn elast. Medium [illegible] ?

Distr. of — between similar systems with angular momenta equal instead of equal energy.

Does ~~it hold~~ if there are relations between the various p without pot. energy? [F. ex. vibrational frequency depending on velocity and not reverse is that possible?]

$$\omega = \frac{a^3}{r^3}\,\omega_0$$

$$\cancel{\frac{8\pi\mu a^3 \omega_0\, 2\pi}{\frac{5}{2}\,\frac{4}{3}\pi\, a^5\, \omega_0^2}}$$

$$u = -\frac{3}{4}\,\frac{u\,a}{r^3}\left(1-\frac{a^2}{r^2}\right)x^2 + u\left(1-\frac{3}{4}\,\frac{a}{r}-\frac{1}{4}\,\frac{a^3}{r^3}\right)$$

$$v = -\frac{3}{4}\,\frac{u\,a}{r^3}\left(1-\frac{a^2}{r^2}\right)xy$$

$$w = - \qquad xz$$

Hullet Z. ph. Chemie 37 p. 385

Jackson Scient. Proc. R. Dublin S. X 1903 part I, 8
tension in liquids without rupture

Ebert Astr. Nachr. 164 Nr 3917 [illegible]

<u>Riecke</u> Beiträge z. Hydrod. Gött. Nachr. 1888 p. 347
Wied. Ann. 36 p. 322 (1889)

Opalescenz (krit. Geb. d. Lösung):

Rothmund Z ph. Ch. 26 p. 446 (1898)

Friedländer 38 p. 385 (1901)

Guthrie Phil Mag. (5) 18 p. 30, 503 (1884

Lehmann Molekularphysik A 2166 II

Tyndall's Versuch

[illegible]

[illegible] Cap. Cont. [illegible] optische [illegible]
([illegible]) 120 [illegible]

[illegible] Lissajous Figuren

[illegible] Pyramiden

[illegible] [Sache [illegible] Stefan]
[[illegible]]

[illegible] inverse Phänomen [illegible]
[illegible] physiol. [illegible]

$\underline{6\pi\mu a u}$ $\qquad \Sigma\delta^2 =$ $\qquad$ Peter! cō gu ?

Stoney

$\cancel{\#}\, a^2\pi$

$$= \frac{6\mu u}{a} = \text{force pro } 1\,cm^2$$

$$: p = x : 760$$

$$: 980 . 1000 = x : 760$$

$$\left(\frac{10^9 . 4}{\pi^2}\right)^2$$

$$x = \frac{760}{980} \frac{.6 . \cancel{0.000}018 . \pi^2 . 3 . 10^{\cancel{8}}}{. 10^3 \cancel{\;} 10^{18} . 4 . \cancel{9.10^6}}$$

$$= \frac{760}{980} . \frac{18 . 18 . \pi^2}{4 . \cancel{\;}} 10^{-20}$$

$$= 10^{-\cancel{18}} . \text{mm}$$

$$\cancel{15}\ \cancel{30}$$

$$\frac{\cancel{10 \cdot 000 \cdot 000}\ \cancel{7}5 . \cancel{3\pi} . 10^5}{\cancel{365 . 24 . 3600}}$$

$$.70 \quad \cancel{8} \quad \cancel{120} \quad 50$$

10

$$= \frac{10^8}{35} = 3 . 10^{\cancel{6}}$$

$$\cancel{900 . 70 . 10^5}$$

$$\frac{6\pi\mu a u . 2R\pi\, T^2}{\frac{M}{2} \cancel{\#} (2R\pi)^2} = \frac{6\pi\mu a \frac{2R\pi}{T} . 2R\pi . T^2}{\frac{M}{2} (2R\pi)^2} = 2 \frac{6\pi\mu a\, T}{M} =$$

$$\cancel{\frac{4}{4}}\ \frac{\;}{2}$$

$$= \frac{12\, \pi \mu T a}{5 . 6 . \frac{4}{3} \pi a^3} = \frac{9}{5 . 6} \mu \frac{T}{a^2}$$

$$a = 10^9 . \frac{2}{\pi}$$

$$= \frac{9}{5 . 6} . \frac{\cancel{0.0000}18 . 365 . \overset{6}{\cancel{24}}\ 3\cancel{600}}{10^{18} . \cancel{\frac{4}{10}}} \cancel{10} = \frac{\cancel{9} . \cancel{18} . \cancel{36} . \cancel{22}}{\cancel{56} . 10^{18}}$$

$$\overset{\cancel{405}}{\cancel{324}} . 400$$

$$\cancel{\;}7$$

$$\frac{1620}{7} 10^{-16} = 23 . 10^{-\cancel{15}}$$

$365 . 24 . 3600 =$

730

1460

$\overline{8760} . 3600$

26280

52560

$\overline{31{,}536000} = 3.15 . 10^7$

[illegible]

CLXVIII S. 24

... chciał się obznajomić z własnościami materyi promieniotwórczej, to żadną miarą nie miałbym odwagi polecić mu książeczki p. Muttermilcha.

W. Żłobicki

Dr. St. Kołłoszko. Co to są elektrony. Dwa [illegible] miesięcznych Wykładów Uniwersyteckich w Krakowie. Kraków; nakładem [illegible]arni D. E. Friedleina. 1905. str. 81.

W miarę tego, jak z rozwojem fizyki zdobywano coraz to nowe tereny umiejętności fizycznych w dziedzinie elektryczności — w miarę tego zmieniały się kilkakrotnie pojęcia o istotnej naturze tego czynnika, który nazywamy elektrycznością. W wielu zjawiskach, na pozór odrębnych, zaczęto się dopatrywać jednej nici przewodniej. Snuła się ta nić, począwszy od praw elektrolizy Faradaya (1834) przez teoryę dyssocyacyi Svante Arrheniusa (1887), przez badania Hittorfa (1869) i Crookesa (1879) aż do

partyjny Blok Pracy
7 do 38 mandatów.
wski Blok Pracy Gos
mandatów.
lski Blok Obrony Chrz
zdobyła 1 do 2 mand
jalistyczna Lista Rob
była 12 mandatów.

cie Nr 2 kandydował reg
ściśle z Bezpartyjnym Blo-
puszczać należy, że
ego Bloku w przyszłej Radzie
liczył 42 radnych na ogólną
czbę 64 radnych.

borów w poszczególnych okrę-
ujące:
dzielnica I) lista Nr 1 uzyskała
nandaty, lista Nr 3 1069 głosów
Nr 4 820 głosów, bez mandatu.
kany w śródmieściu przez Karo-
Rostworowskiego uważać należy
sobisty kandydata a nie za sukces
ckiej, głosujący na tę listę bowiem
i zostali przedewszystkiem nazwi-
zołowego kandydata powołanego tak
wno przez rząd Rzeczypospolitej do Aka-
Literatury i ten wzgląd zapewnił liście
lat.

ta Nr 1 uzyskała 5171 głosów
ta Nr 3 707 głosów, bez man-
443 głosy i 1 mandat. W okrę-
ad którego wchodzą dzielnice
głosy socjalistyczne padały głów-
nicy XII.

III (dzielnica IV) brak jeszcze
ych obliczeń głosów. Wedle prowi-
ych obliczeń lista Nr 1 uzyska 5 do 6
w, lista Nr 4 uzyska 1 mandat a być
lista Nr 3 uzyska w tym okręgu 1

V (dzielnica V) lista Nr 1 uzy-
i wszystkie mandaty, lista

ych,
tami,
gram Be
przeszło

Nie można j
tek głosów padł
nak już można, że tek ten prz
od 100 ogółu oddany głosów, po
przy ostatnich wyborach do sejmu
partyjny (wraz z wyborcami żydowsk
rzy dziś nie głosowali na listę Nr 1) zy
spełna 50 proc. głosów. Przyrost cyfrowy więc
Bezpartyjnego Bloku jest bardzo znaczny
i wskazuje, że cofanie się wpływów partyj
politycznych w Krakowie postępuje coraz
szybciej.

2) Porażka socjalistyczna jest bardzo znacz-
na. Liczba głosów oddanych na kandydatów
socjalistycznych będzie z pewnością o wiele
niższa od cyfry głosów uzyskanej przy ostat-
nich wyborach sejmowych. Już w tej chwili
można mieć wątpliwość, czy ta cyfra głosów
wystarczyłaby socjalistom na uzyskanie 1
z 4 mandatów sejmowych Krakowa. Szczegól-
nie dotkliwą dla socjalistów jest ich porażka
na Grzegórzkach, w Dąbiu i Podgórzu a więc
w dzielnicach o ludności robotniczej. Socjali-
ści z krakowskiej PPS stracili prawo do okre-
ślania siebie jako reprezentacji robotniczej,
gdyż większość głosów robotniczych padła na
listę Nr 1. Niemniej wobec zupełnej klęski li-
sty endeckiej socjaliści są jedynymi liczel
niejszymi przedstawicielami opozycji w
szłej Radzie miejskiej. Tak jak wybory

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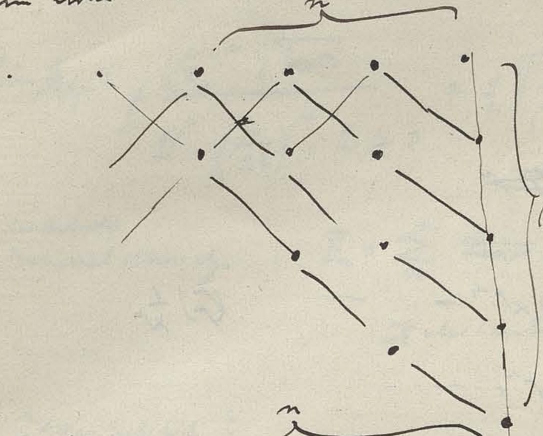
55

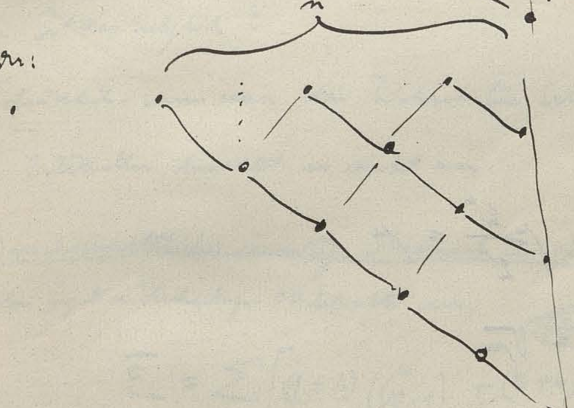
Bellbanders Versuche sollten eigentlich berechnet werden auf Grund d. Formeln für „zum ersten Mal Erreichen"



$$a_{n\mu} = \binom{\mu}{\frac{\mu-n}{2}} \frac{n}{\mu \cdot 2^{\mu}}$$

$$A_\mu = \sum_{n=1}^{n=\mu} a_{n\mu}$$

oder:



$$A_\mu = \sum_{n=1,3,5\ldots}^{n=\mu} a_{n\mu}$$

oder

$$A_\mu = \sum_{n=2,4,6\ldots}^{\mu} a_{n\mu}$$

dasselbe wie ~~Anzahl~~ Wahrsch. „dass ein Teilchen eine vorher noch nicht erlangte Elongation zur Zeit μ erlangt"

welche schon berechnet worden ist:

$$A_\mu = \frac{1}{2}\,\frac{1\cdot3\cdot5\cdot7\ldots(\mu-2)}{2\cdot4\cdot6\ldots(\mu-1)} \quad \text{(für ungerade } \mu\text{)}$$

$$= \frac{1}{2}\,\frac{1\cdot3\cdot5\cdot7\ldots(\mu-1)}{2\cdot4\cdot6\ldots\mu} \quad \text{(für gerade)}$$

also auch lim $A_\mu = \frac{1}{2}\frac{1}{\sqrt{\frac{\mu}{2}\pi}}$ $\longrightarrow$

In Wirklichkeit Superposition der geraden und ungeraden Stellung, so wie obere Fig.,

also: $\lim \sum A_\mu = \sqrt{\frac{\mu}{2\pi}}$ Somit jedenfalls Proportionalität mit $\sqrt{\text{Zeit}}$

Um den Zahlfaktor genauer zu erhalten, muss man d. Zusammenhang mit d. mittleren Elongation fest stellen.

Binomialcoefficienten $\alpha_{n\mu} = \binom{\mu}{\frac{\mu-n}{2}}$ $\qquad \sum_n \alpha_{n\mu} = 2^\mu$

Mittlere Elongation nach μ Intervallen

$$\overline{\varepsilon^2} = \frac{\sum_n n^2 \alpha_{n\mu}}{\sum \alpha_{n\mu}}$$ ~~für geradzahlige μ~~

$$\left(x+\frac{1}{x}\right)^\mu = \binom{\mu}{0} x^\mu + \binom{\mu}{1} x^{\mu-2} + \binom{\mu}{2} x^{\mu-4} + \cdots \binom{\mu}{\mu}\frac{1}{x^\mu}$$

$$\frac{d}{dx}\left[x \frac{d}{dx}\left(\left(x+\frac{1}{x}\right)^\mu\right)\right] = \underbrace{\mu^2\binom{\mu}{0} x^{\mu-1} + (\mu-2)^2\binom{\mu}{1} x^{\mu-3} + \cdots}_{= \sum n^2 \alpha_{n\mu}}$$

für $x = 1$

$$= \mu\, 2^\mu$$

also mittlere Elongation $\qquad \overline{\varepsilon^2} = \mu \frac{2^\mu}{2^\mu} = \mu$

$$\sqrt{\overline{\varepsilon^2}} = \sqrt{\mu}$$

Dagegen ist $\lim A_n = \frac{1}{\sqrt{2\mu\pi}}$

Gewiss nicht, denn z.B. bei der Br. B. unter Einfluss elastischer Kraft, wächst die durchschnittl. Absolut Elongation nur bis zu einer endlichen Grenze, während die durchschn. einmalige maximal Elongation mit der Zeit beliebig gross werden muss.

für gerade μ

$$A_{\mu\mu} = \frac{1}{2}\,\frac{1\cdot 3\cdot 5\cdot 7\cdots(\mu-1)}{2\cdot 4\cdot 6\cdots\cdots(\mu)} = \frac{1}{2}\,\frac{\mu!}{\left(2^{\frac{\mu}{2}}\right)^2\left[\left(\frac{\mu}{2}\right)!\right]^2} = \frac{1}{2}\,\frac{\mu!}{2^\mu\left(\frac{\mu}{2}!\right)^2}$$

$$\lim A_{\mu\mu} = \frac{1}{2}\,\frac{\left(\frac{\mu}{e}\right)^\mu\sqrt{2\mu\pi}}{2^\mu\left(\frac{\mu}{2e}\right)^\mu\cdot 2\frac{\mu}{2}\pi} = \frac{1}{2}\sqrt{\frac{2}{\mu\pi}} = \frac{1}{\sqrt{2\mu\pi}}$$

durchschnittl.
Maximal Abweichung: $\overline{E}_m = \sum_{\mu} A_\mu = \int\frac{d\mu}{\sqrt{2\mu\pi}} = \sqrt{\frac{2\mu}{\pi}}$

ist also identisch mit der durchschnittlichen absoluten Elongation!?

Ist das möglich?

Tatsächlich: wenn man die Wahrsch. für Erreichen einer Maximal Elongation n, innerhalb μ Intervallen, beachtet, so sieht man:

<u>Durchschnittliche ~~einseitige~~ Maximal Elongation</u>, innerhalb μ Intervalle:
(wobei negative Verschiebungen $=0$ betrachtet werden)

für gerade μ

$$\overline{E}_m = \frac{\sum}{2^m}\left[(1+2)\binom{\mu}{\frac{\mu}{2}-1} + (3+4)\binom{\mu}{\frac{\mu}{2}-2} + (5+6)\binom{\mu}{\frac{\mu}{2}-3} + \cdots (\mu-1+\mu)\binom{\mu}{0}\right]$$

Dagegen gewöhnliche „<u>durchschnittliche Absolut Elongation</u>", die am Schluss der μ Intervalle erreicht wird:

$$\overline{\Delta} = \frac{2\sum}{2^m}\left[2\binom{\mu}{\frac{\mu}{2}-1} + 4\binom{\mu}{\frac{\mu}{2}-2} + (6)\binom{\mu}{\frac{\mu}{2}-3} + \cdots\ \mu\binom{\mu}{0}\right]$$

Im Falle grosser Zahlen μ kommt das <u>offenbar auf dasselbe</u> hinaus

Sonst ist $\overline{\Delta} > \overline{E}_m$ $\boxed{\text{Differenz: } \overline{\Delta}-\overline{E}_m = \sum\left[\binom{\mu}{\frac{\mu}{2}-1}+\binom{\mu}{\frac{\mu}{2}-2}+\binom{\mu}{\frac{\mu}{2}-3}+\cdots\binom{\mu}{0}\right] = \frac{1}{2}}$

<u>Also sind obige zwei Ausdrücke für grosse μ identisch</u>!! Genau ist somit $\overline{E}_m = \overline{\Delta} - \frac{1}{2}$

Gilt das vielleicht allgemein auch bei komplizierteren Systemen?

Sedimentations Verteilung

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2} + \mu \frac{\partial n}{\partial x}$$

$$\boxed{\text{für lim } \frac{\partial n}{\partial t}=0 \qquad n = n_0 e^{-\frac{\mu}{D}x}}$$

für $x=0$ $\quad \frac{\partial n}{\partial x}=0$ $\qquad$ $x=\infty$ $\quad \frac{\partial n}{\partial x}=0$

$$n = e^{-kt} f(x)$$

$$D\frac{d^2 f}{dx^2} + \mu \frac{df}{dx} + kf = 0 \qquad D\alpha^2 + \mu\alpha + k = 0$$

$$f = e^{\alpha x} \qquad \alpha^2 + \frac{\mu}{D}\alpha = -\frac{k}{D}$$

$$\alpha = -\frac{\mu}{2D} \pm \sqrt{\frac{\mu^2}{4D^2} - \frac{k}{D}}$$

$$= e^{-\mu x} \frac{\cos}{\sin}(\nu x) \qquad = -\mu \pm i\nu$$

$\frac{\partial n}{\partial x}\Big|_{x=0} = e^{-kt}\alpha = 0$ würde erfordern $\mu=\nu=0$

$$\frac{df}{dx} = -\mu e^{-\mu x}\frac{\cos}{\sin}\nu x \mp \nu e^{-\mu x}\frac{\sin}{\cos}\nu x \Big|^{x=0} = \frac{-\mu}{+\nu} \rightarrow \text{würde fordern } k = \frac{\mu^2}{4D}$$

Vielleicht geht es aber, wenn man annimmt $\left.\begin{matrix}\alpha_1\\ \alpha_2\end{matrix}\right\}$ reell < 0

$$f = Ae^{\alpha_1 x} + Be^{\alpha_2 x}$$

$$= e^{-\frac{\mu}{2D}x}\left[Ae^{\sqrt{\dots}\,x} + Be^{-\sqrt{\dots}}\right]$$

$$\frac{df}{dx}\Big|_{x=0} = \alpha_1 A + \alpha_2 B = 0 \qquad A = \alpha_2 C$$

$$B = -\frac{\alpha_1}{\alpha_2}A \qquad B = -\alpha_1 C$$

$$f = \cancel{e^{-\frac{\mu x}{2D}} \sqrt{\mu^2-4kD}} \left(\cancel{-\mu + \sqrt{\mu^2-4kD}}\right) \cancel{e^{\frac{x\sqrt{\mu^2-4kD}}{2D}}}$$

$$f = C\left[\alpha_2 e^{\alpha_1 x} - \alpha_1 e^{\alpha_2 x}\right]$$

Dabei würde immer noch die Konstante k unbestimmt bleiben

Sie bestimmt sich tatsächlich daraus, dass auch am oberen Ende die Emulsion abgeschlossen sein muss, den wirklich ein streng stationärer Zustand stattfinden soll, kann also nicht

$x=\infty$
$\frac{\partial n}{\partial x}=0$ genügt, sondern man muss haben

$\frac{\partial n}{\partial x}=0$ für $x=h$

$$\frac{\partial n}{\partial x} = e^{-kt}\, C\,\alpha_1\alpha_2\left[e^{\alpha_1 h} - e^{\alpha_2 h}\right] = 0$$ was nicht möglich ist, ausser dass $\alpha_1=\alpha_2$

in diesem Falle ~~…~~ müsste $k=\frac{\mu^2}{4D}$ sein

Also wäre dann:

$$n = e^{-\frac{\mu^2 t}{4D} - \frac{\mu}{2D}x} \qquad -\frac{\mu^2}{4D} = D\frac{\mu^2}{4D^2} - \frac{\mu}{2D}\mu$$

~~…~~ was offenbar nur ein spezielles partik. Integral ist, welches hier nicht brauchbar

---

~~Vielleicht~~ Grenzbedingungen sind falsch!

denn im stationären Endzustand ist offenbar $\frac{\partial n}{\partial x}\Big|_{x=0} = -\frac{\mu}{D} n_0$

Vielleicht sind Grenzbedingungen ersetzbar durch

$$\frac{\partial}{\partial t}\int_0^h n\,dx = 0 \qquad = \int_0^h \frac{\partial n}{\partial t}\,dx = D\frac{\partial n}{\partial x}\Big| + \mu n\Big| = 0$$

$$D\frac{\partial n}{\partial x}\Big|_0 + \mu n\Big|_0 = D\frac{\partial n}{\partial x}\Big|_h + \mu n_h \quad = \text{annähernd} = 0$$

Also wäre (annähernd) $\frac{\partial n}{\partial x}\Big|_0 = -\frac{\mu}{D} n_0$!

Vielleicht nicht annähernd, sondern genau!

Auch könnte man rationeller so ansetzen:

$$n = n_0 e^{-\frac{\mu}{D}x} + \sum e^{-kt} f(x)$$

Denn Bedingung dass nichts durch die Wand durchgeht ist ja jetzt zu ändern! Jetzt ist der Diffusionsstrom nicht $D\frac{\partial n}{\partial x}$ sondern $D\frac{\partial n}{\partial x}+\mu n$

Also lautet die Aufgabe folgendermassen: Grenzbedingungen

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial u}{\partial x}$$

für $x=0$: $D\frac{\partial u}{\partial x} + \mu u = 0$ für $x=h$ $D\frac{\partial u}{\partial x} + \mu u = 0$

$$u = A e^{-\frac{\mu}{D}x} + \sum_k e^{-kt} f(x)$$

Versuch: $f(x) = e^{\alpha x}$ $\alpha = -\frac{\mu}{2D} \pm i\sqrt{\frac{k}{D} - \frac{\mu^2}{4D^2}}$

$$= e^{-\mu x}(A\cos\nu x + B\sin\nu x) \quad = -\mu + i\nu$$

$$u = A e^{-\frac{\mu}{D}x} + z$$

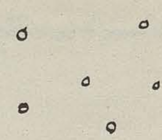
$x=0$; $x=h$; $D\frac{\partial z}{\partial x} + \mu z = 0$

$$\frac{\partial z}{\partial t} = D\left[+A\left(\frac{\mu}{D}\right)^2 e^{-\frac{\mu}{D}x} + \frac{\partial^2 z}{\partial x^2}\right] + \mu\left[-A\frac{\mu}{D} e^{-\frac{\mu}{D}x} + \frac{\partial z}{\partial x}\right]$$

$$\therefore \quad \frac{\partial z}{\partial t} = D\frac{\partial^2 z}{\partial x^2} + \mu\frac{\partial z}{\partial x}$$

Kondensation von Nebel an Kernen

$$\frac{\partial n}{\partial t} = D\,\Delta^2 n \qquad\qquad n = \frac{1}{\sqrt{t^3}}\, e^{-\frac{r^2}{4Dt}}$$



$$n = e^{-kt} f(x\,y\,z)$$

$$-k f = D\,\nabla^2 f$$

$$\nabla u + \underbrace{\frac{k}{D}}_{\kappa^2} u = 0$$

$$t = 0 \qquad n = \text{const} = n_0$$

$$r = r_1 \quad r_2 = r_2 \ldots \qquad n = 0$$

Im Falle sphärischer Symmetrie

$$k = D\sqrt{\kappa}$$

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + \kappa^2 u = 0 \qquad u = \frac{\cos\kappa r}{\kappa r} \qquad \frac{\sin\kappa r}{\kappa r}$$

$$\frac{du}{dr} = -\frac{\sin\kappa r}{r} - \frac{\cos\kappa r}{\kappa r^2}$$

$$\frac{d^2u}{dr^2} = -\frac{\kappa\cos\kappa r}{r} + \frac{2\sin\kappa r}{r^2} + \frac{2\cos\kappa r}{\kappa r^3}$$

$$-\frac{2\sin\kappa r}{r^2} - \frac{2\cos\kappa r}{\kappa r^3} + \frac{\kappa\cos\kappa r}{r} = 0$$

$$n = e^{-kt}\,\frac{\cos\kappa r}{r}$$

$$r = a$$
$$u = 0$$

$$\cos\kappa a = 0 \qquad\qquad \sin\kappa a = 0$$

$$\kappa a = (2m+1)\frac{\pi}{2} \qquad \kappa a = m\pi$$

$$\kappa = \frac{(2m+1)\frac{\pi}{2}}{a}$$

$$\cancel{n = \sum \left[A_k \cos\frac{(2m+1)\pi r}{2a} + B_k \sin\frac{m\pi r}{a}\right]\frac{1}{r}}$$

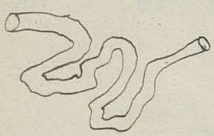
$$n = \sum{}' \frac{1}{r}\left[A_k\, e^{-D\left[\frac{(2m+1)\pi}{2a}\right]t} \cos\frac{(2m+1)\pi r}{2a} + B_k\, e^{-D\left[\frac{m\pi}{a}\right]t} \sin\frac{m\pi r}{a}\right]$$

Bestimmung der Koeffizienten $A_k\ B_k$:

$$\sum\left\{A_k \cos\left(\frac{(2m+1)\pi}{2}\frac{r}{a}\right) + B_k \sin\left(m\pi\frac{r}{a}\right)\right\} = \cancel{\text{const}}\ r c$$

Ein kleineres Korn durchstreicht infolge der Brown'schen Bewegung längere Strecken als ein grosses

aber ob ein grösseres Volum?



Kennzahl:

in welcher Zeit verschiebt sich eine Kugel um Drittel um ihren eigenen Durchmesser?

$$2D\tau = 4a^2 \qquad D = \frac{H\Theta}{N}\frac{1}{6\pi\mu a}$$

$$\tau = \frac{2a^2}{D}$$

also durchstrichenes Volumen pro Zeiteinheit $= \mathrm{Vol} = \frac{a^2\pi \cdot 2a}{\tau} = 2a^3\pi \cdot \frac{D}{2a^2} = a\pi D$

unabhängig vom Radius! $= \frac{H\Theta}{N}\frac{1}{6\mu} = \frac{8{,}3\cdot 10^7 \cdot 300}{6\cdot 10^{23}\cdot 6\cdot 2\cdot 10^{-4}}$

$$= \frac{1}{3}10^{-10} = 30\mu^3$$

Falls es sich aber um kleine Kugeln in Gas handelt, ist dagegen

$$D = \frac{H\Theta}{N}\frac{1}{\cdots a^2 c} \quad \text{also} \quad \mathrm{Vol} \propto \frac{H\Theta}{N}\frac{1}{c\rho_0}!$$

Geschwind. $= \frac{2a}{\tau} = \frac{D}{a}$

$$= \frac{H\Theta}{N}\frac{1}{6\pi\mu a^2}$$

$$\cdots \frac{H\Theta}{N}\frac{1}{a^3 c\rho_0} = \frac{H\Theta}{N}\frac{1}{a^3\varepsilon}$$

somit grösser für kleine Kugeln als grosse

Wenn aber noch dazu die Schwerkraft kommt, setzt sich das resultierende Volum zusammen:

$$v = \frac{\frac{4}{3}\pi a^3\rho g}{\cdots a^2\pi c\rho_0} = \cdot a\frac{\rho}{c\rho_0} = \frac{\frac{4}{3}\pi a^3\rho g}{a^2\varepsilon}$$

$$\mathrm{Volum} = a^2\pi\sqrt{a^2\left(\frac{\rho}{c\rho_0}\right)^2 + \left(\frac{H\Theta}{N}\right)^2\frac{1}{a^6 c^2\rho_0^2}} = \frac{a^2\pi}{\varepsilon}\sqrt{\left(\frac{4}{3}\pi\rho g\right)^2 a^2 + \left(\frac{H\Theta}{N}\right)^2\frac{1}{a^6}}$$

~~Gibt es ein Maximum~~?

$$\mathrm{Vol}^2 = A a^6 + \frac{B}{a^2}$$

$$\frac{d}{da} = 6a^5 A - \frac{2B}{a^3} = 0$$

$$a^8 = \frac{B}{3A}$$

Also gibt es dann eine gewisse Radius Grösse, für welche das durchstrichene Volum maximal ist und zwar beträgt sie:

$$a^8 = \left(\frac{H\theta}{N}\right)^2 \frac{1}{\left(\frac{4}{3}\pi\rho\right)^2} \qquad a = \sqrt[4]{\frac{H\theta}{N}\,\frac{1}{\frac{4}{3}\pi\rho g}} \qquad \begin{aligned} H &= 8{\cdot}3.10^{7} \\ N &= 6\,.10^{23} \end{aligned}$$

$$= \sqrt[4]{\frac{8{\cdot}3.10^{7}.300}{6.10^{23}.4.10^{3}}} = \sqrt[4]{\frac{10^{9}}{10^{26}}} = \sqrt[4]{10^{-17}} = \frac{1}{\sqrt[4]{10}}\,10^{-4}$$

$$= \frac{10^{-4}}{2}\ \text{cm}$$

somit wäre dieser Radius noch ~~merklich~~ 5 mal grösser als d

also ist im ganzen Geltungsbereich der molekularen Reibungsformel das durchstrichene Volum desto grösser je kleiner das Teilchen

[Die Brown'sche Bewegung solcher Teilchen sieht man an den Wilson'schen Photographien für Röntg-

Sobald aber Übersättigung herrscht, muss die Grösse d. durchstrichenen Volumens auf die Anwachsgeschwindigkeit d. Teilchens von Einfluss sein

[Aber macht das etwas merkliches aus im Vergleich zur molekularen Diffusion?]

man müsste berechnen, wie gross d. durchstrichene Volum ist, im Vergleich zu dem mittleren Kondensvolum pro Teilchen, in der Zeit wo die halbe Diffusionskondensation erfolgt.

Bei sehr dichten Nebeln kann d. durchstrichene Volum überwiegen, also ist dann aus obigem ersichtlich dass dann Tendenz zur ~~Gleich~~ Verteilung gleich grosser Tropfen besteht

dagegen muss bei dünnen Nebeln d. letztere überwiegen, dann besteht Tendenz zur Ungleichförmigkeit d. Tropfen, denn Fallgeschw. $v = \frac{2}{9}\frac{a^2\rho g}{\mu}$

also $\frac{\Delta v}{v} = 2\,\frac{\Delta a}{a}$, dadurch wird ein desto grösseres Volum durchstrichen und stärkere Kondensation und das bewirkt ~~ein~~ Zusammenstössen mit d. übrigen langsameren Tropfen, wodurch noch stärkeres Wachstum erfolgt

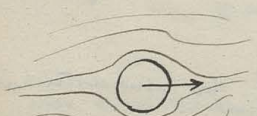
Wie rasch wächst der Radius eines kugelförmigen, in übersättigter Luft sinkenden Tropfens (unter Einfluss der Diffusion und Kondensation)?

Man kann sich die Kugel ruhend vorstellen, aber das Medium in Stokes'scher Weise strömend.
Diffusionsgleichungen für bewegte Medien?

$$\cancel{\frac{\partial c}{\partial t}} \frac{\partial}{\partial x}\left(cu - D\frac{\partial c}{\partial x}\right) + \ldots = \frac{\partial c}{\partial t}$$

Analoges Beispiel: wie rasch löst sich eine ~~Ka~~ Salzkugel in strömendem Wasser?

$$\frac{\partial c}{\partial t} = D.\nabla^2 c + \frac{\partial(cu)}{\partial x} + \frac{\partial(cv)}{\partial y} + \frac{\partial(cw)}{\partial z}$$



Die pro Zeiteinheit sich kondensierende Menge muss offenbar proportional sein D?, überdies abhängen von a und v

also $\frac{m}{t} = D\, f.(a, v)$

Dimensionen von D: $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$

$D = \frac{x^2}{t}$ ||| $m = D\frac{\partial c}{\partial x} a^2 t$ $= x^3$

Das ist nicht möglich, es muss allgemein sein

$$\frac{m}{t} = f.(D, a, v) = f.\left(\frac{x^2}{t}, a, \frac{x}{t}\right)$$

Weiter ist klar: falls verschieden grosse Kugeln Geschwindigkeit v haben, welche ihren Radien proportional sind, und falls $D \sim a^2$, ist der Bewegungsvorgang ähnlich, es werden ähnlich situierte Punkte in denselben ~~es entfällt dieselbe Konzentration~~ Zeiten erreicht. Gradienten sind $\sim \frac{1}{a}$, Oberflächen $\sim a^2$, also Menge $m \sim a$ $\frac{a}{t}$

$$\varepsilon\frac{m}{t} = f(D\varepsilon^2, \varepsilon a, \varepsilon v)$$

$$\varepsilon\frac{m}{t} = D^\alpha a^\beta v^\gamma . \varepsilon = D^\alpha a^\beta v^\gamma \varepsilon^{\beta+\gamma+2\alpha}$$

somit $\beta+\gamma+2\alpha = 1$

$$\frac{m}{t} = D^\alpha \cdot \left(\frac{a}{v}\right)^\beta \cdot v^\gamma$$

$$\frac{x^3}{t} = \left(\frac{x^2}{t}\right)^\alpha t^\beta \cdot \frac{x}{t} \qquad \alpha = \beta = 1$$

unmöglich

Analoges Beispiel: Abkühlung einer Kugel durch eine strömende Flüssigkeit.

$$\frac{m}{t} = D^{\alpha} a^{\beta} v^{1-2\alpha-\beta}$$

$$\frac{x^3}{t} = \left(\frac{x^2}{t}\right)^{\alpha} x^{\beta} \left(\frac{x}{t}\right)^{1-2\alpha-2\beta}$$

$$1 = \alpha + 1 - 2\alpha - 2\beta \qquad \alpha = -2\beta$$

$$3 = \cancel{2\alpha} + \beta + 1 - \cancel{2\alpha} - 2\beta \qquad 2 = -\beta$$

$$\beta = -2$$

$$\alpha = 4$$

$$\frac{m}{t} = D^4 a^{-2} v^1$$

Ist Alles Unsinn! $\frac{m}{t}$ kann gar nicht <u>einer Potenz von v proportional sein</u>, da es für $\lim v = 0$ einen endlichen Wert behält!

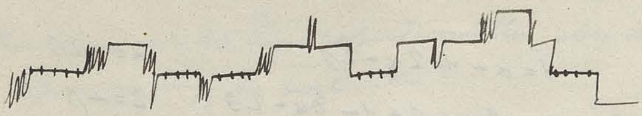
für $\lim v = 0$ haben wir $\frac{m}{t} \sim aD$

Es könnte also z.B. sein:

$$\boxed{\frac{m}{t} = aD \cdot fc.\left(\frac{D}{av}\right)} \qquad m = aD\left(1 + \frac{av}{D}\right)$$

Grenzwerte von $P_n(\pm 1)$ für grosse n

$P_n(+1)$ ist natürlich Funktion der Intervallzeit



es definiert also die Wahrsch. (bezogen auf alle Punkte des n Zustandes) dass zur Zeit t bereits ein $(n+1)$ herrscht

~~Somit erhält man die mittlere Dauer des n Zustandes indem man~~

~~$\int_0^\infty t\, P_n(\pm 1)_t \, dt$ bildet insofern nur Eintreffen in Betracht kommt~~

$P_n(+1) + P_n(-1)$ = Wahrsch., dass zur Zeit t ein $(n+1)$ oder $(n-1)$ herrscht

$P_n(\pm 0)$ definiert die Wahrsch., dass zur Zeit t ~~noch der unveränderte~~ derselbe Zustand n herrscht

Also wenn man von allen möglichen n Punkten sich t Intervalle abschneidet, so wird davon der Bruchteil $P_n(+0)$ noch auf dieselbe n Linie fallen ; $\frac{dP_n(\pm 0)}{dt}$ würde also die Anzahl bedeuten welche aufhören sich in n zu befinden

Also erhält man mittlere Dauer des n Zustandes $= \int t \frac{d}{dt} P_n(0)\, dt$

Das wäre unrichtig, denn es werden in dem Falle auch die Werte (t) mitgezählt, wo bereits eine Unterbrechung durch einen $\frac{n+1}{n-1}$ ten Zustand eingetreten ist.

Also wenn man die Wahrsch. haben will, dass während Zeit t der Zustand n unverändert andauert ist von $P_n(0)$ noch der Bruchteil abzuziehen, der die Wahrsch. angibt, dass ~~bis zur~~ innerhalb der Zeit t bereits ein $\frac{(n+1)}{(n-1)}$ erschienen ist, (und dann wieder n gekommen ist.

also: $P(n_0 (n\pm 1)_t, n_t)$

~~Ist das aber $P_n(\pm 1)$~~ ?

wohl nicht, denn $P_n(\pm 1)$ umfasst nur die Zustände, wo zur Zeit t ein $(n+1)$ herrscht, aber nicht jene ~~Zustände, wo ein $(n+1)$ zwar erschienen~~, aber wieder einer andern Zahl Platz gemacht hat.

~~Also ist $P_n(+1)$ zu vermehren um die Anzahl der Zustände wo zuerst n war, sodann (n+1) dann was anderes n (n+2)~~

Somit braucht man doch die ~~Statist~~ Kenntnis der Anzahl von Tripletts etc

Vielleicht gelingt aber die (genäherte) Berechnung für grosse n?

Vielleicht könnte man sich so helfen, dass man $P_n(0) - T_\infty(n)$ verwendet, was allerdings auch noch zu gross ist — ~~aber~~

$$P_n(0) = e^{-\nu P}\Big[\underbrace{\tbinom{n}{0}(1-P)^n + \tbinom{n}{1}(1-P)^{n-1}P\frac{\nu P}{1!} + \tbinom{n}{2}(1-P)^{n-2}P^2\frac{(\nu P)^2}{2!}}_{S} + \dots \tbinom{n}{n}(1-P)^0 P^n\frac{(\nu P)^n}{n!}\Big]$$

$$\tbinom{n}{0}(1-P)^n x^n + \tbinom{n}{1}(1-P)^{n-1}x^{n-1}P + \tbinom{n}{2}(1-P)^{n-2}x^{n-2}P^2 + \dots \tbinom{n}{n}(1-P)^0 x^0 P^n = \big[x(1-P)+P\big]^n = \Phi(x)$$

$$1 + \frac{\nu P}{1!}x + \frac{(\nu P)^2}{2!}x^2 + \dots \frac{(\nu P)^n}{n!}x^n + \dots = F(x) = e^{\nu P x}$$

$$\Phi(x)\, F(x) = A + Bx^1 + Cx^2 + \dots + Sx^n + Tx^{n+1} + \dots$$

$$\frac{d^n(\overbrace{\Phi F}^{X_n})}{dx^n}\Bigg|_{x=0} = n!\, S$$

$$\frac{d}{dx}\Big\{e^{\nu P x}\big[x(1-P)+P\big]^n\Big\} = e^{\nu P x}\Big\{\nu P\big[x(1-P)+P\big]^n + n(1-P)\big[x(1-P)+P\big]^{n-1}\Big\}$$

$$= \nu P\, X_n + n(1-P)X_{n-1}$$

$$\frac{d^2}{dx^2} \qquad = (\nu P)^2 X_n + 2\nu P\, n(1-P)X_{n-1} + \big[n(1-P)\big]^2 X_{n-2}$$

$$\frac{d^3}{dx^3} \qquad = (\nu P)^3 X_n + 3(\nu P)^2 n(1-P)X_{n-1} + 3\nu P\big[n(1-P)\big]^2 X_{n-2} + \big[n(1-P)\big]^3 X_{n-3}$$

für $x=0$: $X_n = P^n$ ~~stimmt~~ $(\nu P)^n P^n + \tbinom{n}{1}(\nu P)^{n-1} n(1-P)P^{n-1} + \tbinom{n}{2}(\nu P)^{n-2}P^{n-2}\big[n(1-P)\big]^2$

aber das nützt nichts!

$$\lim \tbinom{n}{k}(1-P)^{n-k}P^k\frac{(\nu P)^k}{k!} = \left(\frac{\nu P^2}{1-P}\right)^k\frac{n!}{(n-k)!\,(k!)^2} = \left(\frac{P^2}{1-P}\right)^k\frac{\left(\frac{n}{e}\right)^n \nu^k\sqrt{2n\pi}}{\left(\frac{n-k}{e}\right)^{n-k}\left(\frac{k}{e}\right)^k\left(\frac{k}{e}\right)^k\sqrt{2(n-k)\pi}\,.\,2k\pi}$$

$$= \left[\frac{\nu P^2 e}{(1-P)k^2}\right]^k \frac{1}{\left(1-\frac{k}{n}\right)^{(n-k)}}\cdot\frac{1}{2k\pi}\sqrt{\frac{n}{n-k}}$$

$$\underline{e^{-\nu P}} \left[\frac{\nu P^2 e}{(1-P)k^2}\right]^k \frac{(n-k)^k}{(1-\frac{k}{n})^n} \frac{1}{2\pi k} \sqrt{\frac{n}{n-k}} \quad (1-P)^n$$

$$\cancel{\lim \left(1-\frac{k}{n}\right)^n} \qquad \frac{n^n}{(n-k)^{n-k}}$$

Besser getrennt:

für kurze Zeiten, wo P sehr klein ist

$$P(0) = e^{-\nu P}\left[(1-P)^n + \binom{n}{1}(1-P)^{n-1}\frac{\nu P^2}{1} + \binom{n}{2}(1-P)^{n-2}\frac{(\nu P^2)^2}{2!} + \dots \right]$$

$$= e^{-\nu P}\Big[1 - \binom{n}{1}P$$
$$+\binom{n}{2}P^2 + \binom{n}{1}\nu P^2$$
$$-\binom{n}{3}P^3 - \binom{n}{1}\binom{n-1}{1}\nu P^3$$
$$+\binom{n}{4}P^4 + \binom{n}{1}\binom{n-1}{2}\nu P^4 + \binom{n}{2}\frac{\nu^2 P^4}{2!}$$

$$= \cancel{e^{-\nu P}}\Big[1 - nP$$
$$+\frac{n(n-1)+2n\nu}{2}P^2$$
$$-\frac{n(n-1)(n-2) - 6n(n-1)\nu}{6}P^3$$
$$+\frac{n(n-1)(n-2)(n-3) + 12n(n-1)(n-2)\nu + 6n(n-1)\nu^2}{24}P^4\Big]\Big[1-\nu P + \frac{\nu^2 P^2}{2} - \dots$$

$$= 1-(n+\nu)P$$
$$+\frac{n(n-1)+2n\nu + \nu^2 + 2n\nu}{2}P^2 \quad \dots$$

$$= 1-(n+\nu)P + \frac{n(n-1)+4n\nu+\nu^2}{2}P^2 + \dots$$

für lange Zeiten, wo P beinahe gleich 1:

$$P_n(0) = e^{-\nu P}\frac{(\nu P^2)^n}{n!}\left[1 + \binom{n}{1}\frac{1-P}{\nu P^2} + \binom{n}{2}\frac{1}{n(n-1)}\left(\frac{1-P}{\nu P^2}\right)^2 + \binom{n}{3}\frac{1}{n(n-1)(n-2)}\left(\frac{1-P}{\nu P^2}\right)^3 + \cdots\right]$$

$$= e^{-\nu P}\frac{(\nu P^2)^n}{n!}\left[1 + \binom{n}{1}^2\frac{1-P}{\nu P^2} + \binom{n}{2}^2 2!\left(\frac{1-P}{\nu P^2}\right)^2 + \binom{n}{3}^2 3!\left(\frac{1-P}{\nu P^2}\right)^3 + \cdots\right]$$

$$\neq e^{-\nu P}\frac{(\nu P^2)^n}{n!}\underbrace{\left[1 + n^2\frac{1-P}{\nu P^2} + \frac{n^4}{2!}\left(\frac{1-P}{\nu P^2}\right)^2 + \frac{n^6}{3!}\left(\frac{1-P}{\nu P^2}\right)^3 \cdots\right]}_{e^{\frac{n^2(1-P)}{\nu P^2}}}$$

$$\neq e^{-\nu P + \frac{n^2(1-P)}{\nu P^2}}\frac{(\nu P)^n}{n!} =$$

$$= e^{n-\nu + \frac{n^2}{\nu}(1-P)}\left(\frac{\nu}{n}\right)^n\frac{1}{\sqrt{2\pi n}} = \frac{e^{-\nu}\nu^n}{n!}\cdot e^{\frac{n^2}{\nu}(1-P)}.$$

für genügend lange Zeiten muss sich die Exponentielle entwickeln lassen:

$$\lim P_n(0) = \frac{e^{-\nu}\nu^n}{n!}\left[1 - \frac{n^2}{\nu}(1-P) + \cdots\right]$$

Dann ist also:

$$F = \frac{e^{-\nu}\nu^n}{n!} - P_n(0) \neq \frac{e^{-\nu}\nu^n}{n!}\cdot\frac{n^2}{\nu}(1-P)$$

Frage ob $\int^\infty F\,t\,dt$ endlich ist?

Es scheint nämlich evident zu sein, dass der Anfangszustand n eine Nachwirkung hinterlässt, dass daher bei dieser Berechnungsart, wo $P_n(0) - W_\infty(n)$ verwendet wird, ein zu grosses Resultat erscheint: Wenn also dieses endlich ist, so ist das wirkliche I a fortiori endlich

$$\int(1-P)\,t\,dt =$$

$$\int^\infty t\,dt\left[\frac{2}{\sqrt{\pi}}\underbrace{\int_0^{\frac{1}{2\sqrt{Dt}}} e^{-y^2}dy}_{} - \frac{1}{\beta\sqrt{\pi}}\left[1-e^{-\beta^2}\right]\right] = \int^\infty t\,dt\left\{\frac{\cdots}{\sqrt{Dt}}\right\} = \infty \;!!$$

$$\frac{2}{\sqrt{\pi}}\int\left(1-y^2+\frac{y^4}{2}\right)\cdots = \frac{2}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{3}+\frac{\beta^5}{10}\right] - \frac{1}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{2}\right] = \frac{1}{\sqrt{\pi}}\beta$$

$$P = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\beta} e^{-y^2}dy + \frac{1}{\beta\sqrt{\pi}}\left[1 - e^{-\beta^2}\right] \qquad \beta = \frac{h}{2\sqrt{Dt}}$$

$$\frac{dP}{dt} = \frac{dP}{d\beta}\frac{d\beta}{dt} = \left[-\frac{2}{\sqrt{\pi}}e^{-\beta^2} - \frac{1}{\beta^2\sqrt{\pi}}\left[1-e^{-\beta^2}\right] + \frac{2\beta}{\beta\sqrt{\pi}}e^{-\beta^2}\right]\frac{d\beta}{dt}$$

$$= -\frac{1-e^{-\beta^2}}{\beta^2\sqrt{\pi}}\frac{d\beta}{dt}$$

für kleine β:

$$\lim \frac{dP}{dt} = -\frac{1-\left(1-\beta^2+\frac{\beta^4}{2!}-\cdots\right)}{\beta^2\sqrt{\pi}}\frac{d\beta}{dt} = -\frac{1}{\sqrt{\pi}}\left[1-\frac{\beta^2}{2}\right]\frac{d\beta}{dt}$$

$$= +\frac{1}{\sqrt{\pi}}\left[1-\frac{h^2}{4Dt}\right]\frac{h}{4\sqrt{Dt^3}}$$

$$\int t\,\frac{d(1-P)}{dt}\,dt = \int^{\infty}\frac{dt}{\sqrt{t}} = \infty \;!!$$ Also ist diese Berechnungsweise nicht brauchbar

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Es handelt sich also um die Wahrscheinlichkeit, dass ein „n-Zustand" noch während der Zeit t unverändert weiter andauert.

Das ist die Wahrsch., dass keines der n Moleküle innerhalb dieser Zeit austrete und kein neues hereinkomme. Denn eine Compensation der ein und austretenden ist da äusserst unwahrscheinlich, da sie nur dann gelten würde, falls der Ein und Austritt in genau denselben Zeitmomenten stattfänden. Dies ist ~~ff~~ unendlich wenig wahrscheinlich.

Nun gibt P die Wahrsch. an, dass ein anfangs irgendwo im betrachteten Volum befindliches Molekül zur Zeit t sich ausserhalb jenes Volumens befinde, aber die Wahrsch. dass es schon einmal ausgetreten (eventuell wieder zurückgekommen sei) ist ganz anders, bedeutend grösser.

Würde es sich nur um die Möglichkeit einseitigen Austrittes handeln, so wäre die Frage schon ~~erl~~ erledigt durch frühere Überlegungen (Formel $\frac{n}{\mu 2^{\mu}}\binom{\mu}{\frac{\mu-n}{2}}$)

Hier ist aber die Komplikation, dass der Austritt beiderseitig stattfinden kann (oder auch einseitig, wenn dann noch Reflexion an der anderen festen Wandfläche angenommen wird)

Wenn man die Sache makroskopisch behandelt, so entspricht dies der Diffusionsverteilung in einer Schichte, welche beiderseits constant auf der Konzentration 0 erhalten wird.

~~Als Probe nehmen wir noch die einseitige Verteilung vor.~~

$a_{n\mu}$ gibt die Wahrsch. an dass ein Teilchen aus einer Lage, welche um n Elemente von der Grenzfläche entfernt ist, die Grenzfläche zum ersten Male in der Zeit (μ Intervalle) erreicht

Die Wahrsch., dass ein Teilchen, aus einer beliebigen Anfangslage links von der Grenzfläche, dieselbe zum ersten Male in der Zeit μ erreicht, ist also (wenn ein Teilchen auf ~~[illegible]~~ 2 Distanzen entfällt) $= \sum a_n = A_\mu$

dessen Grenzwert im Falle grosser μ berechnet wird $\lim = \frac{1}{\sqrt{2\mu\pi}}$

dies ist also die relative Anzahl der in ~~[illegible]~~ 2 Intervallen durchtretenden Teilchen

Andererseits würde die übliche Diffusionstheorie liefern

$$c = c_0 + (c_1 - c_0)\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{Dt}}} e^{-y^2}\,dy$$

$$\left.\frac{\partial c}{\partial x}\right|_{x=0} = \frac{2(c_1-c_0)}{\sqrt{\pi}}\,\frac{1}{2\sqrt{Dt}}\,\left. e^{-\frac{x^2}{4Dt}}\right|_{x=0} = \frac{c_1-c_0}{\sqrt{\pi D t}}$$

Dabei ist jedoch der Unterschied, dass für $t=0$ die übliche Theorie ∞ geben würde, während der wirkliche Wert endlich bleibt

Durchtretende Menge $= \frac{c}{\sqrt{\pi D t}}$

Am einfachsten mittels Reihenentwicklung

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$

$$\int_0^l \left[\cos (k+m)\frac{\pi x}{l} - \cos(k-m)\frac{\pi x}{l}\right] dx = 0$$

$$\int_0^l \sin^2\frac{k\pi x}{l}\, dx = \frac{1}{2}\int\left(1 - \cos\frac{2k\pi x}{l}\right) dx$$

$$= \frac{l}{2}$$

$$u = \sum A_k e^{-\frac{Dk^2\pi^2}{h^2}t} \sin\left(\frac{k\pi x}{h}\right)$$

$$\int_0^l \sin\frac{k\pi x}{l}\, dx = -\cos\frac{k\pi x}{l}\Big|_0^l \cdot \frac{l}{k\pi}$$

$$= 0 \quad (k = 2m)$$

$$= \frac{2l}{k\pi} \quad (k = 2m+1)$$

$$\sum A_k \sin\frac{k\pi x}{l} = 1$$

$$A_k = 0 \quad (k = 2m)$$

$$A_k = \frac{4}{k\pi} \quad (k = 2m+1)$$

$$\frac{\partial u}{\partial x}\Big|_0 = \frac{4}{h}\sum e^{-\frac{Dk^2\pi^2}{h^2}t}$$

$$u = \frac{4}{\pi}\sum_{k=1,3,5..}^{\infty} \frac{1}{k} e^{-\frac{Dk^2\pi^2}{h^2}t} \sin\frac{k\pi x}{h}$$

~~in derselben Weise verschwindet~~ Gesammt Inhalt:

$$\int_0^h u\, dx = \ldots \int_0^h \sin\frac{k\pi x}{h} dx = \frac{h}{k\pi}\cos\frac{k\pi x}{h}\Big|_0^h = \frac{2h}{k\pi} \quad k \text{ ungerade}$$

$$= 0 \quad k \text{ gerade}$$

$$\overline{U} = \frac{8}{\pi^2}\sum_{k=1,3,5..}^{\infty} \frac{1}{k^2} e^{-\frac{Dk\pi^2}{h^2}t}$$

also müsste gelten $1 = \frac{8}{\pi^2}\sum\frac{1}{k^2}$

zur Zeit t

in derselben Weise nimmt die Wahrsch. ab, dass ein Teilchen (die Grenzfläche noch nicht erreicht

$$\int_0^\infty t\frac{\partial \overline{U}}{\partial t} dt = \int_0^\infty t \cdot \frac{8}{\pi^2}\sum \frac{D\pi^2}{h^2} e^{-\frac{Dk\pi^2}{h^2}t} = \frac{8D}{h^2}\int_0^\infty \sum t e^{-\frac{Dk\pi^2}{h^2}t} dt$$

$$\int_0^\infty x e^{-\alpha x} dx = \frac{1}{\alpha^2}$$

$$T = 8D \sum_{k=1,3,5\ldots} \left(\frac{h^2}{Dk^2\pi^2}\right)^2 = \cancel{\sum \frac{8h^2}{k^2\pi^2} = \frac{8h^2}{\pi^2}\left(\frac{1}{1} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\right)}$$

$$= \frac{8}{D}\frac{h^2}{\pi^4}\left[1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots\right]$$

Könnte die Rechnung nicht auch so gehen:

$$\left.\frac{\partial u}{\partial x}\right|_h = \frac{4}{h}\sum_{k=1,3,5\ldots}' e^{-\frac{Dk^2\pi^2}{h^2}t} \cos\frac{k\pi x}{h}\Big|_h = \sum' \frac{4}{h}(-1)^k e^{-D\frac{k^2\pi^2}{h^2}t}$$

~~Menge~~ ausgetretene Gesamt-Menge:

$$M = \left[-\left.\frac{\partial u}{\partial x}\right|_0 + \left.\frac{\partial u}{\partial x}\right|_h\right] D = D\sum_{k=1,3,5\ldots}' \frac{8}{h} e^{-D\frac{k^2\pi^2}{h^2}t}$$

$$\int_0^t M\,dt = \sum' \frac{8D}{h}\cdot\frac{h^2}{Dk^2\pi^2}\left(1 - e^{-\frac{Dk^2\pi^2}{h^2}t}\right) = \frac{8h}{\pi^2}\left\{\sum' \frac{1}{k^2} - \sum' \frac{1}{k^2} e^{-\frac{Dk^2\pi^2}{h^2}t}\right\}$$

$$= h - \frac{8h}{\pi^2}\sum \frac{1}{k^2} e^{-\frac{Dk\pi^2}{h^2}t}$$

und übrig bleibender ~~Menge~~ Bruchteil $= \frac{8}{\pi^2}\sum' \frac{1}{k^2} e^{-\frac{Dk\pi^2}{h^2}t}$ so wie früher!

Wenn also n Mol. in dem betrachteten Raume enthalten sind, so ist die Wahrsch., dass irgend eines davon austritt, n mal grösser

Wahrsch., dass ~~irgend eines~~ bis zur Zeit t ~~schon~~ noch keines ausgetreten sei, ist dann die combinierte ~~W = ...~~ Wahrsch. für das gleichzeitige Eintreten von n Ereignissen: dass weder das erste, noch das zweite, noch das dritte usw.... ausgetreten sei

$$= \left[\bar{U}\right]^n$$

$$T = n\int_0^\infty t\left[\bar{U}\right]^{n-1}\frac{dU}{dt}\,dt = \cancel{t\left[\bar{U}\right]^n}\Big|_0^\infty - \int_0^\infty \left[\bar{U}\right]^n dt$$

das ist aber jedenfalls desto kleiner je grösser n

$dW = -W_2(1-\overline{W})\,dt$

Man könnte meinen, es sei $\sum_2$

$W = 1 - n(1-\overline{U})$ $\sum_2$ das wird aber nur dann sein, wenn der Fall wo 2 angetreten

---

Nun ist noch die Möglichkeit des Eintrittes zu berücksichtigen

~~Die~~ Wahrsch. dass ein Teilchen (aus beliebigen Anfangslage) durch die linke Grenzfläche eintrete ist (in der Zeit t -- $t+dt$) (falls die Teilchendichtigkeit ν beträgt) $= \frac{\nu}{\sqrt{\pi D t}}\,dt$

Wahrsch. dass ein solcher durch die linke oder rechte Grenzfläche trete $= \frac{2\nu}{\sqrt{\pi D t}}\,dt$

Bei jener Berechnungsart sind also implicite alle möglichen Molekular-Anordnungen (mit der n Zahl) berücksichtigt, und zwar jede mit solcher Häufigkeit, als es ihrem Vorkommen im stationären Zustand entspricht, also entspricht dies dem Ausdrucke

$$\boxed{T = \tau\,\frac{N_1 + (1+2)N_2 + (1+2+3)N_3 + \dots}{N_1 + 2N_2 + 3N_3}}$$

~~Man~~ Man kann nämlich zweierlei Wahrsch. für die Gesamt-Dauer eines Zustandes entwickeln: 4).

~~Wir~~ Wenn wir wissen, dass einer gewissen molekularen Konstellation (für die n Zahl) eine Dauer t entspricht

~~Also~~ so handelt es sich noch um die Wahrsch. jener Konstellation, welche die Dauer t nach sich zieht.

Jene Wahrsch. kann entweder gemeint sein als:

1). relative Häufigkeit jener Konstellation, in Bezug auf die Gesamtzahl aller Konstellation

$$= \frac{n_k}{n_1 + n_2 + n_3 + \dots}$$

oder als

2). relative Häufigkeit, in Bezug auf das Vorkommen im stationären Zustand, gemessen danach, wie wahrsch. man eine solche Konstellation im station. Zustand antreffen dürfte, wobei dann sämmtliche innerhalb der Zeit t ablaufenden Zustände nur einmal gezählt werden

$$= \frac{N_k}{N_1 + N_2 + N_3 + \dots N_k + \dots}$$

~~$W(k) = \frac{N_1 + 2N_2 + 3N_3 + \dots}{M_1 + 2M_2 + 3M_3 + \dots}$ $\frac{}{N_1 + 2N_2 + 3N_3 + \dots}$~~ $= \frac{T_3}{T_4 (\Theta)}$

Im ersten Falle erhält man als mittlere Wiederkunftszeit:

$$T = \frac{N_1 + 2N_2 + 3N_3 + \dots}{N_1 + N_2 + N_3 \dots} \tau$$

Im zweiten Falle die mittlere Dauer:

$$T = \frac{N_1 + 2^2 N_2 + 3^2 N_3 + 4^2 N_4 + \dots}{N_1 + 2N_2 + 3N_3 + 4N_4 + \dots} \tau$$

Früher hatten wir ~~aufgestellt~~ noch eine dritte ~~Auffassungsmöglichkeit erörtert~~

3). relative Häufigkeit in Bezug auf d. Vorkommen im stationären Zustand
berücksichtigt wird dass bei längerem t der betreffende Zustand häufiger anwesend ist ~~angetroffen wird~~
wobei alle innerhalb t ablaufenden Zustände berücksichtigt werden

Daraus könnte man bilden

$$W(k) = \frac{kN_k}{N_1 + 2N_2 + 3N_3 + 4N_4 \dots} \quad \Bigg\| \quad \frac{T}{2} = \frac{N_1 + 2N_2 + 3N_3 \dots}{N_1 + N_2 + N_3} = \frac{1}{\sum \frac{1}{k} W(k)} = \frac{1}{\int \frac{1}{t} W(t)\, dt}$$

4). Dieser letzte Ausdruck ist nahe verwandt mit der „Nachdauer" eines Zustandes oder der Erwartungszeit für d. Eintritt d. ~~entgegeng.~~ Zustandes bei Gleichberechtigung sämtlicher Ausgangspunkte im stationären Zustand

$$= \frac{\sum k W(k)}{\sum W(k)} = \sum k W(k)$$

Denn ~~\#~~ 3) gilt

$$T_3 = \frac{N_1 + 2^2 N_2 + 3^2 N_3 + \dots}{N_1 + 2N_2 + 3N_3} \tau \qquad = \frac{\cancel{\int \dots dt}}{\cancel{\int \dots dt}} = \int t\, W(t)\, dt = \frac{\int t^2 f(t)\, dt}{\int t f(t)\, dt}$$

also in Falle ~~[illegible]~~ kurzer Intervalle τ und dementsprechend grosser Zahlen k gilt:

$$T_3 = \frac{1}{2} T_4$$

$$T_4 = \frac{N_1 + (1+2)N_2 + (1+2+3)N_3}{N_1 + 2N_2 + 3N_3} \tau \qquad = \frac{1}{2} \frac{\int t^2 f(t)\, dt}{\int t f(t)\, dt}$$

$$\sum (1 + 2 + 3 + \dots n) = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \quad \Bigg\| \quad = \frac{1}{2}\left\{\frac{N_1 + 2^2 N_2 + 3^2 N_3}{\dots} + \frac{N_1 + 2N_2 + 3N_3}{\dots}\right\}$$

Genau aber gilt:

$$T_4 = \frac{1}{2}(1 + T_3)$$

Die vorher berechneten $\bar{U}$ sind nun wohl identisch mit den $W(t)$-Werten

Damit stimmt, dass wir haben

$$\int_0^\infty \frac{1}{t} W(t)\,dt = \infty$$ also $T_2 = 0$ wie schon von vornherein zu erwarten

Berechnet wurde die mittlere Dauer T_3 (oder T_4 ?)

Nun kann man aber nicht setzen

$$\Theta_3 = \frac{T_3}{W(n)}$$

denn $$\Theta_3 \overset{\circ}{W}(n) = \frac{M_1+(1+2)M_2+(1+2+3)M_3+}{M_1+2M_2+3M_3+\dots} \qquad \frac{N_1+2N_2+3N_3}{M_1+2M_2+3M_3} \dots$$

$$T_3 = \frac{N_1+(1+2)N_2+(1+2+3)N_3+\dots}{N_1+2N_2+3N_3+\dots}$$

Es wäre nur dann richtig, falls gelten würde:

$$\frac{M_1+(1+2)M_2+(1+2+3)M_3}{(M_1+2M_2+3M_3+\dots)^2} = \frac{N_1+(1+2)N_2+(1+2+3)N_3}{(N_1+2N_2+3N_3+\dots)^2}$$ ~~[illegible]~~ was wohl unrichtig ist

---

Statistik der 0 bei Svedberg

N_1	N_2	N_3	N_4	N_5
38	18	8	2	1

$111 : 67 = 1.66$
44
4

$1 : 0.679 = 1.47$
321
49
two.

38
36
24
8
5
111

Statistik der 1

$1 : 0{.}646 = 1{.}55$

354
31

	N_1	N_2	N_3	N_4
16	4 2 3 2 5	11111	1111	11111
11	1 5 3 2	111111	1111	
8	11 111111	1111111111		
12	111111111111	1111		
27	7 4 3 3 2 4 4			
74		25	8	5

74
50
24
20

168 : 112 = 1.5
56

$1 : 0{.}722 = 1{.}38$

278
61

2

	N_1	N_2	N_3	N_4
12	7, 2, 1, 11	1 4 1 1 1	11111	1
10	10	11111111		
15	2, 5 6 2	111111		
18	4 2 4 6 2			
12	2 8 2			
67		22	5	1

67
44
15
4

130 : 95 = 1.37
35
65

$1 : 0{.}815 = 1{.}23$

185
22

3

	N_1	N_2	N_3
20	3 7 2 5 3	1111	11
25	2 2 4 5 9 3	1121	
45		9	2

45
18
6

69 : 56 = 1.23
13
18

$1 : 0{.}889 = 1{.}12$

111
22

4

	N_1	N_2	N_3	N_4
	10	111		1
	5, 7			
	22	3	0	1

22
6
4

32 : 26 = 1.23
6

Es handelt sich also bei Bestimmung der T_3 (T_4) Dauer für Wiederkehr um die Wahrscheinlichkeit, dass ein „Nicht n-Zustand" innerhalb des Zeitraumes t Intervalls zum ersten Male einem „n Zustand" Platz macht.

Wir kennen zwar die Wahrsch., dass ein „n Zustand" .... einem „Nicht n Zustand" Platz macht aber von vornherein ist da kein Zusammenhang ersichtlich?

Sollte nicht wegen der Stationarität eine solche Beziehung herrschen, dass:
die Häufigkeit der Fälle, wo innerhalb der Zeit $t \; (\ldots t+dt)$ ein „n-Zustand" zum ersten Male in „Nicht n Zustand" übergeht, ist gleich gross wie umgekehrten Vorgang?

In diesem Falle müsste die Beziehung gelten:

Wahrsch. (eines n Zustands) × Wahrsch (Übergang des n → Nicht n) = Wahrsch (Nicht n) × Wahr (Nicht n → n)

Sicher ist das richtig, wenn man die Worte „zum ersten Male" weglässt — aber darauf kommt es eben an! (das ist einfach die Behauptung von d. Umkehrbarkeit der Zeitfolge)

Unter dieser Hypothese hätte man also:

$$W(m \to n)_t \, dt = W(n \to m)_t \, dt \, \frac{W(n)}{W(m)} = \frac{e^{-\bar{\nu}} \bar{\nu}^n}{n!} \frac{1}{1 - \frac{e^{-\bar{\nu}} \bar{\nu}^n}{n!}} W(n \to m)_t \, dt$$

~~und daraus würde folgen~~ ~~Die mittlere Dauer des „n Zustands" ist nun~~

~~$\int_0^\infty t \, W(m \to n)_t \, dt \neq$~~

~~Da nun $W(n)$, $W(m)$ von der Zeit unabhängig sind, würde das die Folgerung nach sich ziehen, dass die mittlere Dauer des seltenen Zustandes grösser wäre als die des häufigen Zustands.~~

Nun muss aber auf jeden Fall: $\int_0^\infty W(n \to m)_t \, dt = 1$ sein und ebenso: $\int_0^\infty W(m \to n)_t \, dt = 1$

das ist also unmöglich!

~~Es gilt also die Häufigkeit des Übergangs.~~

Es ist eben nur im Allgemeinen gültig: $\underbrace{\text{Häufigkeit } m \to n}$ = $\underbrace{\text{Häufigkeit } n \to m}$

~~darunter~~ Übergang zum ersten, zweiten, dritten Mal (innerhalb t ...) ....

$$\frac{d}{dt}\left[P_n(0)\right] dt = \text{„Wahrsch., dass der } n \text{ Zustand in der Zeit } t \ldots t+dt \text{ in } m \text{ übergeht“}$$

$$= dt \sum (W_1 + W_2 + W_3 + \ldots)$$

zum ersten, zweiten, dritten, ... Male innerhalb der Zeit t

aber die einzelnen $W_k(m \to n)$ und $W_k(n \to m)$ sind ganz verschieden.

W. Wahrsch., dass $\overset{m}{7}$ zum ersten Male in $\overset{n}{2}$ übergeht,

das heisst, dass zur Zeit $t=0$ eine 7 anwesend war

und dass innerhalb $t \ldots t+dt$ eine 2 erschienen ist, aber vor t ~~[illegible]~~ noch niemals 2 erschienen ist.

das kann nur in der Weise geschehen, dass um 5 mehr austreten als eintreten.

also z.B. 6 rote austreten, 1 weisse eintritt, und zwar muss 1. weisse irgendwann eintreten und zum ersten Mal

~~Es können aber~~ in der Zeit $t+dt$ 6 rote austreten

Es können aber auch solche Vorgänge stattgefunden haben:

Dann sind 7 Transporte und zum ersten Male

~~Im allgemeinen müssen aber irgendwann innerhalb t ...~~

$7r$
$6r + 1w$
$5r + 2w$
...
$0r + 7w$
$0r + 6w$
$0r + 5w$
⋮
$0r + 3w$
$0r + 2w$

Im Allgemeinen kann der Eintritt „des ersten Males“ geschehen ~~ist~~ entweder

durch die Bewegung der roten oder der weissen Teilchen

also m geht zum ersten Mal in n über: entweder so dass irgendwann k rote austreten, und innerhalb $t \ldots t+dt$ .. die Zahl $n-m+k$ weisse eintritt, während bis t immer die Zahl der eingetretenen kleiner war als $n-m+k$

Das genügt aber nicht, denn es konnte 2 bereits erreicht werden, als bloss 5 rote ausgetreten ~~[illegible]~~ und $n-m+k-2$ eingetreten waren.

Während, dass $\overset{n=(n-1)}{\cancel{n}}$ zum ersten Mal in $\overset{n}{\cancel{\ldots}}$ übergeht, wobei wir der Zeit t alle möglichen kleinen [illegible]

$(n-1)z + w$

~~Wenn~~ Nur so, dass zu den $(n-1)$ten ein weiteres dazukommt, und zwar erst im Zeitpunkt t –

oder auch ~~dass~~ $(n-1)^z \rightarrow (n-2)z \rightarrow +1w \rightarrow +1w$

oder $(n-1)z - z - z + w + w + w$

oder $(n-1)z - z + w - z + w + w$

~~oder~~ aber $(n-1)z - z + w + w - z + w$ gilt nicht

ebenso $(n-1)z + w - z - z + w - z$ nicht etc.

Vielleicht bekommt man neue Relationen durch Heranziehung der $P_n(0)$ für $t = 2, 3, \ldots \tau$

$$W(n) = \frac{N_1 + 2N_2 + 3N_3}{N_1 + 2N_2 + 3N_3 \ldots +}$$

$$P_n(0)_\tau = \frac{N_2 + 2N_3 + 3N_4 + 4N_5}{N_1 + 2N_2 + 3N_3 + \cdots}$$

$$P_n(0)_{2\tau} = \frac{N_3 + 2N_4 + 3N_5 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

$$P_n(0)_{3\tau} = \frac{N_4 + 2N_5 + 3N_6 + \cdots}{N_1 + 2N_2 + \cdots}$$

$$\varepsilon_1 = \frac{N_1 + N_2 + N_3 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots} = \frac{\tau}{T_2} = 1 - P_n(0)_\tau$$

$$\varepsilon_2 = \frac{N_2 + N_3 + N_4 + \cdots}{N_1 + 2N_2 + 3N_3 + 4N_4} = \cancel{P_n(0)_{2\tau}} \; P_n(0)_\tau - P_n(0)_{2\tau}$$

$$\varepsilon_3 = \frac{N_3 + N_4 + N_5 \cdots}{N_1 + 2N_2 + 3N_3 \cdots} = P_n(0)_{2\tau} - P_n(0)_{3\tau}$$

Nun ist aber:

$$T_2 = \frac{N_1 + 2N_2 + 3N_3 + \cdots}{N_1 + N_2 + N_3}$$

$$= \frac{1}{1 - P_n(0)_\tau}$$

$$T_4 = \frac{N_1 + (1+2)N_2 + (1+2+3)N_3 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

$$T_4 = \varepsilon_1 + 2\varepsilon_2 + 3\varepsilon_3 + 4\varepsilon_4 + \cdots$$

$$= \sum_k k\,\varepsilon_k$$

$$= 1 - P_n(0)_\tau + 2\left[P_n(0)_\tau - P_n(0)_{2\tau}\right] + 3$$

$$+ 3\left[P_n(0)_{2\tau} - P_n(0)_{3\tau}\right] + 4\left[P_n(0)_{3\tau} - P_n(0)_{4\tau}\right] + \cdots$$

$$\boxed{\frac{T_4}{\tau} = 1 + P_n(0)_\tau + P_n(0)_{2\tau} + P_n(0)_{3\tau} + P_n(0)_{4\tau} + \cdots}$$

Im Falle unendlich kurzer Intervalle τ : $\qquad t = \frac{h^2}{4D\beta^2}$

$$I_4 = \int_0^\infty P_2(0)_t \, dt \qquad\qquad \beta = \frac{h}{2\sqrt{Dt}}$$

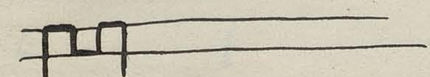
Beispielsweise ist: $P_0(0) = e^{-\nu P}$

Also
$$I_4(0) = \int_0^\infty e^{-\nu P} dt \qquad\qquad P = 1 - \frac{2}{\sqrt{\pi}}\int_0^\beta e^{-y^2} dy + \frac{1}{\beta\sqrt{\pi}}\left[1 - e^{-\beta^2}\right]$$

$$= \frac{h^2}{2D}\int_0^\infty \frac{e^{-\nu P}}{\beta^3}\, d\beta \qquad = \infty!$$

Das ist aber falsch! ! Denn ~~...~~ $P_n(0)_{2\tau} > \frac{N_3 + 2N_4 + \dots}{N_1 + 2N_2 \dots}$

weil dabei auch solche Fälle mitgezählt werden, wo unterdessen eine andere Zahl war und dann erst wieder n gekommen ist! z.B.



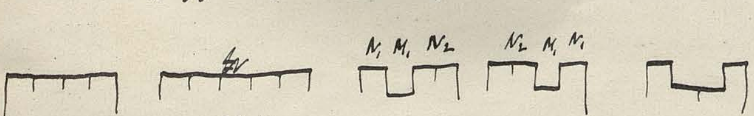
(n m n)

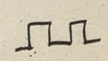
Jedes M_1 bewirkt einen solchen Zuwachs

also sollte sein: $\quad P_n(0)_{2\tau} = P(n\,n\,n) + P(n\,m\,n)$

$$= \frac{N_3 + 2N_4 + 3N_5 + \dots + M_1}{N_1 + 2N_2 + 3N_3}$$

ebenso $\quad P_n(0)_{3\tau} = P(n\,n\,n\,n) + P(n\,m\,n\,n) + P(n\,n\,m\,n) + P(n\,m\,m\,n)$



Nun sind aber dabei ~~die ...~~ nur diejenigen M_1 einzustellen, welche nach N_1 und vor N_2 oder nach N_2 und vor N_1 entfallen aber nicht jene welche z.B. vor N_1 und nach N_1 entfallen 
Da aber die Häufigkeit der M_1 von der umgebenden Konstellation abhängt, lässt sich dies nicht weiter in Rechnung stellen!

$$\cancel{\frac{1}{2^m}}\left(\frac{m}{\cancel{\frac{m-n}{2}}}\right) \quad \cancel{\frac{16}{16}} \quad \frac{64}{16} = 4 = m$$

25
45
10
80 . 2
160 : 32 = 5 = m

$$x_m^2 = m\,\delta^2 = \cancel{2D\frac{t}{m}}$$

$$\cancel{\frac{D\tau}{D+\ldots} = D\frac{\delta}{\ldots}}$$

$$x_t^2 = \frac{t}{\tau}\delta^2 = 2Dt$$

$$\cancel{D \sim \frac{x^2}{t}}$$

$$D = \frac{\delta^2}{2\tau}$$

$$\sqrt{\frac{2\tau}{t\pi}} = \delta\sqrt{\frac{1}{Dt\pi}} = \qquad \frac{1}{\cancel{\delta}}\cancel{\delta}\sqrt{\frac{1}{2\pi t\tau}}$$

$$\frac{1}{\tau}\sqrt{\frac{2}{m\pi}} = \sqrt{\frac{2}{t\tau\pi}}$$

$$\int_0^\infty e^{-\alpha t}\,dt = \frac{1}{\alpha}$$

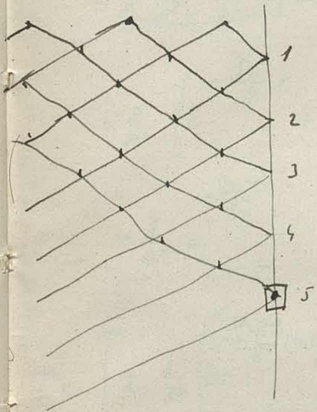
$$\sum \frac{8D}{h}\,\frac{h^2}{Dk^2\pi^2} = \frac{8h}{\pi^2}\sum\frac{1}{k^2}$$

$$\cancel{1+\alpha+\alpha^2+\ldots = \frac{1}{1-\alpha}} \qquad 1 + \frac{1}{9} + \frac{1}{25} + \ldots$$

Wahrsch. dass erst beim n ten Wurf zum ersten Mal ein Teilchen (oder mehr) durch die Grenzfläche durchgeht ist gleich dem Produkt der Wahrsch. dass

$= \sum$

W. Anordnung



für $n = 5$

$$\frac{7}{256}\left[1-\frac{1}{8}-\frac{3}{32}-\frac{5}{128}\right] \cancel{+\frac{3}{128}\left[1-\right.} \left[1-\frac{1}{32}-\frac{5}{128}\right]\left[1-\frac{1}{128}\right]$$

$$+\frac{7}{128}\left[1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}-\frac{5}{128}\right]\left[1-\frac{1}{32}-\frac{5}{128}\right]\left[1-\frac{1}{128}\right]$$

$$+\frac{5}{128}\left[1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}-\frac{5}{128}\right]\left[1-\frac{1}{8}-\frac{3}{32}-\frac{9}{128}\right]\left[1-\frac{1}{128}\right]$$

$$+\frac{7}{4\cdot 128}\left[\quad\cdot\quad\right]\left[\quad\cdot\quad\right]\left[1-\frac{1}{32}-\frac{5}{128}\right]$$

$$+\frac{1}{512}\left[\quad\cdot\quad\right]\left[\quad\cdot\quad\right]\left[\quad\quad\right]\left[1-\frac{1}{128}\right]$$

---

für $n = 1 \quad : \frac{1}{2}$

$n = 2 \quad : \frac{1}{8} + \frac{1}{8}\left[1-\frac{1}{2}\right] = \frac{3}{16}$

$n = 3 \quad : \frac{1}{16}\left[1-\frac{1}{32}\right] + \frac{3}{32}\left[1-\frac{1}{2}-\frac{1}{8}\right] + \frac{1}{32}\left[1-\frac{1}{2}-\frac{1}{8}\right]\left[1-\frac{1}{32}\right]$

$$= \frac{1}{32\cdot 16}\left[31 + 3\cdot 6 + 6\cdot\frac{31}{32}\right] = \frac{1}{32\cdot\cancel{16}\,8}\left[31\cdot\frac{\cancel{38}\,19}{32} + \frac{9}{\cancel{18}}\right]$$

$$\begin{array}{r} 31 \\ 279 \\ 288 \\ \hline 877 \end{array}$$

$$= \frac{877}{8\cdot 32\cdot 32}$$

$W(t)\,dt = n\frac{dP}{dt}dt\; P^{n-1} = \frac{d}{dt}(P^n)\,dt$

$$\rho = \frac{h}{2\sqrt{Dt}}$$

$$\lim_{\substack{h=0\\ \frac{n}{h}=c}} \frac{d}{dt}(P^n) \qquad P = 1-\varepsilon$$

$$P^n = (1-\varepsilon)^n = (1-\varepsilon)^{\frac{n\varepsilon}{\varepsilon}} = e^{-n\varepsilon}$$

$$\lim_{\rho\to\infty} P = \cancel{\ldots}\; 1 - \frac{1}{\rho\sqrt{\pi}} \qquad \varepsilon = \frac{1}{\rho\sqrt{\pi}} \qquad P^n = e^{-\frac{n}{\rho\sqrt{\pi}}} = e^{-\frac{2n\sqrt{Dt}}{h\sqrt{\pi}}} =$$

$$\varphi = \lim P^n = e^{-2\frac{\nu}{h}\sqrt{\frac{Dt}{\pi}}} = \cancel{\ldots}\; e^{-\frac{\nu}{\rho\sqrt{\pi}}}$$

$$n\frac{dP}{dt}dt\,P^{n-1}\cdot\varphi + P^n\frac{d\varphi}{dt}dt = \frac{d}{dt}\left[P^n\cdot\varphi\right]dt$$

$$\int_0^\infty t\,\frac{d}{dt}(P^n\varphi)\,dt = \cancel{t\,P^n\varphi} - \int_0^\infty P^n\varphi\,dt \quad = \text{endlich}$$

$\sqrt{t} = x$

$dt = 2x\,dx$

denn schon $\displaystyle\int_0^\infty \varphi\,dt = \int e^{-\alpha\sqrt{t}}\,dt = 2\int_0^\infty e^{-\alpha x}\,x\,dx = +\frac{2}{\alpha^2}$

$$= \frac{\cancel{4}\,h^2\pi}{2\cancel{4}\,\nu^2 D}$$

Eigentlich, da zwei Grenzflächen:

$$T_3 = \int_0^\infty t\,\frac{d}{dt}\left[P^n\varphi^2\right]dt$$

$$= \int_0^\infty P^n\varphi^2\,dt = \int_0^\infty e^{-\frac{2n\sqrt{Dt}}{h\sqrt{\pi}} - \frac{4\nu}{h}\sqrt{\frac{Dt}{\pi}}}\,dt = \int_0^\infty e^{-\frac{(n+2\nu)}{\rho\sqrt{\pi}}}\,dt$$

$$= \frac{h^2\pi}{2(n+2\nu)^2 D}$$

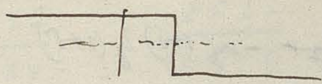
Pierwotne rozumowanie:

$\Delta n = (\nu - n)\,P = 1$

$$P = \frac{1}{\nu - n} = \frac{2\sqrt{Dt}}{h\sqrt{\pi}}$$

$$t = \frac{h^2}{4(\nu-n)^2}\frac{\pi}{D}$$

w razie $n \gg \nu$: $\quad T_3 = \frac{t_2}{2}$



$$\left[\frac{8}{\pi^2}\sum \frac{1}{k^2}\, e^{-\frac{Dk^2\pi^2}{h^2}t}\right]^n \qquad \neq \left[\cancel{\frac{8}{\pi^2}}\, e^{-\frac{D\pi^2 t}{h^2}}\right]^n \neq = e^{-\frac{D\pi^2 t}{h^2}n}$$

$$\cancel{= \frac{8}{\pi^2}\sum \frac{1}{k^2}\left(1-\frac{Dk^2\pi^2}{h^2}t - \dots\right)} = 1 - 8\sum\frac{D}{h^2}t$$

$$=\frac{8}{\pi^2}\left[e^{-\frac{D\pi^2}{h^2}t} + \frac{1}{9}\, e^{-9\frac{D\pi^2}{h^2}t} + \frac{1}{25}\, e^{-25\frac{D\pi^2}{h^2}t} \dots\right.$$

$$=\frac{8}{\pi^2}\, e^{-\frac{D\pi^2 t}{h^2}}\left[1+\frac{1}{9}\, e^{-\frac{8D\pi^2 t}{h^2}} + \frac{1}{25}\, e^{-24\frac{D\pi^2}{h^2}t}\right.$$

$$T_3 = \int_0^\infty t\,\frac{d}{dt}\left(e^{-\frac{D\pi^2 t}{h^2}(n+\nu)}\right)dt = \int_0^\infty e^{-\frac{D\pi^2 t(n+\nu)}{h^2}}\,dt = \frac{h^2}{D\pi^2(n+\nu)}$$

1 : 49 = 1.0204
020
1 / 81 = 12345
190
280
37
1 : 121 =

1 1 1 1 1
400
2041
12345
12

$$p + 2p^2 + 3p^3 + \dots = p\frac{\partial}{\partial p}\left(1+p+p^2+\dots\right)$$

$$= p\frac{\partial}{\partial p}\frac{1}{1-p} = \frac{p}{(1-p)^2}$$

$$\lim = \frac{1}{m\,2^m}\binom{m}{\frac{m-1}{2}} = \frac{1}{2^m}\,\frac{m(m-1)(m-2)\dots\dots(\frac{m}{2}+\frac{1}{2}+1)}{\left(1\cdot2\cdot3\cdots\frac{m-1}{2}\right)^2\frac{m+1}{2}}\left|\frac{(\frac{m}{2}+\frac{1}{2})!}{}\right. = \frac{1}{\cancel{2}}\,\frac{\left(\frac{m}{e}\right)^m\sqrt{2m\pi}}{\left(\frac{m-1}{e}\right)^{m-1}\sqrt{\pi(m-1)}\,\pi\left(\frac{m+1}{e}\right)}$$

$$= \frac{m(m-2)(m-4)\dots}{m(m+1)(m-1)(m-3)\dots}$$

$$= \left(\frac{m}{m-1}\right)^m \frac{\sqrt{m}}{m+1}\,\frac{1}{\sqrt{\pi}}\,e$$

$$= \frac{1}{\sqrt{m\pi}}$$

$$n! = \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

$$\lg W = -\nu \cancel{+ \nu(1+\delta) \lg \nu} + \nu(1+\delta) \cancel{- \nu(1+\delta) \lg \nu} - \nu(1+\delta) \lg(1+\delta)$$

$$- \lg \sqrt{2\pi n}$$

$$= \nu\delta - \nu(1+\delta)\left(\delta - \frac{\delta^2}{2} + \frac{\delta^3}{3}\right) \quad - \lg \sqrt{\ }$$

$$= \nu\delta - \left[\nu\delta - \nu\frac{\delta^2}{2} + \nu\frac{\delta^3}{3} + \nu\delta^2 - \nu\frac{\delta^3}{2} \dots\right.$$

$$= -\nu\frac{\delta^2}{2}$$

0·3247 . $\frac{9}{8}$
29232
3654 . $\frac{9}{16}$
~~32886~~
16443
6166 $\frac{3}{8}$
56
69368 $\frac{9}{32}$
53
216
1953 $\frac{9}{40}$
14647 $\frac{3}{8.62}$
144
24
87

00	·3247 .	359
01	·3654	404
02	·2055	227
03	·077075	8·525
04	·02170	2·40
05	·048825	0·54
06	·00915	0·10

7708
7708
962
8525

217
217
1302
240

488
488
29
5397

915
915
55
1012

216.512
1080
216
43
110·6

3247
325
719
~~3768~~
3591

3654 .
3654
219 .
4043

2055
2055
123
2273

324.512
1620
32
6
1658

$\nu P e^{-\nu\delta} . e^{-\nu}$

$P e^{-\nu P} . e^{-\nu}\nu$

216. $\frac{31}{2}$
108
324
335 . $\frac{31}{4}$

1005
10385 :4 = 2596 . $\frac{31}{6}$
7788
80476 :6 = 1341

1341 . $\frac{31}{8}$
4023
41571 :8 = 5196

52. $\frac{31}{10}$
156
161

416
1248
129 :14 =
3 | 92

161
483
499 :12
416

3247
8527
22729
649
162
26
2357
8561
2357
1414
118
18
39.1

2357
8527
16499
471
118
18
17106

1711
8527
11977
342
86
13
12418

1242
8527
8694
248
62
10
9014

7258 . 9
653

2596 .
215
12980
260
52
132.92

1341
215
6705
134
27
68.66

52 . 512
2560
1024
26.6

805
161
322
8.24

2080
416
83
2.13

512 . 324
1536
1024
204
16589

512 . 335
1536
1536
256
1715

2357
5171
2357
1649
24
11
40.41

1711
9231
1711
513
34
15
22.73

1242
786
745
99
8
8.52

901 . 266
2394

2.4

6.5 . 8.2
520
1
0.53

171·5

(11)
354
2478
35
17
6·5·4

(21) 133
322
966
~~4~~97
42·8

(31) 687
280
1374
850
19·24

(41) 26·6
237
~~253.1~~
26·3. 24
526
105
6·3

(51) 8·2
197
1·6

(22)
278
8~~3~~4
83
37·0

(32)
290
1374
6183
19·92

(42)
287
2~~7~~3. 28
546
2184
7·64

(52)
274
2192
55
2·25

185

(33) 687
5496
343
12·71

214
532
27
11
5·7

33
234

1872
46
1·99

266 p
2394
264. 24
528
105
63

~~2~~
264. 29
528
2376
7·65

111
266
27
3
2·96
02

0·6·8

139
1112
28
1·14

212. 268
424
1272
17
5·7

266
266
26
2·85

266
1064
798
27
1147

n=	0	1	2	3	4	5	
n=0	45 / 39.9	35 / 40.4	19 / 22.7	7 / 8.5	5 / 2.4	0 / 0.5	0.1 | 110.5
1	40 / 40.4	55 / 65.4	40 / 42.8	17 / 19.2	10 / 6.3	1 / 1.6	175.7
2	19 / 22.7	42 / 42.8	35 / 37.0	24 / 19.9	6 / 7.6	2 / 2.2	132.2
3	6 / 8.5	23 / 19.2	22 / 19.9	13 / 12.7	5 / 5.7	0 / 1.9	67.9
4	2 / 2.4	8 / 6.3	10 / 7.6	4 / 5.7	6 / 2.9	2 / 1.1	~~25.7~~ 26.0
5	0 / 0.5	1 / 1.6	2 / 2.2	2 / 1.9	0 / 1.1	0 / 0.7	8.0
6							

0	1	2	3	4	5	
216	335	260	134	052	016	004

168 : 518 = 3.24
126
23

n = 17
ν = 1.55 } 15.45

$$W(17) = \frac{e^{-\nu}\nu^n}{n!} = \frac{e^{-\nu}\nu^n}{\left(\frac{n}{e}\right)^n\sqrt{2n\pi}} = e^{n-\nu}\left(\frac{\nu}{n}\right)^n\frac{1}{\sqrt{2n\pi}}$$

0.4343.
2172
~~19~~6
6.711

0.19033
1.23045
(0.95988 − 2) . 17
671916
1.6218 − 34
6.711.
8.3428 − 34
− 2.0287
6.3141 − 34 =

$\sqrt{27.6859}$

~~0.615~~
1.2305
.3010
.4972
2.0287

16.318
6.711
23.029 − 34
− 2.029
21 − 34 = 13.

$10^{13}\tau = 5.10^5$ Jahr

~~86400 . 39~~
365 . 24 . 60 . 39
= 36 . 24 . 6 . 4 . 10^3
= 9 . 23 . 10^4 = 2.10^7

Kann man die Diskontinuitäten annehmen:

eines Zustandes n   Elongation
Wiederkehrzeit = Zeit, in welcher die durchschn. maximal Abweichung n beträgt

~~Die~~ Wäre

Poisson-Ausgleich   wahrscheinlichste ~~…~~ Maxim.-Abw. $\frac{x}{(\xi)} = \sqrt{2\lg n} = 2{\cdot}15\sqrt{{}^{10}\log n}$

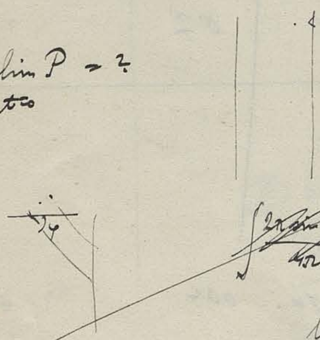
$$n = e^{\frac{x^2}{2\xi^2}}$$

$$dW = \frac{2}{\sqrt{\pi}}\, e^{-\frac{x^2}{2}}\, \frac{dx}{\sqrt{2}}$$

Wieviel diffundiert innerhalb $\lim \tau = 0$ kurzen Zeiten aus der h Schichte nach aussen?

falls Zeit so kurz, dass die Wege als geradlinig betrachtet werden:

$\lim\limits_{t=0} P = ?$



$$P = \cancel{\dots}\ \frac{\nu_x}{\sqrt{\pi} h}\, t = \frac{v\, t}{2\, h \sqrt{6\pi}}$$

Stosszahl pro cm² und sek.

$$\cancel{\int \frac{2\pi \sin\varphi\, d\varphi}{4\pi} \cos\varphi\ \delta\Omega}$$

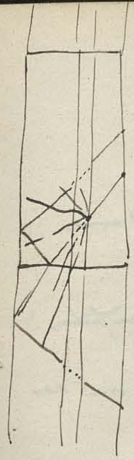
$$\lim T_1 = \frac{h}{W(n)}\,\frac{\tau}{2 v \tau} = \frac{h}{2v}\,\frac{1}{W(n)}$$

Unsinn Verwechslung von P und $P_n(0)$!

$$\lim \Theta_1 = \frac{T_1}{W(n)}$$

$$\lim W_n^{(32)}(0) = 1-(\nu+n)P = 1-(\nu+n)\frac{v\, t}{h\sqrt{6\pi}} \qquad \lim T_1 = \frac{h\sqrt{6\pi}}{2(\nu+n)v}$$

lim T_1 muss allerdings von v oder m und nicht nur von D abhängen; denn D ist gleich gross für verschiedene schwere Teilchen vom gl. Radius; dieselben haben aber sehr verschiedene Geschwindigkeiten und die rascheren müssen häufiger hin und her schwanken als die langsameren, also müssen die rascheren ein kürzeres T_1 verursachen!



Verdünntes Gas

Falls Stoßstand gegeben ist, aber alle Richtungen gleich möglich, wird ~~[illegible]~~ die mittlere ununterbrochene Verweilungsdauer in der Schichte a unendlich gross

$$t = \frac{a}{v\cos\varphi}$$

$$\bar{t} = \frac{1}{2\pi}\int_0^{\frac{\pi}{2}} 2\pi \sin\varphi \, d\varphi \, \frac{a}{v\cos\varphi} = \frac{a}{v} \log\cos\varphi \Big|_{\frac{\pi}{2}}^{0} = \infty$$

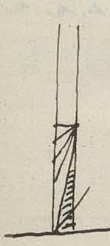
Dabei war vorausgesetzt, dass die Wände regelmässig reflektieren.

Wenn aber die Richtung der reflektierten Bahn ganz zufällig ist (~~[illegible]~~ rauhe Wände)

Sobald nur die beiden parallel-Wände rauh sind, ändert sich nichts

Dagegen ändert sich die Rechnung, falls das Gefäss auch seitlich durch reflectierende Wände begrenzt ist

$$\Sigma > \overbrace{\frac{a}{\sqrt{a^2+b^2}}}^{c}\left(\frac{b}{2} + \frac{a}{\sqrt{a^2+b^2}}\left(\frac{b}{2} \cdots = c + c^2 + c^3 \cdots = \frac{c}{1-c}\right.\right.$$



Es wird $\bar{t} = -\frac{a}{v} \log \frac{a}{\sqrt{a^2+b^2}} + \Sigma$ entsprechend dem gestrichenen Raume

für $\lim a = 0$ wird $\bar{t} = 0$ (?); das Verhältnis $\lim \frac{\bar{t}}{a} = \infty$

aber $\sum_a \bar{t}$ über das ganze Gefäss $= \infty$

Dasselbe gilt auch für die Br. B., denn immer lässt sich die Zeit so kurz wählen, dass im ersten Moment alle Geschw. Richtungen gleich wahrscheinlich sind und alle Bahnstücke als Gerade betrachtet werden können.

Somit nähert sich das Verhältnis $\frac{t}{a}$ keinem bestimmten endlichen Grenzwert; es wächst im Laufe der Zeit fortwährend an.

(innerhalb gewisser Zeit erreichten)

Klar ist der Begriff d. durchschn. Maximal elongation (verschiebung) aus der Anfangslage

aber nur für astatische Systeme, X

sonst muss auch noch d. Verhältnis d. Anfangs- zur Normallage bekannt sein.

Kann man für statische Systeme eine „mittlere (M) durchschn. Maximal elongation (oder) " einführen?

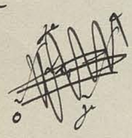
Allerdings, indem man die Anfangslagen nach d. Wahrscheinl. Funktion verteilt und das Mittel der betreffenden Max. El. nimmt

Das ist eben das was ich (~~Gött~~ Vort.) mit der Svedberg'schen Zahlenreihe gemacht habe

$M = f(t)$ so dass zugleich $M = 0$ $t = 0$

Dagegen ist für die Verdichtungszeiten ~~[illegible]~~ endlich für alle Argumente [illegible]

$$\left(\sqrt{\frac{\pi}{\alpha}}\right)^n \cdot e^{-\alpha x_1^2} dx_1 \; e^{-\alpha (x_2 - x_1)^2} dx_2 \; e^{-\alpha (x_3 - x_2)^2} dx_3 \cdots\cdots e^{-\alpha (x_n - x_{n-1})^2} dx_n = W(x_1, \dots dx_1, x_2 \dots \dots)$$



$$\int_{y'}^{\infty} dx_1 \int_{-\infty}^{y} dx_2 \int_{-\infty}^{y} dx_3 \cdots \int_{-\infty}^{y} dx_n + \int_{-\infty}^{y} dx_1 \int_{y}^{\infty} dx_2 \int_{-\infty}^{y} dx_3 \int_{-\infty}^{y} dx_4 \; + -$$

Meine Rechnung Gött. V. würde sich auf folgenden Fall beziehen: wenn wiederholt

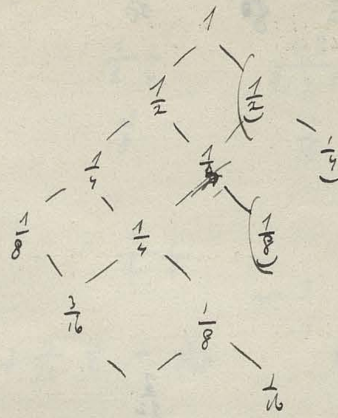
M.B. beobachtet in Sekunden Intervallen, dann 2 Sek I., 3 Sek. Interv., 4 Sek Interv. etc

sodann Maximum gesucht zwischen.
successive

δ_1	δ_2	δ_3	δ_4	max
1.1	1.5	1.7	1.9	1.9
0.9	1.6	1.8	1.7	1.8
1.4	1.4	1.6	2.1	2.1
0.8	1.5	1.7	2.3	2.3
1.0	1.5	1.9	1.8	1.9
1.0	1.6	..		

Mittel . . .

1



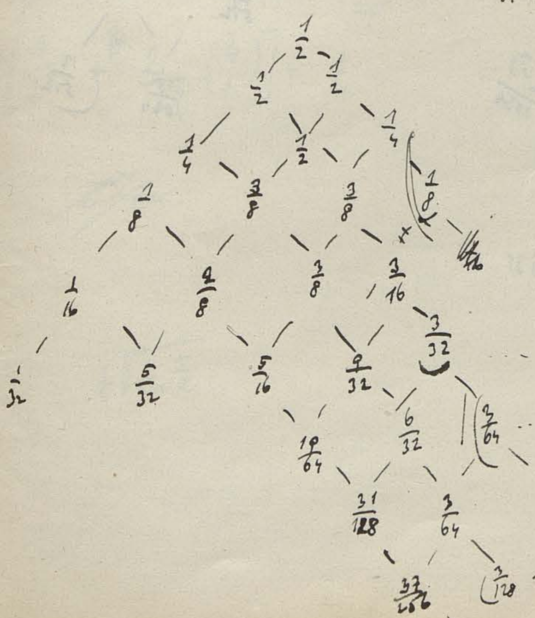
$$\frac{1}{m}\,\frac{1}{2^m}\binom{m}{\frac{m-1}{2}} = \frac{1}{m}\;\frac{m(m-1)(m-2)\cdots\overbrace{\cancel{\left(m-\frac{m-1}{2}+1\right)}}^{\frac{m+3}{2}\cdot\frac{m+1}{2}}\quad\frac{m-1}{2}\cdots 3\cdot 2\cdot 1}{\left(1\cdot 2\cdot 3\cdots\frac{m-1}{2}\right)^2\cdot\frac{m+1}{2}}$$

$$2\cdot 2\cdot 2\cdots\quad 2\quad\cdot\quad 2$$

$$= \frac{m!}{m\left[2\cdot 4\cdot 6\cdots(m-1)\right]^2(m+1)} = \frac{(m-1)!}{\left[2\cdot 4\cdot 6\cdots(m-1)\right]^2}\,\frac{1}{m+1}$$

$$\frac{3}{16} = \frac{1}{2}\,\frac{1\cdot 3}{2\cdot 4}$$

$$= \frac{1\cdot 3\cdot 5\cdots(m-2)}{2\cdot 4\cdot 6\cdots(m-1)}\,\frac{1}{m+1}$$



3/32

$m-k=1$ 3/64

$m-k=2$ 3/64

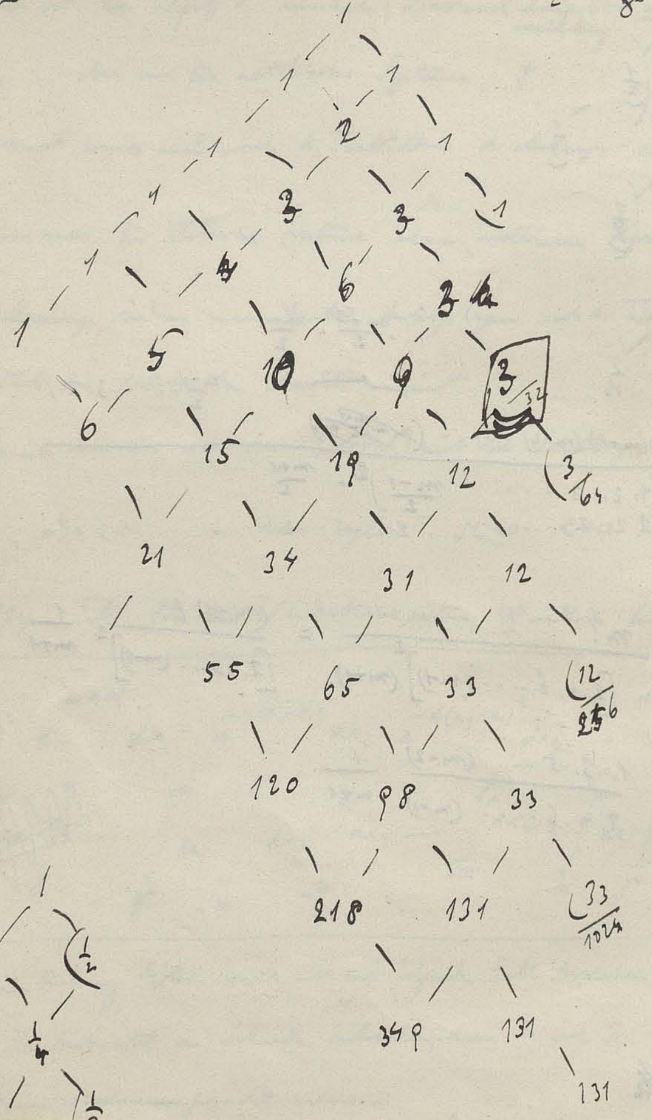
$m-k=3$ 3/128

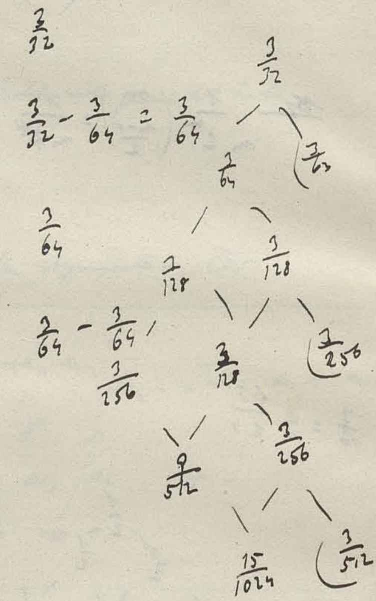
$m-k=4$ 3/128

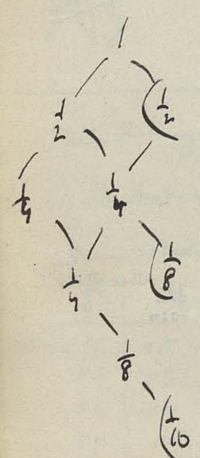
$$\frac{3}{128} - \frac{37}{1024}$$

$m =$ ~~$\frac{1}{2}$~~ $=$

$\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{16}$ $\underline{1}$







$n = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$\times \qquad \frac{1}{2}\frac{1}{4} \qquad \frac{1.3}{2.4.6}$

$\frac{2}{8} \qquad \frac{1}{16}$

$\frac{1}{2}\frac{1}{4}$

$n=3 \quad m=7$

$\frac{1}{8}\left[\frac{1}{2}-\frac{1}{8}\right] + \frac{3}{32}\,\frac{1}{2} + \frac{9}{128} \qquad \binom{7}{2}\frac{1}{2^7}$

$\frac{3}{64} + \frac{3}{64} + \frac{9}{128} = \frac{21}{128} \qquad \frac{7.6}{1.2}\cdot\frac{1}{2^7}$

$21 \quad 8.16$

$\frac{1}{16}\,\frac{3}{8} + \frac{1}{16}\,\frac{1}{2} = \frac{1}{16}\,\frac{7}{8} = \frac{7}{128} \qquad n=4 \quad m=7 \qquad \frac{\partial}{\partial\alpha}\left(\sqrt{\frac{\pi}{\alpha}}\right) = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$

$n=3) \quad 5\,\frac{1}{8} + 1\,\frac{3}{8} = 1$

$$\sqrt{\overline{\Delta^2}} = \frac{\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}dx}{\int_{-\infty}^{\infty} e^{-\alpha x^2}dx} = \sqrt{\frac{\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}}{\sqrt{\frac{\pi}{\alpha}}}} = \frac{1}{\sqrt{2\alpha}}$$

$$2\left[\binom{3}{1}\frac{1}{2^3} + 3\cdot\frac{1}{2^3} = \frac{6}{8}\right] = \frac{3}{2}$$

$\alpha x^2 = z \qquad e^{z}dz$

$2\alpha x\,dx = dz$

$$2\left[\frac{4}{2^4}\cdot + \frac{2}{2^4}\binom{4}{1}\right] = \frac{3}{2}$$

$$|\overline{\Delta}| = \frac{\int_0^\infty x\, e^{-\alpha x^2}dx}{\int_0^\infty e^{-\alpha x^2}dx} = \frac{\frac{1}{2\alpha}}{\frac{\sqrt{\pi}}{2\sqrt{\alpha}}} = \cancel{\ldots} = \frac{1}{\sqrt{\alpha\pi}}$$

$\cancel{\int e^{-x^2}dx}$

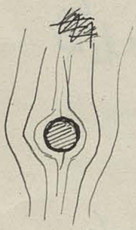
$$|\overline{\Delta}| = \sqrt{\frac{2}{\pi}}\sqrt{\overline{\Delta^2}}$$

$\frac{1}{t}\sqrt{\frac{1}{2\pi}\frac{\tau}{t}} \qquad 2\frac{1}{8}\sqrt{\frac{8^2}{2\tau}\frac{1}{t\pi}}$

$$\frac{1\cdot 3\cdot 5\cdot 7 \cdots (n-1)}{2\cdot 4\cdot 6 \cdots n} = \frac{n!}{2^{n}\left(\frac{n}{2}!\right)^{2}} = \frac{\left(\frac{n}{e}\right)^{n}\sqrt{2n\pi}}{\left[\left(\frac{n}{2e}\right)^{\frac{n}{2}}\sqrt{n\pi}\right]^{2} 2^{n}} = \frac{\sqrt{2n\pi}}{n\pi} = \sqrt{\frac{2}{n\pi}}$$

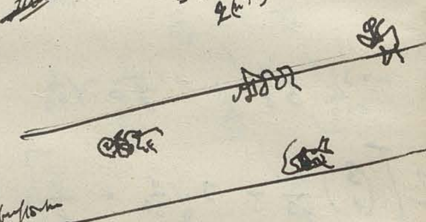
$$\frac{\partial c}{\partial x} = \frac{c_0}{\sqrt{\pi}}\, e^{-\frac{x^2}{4Dt}}\, \frac{1}{\sqrt{Dt}}$$

$$e^{-\frac{\beta(x-x_0 e^{-\beta t})^2}{2D(1-e^{-2\beta t})}}\; e^{-\frac{\beta x_0^2}{2D}} = e^{-\frac{\beta}{2D}\frac{\overbrace{(x-x_0 e^{-\beta t})^2 + x_0^2(1-e^{-2\beta t})}^{x^2-2xx_0 e^{-\beta t}+x_0^2}}{1-e^{-2\beta t}}}$$



$F = k(R, \eta,$

daraus würde die ~~mittlere~~ durchschnittl. Dauer eines n Zustandes folgen $= \frac{h\sqrt{6\pi}}{2(n+1)C}$



Wie viel Zahlenwechsel finden pro Sek statt?

So zu berechnen wie die Stosszahl in Gastheorie für die ganze Oberfläche

Von jenen Zahlenwechseln ist der Prozentsatz $W(n)$ ein günstiger

Also nach Analogie mit Scintillationen wäre die durchschn. Erwartungszeit für die Zahl n:
oder durchschn. Dauer von einer zur andern Sz.
$= \frac{h\sqrt{6\pi}}{\nu C}\frac{1}{W(n)}$

Falls die Schicht
so dünn wäre dass ein merklicher
Bruchteil der auf der einen Seite herausfallenden
auf der andern hinausgehen würde
wäre dies vielleicht ~~nicht~~ nicht unbeachtlich(?)

Einseitige Bewegung gibt Stosszahl: $\frac{\nu C}{h\sqrt{6\pi}}$

Man muss natürlich beide Bewegungsrichtungen und beide Flächen berücksichtigen, also

$$\frac{4\nu C}{h\sqrt{6\pi}}$$

Im Falle vollständiger Unabhängigkeit der Elementar-Ereignisse:

$M_1, M_2 \ldots$ = Wahrsch. für ein einfaches, doppeltes … Intervall („Nichts-n")

$$M_1 = A\, e^{-k\tau} = \alpha$$

$$M_2 = A\, e^{-2k\tau} = M_1^2 = \alpha^2$$

$$M_3 = A\, e^{-3k\tau} \ldots = M_1^3 = \alpha^3$$

$$\Theta_1 = \tau \frac{\alpha + 2\alpha^2 + 3\alpha^3}{\alpha + \alpha^2 + \alpha^3} = \tau\left[1 + \alpha \frac{\alpha + 2\alpha^2 + 3\alpha^3}{\alpha + \alpha^2 + \alpha^3 \ldots}\right] = \tau\left[1 + \alpha\left(1 + \alpha\left(1 + -\right)\right)\right] = \frac{\tau}{1-\alpha}$$

$$\Theta_3 = \tau\left[1 + \underbrace{\frac{\alpha^2 + 2\alpha^3 + 3\alpha^4 + \ldots}{\alpha + 2\alpha^2 + 3\alpha^3 + \ldots}}_{\beta}\left[1 + \beta\left[1 + \beta \ldots\right]\right]\right] = \tau\left(1 + \beta + \beta^2 + \beta^3 + \ldots\right) = \frac{\tau}{1-\beta}$$

Dann wird also wirklich $\Theta_1 = \Theta_3 = \frac{\tau}{1-\alpha} = \tau \frac{1}{1-e^{-k\tau}}$

$$1 - W_{(n)} = V_{(n)}$$

$$T_1 = \Theta_1 \frac{W_{(n)}}{1 - W_{(n)}} = \Theta_1 \frac{1 - V_{(n)}}{V_{(n)}} = \Theta_1\left[\frac{1}{V_{(n)}} - 1\right] \qquad \cancel{\frac{\alpha}{1-\alpha} = \frac{1+\beta}{\beta}}$$

$$\Theta_1 = \tau \frac{V_{(n)}}{1 - V_{(n)}} \qquad \alpha + \alpha^2 + \alpha^3 \ldots = \cancel{(1-\alpha) + (1-\alpha)^2 + (1-\alpha)^3} \ldots = \frac{1-\alpha}{\alpha}$$

$$\alpha + \alpha^2 + \alpha^3 \ldots = \frac{\alpha}{1-\alpha} = \frac{1}{1-\alpha} - 1$$

$$\alpha + 2\alpha^2 + 3\alpha^3 \ldots = \frac{\alpha}{(1-\alpha)^2} \qquad \Theta_1 = \frac{\tau}{1-M} \qquad \Theta_3 = \frac{\tau}{1-M}$$

$$P_n(0) = W_{(n)} \qquad \frac{M_1}{\Sigma(M+N)} = W(1) \qquad \frac{M_2}{\Sigma(M+N)} = W(2) = \left[W(1)\right]^2 = \left(\frac{M_1}{\Sigma(M+N)}\right)^2 \qquad W(3) = \left[W(1)\right]^3$$

$$\frac{M_1 + 2M_2 + 3M_3}{M_1 + M_2 + M_3} = \qquad M_2 = M_1^2 / \Sigma(M+N) \qquad M_3 = M_1^3 / \left(\Sigma(M+N)\right)^2$$

$W(n\,n) = P_n(0)$

$W(n\,n\,n) > P_n(0)$

$$T_3 > \tau\,\underbrace{[1+P(1+P(\ldots\;)}_{\frac{\tau}{1-P}}$$

Wenn N Teilchen verteilt auf $-\infty$ bis $+\infty$ so ist deren stationäre Dichte in der Entfernung x

$$\nu_x dx = N\sqrt{\frac{\beta}{2\pi D}}\; e^{-\frac{\beta x^2}{2D}}\; dx$$

dieselben bewegen sich in jedem Moment $\uparrow\downarrow$ mit mittl. Geschw. $\frac{\Omega}{2}$

also ist die Zahl der von unten aus die Ordinate x (pro Zeiteinheit) überschreitenden Teilchen

$\frac{\nu_x \Omega}{4}$ ebensoviel in umgekehrter Richtung

Also Wahrsch. dass ein (mit gleicher Wahrsch.) irgendwo befindliches Teilchen jene Ordinate in der Zeit dt überschreitet unten oder oben

$$= \frac{2C}{\sqrt{6\pi}}\sqrt{\frac{\beta}{2\pi D}}\; e^{-\frac{\beta x^2}{2D}}\; dt$$

Also mittlere Zeit zwischen zwei Überschreitungen: $\frac{}{2}$

$$T_1 \left(= T_3\;?\right) = \frac{\sqrt{6\pi}}{2C}\sqrt{\frac{2\pi D}{\beta}}\; e^{\frac{\beta x^2}{2D}} = \frac{\sqrt{6\pi}}{2C}\sqrt{2\pi}\;\xi\, e^{\frac{x^2}{2\xi^2}} = \frac{\pi\sqrt{3}}{C}\,\xi\, e^{\frac{x^2}{2\xi^2}}$$

$\frac{\xi^2}{D}\quad \frac{\xi}{x_0} : \frac{\xi}{C}$

$\searrow C\tau \qquad \frac{\xi^2}{x_0\tau} : 1$

Allgemein $T_1 = \frac{1}{\ldots}$? $\qquad 1 - P_n(0) = \ldots$

$W(m\,n) = ?$ $\qquad 1 - W(n\,n) = W(n\,m) \qquad T_1 = \frac{\tau}{W(n\,m)}$

$W(n) = W(m\,n) + W(n\,n) \qquad W(m\,n) + W(n\,m) = 1$

$W(n)\; P_n(0)$

$$P_m(n) = ?$$ $$W(m\,m) = P_m(0)\,W(m)$$

$$P_m(0) + P_m(n) = 1$$ $$W(n\,n) = P_n(0)\,W(n)$$

$$P_n(0) + P_n(m) = 1$$ $$W(m\,n) = W(m)\,P_m(n) = W(n)\,P_n(m) = W(n\,m)$$

$$\cancel{W(n\,m) = W(n)\,P_n(m)}$$

$$\cancel{P_m(n) = 1 - P_n(0) = 1}$$

$$P_m(n) = P_n(m)\,\frac{W(n)}{W(m)} = \left[1 - P_n(0)\right]\frac{W(n)}{W(m)} = \left[1 - P_n(0)\right]\frac{W(n)}{1 - W(n)}$$

$$\Theta_1 = \frac{\tau}{P_m(n)}$$

$$\frac{m(m-1)(m-2)\ldots\left(m-\frac{m-n}{2}+1\right)\left(m-\frac{m-n}{2}\right)!}{1.2.3\ldots\frac{m-n}{2}\quad\left(m-\frac{m-n}{2}\right)!} = \frac{m!}{\frac{m-n}{2}!\,\frac{m+n}{2}!}$$

$$\lim a_{nm} = \frac{n}{m}\;\frac{\left(\frac{m}{\cancel{e}}\right)^m\cancel{\sqrt{2m\pi}}}{\left(\frac{m-n}{\cancel{2}}\right)^{\frac{m-n}{2}}\left(\frac{m+n}{\cancel{2}}\right)^{\frac{m+n}{2}}}\sqrt{\frac{m}{\frac{m-n}{2}\,\frac{m+n}{2}}}\;\frac{1}{\sqrt{2\pi}}$$

$$= \frac{n}{m}\sqrt{\left(\frac{m-n}{m}\right)^{m-n}\cdot\left(\frac{m+n}{m}\right)^{m+n}}\;\sqrt{\frac{2}{\pi m}}$$

$$= \cancel{\frac{n}{m}\sqrt{\left(1-\frac{n}{m}\right)^m}}$$

$$(m-n)\log\left(1-\frac{n}{m}\right) = (m-n)\left(\frac{n}{m} + \frac{n^2}{2m^2} + \frac{n^3}{3m^3}\right) + (m+n)\left(\frac{n}{m} - \frac{n^2}{2m^2} + \frac{n^3}{3m^3}\right)$$

$$= \cancel{n} - \frac{n^2}{2m} + \cancel{\frac{n^3}{3m^2}} + \frac{n^2}{m} - \cancel{\frac{n^3}{2m^2}} + \frac{n^4}{3m^3} \Big\} = \frac{n^2}{m}$$

$$= \frac{n}{m}\,e^{-\frac{n^2}{m}}\sqrt{\frac{2}{\pi m}}$$

$$\cancel{\frac{n^2}{m} = x} \qquad \cancel{m = \frac{n^2}{x}} \qquad \cancel{dm = -\frac{n^2}{x^2}\,dx}$$

$$\sum_{n=0}^{\infty} a_{nm} = \int\sqrt{\frac{2}{\pi}}\,x^{-1/2}e^{-x}\,dx = \sqrt{\frac{2}{\pi}}\,\underbrace{\Gamma(1/2)}_{=\sqrt{\pi}}$$

$$\cancel{\sqrt{\frac{2}{\pi}}\int_{\frac{n^2}{m}}^{\infty}\frac{n^2}{x^2}\,dx\,\frac{x}{n}\,e^{-x}\,\frac{\sqrt{x}}{n}}$$

$$= \sqrt{\frac{2}{\pi}}\int_{\frac{n^2}{m}}^{\infty}\frac{e^{-x}}{\sqrt{x}}\,dx = \sqrt{\frac{2}{\pi}}$$

0

1

1

1 1

2 1

2 3 1

5 4 1

5 9 5 1

14 14 6 1

14 28 20 7 1

1

1

2 1

3 3 1

~~5~~ 6 4 1

10 10 5 1

20 15 6 1

35 21 7 1

$$\left(\frac{\alpha^2}{2}+x\right)^m = x^m + \binom{m}{1}\alpha^2 x^{m-2} + \binom{m}{2}\alpha^4 x^{m-4} + \dots + \alpha^{2m} x^{-m}$$

$\cancel{(1+x)^{\frac{1}{2}}}$ 24

Durch Grenzfläche ~~#~~ von Anfang an durchgetreten

$$e^{-D\pi^2 t \frac{n}{h^2}}$$

~~—~~ seitliche Dimensionen beliebig gross

Wenn nun Dichte $\frac{n}{h}$ constant angenommen wird

aber lim $h = \infty$

so müsste dies die Wahrscheinlichkeit ergeben, dass aus zwei mit gleichmässiger Dichte $\frac{n}{h}$ angefüllten Halbräumen bis zur Zeit t kein einziges Teilchen ausgetreten sei

lim $= 1$ also Unsinn

Wahrsch. dass der positive Überschuss n bei keinem der ~~k~~ k ersten Würfe aufgetreten sei = Wahrsch, dass er zum ersten Mal beim $(k+1)$ oder $(k+2)$ . . . Wurf auftreten werde (da er irgend einmal auftreten muss)

$$\cancel{n=1 \quad m=1 \quad \frac{1}{2} \quad m=3 \quad \frac{3}{8} \quad m=5 \quad \frac{5}{16}}$$

$$\cancel{\frac{5}{16}\left(1-\frac{1}{2}\right)\left(1-\frac{3}{8}\right) = \frac{5}{32}\cdot\frac{5}{8} = \frac{25}{256}}$$

$$W_{(n)k} = \sum_{m=k+1}^{\infty} a_{nm} = 1 - \sum_{m=n}^{m=k} a_{nm}$$

Wahrsch, dass keines der Teilchen bis zur Zeit mT sich an die Wand gestossen habe:

$$= W_{(1)k} \cdot W_{(2)k-1}\, W_{(3)k}\, W_{(4)k-1} \cdots$$

$$W = \frac{8}{\pi^2}\left\{ \frac{1}{1} e^{-\frac{D\pi^2}{h^2}t} + \frac{1}{9} e^{-\frac{D\pi^2}{h^2}\cdot 9t} + \frac{1}{25} e^{-\frac{D\pi^2}{h^2}\cdot 25t} + \cdots \right\}$$

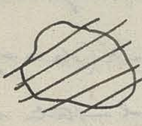
$$\alpha + \frac{\alpha^9}{9} + \frac{\alpha^{25}}{25} + \cdots$$

$$\frac{h^2}{D\pi^2 n} \qquad \cancel{D = \ldots} \quad D = v\lambda \qquad \frac{Dn\pi^2}{h^2} = \frac{v\lambda n\pi^2}{h^2}$$

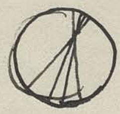
für $\lambda = h$ identisch

$$= \frac{v\nu}{h}$$

(ununterbrochene)
Mittlere (Verweilungszeit in einem Volumen



falls mittlere Weglänge sehr gross im Vergleich zu den Dimensionen des Volumens



~~Zeit~~ Durchschn. Erwartungszeit des Austrittes für ein irgendwo im Innern befindliches Molekül von der Grössenordnung $\frac{a}{v}$

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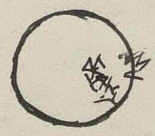
falls mittlere Weglänge klein ist: (Da ist von Unabh. keine Rede, ist doch ein Mol. austritt wird es sehr wahrsch. mehrere Male nacheinander anstossen)



wird sie dementsprechend grösser sein und zwar desto grösser je kleiner d

lässt sich berechnen aus Diffusionstheorie, wird also nur abhängen von D und a, aber nicht von v!

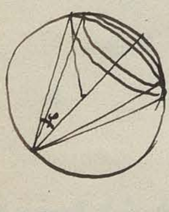
Anzahl der Durchtritts Ereignisse an der Oberfläche ~~ist in~~ (pro Zeit Einheit) beiden Fällen gleich!

(falls Molekülzahl gleich und Geschwind. gleich [die Änderung in d hervorgebracht durch Änderung der Molekül dimensionen]

$= 4a^2\pi n v \ldots$

Also durchschnittl. Dauer einer Anzahl $= \frac{1}{4a^2\pi n v}$

Stosszahl auf die (innere) Oberfläche eines kugelförmigen Gefässes:



Die Oberfläche sei diffus reflektierend, also jede ~~Be~~ Bewegungsrichtung gleichberechtigt?

Wahrsch. einer Richtung $\varphi - \varphi + d\varphi$

$$\frac{2\pi \sin\varphi \, d\varphi}{2\pi}$$

Anzahl der Stösse welche solche Mol pro Sek ausüben:

$$\nu_\varphi = \frac{v}{2a\cos\varphi}$$

$$\cancel{\frac{\partial \rho_0}{\partial x} = -\alpha x} \qquad \frac{\partial^2(\rho-\rho_0)}{\partial x^2} = D\frac{\partial(\rho-\rho_0)}{\partial t}$$

$$\cancel{\rho_0 = -\frac{\alpha x^2}{2} + \beta}$$

$$\frac{\partial^2 \rho}{\partial x^2} + \alpha = D\frac{\partial \rho}{\partial t} \quad ?$$

Also Gesamtzahl der Stösse pro Sek. welche durch ein im Innern befindliches Mol erzeugt wird:

$$\nu = \frac{v}{2a}\int_0^{\frac{\pi}{2}} \frac{\sin\varphi\, d\varphi}{\cos\varphi} = \frac{v}{2a}\,\lg\cos\varphi\Big|_{\frac{\pi}{2}}^{0} = \infty \;!$$

Also ist eine solche Bewegung gar nicht möglich, es müssen die Normalrichtungen bevorzugt werden?

Dagegen ist bei kurzer mittlerer Weglänge in dichten Gasen ohneweiters

$$\nu = 4a^2\pi\,\frac{nC}{\sqrt{6\pi}} = \frac{3N}{a}\,\frac{C}{\sqrt{6\pi}}$$

N = Gesamtzahl der Moleküle in der Kugel

also entfällt auf ein Molekül: $\frac{3}{a}\frac{C}{\sqrt{6\pi}}$ also durchschnittliche Dauer: $\frac{a}{C}\frac{\sqrt{6\pi}}{3}$

Der Fehler steckt darin: gleichförmige Richtungsverteilung gilt für Raumelemente, aber nicht für Flächenelemente.

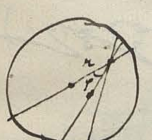
Grundlage: gleichwahrsch. ist der Aufenthalt in irgend einem Raumelement und dasselbe

Es muss hier ein Analogon zum Lambertschen Cosinusgesetz construiert werden.

Auf die Richtung φ — $\varphi + d\varphi$ entfällt

$$2\pi\sin\varphi\cos\varphi\, d\varphi\; d\sigma\; n$$

etwa was ganz das übliche zum $\frac{C}{\sqrt{6\pi}}$ führende Rechnung gibt.



Durchschn. Erwartungszeit wobei alle Bewegungsrichtungen gleich wahrsch.

für die aus einem Punkte in der Entfernung r ausgehenden Moleküle

$$\int_0^{\pi} \frac{2\pi\sin\varphi\, d\varphi}{4\pi}\cdot 2\,\frac{\sqrt{a^2 - r^2\sin^2\varphi}}{v} = -\frac{1}{v}\int^{\pi}\sqrt{a^2 - r^2 + r^2\cos^2\varphi}\; d(\cos\varphi)$$

Es wird jedenfalls einen Ausdruck von der Grössenordnung $\frac{a}{C}$ geben, welcher aber mit durchschnittl. Dauer nicht identisch sein dürfte, denn es besteht da auch keine Unabhängigkeit d. Ereignisse: wenn ein Stoss auf die Wand erfolgt, muss sicher der nächste innerhalb der Zeit $\frac{2a}{v}$ erfolgen!

Kugel :

$$\frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial r^2}$$

$$v = r\theta$$

$\theta = 1$	$t = 0$
$\theta = 0$	$r = a$
$\frac{\partial \theta}{\partial r} = 0$	$r = 0$

$$\boxed{\begin{array}{l} v = 0 \\ \cancel{\frac{\partial v}{\partial r} = r\frac{\partial \theta}{\partial r} + \theta = \frac{v}{r}} \\ \frac{\partial^2 v}{\partial r^2} = 0 \end{array}}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$

für bei $r = 0$ muss also

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{2}{r}\frac{\partial \theta}{\partial r}$$

$$= \frac{1}{r}\frac{\partial^2 v}{\partial r^2}$$

$$v = \sum A_k e^{-\frac{D k^2 \pi^2 t}{a^2}} \sin\left(\frac{k\pi r}{a}\right)$$

$\sin k\pi = 0 \qquad k = 1, 2, 3, 4 \ldots$

$\sin 0 = 0$

$$\sum_{k=1}^{\infty} A_k \sin\frac{k\pi r}{a} = r$$

$$A_k = \frac{2}{a}\int_0^a r \sin\frac{k\pi r}{a}\, dr = \frac{2}{a}\left\{-\frac{a}{k\pi} \cdot r \cos\frac{k\pi r}{a}\Big|_0^a + \frac{a}{k\pi}\cancel{\underbrace{\int_0^a \cos\frac{k\pi r}{a}\, dr}_{\sin\frac{k\pi r}{a}}}\right\}$$

$$= -\frac{2}{k\pi}\left[a \cos k\pi\right]$$

$$A_k = -\frac{2a}{k\pi} \qquad (k = 2m)$$

$$A_k = \frac{2a}{k\pi} \qquad (k = 2m+1)$$

$$\theta = \frac{2a}{\pi r}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k} e^{-\frac{D k^2 \pi^2 t}{a^2}} \sin\frac{k\pi r}{a}$$

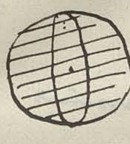
$\cancel{r = 0 \quad \theta = 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots\right)}$

mittlere Dichte : $\rho = \frac{1}{\frac{4}{3}a^3\pi}\int_0^a 4r^2\pi\,\theta\, dr = \frac{3}{a^3}\int_0^a r^2\theta\, dr$

Mittlere Dauer: $T = -\int_0^\infty t \frac{\partial \rho}{\partial t}\, dt = \cancel{-t\rho}\Big|_0^\infty + \int_0^\infty \rho\, dt$

$$\rho = \frac{3}{a^3}\,\frac{2a}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\, e^{-\frac{Dk^2\pi^2 t}{a^2}} \underbrace{\int_0^a r \sin\frac{k\pi r}{a}\, dr}_{(-1)^{k+1}\frac{a^2}{k\pi}} = \frac{6}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}\, e^{-\frac{Dk^2\pi^2 t}{a^2}}$$

$$T = 6\sum\int_0^\infty \frac{dt}{k^2\pi^2}\, e^{-\frac{Dk^2\pi^2 t}{a^2}} = \frac{6a^2}{D\pi^4}\sum_{k=1}^{\infty}\frac{1}{k^4} = \frac{6a^2}{D\pi^4}\left[1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots\right]$$



$$2\int_0^a \frac{2r\pi\, dr \cdot \sqrt{a^2-r^2}}{a^2\pi} = 4\,\frac{\left(\sqrt{a^2-r^2}\right)^{3/2}}{3a^2}\Big|_0^a = \frac{4a}{3}$$

$$\tau = \frac{4a}{3v}$$

$$\bar{\tau} = \frac{\Sigma v\tau}{\Sigma v} = \frac{4a}{3}\,\frac{\Sigma}{\Sigma v} = \frac{4a}{3\Omega}$$

$$\overline{\xi^2} = \frac{C^2}{3}$$

$$\cancel{W(\xi)\, d\xi = \frac{a}{\alpha\sqrt{\pi}}\, e^{-\frac{\xi^2}{\alpha^2}}\, d\xi}$$

$$\cancel{\overline{\xi^2} = \alpha^2 \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} y^2 e^{-y^2}\, dy}$$

$$W(\xi)\, d\xi = e^{-\alpha\xi^2}\, d\xi$$

$$\overline{\xi^2} = \frac{\int_{-\infty}^{+\infty}\xi^2 e^{-\alpha\xi^2}\, d\xi}{\int_{-\infty}^{+\infty} e^{-\alpha\xi^2}\, d\xi} = \frac{\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}}{\sqrt{\frac{\pi}{\alpha}}} = \frac{1}{2\alpha} = \frac{C^2}{3}$$

$$\alpha = \frac{3}{2}\,\frac{1}{C^2}$$

$$\cancel{\bar{\xi} = \frac{\int_0^\infty \xi e^{-\alpha\xi^2}\, d\xi}{\int_0^\infty e^{-\alpha\xi^2}\, d\xi} = \frac{\frac{1}{2\alpha}}{\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}} = \frac{1}{\sqrt{\pi}\sqrt{\alpha}} = \frac{1}{\sqrt{\pi}}\sqrt{\frac{2}{3}}\,\frac{1}{C}}$$

$$\cancel{\bar{\xi} = \sqrt{\frac{2}{3\pi}}\,\frac{1}{C}}$$

$$\bar{\tau} = \frac{4a}{3C}\sqrt{\frac{3\pi}{8}} = \frac{a}{C}\sqrt{\frac{2\pi}{3}}$$

$$\alpha v^2 = y$$

$$\int v^3 e^{-\alpha v^2}\, dv = \frac{1}{2\alpha^2}\int_0^\infty e^{-y}\, y\, dy$$

$$= \frac{1}{2\alpha^2}\left[\cancel{y e^{-y}} - e^{-y}\right]$$

$$\overline{v^2} = \frac{\int_{-\infty}^{+\infty} v^4 e^{-\alpha v^2}\, dv}{\int_{-\infty}^{+\infty} v^2 e^{-\alpha v^2}\, dv} = \frac{\frac{3}{4}\sqrt{\frac{\pi}{\alpha^5}}}{\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}} = \frac{3}{2}\,\frac{1}{\alpha}$$

$$\Omega = \frac{\int_0^\infty v^3 e^{-\alpha v^2}\, dv}{\int_0^\infty v^2 e^{-\alpha v^2}\, dv} = \frac{\frac{1}{2\alpha^2}}{\frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}} = \frac{2}{\sqrt{\alpha\pi}} = C\sqrt{\frac{8}{3\pi}}$$

$$\int dx\sqrt{1-x^2} = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsin x$$

$$4\pi\int_0^a \sqrt{a^2-r^2\sin^2\varphi}\;.\;r^2\,dr = 4\pi\frac{a^4}{\sin^3\varphi}\int\sqrt{1-\left(\frac{r\sin\varphi}{a}\right)^2}\;\left(\frac{r\sin\varphi}{a}\right)^2 d\left(\frac{r\sin\varphi}{a}\right)$$

$$= \frac{4\pi a^4}{\sin^3\varphi}\int_0^{\sin\varphi}\underbrace{\sqrt{1-x^2}\;x^2\,dx}$$

$$= \frac{x^3}{4}\sqrt{1-x^2} + \frac{1}{4}\underbrace{\int\frac{x^2\,dx}{\sqrt{1-x^2}}}_{-\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\arcsin x}$$

$$\frac{x^3\sqrt{1-x^2}}{4} - \frac{x\sqrt{1-x^2}}{8} + \frac{1}{8}\arcsin x$$

$$\frac{3x^2\sqrt{1-x^2}}{4} - \frac{x^4}{4\sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{8} + \frac{x^2}{8\sqrt{1-x^2}} + \frac{1}{8\sqrt{1-x^2}}$$

$$\frac{6x^2-6x^4-2x^4}{8\sqrt{1-x^2}} + \frac{x^2-1+x^2}{8\sqrt{1-x^2}} + \frac{1}{8\sqrt{1-x^2}} = \frac{-8x^4+8x^2}{8\sqrt{1-x^2}} = x^2\,\frac{1-x^2}{\sqrt{1-x^2}}$$

$$= \frac{\frac{4\pi a^4}{\sin^3\varphi}\left[\frac{\sin^3\varphi\cos\varphi}{4} - \frac{\sin\varphi\cos\varphi}{8} + \frac{1}{8}\varphi\right]}{\frac{4\pi}{3}a^3} =$$

$$\frac{3a}{4v}\int\Bigg[\underbrace{\frac{\sin\varphi\cos\varphi}{2} - \frac{\cos\varphi}{2\sin\varphi} + \frac{1}{2}\varphi\Big]\sin\varphi}_{\frac{\varphi\sin^2\varphi\;-\cos^2\varphi}{2\sin\varphi}}\Bigg]d\varphi$$

$\cancel{\int \sqrt{1-\alpha^2 x^2}\, x^2 dx}$

$\frac{\varepsilon r}{a} = x$

$$\int_0^a \sqrt{a^2-\varepsilon^2 r^2}\, r^2 dr = a\int_0^a \sqrt{1-\left(\frac{\varepsilon r}{a}\right)^2}\, r^2 dr = \frac{a^4}{\varepsilon^3}\int_0^\varepsilon \sqrt{1-x^2}\, x^2 dx$$

$$= \frac{a^4}{\varepsilon^3}\left\{ \frac{x^3\sqrt{1-x^2}}{4} - \frac{x\sqrt{1-x^2}}{8} + \frac{1}{8}\arcsin x \right\}$$

$x = \sin\varphi$

$$\int_0^\varphi \sin^2\varphi \cos^2\varphi\, d\varphi = \frac{1}{8}\left[\varphi - \frac{\sin 4\varphi}{4}\right] = \frac{1}{8}\left[\varphi - \frac{\sin 2\varphi \cos 2\varphi}{2}\right]$$

$$= \frac{1}{8}\left[\varphi - \sin\varphi\cos 2\varphi \cos\varphi\right] = \frac{1}{8}\left[\varphi - \sin\varphi\cos^3\varphi + \sin^3\varphi\cos\varphi\right]$$

$\cancel{= \frac{1}{8}\left[\varphi - 2\sin^2\varphi\cos^2\varphi\right]}$

$\cancel{\varphi - \frac{\varphi^3}{3!} - \varphi + \frac{16\varphi^3}{3!}}$

$\cancel{\varphi^3} \quad \cancel{= \frac{15}{3!}} \quad \cancel{\int \frac{\varphi}{\sin^2\varphi}d\varphi}$

$$= \frac{4\pi a^4}{8\,\frac{4\pi a^3}{3}} \int_0^\pi \frac{1}{\sin^2\varphi}\left[\varphi - \sin\varphi\cos^3\varphi + \sin^3\varphi\cos\varphi\right] d\varphi$$

$\cancel{\int \frac{\varphi}{\sin^2\varphi}}$ $\sqrt{m}\, c = \sqrt{3R\theta}$ $= \sqrt{3H\theta\frac{m}{\mu}} = \sqrt{\frac{3H\theta}{N}}$ $\qquad C = \sqrt{\frac{3H\theta}{NM}}$

$$C = \sqrt{\frac{3H\theta}{6.10^{23}\cdot\frac{4}{3}a^3\pi\rho}} = \sqrt{\frac{3.293.\ 8'32.10^7}{8.6.10^{23}\cdot\frac{4}{3}\pi.1'1\ .(1'9.10^{-6})^3}}$$

$\frac{\pi\Omega^2}{8} = \frac{C^2}{3}$

$\frac{9.\cancel{3.2}.\cancel{8'16}\ 1'03}{8.\cancel{17}.2.\cancel{18}.1'9} \quad \frac{10^9}{10^5} = \sqrt{\frac{9'3}{7'6}10^4} = \sqrt{1'22.10^3}$

$\frac{N\Omega}{4} = \frac{NC}{4}\sqrt{\frac{8}{3\pi}} = \frac{NC}{\sqrt{6\pi}}$

2135
854

112

$= 3'5.10^3$

$\cancel{\sqrt{6\pi}}\ \sqrt{18'8}$

2. 18'5. 26

$= \sqrt{1'22.10^2}$

$= 35\ \frac{m}{sek}$

$= \frac{1}{222} = 0'45\ 10^{-2}$

$= \frac{1}{52\sqrt{18'2}} = \frac{1}{52.4'27}$ $\sqrt{\frac{1'1}{19'2}} = \sqrt{\frac{1'14}{20}} = \frac{\sqrt{5'7}}{\sqrt{100}} = \frac{2'4}{10}$

220

$= 1'1\ \frac{m}{sek}$

$= 26\ \frac{cm}{sek}$

4'8
5'6
57'6

$$\frac{h^2}{D\Omega^2(n+\nu)} = \frac{(2.10^{-4})^2}{9{\cdot}86 . 1{\cdot}04 . 10^{-7}(n+\nu)}$$

$$\begin{array}{r} 942 \\ 3{\cdot}14 \\ \hline 1256 \\ \hline 9{\cdot}86 \end{array}$$

$$= \frac{2 . 10^{-2}}{18{\cdot}5} = 10^{-3}$$

$$C = 26$$

$$D = 10^{-7} = \frac{C\Lambda}{3}$$

$$\Lambda = \frac{3.10^{-7}}{26} = 10^{-8} \text{ cm}$$

$$\begin{array}{l} 93 \\ 31 \\ 961 : 4 = 2{\cdot}402 \end{array}$$

$$\Delta^2 = 2\nu P = 1$$

$$\downarrow$$

$$\frac{2}{h}\sqrt{\frac{Dt}{\Omega}}$$

$$\cancel{4}\sqrt{\frac{Dt}{\Omega}} \quad \cancel{4} = \left(\frac{h}{4\nu}\right)^2$$

$$t = \frac{\Omega}{D}\left(\frac{h}{4\nu}\right)^2$$

$$\frac{\Omega\,(2.10^{-4})^2}{10^{-7} . 16 . (1{\cdot}55)^2} = \frac{12{\cdot}6 . 10^{-1}}{16 . \cancel{1{\cdot}55}} = \cancel{1} . 0 \ldots = \frac{1}{30}$$

$$2{\cdot}4 \qquad \frac{1}{3{\cdot}72}$$

$$= 0{\cdot}033$$

$$\frac{\sqrt{G\Omega}}{4\nu C} = 0{\cdot}0045 . \frac{18{\cdot}5}{1{\cdot}55} \quad 12$$

$$0{\cdot}054$$

$$2\, N\, dS\, dt\, \Omega \qquad \sqrt{G\Omega}$$

$$N = \frac{(n+\nu)}{\Omega h}$$

$$= \frac{2\, dS\, dt\, (n+\nu)}{h\sqrt{G\Omega}}$$

$$\frac{10^{\cancel{4}\,7}}{\cancel{220}} . \cancel{2} . \cancel{10} . \frac{\cancel{40}}{39} . \cancel{4} . \frac{1}{60} . \frac{1}{24} = \frac{10^4}{11 . 39 . 6 . 24} =$$

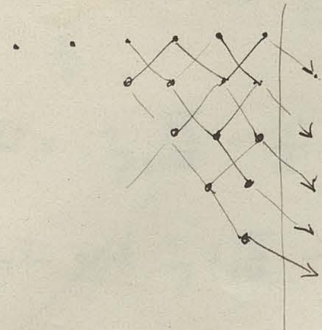
$$11$$

$$\begin{array}{r} 78 \\ 156 \\ \hline 9{\cdot}36 \quad . 6{\cdot}6 \\ 5616 \\ 562 \\ \hline 618 \end{array}$$

$$= \frac{10^3}{6{\cdot}2} = 161^{\underline{d}}$$

$$38$$

Wahrsch., dass kein Teilchen die Grenzfläche (bis t) überschritten habe:



$$W_k = \cancel{(1-a_{1m})(1-}$$

$$= \cancel{\left(1-\sum_{m=1}^{m=k} a_{1m}\right)\left(1-\sum_{m=2}^{m=k-1} a_{2m}\right)\left(1-\sum_{m=3}^{m=k-2} a_{3m}\right)\dots}$$

$$\left(1-\sum_{m=1}^{m=k}{}' a_{1m}\right)\left(1-\sum_{m=2}^{m=k-1}{}' a_{2m}\right)\left(1-\sum_{m=1}^{m=k-2}{}' a_{3m}\right)\cancel{(\dots}$$

$$\cancel{\sum_{m=k+1}^{\infty}{}' a_{1m}\cdot\sum_{m=k}^{\infty}{}' a_{2m}\cdot\sum_{m=k-1}^{\infty}{}' a_{3m}\cdot\sum_{m=k-2}^{\infty}{}' a_{4m}\cdot\sum_{m=k-4}^{\infty} a_{5m}}$$

$$\log W_k = \sum{}' \log\left(1-\sum_{m=k}^{m=k-n} a_{nm}\right)$$

$$\sum_{n=m}^{\infty} a_{nm}$$

$$\sum{}' a_{33} + a_{53} + a_{73} + \dots = \sum_{n=m}^{\infty}\frac{n}{m}e^{-\frac{n^2}{m}}\sqrt{\frac{2}{\pi m}} = \frac{1}{2}\sqrt{\frac{1}{\pi k}}\sum_{n=2k}^{\infty}\frac{n}{k}e^{-\frac{n^2}{2k}}$$

$$\cancel{\frac{n^2}{m}} \quad \cancel{m=2k}$$

$$\frac{n^2}{2k} = x \qquad \frac{1}{k} = \frac{2x}{n^2} \qquad = \frac{1}{2\sqrt{\pi}}\int_{\frac{n^2}{2k}}^{\infty}\frac{dx}{x}\sqrt{\frac{x}{2}}\,e^{-x}$$

$$-\frac{n^2}{2k^2}\,dk = dx = -\frac{dk}{k}\cdot x$$

$$= \frac{1}{2\sqrt{2\pi}}\int_{?}^{\infty} x^{-1/2}e^{-x}\,dx$$

$$\frac{dW_k}{dk} =$$

$$\sum_{m=k-n}^{\infty} a_{nm} = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{\frac{n^2}{2k}}}^{\infty} e^{-z^2}dz = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{\frac{n^2}{m}}}^{\infty} e^{-z^2}dz.$$

$$\cancel{= \frac{1}{\sqrt{2\pi}}\int_{\sqrt{\frac{n^2}{k-n}}}^{\infty} e^{-z^2}dz \mp \frac{1}{\sqrt{2\pi}}\,\frac{e^{-\frac{n^2}{k-n}}}{2\frac{n}{\sqrt{k-n}}}}$$

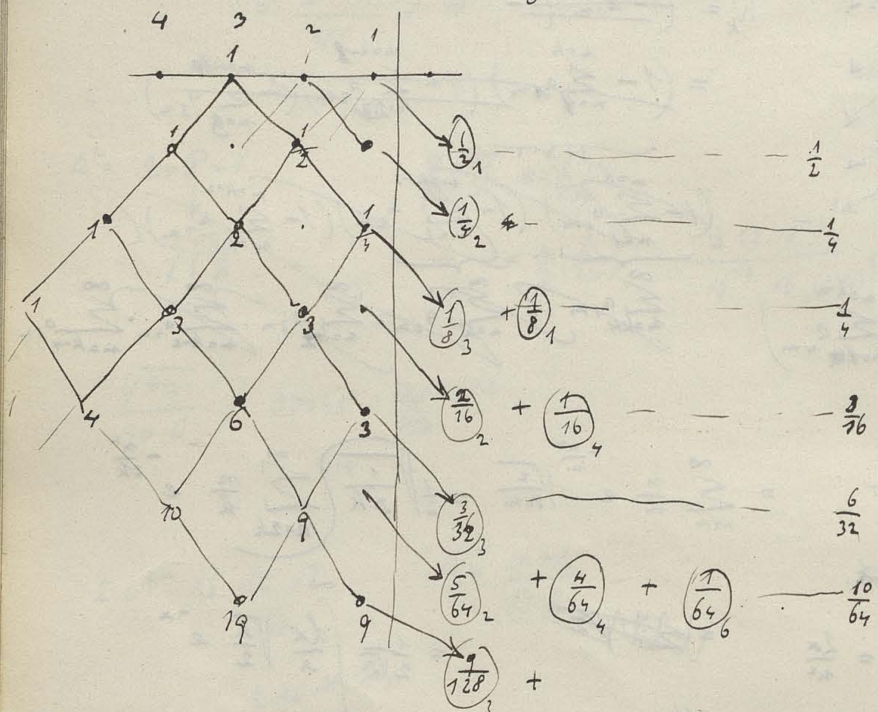
$$\cancel{1 < n < k}$$

$$\cancel{\int_0^{dk}\left[\log\frac{1}{\sqrt{2\pi}} - \frac{n^2}{k-n} - 2\log\frac{n}{\sqrt{k-n}}\right]dn}$$

$$\frac{n^2}{k-n} = z \qquad nz + n^2 = kz$$

$$n = -\frac{z}{2} \pm \sqrt{kz + \frac{z^2}{4}}$$

$$\int_0^k \frac{n^2}{k-n}\,dn = \cancel{\int_0^{k-n} \frac{(k+y)^2}{y}\,dy} = \int_0^k \frac{(k-y)^2}{y}\,dy \quad \infty$$



$\frac{7}{8}\;\frac{3}{28} = \frac{3}{32}$

$\frac{7}{8}\cdot\left(1-\frac{3}{28}\right) = \frac{25}{32}$

$= 1-\frac{1}{8}-\frac{3}{32}$

Zweierlei Arten Überlegung:

1). Wahrsch. dass bis zum k ten Mal kein einziges Teilchen übergegangen sei = dass weder das erste bis zum k ten Mal noch das zweite bis zum k ten Mal, ... noch k ten Teilchen bis zum k ten Mal übergegangen

$$\overbrace{1-\sum_{m=1}^{k} a_{1m}} \qquad \overbrace{1-\sum_{m=2}^{k} a_{2m}} \qquad \overbrace{1-\sum_{m=k}^{k} a_{km}}$$

$$\underbrace{\quad} = \sum_{m=k+1}^{\infty} a_{1m} \qquad \sum_{m=k+1}^{\infty} a_{2m} \qquad \sum_{m=k+1}^{\infty} a_{km}$$

$k = \frac{t_1}{\tau}$

$$\downarrow$$

$$\frac{1}{\sqrt{2\pi}}\int_{\frac{1}{\sqrt{k+1}}}^{\infty} e^{-z^2}\,dz \cdot \frac{1}{\sqrt{\pi}}\int_{\frac{2}{\sqrt{k+1}}}^{\infty} e^{-z^2}\,dz \cdots \frac{1}{\sqrt{2\pi}}\int_{\frac{k+1}{\sqrt{k+1}}}^{\infty} e^{-z^2}\,dz$$

$$W = \frac{1}{\sqrt{2\pi}} \int_{\frac{n}{\sqrt{k}}}^{\infty} e^{-z^2} dz = \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{n^2}{k}}}{2\frac{n}{\sqrt{k}}}$$

$$k! = \left(\frac{k}{e}\right)^k \sqrt{2\pi k}$$

$$\lg W = \lg \frac{\sqrt{k}}{2\sqrt{2\pi}} - \frac{n^2}{k} - \lg n$$

$$\sum_{n=1}^{k} \lg W = \frac{v}{\delta} k \lg \frac{1}{2}\sqrt{\frac{k}{2\pi}} - \frac{v}{\delta}\frac{k^2}{3} - \lg k!$$

$$e^{-}$$

2). = Wahrsch. dass weder beim ersten Mal ein Teilchen übergegangen ist

noch beim zweiten Mal

noch beim dritten Mal usw.

$$W(0) + W(1) + W(2) + \dots = 1 \qquad W(2) = W(1)^2 \dots$$

$$W(1) + 2W(2) + 3W(3) \dots = A$$

$$W(0) + \alpha + \alpha^2 + \alpha^3 \dots = 1 = W(0) + \frac{\alpha}{1-\alpha} = W(0) + \frac{1-\beta}{\beta} = W(0) + \beta A$$

$$\alpha + 2\alpha^2 + 3\alpha^3 \dots = A = \frac{\alpha}{(1-\alpha)^2} = \frac{1-\beta}{\beta^2}$$

$$1 + \alpha + \alpha^2 \dots = \frac{1}{1-\alpha}$$

$$1 + 2\alpha + 3\alpha^2 \dots = \frac{1}{(1-\alpha)^2}$$

$$A\beta^2 = 1 - \beta$$

$$\beta^2 + \frac{\beta}{A} = \frac{1}{A}$$

$$\beta = -\frac{1}{2A} \pm \sqrt{\left(\frac{1}{2A}\right)^2 + \frac{1}{A}}$$

$$W(0) = 1 - \beta A = 1 + \frac{1}{2} \mp \sqrt{\left(\frac{1}{2}\right)^2 + A}$$

U_{Ht} = Wahrsch., d. ein anfangs irgendwo innerhalb H befindliches Teilchen bis zur Zeit t nicht kein einziges mal die Grenzfläche nicht überschritten habe

U^N_{Ht} = W. d. keines von N innerhalb H bef. . . . –

$$N = \frac{v}{h} H$$

Wenn es unabhängig von H sein soll, so ist dies nur möglich falls $U_{Ht} = V^{\frac{1}{H}}$

V unabhängig von H

$$\frac{1}{m}\left[m - \sqrt{\frac{2m}{\pi}}\right] = 1 - \sqrt{\frac{2}{m\pi}}$$

$$\left[1 - \sqrt{\frac{2}{m\pi}}\right]^{N\sqrt{\frac{2}{m\pi}} / \sqrt{\frac{2}{m\pi}}} = e^{-\frac{v}{h} H \sqrt{\frac{2}{m\pi}}}$$

$$\left(1 - \sum_{m=1}^{k} a_{1m}\right) + \left(1 - \sum_{m=2}^{k} a_{2m}\right) + \left(1 - \sum_{m=3}^{k} a_{3m}\right) + \quad \underbrace{\left(1 - \sum_{m=n}^{k} a_{nm}\right)}_{\sum_{m=k+1}^{\infty} a_{nm}}$$

$$\frac{1}{\sqrt{2\pi}} \int_{\frac{1}{\sqrt{k}}}^{\infty} e^{-z^2} dz \quad - \quad - \quad - \quad - \quad \frac{1}{\sqrt{2\pi}} \int_{\frac{n}{\sqrt{k}}}^{\infty} e^{-z^2} dz$$

$$\frac{1}{n}\sqrt{\frac{k}{2\pi}} \int_1^n dz \int_z^\infty e^{-kx^2} dx = z \int_z^\infty \left(\frac{z}{\sqrt{k}} = x\right) - \int z e^{-kz^2} dz = \frac{e^{-kz^2}}{2k} \Big|_0^{\frac{n}{\sqrt{k}}} \frac{1}{n}\sqrt{\frac{k}{2\pi}}$$

$$\frac{1}{n}\sqrt{\frac{k}{2\pi}} \int_1^n dy \int_{\frac{y}{\sqrt{k}}}^\infty e^{-x^2} dz = y \int_{\frac{y}{\sqrt{k}}}^\infty e^{-x^2} dz \Big|_{y=0}^{y=n} - \frac{1}{\sqrt{k}} \int_{\frac{y}{\sqrt{k}}}^\infty y e^{-\frac{y^2}{k}} dy = \frac{1}{2n}\sqrt{\frac{k}{2\pi}} e^{-\frac{y^2}{k}} \Big|^n$$

$$\frac{1}{n\sqrt{2\pi}} n \frac{e^{-\frac{n^2}{k}}}{2\frac{n}{\sqrt{k}}}$$

$$= \frac{1}{2\sqrt{2\pi}} \frac{e^{-\frac{n^2}{k}} - 1}{\sqrt{\frac{n^2}{k}}} \neq \frac{1}{2\sqrt{2\pi}} \sqrt{\frac{n^2}{k}}$$ für kleines $\frac{n^2}{k}$

$$= \frac{1}{2\sqrt{2\pi}} \frac{\sqrt{k}}{n}$$ für grosses $\frac{n^2}{k}$

$$= \frac{n}{2\sqrt{2\pi}} \frac{1}{\sqrt{k}}$$

$a_{nm} = \frac{n}{m}\sqrt{\frac{2}{\pi m}}\, e^{-\frac{n^2}{m}}$ $\qquad \frac{1}{9}\sqrt{\frac{2}{9\pi}}\; e^{-\frac{1}{9}} = \frac{1}{9\sqrt{14}} = \frac{1}{34} \qquad \frac{14}{512} = \frac{7}{256} = \frac{1}{36}$

88

$n = m = 9$

$\sum_{m=n}^{m=\infty} a_{nm} =$ $\qquad \cancel{\sqrt{\frac{2}{9\pi}}\, e^{\frac{1}{9}} = \frac{1}{\sqrt{14}}\, e^{\frac{1}{9}} = \frac{1}{3\cdot 7\cdot 28}}$

$\frac{1}{2}\int \frac{\sqrt{2}}{\sqrt{\pi m}}\, \frac{n}{m}\, e^{-\frac{n^2}{m}}\, dm \qquad \frac{n^2}{m} = z = y^2 \qquad m = \frac{n^2}{z}$

$dm = -\frac{n^2}{z^2}\, dz$

$$-2\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{z}\,\frac{z}{n^2}\, e^{-z}\,\frac{n^2}{z^2}\, dz = \frac{\sqrt{2}}{2\sqrt{\pi}}\int \frac{e^{-z}}{\sqrt{z}}\, dz = -\sqrt{\frac{2}{\pi}}\int_{\sqrt{n}}^{0} e^{-y^2} dy = \sqrt{\frac{2}{\pi}}\int_0^{\sqrt{n}} e^{-y^2} dy$$

$$\cancel{(1+x)^{\frac{m}{2}} = \sum \binom{\frac{m}{2}}{k} x^k \qquad \frac{\frac{m}{2}\left(\frac{m}{2}-1\right)\left(\frac{m}{2}-2\right)\cdots\left(\frac{m}{2}-k+1\right)}{1.\,2.\,\cdots\, k} = \frac{m(m-2)(m-4)\cdots(m-2k+2)}{2.\,4.\,6\cdots 2k}}$$

$$\cancel{(1+x)^m = \binom{m}{0}x^{-m} + \binom{m}{1}x^{-m+2} + \binom{m}{2}x^{-m+4} \cdots \binom{m}{\frac{m}{2}}x^0} +$$

$\left\{\begin{matrix} n \text{ gross} \\ \frac{n^2}{m} \text{ klein}\end{matrix}\right.$

$\left(1 - \frac{1}{2\sqrt{2\pi}}\sqrt{\frac{tD}{H}}\right)^{\frac{\nu}{h}H}$

$$\sum_{m=k}^{\infty} a_{nm} = \cancel{\sqrt{\frac{2}{\pi}}\int_0^{\sqrt{k}} e^{-y^2} dy} = \sqrt{\frac{2}{\pi}}\int_0^{\frac{n}{\sqrt{k}}} e^{-y^2} dy$$

$$\left[1 - \frac{1}{2\sqrt{2\pi}}\,\frac{\sqrt{k}}{n}\right]^{N = \frac{\nu}{h}H} = e^{-\frac{1}{2\sqrt{2\pi}}\frac{\sqrt{k}}{n}N} \qquad k = \frac{t}{\tau}$$

$\frac{\sqrt{k}}{n} = \frac{H}{\delta}$ $\qquad \frac{\delta}{\sqrt{2\tau}} = \sqrt{D}$

$$e^{-\frac{1}{2\sqrt{2\pi}}\frac{\nu}{h}\frac{\delta\sqrt{t}}{\sqrt{\tau}}} = e^{-\frac{1}{2\sqrt{\pi}}\frac{\nu}{h}\sqrt{Dt}}$$

$\cancel{-\frac{x e^{-\alpha x}}{\alpha} - \frac{e^{-\alpha x}}{\alpha^2}}$

$\cancel{-\frac{e^{-\alpha x}}{\alpha} + x e^{-\alpha x} + \frac{e^{-\alpha x}}{\alpha}}$

$$\int_0^\infty e^{-\alpha\sqrt{t}}\, dt = 2\int_0^\infty e^{-\alpha x}\, x\, dx = -x\cancel{\frac{e^{-\alpha x}}{\alpha}} + \frac{2}{\alpha^2} \qquad T = \frac{1}{\alpha^2} = \frac{h^2\, 4\pi}{\nu^2 D}$$

$\alpha x + \frac{1}{2} = z$

$\cancel{-\alpha^2 x^2 - 2\alpha x} \qquad e^{-\alpha^2 t - \alpha\sqrt{t}}$

$$\int e^{-\alpha^2 t - \alpha\sqrt{t}}\, dt = \int_0^\infty e^{-\alpha^2 x^2 - \alpha x}\, x\, dx = \frac{e^{\frac{1}{4}}}{\alpha^2}\int_{\frac{1}{2}}^{\infty} e^{-z^2}\left(2 - \tfrac{1}{2}\right) dz$$

$$\int_0^\infty \left(\alpha^2 + \frac{\alpha}{2\sqrt{t}}\right) e^{-\alpha^2 t - \alpha\sqrt{t}}\, t\, dt = \int \cancel{\alpha^2 + \alpha}$$

Nach Diffusionstheorie: übergegangene Menge

$$\int_0^t M\,dt = c\sqrt{\frac{D}{\pi}}\int\frac{dt}{\sqrt{t}} = 2c\sqrt{\frac{Dt}{\pi}}$$

Wahrsch., dass das eine Teilchen nicht übergegangen sei:

$$\frac{Hc - 2c\sqrt{\frac{Dt}{\pi}}}{Hc} = 1-\frac{2}{H}\sqrt{\frac{Dt}{\pi}}$$

Falls nun $N = \frac{\nu}{h}H$ von einander unabhängige Teilchen vorhanden sind, Wahrsch. dass keines derselben übergegangen sei:

$$W_t = \left[1-\frac{2}{H}\sqrt{\frac{Dt}{\pi}}\right]^{\frac{\nu}{h}H}$$

Nun lässt sich H beliebig gross wählen, so dass man immer setzen kann:

$$W_t = \lim \uparrow = e^{-\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}}$$

Für Schichten von endlichem H ist dies nicht anwendbar für Zeiten ~~der~~ Grössenordnung $t \sim \frac{H^2}{D}$

Darum müssen die inneren Teilchen anders berücksichtigt werden

Wahrsch. dass keine Zahlenänderung eingetreten sei:

$$W_t = e^{-\frac{D\pi^2 n t}{h^2} - \frac{4\nu}{h}\sqrt{\frac{Dt}{\pi}}} = e^{-\pi^2 n z - \frac{4\nu}{\sqrt{\pi}}\sqrt{z}}$$

$$\frac{Dt}{h^2} = z \qquad T = \int_0^\infty W_t\,dt \qquad \nu\pi^2 z = x^2$$

Wenn $n = \nu$:

$$T = \frac{h^2}{D}\int_0^\infty e^{-(\pi^2 z + 4\sqrt{\frac{z}{\pi}})\nu}\,dz = \frac{2h^2}{\pi^2 D\nu}\int_0^\infty e^{-(x^2 + \frac{4\sqrt{\nu}}{\sqrt{\pi^3}}x)}\,x\,dx$$

Falls nur die äusseren Teilchen berücksichtigt würden:

$$W = e^{-\frac{4\nu}{h}\sqrt{\frac{Dt}{\pi}}}$$

$$T = 2\int e^{-\frac{4\nu}{h}\sqrt{\frac{D}{\pi}}\,x}\,x\,dx \qquad \cancel{T = \frac{2h^2}{\pi^2 D\nu}\int_0^\infty e} = \frac{2}{\left(\frac{4\nu}{h}\sqrt{\frac{D}{\pi}}\right)^2} = \frac{\pi h^2}{8\nu^2 D}$$

dasselbe wie wenn $n=0$

$$J = \int_0^\infty e^{-x^2-\beta x}\, x\,dx = e^{\frac{\beta^2}{4}}\int_0^\infty e^{-\overbrace{(x+\frac{\beta}{2})}^{z}{}^2}\, x\,dx = e^{\frac{\beta^2}{4}}\int_{\frac{\beta}{2}}^\infty e^{-z^2}\left(z-\frac{\beta}{2}\right)dz$$

$$= e^{\frac{\beta^2}{4}}\left[\frac{e^{-z^2}}{2}\Big|_\infty^{\frac{\beta}{2}} - \frac{\beta}{2}\int_{\frac{\beta}{2}}^\infty e^{-z^2}dz\right] = \frac{1}{2} - \frac{\beta}{2}\, e^{\frac{\beta^2}{4}}\underbrace{\int_{\frac{\beta}{2}}^\infty e^{-z^2}dz}$$

für grosse $\frac{\beta}{2}$

$$= \frac{e^{-\frac{\beta^2}{4}}}{\beta}\left[1-\frac{2}{\beta^2}\right]$$

Jedenfalls muss J desto kleiner sein, je grösser β $\quad \therefore \quad = \frac{1}{\beta^2} = \frac{\pi^3}{16\nu}$

also ist J enthalten zwischen den Grenzen: $\frac{1}{2} > J > 0$ (($I = \frac{2h^2}{\pi^2 D\nu}\cdot\frac{\pi^3}{16\nu} = \frac{h^2\pi}{8D\nu^2}$

Sonst, wenn $n \gtrless \nu$

$$I = \frac{2h^2}{\pi^2 D n}\int_0^\infty e^{-\left[x^2+\frac{4\nu}{\sqrt{n\pi^3}}x\right]}\, x\,dx \qquad \beta = \frac{4\nu}{\sqrt{n\pi^3}} \qquad I =$$

Dagegen aus $\bar{\Delta}^2 = 2\nu P = 1 \quad = \frac{4\nu}{h}\sqrt{\frac{Dt}{\pi}}$

$P \downarrow \frac{2}{h}\sqrt{\frac{Dt}{\pi}}$

$$I = \frac{h^2\pi}{16\nu^2 D}$$

$\to$ I muss desto kleiner sein, je grösser n; dabei kommt es auf das Verhältnis $\frac{n\sqrt{\pi^3}}{\nu^2 \cdot 4}$ an; falls dieses gross

gilt angenähert: $I = \frac{2h^2}{\pi^2 D n}$

Es ist doch unwahrscheinlich, dass im Falle grosser ν, n die I-Zeit unabhängig davon wird, ob in der Schichte selbst Teilchen sich befinden oder nicht!

$$\frac{8}{\pi^2}\Big\{1-\frac{D\pi^2}{h^2}t+\Big(\frac{D\pi^2}{h^2}t\Big)^2\frac{1}{2!}-$$

$$+\frac{1}{9}\Big[1-9\frac{D\pi^2 t}{h^2}+\Big(9\frac{D\pi^2}{h^2}t\Big)^2\frac{1}{2!}-\cdots\Big]$$

$$+\frac{1}{25}\Big[1-25 \qquad\qquad \Big\}$$

$$\frac{dW}{dt}=\frac{8D}{h^2}\sum e^{-\frac{Dk^2\pi^2}{h^2}t}\Bigg|_{t=0}=\infty$$

$$\int_0^c \sin\frac{n\pi\alpha}{c}\,d\alpha=\frac{c}{n\pi}(\cos n\pi-1)=\begin{cases}0 & n=2k\\ -\frac{2c}{n\pi} & (n=2k+1)\end{cases}$$

$$u=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{c}\,e^{-D\left(\frac{n\pi}{c}\right)^2 t}$$ stimmt $$\Big\|\ \frac{\partial u}{\partial x}\Big|_{x=0}=\frac{4}{c}\sum_{k=1}^{\infty}e^{-D\left(\frac{k\pi}{c}\right)^2 t}$$

für $t=0$
divergente Reihe
also kann man nicht gliedweise diff.?

Das Halbraum-Problem:

Wahrsch. dass gerade eines übergegangen sei:

$$N\frac{2}{H}\sqrt{\frac{Dt}{\pi}}\Big[1-\frac{2}{H}\sqrt{\frac{Dt}{\pi}}\Big]^{N-1}\neq\underbrace{\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}}_{=\alpha}\,e^{-\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}}=\alpha e^{-\alpha}$$

Wahrsch. dass gerade zwei übergeg. sind:

$$\frac{N(N-1)}{1.2}\Big(\frac{2}{H}\frac{Dt}{\pi}\Big)^2\Big[\qquad\Big]^{N-2}\neq\frac{\alpha^2}{2}e^{-\alpha}$$

etc.

Gesammte Wahrsch. $e^{-\alpha}\Big[1+\alpha+\frac{\alpha^2}{2}+\frac{\alpha^3}{3!}+\cdots\Big]=1$

~~[illegible]~~ Durchschn. übergehende Menge: $e^{-\alpha}\Big[0.1+\alpha.1+2.\frac{\alpha^2}{2}+3\frac{\alpha^3}{3!}+\cdots\Big]$

$$=e^{-\alpha}\Big[\alpha+\frac{\alpha^2}{1}+\frac{\alpha^3}{2!}+\frac{\alpha^4}{3!}+\cdots\Big]=\alpha=\frac{2\nu}{h}\sqrt{\frac{Dt}{\pi}}$$

$$\frac{h^2 \pi}{3\cancel{2} \nu^2 D} \cdot \cancel{\#} \sqrt{2\nu\pi}\; e^{\frac{\nu\delta^2}{2}}$$

$$= \frac{h^2 \pi^{3/2}}{2\sqrt{\cancel{2}}\, D} \frac{1}{\nu^{3/2}}\, e^{\frac{\nu\delta^2}{2}} = \frac{h^2}{\cancel{4}D}\left(\frac{\pi}{2}\right)^{3/2} e^{\frac{\delta^2}{2\xi^2}} \quad \xi^3$$

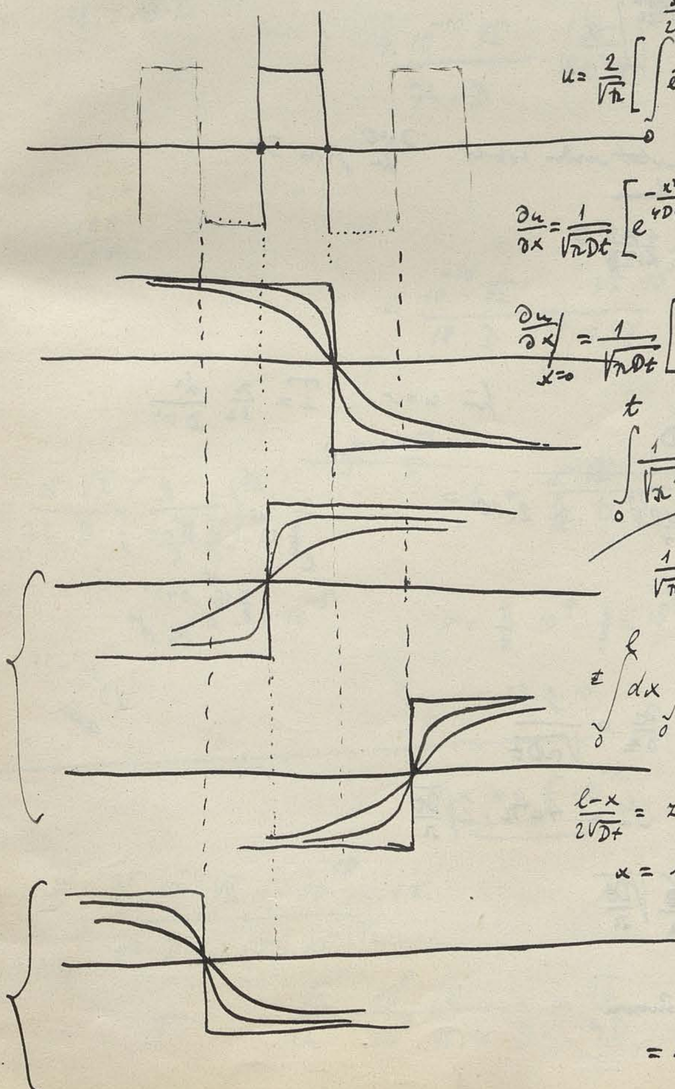
$$\lim W_{(n)} = \lim \frac{e^{-\nu}\nu^n}{n!} = e^{n-\nu}\left(\frac{\nu}{n}\right)^n \frac{1}{\sqrt{2n\pi}} = e^{\nu\delta}\left(\frac{1}{1+\delta}\right)^{\nu(1+\delta)} \frac{1}{\sqrt{2\nu\pi}}$$

$$\lg W_{(n)} = \nu\delta - \nu(1+\delta)\left(\delta - \frac{\delta^2}{2}\right) \ldots$$

$$= \cancel{\nu\delta} - \cancel{\nu\delta} + \nu\frac{\delta^2}{2} - \nu\delta^2$$

$$= -\nu\frac{\delta^2}{2}$$

$$= \frac{e^{-\frac{\nu\delta^2}{2}}}{\sqrt{2\nu\pi}}$$



$$u = \frac{2}{\sqrt{\pi}}\left[\int_0^{\frac{x}{2\sqrt{Dt}}} e^{-y^2} dy + \int_0^{\frac{l-x}{2\sqrt{Dt}}} - \int_0^{\frac{l+x}{\cdots}} - \int_0^{\frac{2l-x}{\cdots}} + \int_0^{\frac{2l+x}{}} + \int_0^{\frac{3l-x}{\cdots}}\right.$$

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{\pi D t}}\left[e^{-\frac{x^2}{4Dt}} - e^{-\frac{(l-x)^2}{4Dt}} - e^{-\frac{(l+x)^2}{4Dt}} + e^{-\frac{(2l-x)^2}{4Dt}} + e^{-\frac{(2l+x)^2}{4Dt}} - e^{-\frac{(3l-x)^2}{4Dt}}\right.$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \frac{1}{\sqrt{\pi D t}}\left[1 - e^{-\frac{l^2}{4Dt}} - e^{-\frac{l^2}{4Dt}} + e^{-\frac{4l^2}{4Dt}} + e^{-\frac{4l^2}{4Dt}} \ldots - \; 9l^2\right.$$

$$\int_0^t \frac{1}{\sqrt{\pi D t}}\, e^{-\left(\frac{nl}{\cancel{2}Dt}\right)^2} dt \qquad \frac{1}{\sqrt{Dt}} = z \qquad t = \frac{1}{z^2 D}$$

$$\frac{1}{\sqrt{\pi D}}\int_\infty^{\frac{1}{\sqrt{Dt}}} \frac{e^{-\frac{(nl)^2 z^2}{4}}}{z^2}\, dz = -\frac{1}{z}\, e^{\cdots} - \int e^{\cdots}\, dz$$

$$\int_0^l dx \int_0^{\frac{l-x}{2\sqrt{Dt}}} e^{-y^2} dy = x \int_0^{\frac{l-x}{2\sqrt{Dt}}} e^{-y^2} dy \Bigg|_0^l - \int_0^l x\, e^{-\left(\frac{l-x}{2\sqrt{Dt}}\right)^2} dx$$

$$\frac{l-x}{2\sqrt{Dt}} = z$$

$$x = l - 2z\sqrt{Dt}$$

$$= \int_0^{\frac{l}{2\sqrt{Dt}}} \left[2z\sqrt{Dt} - l\right] e^{-z^2} dz \cdot 2\sqrt{Dt}$$

$$= \underbrace{-2Dt\; e^{-z^2}\Big|_0^{\frac{l}{2\sqrt{Dt}}}} - 2l\sqrt{Dt}\int_0^{\frac{l}{2\sqrt{Dt}}} e^{-z^2} dz$$

$$= 2Dt\left[1 - e^{-\frac{l^2}{4Dt}}\right] -$$

Jedenfalls gibt es einen Bereich genügend kurzer t wo angenähert gilt:

$$W = 1 - \frac{1}{h}\frac{4\sqrt{D}\sqrt{t}}{\sqrt{\pi}}$$

Solange nämlich $\frac{4Dt}{h^2}$ sehr klein ist

Dagegen zweite Art der Darstellung:

$$W = \frac{8}{\pi^2}\Big[e^{-\frac{D\pi^2}{h^2}t} + \frac{1}{9}e^{-9\frac{D\pi^2}{h^2}t} + \frac{1}{25}e^{-25\frac{D\pi^2}{h^2}t} + \dots$$

$$= 1 - \frac{8}{\pi^2}\sum_{k=1,3,5\dots}\frac{1}{k^2}\left[1 - e^{-\frac{k^2 D\pi^2 t}{h^2}}\right]$$

W kann gleich $= \frac{8}{\pi^2}e^{-\frac{D\pi^2 t}{h^2}}$ gesetzt werden, sobald $\frac{D\pi^2 t}{h^2}$ gross ist

Für kürzere Zeiten dagegen erstere Darstellung besser

$$W^n = \left[1 - \frac{1}{h}\frac{4\sqrt{D}}{\sqrt{\pi}}\sqrt{t}\right]^n = e^{-\frac{4n\sqrt{D}}{h\sqrt{\pi}}\sqrt{t}}$$

Dann wird also im Ganzen:

$$T = \int e^{-\frac{4(n+\nu)}{h\sqrt{\pi D}}\sqrt{t}}\,dt = \frac{\pi h^2}{8(n+\nu)^2 D}$$

für $n=\nu$ $\quad T = \frac{\pi}{32}\frac{h^2}{D\nu^2}$

$$\frac{h^2\pi}{32\nu^2 D}\sqrt{2\nu\pi}\,e^{\frac{\nu\delta^2}{2}}$$

$\nu = \frac{3.10^{19}}{4}$ $\quad \| \; 2.10$

$\delta = \frac{1}{100}$

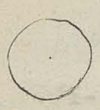
$e^{\frac{3}{8}\cdot 10^{15}}$

$\frac{3}{8}10^{15}\cdot 0{,}434$

$1{,}6\cdot 10^{14}$

$10^{10^{14}-30}$

Kugelförmiger Raum



$$\frac{\partial(cr)}{\partial t} = \frac{\partial^2(cr)}{\partial r^2}$$

$$cr = \frac{2}{\sqrt{\pi}}\int_0^{\frac{a-r}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

$$\frac{\partial c}{\partial r} = \frac{1}{\sqrt{\pi D t}}$$

$$M = 4a^2\pi\cdot 2\sqrt{\frac{Dt}{\pi}}$$

$$W = 1 - \frac{4a^2\pi}{\frac{4}{3}a^3\pi}2\sqrt{\frac{Dt}{\pi}} = 1 - \frac{6}{a}\sqrt{\frac{Dt}{\pi}}$$

$$W^\nu = e^{-\frac{6\nu}{a}\sqrt{\frac{Dt}{\pi}}}$$

ebensoviel Äussere

$$\int_0^\infty e^{-\alpha\sqrt{t}}\,dt = 2\int_0^\infty e^{-\alpha x}\,x\,dx = \frac{2}{\alpha^2}$$

$$I_3 = \frac{2}{\left[\frac{6(n+\nu)}{a}\sqrt{\frac{D}{\pi}}\right]^2} = \frac{a^2\pi}{18(n+\nu)^2 D} \cdot \sqrt{2\nu\pi}\; e^{\frac{\nu\delta^2}{2}}$$

$a = 2\,10^{-5}$; $\nu = \frac{\cancel{3}.10^{19}}{4} . 10^{-15} \frac{4}{\cancel{3}}\pi$

$= 10^4.\pi.8$

$$\frac{a^2 \cancel{\pi}\sqrt{2}}{18.4.\cancel{\pi} D}\left(\frac{\pi}{\nu}\right)^{3/2} \cdot e^{\frac{\nu\delta^2}{2}} =$$

$$\frac{4.10^{-10}\sqrt{2}}{72.D}\left(\frac{\cancel{\pi}}{8.10^4\cancel{\pi}}\right)^{3/2} e^{\frac{\cancel{\quad}\pi}{2}.8}$$

$3{\cdot}14$

$e^{4\pi} = e^{12{\cdot}4}$

$12{\cdot}4 . 0{\cdot}434$
496
372
49
$5{\cdot}38$

18.16
108
288

$$= \frac{10^{-10}\sqrt{2} \quad . \quad 3{\cdot}5.10^5}{18.D.10^6.8\sqrt{8}} = \frac{10^{-11}\cancel{\quad}}{18.8.2.D} = \frac{10^{-13}}{2{\cdot}88.D}$$

$$\frac{a^2\sqrt{2}}{72.D}\left(\frac{\cancel{\pi}}{\frac{4a^3\cancel{\pi}}{3}N}\right)^{3/2} e^{\frac{4a^3\pi}{3}N\frac{\delta^2}{2}} = 1 = \frac{\cancel{\quad}\; 3\sqrt{6}}{72.D.8.a^2\sqrt{a}\,N^{3/2}}\; e^{\frac{2}{3}a^3\pi N\delta^2}$$

$a = 3.10^{-5}$ $\qquad \nu = \frac{\cancel{3}}{\cancel{4}}.10^{19}.\frac{\cancel{4}}{\cancel{3}}\pi\,10^{-15}.\,27$

$= 27.\pi.10^4$

$3{\cdot}141.\ 27$
6283
2199
$84{\cdot}8$

$e^{27\pi}$

$0{\cdot}434.\ 85$
217
3472
$36{\cdot}9$

$\boxed{a = \frac{5}{2}.10^{-5}}$ $\qquad \nu = 10^4\pi.\frac{125}{8}$

$= 10^4\pi.15{\cdot}6$

$15{\cdot}6.\ 3{\cdot}14$
468
156
62
$49{\cdot}0$

$0{\cdot}434.\ 49$
1736
3906
$18{\cdot}27$

$$\cancel{=} \frac{\frac{25}{4}.10^{-10}\sqrt{2} \quad . \quad 10^{18} \quad . 1{\cdot}8}{72.D \quad . (15{\cdot}6.10^4)^{3/2}}$$

$$T = \frac{\sqrt{2}}{12}\,\frac{10^8.\,1{\cdot}8}{10^6.15.4.D} = \frac{\sqrt{2}}{4.D}$$

Svedberg $\quad k = 2.10^{-4}$

$D = 10^{-7}$

$v = 1'55$

$$T = \frac{\pi}{32}\frac{k^2}{Dv^2} = \frac{\pi .\ 4.10^{-8}}{32.\ 10^{-7}.\ (1'55)^2} = \frac{\pi}{8}\,\frac{10^{-1}}{2'4}$$

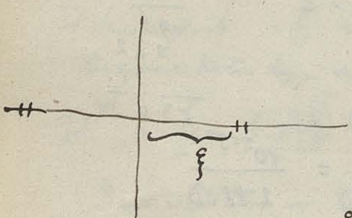
$$= \frac{3'14}{19'2}\,10^{-1}$$

$$= \underline{\underline{1'6\ .10^{-2}}}$$

$n = 17$

$$T = \frac{\pi}{8}\,\frac{4.10^{-8}}{10^{-7}\,(18'5)^2} = \frac{\pi .10^{-1}}{2.18.19} = \frac{\pi .10^{-1}}{684} = 3142:684 = 4'61\ .10^{-3}$$

Wenn ein Teilchen aus einer Lage ξ ($\cdots \xi + d\xi$) ausgeht, was ist die Wahrsch., dass es bis zur Zeit t ~~nicht~~ (= kein einziges mal) durch die Null-Ebene gegangen sei?



Verteilung

$$dx . u = \frac{1}{\sqrt{\pi D t}}\left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{(x+\xi)^2}{4Dt}}\right] dx \qquad \Big\| \frac{d\xi}{\alpha}$$

$$U = \int_0^\infty u\,dx = \frac{1}{\sqrt{\pi D t}}\int_0^\infty \Big[\qquad\Big] dx$$

$$T = -\int_0^\infty t\,\frac{dU}{dt}\,dt = \int_0^\infty U\,dt$$ = ~~mittlere~~ durchschnittl. Wartez., falls ein Teilchen ursprünglich in ξ

$$\overline{T} = \int_0^H T\,\frac{d\xi}{H} = \int_0^\infty dt \int_0^H U\,\frac{d\xi}{H} = \cancel{\int_0^H dt}$$ = falls ein Teilchen im Bereich von Null bis H

gefährlicher Sprung? von einem auf viele Teilchen

$$\overline{\overline{T}} = \lim_{\substack{H=\infty \\ \frac{V}{H}=\text{const}}} \int_0^\infty dt\left[\int_0^H U\,\frac{d\xi}{H}\right]^\nu$$

Ist die Mittelbildung überall richtig? Es sollten anfangs die verschiedenen ~~Zeiten~~ Warte T gebildet werden, welche den verschiedenen örtlichen Teilchen Anordnungen entsprechen. Jedes sollte mit der Wahrscheinlichkeit der betreffenden Teilchen Anordnung multipliziert werden und daraus erst das Mittel zu nehmen

$$\sqrt{\frac{8\pi}{3}}\ \frac{a}{\cancel{4}\nu C}\sqrt{\nu\pi}\ e^{\frac{\nu\delta^2}{2}}$$

91

$$160 \cdot \frac{\cancel{4 \cdot 9 \cdot 10^{-3}}\ (10^4)}{\cancel{24 \cdot 10^{-7}}} = 80 \cdot 10^4 = \frac{8 \cdot 10^5}{360}\ d. = 2200\ J.$$

$$= \sqrt{\frac{2\pi}{3}}\ \frac{a}{(n+\nu)\,C} \qquad = \frac{a\pi}{\sqrt{3\nu}\,C}\,e^{\frac{\nu\delta^2}{2}}$$

Zeiten Θ_1, T_1 für kugelförmigen Raum $O_2 - N_2$

$$T_1 = \frac{\sqrt{6\pi}}{\cancel{4\rho^2\pi}(n+\nu)\,C} \cdot \cancel{\tfrac{4}{3}a^3\pi} \qquad \Theta_1 = \sqrt{2\nu\pi}\,e^{\frac{\nu\delta^2}{2}} \cdot T_1 = \frac{\sqrt{6\pi}\ \sqrt{2\nu\pi}}{8a^2\pi\,\nu\,C}\,e^{\frac{\nu\delta^2}{2}} \cdot \frac{4}{3}a^3\pi$$

wenn n u. ν die im Innern Raum befindliche Anzahl bedeuten)

$$= \frac{T_1}{W(n)} \qquad = \frac{\sqrt{3\nu}}{4a^2\nu C}\,e^{\frac{\nu\delta^2}{2}} = \frac{\sqrt{3}}{\cancel{4}aC\sqrt{\nu}}\,e^{\frac{\nu\delta^2}{2}} \cdot \cancel{\tfrac{4}{3}a^3\pi}$$

$$= \frac{\pi}{\sqrt{3}}\ \frac{a}{C\sqrt{\nu}}\,e^{\frac{\nu\delta^2}{2}}.$$

Falls zwei Teilchen zwischen 0 und H:

Wahrsch., dass keines von beiden abgegangen sei, wenn das eine von ξ_1, das andere von ξ_2 ausgegangen ist:

$$= U_1\,U_2$$

W. d. ...., wenn ξ_1 und ξ_2 beliebig innerhalb Null und H:

$$= \iint_0^H U_1\,U_2\,\frac{d\xi_1}{H}\,\frac{d\xi_2}{H} = \left[\frac{1}{H}\int_0^H U\,d\xi\right]^2$$

$$\overline{\overline{T}} = -\iint_0^H \frac{d\xi_1}{H}\,\frac{d\xi_2}{H}\int_0^\infty t\,\frac{d(U_1U_2)}{dt}\,dt = \int_0^\infty dt\left[\frac{1}{H}\int_0^H U\,d\xi\right]^2$$

also ist meine Methode ganz richtig! Ebenso für n Teilchen!

$$\frac{e^{-\frac{\nu\delta^2}{2}}}{\sqrt{2\nu\pi}} = \frac{e^{-\frac{(n-\nu)^2}{2\nu}}}{\sqrt{2\nu\pi}} \qquad \sum_{n=-\infty}^{+\infty} e^{-\frac{(n-\nu)^2}{2\nu}} = 2\sum_{n=\nu}^{\infty} e^{-\frac{(n-\nu)^2}{2\nu}} = 2\sum_{n=0}^{\infty} e^{-\frac{n^2}{2\nu}} = ?$$

$$\cancel{x^{-1} + x^{-4} + x^{-9} + x^{-16} \ldots}$$

$$T = \frac{\sqrt{3}}{4}\,\frac{1}{a^2 C\sqrt{\nu}}\,e^{\frac{\nu\delta^2}{2}}$$

$C = 480 \frac{m}{sek} = 4{,}8.10^4$

$\nu = \frac{4}{3}a^3\pi\,\frac{N}{4}$ $\quad N = 3.10^{19}$

$\delta = 10^{-2}$

$a = 1$ cm

$$\Theta_1 = \frac{T}{\Delta t} = \frac{\sqrt{3}\;\sqrt{\cancel{3}}}{4.\,4{,}8.10^4\sqrt{\pi}\sqrt{\cancel{3}\,10^{19}}}\,e^{+\frac{10^{19}.\pi.10^{-4}}{2}} \quad \text{(line through the exponent)} \; + \frac{10^{15}\pi}{2}$$

$$= \sqrt{\frac{3}{10\pi}}\,\frac{10^{-13}}{4.\,4{,}8.}\,e$$

3'1415

0'4343 . 1'57
628
47
6
0'681 .10^14

$$\frac{4{,}8.10^{10^{14}}}{10^{13}} = 4{,}8.\,10^{10^{14}-13}.4$$

$$\neq 10^{10^{14}}$$

$a = 2.10^{-5}$

$C = 4{,}8.10^4$

$\nu = 27.10^{-15}.10^{19}\pi = 2{,}7.10^5.\pi = 8{,}5.10^5$

$\delta = 10^{-2}$

49715
43136
9285

8'48

$$\Theta_1 = \frac{T}{t} = \frac{\sqrt{3}}{4}\,\frac{1}{9.10^{-10}.\,4{,}8.10^4\sqrt{85.10^4}}\,e^{42.}$$

$$= \frac{\cancel{\sqrt{3}}.10^4.10^{18}.\cancel{1{,}7}\,3'14.\frac{\cancel{3}}{\cancel{3}}\,\cancel{27}.10^{-15}\,\cancel{\pi}\;5'46}{\cancel{4}\,\cancel{4}\,4{,}8\,\cancel{\sqrt{85}}\,\cancel{\sqrt{20}}\;5'32} = \frac{10^7}{4{,}7} = 2.10^6$$

0'4343. 42
17372
8686
18'24

314
220
12
546

$a = 2.10^{-5}$ $\quad C = 4{,}8.10^4$

5'429

807
27
70

$\nu = \cancel{3{,}1}\;25.10^4$

$$\frac{T}{t} = \frac{\sqrt{3}}{\cancel{4}.\cancel{4}.10^{-10}.\,4{,}8.10^4.\cancel{5}.10^2}\,e^{12'5} = \frac{\cancel{\sqrt{3}}.10^3.\;2'69.10^5.\cancel{4}\pi.\cancel{4}10^{-15}}{\cancel{8.4{,}8}\;\cancel{1{,}7.1{,}73}\;\cancel{\sqrt{3}}} = \frac{8'44}{2'08}.10^{-7}$$

80 $\quad$ 3'14 $\quad$ 2'08

$= 4.10^{-7}$

$$\frac{\Theta_1}{\Theta_3} = \cancel{\frac{3}{2}}\ \frac{\sqrt{6\pi}}{8 a^2 \pi \cancel{N} C} \cdot \frac{9 \cdot \cancel{72} \cdot 4 \nu^2 D}{a^2 \pi} = \frac{9\sqrt{6}}{\pi\sqrt{\pi}}\ \frac{D\nu}{a^4 C} \qquad D = \frac{C\lambda}{3}$$

$$= \frac{9\sqrt{6}}{\pi\sqrt{\pi}}\ \frac{\nu\lambda}{3a^4} = \cancel{\frac{3\sqrt{6}}{\pi\sqrt{\pi}}\ \frac{\lambda}{a^4}\ \frac{4}{3}\ \frac{N}{4}}$$

$$\Theta_1 = \frac{\sqrt{6\pi}}{8 a^2 \pi \frac{N}{4} C} \quad \nu = \frac{\sqrt{6\pi} \cdot 3a}{2 \cdot \nu C}$$

$$= \frac{\sqrt{6}}{\pi}\ \frac{\lambda N}{\cancel{a}} \cdot \frac{4}{3} a^2 \pi$$

$$= \frac{4}{3}\sqrt{6\pi} \cdot a^2 \lambda N$$

$$4 \cdot 10^{-10} \cdot 10^{-5} \cdot 3 \cdot 10^{19}$$

$$\cancel{\Theta_1 = \sqrt{3}}$$

$$D = \frac{R\Theta}{N}\ \frac{1}{3\pi a \mu} = \frac{\lambda}{3}\sqrt{\frac{R\Theta}{}}$$

$$T_3 = \frac{a^2 \pi}{72 \cdot \nu^2 D} = \frac{a^2 \pi}{72 \cdot \left[\frac{4}{3}\pi a^3 \frac{N}{4}\right]^2 D}$$

$$\frac{\nu}{h} =$$

für Schichten:

$$T_1 = \frac{\sqrt{6\pi}\, h}{4\nu C} \qquad \frac{T_1}{T_3} = \frac{\sqrt{6\pi}\, h}{4\nu C}\ \frac{32 \nu^2 \frac{C\lambda}{3}}{\pi h^2} = \frac{\sqrt{6\pi} \cdot 8}{3\pi}\ \frac{\nu\lambda}{h}$$

$$T_3 = \frac{\pi h^2}{32 \nu^2 D}$$

Man kann doch ν beliebig gross machen, indem man (bei constantem h und const. Raumdichte) den Flächeninhalt der Schichte beliebig vermehrt. Warum soll da das Verhältnis $\frac{T_1}{T_3}$ beliebig wachsen?

$$a = 1 \cdot 10^{-5} \qquad \nu = 3{,}14 \cdot 10^4 \qquad e^{1{,}57}$$

$$\cancel{0{,}779} \quad 0{,}6818$$

$$\Theta_1 = \frac{\sqrt{3}}{4 \cdot 10^{-10} \cdot 48 \cdot 10^4 \cdot \sqrt{3{,}14 \cdot 10^2}}\ \frac{10^4}{\cancel{4}} \cdot \frac{4}{3} \cdot 10^{-15} \cdot \pi = \underline{\underline{10^{-11}}}$$

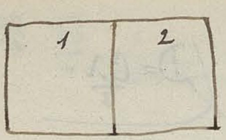
4343
2171
304
6818

$$\underline{a = \frac{5}{2} \cdot 10^{-5}} \qquad \nu = \frac{125}{8} \cdot 3{,}14 \cdot 10^4 = \frac{13}{8} \cdot 3 \cdot 10^4 = 49 \cdot 10^4 \qquad e^{24{,}5} =$$

8686
1737
217
10,64

$$\Theta_1 = \frac{\cancel{\left(\frac{5}{2}\right)}\frac{3}{2}}{\cancel{\left(\frac{5}{2}\right)}} \cdot \frac{4{,}37 \cdot 10^{10} \cdot 10^{-11}}{\cancel{\sqrt{5}}\ 1{,}58} = \underline{\underline{0{,}3}} \text{ sek}$$

$$\cancel{\frac{10^{-5}}{13 \cdot 10^{2} \sqrt{10} \cdot 10^{2}}}\ \cancel{48}\ 3 \cdot 10^{-16}\ ?$$



Wahrsch., dass eine Zahlyänderung um 1 eintritt

a) bei einer ~~Halbfläche~~ W

b) bei der ganzen: entweder Änderung in 1 und nicht in 2: $W(1-W)$

oder " 2 1: "

also Wahrsch. einer Änderung um 1 bei der ganzen Fläche $= 2W(1-W)$

Wenn W die Wahrsch. einer Änderung überhaupt bedeutet, so ist

$$W_{ganz} = 2 W_{halb}$$

Somit sollte man erwarten, dass die Wahrsch. einer Zahlenänderung bei konstanter Raumdichte proportional sein wird der Oberfläche der Schichte, also proportional v (resp. n)

$$\frac{D}{C} = \frac{\frac{1}{6\pi a \mu}\frac{R\theta}{N}}{\sqrt{\frac{3R\theta}{N\frac{4}{3}a^3\pi\rho}}} = \frac{\sqrt{\frac{R\theta}{N}}}{\sqrt{\frac{9\cdot 9\pi^2 a^2\mu^2}{\frac{4}{3}a^3\pi\rho}}} = \sqrt{\frac{R\theta}{N}}\cdot\frac{2}{9}\cdot\frac{1}{\mu}\sqrt{\frac{a\rho}{\pi}}$$

$$6\frac{D}{C^2} = \frac{2}{a\pi\mu}\,\frac{4}{9}a^3\pi\rho = \frac{8a^2\rho}{9\mu}$$

So kurze Zeiten dass man ~~das experimentell~~ die Ungiltigkeit der Diffusionstheorie nachweisen könnte und wohl schwer experimentell herzustellen

Vielleicht bei Auflösungsgeschwindigkeit fester Körper?

Einfachstes Beispiel: Halbraum-Problem

Anzahl der Stösse pro Flächeneinheit : $2\frac{NC}{\sqrt{6\pi}}$

also durchschn. ~~mittlere~~ Dauer zwischen zwei Stössen auf Fläche F: $T_1 = \frac{\sqrt{6\pi}}{2FNC}$

Wahrsch., dass noch kein Teilchen übergegangen sei:

$$W = e^{-2NF\sqrt{\frac{Dt}{\pi}}}$$

Warsch. dass einer oder mehrere übergeg. sei: $1-e^{-\frac{2NF}{3}\sqrt{\frac{Dt}{\pi}}}$; Wahrsch., dass gerade eines übergeg. sei: $\alpha e^{-\alpha}$

~~Ab~~ Durchschn. Zeit bis zum Übergange:

$$T_3 = -\int_0^\infty t\,\frac{dW}{dt}\,dt = \frac{2\pi}{4(NF)^2 D} = \frac{\pi}{2N^2 D F^2}$$

$$\int_0^\infty \frac{\alpha\sqrt{t^3}\,e^{-\alpha\sqrt{t}}\,dt}{\int \alpha\sqrt{t}\,e^{-\alpha\sqrt{t}}\,dt} = 2\int_0^\infty \frac{\alpha x^4 e^{-\alpha x}\,dx}{2\int \alpha x^2 e^{-\alpha x}\,dx} = \frac{\frac{2}{\alpha^4}\int_0^\infty y^4 e^{-y}\,dy}{\frac{2}{\alpha^2}\int_0^\infty y^2 e^{-y}\,dy} = \frac{\cdots}{\alpha^2} \qquad ?$$

$\sqrt{t} = x$

$t = x^2$

Aber Diffusionstheorie ist nur anwendbar für Zeiten welche eine gewisse untere Grenze überschreiten! Denn Übergangsgeschwindigkeit:

$$\frac{NC}{\sqrt{6\pi}} \geqq N\sqrt{\frac{D}{\pi t}}$$

$$t \geqq \frac{6D}{C^2} \qquad \text{d.h.} \qquad T_3 = \frac{\pi h^2}{32\nu^2 D} \gg \frac{6D}{C^2}$$

$$\therefore \quad \frac{h}{\nu} \gg \sqrt{\frac{32.6}{\pi}}\cdot\frac{D}{C}$$

Dann ist für = : $T_1 = \frac{\sqrt{6\pi}}{4C}\sqrt{\frac{32.6}{\pi}}\,\frac{D}{C} = \frac{6.\sqrt{32}}{4}\,\frac{D}{C^2}$ also von derselben Grössenordnung sonst wird aber T_1 kleiner sein

Wahrsch., dass bei $m=$ 8 Würfen der Maximal überschuss ~~gerade~~ $n=$ 4 ~~beträgt~~ auftritt:

$$\frac{1}{16} + \frac{4}{64} + \frac{14}{256} - \frac{1}{32} - \frac{5}{128} = \frac{3}{32} + \frac{1}{64} = \frac{7}{64}$$

$$\binom{8}{2}\frac{1}{2^8} = \frac{8.7}{1.2}\,\frac{1}{2^8} = \frac{7}{2^6} = \frac{7}{64}$$

$n=8$ $\quad n=3$

$$\frac{1}{8} + \frac{3}{32} + \frac{9}{128} - \frac{1}{16} - \frac{2}{32} - \frac{7}{128} = \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = \frac{7}{64}$$

$m=8$ $\quad n=2$

$$\frac{1}{4} + \frac{2}{16} + \frac{5}{64} + \frac{7}{128} - \frac{1}{8} - \frac{6}{64} - \frac{9}{128} = \frac{2}{8} - \frac{1}{32} = \frac{7}{32}$$

$$\binom{8}{3}\frac{1}{2^8} = \frac{8.7.6}{1.2.3}\,\frac{1}{8.8.4} = \frac{7}{32}$$ stimmt

---

[illegible] Intervall [illegible]

$$\frac{N_1 + N_2 + N_3 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots} = W(\tau)$$

Also $T_1 = \frac{\tau}{W(\tau)}$!

$$W(k\tau) = \frac{N_k + N_{k+1} + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

$$T_3 = \tau \sum_{k=1}^{\infty} k\, W(k\tau)$$

$$W(k\tau) - W((k+1)\tau) = \frac{N_k}{N_1 + 2N_2 + 3N_3 + \cdots}$$

$1 - W(\tau) = $ [illegible]

also muss sein:

$$\sum_{k=1}^{\infty} k\left[W(k\tau) - W((k+1)\tau)\right] = 1$$

Wahrsch. dass auf n im nächsten Intervalle wieder n folge (und später ein beliebiger Wert)

$$= P_n(0) = W(n,n)$$

$$W(n,n) = \frac{N_2 + 2N_3 + 3N_4 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

Wahrsch., dass auf n im nächsten Intervalle wieder n folge (aber später ein „nicht-n“) dass also n ~~…~~ nur durch zwei Intervalle unverändert andauere

$$W(2\tau) = \frac{N_2 + N_3 + N_4 + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

~~Wahrsch. dass auf n im nächsten Intervalle kein n folge, also dass nur durch 1 Intervall unverändert ~~W(1τ)~~ ~~folge~~~~ $W(1\tau) = \frac{N_1 + N_2 + N_3 + \cdots}{}$

~~bezogen auf alle Zustände [ein Intervall …]~~

~~Wahrsch. dass ein (k-faches Intervall auftritt = … k-mal $\frac{k N_k}{N_1 + 2N_2 + 3N_3 \cdots}$~~

Wahrsch. dass der Zustand (durch …) k Intervalle noch weiter dauere

$$= \frac{N_k + N_{k+1} + \cdots}{N_1 + 2N_2 + 3N_3 + \cdots}$$

$$T_3 = \tau \frac{N_1 + (1+2)N_2 + (1+2+3)N_3 + \cdots}{N_1 + 2N_2 + 3N_3 \cdots}$$

$$= \tau \left[\cancel{1 + \frac{N_2 + (1+2)N_3 + (1+2+3)N_4}{N_1 + 2N_2 + 3N_3 + \cdots}} \right] = \tau \left[\cancel{1 +} \right.$$

$$= \tau \left[\frac{N_1 + N_2 + N_3 \cdots}{N_1 + 2N_2 + 3N_3} + \frac{2N_2 + (2+3)N_3 + (2+3+4)N_4}{\cdots} \right]$$

$$= \tau \left[\frac{N_1 + N_2 + N_3}{\cdots} + 2\frac{(N_2 + N_3 + N_4)}{\cdots} + 3\frac{(N_3 + N_4 + N_5 + \cdots)}{} \right]$$

$$= \tau \left[W(1\tau) + 2W(2\tau) + 3W(3\tau) + \cdots \right] = \sum_{k=1}^{\infty} k\tau\, W(k\tau)$$

Nun ist offenbar: $W(1\tau) = 1 - W(n\,n) = 1 - P_n(0)$

Somit kann man setzen $T_1 = \frac{\tau}{W(1\tau)} = \frac{\tau}{1 - P_n(0)}$

$$\lim T_1 = \frac{1}{\lim\left(\frac{W(1\tau)}{\tau}\right)} = \frac{1}{(\text{Wahrsch., dass innerhalb } dt \text{ der } n \text{ Zustand (zum ersten Male) in Nicht-}n \text{ übergehe}) : dt}$$

$$\bar{W}(nnn) + \bar{W}(nnm) = \bar{W}_1(nn) \qquad \cancel{\bar{W}(nnm)}$$

$$\bar{W}(nmn) + \bar{W}(nmm) = \bar{W}_1(nm) = \bar{W}(n) - \bar{W}_1(nn)$$

$$\bar{W}(nnm) + \bar{W}(mnm) = \bar{W}_1(nm) =$$

$$\bar{W}(nmn) + \bar{W}(mmm) = \bar{W}_1(mm) = 1 - 2\bar{W}(n) + \bar{W}_1(nn)$$

$$\bar{W}(nnn) + \bar{W}(nmn) = \bar{W}_2(nn)$$

$$\cancel{\bar{W}(nnm) + \bar{W}(nmm) = \bar{W}_2(nm)}$$

$$\bar{W}(mnm) + \bar{W}(mmm) = \bar{W}_2(mm) = 1 - 2\bar{W}(n) + \bar{W}_2(nn)$$

$$\bar{W}(nmm) - \bar{W}(mnm) = \bar{W}_1(nn) - \bar{W}_2(nn)$$

$$\bar{W}(nnm) + \bar{W}(nmm) = \bar{W}(n) - \bar{W}_2(nn)$$

$$\bar{W}(nmn) - \bar{W}(nnm) = \bar{W}_2(nn) - \bar{W}_1(nn)$$

es fehlt eine Gleichung

Brillouin setzt: $N = \frac{n\xi}{2} \qquad \xi = \sqrt{2Dt}$

$$= n\sqrt{\frac{Dt}{2}}$$

$$\frac{dN}{dt} = \frac{n}{2}\sqrt{\frac{D}{2t}}$$

$\frac{1}{2} \ldots\ldots \frac{4}{\pi}$

in korrekter Weise wäre zu setzen: $\xi = \sqrt{\frac{2}{\pi}}\sqrt{2Dt}$

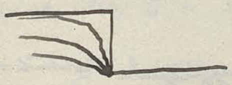
$$= 2\sqrt{\frac{Dt}{\pi}}$$

$\frac{276}{157} = $

$N = n\sqrt{\frac{Dt}{\pi}}$

$\frac{dN}{dt} = \frac{n}{2}\sqrt{\frac{D}{\pi t}}$

Nach Diffusionstheorie:



$$c = c_0 \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{Dt}}} e^{-y^2}\,dy$$

für $t = 0$: $c = c_0$

für $x = 0$: $c = 0$

Also gilt das bei Erhaltung der Null-Konzentration im 0 Punkt

$$\frac{\partial c}{\partial x} = \frac{2c_0}{\sqrt{\pi}} \frac{1}{2\sqrt{Dt}} \cancel{e^{-\frac{x^2}{4Dt}}} = \cancel{\frac{c_0}{\sqrt{\pi Dt}}} \quad \frac{c_0}{\sqrt{\pi D t}}$$

$$D \frac{\partial c}{\partial x} = c_0 \sqrt{\frac{D}{\pi t}}$$

Somit sollte sein: $\frac{dN}{dt} = n\sqrt{\frac{D}{\pi t}}$ =

Somit: $\boxed{D = \frac{A^2 \pi}{4 n^2 t} = \frac{RT}{N} \frac{1}{6\pi\eta P}}$

$$N = 2n\sqrt{\frac{Dt}{\pi}} = A$$

Dagegen würde bei Verteilung

$$c = \frac{c_0}{2}\left[1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{Dt}}} \cdots \right]$$

$t = 0$: $c = c_0$ $\quad x > 0$; $\boxed{c = 0 \quad x < 0}$

$x = 0$: $c = \frac{c_0}{2}$

resultieren:

$$D \frac{\partial c}{\partial x} = \frac{c_0}{2}\sqrt{\frac{D}{\pi t}} \qquad N = n\sqrt{\frac{Dt}{\pi}}$$

$$\boxed{D = \frac{A^2\pi}{n^2 t} = \frac{RT}{N}\frac{1}{6\pi\eta P} \qquad N = RT\,\frac{n^2 t}{A^2 \pi}\,\frac{1}{6\pi\eta P}}$$

Diese Verteilung resultiert in gewöhnlicher Weise durch Superposition der Br. B. (~~zu~~ aller c_0-Stellen). Daher ist es ganz natürlich, dass Brillouin's korrigierte Berechnungsweise eben diesen Wert ergibt.

Zu setzen $\xi = \Delta x$ ist offenbar falsch, da ja Δx das Mittel der weiter und weniger weit vorgerückten Verschiebungen bedeutet. ~~Es setzen wir~~ Falls die Wand ein durchlässiges Gitter wäre, welches den Teilchen wieder den Rückweg gestattet, wäre dies ganz richtig. Aber da ein schon an der Wand einmal angestossenes Teilchen daran kleben bleiben ~~muss~~, wird so die betreffende Anzahl weit unterschätzt.

Also ist wirklich: $N = RT\,\frac{4n^2 t}{A^2\pi}\,\frac{1}{6\pi\eta P}$ es wäre also: $N = 4 \cdot 44 \cdot 10^{22}$! !

Diffusion (von Hg Dampf) lässt sich die Sache so führen, dass die Abhängigkeit von Molekulardichte wahrnehmbar wird?

Es muss immer die diffundierende Menge $\leqq$ Gesamtzahl der ~~stoss~~ verfliegenden Moleküle sein

$$D \frac{\partial c}{\partial x} \leqq c\,\Omega$$

Nun kann D bei Gasen beliebig gross gemacht werden durch Abnahme des Drucks.

(Gesamt oder Partial-?)!

Also z.B. wenn $\Omega = 5.10^4$ $\quad \frac{1}{c}\frac{\partial c}{\partial x} = 1$

so würden bei $p = \frac{1}{100}$ mm Hg schon Divergenzen auftreten

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Menge welche im Vacuum verdampfen würde:

$$c\,\Omega = \rho\,\Omega$$

z.B. $\vartheta = 18^0 \quad p = 0.001$ mm Hg

$$\rho = \frac{10^{-3}}{760}\;\frac{200}{28}\;0.0013 = 1.2.10^{-8}$$

152
608
2128

$$\Omega = 4.8.10^4 \sqrt{\frac{28}{200}} = 4.8.\sqrt{14}\,.10^3$$

$$= 1.4.10^4 \cdot \frac{1}{\sqrt{6\pi}}$$

$$= 3.10^3$$

$$\rho\,\Omega = 3.6.10^{-5}\,\frac{gr}{cm^2\,sek} = 0.12\,\frac{gr}{cm^2.h}$$

Ebenso ist auch

$$k\frac{\partial \theta}{\partial x} \leqq c_v.\rho.\Omega.\theta$$

$c_v\, c\, \rho\, \lambda$

$$\frac{\lambda}{\partial x} \leqq 1$$

Könnte vielleicht in sehr dünnen Schichten (Oberfläche von Flammen, galvanisch erwärmte Drähte in fliessendem Wasser) merklich werden

Dabei würde aber Temperatursprung stören

Fokker Ann. 43 p. 812 Anwendung auf mein Beispiel (Süt.)

$$W(x)\, f(x)\, \tau - \overline{W(x)\, R} + \frac{1}{2}\frac{\partial}{\partial x}\left\{W(x)\, \overline{R^2}\right\} = 0$$

R = Änderung von x infolge unregelm. äusseren Einwirkung

$f(x)$ = Geschw. mit welcher x abnehmen würde, wenn sich selbst überlassen

$$f(x) = +\alpha y^{\mu} x = \beta x$$

$$\overline{R} = 0$$

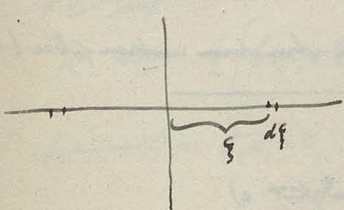
$$\overline{R^2} = 2Dt$$

$$\beta x\, W(x) + D\frac{\partial}{\partial x} W(x) = 0$$

$$\frac{dW}{W} = -\frac{\beta}{D} x\, dx$$

$$W(x) = e^{-\frac{\beta}{2D}x^2}$$ stimmt

Wenn ein Teilchen aus einer Lage $\xi \ldots \xi + d\xi$ ausgeht, so ist die Wahrsch., dass es bis zur Zeit t kein einziges mal durch die Null-Ebene gegangen sei:



$$U = \int_0^\infty \omega\, dx = \frac{1}{2\sqrt{\pi D t}}\int_0^\infty \left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{(x+\xi)^2}{4Dt}}\right] dx$$

$$= \frac{1}{\sqrt{\pi}}\left[\int_{-\frac{\xi}{2\sqrt{Dt}}}^{\infty} e^{-y^2} - \int_{\frac{\xi}{2\sqrt{Dt}}}^{\infty} e^{-y^2}\, dy\right] = \frac{1}{\sqrt{\pi}}\int_{-\frac{\xi}{2\sqrt{Dt}}}^{+\frac{\xi}{2\sqrt{Dt}}} e^{-y^2}\, dy$$

und die Wahrsch., dass es innerhalb der Zeit $t \ldots t + dt$ zum ersten Male durch jene Ebene treten ist:

$$-\frac{dU}{dt}\, dt =$$

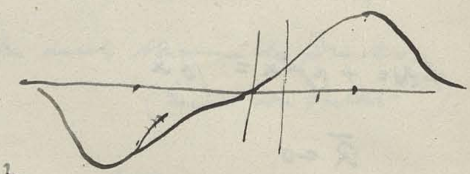
$$= \frac{2}{\sqrt{\pi}}\, \frac{\xi}{4\sqrt{Dt^3}}\, e^{-\frac{\xi^2}{4Dt}}\, dt$$

Maximum: für $-\frac{3}{2\sqrt{t^5}} + \frac{\xi^2}{4Dt^2}\frac{e^{\cdots}}{\sqrt{t^3}} = 0$ ∴ $z^2 e^{-z^2} = \frac{3}{2}$

Wie ist die Verteilung (in einer Säule) in welcher zur Zeit $t=0$ im Punkte $x=0 \ldots x+dx$ ~~$\xi \ldots \xi+d\xi$~~ die Konzentration 1 herrscht und in der Ebene $\cancel{x}\ \xi = a - ct$ die Konzentration Null?

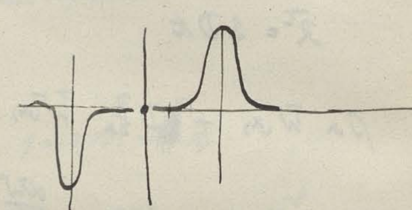
Also die Null-Konzentrations-Ebene soll sich mit konstanter Geschwindigkeit nähern!

Mit der Zeit veränderliche Grenzbedingung!



Darf man setzen: ?

$$\cancel{u = \frac{1}{\sqrt{\pi D t}}\left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{(x+\xi+2(a-ct))^2}{4Dt}}\right]} \ ?$$



$$u = \frac{1}{\sqrt{\pi D t}}\left[e^{-\frac{x^2}{4Dt}} - e^{-\frac{[2(a-ct)-x]^2}{4Dt}}\right]$$

Dann wäre allerdings $u=0$ für

$$[2(a-ct)]^2 - 2x[2(a-ct)] = 0$$

$$x = a - ct$$

aber die Differentialgleichung $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$.

wäre wohl nicht erfüllt! Also ist jene Annahme unbrauchbar!

Es ist dies dieselbe Aufgabe wie:

Diffusion verbunden mit konvektiver Strömung (von Geschwindigkeit c)

$$x = \xi - ct$$

$$du = \frac{\partial u}{\partial x}\underset{\substack{\downarrow \\ = d\xi - c\,dt}}{dx} + \left(\frac{\partial u}{\partial t}\right)_{x=\text{const}} dt = \frac{\partial u}{\partial \xi} d\xi + \left(\frac{\partial u}{\partial t}\right)_{\xi=\text{const}} dt$$

$$\left(\frac{\partial u}{\partial t}\right)_{\xi=\text{const}} = \left(\frac{\partial u}{\partial t}\right)_{x=\text{const}} - c\frac{\partial u}{\partial x}$$

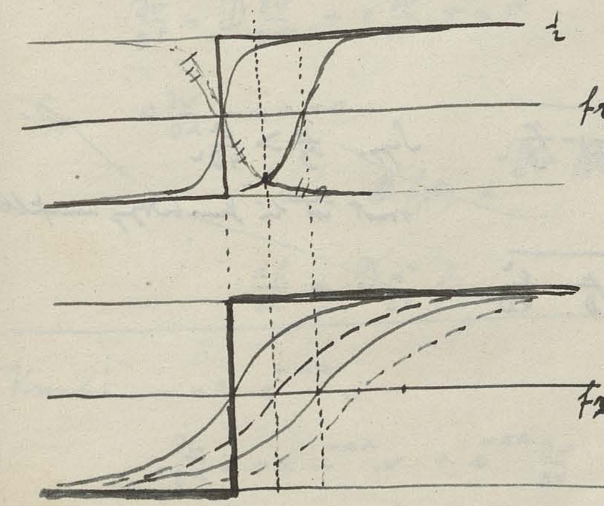
$$\left(\frac{\partial u}{\partial x}\right) = \frac{\partial u}{\partial \xi}$$

Also Transformation auf neue Koordinaten:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial \xi^2} - c\frac{\partial u}{\partial \xi}$$ (Bewegung der Masse in Richtung positiver $\xi \longrightarrow$)

mit Grenzbedingung $u=0$ für $\xi=0$

Partikuläre Lösung: $u = A(ct - \xi)$



$f_0 = f(x) =$

$$f_1 = \frac{1}{2}\left[f(x) + f(x - 2c\tau)\right]$$

$$f_2 = \frac{1}{4}\left[f(x) + 2f(x - 2c\tau) + f(x - 4c\tau)\right]$$

$$f_3 = \frac{1}{2}\left[f_2 + f_2(-2c\tau)\right]$$

$$= \frac{1}{8}\left[f(x) + 3f(x - 2c\tau) + 3f(x - 4c\tau) + f(x - 6c\tau)\right]$$

~~f(x+n\tau)~~

$$f(x, n+1) = \frac{1}{2}\left[f(x, (n)\tau) + f(x - 2c\tau, (n)\tau)\right]_{t=(n+1)\tau}$$

$$\cancel{f(x,(n+1)\tau) = f(x, n\tau) - c\tau\left.\frac{\partial f}{\partial x}\right|_{n\tau} + c^2\tau^2\left.\frac{\partial^2 f}{\partial x^2}\right|_{n\tau}}$$

$$\cancel{\frac{\partial f}{\partial t} = -c\frac{\partial f}{\partial x} + c^2\tau\frac{\partial^2 f}{\partial x^2}}$$

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} \qquad \xleftarrow{c}$$

$u = 0$

$x = 0$

$$u = e^{-\gamma t} f(x)$$

$$D\frac{\partial^2 f}{\partial x^2} + c\frac{\partial f}{\partial x} + \gamma f = 0$$

$$f = e^{\alpha x}$$

$$D\alpha^2 + c\alpha + \gamma = 0$$

$$\alpha = -\frac{c}{2D} \pm \sqrt{\frac{c^2}{4D^2} - \frac{\gamma}{D}}$$

Supp. $\frac{\gamma}{D} > \frac{c^2}{4D^2}$

sonst ist die Grenzbedingung unerfüllbar

$$\cancel{u = e^{-\gamma t}}$$

$$\alpha = -\frac{c}{2D} \pm i\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}$$

$$u = e^{-\gamma t - \frac{c}{2D}x}\ \sin\left(x\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right)$$

$$u = e^{-\gamma t - \frac{cx}{2D}}\ \sin\left[x\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right]$$

$$\rho^2 = \frac{\gamma}{D} - \frac{c^2}{4D^2}$$

$$\gamma = \rho^2 D + \frac{c^2}{4D}$$

~~Lässt sich auch so darstellen~~

$$\cancel{\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial u}{\partial x} + cu\right)}$$

$$U = \frac{e^{-\frac{(x-\xi)^2}{4Dt}}}{2\sqrt{Dt\pi}}$$

$$u = \frac{e^{-\frac{cx}{2D} - \frac{c^2 t}{4D} - \frac{(x-\xi)^2}{4Dt}}}{2\sqrt{\pi D t}}$$

ist ein partikuläres Integral

Damit $\int u\,dx = 1$ wird, muss man nehmen

$$u = \frac{e^{\frac{c(\xi - x)}{2D} - \frac{c^2 t}{4D} - \frac{(x-\xi)^2}{4Dt}}}{2\sqrt{t}\sqrt{\pi D}}$$

$$= \frac{e^{-\frac{(x-\xi+ct)^2}{4Dt}}}{2\sqrt{\pi D t}}$$

also nichts Neues!

$$\int_{-\infty}^{+\infty} u\,dx = \cancel{e^{-\frac{c^2 t}{4D}}} = e^{-\frac{c}{2D}\xi}$$

= constans

zur Zeit $t = 0$ und $x = -\infty$

$u = 0$ überall mit Ausnahme $x = \xi$

zur Zeit $t = \infty$ $u = 0$

Versuch: $u = e^{-\alpha t} v$

$$\frac{\partial u}{\partial t} = -\alpha u + e^{-\alpha t}\frac{\partial v}{\partial t}$$

$$\frac{\partial u}{\partial x} = e^{-\alpha t}\frac{\partial v}{\partial x}$$

$$-\alpha e^{-\alpha t}.v + e^{-\alpha t}\frac{\partial v}{\partial t} = D e^{-\alpha t}\frac{\partial^2 v}{\partial x^2} + c e^{-\alpha t}\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial x^2} + c\frac{\partial v}{\partial x} + \alpha v$$

~~Supp~~ $c\frac{\partial v}{\partial x} + \alpha v = 0$

$$v = f_1(t)\, e^{-\frac{\alpha}{c}x}$$

$$\frac{df}{dt} = D\frac{\alpha^2}{c^2} f$$

Versuch: $u = e^{-\alpha x} v$ $\qquad u = v.e^{-\frac{cx}{2D}}$

$$\frac{\partial u}{\partial x} = -\alpha e^{-\alpha x}.v + e^{-\alpha x}\frac{\partial v}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \alpha^2 e^{-\alpha x}.v - 2\alpha e^{-\alpha x}\frac{\partial v}{\partial x} + e^{-\alpha x}\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = D\left[\frac{\partial^2 v}{\partial x^2} - 2\alpha\frac{\partial v}{\partial x} + \alpha^2 v\right] + c\left[\frac{\partial v}{\partial x} - \alpha v\right]$$

$$= D\frac{\partial^2 v}{\partial x^2} + \underbrace{(c - 2\alpha D)}_{=0}\frac{\partial v}{\partial x} + \underbrace{(\alpha^2 D - c\alpha)}_{\frac{c^2}{4D} - \frac{c^2}{2D} = -\frac{c^2}{4D}} v$$

$$\alpha = \frac{c}{2D}$$

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial x^2} - \frac{c^2}{4D} v \qquad u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}}.\mathcal{U}$$

$$v = e^{-\frac{c^2}{4D}t}.\mathcal{U} \qquad \frac{\partial \mathcal{U}}{\partial t} = D\frac{\partial^2 \mathcal{U}}{\partial x^2} \qquad = e^{-\frac{c}{4D}(2x+ct)}.\mathcal{U}$$

$$\cancel{-\gamma\mathcal{U} + \frac{\partial\mathcal{U}}{dt} = D\frac{\partial^2\mathcal{U}}{\partial x^2} - \gamma\mathcal{U}}$$

$$u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \cdot U$$

wenn nun $U = \frac{1}{\sqrt{\pi}} \int_{0}^{\frac{x}{2\sqrt{Dt}}} e^{-y^2}\, dy$

angenommen wird

so ist für: $t=0$, $x>0$:

$$u = \frac{1}{2} e^{-\frac{cx}{2D}}$$

für $x=0$, t beliebig

$$u=0$$

Andrer Versuch:

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + c \frac{\partial u}{\partial x}$$

$$u = \varphi\Big[(\overbrace{ct+x}^{z}), t\Big]$$

$$\cancel{\frac{\partial u}{\partial t} = \frac{\partial \varphi}{\partial z} c}$$

$$\cancel{\frac{\partial u}{\partial x} = -\frac{\partial \varphi}{\partial z} + \frac{\partial \varphi}{\partial x}}$$

$$\cancel{\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 \varphi}{\partial x^2} - 2\frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \varphi}{\partial z^2}}$$

$$\frac{\partial u}{\partial t} = c \frac{\partial \varphi}{\partial z} + \frac{\partial \varphi}{\partial t}$$

$$\frac{\partial u}{\partial x} = + \frac{\partial \varphi}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 \varphi}{\partial z^2}$$

$$\cancel{c\frac{\partial \varphi}{\partial z}} + \frac{\partial \varphi}{\partial t} = D \frac{\partial^2 \varphi}{\partial z^2} + \cancel{c \frac{\partial \varphi}{\partial z}}$$

Also ist jede Lösung von: $\frac{\partial \varphi}{\partial t} = D \frac{\partial^2 \varphi}{\partial z^2}$

anwendbar, wenn statt z eingesetzt wird: $z = ct+x$

$$\frac{1}{2a\sqrt{t}} \int_{-\infty}^{0} \Phi(z)\, e^{-\frac{(z-ct)^2}{4a^2 t}}\, dz = -\int_{0}^{\infty} e^{-\frac{(z-ct)^2}{4a^2 t}} \frac{dz}{2a\sqrt{t}} = -\int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2}\, d\beta$$

$$u_x = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(x + 2a\beta\sqrt{t})\, e^{-\beta^2} d\beta$$

$$\int_{-\infty}^{+\infty}$$

für ~~...~~ $x > \cancel{...}0$ $u = \text{const} = 1$ <u>für $t=0$</u>

$x = ct \qquad u = 0$

$x=0 \qquad u=0$

$$\int_{-\infty}^{+\infty} \underbrace{\Phi(2a\beta\sqrt{t})}_{\cancel{+}} e^{-\beta^2} d\beta = 0$$

$$0 = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(\underbrace{ct + 2a\beta\sqrt{t}}_{z})\, e^{-\beta^2} d\beta$$

$$\Phi(+x) \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\beta^2} d\beta = 1 \quad \therefore \Phi(+x) = 1$$

$$ct + 2a\beta\sqrt{t} = z$$

$$\beta = \frac{z - ct}{2a\sqrt{t}}$$

$$0 = \frac{1}{\sqrt{\pi}} \int_{\beta=-\infty}^{\beta = -\frac{c}{2a}\sqrt{t}} \Phi(-z)\, e^{-\beta^2} d\beta + \frac{1}{\sqrt{\pi}} \int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2} d\beta$$

$$\int_{\beta=-\infty}^{\beta=-\frac{c}{2a}\sqrt{t}} \Phi(\underbrace{ct +}_{<0} 2a\beta\sqrt{t})\, e^{-\beta^2} d\beta = -\int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2} d\beta$$

$$\int_{z=-\infty}^{z=0} \Phi(z)\, e^{-\left(\frac{z-ct}{2a\sqrt{t}}\right)^2} \frac{dz}{2a\sqrt{t}} = -\int_{-\frac{c}{2a}\sqrt{t}}^{\infty} e^{-\beta^2} d\beta$$

$$u_x = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\beta = -\frac{x}{2a\sqrt{t}}} \underset{<0}{\Phi(z)}\, e^{-\beta^2} d\beta + \int_{z=0}^{\infty} e^{-\beta^2} d\beta$$

$$z=0 \to \beta = -\frac{x}{2a\sqrt{t}}$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{0} \Phi(z)\, e^{-\left(\frac{z-x}{2a\sqrt{t}}\right)^2} \frac{dz}{2a\sqrt{t}} + \int_{-\frac{x}{2a\sqrt{t}}}^{\infty} e^{-\beta^2} d\beta$$

$$u_x = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(x + 2a\beta\sqrt{t})\, e^{-\beta^2} d\beta \quad = \int_{-\infty}^{-\frac{x}{2a\sqrt{t}}} + \int_{-\frac{x}{2a\sqrt{t}}}^{\infty}$$

Annahme:

$x > 0 \qquad t = 0 \qquad u = 1$

$x = 0 \qquad t = t \qquad u = 0$

$$\int_{-\infty}^{+\infty} \Phi(2a\beta\sqrt{t})\, e^{-\beta^2} d\beta = 0 \quad = \int_{-\infty}^{0} + \int_{0}^{\infty} \quad = \frac{\sqrt{\pi}}{2} + \int_{-\infty}^{0} \quad = \frac{\sqrt{\pi}}{2} + \int_{0}^{\infty} \Phi(-2a\beta\sqrt{t})\, e^{-\beta^2} d\beta$$

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \Phi(+x)\, e^{-\beta^2} d\beta = 1 \quad = \Phi(+x) \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\beta^2} d\beta = \Phi(x) = 1$$

$$\frac{1}{2a\sqrt{t}} \int_0^{\infty} \Phi(-z)\, e^{-\frac{z^2}{4a^2 t}} dz = +\frac{\sqrt{\pi}}{2} \longrightarrow$$

Czy z tego z konieczności wynika że

$\Phi(-z) = 1$?

$$u_x = \frac{1}{\sqrt{\pi}} \int_0^{\infty} \Phi(-z)\, e^{-\frac{(z-x)^2}{4a^2 t}} \frac{dz}{\cdots} - \frac{1}{\sqrt{\pi}} \int_0^{\infty} \Phi(z)\, e^{-\frac{(z+x)^2}{4a^2 t}} \frac{dz}{\cdots}$$

$$\cancel{\frac{1}{2a\sqrt{\pi t}} \int_0^{\infty} \left[e^{-\frac{(\alpha-x)^2}{4a^2 t}} + e^{-\frac{(\alpha+x)^2}{4a^2 t}} \right] d\alpha = \frac{1}{2a\sqrt{\pi t}} \int_{-\frac{x}{2a\sqrt{t}}}^{+\frac{x}{2a\sqrt{t}}} e^{-\beta^2} d\beta}$$

$$\alpha \int_0^{\infty} \Phi(-z)\, e^{-\alpha^2 z^2} dz = \frac{\sqrt{\pi}}{2}$$

$$\int_0^{\infty} \Phi(yz)\, e^{-z^2} dz = \frac{\sqrt{\pi}}{2}$$ dla jakiegobądź y

czy to koniecznie powoduje że aby $\Phi = 1$?

~~Tak też pisze w którym równaniem ... ostatnie wyst...~~

$$\int_0^\infty \Phi^{(n)}_{y(z)}\, z^n e^{-2z}\, dz = 0 \qquad \text{przy czym } \Phi^{(n)}_{y\,z} \qquad \int e^{i\alpha(x-\xi)}\, d\xi = \frac{e^{i\alpha(x-\xi)}}{i\alpha}\Big|_0^\infty = \frac{e^{i\alpha x}}{i\alpha}$$

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x}$$

$$u = e^{i\alpha x - \gamma t} \qquad\qquad u = e^{-i\alpha x - \gamma t}$$

$$-D\alpha^2 + c\alpha i + \gamma = 0$$

$$\gamma = D\alpha^2 - c\alpha i \qquad\qquad \gamma = D\alpha^2 + c\alpha i$$

$$u = \frac{e^{i\alpha x + c\alpha i t - D\alpha^2 t} - e^{-i\alpha x - c\alpha i t - D\alpha^2 t}}{2i}$$

$$= e^{-D\alpha^2 t}\,\cancel{\cos(\alpha x + c\alpha t) + i}\,\sin(\alpha x + c\alpha t) + \ldots$$

$$= e^{-D\alpha^2 t}\,\underset{\cos}{\sin}\,\alpha(x + ct)$$

Ale w takim razie nie byłoby $u_{x=0} = 0$?

W każdym razie warunki:

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\alpha \int_{-\infty}^{+\infty}$$

$$u_{\substack{x\\t=0}} = \int_{-\infty}^{+\infty}\left[f(\alpha)\, e^{i\alpha x}\, d\alpha + g(\alpha)\, e^{-i\alpha x}\, d\alpha\right] = 1 \quad ((\; x > 0$$

$$u_{x=0} = \int_{-\infty}^{+\infty} e^{-D\alpha^2 t}\left[f(\alpha)\, e^{ic\alpha t}\, d\alpha + g(\alpha)\, e^{-ic\alpha t}\, d\alpha\right] = 0$$

z tego:

$$u_{x\,t} = \int_{-\infty}^{+\infty}\left[f(\alpha)\, e^{i\alpha(x+ct)} + g(\alpha)\, e^{-i\alpha(x+ct)}\right] e^{-D\alpha^2 t}\, d\alpha$$

Jest zamo... z równaniem pierwszym Kirchhoffa

~~Więc porównanie do Fourier planowanej~~

$$\cancel{\frac{\partial u}{\partial t} = D \frac{\partial u}{\partial x^2}}$$

$$u = \int_{\gamma = \frac{c^2}{4D}}^{+\infty} e^{-\gamma t - \frac{cx}{2D}} \sin\left[x \sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right] f(\gamma)\, d\gamma$$ erfüllt die Bedingungen $x=0$ $u=0$ für beliebige t

für $t=0$ soll sein:

$$\underbrace{u = \int_{-\infty}^{+\infty} e^{-\frac{cx}{2D}} \sin\left(x \sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}\right) f(\gamma)\, d\gamma}_{} = 1 \quad \text{für } x > 0$$

$$= e^{-\frac{cx}{2D}} \int_{-\infty}^{+\infty} f(\gamma) \sin\left(x \overbrace{\sqrt{\frac{\gamma}{D} - \frac{c^2}{4D^2}}}^{\beta}\right) d\gamma = 1$$

$$f(x) = \frac{1}{\pi} \int_0^{\infty} d\alpha \int_{-\infty}^{+\infty} f(\gamma) \cos \alpha(\gamma - x)\, d\gamma$$

$$\beta^2 = \frac{\gamma}{D} - \frac{c^2}{4D^2}$$

$$\gamma = \beta^2 D + \frac{c^2}{4D}$$

Übrigens könnte man auch setzen:

$$u = \int_{\beta=0}^{\infty} e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t - \frac{cx}{2D}} \sin \beta x \cdot f(\beta)\, d\beta$$

$$u_0 = 1 = e^{-\frac{cx}{2D}} \int_0^{\infty} f(\beta) \sin \beta x \, d\beta = \Phi(x)$$

$$f(y) = \frac{2}{\pi}\int_0^\infty \sin xy \, dx \int_0^\infty f(\beta) \sin \beta x \, d\beta$$

$$f(y) = e^{-ny} \qquad \int_0^\infty e^{-n\beta} \sin\beta x \, d\beta = \frac{x}{x^2+n^2}$$

Gdyby było c ze znakiem przeciwnym

$$1 = e^{\frac{cx}{2D}} \int_0^\infty f(\beta) \sin\beta x \, d\beta$$ można by pomocą założyć $f(\beta) = \frac{\beta}{\beta^2 + \left(\frac{c}{2D}\right)^2}$

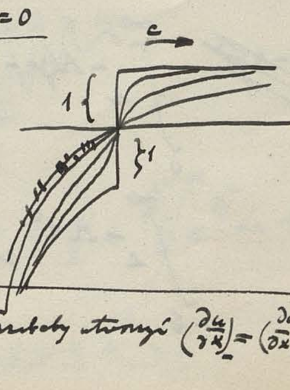
Więc byłoby:

$$u = \int_{\beta=0}^\infty e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t + \frac{cx}{2D}} \frac{\beta}{\beta^2 + \frac{c^2}{4D^2}} \cdot \sin\beta x \cdot d\beta$$

Spełnia równanie $\frac{\partial u}{\partial t} - D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} = 0$

oraz warunki $t=0$ $x>0$ $u=1$

$x=0$ $u=0$



Dla $x<0$ będzie $u = -e^{\frac{cx}{D}}$

dla $t=0$

Czy jest ciągłość dla $x=0$ między jednym a drugim rozwiązaniem? Trzeba by utworzyć $\left(\frac{\partial u}{\partial x}\right)_- = \left(\frac{\partial u}{\partial x}\right)_+ \Big|_{x=0}$?

$$f(x) = \frac{1}{\pi}\int_0^\infty d\alpha \int_{-\infty}^{+\infty} f(\mu)\left[\cos\alpha\mu \cos\alpha x + \sin\alpha\mu \sin\alpha x\right] d\mu$$

jeśli $f(-\mu) = -f(\mu)$

$$f(x) = \frac{2}{\pi}\int_0^\infty d\alpha \int_{\infty}^{+\infty} f(\mu) \sin\alpha\mu \sin\alpha x \, d\mu \qquad = \sqrt{2}\cdot\frac{e^{-\frac{(x-x_0)^2}{2}}}{\sqrt{\pi}}\cdot e^{-\frac{cx}{2D}} = \Phi(x)$$

$$\int_{-\infty}^{+\infty} e^{-\frac{(\mu-x_0)^2}{2}+\cancel{\frac{c\mu}{2D}}} \sin\alpha\mu \, d\mu =$$

$$f(\beta) = \frac{2}{\pi}\int_0^\infty e^{-\frac{(\mu-x_0)^2}{2}+\frac{c\mu}{2D}} \sin\beta\mu \, d\mu$$

$$u = \frac{2}{\pi}\sqrt{\frac{2}{\pi}}\int_0^\infty\int_0^\infty e^{-\left(\beta^2 D+\frac{c^2}{4D}\right)t - \frac{cx}{2D} - \frac{(\mu-x_0)^2}{2} + \frac{c\mu}{2D}} \sin\beta x \sin\beta\mu \, d\mu \, d\beta$$

~~dla~~ $t=0$: $$u = \frac{2}{\pi}\sqrt{\frac{2}{\pi}}\iint e^{-\frac{cx}{2D}} e^{-\frac{(\mu-x_0)^2}{2}+\frac{c\mu}{2D}} \sin\beta\mu \, d\mu \cdot \sin\beta x \, d\beta$$

$$\cancel{= e^{-\frac{cx}{2D}}\sqrt{\frac{2}{\pi}}\, e^{\frac{cx}{2D}-\frac{(x-x_0)^2}{2}}}$$

$$\cancel{\int e^{-A\mu^2}} \int_0^\infty e^{-A(\mu-b)^2} \sin a\mu \, d\mu = ?$$

$$\int_{-b}^\infty e^{-Az^2} \sin a(z+b)\, dz = \cos ab \int_{-b}^\infty e^{-Az^2} \sin az \, dz$$

$$+ \sin ab \int_{-b}^\infty e^{-Az^2} \cos az \, dz$$

W każdym razie

$$\lim_{\varepsilon=0} \sqrt{\frac{\varepsilon}{\pi}} \int_0^\infty e^{-\frac{(y-x_0)^2}{\varepsilon} + \frac{cy}{2D}} \sin \beta y \, dy = \sin \beta x_0$$

$$u = \frac{2}{\pi} \int_0^\infty e^{-\left(\beta^2 D + \frac{c^2}{4D}\right) t - \frac{cx}{2D}} \sin \beta x_0 \sin \beta x \, d\beta$$

$t=0$:

$$u = e^{-\frac{cx}{2D}} \frac{2}{\pi} \int_0^\infty \sin \beta x \sin \beta x_0 \, d\beta$$

$x \lessgtr x_0 \quad = ?$

$x = x_0 \quad \int_0^\infty (\sin \beta x_0)^2 d\beta$

$$\omega = \int_0^\infty e^{-\alpha^2} \sin \alpha x \, d\alpha$$

$$\frac{\partial \omega}{\partial x} = \int_0^\infty \alpha e^{-\alpha^2} \cos \alpha x \, d\alpha \quad \cancel{= \frac{1}{2\pi} \sqrt{\frac{\pi}{\beta}} e^{-\frac{x^2}{4\beta}}}$$

$\cancel{\omega = \int \frac{\partial \omega}{\partial x} dx}$

$$(e^{-\alpha^2} \cos \alpha x)\Big|_0^\infty = -2 \int_0^\infty \alpha e^{-\alpha^2} \cos \alpha x \, d\alpha - x \int_0^\infty e^{-\alpha^2} \sin \alpha x \, d\alpha$$

$$-1 = -2 \frac{d\omega}{dx} - x\omega$$

$$\int_0^\infty e^{-\alpha^2} \sin \alpha x \, d\alpha = \frac{e^{-\frac{x^2}{4}}}{2} \int_0^x e^{\frac{y^2}{4}} dy$$

$$2\frac{d\omega}{dx} + \omega x = 1 \quad \Big| \; y$$

$$2\frac{dy}{dx} + yx = 0 \quad \Big| \; \omega$$

$$2\frac{\left(y \frac{d\omega}{dx} - \omega \frac{dy}{dx}\right)}{y^2} = \frac{1}{y}$$

$$2 \frac{d}{dx}\left(\frac{\omega}{y}\right) = \frac{1}{y}$$

$$\omega = \frac{y}{2} \int \frac{1}{y} dx = \frac{1}{2} e^{-\frac{x^2}{4}} \left[\int^x e^{\frac{x^2}{4}} dx + C\right]$$

. . . .

$$-\frac{x}{2} e^{-\frac{x^2}{4}} \int + 1 + \frac{x}{2} \ldots$$

Versuch

$$u = A\frac{2}{\pi}\int e^{-\frac{c^2}{4D}t-\frac{cx}{2D}}\int_0^\infty e^{-\beta^2 Dt}\underbrace{\cos\beta x\cos\beta x_0\,d\beta}$$

$$\left[-\cos\beta(x+x_0)+\cos\beta(x-x_0)\right].$$

$$=A\frac{2}{\pi}\underbrace{e^{-\frac{c^2t}{4D}-\frac{cx}{2D}}}_{U}\cdot\underbrace{\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\left\{e^{-\frac{(x-x_0)^2}{4Dt}}-e^{-\frac{(x+x_0)^2}{4Dt}}\right\}}_{V}$$

$$\frac{\partial u}{\partial t}-D\frac{\partial^2 u}{\partial x^2}-c\frac{\partial u}{\partial x}=U\Big(\underbrace{\frac{\partial}{\partial t}-D\frac{\partial^2}{\partial x^2}}_{=0}-c\frac{\partial}{\partial x}\Big)V+V\Big(\frac{\partial}{\partial t}-D\frac{\partial^2}{\partial x^2}-c\frac{\partial}{\partial x}\Big)U-2D\frac{\partial U}{\partial x}\frac{\partial V}{\partial x}$$

$$=+c\,\frac{2(x-x_0)}{4Dt}V_1-c\,\frac{2(x+x_0)}{4Dt}V_2+\cancel{(V_1-V_2)\left(-\frac{c^2}{4D}-\frac{c^2}{4D}+\frac{c^2}{2D}\right)}+\cancel{2D}\frac{c}{\cancel{2D}}\left[-\frac{(x-x_0)}{2Dt}V_1+\frac{(x+x_0)}{2Dt}V_2\right]$$

$$=0.$$

Also Differentialgleichung ist erfüllt.

Anfangsbedingungen:

für $x=0$ $\quad u=0$ bei beliebigem t

für $t=0$ $\quad u=0$ für alle x, mit Ausnahme $x=x_0$

dann ist $\int_0^\infty u\,dx=\frac{2}{\cancel{\pi}}e^{-\frac{cx_0}{2D}}\cdot A=1$

$$A=\frac{1}{2}e^{\frac{cx_0}{2D}}$$

$$\lim_{x_0=\infty} u = e^{-\frac{c^2t}{4D}}\left[e^{\frac{x x_0}{2Dt}} - e^{-\frac{x x_0}{2Dt}}\right] =$$

Somit ist:

$$u = \frac{e^{-\frac{c^2t}{4D} - \frac{c(x-x_0)}{2D}}}{2\sqrt{\pi D t}}\left\{e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}}\right\}$$

unter Bedingung: $= e^{-\frac{[ct+x-x_0]^2}{4Dt}}$

$x+x_0 \gg x-x_0$ also wenn x und x_0 nahe gleich sind

$$= \frac{1}{2\sqrt{\pi D t}}\left\{e^{-\frac{(x-x_0+ct)^2}{4Dt}} - e^{-\frac{(x+x_0)^2+2ct(x-x_0)+c^2t^2}{4Dt}}\right\}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \frac{e^{-\frac{c^2t}{4D}+\frac{cx_0}{2D}}}{2\sqrt{\pi D t}}\underbrace{\left[-\frac{2(x-x_0)}{4Dt}+\frac{2(x+x_0)}{4Dt}\right]_{x=0}}_{\frac{x_0}{Dt}} e^{-\frac{x_0^2}{4Dt}} = \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi(Dt)^3}}\cdot x_0$$

Wahrsch. dass ein (von x_0 ausgehendes) Teilchen „zum ersten Mal" zwischen t und $t+dt$ durch die Null Ebene durchtritt:

$$D\left(\frac{\partial u}{\partial x}\right)_{x=0} dt = x_0 \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi D t^3}}\,dt$$

Vielleicht aber $D\left.\frac{\partial u}{\partial x}\right|_0 + c\,u|_0 =$
ist identisch damit da $u_0=0$

Dagegen Fletcher: $b = x_0$ $\quad h = \frac{1}{4D}$

$$\frac{1}{2}\sqrt{\frac{1}{4\pi D}}\,\frac{1}{\sqrt{t^3}}(x_0+ct)\,e^{-\frac{(x_0-ct)^2}{4Dt}}\,dt = \frac{(x_0+ct)\,e^{-\frac{(x_0-ct)^2}{4Dt}}}{4\sqrt{\pi D t^3}}$$

Also ist bei raschem Fall, wo annähernd $x_0=ct$, das Resultat dasselbe, dagegen im Grenzfalle verschwindend langsamen Falles (im Vergl. zur Br. B.) Resultat $\frac{1}{2}$ mal.

Durchschnittliche Fallzeit:

$$T = x_0 \int_0^\infty \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\, dt$$

$$\frac{x_0}{2\sqrt{D}\,c} - \frac{ct}{2\sqrt{D}} = \frac{z\sqrt{t}}{c}$$

$$\frac{x_0 - ct}{2\sqrt{Dt}} = z \qquad t + \frac{2z}{c}\sqrt{Dt} = \frac{x_0}{c}$$

$$\frac{x_0}{2c\sqrt{D}} = \frac{t}{2\sqrt{D}} + \frac{z}{c}\sqrt{t}$$

$$\cancel{\sqrt{t} = -\frac{z}{2c} \pm \sqrt{\frac{x_0}{2c\sqrt{D}} + \frac{z^2}{4c^2}}}$$

$$\sqrt{t} = -\frac{z\sqrt{D}}{c} \pm \sqrt{\frac{x_0}{c} + \frac{z^2 D}{c^2}}$$

$$\frac{1}{2}\frac{dt}{\sqrt{t}} = dz\left\{-\frac{1}{2c} \pm \frac{z}{4c^2}\frac{1}{\sqrt{\frac{z^2}{4c^2} + \frac{x_0}{2c\sqrt{D}}}}\right\}$$

$$T = \frac{-x_0}{2c\sqrt{\pi D}}\left\{\int_{z=+\infty}^{z=0}\left[1 \pm \frac{z}{2c}\frac{1}{\sqrt{\frac{z^2}{4c^2}+\frac{x_0}{2c\sqrt{D}}}}\right]e^{-z^2}dz + \int_{z=0}^{z=-\infty}\left[1 - \frac{z}{2c}\frac{1}{\sqrt{\frac{z^2}{4c^2}+\frac{x_0}{2c\sqrt{D}}}}\right]e^{-z^2}dz\right\}$$

$$= \frac{x_0}{2c\sqrt{\pi D}}\left\{\underbrace{\int_{-\infty}^{+\infty} e^{-z^2}dz}_{\sqrt{\pi}} \underbrace{- \int_{-\infty}^{0} + \int_0^{+\infty} \frac{z\, e^{-z^2}}{2c\sqrt{\frac{z^2}{4c^2}+\frac{x_0}{2c\sqrt{D}}}}\,dz}_{}\right\}$$

$$= 2\int_0^{+\infty} \frac{z\, e^{-z^2}}{\sqrt{z^2 + \frac{2c x_0}{\sqrt{D}}}}\,dz$$

$$= \int_0^\infty \frac{e^{-y}\,dy}{\sqrt{y + \frac{2cx_0}{\sqrt{D}}}} = e^{\frac{2cx_0}{\sqrt{D}}}\int_{\frac{2cx_0}{\sqrt{D}}}^\infty \frac{e^{-z}dz}{\sqrt{z}} = 2e^{\frac{2cx_0}{\sqrt{D}}}\int_{x=\sqrt{\frac{2cx_0}{\sqrt{D}}}}^\infty e^{-x^2}dx$$

$$\frac{1}{2}\frac{dt}{\sqrt{t}} = \left\{ -\frac{\sqrt{D}}{c} \overset{}{\underset{?}{\pm}} \frac{\frac{zD}{c^2}}{\sqrt{\frac{x_0}{c} + \frac{z^2 D}{c^2}}} \right\} dz$$

$$T = \frac{+x_0}{c\sqrt{\pi}} \left\{ \int_{-\infty}^{+\infty} e^{-z^2} dz + 2\int_0^\infty \frac{z\, e^{-z^2} dz}{\sqrt{z^2 + \frac{x_0 c}{D}}} \right\}$$

$$T = \frac{x_0}{c}\left\{1 + \frac{1}{\sqrt{\pi}}\, e^{\frac{cx_0}{D}} \underbrace{\int_{x=\sqrt{\frac{cx_0}{D}}}^{\infty} e^{-x^2} dx}\right\} \; ?$$

$$= e^{-x^2}\left[\frac{1}{2x} - \frac{2}{(2x)^3} \cdots\right] = \frac{e^{-\frac{cx_0}{D}}}{2\sqrt{\frac{cx_0}{D}}}\left[1 - \frac{D}{2cx_0}\right]$$

also für kleines $\frac{cx_0}{D}$: $T = \frac{3x_0}{2c}$

grosses : $T = \frac{x_0}{c}$

$$= \frac{x_0}{c}\left\{1 + \frac{1}{2\sqrt{\pi}}\sqrt{\frac{D}{cx_0}}\left[1 - \cdots\right]\right\} \; ?$$

Dagegen bei Fletcher: $T = \frac{x_0}{c} + \frac{D}{c^2} = \frac{x_0}{c}\left\{1 + \frac{D}{cx_0}\right\}$ ‖ für kleines $\frac{cx_0}{D}$: $T = \infty$!

grosses $T = \frac{x_0}{c}$

$$\frac{TD}{x_0^2} = \varepsilon + \frac{1}{2\sqrt{\pi}}\sqrt{\varepsilon^3}$$

~~…~~ Argument für kleine ε:

$$\varepsilon = \frac{TD}{x_0^2}\left[1 - \frac{1}{2\sqrt{\pi}}\sqrt{\frac{TD}{x_0^2}}\right] \; ?$$

Somit: $c = \frac{D}{\varepsilon x_0} = \frac{x_0}{T}\cdot\frac{1}{1 - \frac{1}{2x_0}\sqrt{\frac{TD}{\pi}}}$

$$c = \frac{x_0}{T}\left(1 + \frac{1}{2x_0}\sqrt{\frac{TD}{\pi}}\right) = \frac{x_0}{T} + \frac{1}{2}\sqrt{\frac{D}{T\pi}}$$

während Fletcher: $c = \frac{x_0}{T}\left(1 + \frac{TD}{x_0^2}\right)$

Im extremen Fall $c=0$:

$$I = \frac{\xi}{\sqrt{\pi}} \int_{t=0}^{\infty} \frac{e^{-\frac{\xi^2}{4Dt}}}{2\sqrt{Dt}}\, dt$$

$$\frac{\xi^2}{4Dt} = y^2$$

$$\sqrt{\frac{t}{D}} = \frac{\xi}{2D y}$$

$$\frac{dt}{2\sqrt{Dt}} = -\frac{dy}{2D\, y^2} \cdot \xi$$

$$= -\frac{\xi^2}{\sqrt{\pi}} \frac{1}{2D} \int_{\infty}^{0} \frac{e^{-y^2}\, dy}{y^2} = \infty$$

Weiss und Ehrenhaft setzen:

$$\overline{\lambda^2} = \frac{\overline{\Lambda^2}}{t} = \frac{1}{n} \sum \frac{\Lambda^2}{t} = \frac{1}{n} \sum \frac{(l - v_m t)^2}{t} = \frac{l^2}{t_m^2} \left[\overline{\frac{(t_m - t)^2}{t}} \right] =$$

$$= \frac{l^2}{n} \sum \frac{\left(1 - \frac{t}{t_m}\right)^2}{t} = l^2 \left[\left(\overline{\frac{1}{t}}\right) - \frac{2}{t^m} + \frac{\overline{t}}{(t^m)^2} \right]$$

$$= l^2 \left[\left(\overline{\frac{1}{t}}\right) - \frac{1}{t_m} \right] \qquad (\overline{t} = t_m$$

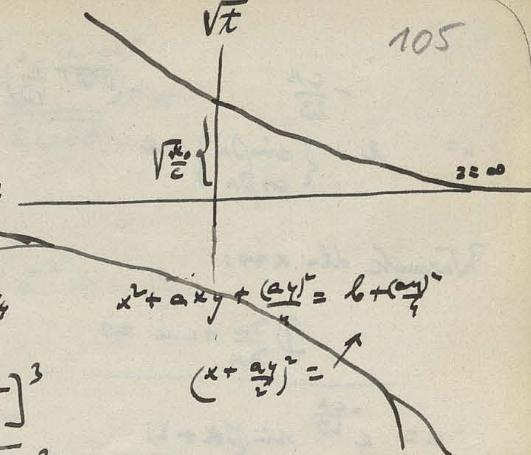
$$\left(\overline{\frac{1}{t}}\right) = x_0 \int_0^{\infty} \frac{1}{t^2} \frac{e^{-\frac{(x_0 - ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\, dt$$

$$\sqrt{t} = -\frac{z\sqrt{D}}{c} \pm \sqrt{\frac{x_0}{c} + \frac{z^2 D}{c^2}}$$

$$= \frac{\sqrt{D}}{c} \left[-z \pm \sqrt{z^2 + \frac{c x_0}{D}} \right]$$

$$\frac{1}{2} \frac{dt}{\sqrt{t}} = \frac{\sqrt{D}}{c} \left[-1 \pm \frac{z}{\sqrt{}} \right] dz = \frac{\mp\sqrt{D}}{c} \frac{(-z \pm \sqrt{})}{\sqrt{}}\, dz$$

$$\overline{\left(\frac{1}{t}\right)} = \frac{x_0}{\sqrt{\pi D}} \cancel{\int_{\infty}^{0}} + \int_{0}^{-\infty}$$



$$x^2 + axy + \frac{(ay)^2}{4} = b + \frac{(ay)^2}{4}$$

$$\left(x + \frac{ay}{2}\right)^2 =$$

$$= \frac{x_0}{\sqrt{\pi D}} \frac{\sqrt{D}}{c} \int_{-\infty}^{+\infty} \frac{\left(z - \sqrt{z^2 + \frac{cx_0}{D}}\right) e^{-z^2} dz}{\sqrt{z^2 + \frac{cx_0}{D}} \left(\frac{\sqrt{D}}{c}\right)^4 \left[z - \sqrt{}\right]^4}$$

$$= -\frac{x_0 c^3}{\cancel{2}\sqrt{\pi} D^2} \int_{-\infty}^{+\infty} \frac{e^{-z^2} dz}{\sqrt{z^2 + \frac{cx_0}{D}} \left[z - \sqrt{z^2 + \frac{cx_0}{D}}\right]^3} \frac{\left[z + \sqrt{}\right]^3}{\left[z + \sqrt{}\right]^3}$$

$$= \frac{x_0 c^3}{D^2 \sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{e^{-z^2} dz \left[\cancel{z^3} + 3z^2 \sqrt{} + 3z\cancel{\left(z^2 + \frac{cx_0}{D}\right)} + \sqrt{}^{\,3}\right]}{\left(\frac{cx_0}{D}\right)^3 \sqrt{z^2 + \frac{cx_0}{D}}}$$

$$= \frac{x_0 c^3}{D^2 \sqrt{\pi}} \int \frac{e^{-z^2} dz}{\left(\frac{cx_0}{D}\right)^3} \left(3z^2 + z^2 + \frac{cx_0}{D}\right) = \frac{D}{x_0^2 \sqrt{\pi}} \int_{-\infty}^{+\infty} \left[4z^2 + \frac{cx_0}{D}\right] e^{-z^2} dz$$

$$= \frac{c}{x_0} + \frac{4D}{x_0^2 \sqrt{\pi}} \int_{-\infty}^{+\infty} \underbrace{z^2 e^{-z^2} dz}_{\frac{\sqrt{\pi}}{2}}$$

$$= \frac{c}{x_0} + \frac{2D}{x_0^2}$$

$$\overline{\left(\frac{1}{t}\right)} - \frac{1}{t_m} = \frac{2D}{x_0^2}$$

$\overline{\lambda^2} = \cancel{t}\, 2D$! Also wäre Weiss' Berechnungs-Methode ganz richtig!

$$u = e^{-\frac{cx}{2D}} \left\{ \begin{matrix} \sin\beta x \\ \cos\beta x \end{matrix} \right\} \cdot e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t} f(\beta)\, d\beta$$

Warunek dla $x=0$:

$$D\frac{\partial u}{\partial x} + cu = 0$$

$$u = e^{-\frac{cx}{2D}} \sin(\beta x + \varepsilon)$$

$$\frac{\partial u}{\partial x} = e^{-\frac{cx}{2D}} \beta\cos(\beta x+\varepsilon) - \frac{c}{2D} u$$

$$D\beta\cos\varepsilon + \frac{c}{2}\sin\varepsilon = 0$$

$$tg\,\varepsilon = -\frac{2D\beta}{c}$$

$$u = e^{-\frac{cx}{2D}} \int \frac{\sin\beta x \mp \cos\beta x \cdot \frac{2D\beta}{c}}{\sqrt{1+\frac{4D^2\beta^2}{c^2}}} e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t} f(\beta)\, d\beta$$

$$= e^{-\frac{cx}{2D} \mp \frac{c^2}{4D}t} \int_0^\infty e^{-\beta^2 Dt} \left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\, d\beta$$

$$+ D\beta\cos\beta x + \frac{2D^2\beta^2}{c}\sin\beta x$$

$$\cancel{\frac{d}{dz}\left[\frac{e^{-z^2}}{\sqrt{z^2+\alpha}}\right] = -z\frac{2e^{-z^2}}{\sqrt{z^2+\alpha}} - z\frac{e^{-z^2}}{\sqrt{z^2+\alpha}^3}}$$

$$\cancel{\int_\alpha^\infty \frac{e^{-x}\,dx}{\sqrt{x+\alpha}} = -\frac{e^{-x}}{\sqrt{x+\alpha}} + \frac{1}{2}\int\frac{e^{-x}}{\sqrt{x+\alpha}^3}}$$

$$\cancel{= \frac{x\,e^{-x}}{\sqrt{x+\alpha}} + \int x\frac{e^{-x}}{\sqrt{x+\alpha}}dx + \frac{1}{2}\int\frac{x\,e^{-x}}{\sqrt{}^3}dx}$$

$$\frac{x_0^2}{Dt} \quad x c \quad \sqrt{\frac{t}{D}} \quad \frac{c^2 t}{D} \quad \frac{xc}{D}$$

$$\bar{t^n} = -\frac{b}{c\sqrt{\pi}} \int \frac{e^{-z^2} dz}{\sqrt{z^2+\frac{cb}{D}}} \left[-z + \sqrt{z^2+\frac{cb}{D}}\right]^{-1+2n} \cdot \frac{D^{n-1}}{c^{2n-2}}$$

$$= +\frac{b D^{n-1}}{c^{2n-1}\sqrt{\pi}} \int \left(e^{-z^2}dz\right)\sqrt{z^2+\frac{cb}{D}}^{\,2n-2}$$

$$e^{-\left(\beta^2 D + \frac{c^2}{4D}\right)t} \sin\left(\beta x - \operatorname{arctg}\frac{2D\beta}{c}\right) f(\beta)\, d\beta$$

$$y_2^2 = \left[\frac{x_0 - ct_2}{2\sqrt{Dt_2}}\right]^2 + \frac{cx_0}{D}$$

$$\frac{dt}{\sqrt{t}} = \frac{2\sqrt{D}}{c}\left[-1 + \frac{z}{\sqrt{z^2+\frac{cx_0}{D}}}\right]dz \qquad \bar{t} = x_0\int_0^\infty \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\,dt$$

$$\sqrt{t} = \frac{\sqrt{D}}{c}\left[-2 + \sqrt{z^2+\frac{cx_0}{D}}\right]$$

$$\bar{t} = \frac{x_0}{c\sqrt{\pi}}\int_{\infty}^{-\infty} e^{-z^2}\left[-1+\frac{z}{\sqrt{z^2+\frac{cx_0}{D}}}\right]dz = \frac{x_0}{c\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-z^2}dz = \frac{x_0}{c}$$

Probe ob $\int W = 1$:

$$\bar{W} = \frac{x_0 c}{D\sqrt{\pi}}\int_{\infty}^{-\infty} e^{-z^2}\frac{-1+\frac{z}{\sqrt{z^2+\frac{cx_0}{D}}}}{\left[-z+\sqrt{\quad}\right]^2}dz = \frac{x_0 c}{D\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{e^{-z^2}}{\sqrt{z^2+\frac{cx_0}{D}}}\,\frac{z+\sqrt{\quad}}{\frac{cx_0}{D}}dz$$

$$= \frac{1}{\sqrt{\pi}}\left\{\int_{-\infty}^{+\infty} e^{-z^2} + \cancel{\int_{-\infty}^{+\infty}\frac{z e^{-z^2}}{\sqrt{z^2+\dots}}dz}\right\} = 1 \quad \text{stimmt}$$

$$W\Big|_{t_1}^{t_2} = \frac{1}{\sqrt{\pi}}\int_{\frac{x_0-ct_2}{2\sqrt{Dt_2}}}^{\frac{x_0-ct_1}{2\sqrt{Dt_1}}}\frac{e^{-z^2}}{\sqrt{z^2+\frac{cx_0}{D}}}\left[z+\sqrt{\quad}\right]dz$$

$$= \frac{1}{\sqrt{\pi}}\int_{z_2}^{z_1} e^{-z^2}dz + \frac{1}{\sqrt{\pi}}\int\frac{z e^{-z^2}}{\sqrt{z^2+\frac{cx_0}{D}}}dz$$

Bletcher: $\frac{1}{2\sqrt{\pi}}\int\frac{e^{-z^2}}{\sqrt{\quad}}\left[z+\sqrt{\quad}\right]\left\{1+\frac{\cancel{c}}{x_0}\frac{D}{\cancel{c}}\left[-z+\sqrt{\quad}\right]^2\right\}$

$$= \frac{1}{2\sqrt{\pi}}\int e^{-z^2}\left[\frac{z}{\sqrt{\quad}}+1\right] +$$

$$+\frac{1}{2\sqrt{\pi}}\cancel{\frac{c}{x_0}}\int\frac{e^{-z^2}}{\sqrt{\quad}}\cancel{\frac{x_0}{c}}\left[-z+\sqrt{\quad}\right] = \frac{1}{\sqrt{\pi}}\int e^{-z^2}dz$$

wie Bletcher angibt (4.91)

$$\frac{cx_0}{D} = 4bVh \qquad y^2 = z^2+\frac{cx_0}{D}$$

$$\frac{1}{\sqrt{\pi}}\int_{z_2}^{z_1}\frac{z e^{-z^2}dz}{\sqrt{\underbrace{z^2+\frac{cx_0}{D}}_{y^2}}} = \frac{1}{\sqrt{\pi}}\int_{y_2}^{y_1}\frac{\cancel{y}e^{-y^2}dy}{\cancel{y}}\cdot e^{+\frac{cx_0}{D}} = \frac{e^{\frac{cx_0}{D}}}{\sqrt{\pi}}\int_{\sqrt{z_2^2+\frac{cx_0}{D}}}^{\sqrt{z_1^2+\frac{cx_0}{D}}} e^{-y^2}dy$$

$$\bar{W}\Big|_{t=0}^{t_1} = \frac{1}{\sqrt{\pi}}\int$$

Für $0 < t < t_g$ | $t_g < t < \infty$

$z = \infty \quad z = 0$ | $z = 0 \quad z = -\infty$

$$\overline{W} = \frac{1}{2} \pm \frac{e^{\frac{cx_0}{D}}}{\sqrt{\pi}} \int_{y^2 = \frac{cx_0}{D}}^{\infty} e^{-y^2}\, dy \doteq \frac{1}{2} \pm \frac{1}{\sqrt{\pi}} \quad ?$$

$$\overline{t_a} = x_0 \int_0^{t_g} \frac{e^{-\frac{(x_0 - ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\, dt = \frac{x_0}{2\sqrt{\pi D}} \int_{\infty}^{0} e^{-z^2}\, \frac{2\sqrt{D}}{c} \left[-1 + \frac{z}{\sqrt{z^2 + \frac{cx_0}{D}}} \right] dz$$

$$= \frac{x_0}{c} \left[\frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{z\, e^{-z^2}\, dz}{\sqrt{z^2 + \frac{cx_0}{D}}} \right] \qquad t_a^+ = x_0 \int_{t_g}^{\infty} \cdots = \frac{x_0}{c} \left[\frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\infty} \cdots \right]$$

$$\overline{W_a} = \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{z\, e^{-z^2}\, dz}{\sqrt{\cdots}}$$

$$\overline{\left(\frac{t_a}{W_a} \right)} \neq \frac{x_0}{c} \left[1 - \frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{z\, e^{-z^2}\, dz}{\sqrt{z^2 + \frac{cx_0}{D}}} \right] \qquad \frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{z\, e^{-z^2}\, dz}{\sqrt{z^2 + \frac{cx_0}{D}}} = \frac{e^{\frac{cx_0}{D}}}{\sqrt{\pi}} \int_{y^2 = \frac{cx_0}{D}}^{\infty} e^{-y^2}\, dy$$

Ex.: $c = 0{\cdot}00211$

$x_0 = 0{\cdot}0373$

$$\frac{cx_0}{D} = 0{\cdot}0021 \cdot 0{\cdot}037 \cdot 142 \cdot 10^4 = 0{\cdot}21 \cdot 3{\cdot}7 \cdot 142 = 100$$

$$\frac{1}{D} = 4h = 142{\cdot}4 \cdot 10^4 \qquad \int_{\beta}^{\infty} e^{-y^2}\, dy \neq \frac{e^{-\beta^2}}{2\beta} = \frac{e^{-\frac{cx_0}{D}}}{2\sqrt{\frac{cx_0}{D}}}$$

$$t^- = \frac{x_0}{c} \left[1 - \frac{2}{\sqrt{\pi}} \sqrt{\frac{D}{cx_0}} \right]$$

Fletcher $\frac{t^+ - t^-}{2} = t_g \frac{2}{\sqrt{\pi}} \sqrt{\frac{x_0 c}{D}} =$

$$\frac{t^+ - t^-}{2} = \frac{x_0}{c} \frac{2}{\sqrt{\pi}} \sqrt{\frac{D}{cx_0}} = \frac{2}{\sqrt{\pi}} \frac{1}{c} \sqrt{\frac{Dx_0}{c}}$$

$= \cancel{\ldots}$

also scheinbar dasselbe Resultat
aber Unterschied darin, dass hier t_g dem wirklichen Durchschnitt
entspricht, bei Fletcher aber noch eine (falsche) Korrektur
dazukommt

$$\tau = \frac{t^+ - t^-}{2} = \frac{2}{\sqrt{\pi}} \frac{1}{c}\sqrt{D t_g} = \frac{2}{\sqrt{\pi}}$$

$$\frac{x_0+ct}{2\sqrt{\pi D t}} = 1$$

$$\frac{\tau^2 \pi c^2}{4 t_g} = D$$

$$\Delta = e^{\frac{cx_0}{D}} \int_{\sqrt{z_1^2+\frac{cx_0}{D}}}^{\sqrt{z_2^2+\frac{cx_0}{D}}} e^{-z^2} dz = \cancel{e^{\frac{cx_0}{D}}} \cdot \left(\frac{e^{-z_2^2-\cancel{\frac{cx_0}{D}}}}{2\sqrt{z_2^2+\frac{cx_0}{D}}} - \frac{e^{-z^2-\cancel{\frac{cx_0}{D}}}}{2\sqrt{\cdots}} \right)$$

Maximalwert für $z_2 = 0$, $z_1 = \infty$: $\Delta = \dfrac{1}{2\sqrt{\frac{cx_0}{D}}} = \dfrac{1}{2\sqrt{43\cdot 6}} \neq \dfrac{1}{13}$

$$\int \frac{(x_0+ct)\, e^{-\frac{(x_0-ct)^2}{4Dt}}}{\sqrt{\pi D t}} dt = \frac{1}{2\sqrt{\pi}} \int_0 \left[z + \frac{c\sqrt{b}}{\sqrt{D}}\right] e^{-z^2} dt = \frac{1}{2\sqrt{\pi}} \int_{\infty} \left[\sqrt{z^2+\frac{cx_0}{D}}\right] e^{-z^2} dt$$

$$\bar{t} = \frac{-1}{2\sqrt{\pi}} \cdot \frac{2D}{c^2} \int_{\infty} \left[z - \sqrt{z^2+\frac{cx_0}{D}}\right]^2 e^{-z^2} dz = \frac{D}{c^2\sqrt{\pi}} \int_{0}^{-\infty} \left[z - \sqrt{z^2+\frac{c x_0}{D}}\right]^2 e^{-z^2} dz$$

$$t^+ = \frac{D}{c^2\sqrt{\pi}} \int_0 \left[z - \sqrt{}\right]^2 e^{-z^2} dz$$

$$\frac{t^+ + t^-}{2} = \frac{D}{c^2\sqrt{\pi}} \int_0^{\infty} 2z\sqrt{z^2+\frac{cx_0}{D}}\, e^{-z^2} dz$$

$$= \frac{-D}{c^2\sqrt{\pi}} \int_0^{-\infty} \left[z - \sqrt{}\right]^2 e^{-z^2} dz$$

$$\bar{W}^+ = \frac{1}{\sqrt{\pi}} \int e^{-z^2} \left[1 + \frac{2}{\sqrt{z^2+\frac{cx_0}{D}}}\right] dz$$

$$\tau = \frac{2b}{c} \frac{J}{1-J^2} \quad \Big\| \quad J = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \frac{z\, e^{-z^2}}{\sqrt{z^2+\frac{cb}{D}}} dz$$

$$= \frac{2e^{\frac{cb}{D}}}{\sqrt{\pi}} \underbrace{\int_{\sqrt{\frac{cb}{D}}}^{\infty} e^{-y^2} dy}_{\frac{e^{-\frac{cb}{D}}}{2\sqrt{\frac{cb}{D}}}\left[1 - \frac{D}{2cb}\right]}$$

$$\tau = \frac{2b}{c} \underline{\sqrt{\frac{D}{cb\pi}} \left[1 - \frac{D}{2cb}\right]} \left[1 + \frac{D}{cb\pi}\right]$$

$$\neq \frac{2}{c}\sqrt{\frac{Db}{c\pi}} = 2\cancel{t}\sqrt{\frac{D}{cb\pi}}$$

$$\bar{t}_{\frac{1}{2}+\delta} = \bar{t}_{\frac{1}{2}} \left(1 + \frac{2\delta}{\sqrt{\pi}}\right) \quad \Big\| \quad \frac{\delta t_{1/2}}{t_{1/2}} = \frac{D}{cb}$$

$$\delta z = \frac{c\,\delta t}{2\sqrt{Dt}} = \frac{D}{2b} \frac{t}{\sqrt{Dt}} = \frac{\sqrt{Db}}{2b\sqrt{c}} = \frac{1}{2}\sqrt{\frac{D}{bc}}$$

$$\Delta\tau = \bar{t}_{1/2} \frac{2\delta}{\sqrt{\pi}} = \frac{t_g}{\sqrt{\pi}} \sqrt{\frac{D}{bc}}$$

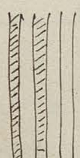
$$W(n) = \frac{e^{-\nu}\nu^n}{n!}$$

$$\overline{\sum n \lg n}$$

$$= \frac{N}{\nu} e^{-\nu}\left[1\frac{\nu^1}{1!}\lg(1) + 2\frac{\nu^2}{2!}\lg 2 + 3\frac{\nu^3}{3!}\lg 3 + \dots \quad \dots\right]$$

$$H = N e^{-\nu}\left[\frac{\nu}{1}\lg 2 + \frac{\nu^2}{2!}\lg 3 + \frac{\nu^3}{3!}\lg 4 + \dots\right]$$

$$\lim_{\nu=0} H = 0$$



$$\cancel{\Delta\{\underset{\lg W}{n \lg n} + (N-n)\lg(N-n)\}}$$

$$\cancel{W = e^{n\lg n + (N-n)\lg(N-n)} = n^n (N-n)^{N-n}}$$

$$\lg W = -\sum n \lg n = \cancel{n \lg n + \dots} - \sum \nu(1+\delta)[\lg\nu + \lg(1+\delta)] = \overset{W_0}{\sum \lg\nu \cdot N} + \cancel{\nu\lg\nu\sum\delta}^{=0}$$

$$+ \nu\underbrace{\sum(1+\delta)\lg(1+\delta)}_{= \nu\left[\delta + \frac{\delta^2}{2} + \frac{\delta^3}{3} + \delta^2 - \frac{\delta^3}{2}\right]}$$

$$= \lg W_0 - \nu \sum \frac{\delta^2}{2} \dots$$

$$W = A e^{-\nu\sum\frac{\delta^2}{2}}$$

$$\cancel{n\lg n + \dots \frac{N-n}{\dots}\lg\left(\frac{N-n}{\dots}\right)\nu}$$

$$\cancel{\lg W = \sum n \lg n = n_0 \lg n_0 + \sum_{n_1}^{\infty} n \lg n}$$

$$\cancel{\lg W_1 \quad \sum_2^{\infty} n \lg n = \int^{\infty} \lg n \, dn}$$

$$\Delta H = \lg \frac{W_2}{W_1}$$

$$W(v)\,dv = c\,v^2 e^{-\frac{v^2}{\alpha^2}}\,dv$$

$$\frac{\partial W}{dv^2} = 0 \qquad e^{-\frac{v^2}{\alpha^2}}\left[1 - \frac{v^2}{\alpha^2}\right] = 0$$

$$v^2 = \alpha^2$$

$$W(\xi)\,d\xi = c\,e^{-\frac{\xi^2}{\alpha^2}}\,d\xi$$

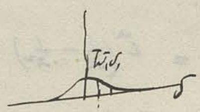
$$W(\xi^2)\,d(\xi^2) = 2\xi\,c\,e^{-\frac{\xi^2}{\alpha^2}}\,d\xi$$

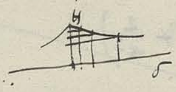
$$\frac{\partial}{\partial \xi^2}\left(e^{-\frac{\xi^2}{\alpha^2}}\right) =$$

$$H = \sum n \lg n$$

$$n = \nu(1+\delta)$$

$$W(\delta)\,d\delta = \sqrt{\frac{2}{\nu\pi}}\; e^{-\frac{\nu\delta^2}{2}}\,d\delta$$





$$dH = (1 + \lg n)\,dn$$

$$W(H)\,dH = \;?$$

Max: $\frac{\partial W(H)}{\partial H} = 0$

$$\frac{dW}{d\delta}\frac{d\delta}{dH}$$

$$= -\nu\delta \cdot e^{-\frac{\nu\delta^2}{2}} \cdot \underbrace{\frac{d\delta}{dn}}_{\frac{1}{\nu}}\frac{dn}{dH} = \frac{-\delta\, e^{-\frac{\nu\delta^2}{2}}}{1+\lg n} = 0$$

$$\delta = 0$$

Ebenso wenn es sich um den Absolutwert der $|\xi|$ handelt, so ist der $\underbrace{\text{wahrscheinlichste Wert}}$ $= 0$

aber der ~~wahrsch.~~ durchschnittliche ist endlich

denn Diagramm:

$\xi = 0$

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p. 119 el. u. $\int d\omega \int d\lambda$ $\varphi' - \varphi$ in $\int d\lambda$ $\sum_\omega d\omega\,\varphi$

ges. W. $\int d\omega$ $\nu_3 = \ldots$

$$\Delta \sum_{\omega d\omega} \varphi = \mathfrak{B}_\xi(\varphi) = \iiiint_0^{\infty\;2\pi} (\varphi' - \varphi)\, f F_1 g b\, d\omega\, d\omega_1\, db\, d\varepsilon$$

ungenau, denn es ist zwar $f F_1 g b\, d\omega\, d\omega_1\, db\, d\varepsilon$ die durchschnittliche Anzahl von $\int d\omega$

welche die f .. erleiden, aber

Wirkliche Dichte der Elemente $d\sigma\, d\omega \ldots\ldots\; \psi$ dann ist allerdings $f = \overline{\psi}$

und $\sum f\, d\sigma\, d\omega = \sum \psi\, d\sigma\, d\omega = \sum f(1+\delta)\, d\sigma\, d\omega$

Es ist aber das wirkliche $H = \sum \psi \lg \psi\, d\sigma\, d\omega < \sum f \lg f\, d\sigma\, d\omega$

Wenn man Molekülraum in genügend kleine Teile teilt, so dass immer nur 0, manchmal 1 Molek. vorkommen, wie soll Stosszahl berechnet werden?

$$\sum n^2 = e^{-\nu}\left[0 + \frac{\nu}{1}.0 + \frac{\nu^2}{2}2 + \frac{\nu^3}{3!}3.2 + \frac{\nu^4}{4!}4.3 + \frac{\nu^5}{5!}5.4 + \cdots\right]$$

$$\cancel{\frac{d}{dx}\left(\frac{e^x}{x}\right) = \frac{d}{dx}\sum \frac{x^{n-1}}{n!} = n-1\,\frac{x^{n-2}}{n!}} = e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)$$

$$\frac{1}{x} + \cancel{1} + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!}$$

$$\frac{d^2}{dx^2}\left(\frac{e^x}{x}\right) = \sum (n-1)(n-2)\frac{x^{n-2}}{n!} = e^x\left(\frac{1}{x} - \frac{2}{x^2} + \frac{2}{x^3}\right)$$

$$-\frac{1}{x^2} + \frac{1}{2!} + \frac{2x}{3!} + \frac{3x^2}{4!}$$

$$+\frac{2}{x^3} + \frac{1.2}{3!} + \frac{2.3x}{4!} + \cdots$$

$$2 + \frac{1.2}{3!}x^3 + \frac{2.3}{4!}x^4 + \frac{3.4}{5!}x^5 + \cdots = e^x\left(2 - 2x + x^2\right)$$

$$\sum n^2 = e^{-\nu}\left[e^{\nu}\left(2 - 2\nu + \nu^2\right) - 2\right] + \frac{\nu^2}{2}e^{-\nu} = 2 - 2\nu + \nu^2 - 2e^{-\nu} + \frac{\nu^2}{2}e^{-\nu}$$

$$= e^{-\nu}\left[\frac{\nu^2}{1!} + \frac{\nu^3}{1} + \frac{\nu^4}{2!} + \frac{\nu^5}{3!} + \cdots\right] = \underline{\underline{\nu^2}}$$

$$\bar{y} = \frac{\int_0^\infty y\, e^{-\alpha y}\,dy}{\int_0^\infty e^{-\alpha y}\,dy} = \frac{-\frac{e^{-\alpha y}}{\alpha}\cdot y + \frac{1}{\alpha}\int e^{-\alpha y}\,dy}{\int} = \frac{1}{\alpha}$$

$$\alpha = \frac{g}{R\theta} = \frac{\mu g}{H\theta} = \frac{\frac{4}{3}a^3\pi(\rho - \rho')g N}{H\theta}$$

$$\frac{\bar{y}F}{T} = \bar{y}\cdot\frac{4}{3}\pi a^3(\rho - \rho')g =$$

$$D\frac{\partial u}{\partial x} + cu = \frac{cu}{2} + \frac{x_0}{2t}(A+B)\,e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}}\,\frac{1}{2}\sqrt{\frac{\pi}{Dt}}$$

$$= \frac{1}{2}\sqrt{\frac{\pi}{Dt}}\,e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}}\Big[\underbrace{\frac{c}{2}(A-B) + \frac{x_0}{2t}(A+B)}_{\text{falls } = 0}\Big] = 0 \text{ für alle } t$$

$t=0$:

$$\boxed{u = e^{-\frac{cx}{2D}+\frac{c^2}{4D}t}\int_{-\infty}^{\infty} e^{-\beta^2 Dt}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\,d\beta}$$

$$u = \cancel{e^{\frac{cx}{2D}}}\int_{-\infty}^{\infty}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\,d\beta = \sqrt{\frac{2}{\pi}}\, e^{-\frac{(x-x_0)^2}{2}\cancel{-\frac{cx}{2D}}}$$

$$= \frac{1}{\pi}\int_0^{\infty} d\alpha \int_{-\infty}^{+\infty}\varphi(y)\left[\cos\alpha y\cos\alpha x + \sin\alpha y\sin\alpha x\right] dy$$

$$= \frac{1}{\pi}\iint \sin\beta x \cdot \varphi(y)\sin\beta y\, d\beta\, dy$$

$$+ \iint \cos\beta x\, \varphi(y)\cos\beta y\, d\beta\, dy$$

$$f(\beta) = \int \varphi(y)\sin\beta y\, dy$$

$$-\frac{2D\beta}{c} f(\beta) = \int\varphi(y)\cos\beta y\, dy$$

$$\cancel{f(\beta) = \sqrt{\tfrac{2}{\pi}}\, e^{-\frac{(x-y)^2}{2}}\cos\beta y\, dy}$$

$$\frac{\partial u}{\partial x} = -\frac{c}{2D}u - \cancel{\frac{(x-x_0)}{2Dt}\mathcal{U}V_1} + \frac{(x+x_0)}{2Dt}\mathcal{U}V_2$$

Versuch:

$$u = \underbrace{e^{-\frac{c^2t}{4D}-\frac{cx}{2D}}}_{\mathcal{U}}\ \frac{1}{2}\sqrt{\frac{\pi}{Dt}}\left\{\underbrace{A\, e^{-\frac{(x-x_0)^2}{4Dt}}}_{V_1} - \underbrace{B\, e^{-\frac{(x+x_0)^2}{4Dt}}}_{V_2}\right\} = \mathcal{U}\,(V_1 - V_2)$$

$$\frac{\partial u}{\partial t} - D\frac{\partial^2 u}{\partial x^2} - c\frac{\partial u}{\partial x} = -c\,\mathcal{U}\frac{\partial(V_1-V_2)}{\partial x} - 2D\frac{\partial\mathcal{U}}{\partial x}\frac{\partial(V_1-V_2)}{\partial x} = -\underbrace{\left[c\,\mathcal{U} + 2D\frac{\partial\mathcal{U}}{\partial x}\right]}_{\left[\cancel{c - 2D\frac{c}{2D}}\right]\mathcal{U}}\frac{\partial(V_1-V_2)}{\partial x} = 0$$

also Diff-Gleichung ist erfüllt

Anfangsbedingungen: für $t=0$

$u=0$ für alle x mit Ausnahme $x=x_0$

für $x=0$:

$$\begin{cases} u = e^{-\frac{c^2t}{4D}}\,\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\,(A-B)\, e^{-\frac{x_0^2}{4Dt}} \\ \frac{\partial u}{\partial x} = -\frac{c}{2D}u + \frac{x_0}{2Dt}\, e^{-\frac{c^2t}{4D}}(A+B)\, e^{-\frac{x_0^2}{4Dt}}\,\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \end{cases}$$

$$\varphi(y) = \sqrt{\frac{\varepsilon}{\pi}}\; e^{-\frac{(x_0-y)^2}{\varepsilon}}$$

$$f(\beta) = \int_{-\infty}^{+\infty} \varphi(y) \sin y\beta \, dy \quad \Big| \quad -\frac{2D\beta}{c} f(\beta) = \int_{-\infty}^{\infty} \varphi(y) \cos y\beta \, dy$$

$$u_0 = \int \left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\, d\beta$$

$$u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \int_{-\infty}^{\infty} e^{-D\beta^2 t}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\, d\beta \rightarrow \boxed{\text{erfüllt Diff. Gleichung } (\text{~~[illegible]~~ } \text{[illegible]}) \text{ und Bedingung } D\frac{\partial u}{\partial x} + cu = 0 \Big|_{x=0}}$$

$$= e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \int_{-\infty}^{+\infty}\left[e^{-D\beta^2 t} \sin\beta x \, d\beta \int_{-\infty}^{\infty} \varphi(y) \sin y\beta\, dy + e^{-D\beta^2 t} \cos\beta x \, d\beta \int_{-\infty}^{\infty} \varphi(y)\cos y\beta\, dy\right]$$

$$= e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \int_0^{\infty}\int_{-\infty}^{\infty} e^{-D\beta^2 t} \cos\beta(y-x)\, \varphi(y)\, dy$$

$$= e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \int_0^{\infty} d\beta \int_{-\infty}^{\infty} dy\; e^{-D\beta^2 t}\; e^{-\frac{(x_0-y)^2}{\varepsilon}} \sqrt{\frac{\varepsilon}{\pi}} \cos\beta(y-x)$$

$$\lim_{\varepsilon=0} \sqrt{\frac{\varepsilon}{\pi}} \int_{-\infty}^{+\infty} e^{-\frac{(x_0-y)^2}{\varepsilon}} \cos\beta(y-x)\, dy = \cos\beta(x_0-x)$$

$$u = e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \int_0^{\infty} \underbrace{e^{-D\beta^2 t}\cos\beta(x_0-x)\, d\beta} \qquad \alpha = \beta\sqrt{Dt}$$

$$\int_{-\infty}^{+\infty} e^{-\alpha^2}\cos\alpha x\, d\alpha = \sqrt{\pi}\, e^{-\frac{x^2}{4}} \quad \Bigg\| \quad = \frac{1}{\sqrt{Dt}} \int_0^{\infty} e^{-\alpha^2} \cos\frac{\alpha(x_0-x)}{\sqrt{Dt}}\, d\alpha$$

$$\int_0^\infty e^{-\alpha^2}\cos\alpha x\,d\alpha = \frac{\sqrt{\pi}}{2}\,e^{-\frac{x^2}{4}}$$

$$\int_0^\infty e^{-\alpha^2}\sin\alpha x\,d\alpha = \frac{1}{2}\,e^{-\frac{x^2}{4}}\int_0^x e^{\frac{y^2}{4}}dy$$

$$\cancel{u = \frac{e^{-\frac{cx}{2D}-\frac{c^2t}{4D}-\frac{(x_0-x)^2}{4Dt}}}{2\sqrt{Dt}\sqrt{\pi}}}$$

$$u = e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\int_{-\infty}^{+\infty} e^{-D\beta^2 t}\left[\sin\beta x-\frac{2D\beta}{c}\cos\beta x\right]d\beta\;\underbrace{\int_{-\infty}^{+\infty}\sqrt{\frac{\varepsilon}{\pi}}\,e^{-\frac{(x_0-\mu)^2}{\varepsilon}}\sin\mu\beta\,d\mu}$$

Versuch ↓

$\lim = \sin x_0\beta$

$f(\beta) = \sin\beta x_0$

$$= 2e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\int_{0}^{+\infty} e^{-D\beta^2 t}\left[\sin\beta x\sin\beta x_0-\frac{2D\beta}{c}\cos\beta x\sin\beta x_0\right]d\beta$$

$$\cancel{\int_0^\infty e^{-D\beta^2 t}\sin\beta z\,d\beta = -\frac{e^{-D\beta^2 t}}{2Dt}\sin\beta z+\int\frac{e^{-D\beta^2 t}}{2Dt}\cos\beta z\,d\beta}$$

$$2\sin\beta x_0\cos\beta x = \sin\beta(x_0-x)+\sin\beta(x_0+x)$$

$$\int_0^\infty e^{-\alpha^2}\cos\alpha x\,d\alpha = \frac{\sqrt{\pi}}{2}\,e^{-\frac{x^2}{4}}$$

$$\int_0^\infty \alpha\,e^{-\alpha^2}\sin\alpha x\,d\alpha = \frac{\sqrt{\pi}}{4}\,x\,e^{-\frac{x^2}{4}}$$

$$\frac{2\alpha\sqrt{D}}{c\sqrt{t}}$$

$$u = \frac{e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}}{\sqrt{Dt}}\int e^{-\alpha^2}\left[\cos\frac{\alpha(x-x_0)}{\sqrt{Dt}}-\cos\left(\alpha\frac{(x+x_0)}{\sqrt{t}}\right)-\frac{2D}{c}\frac{\alpha}{\sqrt{Dt}}\left[\sin\left(\alpha\frac{(x_0-x)}{\sqrt{Dt}}\right)+\sin\left(\alpha\frac{(x_0+x)}{\sqrt{Dt}}\right)\right]\right]$$

$$\cancel{= e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\left\{e^{-\frac{(x-x_0)^2}{4Dt}}-e^{-\frac{(x+x_0)^2}{4Dt}}\right\}-\frac{2\sqrt{\pi}}{c\,t}\left\{e^{-\frac{(x-x_0)^2}{4Dt}}\int_0^{\frac{x-x_0}{2\sqrt{Dt}}}e^{z^2}dz+e^{-\frac{(x+x_0)^2}{4Dt}}\int_0^{\frac{x+x_0}{2\sqrt{Dt}}}e^{z^2}dz\right\}\right]}$$

$$\cancel{u_1\Big|_{x=0} = e^{-\frac{c^2t}{4D}}\left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\;-\frac{2\sqrt{\pi}}{c\,t}\left\{e^{-\frac{x_0^2}{4Dt}}\int_0^{\frac{x_0}{2\sqrt{Dt}}}e^{z^2}dz\right\}\right]}$$

$$u = e^{-\frac{cx}{2D} - \frac{c^2t}{4D}} \left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \left\{ e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}} \right\} - \frac{\sqrt{\pi}}{2ct} \left\{ \frac{x_0 - x}{\sqrt{Dt}} e^{-\frac{(x_0-x)^2}{4Dt}} + \frac{x_0+x}{\sqrt{Dt}} e^{-\frac{(x_0+x)^2}{4Dt}} \right\} \right]$$

$$u\Big|_{x=0} = e^{-\frac{c^2t}{4D}} \left[\cancel{\frac{1}{2}\sqrt{\frac{\pi}{Dt}}} - \frac{\sqrt{\pi}}{ct} \frac{x_0}{\sqrt{Dt}} e^{-\frac{x_0^2}{4Dt}} \right]$$

$$\frac{\partial u}{\partial x}\Big|_0 = -\frac{c}{2D} u + e^{-\frac{cx}{2D} - \frac{c^2t}{4D}} \left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \left[-\frac{(x-x_0)}{2Dt} e^{\cdots} + \frac{(x+x_0)}{2Dt} e^{\cdots} \right] - \frac{\sqrt{\pi}}{2ct\sqrt{Dt}} \left[+ \frac{(x_0-x)^2}{2Dt} \quad \frac{(x_0+x)^2}{2Dt} \right] e^{-\frac{x_0^2}{4Dt}} \right]$$

$$= -\frac{c}{2D} u + e^{-\frac{c^2t}{4D}} \left[\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \frac{x_0}{Dt} e^{-\frac{x_0^2}{4Dt}} \right] = \frac{\sqrt{\pi}\, x_0}{Dt\sqrt{Dt}} e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}}$$

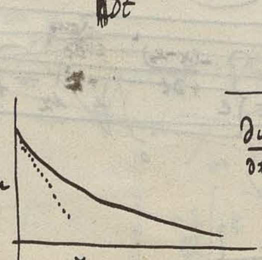
$$u = \boxed{u} - \frac{2}{ct} e^{-\frac{cx}{2D} - \frac{c^2t}{4D}} \cdot \int_0^\infty e^{-\alpha^2} \cdot \alpha \left[\sin\left(\alpha \frac{x_0 - x}{\sqrt{Dt}}\right) + \sin\left(\alpha \frac{x_0+x}{\sqrt{Dt}}\right) \right] d\alpha$$

$$u\Big|_{x=0} = \cancel{\boxed{u}} - \frac{4}{ct} e^{-\frac{c^2t}{4D}} \underbrace{\int_0^\infty e^{-\alpha^2} \cdot \alpha \sin\left(\frac{\alpha x_0}{\sqrt{Dt}}\right) d\alpha}_{\frac{\sqrt{\pi}}{4} \frac{x_0}{\sqrt{Dt}} e^{-\frac{x_0^2}{4Dt}}} = -\frac{\sqrt{\pi}\, x_0}{ct\sqrt{Dt}} e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}}$$

$$\frac{\partial u}{\partial x}\Big|_0 = \frac{\partial \boxed{u}}{\partial x}\Big|_0 + \frac{1}{2D} \frac{\sqrt{\pi}\, x_0}{t\sqrt{Dt}} e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}} - \frac{2}{ct} e^{-\frac{c^2t}{4D}} \int_0^\infty e^{-\alpha^2} \frac{\alpha^2}{\sqrt{Dt}} \cancel{\left[-\cos \frac{\alpha x_0}{\sqrt{Dt}} + \cos \frac{\alpha x_0}{\sqrt{Dt}} \right]}$$

$$-\frac{c}{2D}\cancel{u} + \frac{1}{2}\sqrt{\frac{\pi}{Dt}}\, e^{-\frac{c^2t}{4D}} \frac{x_0}{Dt} e^{-\frac{x_0^2}{4Dt}} = \sqrt{\frac{\pi}{Dt}} \frac{x_0}{Dt} e^{-\frac{c^2t}{4D} - \frac{x_0^2}{4Dt}}$$

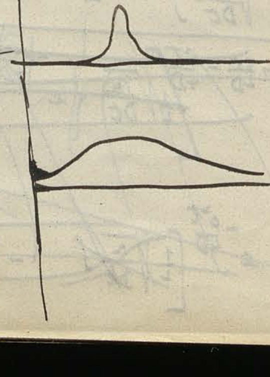
$$cu + D\frac{\partial u}{\partial x} = 0 \Big|_{x=0}$$



$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x}$$

$$\lim \frac{\partial u}{\partial t} = 0$$

um: $u = u_0 e^{-\frac{c}{D}x}$



$$u = A e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}} \cdot \frac{1}{2}\sqrt{\frac{\pi}{Dt}} \left\{ e^{-\frac{(x-x_0)^2}{4Dt}} \left[1 - \frac{x_0 - x}{ct}\right] - e^{-\frac{(x+x_0)^2}{4Dt}} \left[1 + \frac{x_0 + x}{ct}\right] \right\}$$

$\lim_{t=0} u = 0$ mit Ausnahme von $x = x_0$

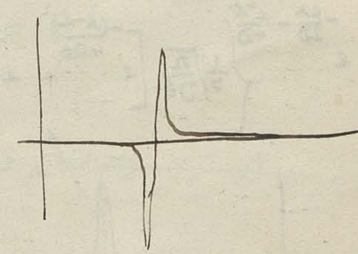
$$\int_0^\infty \lim_{t=0} u \, dx = A e^{-\frac{c x_0}{2D}} \left\{ \underbrace{\frac{1}{2}\sqrt{\frac{\pi}{Dt}} \int_{-\infty}^{+\infty} e^{-\frac{x^2}{4Dt}} dx}_{=1} \right\} - \cancel{\frac{\sqrt{\pi}}{ct} \int_{-\infty}^{+\infty} \frac{x}{2\sqrt{Dt}} e^{-\frac{(\ldots x)^2}{4Dt}} dx} \Big\}$$

$$A = e^{\frac{c x_0}{2D}}$$

$$u = \frac{1}{2}\sqrt{\frac{\pi}{Dt}} \, e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} \left[e^{-\frac{(x-x_0)^2}{4Dt}} - e^{-\frac{(x+x_0)^2}{4Dt}} - \frac{x_0 - x}{ct} e^{-\frac{(x_0-x)^2}{4Dt}} - \frac{x_0 + x}{ct} e^{-\frac{(x_0+x)^2}{4Dt}} \right]$$

$$u = \frac{1}{2}\sqrt{\frac{\pi}{Dt}} \, e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} \left[e^{-\frac{(x-x_0)^2}{4Dt}} \left(1 - \frac{x_0 - x}{ct}\right) - e^{-\frac{(x+x_0)^2}{4Dt}} \left(1 + \frac{x_0 + x}{ct}\right) \right]$$

Entspricht einem andern Anfangszustand nämlich:



sodass $\int_0^\infty u \, dx = 0$

daher auch $\lim u = 0$

$$-\frac{2D\beta}{c} f(\beta) = \int_{-\infty}^{+\infty} \varphi(y)\cos\beta y\,dy \qquad f(\beta) = -\frac{c}{2D\beta}\int_{-\infty}^{+\infty}\varphi(y)\cos\beta y\,dy$$

$$\lim \int \varphi(y)\cos\beta y\,dy = \lim_{\varepsilon\to 0}\sqrt{\frac{\varepsilon}{\pi}}\int e^{-\frac{(x_0-y)^2}{\varepsilon}}\cos\beta y\,dy = \cos\beta x_0$$

Versuch

$$\boxed{f(\beta) = \frac{c}{2D}\,\frac{\cos\beta x_0}{\beta}}$$

$$u = e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\int_{-\infty}^{+\infty} e^{-D\beta^2 t}\cos\beta x_0\left[\cos\beta x - \frac{c}{2D\beta}\sin\beta x\right]d\beta$$

$$D\beta^2 t = \alpha^2 \qquad \frac{1}{\beta} = \frac{\sqrt{Dt}}{\alpha}$$

$$J(x) = \int_0^\infty e^{-\alpha^2}\frac{\sin\alpha x}{\alpha}\,d\alpha = ? \qquad \frac{\partial J}{\partial x} = \int_0^\infty e^{-\alpha^2}\cos\alpha x\,d\alpha = \frac{\sqrt{\pi}}{2}e^{-\frac{x^2}{4}}$$

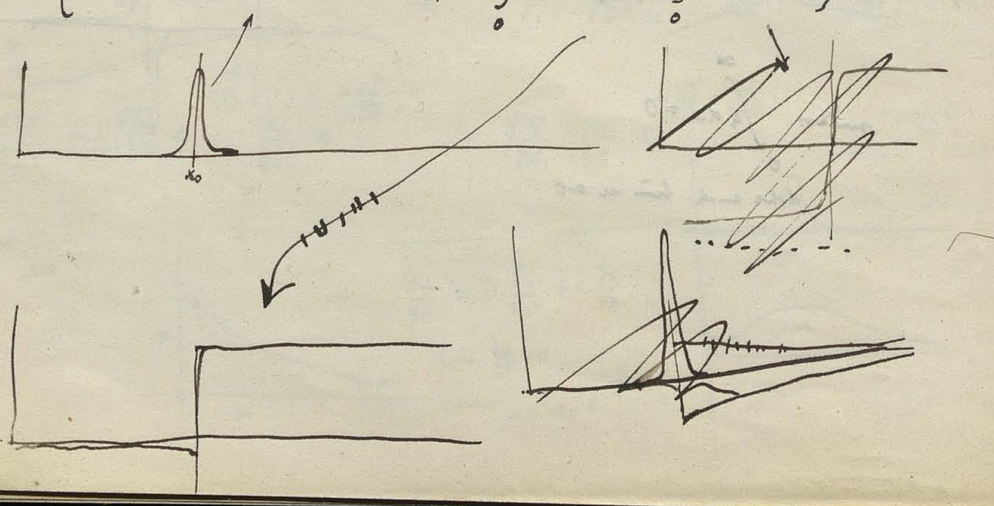
$$J = \frac{\sqrt{\pi}}{2}\int_0^x e^{-\frac{x^2}{4}}dx = \sqrt{\pi}\int_0^{\frac{x}{2}} e^{-z^2}dz$$

$$\boxed{\begin{aligned}&\int_0^\infty e^{-px^2}\cos qx\,dx = \frac12 e^{-\frac{q^2}{4p}}\sqrt{\frac{\pi}{p}}\\ &\int_0^\infty e^{-px^2}\sin qx\,dx = \sum_0^\infty(-1)^n\frac{1}{(n+2)^{2n+1}}\frac{q^{2n+1}}{p^{n+1}}\end{aligned}}$$

$$u = e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\left\{\int_0^\infty \frac{e^{-\alpha^2}}{\sqrt{Dt}}\left[\cos\frac{\alpha(x-x_0)}{\sqrt{Dt}}+\cos\frac{\alpha(x+x_0)}{\sqrt{Dt}}\right]d\alpha - \frac{c}{2D}\int_0^\infty\frac{e^{-\alpha^2}}{\alpha}\left[\sin\frac{\alpha(x-x_0)}{\sqrt{Dt}}+\sin\frac{\alpha(x+x_0)}{\sqrt{Dt}}\right]d\alpha\right\}$$

$$= e^{-\frac{cx}{2D}-\frac{c^2t}{4D}}\left\{\frac12\sqrt{\frac{\pi}{Dt}}\left[e^{-\frac{(x-x_0)^2}{4Dt}}+e^{-\frac{(x+x_0)^2}{4Dt}}\right] - \frac{c\sqrt{\pi}}{2D}\left[\int_0^{\frac{x-x_0}{2\sqrt{Dt}}}e^{-z^2}dz + \int_0^{\frac{x+x_0}{2\sqrt{Dt}}}e^{-z^2}dz\right]\right\}$$

$t=0$



$$u_0 = \int_{-\infty}^{+\infty} \left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\, d\beta = \varphi(x) \qquad f(\beta) = ?$$

$$= \frac{1}{\pi}\int_0^\infty d\alpha \int_{-\infty}^{+\infty} \varphi(\gamma)\left[\cos\alpha\gamma\cos\alpha x + \sin\alpha\gamma\sin\alpha x\right] d\gamma$$

$$= \frac{1}{\pi}\int_0^\infty d\alpha \left[\cos\alpha x\int_{-\infty}^{+\infty}\varphi(\gamma)\cos\alpha\gamma\, d\gamma + \sin\alpha x\int_{-\infty}^{+\infty}\varphi(\gamma)\sin\alpha\gamma\, d\gamma\right]$$

Solche Funktion dass $\varphi(x) = 0$ für alle x mit Ausnahme $x = x_0$ und

$$\int_0^\infty \varphi(x)\,dx = 1 = \int_{-\infty}^{+\infty}\left\{-\frac{\cos\beta x}{\beta}\Big|_0^\infty - \frac{2D}{c}\sin\beta x\Big|_0^\infty\right\} f(\beta)\,d\beta = \int_{-\infty}^{+\infty}\frac{f(\beta)}{\beta}\,d\beta = 1$$

Verschiedenes Resultat je nach Reihenfolge der Limes!

Jedenfalls muss die richtige Lösung die Form besitzen:

$$u = e^{-\frac{cx}{2D} - \frac{c^2}{4D}t}\int_{-\infty}^{+\infty} e^{-\beta^2 Dt}\left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\,d\beta$$

$$\int_0^\infty e^{-mx}\sin\beta x\,dx = \frac{\beta}{\beta^2+m^2}$$

$$\int_0^\infty e^{-mx}\cos\beta x\,dx = \frac{m}{\beta^2+m^2}$$

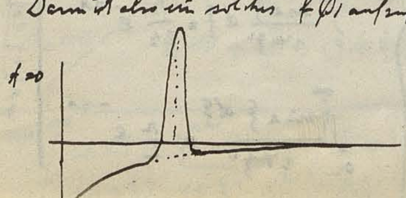
$$\int_0^\infty u\,dx = e^{-\frac{c^2}{4D}t}\int_{-\infty}^{+\infty} e^{-\beta^2 Dt}\left[\frac{\beta}{\beta^2+\frac{c^2}{4D^2}} - \frac{2D\beta}{c}\,\frac{\frac{c}{2D}}{\beta^2+\frac{c^2}{4D^2}}\right]f(\beta)\,d\beta = \underline{\underline{0!}}$$

Also ist obige Form ~~obige~~ nicht genügend (nicht allgemein genug)

Vielleicht so, dass man die Lösung

$u = e^{-\frac{cx}{D}}$ hinzuaddiert, welche sowohl Dgl. wie Grenzbedingung erfüllt und $\int_0^\infty u\,dx = \text{const}$ macht

Dann ist also ein solches $f(\beta)$ aufzusuchen, welches für $t = 0$ ~~[illegible]~~ eine u-Verteilung ergibt:



Wenn:

$$\int_{-\infty}^{\infty}\left[\sin\beta x - \frac{2D}{c}\beta\cos\beta x\right] f(\beta)\, d\beta = \varphi(x)$$

$$= \frac{1}{2\pi}\left\{\int_{-\infty}^{+\infty} d\beta \cos\beta x \int_{-\infty}^{+\infty}\varphi(y)\cos\beta y\, dy + \int_{-\infty}^{+\infty} d\beta \sin\beta x \int_{-\infty}^{+\infty}\varphi(y)\sin\beta y\, dy\right\}$$

Kann man daraus schliessen, dass:

$$f(\beta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\varphi(y)\sin\beta y\, dy \qquad -\frac{2D}{c}\beta f(\beta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\varphi(y)\cos\beta y\, dy$$

sein muss?
(Riem.-Weber II p. 91)

~~Offenbar nicht, denn~~ Dann wäre:

$$u = \frac{e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}}}{2\pi}\int_{-\infty}^{+\infty} e^{-\beta^2 Dt}\left[\sin\beta x \int_{-\infty}^{+\infty}\varphi(y)\sin\beta y\, dy + \cos\beta x \int \varphi(y)\cos\beta y\, dy\right] d\beta$$

Erfüllt dies die Bdg. $D\frac{\partial u}{\partial x} + cu\Big|_{x=0} = 0$? welche durch obige Gleichg erfüllt wird?

$$u_0 = \frac{e^{-\frac{c^2 t}{4D}}}{2\pi}\int e^{-\beta^2 Dt}\, d\beta \int \varphi(y)\cos\beta y\, dy$$

$$\frac{\partial u}{\partial x}\Big|_0 = -\frac{c}{2D}u_0 + \frac{e^{-\frac{c^2 t}{4D}}}{2\pi}\int e^{-\beta^2 Dt}\,\beta\, d\beta \int \varphi(y)\sin\beta y\, dy$$

Dann müsste sein:

$$\int_{-\infty}^{+\infty} e^{-\beta^2 Dt}\, d\beta\left\{\beta\int_{-\infty}^{\infty}\varphi(y)\sin\beta y\, dy + \frac{c}{2D}\int_{-\infty}^{\infty}\varphi(y)\cos\beta y\, dy\right\} = 0$$

Blödsinn!

(Burkhardt)
Encykl. II A 12
(1043)

$$\int_0^\infty e^{-x\xi}\,\xi^{a-1}\frac{[\cos x\xi - \xi\sin x\xi]}{1+\xi^2}\, d\xi = \Gamma(a)\cos\frac{a\pi}{2}\int_x^\infty x^{-a}e^{-x}\, dx$$

$$\int_0^\infty \frac{\cos x\xi + \xi\sin x\xi}{1+\xi^2}\, d\xi = \pi e^{-x} \quad (x>0)$$

(847) (848)

$$\int_0^\infty \frac{\cos x\xi}{r^2+\xi^2}\, d\xi = \frac{\pi}{2r}e^{-rx}$$

$$\int_0^\infty \frac{\xi\sin x\xi\, d\xi}{r^2+\xi^2} = \frac{\pi}{2}e^{-rx}$$

$$\int_0^\infty \frac{e^{-\xi^2}}{r^2+\xi^2}\left\{\begin{matrix} r\cos x\xi \\ \xi\sin x\xi\end{matrix}\right\} d\xi = \frac{1}{2}\sqrt{\pi}\, e^{-r^2}\left[e^{-rx}\int_{-\infty}^{\frac{x}{2}-r} e^{-z^2}\, dz \pm e^{rx}\int_{\frac{x}{2}+r}^{\infty} e^{-z^2}\, dz\right]$$

Jedenfalls muss der Voraussetzung gemäss auch sein $\frac{\partial\varphi}{\partial x} + \frac{c}{D}\varphi = 0 \Big|_{x=0}$

$$\therefore \int_{-\infty}^{+\infty}\left[\beta\cos\beta x - \frac{2D}{c}\beta^2\sin\beta x + \frac{c}{D}\sin\beta x - 2\beta\cos\beta x\right] f(\beta)\, d\beta \Big|_{x=0} = 0 \qquad = \int_{-\infty}^{+\infty}\beta f(\beta)\, d\beta$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\beta \left\{\left[\beta\cos\beta x + \frac{c}{D}\sin\beta x\right]\int\varphi(y)\sin\beta y\, dy + \left[-\beta\sin\beta x + \frac{c}{D}\cos\beta x\right]\int\varphi(y)\cos\beta y\, dy\right\}\Big|_{x=0}$$

$$= \int\beta\, d\beta \int\varphi(y)\sin\beta y\, dy + \int\frac{c}{D}\, d\beta\int\varphi(y)\cos\beta y\, dy \quad = 0$$

Also wird jene Bedingung tatsächlich für $t = 0$ erfüllt sein, ob aber auch für später?

$$u = \frac{e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}}}{2\pi}\int e^{-\beta^2 Dt}\, d\beta\int\varphi(y)\cos\beta(x-y)\, dy = \frac{e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}}}{2\pi}\int\varphi(y)\cdot\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\, e^{-\frac{(x-y)^2}{4Dt}}\, dy$$

$$\frac{\partial u}{\partial x} = -\frac{c}{2D}u - \frac{e^{-\frac{cx}{2D} - \frac{c^2 t}{4D}}}{2\pi}\int\varphi(y)\,\tfrac{1}{2}\sqrt{}\cdot\frac{x-y}{2Dt}\, e^{-\frac{(x-y)^2}{4Dt}}\, dy$$

$$\int_{-\infty}^{\infty}\left[\frac{y}{2Dt} + \frac{c}{2D}\right]\varphi(y)\, e^{-\frac{y^2}{4Dt}}\, dy = 0\;?$$

Vielleicht liegt die Schwierigkeit vielleicht darin dass schon die Anfangsbedingung $u = \varphi(x)\big|_{t=0}$ ~~mit der~~ der Bed. verträglich sein muss und dabei $\int_0^\infty \varphi\, dx = 0$

$$\sin\beta x - \frac{2D}{c}\beta\cos\beta x \quad \Big|\; a_0 + b_0 x$$

$$\frac{\partial\varphi}{\partial\beta} = x\cos\beta x - \frac{2D}{c}\cos\beta x + \frac{2Dx}{c}\beta\sin\beta x \quad \Big|\; a_1 + b_1 x$$

$$\frac{\partial^2}{\partial\beta^2} = -x^2\sin\beta x + \frac{4Dx}{c}\sin\beta x + \cancel{\frac{2Dx}{c}\sin\beta x} + \frac{2Dx^2}{c}\beta\cos\beta x \quad \Big|\; a_2$$

$$\cancel{b_0 = \frac{2D}{c}\beta} \;\Big\|\; \cancel{\left(\frac{2D}{c}\right)^2 y\cos y x}$$

$$a_0 = 0 \qquad b_0 + \frac{2D}{c}\beta\, a_1 + \frac{4D}{c}a_2 = 0 \;\Big\|\; \frac{2D}{c}\beta\, b_0 + a_1 = 0$$

$$b_1 = 0$$

$$(1033)\quad \int_0^\infty \begin{Bmatrix}\cos x\xi \\ \sin x\xi\end{Bmatrix}\frac{\xi^{a-1}\, d\xi}{1+\xi^2} = \frac{\pi}{2}\begin{Bmatrix}\frac{\operatorname{Cos} x}{\sin\frac{a\pi}{2}} \\ \frac{\operatorname{Sin} x}{\cos\frac{a\pi}{2}}\end{Bmatrix} + \frac{1}{2}\Gamma(a)\begin{Bmatrix}\cos\frac{a\pi}{2} \\ \sin\frac{a\pi}{2}\end{Bmatrix}\left[e^{-x}\int_0^x x^{-m}e^{x}\, dx + e^{x}\int_0^x x^{-m}e^{-x}\, dx\right]$$

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x}$$

$$u = e^{-\gamma t} f(x) \qquad f = e^{\alpha x}$$

$$D\alpha^2 + c\alpha + \gamma = 0 \qquad \begin{array}{l|l} \alpha = \alpha_1 + i\alpha_2 & \alpha_1 - i\alpha_2 \\ \gamma = \gamma_1 + i\gamma_2 & \gamma_1 - i\gamma_2 \end{array}$$

$$\left.\begin{array}{l} D(\alpha_1^2 - \alpha_2^2) + c\alpha_1 + \gamma_1 = 0 \\ 2D\alpha_1\alpha_2 + c\alpha_2 + \gamma_2 = 0 \end{array}\right\}$$

$$u = e^{-\gamma_1 t + \alpha_1 x} \underbrace{\left[e^{i(\gamma_2 t + \alpha_2 x)} + e^{-i(\gamma_2 t + \alpha_2 x)} \right]}_{\substack{\cos \\ \sin}(\gamma_2 t + \alpha_2 x)}$$

$$u = e^{\alpha_1 x + t[c\alpha_1 + D(\alpha_1^2 - \alpha_2^2)]} \begin{array}{l}\cos \\ \sin\end{array} \left[t(2D\alpha_1\alpha_2 + c\alpha_2) + \alpha_2 x \right]$$

1). Spezialfall

$$\alpha_1 = -\frac{c}{2D} \quad ! \; \gamma_2 = 0$$

$$\alpha_2 = +\beta$$

$$e^{-\frac{cx}{2D} + \frac{c^2}{4D}t - Dt\beta^2}$$

$$t = 0 \qquad u = e^{\alpha_1 x} \begin{array}{l}\cos \\ \sin\end{array}(\alpha_2 x)$$

2). Spezialfall

$\alpha_1 = 0$ gibt gedämpfte Wärmewelle für $x = ct$ fortschr.

$$\bar{u} = \frac{D}{c} + \frac{1}{\sqrt{\pi}}\left[\sqrt{2Dt}\int_{-\frac{x_0+ct}{2\sqrt{Dt}}}^{\infty} u e^{-u^2} du + (x_0 - ct)\int_{-\frac{x_0+ct}{2\sqrt{Dt}}}^{\infty} e^{-u^2} du\right] - \frac{Dt}{c^2\sqrt{\pi}}\int_{-\infty}^{\frac{x_0+ct}{2\sqrt{Dt}}} e^{-u^2} du - \frac{Dt}{c^2\sqrt{\pi}} e^{\frac{cx_0}{D}} \int_{\frac{x_0+ct}{2\sqrt{Dt}}}^{\infty} e^{-u^2} du$$

und die durchschn. Entropie!

Also gilt auch für den durchschnittl. Wert nicht das gewöhnl. Fallgesetz

$$\int_0^\infty u\,dx = e^{+t[c\alpha_1 + D(\alpha_1^2-\alpha_2^2)]} \,\frac{\cos}{\sin}\, t(2D\alpha_1\alpha_2 + c\alpha_2)\int_0^\infty e^{\alpha_1 x}\,\frac{\cos}{\sin}\,\alpha_2 x\,dx$$

Aber alle diese Lösungen müssen $\lim_{t=\infty}\int_0^\infty u\,dx = 0$ geben, falls Exponent (bei t) negativ

Somit wenn Bedingung $D\frac{\partial u}{\partial x} + cu = 0$

auch erfüllt sein soll (d. h. dass nichts durch die Anfangsebene hindurchtritt) so muss schon von allem Anfang an $\int_0^\infty u\,dx = 0$ sein und demgemäss muss φ gewählt werden.

Falls die wirkliche Verteilung dieser Bedingung nicht Genüge leistet, muss Lösung zusammengesetzt werden aus $u = Ae^{-\frac{cx}{D}}$ und der übrigen welche $\int_0 u\,dx = 0$ erfüllt.

$\boxed{\text{Für } c=0 \text{ reduziert sich auf die Lösung welche ich früher angegeben habe}}$

Richtige Lösung:

$$u = \frac{C}{D}e^{-\frac{cx}{D}} + \frac{\cancel{D}\,1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x-x_0)^2}{4Dt}} + e^{-\frac{(x+x_0)^2}{4Dt}}\right]e^{-\frac{c(x-x_0)}{2D}-\frac{c^2t}{4D}} - \frac{C}{\sqrt{\pi}\,D}e^{-\frac{cx}{D}}\int_{-\infty}^{\frac{x+x_0-ct}{2\sqrt{Dt}}} e^{-z^2}dz$$

$\boxed{\text{stimmt}}$ Dgl. u. Grbd.

Das entspricht auch dem Falle dass ein Niederschlag auf einem Filter durch einen Wasserstrom gewaschen wird

So kann man durch entsprechende Wahl der Durchflussgeschwindigkeit alle möglichen Fälle realisieren, welche sich bei Schwere nicht gut verwirklichen lassen (vorausgesetzt dass der Niederschlag nicht zusammenklebt!)

$$\cancel{u = e^{-\frac{cx}{D}}\left[1-\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\frac{x+x_0-ct}{2\sqrt{Dt}}} e^{-z^2}dz\right]} + \frac{D}{C}\frac{1}{2\sqrt{\pi Dt}}\left[e^{-\frac{(x-x_0+ct)^2}{4Dt}} + e^{-\frac{(x+x_0+ct)^2}{4Dt}+\frac{cx_0}{D}}\right]$$

$\boxed{\text{Grenzfälle für } t=0,\ t=\infty \text{ stimmen}}$

Nun sieht man wenn $\frac{\partial u}{\partial x} = 0$ gesetzt wird (wahrscheinlichster Wert) so resultiert dafür nicht

~~$x = x_0 + ct$~~

$x = x_0 - ct$ also gibt das gewöhnliche Fallgesetz nicht den wahrscheinlichsten Fall an.

$$\bar{u} = \int_0^\infty x e^{-\frac{cx}{D}}dx + \frac{D}{C}\frac{1}{2\sqrt{\pi Dt}}\left[\int_0^\infty x e^{-\frac{(x-x_0+ct)^2}{4Dt}}dx + e^{\frac{cx_0}{D}}\int_0^\infty x e^{-\frac{x^2+2x(x_0+ct)+(x_0+ct)^2}{4Dt}}dx\right] - \frac{1}{\sqrt{\pi}}\int^\infty x e^{-\frac{cx}{D}}dx\int_{-\infty}^{\frac{x+x_0-ct}{\sqrt{\ldots}}} e^{-z^2}dz$$

$$= -\frac{1}{\sqrt{\pi}}\left(\frac{D}{C}\right)^2\int_{-\infty}^{\frac{x_0-ct}{2\sqrt{Dt}}} e^{-z^2}dz - \frac{D}{2C\sqrt{D\pi t}}\,e^{\frac{cx_0}{D}}\int\left(x+\frac{D}{C}\right)e^{-\frac{(x+x_0+ct)^2}{4Dt}}dx$$

$$\lim a_{mn} = \frac{n}{m\,2^m}\;\frac{m(m-1)(m-2)\cdots\overbrace{\left(m-\frac{m-n}{2}+1\right)}^{\frac{m+n}{2}+1}\;\frac{m+n}{2}!}{1\cdot2\cdot3\cdots\frac{m-n}{2}\qquad\frac{m+n}{2}!} = \frac{n}{m\,2^m}\;\frac{m!}{\frac{m-n}{2}!\,\frac{m+n}{2}!}$$

$$\log\frac{m!}{\frac{m-n}{2}!\,\frac{m+n}{2}!} = \frac{\left(\frac{m}{e}\right)^m \cancel{\quad}\;2\sqrt{m}}{\left(\frac{m-n}{2e}\right)^{\frac{m-n}{2}}\left(\frac{m+n}{2e}\right)^{\frac{m+n}{2}}\sqrt{m^2-n^2}\sqrt{2\pi}}$$

$$= m\log m - \frac{m(1-\delta)}{2}\log\frac{m}{2}(1-\delta) - \frac{m}{2}(1+\delta)\log\frac{m}{2}(1+\delta) + \cdots$$

$$\cancel{= m\log m - \frac{m}{2}(1-\delta)\log m + \frac{m}{2}(1+\delta)\log 2 \quad + \frac{m}{2}(1-\delta)\left(\delta+\frac{\delta^2}{2}\right)}$$

$$\cancel{-\frac{m}{2}(1+\delta)\log m} + \frac{m}{2}(1+\delta)\log 2 - \frac{m}{2}(1+\delta)\left(\delta-\frac{\delta^2}{2}\right)$$

$$= m\log 2 + \left[\begin{matrix}\delta-\delta^2+\frac{\delta^2}{2}\\ -\delta-\cancel{\delta^2}+\frac{\delta^2}{2}\end{matrix}\right] =$$

$$\lim a_{mn} = \frac{n}{m}\,e^{-\frac{\delta^2 m}{2}}\sqrt{\frac{2}{\pi}}\;\frac{1}{\sqrt{m}}$$

$$\frac{m+n}{2}\qquad\frac{m-n}{2}$$

$$+\qquad -$$

$$= \frac{n}{m}\sqrt{\frac{2}{m\pi}}\;e^{-\frac{1}{2}\left(\frac{n^2}{m}\right)}$$

$$m = \frac{t}{\tau}\qquad D = \frac{\delta^2}{2\tau}\qquad e^{-\frac{x^2\tau}{2\delta^2 t}} = e^{-\frac{x^2}{4Dt}}.$$

$$n\delta = x$$

$$\lim\frac{a_{mn}}{2\tau} = \frac{x}{2t\delta}\sqrt{\frac{2\tau}{\pi t}}\;e^{-\frac{x^2}{2Dt}} = \frac{x}{2t\sqrt{\pi D t}}\,e^{-\frac{x^2}{4Dt}}$$

$$\frac{1}{\sqrt{\pi}}\int_0^\infty e^{-\frac{x^2}{4Dt}}\frac{x}{t\cdot2\sqrt{Dt}}\,dt = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-z^2}dz = 1$$

$$z = \frac{x}{2\sqrt{Dt}}\qquad dz = -\frac{dt}{4\sqrt{Dt^3}}\cdot x$$

$$\int_0^\infty\frac{b}{2\sqrt{\pi D t^3}}\,e^{-\frac{b^2}{4Dt}}\,db = \frac{\sqrt{D}}{\sqrt{\pi t}}\int_0^\infty e^{-x}dx = \sqrt{\frac{D}{\pi t}}$$

$$\int \frac{e^{-z^2}dz}{\sqrt{z^2+\frac{cb}{D}}}\left[\sqrt{z^2+\frac{cb}{D}}-z\right]^{2n-1} = \int e^{-z^2}dz\left\{\sqrt{z^2+\alpha}^{\,2n-2} - \binom{2n-1}{1} z\sqrt{z^2+\alpha}^{\,2n-3} + \binom{2n-1}{2} z^2\sqrt{z^2+\alpha}^{\,2n-4} + \cdots\right\}$$

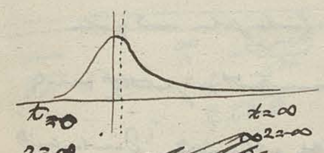
$$\frac{1}{\sqrt{\pi}}\int_0^\infty \frac{2e^{-z^2}dz}{\sqrt{z^2+\alpha}} = \frac{e^{\alpha}}{\sqrt{\pi}}\int_{\sqrt{\alpha}}^\infty e^{-y^2}dy$$

$\beta z^2+\alpha = y^2$

$$\int_0^\infty \frac{z\, e^{-z^2}dz}{\sqrt{\beta z^2+\alpha}} = \frac{1}{\beta}\int_{\sqrt{\alpha}}^\infty e^{-\frac{y^2-\alpha}{\beta}}dy = \frac{e^{\frac{\alpha}{\beta}}}{\beta}\int_{\sqrt{\alpha}}^\infty e^{-\frac{y^2}{\beta}}dy$$

$$\frac{1-\alpha}{1+\alpha}-\frac{1+\alpha}{1-\alpha} = \frac{(1-\alpha)^2-(1+\alpha)^2}{1-\alpha^2} = \frac{-4\alpha}{1-\alpha^2}$$

Platina $\quad \bar{t} = \frac{b}{c}+\frac{D}{c^2} = \frac{b}{c}\left[1+\frac{D}{bc}\right]$



$$\Delta\tau = \frac{\partial \tau}{\partial \bar{t}}\,\Delta\bar{t}$$

$$\cancel{t = \frac{x_0}{c}\frac{1}{\sqrt{\pi}}\int \frac{e^{-z^2}\,2z\,dz}{\sqrt{z^2+\frac{cb}{D}}}}$$

$$\tau' = \frac{x_0}{c\sqrt{\pi}}\left[\int e^{-z^2}dz - \int \frac{2e^{-z^2}}{\sqrt{z^2+\frac{cx}{D}}}dz\right]$$

$$= \frac{1}{2}\left\{\frac{\int_{-\Delta z} \; \int_{-\Delta z}}{\left[\int_{-\Delta z}^\infty e^{-z^2}dz + \int_{-\Delta z}^\infty \cdots\right]} - \frac{\int + \int}{\int - \int}\right\}$$

$$= \frac{x_0}{2c}\, 4\cdot \frac{\int_{-\Delta z}^\infty \frac{2e^{-z^2}dz}{\sqrt{\;}}}{\int_{-\Delta z}^\infty e^{-z^2}dz} = \frac{2x_0}{c}\,\frac{\left[\frac{1}{2\sqrt{\frac{cx_0}{D}}} - \frac{(\Delta z)^2 e^{-(\Delta z)^2}}{2\sqrt{(\Delta z)^2+\frac{cx_0}{D}}}\right]}{\frac{\sqrt{\pi}}{2}+e^{-(\Delta z)^2}\Delta z} \quad \cancel{\left[\frac{\sqrt{\pi}}{2}+e^{-(\Delta z)^2}\Delta z\right]}$$

$$\mp \frac{2x_0}{c\sqrt{\pi}}\sqrt{\frac{D}{cx_0}}\left\{1-\underline{\underline{\Delta z}} + \left(\Delta z^2 \cdots\right)\right\}$$

$$\tau = \frac{2t_0}{\sqrt{\pi}}\sqrt{\frac{D}{cb}}\left\{1-\underline{\underline{\frac{1}{2}\sqrt{\frac{D}{bc}}}}\cdots\right\}$$

$$z = \frac{x_0-ct}{\sqrt{4Dt}}$$

$$\Delta z = \frac{c\,\Delta t}{\sqrt{4Dt}} = \Delta t\cdot\frac{c\sqrt{c}}{2\sqrt{Db}} = \frac{D}{c^2}\,\frac{c}{2}\sqrt{\frac{c}{Db}}$$

$$\Delta z = \frac{1}{2}\sqrt{\frac{D}{bc}}$$

$$\frac{t^+ + t^-}{2} = \frac{x_0}{2c} \left\{ \frac{1-\alpha}{1+\alpha} + \frac{1+\alpha}{1-\alpha} \right\} = \frac{x_0}{c} \frac{1+\alpha^2}{1-\alpha^2} \neq \frac{x_0}{c}\left[1+2\alpha^2\right]$$

$$= \frac{b}{c}\left[1 + 2\frac{D}{\pi b c}\right] = \frac{b}{c} + \frac{2}{\pi}\frac{D}{c^2}$$

$$= \frac{b}{c}\left[1 + \frac{2}{\pi}\left(\frac{1}{Z_{\text{Teilchen}}}\right)^2\right]$$

~~Ein Widerspruch?~~

Wenn Teilchen von x_0 ausgehen und man beobachtet Zeit wann sie zum ersten Mal die Null Ebene überschreiten, so muss es darunter auch solche geben welche anfangs nach links gehen und später abermals durch die ~~Null~~ x_0 Ebene treten. Für solche muss die Gesamtzeit eingestellt werden (nicht die Zeit vom zweiten Durchtritt durch die x_0 Ebene!), wenn der allgemeine Durchschnitt der Fallzeiten $= \frac{b}{c}$ sein soll. Damit geschieht automatisch wenn bei einer Skala die Zeit von Anfang bis Schluss beobachtet wird.

Fletcher Ph. Rev. 4

Drop Nr 12: $t^+ = 4.365$
$t^- = 3.078$
7.443 3.721_5
~~t~~ $t_0 = 3.672$
$\Delta = 50 = 1.3\,\%$
Theor. $1.7\,\%$

8.349
6.480
14.829 7.414_5
7.344
70
$0.9\,\%$
theor. $0.8\,\%$

Drop Nr 9 4.708
3.109
7.817 3.909
3.795
$\Delta = 104 = 2.8\,\%$
76
28
Theor: $2.2\,\%$

Nr 6: 2.410
3.410
5.820 2.910
2.844
$\Delta = 66 = 2.22$
theor. $= 1.7\,\%$

Nr 5 2.735
1.997
4.732 2.366
2.325
$41 = 1.8\,\%$
theor.: $1.3\,\%$

$$\tau = \frac{2b}{c}\frac{J}{1-J^2} = \frac{2b}{c} J\left[1 + J^2 + J^4 \dots\right]$$

$$= \frac{2b}{c\sqrt{\pi}}\frac{\cancel{\sqrt{}}}{2}\underbrace{\left[1 - \frac{2^2}{2}\right]\left[1 + \frac{2^2}{\pi}\right]}_{\left(1 - 2^2\cdot\frac{\pi-2}{2\pi}\right)}$$

$$J = \frac{2e^{\frac{cb}{D}}}{\sqrt{\pi}}\int_{\sqrt{\frac{cb}{D}}}^{\infty} e^{-y^2}dy =$$

$$= \frac{1}{\sqrt{\pi}}\sqrt{\frac{D}{cb}}\left[1 - \frac{D}{2cb} + \dots\right]$$

$$\tau = \frac{M}{W} = \frac{3H\theta}{N}\frac{1}{WC^2} \qquad \frac{1}{W} = B = \frac{D}{\frac{H\theta}{N}}$$

$$= \frac{3D}{C^2}$$

$$\overline{\sqrt{t}} = x_0\int_0^\infty \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2t\sqrt{\pi D}}dt \qquad \overline{\left(\frac{1}{\sqrt{t}}\right)} = x_0\int_0^\infty \frac{e^{-\frac{(x_0-ct)^2}{4Dt}}}{2t^2\sqrt{\pi D}}dt$$

$$= \frac{x_0}{2\sqrt{\pi D}}\int_{+\infty}^{-\infty} 2e^{-z^2}dz\,\frac{\left(-1 + \frac{2}{\sqrt{}}\right)}{\left(-2 + \sqrt{}\right)} = \frac{x_0}{\sqrt{\pi D}}\int_{-\infty}^{+\infty}\frac{e^{-z^2}dz}{\sqrt{z^2 + \frac{cx_0}{D}}}$$

$$\overline{\left(\frac{1}{\sqrt{t}}\right)} = \frac{x_0}{\sqrt{\pi D}}\frac{c^2}{D}\int\frac{e^{-z^2}dz}{\sqrt{\dots}}\frac{\left(\cancel{}\right)}{\left(-2+\sqrt{\dots}\right)^2} = \frac{\cancel{x_0}}{\sqrt{\pi D}}\frac{c}{\cancel{D}}\int\frac{e^{-z^2}dz}{\sqrt{}}\frac{\left(2+\sqrt{}\right)^2}{\frac{\cancel{cx_0}}{D}} = \frac{c}{\sqrt{\pi D}}\int_{-\infty}^{+\infty}\frac{\left(2z^2 + \frac{cx_0}{D}\right)e^{-z^2}dz}{\sqrt{}}$$

$$\bar{t} = \frac{x_0}{c} \qquad \bar{t}\,\overline{\left(\frac{1}{\sqrt{t}}\right)} - \overline{\left(\sqrt{t}\right)} = \frac{x_0}{\sqrt{\pi D}}\left\{\int_{-\infty}^{+\infty}\frac{2z^2 + \frac{cx_0}{D} - 1}{\sqrt{z^2 + \frac{cx_0}{D}}}e^{-z^2}dz\right\}$$

$$b\,\frac{e^{-\frac{(b-ct)^2}{4Dt}}}{2\sqrt{\pi D t^3}}\,dt\;\frac{e^{-\frac{[b-c(T-t)]^2}{4D(T-t)}}}{2\sqrt{\pi D(T-t)^3}}$$

$$\cancel{(b^2-2bct+c^2t^2)(T-t)+[b^2-2bc(T-t)+c^2(T-t)^2]t=b^2T-2bc(tT-t^2+Tt-t^2)}$$

$$\frac{b^2}{t}-2bc+c^2t+\frac{b^2}{T-t}-2bc+c^2(T-t)=\frac{b^2T}{t(T-t)}-4bc+c^2T$$

$$\int_0^\infty\frac{e^{-\frac{b^2T}{t(T-t)}}}{\sqrt{t(T-t)}^{\,3}}\,dt=$$

$$\frac{b\sqrt{T}}{\sqrt{t(T-t)}}=2u\frac{b}{\sqrt{T}}\qquad\frac{T^2}{4}=u^2tT-u^2t^2$$

$$(ut-\tfrac{T}{2})^2=0$$

$$\frac{1}{2}\frac{b\sqrt{T}(T-2t)}{\sqrt{}^{\,3}}\,dt=du\qquad t=\frac{T}{2u}$$

$$=\int_0^\infty\frac{e^{-\frac{4u^2b^2}{T}}}{\left[\frac{T^2}{2u}(1-\frac{1}{2u})\right]^{3/2}}\frac{T}{2u^2}du=4\int\frac{e^{-\frac{4u^2b^2}{T}}}{T^2(2u-1)^{3/2}}u\,du$$

$$2u-1=z$$
$$2u=z+1$$

$$\downarrow$$

$$=\frac{4}{T^2}\int\frac{e^{-\frac{(z+1)^2b^2}{T}}}{z^{3/2}}dz\,(z+1)$$

$$q\frac{\partial U}{\partial q}\qquad\dot q\frac{\partial T}{\partial\dot q}=2T$$

Schaefer u. Jentzsch zur Th. d. [illegible] Phys. Z. 10, 630, 1909

Analog für unser Problem:

$$\frac{\partial^2 u}{\partial t^2} - D\frac{\partial u}{\partial x^2} - c\frac{\partial u}{\partial x} = 0$$

$$\left.\begin{matrix} x=0 \\ x=\infty \end{matrix}\right\} u = 0$$

$$u = v.e^{int}$$

$$D\frac{\partial^2 v}{\partial x^2} + c\frac{\partial v}{\partial x} + n^2 v = 0 \quad \Big| \; K$$

$$D\frac{\partial^2 K}{\partial x^2} + c\frac{\partial K}{\partial x} \qquad = 0 \quad \Big| \; v$$

$$v^{0,\infty} = 0$$

$$\frac{\partial K}{\partial x}\Big|_{\xi-}^{\xi+} = 1 \qquad K^{0,\infty} = 0$$

$$D\int_0^\infty \left(K\frac{\partial v}{\partial x^2} - v\frac{\partial K}{\partial x^2}\right)dx + c\int\left(K\frac{\partial v}{\partial x} - v\frac{\partial K}{\partial x}\right) + n^2\int v\, K_{dx} = 0$$

$$\pm \frac{\partial K}{\partial x}\frac{\partial v}{\partial x}$$

$$D\cancel{\left(K\frac{\partial v}{\partial x}\right)\Big|_0^\infty} - \cancel{D\frac{\partial v}{\partial x}\Big|_\xi} + Dv_\xi = n^2\int_0^\infty v(x)\,K(x,\xi)\,dx + c\int_0^\infty \underbrace{\left[\frac{\partial v}{\partial x}K(x,\xi) - v(x)\frac{\partial K(x,\xi)}{\partial x}\right]}_{\cancel{\partial K} - \int\frac{\partial v}{\partial x}K_-}dx$$

$$\lim_{t=0}\int_0^{\frac{x-x_0}{2\sqrt{Dt}}} e^{-z^2}dz = \frac{\sqrt{\pi}}{2} - \frac{e^{-\frac{(x-x_0)^2}{4Dt}}}{\frac{x-x_0}{\sqrt{Dt}}} \quad (x>x_0)$$

$$\lim_{t=0}\int_0^{\frac{x+x_0}{2\sqrt{Dt}}} e^{-z^2}dz = \frac{\sqrt{\pi}}{2} - \frac{e^{-\frac{(x+x_0)^2}{4Dt}}}{\frac{x+x_0}{\sqrt{Dt}}}$$

$$\lim\int + \int = \sqrt{\pi} - \sqrt{Dt}\left(\frac{1}{x-x_0} + \frac{1}{x+x_0}\right) = \sqrt{\pi} - \sqrt{Dt}\cdot\frac{2x}{x^2-x_0^2}$$

$$\lim\int + \int \quad (x<x_0) \quad = \sqrt{Dt}\left(\frac{1}{x-x_0} - \frac{1}{x+x_0}\right) = \sqrt{Dt}\,\frac{2x_0}{x^2-x_0^2} = -\sqrt{Dt}\,\frac{2x_0}{x_0^2-x^2}$$

$$v_\xi = \frac{n^2}{D}\int_0^\infty v(x)\,K(x,\xi)\,dx + 2c\int_0^\infty \frac{\partial v}{\partial x}K(x,\xi)\,dx = \int_0^\infty\left[\frac{n^2}{D}v(x) + 2c\frac{\partial v}{\partial x}\right]K(x,\xi)\,dx$$

$$W = \sqrt{\frac{\beta}{2\pi D(1-e^{-2\beta t})}}\; e^{-\frac{\beta(x-x_0e^{-\beta t})^2}{2D[1-e^{-2\beta t}]}}$$

Für gegebenes t <u>wahrscheinlichste</u> Änderungsgrösse, welche $(x-x_0 e^{-\beta t})^2$ zum Minimum macht

also $x = x_0 e^{-\beta t}$

also ist hier die durchschnittliche auch die wahrscheinlichste

$$\frac{\partial W}{\partial x} = -\sqrt{\ldots}\; \frac{\beta\,(x-x_0e^{-\beta t})}{D\,(1-e^{-2\beta t})}\, e^{-\frac{\beta \cdots}{2D\cdots}}$$

$$\frac{\partial^2 W}{\partial x^2} = \sqrt{\ldots}\left[\frac{\beta^2(x-x_0e^{-\beta t})^2}{D^2(1-e^{-2\beta t})^2} - \frac{\beta}{D(1-e^{-2\beta t})}\right] e^{-\frac{\beta}{2D}\cdots}$$

$$\frac{\partial W}{\partial t} = \cancel{\frac{\beta^{3/2}\,e^{-2\beta t}}{2\sqrt{2\pi D}\sqrt{1-e^{-2\beta t}}^{\,3}}}\left\{-\frac{\beta\, e^{-2\beta t}}{1-e^{-2\beta t}} - \frac{\beta}{D}\frac{(x-x_0e^{-\beta t})x_0e^{-\beta t}}{1-e^{-2\beta t}} + \frac{\beta}{D}\frac{(x-x_0e^{-\beta t})^2}{[1-e^{-2\beta t}]^2}e^{-2\beta t}\right\}$$

$$= \left\{\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})}{1-e^{-2\beta t}}\underbrace{\left[\frac{(x-x_0e^{-\beta t})e^{-2\beta t}}{1-e^{-2\beta t}} - x_0e^{-\beta t}\right]}_{\frac{x e^{-2\beta t} - x_0 e^{-\beta t}}{1-e^{-2\beta t}}} - \frac{\beta\, e^{-2\beta t}}{1-e^{-2\beta t}}\right\}\sqrt{}\; e^{\cdots}$$

$$\frac{\partial W}{\partial t} - D\frac{\partial^2 W}{\partial x^2} - x\beta\frac{\partial W}{\partial x} = \left\{\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})(x-x_0e^{+\beta t})e^{-2\beta t}}{(1-e^{-2\beta t})^2} - \frac{\beta\, e^{-2\beta t}}{1-e^{-2\beta t}} - \right.$$

$$-\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})^2}{(1-e^{-2\beta t})^2} + \frac{\beta}{1-e^{-\beta t}} -$$

$$\left. +\frac{\beta^2}{D}\frac{x(x-x_0e^{-\beta t})}{1-e^{-2\beta t}}\right\}\sqrt{}\; e^{\cdots}$$

$$= \left[\frac{\beta^2}{D}\frac{(x-x_0e^{-\beta t})}{(1-e^{-2\beta t})^2}\left\{\cancel{x e^{-2\beta t}} - \cancel{x_0e^{-\beta t}} - \cancel{x} + \cancel{x_0e^{-\beta t}} + \cancel{x} - \cancel{x e^{-2\beta t}}\right\}\right..$$

$$= \frac{\beta}{1-e^{-2\beta t}}\left[1 - e^{-2\beta t}\right]\sqrt{}\; e^{\cdots} = \cancel{\frac{\ldots}{2\sqrt{1-e^{-2\beta t}}}\,2 + 2e^{-\beta t} - 3e^{-2\beta t}}$$

$$= \beta W$$

$$\frac{\partial}{\partial x}(xW) = W + x\frac{\partial W}{\partial x}$$
$$\frac{\partial^2}{\partial x^2} \quad = 2\frac{\partial W}{\partial x} + x\frac{\partial^2 W}{\partial x^2}$$

$$= \beta\left[\cancel{1} - \frac{2}{1-e^{-2\beta t}}\right] W$$

$$\frac{\partial W}{\partial t} - D\frac{\partial^2 W}{\partial x^2} - \beta\left(x\frac{\partial W}{\partial x} + W\right) = 2\beta\left[1 - \frac{1}{1-e^{-2\beta t}}\right] W$$

$$\log W = \frac{1}{2}\log\left(\frac{\beta}{2\pi D}\right) - \frac{1}{2}\log\left(1-e^{-2\beta t}\right) - \frac{\beta}{2D}\,\frac{(x-x_0 e^{-\beta t})^2}{1-e^{-2\beta t}}$$

$$\frac{1}{W}\frac{\partial W}{\partial t} = \qquad - \frac{\beta e^{-2\beta t}}{1-e^{-2\beta t}}$$

Falls die kinetische Energie als determinierendes Moment der Br. B. beibehalten würde, (so weit sie für Zeiten $t < \tau$ überwiegt) würden wir w. Wahrsch. die anscheinend Schwingungsgleichung erhalten:

$$\frac{\partial^2 W}{\partial t^2} - \alpha\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + \beta\frac{\partial}{\partial x}(xW)$$

und allgemeine Verteilung: $\underbrace{\bar{W} = \int f(x_0)\, W(x, x_0, t)\, dx_0}$

Das ist ermöglicht durch Unabhängigkeit der zusammensetzenden Partialprozesse. Es wäre aber nicht erlaubt z.B. für $W(x)$ auszuwenden, weil hier der Wert von $W(x)$ durch Existenz eines benachbarten x Datum beeinflusst wird.

$$x = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$
$$\frac{\partial z}{\partial x} = -\frac{1}{2}$$

Somit ist die Gleichung erfüllt:

$$\frac{\partial W}{\partial t} - D\frac{\partial^2 W}{\partial x^2} - \beta\frac{\partial}{\partial x}(xW) = 0$$

Allgemein: Brown'sche Bewegung unter Einwirkung einer Kraft: $F = f(x)$ wird definiert durch

Spezielles Integral d. DGl.

$$\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + \beta\frac{\partial}{\partial x}\left[W\cdot f(x)\right]$$

welche sich nach d. Methode ... lösen lassen sollte?

Im Raume:

$$\frac{\partial W}{\partial t} = D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) W + \beta\underbrace{\left\{\frac{\partial}{\partial x}\left[W f(x)\right] + \frac{\partial}{\partial y}\left[W f(y)\right] + \frac{\partial}{\partial z}\left[W f(z)\right]\right\}}$$

$$= -\left(\frac{\partial W}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial W}{\partial y}\frac{\partial U}{\partial y} + \frac{\partial W}{\partial z}\frac{\partial U}{\partial z}\right) - W\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) U$$

Dasselbe bezieht sich auf Diffusion einer Substanz welche von äußeren Kräften beeinflusst wird

Gouy!

z.B. $O_2 \leftrightarrow N_2$ in Erdatmosphäre etc.

Das ist eigentlich ein Spezialfall der Boltzmann'schen Gleichung der Gastheorie

$$\frac{\partial F}{\partial t} + \xi\cdot\frac{\partial F}{\partial x} + \eta\cdot\frac{\partial F}{\partial y} + \zeta\cdot\frac{\partial F}{\partial z} + X\frac{\partial F}{\partial \xi} + Y\frac{\partial F}{\partial \eta} + Z\frac{\partial F}{\partial \zeta} = \iint (F_1 F_1' - F F')\, d\omega \ldots$$

welche genau genommen doch wohl so lauten sollte: (?)

$$\frac{\partial F}{\partial t} + \frac{\partial(\xi F)}{\partial x} + \frac{\partial}{\partial y}(\eta F) + \frac{\partial}{\partial z}(\zeta F) + X\frac{\partial F}{\partial \xi} + Y\frac{\partial F}{\partial \eta} + Z\frac{\partial F}{\partial \zeta} = \ldots$$

Durch Abbildung (oder Ätzung ?) erhält man nämlich wenn

~~Also scheint elast. Kraft zusammengesetzt aus zwei Teilen:~~

~~$f = x$~~

Für die Schwankungen der Teilchenzahl muss im Grenzfall genau dieselbe Formel gelten wie für Br. B. mit elastischer Kraft, sofern man sich auf geringe Unterschiede δ beschränkt.

Also sollte die Diff. Gl. herauskommen:

$$\frac{\partial W}{\partial t} = a\frac{\partial^2 W}{\partial x^2} + b\left[\frac{\partial W}{\partial x}(x-\nu) + W\right]$$ oder besser:

$$\frac{\partial W(\delta)}{\partial t} = a\frac{\partial^2 W(\delta)}{\partial \delta^2} + b\frac{\partial}{\partial \delta}(W\delta)$$

Das ist wohl nicht der Fall, denn für $\overline{\Delta_x^2}$ erhält man nicht das Analogon der Einst. Formel.

Versuch ob ein lösbares Beispiel sich ergibt durch Annahme $f(x) = a - \frac{b}{x}$

$$\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + a\frac{\partial W}{\partial x} - b\frac{\partial}{\partial x}\left(\frac{W}{x}\right)$$

Falls $a = 0$:

Versuch: $\frac{x^2}{t} = z^2$

$x = 2\sqrt{t}$

$z = \frac{x}{\sqrt{t}}$

$$\cancel{dW = \frac{\partial W}{\partial x}dx + \frac{\partial W}{\partial t}dt = \frac{\partial W}{\partial z}}$$

$$\frac{\partial W}{\partial t} = \frac{dW}{dz}\frac{\partial z}{\partial t} = -\frac{1}{2}\frac{x}{\sqrt{t^3}}\frac{dW}{dz} = -\frac{1}{2}\frac{z}{t}\frac{dW}{dz}$$

$$\frac{\partial W}{\partial x} = \frac{dW}{dz}\frac{\partial z}{\partial x} = \frac{1}{\sqrt{t}}\frac{dW}{dz}$$

$$\frac{\partial^2 W}{\partial x^2} = \frac{d}{dz}\left(\frac{\partial W}{\partial x}\right)\frac{\partial z}{\partial x} = \frac{1}{t}\frac{d^2 W}{dz^2}$$

$$-\frac{1}{2}\frac{z}{t}\frac{dW}{dz} = \frac{D}{t}\frac{d^2W}{dz^2} - \frac{b}{z\sqrt{t}}\frac{1}{\sqrt{t}}\frac{dW}{dz} + \frac{bW}{z^2 t}$$

$$D\frac{d^2W}{dz^2} + \left(\frac{1}{2}z - \frac{b}{z}\right)\frac{dW}{dz} + \frac{b}{z^2}W = 0$$

$$\frac{\partial W}{\partial t} = D\,\frac{\partial^2 W}{\partial x^2} + \beta\,\frac{\partial}{\partial x}\left[W\cdot f(x)\right] \qquad \frac{\partial^2}{\partial x^2}(xW) = x\,\frac{\partial^2 W}{\partial x^2} + 2\,\frac{\partial W}{\partial x}$$

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} xW\,dx = D\,\frac{\partial}{\partial x}(xW)\Big| + \int\Big(\underbrace{\beta x\,\frac{\partial}{\partial x}[Wf]}_{\beta x Wf\big| - \int \beta W f\,dx} - \underbrace{2D\,\frac{\partial W}{\partial x}}_{-2DW\big|}\Big)\,dx$$

$$\therefore\quad \frac{\partial}{\partial t}\cdot\int_{-\infty}^{+\infty} xW\,dx = -\beta\cdot\int_{-\infty}^{+\infty} W\cdot f(x)\cdot dx$$

(Dasselbe auch direkt ~~aus~~ aus D Gl. durch zweimalige Integration nach x)

Bei Vernachlässigung der Br. B. und Behandlung ~~der~~ gemäss irreversibler Dynamik hätte man:

$$\frac{dx}{dt} = -\beta\, f(x)$$

Hier gilt also in Wahrscheinlichkeit

$$\frac{\partial(\bar{x})}{\partial t} = -\beta\,\bar{f}$$

das ist aber nicht dasselbe denn $\bar{f} = \int_{-\infty}^{+\infty} W f(x)\,dx$

braucht nicht identisch zu sein mit $f(\bar{x}) = f\left(\int xW\,dx\right)$

Es ~~wird aber nur~~ muss in dem Falle identisch sein wenn $f(x) = \alpha x$

aber ob auch sonst, ist von vornherein nicht ersichtlich

Für $t=\infty$

$\frac{\partial W}{\partial t} = 0$ $\qquad D\,\frac{\partial^2 W}{\partial x^2} + \beta\,\frac{\partial}{\partial x}(Wf) = 0$

$$D\,\frac{dW}{dx} = -\beta W f(x) + c$$

~~also~~ $c=0$ denn für $f(x)=0$ muss $W=\text{const}$

$$W = A\,e^{-\frac{\beta}{D}\int f(x)\,dx} = A\,e^{-\frac{\beta}{D}U}$$

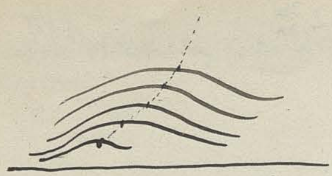
Für $t=0$

bleibt aus physikalischen Gründen nur $\frac{\partial W}{\partial t} = D\,\frac{\partial^2 W}{\partial x^2}$ übrig

---

Wie muss der Punkt wandern, für welchen $\frac{\partial W}{\partial x}\Big|_t = 0$, also der Punkt W_{max} ?

$$\frac{d}{dt}(W_m) = ?$$



$$W = \varphi(x,t)$$

$$dW = \frac{\partial \varphi}{\partial x}\,dx + \frac{\partial \varphi}{\partial t}\,dt$$

$$\cancel{dW = 0 \text{ für solche } dx,\, dt \text{ für welche } \frac{dx}{dt} = -\frac{\frac{\partial W}{\partial x}}{\frac{\partial W}{\partial t}}}$$

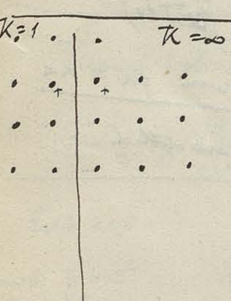
$$W = \varphi(x,t)$$

$$\frac{\partial \varphi}{\partial x} = 0$$

$$W = \varphi(x_0 t_0) + \Delta x \underbrace{\left(\frac{\partial \varphi}{\partial x}\right)_0}_{=0} + \Delta t \left(\frac{\partial \varphi}{\partial t}\right)_0 + \tfrac{1}{2}\Delta x^2 \left(\frac{\partial^2 \varphi}{\partial x^2}\right)_0 + \Delta x \Delta t \frac{\partial^2 \varphi}{\partial x \partial t} + \frac{\Delta t^2}{2}\frac{\partial^2 \varphi}{\partial t^2} + \ldots$$

$$\frac{\partial W}{\partial(\Delta x)} = \Delta x \left(\frac{\partial^2 \varphi}{\partial x^2}\right)_0 + \Delta t \left(\frac{\partial^2 \varphi}{\partial x \partial t}\right)_0 = 0 \quad (\max)$$

$$\left(\frac{\Delta x}{\Delta t}\right)_{\max} = -\frac{\frac{\partial^2 \varphi}{\partial x \partial t}}{\frac{\partial^2 \varphi}{\partial x^2}} = -\frac{D\frac{\partial^3 W}{\partial x^3} + \beta \frac{\partial}{\partial x}\left(W f(x)\right)}{\frac{\partial^2 W}{\partial x^2}}$$



Sobald Entfernung $= \frac{1}{2}$ Netzdistanz, bleibt die Anordnung ungeändert so als ob von Wand überhaupt keine Unterbrechung gäbe

Dagegen für kleinere ~~Abstände~~ Entfernung resultiert scheinbar abstossende } Kraft da erstere grösser als letztere

grösser ... anziehende

daher im Ganzen eine abstossende Kraft; es wird also die mittlere Konzentration der + Ionen in den Oberflächenschichten geringer sein als normal

und das müsste ~~sich~~ in noch höherem Grad für mehrwertige Ionen gelten, da die ~~K~~ elektr. Kräfte $\propto e^2$

Somit müssten die Randschichten vor Allem Mangel zeigen an mehrwertigen Ionen

Wenn also, wie in Wirklichkeit der Fall ist, die Wandschichten (in Wasser) negativ sind gegen die inneren Schichten so heisst das dass ~~da~~ darin mehrwertige positive Ionen fehlen.

Das ist schwer begreiflich, ebenso wäre der Einfluss von NaOH, HCl Zusätzen unbegreiflich wenn es nur auf die Wertigkeit ankäme.

Ob aber der Einfluss mehrwertiger Ionen überhaupt in dieser Richtung geht?

Enormer Einfluss von $Th(NO_3)_4$ Zusatz; ~~wirkt~~ wirkt im Sinne einer Umkehrung jener Doppelschicht, also so dass Wand~~[illegible]~~ positiv wird gegen Flüssigkeit; dagegen wäre Obigem zufolge gerade das Umgekehrte zu erwarten.

$$\frac{0{\cdot}2\,mg}{1\,l} = \qquad \begin{array}{r} 14 \\ 48 \\ \hline 62.4 \\ 248 \\ 232 \\ \hline 480 \end{array}$$

absolutes Mol. gewicht $= \frac{480}{N}$

auf $0{\cdot}2\,mg$ entfallen daher $\underbrace{\frac{0{\cdot}2}{480}}\,N$ · Th Molek.

$$n = 10^{-3}\,\frac{0{\cdot}2}{500}\cdot 6.10^{23} = 0{\cdot}3\,.10^{18}$$

also Abstand der Molek. $= \frac{1}{\sqrt[3]{n}} = \sqrt[3]{3{\cdot}3.10^{-18}} = \underline{\underline{1{\cdot}5.10^{-6}\,cm}}$!

Allerdings bewirkt dies nichts betreffs der Abstände in der Grenzschichten selbst!

Experimentum crucis für oder gegen den Einfluss der DK (Ionentheorie) auf die Doppelschicht:

Ist die Wirkung mehrwertiger Ionen zurückzuführen auf die durch die DK der Wand bewirkten Anziehungs und Abstossungskräfte so müsste die Wirksamkeit derselben sich gerade umkehren wenn die DK umgekehrt werden also für den Fall $K_1 > K_2$

müsste Kationen so wirken wie

Anionen im Falle $K_1 < K_2$

Gibt es aber ein ionisierendes Lösungsmittel mit hinreichend kleinem K?

$$u_{t=0} = e^{-\frac{cx}{2D}} \int_{-\infty}^{+\infty} \left[\sin\beta x - \frac{2D\beta}{c}\cos\beta x\right] f(\beta)\, d\beta$$

z.B. $f(\beta) = e^{-m\beta}$ $\qquad f(-\beta) = e^{-m\beta}$

$$\int_0^\infty e^{-m\beta}\sin\beta x\, d\beta = \frac{x}{x^2+m^2} \qquad \int_0^\infty \beta e^{-m\beta}\cos\beta x\, dx = \frac{1}{x^2+m^2} - \frac{2x^2}{(x^2+m^2)^2}$$

$$= \frac{m^2-x^2}{(x^2+m^2)^2}$$

$$u_0 = e^{-\frac{cx}{2D}}\left[\frac{x}{x^2+\frac{c^2}{4D^2}} - \frac{2D}{c}\frac{\frac{c^2}{4D^2}-x^2}{\left(\frac{c^2}{4D^2}+x^2\right)^2}\right] = e^{-\frac{cx}{2D}}\frac{2D}{c}\frac{\frac{xc}{2D}\left(\frac{c^2}{4D^2}+x^2\right) - \frac{c^2}{4D^2}+x^2}{\left(\frac{c^2}{4D^2}+x^2\right)^2}$$

$$e^{-\frac{cx}{2D}}\left[F(x) - \frac{2D}{c}F'(x)\right] = \varphi(x)$$

wie kann man daraus f bestimmen, wenn φ gegeben ist?

$$F - \frac{2D}{c}F' = e^{+\frac{cx}{2D}}\varphi(x) = \Phi(x)$$

$$y' + ay = X$$

$$z' + az = 0$$

$$\frac{(y'z - z'y)}{z^2} = \frac{X}{z}$$

$$\frac{d}{dx}\left(\frac{y}{z}\right) =$$

$$y = z\int \frac{X}{z}\, dx$$

$$= e^{-ax}\int X\, dx$$

$$F' - \frac{c}{2D}F = -\frac{c}{2D}\Phi$$

$$F = -\frac{c}{2D}e^{\frac{cx}{2D}}\int e^{-\frac{cx}{2D}}\Phi(x)\, dx = -\frac{c}{2D}e^{\frac{cx}{2D}}\int \varphi(x)\, dx$$

$$= \int_{-\infty}^{+\infty}\sin\beta x \cdot f(\beta)\, d\beta$$

$$= \frac{1}{\pi}\int_0^\infty d\alpha\,\sin\alpha x\int_{-\infty}^{+\infty}\Phi(y)\sin\alpha y\, dy$$

$$f(\beta) = \frac{1}{2}\int_{-\infty}^{+\infty}\Phi(y)\sin\beta y\, dy$$

$$= -\frac{c}{2D}\int_0^\infty e^{\frac{cy}{2D}}\sin\beta y\cdot\int^y\varphi(x)\,dx\cdot dy$$

Tatsächlich so wie früher: $\int_0^{\infty} \varphi(x)\,dx = -\frac{2D}{c} e^{-\frac{cx}{2D}} f(x) \Big|_0^{\infty} + \frac{2D}{c}\int e^{-\frac{cx}{2D}} f'(x)\,dx - \cancel{\frac{2D}{c}\int e^{-\frac{cx}{2D}} f(x)\,dx} = -\frac{2D}{c} f(0)$

$$= -\frac{2D}{c}\int \sin(\beta) f(\beta)\,d\beta = 0$$

$\cancel{e^{-\frac{cx}{2D}}\int_{-\infty}^{\infty} \sin\beta x}$ Probe: $\varphi(x) = -\frac{c}{2D}\int \varphi(x)\,dx + \frac{2D}{A} e^{-\frac{cx}{2D}} \left[\frac{c}{2D} e^{\frac{cx}{2D}}\int \varphi(x)\,dx + e^{\frac{cx}{2D}}\varphi(x)\right]$

stimmt

wenn $u = A e^{-\frac{cx}{D}}$:

$$\int_0^{\infty} u\,dx = A\int_0^{\infty} e^{-\frac{cx}{D}}\,dx = \frac{AD}{c}$$

$$u = \frac{C}{D} e^{-\frac{cx}{D}} - \lim \sqrt{\frac{\varepsilon}{\pi}}\, e^{-\frac{(x_0-x)^2}{\varepsilon}} = \varphi(x)$$

$$\int e^{-mx}\sin\beta x\,dx = -\frac{e^{-mx}}{m}\sin\beta x + \frac{\beta}{m}\int e^{-mx}\cos\beta x\,dx$$

$$\int e^{-mx}\cos\beta x\,dx = -\frac{e^{-mx}}{m}\cos\beta x - \frac{\beta}{m}\int e^{-mx}\sin\beta x\,dx$$

$$J = -\frac{e^{-mx}\sin\beta x}{m} - \frac{\beta}{m^2} e^{-mx}\cos\beta x - \frac{\beta^2}{m^2} J$$

$$J = -\frac{[m\sin\beta x + \beta\cos\beta x]\,e^{-mx}}{m^2+\beta^2}$$

$$-\frac{2D}{c} f(\beta) = \int_0^{\infty} e^{\frac{cy}{2D}}\sin\beta y\int_0^{y}\varphi(x)\,dx\cdot dy =$$

$$= \cancel{\int_0^{y}\varphi(x)\,dx\cdot\int_1^{y} e^{\frac{cy}{2D}}\sin\beta y\,dy}\Big|_0^{\infty} - \int_0^{\infty}\varphi(y)\,\frac{\left[\frac{c}{2D}\sin\beta y + \beta\cos\beta y\right] e^{-\frac{c}{2D}y}}{\beta^2+\frac{c^2}{4D^2}}\,dy$$

$$\int_0^{y}\varphi(x)\,dx = \left[1 - e^{-\frac{cy}{D}}\right] \quad \| \quad y < x_0$$

$$= -e^{-\frac{cy}{D}} \quad \| \quad y > x_0$$

$$-\frac{2D}{c} f(\beta) = \int_0^{\infty} e^{\frac{cy}{2D}}\sin\beta y\int_0^{y}\varphi(x)\,dx\,dy = \int_0^{x_0} e^{\frac{cy}{2D}}\sin\beta y\left[1 - e^{-\frac{cy}{D}}\right]dy - \int_{x_0}^{\infty} e^{\frac{cy}{2D}}\sin\beta y\cdot e^{-\frac{cy}{D}}\,dy$$

$$= \int_0^{x_0} e^{\frac{cy}{2D}}\sin\beta y\,dy - \int_0^{\infty} e^{-\frac{cy}{2D}}\sin\beta y\,dy$$

$$= \frac{\left[\frac{c}{2D}\sin\beta x_0 - \beta\cos\beta x_0\right] e^{\frac{cx_0}{2D}} - \beta}{\frac{c^2}{4D^2}+\beta^2}$$

Grenzfall für $x_0 = 0$:

$$u = \frac{C}{D} e^{-\frac{cx}{D}} + \frac{e^{-\frac{x^2}{4Dt} - \frac{cx}{2D} - \frac{c^2 t}{4D}}}{\sqrt{D\pi t}} - \frac{C}{D}\,\frac{e^{-\frac{cx}{D}}}{\sqrt{\pi}} \int_{-\infty}^{\frac{x-ct}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

$$= \frac{C}{D} e^{-\frac{cx}{D}} \left[1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x-ct}{2\sqrt{Dt}}} e^{-z^2}\,dz\right] + \frac{e^{-\frac{(x+ct)^2}{4Dt}}}{\sqrt{\pi D t}}$$

$$\bar{x} = \underbrace{\frac{C}{D}\int_0^\infty x e^{-\frac{cx}{D}}\,dx}_{-\frac{e^{-\frac{cx}{D}}}{\frac{c}{D}}\left(x+\frac{D}{c}\right)\Big|_0^\infty} - \underbrace{\frac{C}{D}\,\frac{1}{\sqrt{\pi}} \int_0^\infty x e^{-\frac{cx}{D}}\,dx \int_{-\infty}^{\frac{x-ct}{2\sqrt{Dt}}} e^{-z^2}\,dz}_{\underbrace{-\frac{e^{-\frac{cx}{D}}}{\frac{c}{D}}\left(x+\frac{D}{c}\right)\int_{-\infty}^{\frac{x-ct}{2\sqrt{Dt}}}\Big|_0^\infty}_{+\frac{D^2}{c^2}\int_{-\infty}^{-\frac{c}{2}\sqrt{\frac{t}{D}}} e^{-z^2}\,dz} \underbrace{+\frac{D}{c}\int_0^\infty e^{-\frac{cx}{D}}\left(x+\frac{D}{c}\right)\frac{e^{-\frac{(x-ct)^2}{4Dt}}}{2\sqrt{Dt}}\,dx}_{+\frac{D}{c}\int_0^\infty \left(x+\frac{D}{c}\right)\frac{e^{-\frac{(x+ct)^2}{4Dt}}}{2\sqrt{Dt}}\,dx}} + \frac{1}{\sqrt{\pi D t}} \int_0^\infty x e^{-\frac{(x+ct)^2}{4Dt}}\,dx$$

$\cancel{\frac{x+ct}{2\sqrt{Dt}} = u}$

$\cancel{x = 2u\sqrt{Dt} - ct}$

$\cancel{dx = 2du\sqrt{Dt}}$

$$\cancel{\frac{D}{c}\int \left(2u\sqrt{Dt} - ct + \frac{D}{c}\right) e^{-u^2}\,du}$$

$$\bar{x} = \frac{D}{c} - \frac{D}{c}\,\frac{1}{\sqrt{\pi}} \int_{-\infty}^{-\frac{c}{2}\sqrt{\frac{t}{D}}} e^{-u^2}\,du + \underbrace{\frac{1}{2\sqrt{\pi D t}} \int_0^\infty x\, e^{-\frac{(x+ct)^2}{4Dt}}\,dx - \frac{D}{c}\int_0^\infty \frac{e^{-\frac{(x+ct)^2}{4Dt}}}{2\sqrt{\pi D t}}\,dx}$$

$\frac{x+ct}{2\sqrt{Dt}} = u$

$x = 2u\sqrt{Dt} - ct$

$dx = 2du\sqrt{Dt}$

$$= \int_{+\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} \left[2u\sqrt{Dt} - ct - \frac{D}{c}\right] \frac{e^{-u^2}}{\sqrt{\pi}}\,du$$

$$\cancel{= 2\sqrt{\frac{Dt}{\pi}} \int_{-\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} u e^{-u^2}\,du + \frac{ct}{\sqrt{\pi}} \int e^{-u^2}\,du}$$

$$\cancel{\bar{x} = \sqrt{\frac{Dt}{\pi}}\, e^{-\frac{c^2 t}{4D}} + \frac{ct}{\sqrt{\pi}} \int_{-\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} e^{-u^2}\, du}$$

$$\cancel{\bar{x} = \frac{D}{c} - \frac{2D}{c} \frac{1}{\sqrt{\pi}} \int_{\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} e^{-u^2}\, du}$$

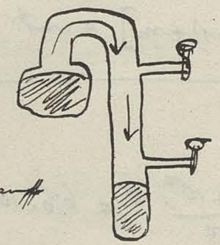
$$\bar{x} = \frac{D}{c} - \frac{1}{\sqrt{\pi}}\left(ct + \frac{2D}{c}\right) \int_{\frac{c}{2}\sqrt{\frac{t}{D}}}^{\infty} e^{-u^2}\, du + \sqrt{\frac{Dt}{\pi}}\, e^{-\frac{c^2 t}{4D}}$$

$\lim\limits_{t=0} \bar{x} = \sqrt{\frac{Dt}{\pi}}$ ebenso wie Brown'sche Bewegung

$\lim\limits_{t=\infty} \bar{x} = \frac{D}{C}$

Methode zur Bestimmung d. Diffusionscoeff.

z.B. Luft – Hg-Dampf oder andere Flüssigkeits-Dampf



Auch Bestimmung d. Diffusionscoeff. von Gummigutt-Teilchen:

1). Verteilung im Schwerefeld : $u = \frac{c}{D} e^{-\frac{cx}{D}}$

2). " " bei gleichzeitiger entgegengesetzter convectiver Strömung (indem poröse Zwischenwand verwendet wird): $u = \frac{C'}{D} e^{-\frac{c'x}{D}}$

Oder Diffusionscoeff. von ~~[illegible]~~ Teilchen die spezif. leichter sind als Wasser:

1). Verteilung im Schwerefeld an der Oberfläche

2). " " " bei gleichzeitiger Verdampfung oder Kondensation

was denselben Effekt hat wie Überlagerung einer convectiven Strömung

Dasselbe ginge auch bei Gummigutt-Teilchen falls sie hinreichend klein sind

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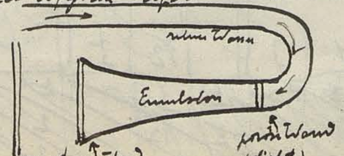
Fraktioniertes Zentrifugieren ersetzt durch einmaliges Zentrifugieren bei gleichzeitigem Gegenstrom



Querschnitt $q = \frac{A}{x}$

Dabei wirkt der Apparat unabhängig von Rotationsgeschwindigkeit falls Wasser~~[illegible]~~ und Zentrifugalkraft ~~[illegible]~~ verursacht

$$W(n,m) = P\,W(n-1,m) + (1-P)\,W(n-1,m-1)$$

$$W(n,m) - W(n-1,m) = (1-P)\left[W(n-1,m-1) - W(n-1,m)\right]$$

Für grosse n, m Grenzfall:

$$\frac{\partial W}{\partial n} + (1-P)\frac{\partial W}{\partial m} = 0$$

$$W\left[n(P-1)+m\right] = \text{const} = W\left[nP - (n-m)\right]$$

$$\lambda_m \theta = 2950$$

$$\varepsilon = \frac{hc}{\lambda} = \frac{6{,}5 \cdot 10^{-27} \cdot 3 \cdot 10^{10}}{2950} \cdot \theta = 6{,}6 \cdot 10^{-20} \cdot \theta$$

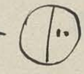
$$E = \frac{m c^2}{2} = \frac{3}{2} k\theta = \frac{3}{2}\,\frac{8 \cdot 10^{7}}{6 \cdot 10^{23}}\,\theta = 2 \cdot 10^{-16} \cdot \theta$$

$$\frac{\varepsilon}{E} = 3{,}3 \cdot 10^{-4}$$

also sind Quanten klein im Verh. zur m. kin. En. derselben Temperatur

---

Rotierende Scheibe, Wahrscheinlichkeit gewisser Trefferlagen

Wenn in zwei Kammern geteilt so ist Wahrsch.  ebenso gross wie  + 

$$\frac{2!}{1!\,1!} \qquad \frac{2!}{0!\,2!} \qquad \frac{2!}{2!\,0!}$$

denn

I	II
12	
1	2

2	1
	12

sind gleichmögliche Fälle

Wenn in drei ~~Kammern~~ Zellen geteilt

123		
12	3	
12		3
13	2	
1	23	
1	2	3
13		2
1	3	2
1		23
23	1	
2	13	
2	1	3
3	12	
	123	
	12	3
3	1	2
	13	2
	1	23

, ...

Alle drei in einer $\frac{3}{27} = \frac{1}{9}$ $: 3 = \frac{1}{27}$

zwei in einer und eine extra $\frac{18}{27} = \frac{2}{3} : 6 = \frac{1}{9}$

drei extra $\frac{6}{27} = \frac{2}{9} : 1 = \frac{2}{9}$

Dagegen wenn die Zellen individuell betrachtet

$\frac{3!}{3!0!0!} = 1$ [3 Arten]

$\frac{3!}{1!2!0!} = 3$ [6 Arten]

$\frac{3!}{1!1!1!} = 6$ [1 Art]

$\sum'[(x_n-x_k)^2+(y_n-y_k)^2+(z_n-z_k)^2] = \delta$ soll Minimum werden während gleichzeitig

$\sum[x_n^2+y_n^2+z_n^2] = C$ konstant bleibt

Also $\underbrace{\sum(x_n x_k + y_n y_k + z_n z_k)}_{\cancel{\cos}}$ soll Maximum werden

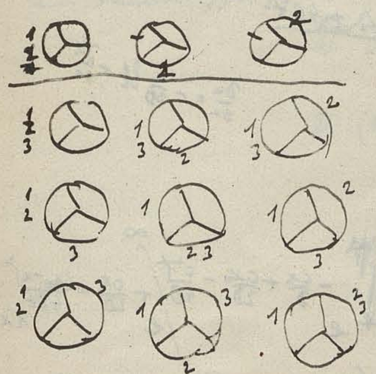
$$\sum x_n\delta x_n - x_n\delta x_k - x_k\delta x_n + x_k\delta x_k$$
$$\lambda(x_n\delta x_n + x_k\delta x_k)$$

$$\sum\{\delta x_n[x_n(1+\lambda)-x_k] + \delta x_k[x_k(1+\lambda)-x_n]\} = 0$$

$$\lambda = \frac{x_k - x_n}{x_n}$$

Es ist also (bei individueller Zellenteilung) am wahrscheinlichsten, dass alle Zellen gleich viel Treffer erhalten, also sollte ein gewisses Trefferbild eine Wahrscheinlichkeit besitzen unabhängig von der Zellenteilung und kontinuierlich variabel. Gibt es nicht eine derartige Wahrsch.-Funktion?

Vielleicht ~~quadratische Abweichung von~~ Quadratsumme der Differenzen?



Andere Form jener Lösung:

$$u = \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x-x_0)^2}{4Dt}} + e^{-\frac{(x+x_0)^2}{4Dt}}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} + \int_{\frac{x+x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}\,dz \cdot \frac{c}{D\sqrt{\pi}}\, e^{-\frac{cx}{D}}$$

Dagegen bei [illegible] für den Fall $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$:

$$u = \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x_0-x)^2}{4Dt}} + e^{-\frac{(x_0+x)^2}{4Dt}} - 2h\, e^{hx} \underbrace{\int_x^{\infty} e^{-hx - \frac{(x_0+x)^2}{4Dt}}\,dx}_{e^{\frac{cx_0}{D} + \frac{c^2 t}{D}}}\right]$$

~~Versuch:~~ $u = U \cdot e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$

$$\frac{\partial u}{\partial x} = \left[-\frac{c}{2D}U + \frac{\partial U}{\partial x}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$$

$$D\frac{\partial u}{\partial x} + cu = \left[\frac{c}{2}U + D\frac{\partial U}{\partial x}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$$

$$D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = \left[D\frac{\partial^2 U}{\partial x^2} - \frac{c^2}{4D}U - \frac{\partial U}{\partial t} + \frac{c^2}{4D}U\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$$

Somit erhält man Lösung von $D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$

mit: $D\frac{\partial u}{\partial x} + cu = 0 \;\|\; x=0$

wenn man die Lösung von

$$D\frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial t} = 0$$

mit: $D\frac{\partial U}{\partial x} + \frac{c}{2}U = 0 \;\|\; x=0$

multipliziert mit $e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$

$$\frac{\partial u}{\partial x} = -\frac{c}{2D}U + \frac{\partial U}{\partial x}$$

Also erhält man U nach [illegible] durch Einsetzen $h = -\frac{c}{2D}$ und

$$u = \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x_0-x)^2}{4Dt}} + e^{-\frac{(x_0+x)^2}{4Dt}}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}} + \frac{1}{2\sqrt{\pi D t}}\,\frac{c}{D}\, e^{-\frac{cx}{D} + \frac{cx_0}{2D} - \frac{c^2 t}{4D}} \underbrace{\int_x^{\infty} e^{+\frac{cx}{2D} - \frac{(x_0-x)^2}{4Dt}}\,dx}$$

$$e^{-\frac{cx}{D}} \int_{\frac{x-x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}\,dz$$

$\boxed{\text{stimmt}}$!

Allgemein ~~Versuch~~ $u = U\,V$ $\quad D\frac{\partial^2 u}{\partial x^2} + f\frac{\partial u}{\partial x} + u\frac{\partial f}{\partial x} - \frac{\partial u}{\partial t} = 0$

$$D\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(u f) - \frac{\partial u}{\partial t} = 0$$

$$V\left[D\frac{\partial^2 U}{\partial x^2} + U\frac{\partial f}{\partial x} + f\frac{\partial U}{\partial x} - \frac{\partial U}{\partial t}\right] + 2D\frac{\partial V}{\partial x}\frac{\partial U}{\partial x} + D\frac{\partial^2 V}{\partial x^2}U + fU\frac{\partial V}{\partial x} - U\frac{\partial V}{\partial t} = 0$$

für $f = \text{const}$:

$$V\left[D\frac{\partial^2 U}{\partial x^2} + c\frac{\partial U}{\partial x} - \frac{\partial U}{\partial t}\right] + 2D\frac{\partial V}{\partial x}\frac{\partial U}{\partial x} + D\frac{\partial^2 V}{\partial x^2}U + cU\frac{\partial V}{\partial x} - U\frac{\partial V}{\partial t} = 0$$

$$D\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t} \qquad c\frac{\partial}{\partial x}(UV) + 2D\frac{\partial U}{\partial x}\frac{\partial V}{\partial x} = 0 \quad ?$$

$$D\frac{\partial^2 U}{\partial x^2} = \frac{\partial U}{\partial t}$$

oder:

Allgemein erhält man Lösung von $D\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$

mit Grenzbedingung für $x = 0$: I). $u = 0$

II). $\frac{\partial u}{\partial x} = 0$

III). $\frac{\partial u}{\partial x} + h u = 0$

indem man die Lösung von

$$D\frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial t} = 0$$

mit Grenzbedingung I). $u = 0$

II). $U - \frac{2D}{c}\frac{\partial U}{\partial x} = 0$

III). $\left(h - \frac{c}{2D}\right)U + \frac{\partial U}{\partial x} = 0$

mit $e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$ multipliziert

$$W(x_0 x)_{(k+1)\tau}\,dx = \int_{-\infty}^{+\infty} W(x_0\alpha)_{k\tau}\, W(\alpha x)_\tau \, d\alpha\, dx$$

$$W(\alpha x)_\tau = \frac{1}{2\sqrt{\pi D t}}\, e^{-\frac{\left(x-\alpha-\frac{\beta}{\alpha}\right)^2}{4Dt}}\, dx \quad \text{od. auch: } e^{-\frac{\left(x-\alpha-\frac{\beta}{x}\right)^2}{4Dt}} \qquad \beta = \frac{}{t_e}$$

$$W(x_0\alpha)_t = \frac{1}{2\sqrt{\pi D t}}\, e^{-\frac{\left(\alpha - x_0 - \frac{\beta}{x_0}\right)^2}{4Dt}}\, d\alpha$$

$$W(x_0 x)_{2\tau}\, dx = \left(\frac{1}{2\sqrt{\pi D t}}\right)^2 e^{-\frac{\left(x_0+\frac{\beta}{x_0}\right)^2 - 2\alpha\left(x_0 - \frac{\beta}{x_0}\right) + \alpha^2 + x^2 - 2\alpha x - \frac{2\beta x}{\alpha} + \alpha^2 + \frac{\beta^2}{\alpha^2} + 2\beta}{4Dt}}$$

$$\lim_{\beta=0} P = 1 - \frac{2}{\sqrt{\pi}}\left[\beta - \frac{\beta^3}{3}\right] + \frac{1}{\beta\sqrt{\pi}}\left[\frac{\beta^2}{1} - \frac{\beta^4}{2}\right]$$

$$= 1 - \frac{2\beta}{\sqrt{\pi}} + \frac{2\beta^3}{3\sqrt{\pi}} + \frac{\beta}{\sqrt{\pi}} - \frac{\beta^3}{2\sqrt{\pi}} = 1 - \frac{\beta}{\sqrt{\pi}} + \frac{\beta^3}{6\sqrt{\pi}}$$

$$t \sim \frac{1}{\beta^2} \qquad z = e^{-\frac{1}{\beta^2}}$$

$$-\log z = \frac{1}{\beta^2}$$

$$\omega = 2n\pi$$

$$P = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{1}{\sqrt{\log z}}} e^{-y^2}\,dy + \frac{\sqrt{\log z}}{\sqrt{\pi}}\left[1 - e^{+\frac{1}{\log z}}\right]$$

$$U = i\omega = 4\pi\left(1 - \frac{r}{\sqrt{r^2+a^2}}\right)\overbrace{\frac{}{2\varepsilon n}}^{i} = 4\varepsilon\omega\left(1 - \frac{r}{\sqrt{a^2+r^2}}\right)$$

$$F = 4\pi i\left[\frac{1}{\sqrt{a^2+r^2}} - \frac{1}{2}\frac{r^2}{\sqrt{}^3}\right] = 4\pi i \,\frac{2a^2+r^2}{\sqrt{a^2+r^2}^{3/2}}$$

$$W = i t^{\prime\prime} M$$

$$M \neq \frac{a^2\pi\, 4\pi}{r}\left(1 + \frac{a^2}{2r^2}\right) \qquad \neq \frac{4\pi i}{r}\left[1 + \frac{2a^2}{r^2}\right]\left[1 + \frac{3}{2}\frac{a^2}{r^2}\right]$$

$$= \frac{4\pi^2 a^2}{r} \qquad = \frac{4\pi i}{r}\left[1 + \frac{a^2}{2r^2}\right]$$

Bohr'sches -Debye Modell H_2

$$\frac{e^2}{r} = 6 \cdot 10^{-14}$$

$$r = \frac{(4.7)^2\, 10^{-20}}{6 \cdot 10^{-14}} = 4 \cdot 10^{-6}$$

$$f_{el} = \cancel{16}\; 6\, e\, d^2$$

$$+ = + \qquad + = +$$

$$\cancel{kt}\; f_{el} = 6\,\frac{e^2}{r^4}\, d^2$$

$$U_{el} = \frac{3}{2}\,\frac{e^2 d^2}{r^3} = 6 \cdot 10^{-14}$$

$$r^3 = \frac{3}{2}\,\frac{e^2}{6\cdot 10^{-14}} \cdot d^2 = 6\cdot 10^{-6} \cdot 0.09 \cdot 10^{-16}$$

$$= \frac{1}{2} \cdot 10^{-22} = 50 \cdot 10^{-24}$$

$$\underline{\underline{r = 4 \cdot 10^{-8}}}$$

$$\frac{d^2}{dr^2}\left(2\,\frac{e^2 d^2}{r^3}\right) = 24\,\frac{e^2 d^4}{r^5} = 6 \cdot 10^{-14}$$

$$r^5 = 24 \cdot 4 \cdot 10^{-6} \cdot (0.3)^4 \cdot 10^{-32} = 10^{-38} = 100 \cdot 10^{-40}$$

$$r = \sqrt[5]{100} \cdot 10^{-8} = 2 \cdot 10^{-8}$$

$$W = (2\varepsilon_n n)^2\,\frac{4\pi^2 a^2}{r} = \; = \cancel{4}\pi^2 \cdot \frac{10^{-16}}{\cancel{4}} \cdot 4 \cdot \left(\frac{4.7 \cdot 10^{-10}}{3 \cdot 10^{10}}\right)^2 \cdot \frac{4}{6} \cdot 10^{32}\;\frac{1}{r}$$

$$\begin{array}{r} 18 \cdot 18 \\ 144 \\ \hline 324 \end{array}$$

$$= 10^{-15}\;\frac{16 \cdot 3.2}{\cancel{16}\; 6} \cdot \cancel{}\; 10^{-8}\;\frac{1}{r}$$

$$r = \frac{10^{-15} \cdot \frac{8 \cdot 3.2}{3} \cdot 10^{-8}}{6 \cdot 10^{-14}} = 10^{-9} \cdot 1.5$$

Also sind magnet. Anziehungskräfte bei kleinen Distanzen viel geringer wie die elektr. Abstossung, doch muss vielleicht letztere in grösseren Distanzen überwiegen, da die gegenseitige magnet. Energie abnimmt wie $\frac{1}{r}$, die elektrostat. wie $\frac{1}{r^5}$

~~Möglichkeit begreiflich dass mit (Anzahl d. Elektronen im Ringe)^2 wächst Anziehung~~

Bei einatomigen Molek. wird sich der Ring immer der wirksamen magn. Kraft anpassen welche von d. benachbarten ~~Ring~~ Atom ausgeübt wird.

Born u. Karman Ph. Z. 13, 297, 1912

$m\ddot{u}_n = \alpha(u_{n+1} - 2u_n + u_{n-1})$ $\qquad -m\nu^2 = \alpha\left[e^{i\varphi} + e^{-i\varphi} - 2\right]$

$u_n = u\, e^{i(\nu t + n\varphi)}$ $\qquad = -4\alpha \sin^2\frac{\varphi}{2}$

$\boxed{na = \lambda = \frac{2\pi a}{\varphi}}$

$\nu = \nu_0 \sin\frac{\varphi}{2} \qquad \nu_0 = 2\sqrt{\frac{\alpha}{m}}$

Wellengeschw. $\omega = \frac{\nu\lambda}{2\pi} = \frac{\nu_0\lambda}{2\pi}\sin\frac{a\pi}{\lambda} = \frac{a\nu}{2\arcsin\frac{\nu}{\nu_0}}$

Also ν hat Maximalwert ν_0 Grenzfrequenz wobei  und $\omega = \frac{a\nu_0}{\pi}$ $\left(= \frac{2}{\pi}c\right.$

Minimalwert $\nu = 0$ und $\omega = \frac{a\nu_0}{2}$

p. 305:

$T = \frac{m}{2}\left[\cdots \dot{u}_{-2}^2 + \dot{u}_{-1}^2 + \dot{u}_0^2 + \dot{u}_1^2 + \cdots\right]$

$V = \frac{\alpha}{2}\left[\cdots (u_{-1} - u_0)^2 + (u_0 - u_1)^2 + \cdots\right] = \alpha\left[\cdots u_1^2 + u_0^2 + u_1^2 + u_2^2 + \cdots - u_{-1}u_0 - u_0u_1 - u_1u_2 - \right]$

Normalkoordinaten:

$U_n = \cdots k_{-1n}u_{-1} + k_{0n}u_0 + k_{1n}u_1 + k_{2n}u_2 + \cdots$

so dass:

$T = \frac{m}{2}\left[\cdots \dot{U}_{-2}^2 + \dot{U}_{-1}^2 + \dot{U}_0^2 + \dot{U}_1^2 + \cdots\right]$

$V = \frac{\alpha}{2}\left[\cdots p_{-1}U_{-1}^2 + p_0U_0^2 + p_1U_1^2 + \cdots\right]$

Dann jede Koordinate unabhängig von übrigen.

$$m\ddot{U}_n + \alpha p_n U_n = 0$$

$$U_n = A_n \sin(\nu_n t + \varepsilon_n) \qquad \nu_n = \sqrt{p_n\frac{\alpha}{m}}$$

Auffindung der Normalkoordinaten:

Da U sich linear aus u zusammensetzen, muss auch gelten:

$$m\ddot{U}_n = \alpha(U_{n+1} - 2U_n + U_{n-1})$$

und wenn U_n wirklich Normalkoord. sind, so heisst das, dass jede unabhängig von d. übrigen schwingen kann also

$U_n = \cancel{\ldots} \; A_n \sin(\nu_n t + \varepsilon_n)$ also

$\ddot{U}_n = -\nu_n^2 U_n = - p_n \frac{\alpha}{m}$

$\therefore$

(33) $\quad - p_n U_n = U_{n+1} - 2 U_n + U_{n-1}$ Dies Gleichungssystem besitzt im Allg. keine Lösung (für beliebige p) dann ist nötig dass Det

$$\begin{vmatrix} \ldots & & & & & & & \\ \ldots 0 & -1 & 2-p & -1 & 0 & 0 & \ldots \\ \cdot\; 0 & 0 & -1 & 2-p & -1 & 0 & \ldots \\ \vdots\; 0 & 0 & 0 & -1 & 2-p & -1 & 0 \ldots \\ & & \ldots & & & & \end{vmatrix} = 0$$

Wurzeln davon $p_1\, p_2\, p_3 \ldots$

Wenn diese bei (33) eingesetzt so folgen n verschiedene ~~Lös~~ Lösungssysteme welche der Relation erfüllen [illegible]

$$\sum_m k_{mn} k_{mn'} = 0 \qquad (n \neq n')$$

und bei entsprechender Wahl d. willkürlichen Faktoren:

$$\sum_m k_{mn}^2 = 1$$

dann sind dies gerade die gesuchten Koeffizienten der k für U

Anzahl der in einem Frequenzintervall $d\nu$ befindlichen Normalschwingungen:

$$N(\nu)\, d\nu = N L \frac{d\varphi}{2\pi} \qquad \left(\nu = \nu_0 \sin\frac{\varphi}{2}\right)$$

L = Länge der Saite

N = Anzahl Massenpunkte pro Längeneinheit

$$u = \frac{c}{D}\,\frac{e^{-\frac{cx}{D}}}{\sqrt{\pi}} \int_{\frac{x+x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}\,dz + \frac{1}{2\sqrt{\pi D t}}\left[e^{-\frac{(x-x_0)^2}{4Dt}} + e^{-\frac{(x+x_0)^2}{4Dt}}\right] e^{-\frac{c(x-x_0)}{2D} - \frac{c^2 t}{4D}}$$

$x = x_0$:

$$u = \frac{c}{D}\,\frac{e^{-\frac{cx_0}{D}}}{\sqrt{\pi}} \int_{\frac{2x_0-ct}{2\sqrt{Dt}}}^{\infty} e^{-z^2}\,dz + \frac{1}{2\sqrt{\pi D t}}\left[1 + e^{-\frac{x_0^2}{Dt}}\right] e^{-\frac{c^2 t}{4D}}$$

$$z = \frac{x+x_0-ct}{2\sqrt{Dt}}$$

$$\frac{1}{\sqrt{\pi}}\int_0^\infty \frac{e^{-\frac{(\xi-\alpha t)^2}{4Dt}}\,d\xi}{2\sqrt{Dt}} - \int_0^\infty \frac{e^{-\frac{(\xi+\alpha t)^2}{4Dt}}\,d\xi}{\sqrt{Dt}} = \frac{1}{\sqrt{\pi}}\left[\int_{-\frac{\alpha t}{\sqrt{Dt}}}^{\infty} e^{-u^2}\,du - \int_{\frac{\alpha t}{\sqrt{Dt}}}^{\infty} e^{-u^2}\,du\right] = \frac{1}{\sqrt{\pi}}\,\frac{\alpha t}{\sqrt{Dt}}$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\frac{\alpha t}{\sqrt{Dt}}} e^{-u^2}\,du$$

$$W(r) = 1 - 2\int_r^\infty \frac{e^{-\frac{\xi^2}{4Dt}}}{\sqrt{\pi D t}}\,d\xi = 1 - \frac{2}{\sqrt{\pi}}\int_{\frac{r}{2\sqrt{Dt}}}^{\infty} e^{-z^2}\,dz = 1 - \frac{2}{\sqrt{\pi}}\,\frac{e^{-\frac{r^2}{4Dt}}}{\frac{r}{\sqrt{Dt}}} = 1 - \frac{2}{r}\sqrt{\frac{Dt}{\pi}}\,e^{-\frac{r^2}{4Dt}}$$

$$W^n = e^{-\frac{2n}{r}\sqrt{\frac{Dt}{\pi}}\left(e^{-\frac{r^2}{4Dt}}\right)}$$

$$W(x)\int \frac{e^{-\frac{(\xi+\eta-\alpha t)^2}{4Dt}}\,d\eta\,d\xi}{\sqrt{\quad Dt}}$$

$$\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2} + \beta\frac{\partial}{\partial x}\left[Wf\right]$$

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty} x^2 W\,dx = D\underbrace{\int x^2\frac{\partial^2 W}{\partial x^2}\,dx} + \beta\underbrace{\int x^2\frac{\partial}{\partial x}(Wf)\,dx}$$

$$\cancel{x^2 Wf} - 2\int \underbrace{x.(W.f)\,dx}$$

$$\cancel{x^2\frac{\partial W}{\partial x}} - 2\int x\underbrace{\frac{\partial W}{\partial x}\,dx}$$

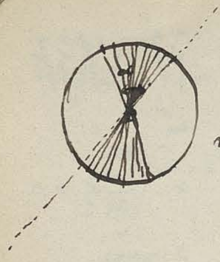
$$\cancel{xW} - \int \underbrace{W\,dx}_{=1}$$

$$\frac{\partial}{\partial t}(\overline{x^2}) = 2D - 2\beta\int_{-\infty}^{+\infty} x\,W(x)\,f(x)\,dx \qquad \frac{\partial}{\partial t}(\bar{x}) = -\beta\int_{-\infty}^{\infty} Wf\,dx$$

$$\frac{\partial}{\partial t}\overline{(x-x_0)^2} = \frac{\partial(\overline{x^2})}{\partial t} - 2x_0\frac{\partial(\bar{x})}{\partial t} = 2D - 2\beta\int_{-\infty}^{+\infty} x.W.f.\,dx + 2x_0\,\beta\int Wf\,dx$$

$$= 2D + 2\beta\int_{-\infty}^{+\infty}(x_0 - x)\,W(x)\,f(x)\,dx$$

$$= 2D - 2\beta\,\overline{\left[(x_0 - x)\,f(x)\right]}$$



Wahrscheinlichkeit gewisser Trefferlagen auf der rotierenden Scheibe:

relat. Wahrsch. dass zwei Treffer in A, keiner in B : $\frac{2!}{2!0!} = 1 \quad = \frac{1}{4}$

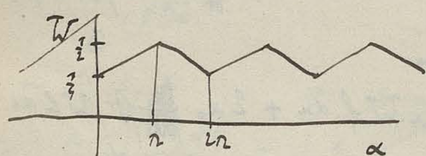
" ein Treffer in A, einer in B : $\frac{2!}{1!1!} = 2 \quad = \frac{1}{2}$

" kein ~~[illegible]~~ Treffer in A, zwei in B : $\frac{2!}{0!2!} = 1 \quad = \frac{1}{4}$

Kann man nicht daraus die Wahrsch. eines Winkelabstands α, unabhängig von der Lage der Zellenteilung ~~[illegible]~~ ableiten? Indem man die Zelle rotieren lässt

$$\overline{W} = 2\left(\frac{1}{2}\frac{\alpha}{2\pi} + \frac{1}{4}\frac{\pi-\alpha}{2\pi}\right) = \frac{\alpha}{\pi}\left(\frac{1}{2}-\frac{1}{4}\right) + \frac{1}{4} = \frac{1}{4}\left(1+\frac{\alpha}{\pi}\right) \qquad \alpha < \pi$$

$$\overline{W} = 2\left(\frac{1}{4}\frac{\alpha-\pi}{2\pi} + \frac{1}{2}\frac{2\pi-\alpha}{2\pi}\right) = \frac{\alpha}{\pi}\left(\frac{1}{4}-\frac{1}{2}\right) + 1 - \frac{1}{4} = 1 - \frac{1}{4}\left(1+\frac{\alpha}{\pi}\right) \qquad \alpha > \pi$$



Dabei würde aber doch W von der Zahl der Zellen abhängen, denn würde man z.B. 4 Zellen anbringen, so wäre z.B.

~~[illegible]~~ W. ~~[illegible]~~ für	2 in A,	0 B,	0 C,	0 D	$= \frac{2!}{2!0!0!0!}$	$= 1$	$= \frac{1}{16}$
	1 A,	1 B,	0	0	$= \frac{2!}{1.}$	$= 2$	$= \frac{1}{8}$
	1	0	1	0		$= 2$	
	1	0	0	1		$= 2$	
	0	2	0	0		$= 1$	
	0	1	1	0		$= 2$	
	0	1	0	1		$= 2$	
	0	0	1	1		$= 2$	
	0	0	2	0		$= 1$	
	0	0	0	2		$= 1$	

$$W = 4\left[\frac{\alpha}{2\pi}\,\frac{1}{8} + \left(\frac{\pi}{2}-\alpha\right)\frac{1}{2\pi}\,\frac{1}{16}\right] = \frac{\alpha}{4\pi} + \frac{1}{16} - \frac{\alpha}{8\pi} = \frac{1}{16}\left(1+\frac{2\alpha}{\pi}\right)$$

Offenbar ist W abhängig von Zellenteilung und es hat gar keinen Sinn von der ~~[illegible]~~ Wahrscheinlichkeit einer gewissen Trefferanordnung zu sprechen, ohne die Zellenteilung zu definieren. An und für sich ist jede Trefferanordnung gleich wahrscheinlich?

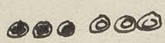
Andererseits wenn es sich z.B. um Vermischen weisser und schwarzer Kugeln handelt, so entspricht das unzweifelhaft einem Wachsen der Wahrscheinlichkeit (als einem reellen Vorgang). Wie ist dieser Begriff aber zu definieren ohne Bezug auf eine (willkürliche) Zellenteilung?

Eindimensionaler Vorgang: 

~~[illegible]~~ sorge ich wie

gleichberechtigt ist jede bestimmte Kombination

gleichberechtigt sind alle einzelnen Permutationen der individuellen Elemente; darunter sind aber viele gleichwertig wenn wir die Individuen nicht unterscheiden

z.B.:  und: 

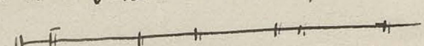
Zu untersuchen auf ihren W-Zustand

Dagegen wenn wir eine Statistik der Abstände von einem ~~w~~ weiss bis zum nächsten weiss untersuchen, wird die Formel $\left. e^{-\nu}\frac{\nu^n}{n!}\right|_{\nu=1} = \frac{1}{n!\,e} = W(n)$ die Wahrsch. eines $n$fachen Abstandes bedeuten

? (falls durchschnittlich ebenso viel weiss wie schwarz)

Bei kontinuierlicher Wahrscheinlichkeit z.B.: wenn Scintillationen auf der t Axe registriert werden

$W(t)\,dt = k\,e^{-kt}\,dt$



Hiernach lässt sich die Wahrsch. verschiedener Distanzen abschätzen; aber Frage ob eine bestimmte Verteilung wahrsch. oder unwahrsch. ist?

$W(t_1)\,dt_1\, W(t_2)\,dt_2\, W(t_3)\,dt_3 \ldots = k^n e^{-k(t_1+t_2+\ldots t_n)}\,dt_1\,dt_2 \ldots dt_n$

also kommt es nur auf die Summe $t_1+t_2+\ldots t_n$ an und die Verteilung ist ganz gleichgültig!

Nadzwyczajne wydanie

„CZAS" wychodzi codziennie o godzinie 6-tej wieczór.
Numer pojedynczy 25 gr.

PRENUMERATA MIESIĘCZNIE WYNOSI:

Krakowie bez odnoszenia do domu	zł. 5.40
Krakowie z odnoszeniem do domu	zł. 6.—
prowincję z przesyłką pocztową	zł. 6.—
anicę z przesyłką pocztową	zł. 10.—

Za każdą zmianę adresu dolicza się zł. 0.50.

lamacje niezapieczętowane nie podlegają opłacie pocztowej.

Listów nieopłaconych nie przyjmuje się.

renumeratę przyjmują: Adm istracja „Czasu", wszystkie urzędy pocztowe, wszystkie miejscow i zamiejscowe Biura dzienników.

Redakcja r isów nie zwraca.

C

KO

Telefon Redak
Adres Redakcji
Godziny biur

Z Bezpartyjnego Bloku w

12 żydów — 12 socjalistów —

131

$$\frac{1}{\varepsilon}\frac{r}{m} \equiv \frac{q}{n} \equiv \frac{q^2}{m} \equiv \frac{q\,K}{\varepsilon m^2} \equiv 1$$

~~100 m~~

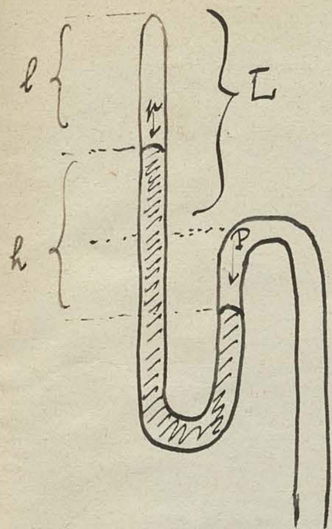
$m = \frac{1}{100}$

$v = \frac{1}{10}$

$$r \equiv \varepsilon m$$

$$q \equiv n \equiv \sqrt{m}$$

$$K = \frac{\varepsilon m^2}{q} = \varepsilon m^{3/2}$$



$$pv = R\theta$$

$$p\,l = c\theta \qquad \Big| \qquad \cancel{p_0 l_0 = c\theta_0}$$

$$P = p + h \qquad \Big| \qquad \cancel{P_0 = p_0 + h_0}$$

$$l + \frac{h}{2} = L$$

$$p\left(L - \frac{h}{2}\right) = c\theta$$

$$P = p + h$$

$$P = h + \frac{c\theta}{L - \frac{h}{2}} \qquad \Big\| \qquad P_0 = h_0 + \frac{c\theta_0}{L - \frac{h_0}{2}}$$

$$2PL - P\frac{h}{\cancel{2}} = 2hL - \frac{h^2}{\cancel{2}} + 2c\theta$$

$$h^2 - h(P + 2L) = 2(c\theta - PL)$$

$$h = \frac{P + 2L}{2} \pm \sqrt{\left(\frac{P+2L}{2}\right)^2 + 2(c\theta - PL)}$$

$$P = h + \frac{\theta}{\theta_0}(P_0 - h_0)\,\frac{L - \frac{h_0}{2}}{L - \frac{h}{2}}$$

$$c = \frac{(P_0 - h_0)\left(L - \frac{h_0}{2}\right)}{\theta_0}$$

$$h = \frac{P + 2L}{2} - \sqrt{\left(\frac{P - 2L}{2}\right)^2 + 2\frac{\theta}{\theta_0}(P_0 - h_0)\left(L - \frac{h_0}{2}\right)}$$

$$= L + \frac{P}{2} - \sqrt{\left(\frac{P}{2} - L\right)^2 + \frac{\theta}{\theta_0}(P_0 - h_0)(2L - h_0)}$$

Dla wszystkich θ

$$h = L + \frac{P}{2} - \sqrt{\left(\frac{P}{2} - L\right)^2 + (P_0 - h_0)(2L - h_0)}$$

$\rho\left(u\frac{\partial u}{\partial x}+\ldots\right)=\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\mu}{\rho}\Delta^2 u$

. . . . .

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}=0$

~~[illegible]~~ m u
n x
α μ
b p
r ϱ

Podobieństwo dynamiczne

$$\frac{m^2}{n}\equiv\frac{b}{rn}\equiv\frac{\alpha}{r}\frac{m}{n^2}$$

$$m^2\equiv\frac{b}{r}\equiv\frac{\alpha m}{rn}$$

$$\boxed{m^2\equiv\frac{b}{r}\quad\Big|\quad m\equiv\frac{\alpha}{rn}}$$

$n=1$ $m=\frac{\alpha}{r}$ $b=rm^2=\frac{\alpha^2}{r}$

Symetrya prądu i tworzenie się promieni będzie zatem zależeć (dla różnych cieczy) od $b=\frac{\alpha^2}{r}$

wtedy: $m=\frac{\alpha}{r}$

|| Tem większa symetrya czem większe α

|| z drugiej strony podobieństwo między ~~[illegible]~~

przy stałem b r m/: α∞ n t.j. tworzenie promieni

tem łatwiejsze czem większy otwór (jeśli równa stała)

Przy stałem α, r:

~~m~~ $m=\sqrt{b}$ $n=\frac{1}{\sqrt{b}}$

Podobne przy zwiększeniu ciśnienia a zmniejszeniu rozmiarów

$$\frac{b}{h}\frac{m^2}{n} \equiv \frac{b}{n} \equiv \beta\frac{n}{n^2} \equiv \frac{b}{h}$$

$$\frac{mb}{n} \equiv \beta\frac{m^2}{n^2} \equiv \gamma\frac{h}{n^2}$$

$$m^2 = h = n\,; \quad \beta = \gamma\,; \quad nb = m\beta$$

$$\frac{1}{\eta}\frac{d\eta}{d\theta} = \frac{1}{\eta}\left(\frac{\partial\eta}{\partial\theta}\right)_{v=const} + \frac{1}{\eta}\left(\frac{\partial\eta}{\partial v}\right)_{\theta=const}\frac{\partial v}{\partial\theta} = \frac{1}{\eta}\frac{\partial\eta}{\partial\theta} + \frac{1}{\eta}\frac{\partial\eta}{\partial p}\,\frac{\frac{1}{v}\frac{\partial v}{\partial\theta}}{\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_{\theta=const}}\,.$$

$$= \frac{1}{\sqrt{T}} - \frac{\alpha}{\beta}\cdot\frac{1}{\eta}\frac{\partial\eta}{\partial p}$$

Co do $\left(\frac{\partial\eta}{\partial v}\right)_{\theta=const}$:

Pod założeniem kul różnią się rozmaite gazy tylko co do wielkości tychże i mas
σ μ

Porównajmy dwa gazy o wielkościach σ_1 σ_2.
(o równej ilości cząsteczek) $N_1 = N_2$ μ_1 μ_2

Ruchy ich będą dynamicznie podobne, jeżeli:

$$V_1 : V_2 = \sigma_1^3 : \sigma_2^3 \qquad \text{I).}$$

$$\rho_1 : \rho_2 = \frac{\mu_1}{\sigma_1^3} : \frac{\mu_2}{\sigma_2^3}$$

i jeżeli $c_1 : c_2 = \sigma_1 : \sigma_2$

zatem $T_1 : T_2 = \mu_1 c_1^2 : \mu_2 c_2^2 = \mu_1\sigma_1^2 : \mu_2\sigma_2^2$ II).

Wtedy I II

$$\frac{\partial u_1}{\partial y_1} = \frac{\partial u_2}{\partial y_2}$$

$$X_{y_1} : X_{y_2} = \mu_1 c_1 : \mu_2 c_2$$

$$\omega_1 : \omega_2 = \sigma_1^2 : \sigma_2^2$$

$$\eta_1 : \eta_2 = \frac{X_{y_1}}{\omega_1} : \ldots = \frac{\mu_1 c_1}{\sigma_1^2} : \frac{\mu_2 c_2}{\sigma_2^2} = \frac{\mu_1}{\sigma_1} : \frac{\mu_2}{\sigma_2}$$

podobne są V_1 o dV_1 $\qquad dV_1 : dV_2 = \sigma_1^{\,3} : \sigma_2^{\,3} = V_1 : V_2$

V_2 o dV_2

$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial v}\right)_1 : \left(\frac{1}{\eta}\frac{\partial\eta}{\partial v}\right)_2 = \frac{1}{\sigma_1^{\,3}} : \frac{1}{\sigma_2^{\,3}} \quad \Big\| \quad \left(\frac{v}{\eta}\frac{\partial\eta}{\partial v}\right)_1 : \left(\quad\right)_2 = 1 : 1$$

$$\frac{T_1}{\eta_1} : \frac{T_2}{\eta_2} = \sigma_1^{\,3} : \sigma_2^{\,3}$$

$$v \cdot \frac{1}{\eta}\frac{\partial\eta}{\partial v} = \frac{\frac{1}{\eta}\frac{d\eta}{d\theta} - \frac{1}{T}}{\alpha}$$

$\eta = fc.(v, \theta)$ $\qquad$ Normalsubstanz: σ_0, μ_0, η_0

$$\eta_{\theta=\theta_0} = fc.(v, \theta_0) = \varphi(v)$$

$\eta(v,\theta)$ porównać z $\eta(v, \theta_0)$ $\Big\|$: $\eta(v,\theta) : \eta(v,\theta_0) = \sqrt{\theta} : \sqrt{\theta_0}$

$$\eta(v,\theta) = \sqrt{\frac{\theta}{\theta_0}}\,\eta(v,\theta_0) = \sqrt{\frac{\theta}{\theta_0}}\,\varphi(v)$$

$$\varphi(v) = \sqrt{\frac{\theta_0}{\theta}}\,\eta(v,\theta)$$

Przy gęstości $\qquad \rho_1 = \overbrace{\frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^3}^{\alpha} \rho_2$

i temperaturze: $\qquad \theta_1 = \overbrace{\frac{\mu_1}{\mu_2}\left(\frac{\sigma_1}{\sigma_2}\right)^2}^{\beta} \theta_2$

będą lepkości w stosunku: $\qquad \eta_1 = \underbrace{\frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)}_{\gamma} \eta_2$

$\cancel{\sqrt{\alpha\gamma} = \frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^2}$

$\cancel{\beta\sqrt{\alpha\gamma} = \left(\frac{\mu_1}{\mu_2}\right)^2} \cdot \qquad \cancel{\alpha\beta = \left(\frac{\mu_1}{\mu_2}\right)^2 \frac{\sigma_2}{\sigma_1}}$

$\cancel{\frac{\alpha}{\beta} = \frac{\sigma_1}{\sigma_2}}$

$$\frac{\alpha}{\gamma} = \left(\frac{\sigma_2}{\sigma_1}\right)^2$$

$$\sqrt{\alpha\gamma} = \frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^2$$

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{\alpha\gamma^3}}{\alpha}$$

$$\beta = \sqrt{\frac{\gamma^3}{\alpha}} \cdot \frac{\gamma}{\alpha} = \sqrt{\frac{\gamma^5}{\alpha^3}}$$

$$\eta_1\left[\frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^3 \rho_2,\ \frac{\mu_1}{\mu_2}\left(\frac{\sigma_1}{\sigma_2}\right)^2\theta_2\right] = \frac{\mu_1}{\mu_2}\frac{\sigma_2}{\sigma_1}\cdot\eta_2\left[\rho_2,\theta_2\right] \qquad \gamma^5 = \beta^2\alpha^3$$

$$\underbrace{\eta_1\left[\alpha\rho_2,\ \beta\theta_2\right]} = \gamma\cdot\eta_2\left[\rho_2,\theta_2\right] = \sqrt[5]{\beta^2\alpha^3}\cdot\eta_2\left[\rho_2,\theta_2\right]$$

$$= \sqrt{\beta}\cdot\eta_1\left[\alpha\rho_2,\ \theta_2\right] = \sqrt[5]{\beta^2\alpha^3}\cdot\eta_2\left[\rho_2,\ \theta_2\right]$$

$$\eta_1\left[\alpha\rho_2,\ \theta_2\right] = \sqrt[10]{\frac{\alpha^6}{\beta}}\cdot\eta_2\left[\rho_2,\ \theta_2\right] \qquad \sqrt[10]{\frac{\alpha^6}{\beta}} = \sqrt{\frac{\mu_1}{\mu_2}}\cdot\left(\frac{\sigma_2}{\sigma_1}\right)^2$$

ważne przy dowolnej temperaturze, ale przy oznaczonym stosunku objętości

$$\eta_2\left[\rho_2;\theta_2\right] = \sqrt{\frac{\mu_2}{\mu_1}}\left(\frac{\sigma_1}{\sigma_2}\right)^2\cdot\eta_1\left[\frac{\mu_1}{\mu_2}\left(\frac{\sigma_2}{\sigma_1}\right)^3\rho_2;\ \theta_2\right]$$

$$\eta_2\left[\rho_2,\theta_2\right] = \varepsilon\cdot\eta_1\left[\alpha\rho_2,\ \theta_2\right]$$

$$\eta_2\left[\rho_1\,\theta\right] = \varepsilon\ \eta_1\left[\alpha\rho_1,\ \theta\right]$$

Wartości Sachs : Benzol$_{20°}$: $\frac{1}{\eta}\frac{\partial\eta}{\partial p} = 0{\cdot}000930$ (atm.)

$$\frac{1}{v}\frac{\partial v}{\partial p} = 0{\cdot}000091$$

$\eta_{10} = 42{\cdot}4$
$\eta_{30} = 31{\cdot}5$ } $73{\cdot}9$

$10{\cdot}9 : 37 = 0{\cdot}295 : 20$
$35/2$ $\quad 0{\cdot}0147$

$$\frac{1}{\eta}\frac{d\eta}{d\theta} = 0{\cdot}0147$$

$$\alpha_{20} = 0{\cdot}00123$$

$$\frac{\alpha}{\beta}\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{0{\cdot}00123\cdot 0{\cdot}000093}{0{\cdot}000091} = 0{\cdot}0123\cdot\frac{93}{91}$$

25 $\quad 124$

$0{\cdot}0000012$ $\quad 40$
$0{\cdot}0000511$
$0{\cdot}0011763$
$0{\cdot}0012274$

$-\ \boxed{0{\cdot}0148}$
$+\ 0{\cdot}0017$
$-\ 0{\cdot}0131$ $\quad \frac{1}{\eta}\frac{d\eta}{d\theta}$ oblicz.

$$\frac{\partial}{\partial t}(\sqrt{T}) = \frac{1}{2}\frac{1}{\sqrt{T}} = \frac{1}{2}\frac{1}{293} = \frac{1}{586} = .166$$

4

$\boxed{(C_2 \cancel{H_5})_2 O}$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = 0{\cdot}000730$

$\frac{1}{v}\frac{\partial v}{\partial p} = 0{\cdot}000173$

$\alpha = 0{\cdot}00155$

$\frac{d}{\beta}\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{0{\cdot}00155 . \cancel{0{\cdot}000}73}{\cancel{0{\cdot}000}173}$

1903
8633
0536
2380
8156

$= 0{\cdot}006_{54}$

obliua

0000035, 20
·00007
148
0·00155

47
151·3
0·00156

$\eta = A\,\frac{t_0 - t}{t - T}$ $\quad t_0 = 194$

$$\eta_1 t_1 - \eta_1 T = A(t_0 - t_1) \quad \Big| \; \eta_2 \; \Big| \; t_0 - t_2$$
$$\eta_2 t_2 - \eta_2 T = A(t_0 - t_2) \quad \Big| \; \eta_1 \; \Big| \; t_0 - t_1$$

$\cancel{\eta_1 t_1 - \eta_2 t_2}$ $\quad \eta_1\eta_2(t_1 - t_2) = A\{\eta_2(t_0 - t_1) - \eta_1(t_0 - t_2)\}$

$\eta_1 t_1 (t_0 - t_2) - \eta_2 t_2 (t_0 - t_1) = [\eta_1 (t_0 - t_2) - \eta_2 (t_0 - t_1)]\, T$

$t_0 = 193{\cdot}5$ $\qquad 2{\cdot}392 . 191{\cdot}1 \qquad - 2{\cdot}871 . 175{\cdot}1$

$t_1 = 2{\cdot}4$ $\qquad \eta_1 = 2{\cdot}871$

$t_2 = 18{\cdot}4$ $\qquad \eta_2 = 2{\cdot}392$

0·479 : (2·6.16)

3788
2812
6600

4581
2433
~~2433~~
7014

4571

5028
4571
45·7

0·3788
0·4581
1·2041
2·0410
1·6599
0·3811

$A = 2{\cdot}405$

$$\begin{array}{r} \bar{1}\cdot 4581 \\ 1\cdot 3802 \\ 2\cdot 2433 \\ \hline \bar{3}\cdot 0816 \\ 24 \end{array} \qquad \begin{array}{r} 0\cdot 3788 \\ 1\cdot 2648 \\ 2\cdot 2812 \\ \hline \bar{3}\cdot 9248 \end{array} \qquad \begin{array}{r} 3857\cancel{5} \\ -1\cdot 6599 \\ \hline 2\cdot 1976 \\ 759 \\ 17 \quad :20 \end{array}$$

$$T = -157\cdot 6$$

$$12070 \qquad \begin{array}{r} 84100 \\ 12070 \\ \hline 72030 \end{array}$$

$$\eta = 2\cdot 405 \; \frac{193\cdot 5 - t}{t + 157\cdot 6}$$

$$\log \eta = \log A + \log(t_0 - t) - \log(t - T)$$

$$\frac{1}{\eta}\frac{d\eta}{dt} = -\frac{1}{t_0 - t} - \frac{1}{t - T} = -\frac{\cancel{t} - T + t_0 - \cancel{t}}{(t_0 - t)(t - T)}$$

$$t_0 - T = \left.\begin{array}{r} 193\cdot 5 \\ +157\cdot 6 \end{array}\right\} = 351\cdot 1$$

$$\begin{array}{l} t_0 - t = 173\cdot 5 \\ t - T = 177\cdot 6 \end{array} \quad \begin{array}{r} 23925 \\ \\ 2495 \\ \hline 48875 \end{array} \qquad \begin{array}{r} 5454 \\ 48875 \\ \hline 05665 \end{array} \qquad = 0\cdot 0114$$

2nolez. (Hydrauliu)

$$\begin{array}{r} 19\cdot 3 \\ 18\cdot 9 \\ \hline 0\cdot 4 \end{array} : 19\cdot 1 = \frac{0\cdot 021}{20} = \qquad 0\cdot 001$$

Rellstab (?) Diss. Bonn 1868

$$\left.\begin{array}{r} 14\cdot 5 \\ 11\cdot 7 \end{array}\right\} \; 26\cdot 2$$

$$\begin{array}{r} 2\cdot 8 \\ 18 \end{array} : 13\cdot 1 = \frac{0\cdot 214}{20} = \qquad 0\cdot 0107$$

Pribram & Handl

$$P = p + a\rho^2 = R\theta\,\rho\, f(\rho) \qquad f(0) = 1$$

$$\cancel{2a\rho = R\theta\left[f(\rho) + \rho f'(\rho)\right]}$$ — ~~dla $\rho = 0$ [illegible]~~

VdW:

$$\frac{\rho}{f(\rho)} = \frac{1}{\frac{1}{\rho} - b}$$

$$f = \frac{1}{1 - b\rho}$$

$$1 = \left\{R\theta\left[f(\rho) + \rho f'(\rho)\right] - 2a\rho\right\}\frac{\partial\rho}{\partial p}$$

$$2a\rho\frac{\partial\rho}{\partial\theta} = R\rho f(\rho) + R\theta\left[f(\rho) + \rho f'(\rho)\right]\frac{\partial\rho}{\partial\theta}$$

$$\frac{1}{\rho^2} = \left\{R\theta\frac{f + \rho f'}{\rho} - 2a\right\}\frac{1}{\rho}\frac{\partial\rho}{\partial p}$$

$$2a\frac{1}{\rho}\frac{\partial\rho}{\partial\theta} = \frac{Rf}{\rho} + R\theta\frac{f + \rho f'}{\rho}\frac{1}{\rho}\frac{\partial\rho}{\partial\theta}$$

$$\frac{1}{\rho^2} = \left\{R\theta\frac{f + \rho f'}{\rho} - 2a\right\}\beta \qquad \alpha$$

$$-\frac{Rf}{\rho} = -\left\{R\theta\frac{f + \rho f'}{\rho} - 2a\right\}\alpha \qquad \beta$$

$$\frac{\alpha}{\rho^2} - \frac{Rf\beta}{\rho} = 0 \qquad\qquad \frac{\alpha}{\beta} = Rf\rho$$

$$\alpha = +Rf\rho\cdot\beta = \frac{p + a\rho^2}{\theta}\cdot\beta$$

$$\alpha_0 = \frac{a\rho_0^2}{\theta}\beta_0 \qquad \left[\!\left[\; a = \frac{\alpha_0}{\beta_0}\frac{\theta}{\rho_0^2}\right.\right.$$

powinno być niezależne od temperatury

przy absol. zero : ściśliwość = 0

$(C_2H_5)_2O$: $\beta_{14°} = 0{\cdot}000168$ | $\alpha_{14} = 0{\cdot}00153$

$\beta_{100°} = 0{\cdot}000560$ | $\alpha_{100} = 0{\cdot}002976$

0₃ 13489
0₂ 131074
135088
0₂ 279651
− 10347
0·0₂17618

13489
13107
13509
40105
− 10347
0·0₂29758 (Hirn)
29911 (Kopp extrap.)
31865 (Pierre extrapol.)

0·13489
6554
3377
23420
− 3449
1·19971

$$\frac{\rho_{100°}}{\rho_{14°}} = \frac{1{\cdot}0021}{1{\cdot}1997} \cdot 0{\cdot}7366$$

$$\frac{0{\cdot}00153}{0{\cdot}000168} \cdot \frac{287}{(0{\cdot}7366)^2} \cancel{(0{\cdot}7366)^2} \cdot (1{\cdot}0021)^2$$

0·8673 − 1

0·1847 − 3
2·4579
0·0018
0·6444 − 1
− 0·9598 + 5
0·6846 + 3

0·2253 − 4
0·7345 − 1
0·9598 − 5

$a_{14°} =$ 4840 (Atmosph.)

$$\frac{2976}{560} \cdot \frac{373}{(0{\cdot}7366)^2} (1{\cdot}1997)^2$$

4737
5717
0·1784
0·2238
− 0·4827
0·7411 + 2

7482
7345
4827

$a_{100°} =$ 5510 (Atmosph.)

ważne tylko przy zawiadamianiu gęstości pary nasyconej 100°, ponieważ α_0 β_0

vdW: $a = \frac{\vartheta^2}{\pi} \frac{27}{64} R^2 = \frac{27}{64} \frac{\vartheta^2}{\pi} \left(\frac{p_0}{\rho_0 \vartheta_0}\right)^2 \left(\frac{\mu_0}{\mu}\right)^2$

$= p_0 \cdot \frac{27}{64} \frac{\vartheta^2}{\pi} \frac{p_0}{\rho_0^2 \vartheta_0^2} \left(\frac{\mu_0}{\mu}\right)^2$

$\vartheta = 193.3$
$\frac{273}{466.3}$

$\pi = 37.1$

$(C_2H_5)_2O = \left.\begin{matrix}24\\5\end{matrix}\right\} \quad 29.$
$\frac{\;}{58}$
$\frac{16}{74}$

$N_2 \ldots 29.$

$\frac{27.}{64} \frac{(466)^2}{37.1} \frac{1}{(0.001293 \cdot 273)^2} \left(\frac{29}{74}\right)^2$

1116

$\rho_k = \frac{1}{3b}$

$\frac{2a^2}{(1-b\rho)^3}$

$\frac{4}{\rho^2}\frac{(1-b\rho)^3}{7 b^2}$

$\left(\frac{2}{3}\right)^3$
$\frac{1}{9}$
$= \frac{8}{3}$

0.4314
1.3368
0.9248
2.6930
−0.2097+1
3.4833

0.8062
1.5694
0.2233−6
0.8724
1.7384
5.2097−6

$a = 3040$ Atm | 2 punkty krytyczne

$\frac{\partial p}{\partial \rho} = \left| R\theta \left[f + \rho f'\right] = 2a\rho \right\| = 0$

$\frac{\partial^2 p}{\partial \rho^2} = \left| R\theta \left[2f' + \rho f''\right] = 2a \right\| = 0$

$\frac{f + \rho f'}{2f' + \rho f''} = \rho \Big|$

$f = \rho f' + \rho^2 f'' \Big|_{\text{kryt.}}$

w punkcie krytycznym

$\frac{R\theta_k}{p_k v_k} = \left(\frac{R.\theta}{p} \rho\right)_k$

$= \frac{2a\rho^2}{[f + \rho f']\,\cancel{2a\rho^2}}$

$= \frac{R\theta}{-a\rho + R\theta . f} = \frac{1}{f - \frac{a\rho}{R\theta}}$

$= \frac{1}{f - \frac{f + \rho f'}{2}} = \frac{2}{f - \rho f'} = \frac{2}{\rho^2 f''}$

$c = c_{gaz} + \frac{\alpha \, \cancel{\rho^2} \alpha}{\rho} = c_{gaz} + \frac{a \alpha \rho}{}$

86636
59,91

0·6848
0·1847-3
6·869 5

~~6232~~
~~5991 -1~~
~~2 2 23~~

Nyp. Eter $a_{14°} = 4840 \cdot 10^6$

$\boxed{\text{V. d. W.: } a\rho^2 = 1430 \text{ Atm}}$

$\alpha_{14} = 0·00153$

$\rho_{14} = \frac{0·7366}{1·0021} = 0·7351$

$\frac{4840 \cdot \cancel{10^6} \cdot 0·00153 \cdot 0·7^{1.7}}{\cancel{(0·7351)^2} \; 42 \cdot \cancel{10^6}} = \frac{4·84 \cdot 0·153 \cdot 0·7351}{4·2 \cdot (0·7351)^2} = 0·444$
$+ c_{gaz}$ } = c oblicz.

c obserw. = 0·56

c_{gaz}? jednoatomowy:

$c_p - c_v = AR$

$c_v (k-1) = AR$

$c_v = \frac{AR}{k-1}$ ‖ ~~$c_1 M_1 = c_2 M_2 = R_1 R_2$~~

$c_{Hg} = 0·0246 \cdot \frac{3}{5}$
$= 0·01476$

$\mu_{Hg} = 200$

$\mu_{..} = 74$

$0·0147 \cdot \frac{200}{74} = 0·040$

podczas gdy różnica oblicz. 0·12

$c_{gaz} = \underline{0·37}$! Winkelm. p. 391

6·8659
+ 0·8664 -1
6·7323
− 7·6232
0·1091 -1

0·000854.14
3316
0·01186 : 1·08

0·1286
$+ c_v$ gaz } = c oblicz.

c_v gaz 14° = ~~0·13~~ 0·3455
+ 0110
0·3565

c obserw. 14° = 0·529
+ 008
0·537

c oblicz.		c obserw.
0·485	>	0·537

Hg. $\beta = 0{\cdot}00000295$

$\alpha_{0^\circ} = 0{\cdot}0001818$ $\alpha_{100^\circ} = 0{\cdot}0001822$

$v_0 = \frac{1}{13{\cdot}59}$ $v_{100} = 1{\cdot}0182$

$a = \frac{\alpha}{\beta}\frac{\theta}{\rho^2} = \frac{0{\cdot}000182}{0{\cdot}00000295} \cdot \frac{273.}{(13{\cdot}59)^2}$

2601	4698
4362	2664
6963	7362
7362	
9601	2265

$a = 91{\cdot}2$ (Atm.)

$a\rho^2 = 16.830$ Atm.

$a\alpha\rho_{0^\circ} =$

9601	9601
1332	1332
2596	2605
3529	3538
	−6078
	3460

$\frac{91 . 13{\cdot}6 . 0{\cdot}00018}{42}$

2254 322

$:42 = 0{\cdot}005367$

2218 $:42$ 31686 $0{\cdot}005331$

$C_{v\,gaz} = 0{\cdot}0246 \cdot \frac{3}{5} = 0{\cdot}01476$

$C_{liq} = \underset{(0^\circ)}{0{\cdot}03333}$ $\underset{(100^\circ)}{0{\cdot}03262}$

wzg. ilościowo wcale nie ma zgodności, tylko znak dobry.

Podobieństwo różnych gazów:

$$p + a\rho^2 = R\theta\rho f(\rho)$$

Różnice pod względem: μ, σ, a

$n : n_1 =$

Podobieństwo istnieje, jeśli $\sigma_1^3 : \sigma^3 = \frac{\mu_1}{\rho_1} : \frac{\mu}{\rho}$

wtedy jeszcze różnica co do masy i prędkości (temp.)

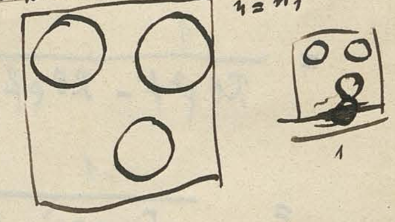
$$p + a\rho^2 : p_1 + a_1\rho_1^2 = \frac{\mu c^2}{\sigma^3} : \frac{\mu_1 c_1^2}{\sigma_1^3} = \frac{\theta}{\sigma^3} : \frac{\theta_1}{\sigma_1^3}$$

$$\theta : \theta_1 = \mu c^2 : \mu_1 c_1^2 \qquad = \theta\left(\frac{\rho}{\mu}\right) : \theta_1\left(\frac{\rho_1}{\mu_1}\right)$$

$$= \frac{\mu c^2}{\sigma^3}$$

$$p + a_i\rho = \frac{R}{\mu}\theta\rho f\left(\frac{\rho\sigma_i^3}{\mu_i}\right)$$

$n = n_1$



1

$$p_1 + a_k\rho_1 = \underbrace{(p + a_i\rho)}_{R\theta\rho f\left(\frac{\rho\sigma_i^3}{\mu_i}\right)}\frac{\theta_1}{\theta}\left(\frac{\rho_1}{\rho}\right)\left(\frac{\mu}{\mu_1}\right) = R\theta_1\rho\left(\frac{\rho_1}{\rho}\right) f\left(\frac{\rho_1\sigma_k^3}{\mu_1}\right)$$

$$\rho_1\left(\frac{\sigma}{\sigma_1}\right)^3\left(\frac{\sigma}{\sigma_1}\right)^2$$

$$\frac{\rho_1}{\rho} = \left(\frac{\sigma}{\sigma_1}\right)^3\frac{\mu_1}{\mu}$$

$$= R\theta_1\rho\left(\frac{\rho_1}{\rho}\right)\left(\frac{\mu}{\mu_1}\right) f\left(\frac{\rho_1\sigma_k^3}{\mu_1}\right) = R\theta_1\rho_1 f\left(\frac{\rho_1\sigma_k^3}{\mu_1}\right)\cdot\left(\frac{\mu}{\mu_1}\right)$$

$p+$

$p + a_i \rho^2 =$

$= \frac{r}{\mu_i}\,\theta\rho \cdot \varphi\left(\frac{\rho b_i^3}{\mu_i}\right)$

$R = \frac{r}{\mu_1}$

$f(\rho) = \varphi\left(\frac{\rho b^3}{\mu}\right)$

$f'(\rho) = \frac{b^3}{\mu}\,\varphi'$

$f''(\rho) = \frac{b^6}{\mu^2}\,\varphi''\left(\frac{\rho b^3}{\mu}\right)$

$$\beta = \frac{1}{\rho^2\left[R\theta\,\frac{f+\rho f'}{\rho} - 2a\right]} = \frac{1}{R\theta\rho f + R\theta\rho^2 f' - 2a\rho^2}$$

$$= \frac{1}{R\theta\rho^2 f' - R\theta\rho f + 2p}$$

$$= \frac{1}{\rho^2\left[\frac{p+a\rho^2}{\rho f}\,\frac{f+\rho f'}{\rho} - 2a\right]} = \frac{1}{\cancel{p + a\rho^2 f +}}$$

$$= \frac{1}{p + a\rho^2 + \rho(p+a\rho^2)\frac{f'}{f} - 2a\rho^2} = \frac{1}{p\left(1+\frac{\rho f'}{f}\right) + a\rho^2\left(\frac{\rho f'}{f} - 1\right)}$$

$$= \frac{1}{p - a\rho^2 + (p+a\rho^2)\frac{\rho f'}{f}}$$

Jestli $p \gg a\rho^2$: $\beta = \frac{1}{p\left(1+\frac{\rho f'}{f}\right)}$ $\qquad p\beta = \frac{1}{1+\frac{\rho f'}{f}}$

$$\alpha = \frac{R f \qquad \cdot \theta_p}{\rho\left\{R\theta\, \frac{f+\rho f'}{\rho} - 2a\right)\cdot \theta_p} = \frac{p + a\rho^2}{\theta\left\{(p + a\rho^2)(1+\rho \frac{f'}{f}) - 2a\rho^2\right\}}$$

$$= \frac{p + a\rho^2}{\theta\left\{p - a\rho^2 + (p+a\rho^2)\,\rho\frac{f'}{f}\right\}}$$

Jeśli $p \gg a\rho^2$: $\quad \alpha\theta = \beta p$

$$\alpha : \beta = p : \theta$$

$$\alpha = \frac{1}{\theta\left(1+\rho\frac{f'}{f}\right) - \underbrace{\frac{2a\rho^2}{\rho R f}}_{= \frac{2 R\theta\rho f - 2p}{R\rho f}}} = \frac{1}{\theta\left(\rho\frac{f'}{f} - 1\right) + \frac{2p}{R\rho f}}$$

Dla bardzo wielkich p: $\quad p = R\theta\rho f$

$$\beta = \frac{1}{p + R\theta\rho^2 f'}$$

$p\beta \overset{= 3000\text{ atm.}}{} = 0{\cdot}078$ ~~000~~ $\qquad$ więc głównie część jest: $\rho\frac{f'}{f}$

$$p = R\theta\rho f \qquad \cancel{\lg p = \lg R\theta + \lg\rho + \lg f}$$

~~[illegible]~~

Czemu dla rozmaitych ciały podobne $\beta_{p=\infty}$?

$$\rho\,\frac{\partial p}{\partial \rho} = \left(R\theta f + R\theta\rho f'\right)\rho$$

$$\frac{1}{\beta} = p\left(1 + \rho\frac{f'}{f}\right) = p\left(1 + \frac{\rho\sigma^3}{\mu}\cdot\frac{\varphi'\left(\frac{\rho\sigma^3}{\mu}\right)}{\varphi\left(\frac{\rho\sigma^3}{\mu}\right)}\right) = p\ \text{fc.}\left(\frac{\rho\sigma^3}{\mu}\right)$$

Jiní $\frac{\rho f'}{f} \gg 1$:

$$\beta = \frac{1}{(p+a\rho^2)\frac{\rho f'}{f}} = \frac{1}{R\rho\theta f \frac{\rho f'}{f}} = \frac{1}{R\rho^2\theta f'} = \frac{1}{\frac{R}{\mu}\rho^2\theta\frac{b^3}{\mu}\varphi'}$$

$$\alpha = \frac{1}{\theta\frac{\rho f'}{f}} =$$

Zavrem:

$$\alpha : \beta = \frac{1}{\theta} : \frac{1}{p+a\rho^2} = p+a\rho^2 : \theta$$

$$\alpha\theta = \beta(p+a\rho^2)$$

P. ex. C_2H_5OH (Amagat)

$p=p|$ $\alpha_{10^\circ} = 0{\cdot}001080$ $\qquad a = \frac{280 \cdot 0{\cdot}00108}{0{\cdot}000092 \cdot (0{\cdot}807)^2}$

$\beta = 0{\cdot}0000919$

$\rho = 0{\cdot}80686$ $\qquad = 5052$ (at).

```
 4472    9633
 0334    8138
 ----    ----
 4806    7771
-7771
 ----
 7035
```

9844

$p = 3000$ $\quad \rho = 0{\cdot}9647$

```
7035
9688
----
6723
```

při

$a\rho^2 = 4702$

$p + a\rho^2 = 7702$

$\beta = < 0{,}0000300$

$\frac{\beta(p+ap^2)}{\theta} = \frac{0{,}231}{280} = 0{,}0008$

$\alpha_{3000} = 0{,}000535$

Povšechně Amagat

$\alpha_{2500\text{ atm}, 280} = 0{,}00113$

$\beta = 0{,}000106$

$\rho = 0{,}849$

$\frac{\alpha\theta - \beta p}{\beta \rho^2} = a$

$\frac{0{,}00113 \cdot 280}{226}$ $\quad$ $\frac{0{,}000106 \cdot 2500}{0{,}265}$

$\frac{904}{0{,}3164}$

$\frac{0{,}2650}{0{,}0514} : 0{,}764 = \begin{vmatrix} 7110 \\ 8831 \\ 8279 \end{vmatrix}$ $\quad 0{,}0673 \cdot 10^4 = 673 = a$

1500 atm: $\rho = 0{,}724$

$\beta = 0{,}000197$

8597 . 2
7194
8280
5474
1500
2047 atm.

3112
2945
6057
− 4472
1585

1233

$\alpha = 0{,}00144$
Mittel

$\rho_{100} = 0{,}1328$

2466
8280
0746

1187,5
100
111,88

$\alpha_{\text{atm } 1500} = 0{,}00161$

$\alpha = 0{,}00478$
$\theta = 280$

6794
4472
1266

1266
− 0488
0778

$\beta_{\text{ber.}} = 0{,}01196$

$$\frac{p_1}{\rho_1} + a^2\rho_1 = R\theta$$
$$\frac{p_2}{\rho_2} + a^2\rho_2 = R\theta$$

jeśli $\rho_2 = 2\rho_1$

$$\frac{p_1 v_1}{p_2 v_2} = \frac{R\theta - a^2\rho_1}{R\theta - a^2\rho_2} = 1 + a^2\frac{(\rho_2 - \rho_1)}{R\theta}$$

$$= 1 + a^2 \frac{\rho^2}{p}$$

$$673.(0{\cdot}00129)^2 = \begin{array}{r}2212\\8280\\\hline 0492\end{array} = 0{\cdot}00112 \quad = 1{\cdot}00112 \text{ oblicz.}$$

$1{\cdot}0014$, obserw.

$$pv = R\theta f - a^2\rho$$

$$\frac{\partial}{\partial\rho}(pv) = R\theta f' - a^2 = 0 \text{ dla Minim.}$$

$$f' = \frac{a^2}{R\theta}$$

wtedy byłoby $\beta = \dfrac{1}{R\theta\rho f - a\rho^2} = \dfrac{1}{p}$

$$\alpha\theta = 1 + \frac{a\rho^2}{p}$$

$$pv = a(p_0 v_0)$$

$$v + p\frac{\partial v}{\partial p} = 0$$

$$\frac{1}{v}\frac{\partial v}{\partial p} = -\frac{1}{p}$$

to się rozumie samo przez się

$$\alpha\theta = \beta(p + a\rho^2)$$
$$= \left(\frac{1}{p} - \frac{1}{\eta}\frac{\partial\eta}{\partial p}\right)(p + a\rho^2)$$

$$\frac{pv}{p_0 v_0} = \eta(p)$$

$$v + p\frac{\partial v}{\partial p} = p_0 v_0 \cdot \frac{\partial\eta}{\partial p}$$

$$-\frac{1}{v}\frac{\partial v}{\partial p} = \frac{1}{p} - \frac{p_0 v_0}{pv}\frac{\partial\eta}{\partial p}$$

$$\beta = \frac{1}{p} - \frac{p_0 v_0}{pv}\frac{\partial\eta}{\partial p} = \frac{1}{p} - \frac{1}{\eta}\frac{\partial\eta}{\partial p}$$

Air Wittkowski

$\theta = 0$: $p = 1$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = -0{\cdot}00051$

~~$\theta = 58$~~

$p = 50$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{-0{\cdot}00035}{0{\cdot}9754}$

$p = 100$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = 0$

$p = 127$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{0{\cdot}0003}{0{\cdot}9730}$

$\theta = -103{\cdot}5$ $p = 1$ $\alpha =$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} =$

$p = 50$ $\alpha = 487$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{-000027}{0{\cdot}4839}$

~~$p =$~~ 100 $\alpha = 579$

$\frac{1}{\eta}\frac{\partial\eta}{\partial p} = \frac{0{\cdot}00029}{0{\cdot}3881}$

W każdym razie według tej hipotezy c_v powinno być niezależne od temperatury i od ciśnienia co jest (zupełnie w) sprzeczności z rezultatami Witkowskiego.

zmienności $\frac{\partial c_v}{\partial v}$ wskazują na $\frac{\partial c_v}{\partial \theta}$

Już z tej przyczyny teorya ta nie może być ścisła bo nawet przy wzroście miękkości kuli zgęszczenia c_v powinno być równe zeroi (podczas gdy Rosheu ... -

Compressibilität fester Körper:

		$\alpha = 3.$	$\rho =$	c_a
Pb (Amagat)	0·00000276	0·0000292	11·3	0·00164
Cu "	0· 0857 (10...)	168	8·7	0·00195
(Regnault)	0· 123			
Stahl (Amagat)	0·00000068	11	7·8	0·00152
SiO_2 (Voigt)	0· 2675			
Topas "	0·000000061			

$$a = \frac{\alpha\theta}{\beta\rho^2} \qquad c_a = a\alpha\rho = \frac{\alpha^2\theta}{42.\beta\rho}$$

876
0·9425

1·8850 − 10
2·4472
0·3322 − 6
− 0·4941 + 5
0·8381 − 2
− 1·6232
0·2149 − 3

0·4410 − 6
1·0531
0·4941 − 5

504
7024

1·4048 − 10
2·4472
0·8520 − 7
5627
2893

9395
6232
5627

0828
4472
5300
3478
~~0433~~
1822

~~8419~~
~~8921~~
~~6232~~
~~5567~~

8325
8921
6232
3478

~~0~~ 152

$c_{Pb} = 0.0314_{164} = 5\%$

$c_{Cu} = 0.094_{195} = 2\%$

$c_{Fe} = 0.113_{152} = 1.4\%$

$c_{Hg} = 0.0333_{54} = 16\%$

więc u ciał stałych wpływ nieznaczny

wyłomaczeniu R-masa nie poruszam.

ρ

0.4750 : 7

0.0679

Air Wrób. $\theta = -103.5$

$p = 40.88$

$\cancel{c_p = 0.371}$ } $k = 2.16$

$\cancel{c_v = 0.172}$

$c_p = 0.345$

$c_v = 0.189$

0.156

$k = 1.83$

$\alpha = 0.00463$

$p = 40.88$

$\alpha p^2 = +$ 7.235

48.116

$= \cancel{0.005304}$

$= 0.0516$

6656
6822
3478
− 6232
7246
− 0157 −1
7089

$\frac{p}{\rho}\,\frac{\rho_0}{p_0} = 0.5098$

$\rho = \frac{40.88}{0.5098}\ 0.001293$

6115
1116
7231
− 7074
0.0157 − 1
2.8280
1.8437
157 −1
0.8594

$a = 673$

$\rho = 0.104$

tyle przypadło by na pracę zewnętrzną teoretycznie, podczas gdy w rzeczyw. 0.156!

a musiało by 20 razy tyle wyn. —

$\theta = 0°$

~~$p = 96.8$~~

$c_p = 0.282$

~~$c_v = 0.177$~~

~~0.105~~

$k = 1.60$

$\alpha =$

$\theta = -35°$

$p = \cancel{\quad}\ 65.5$

$\alpha = 0.004486$

$c_p = 0.294$

$\underline{190}$

0.104

65.5

$\underline{7.3}$

72.8

$\rho = \dfrac{65.5 \cdot 0.001293}{0.8184}$

8162

$\underline{1116}$

9278

$- \underline{9130}$

0148

$\rho = 0.1035$

1.8621

$\underline{6518}$

5139

$- \underline{6380}$

8759

6232

$\underline{0148}$

6380

Δ dlica $= 0.07515$

ażeby otrzymać 0'104 musiałoby <u>a</u> być <u>4</u> razy tak duże!

podnoś gdy 0'104

C_6H_6 $\quad c = 0.3834 + 0.001043\ \theta$

$\alpha = 0.0011\dot{7}626 + 0.0000025551\ \theta + 0.0_7 2419\ \theta^2$

$\rho_0 = 0.899$

$\beta = 0.000059$

$a = \dfrac{\alpha \theta}{\beta \rho^2}$

$c_a = a \alpha \rho$

$$\frac{\partial c_a}{\partial \theta} = a\left[\frac{\partial \alpha}{\partial \theta} \rho + \alpha \frac{\partial \rho}{\partial \theta}\right] = a\rho\left[\frac{\partial \alpha}{\partial \theta} - \alpha^2\right]$$

$$\frac{\partial\alpha}{\partial\theta} = 0{\cdot}256 .\ 10^{-5}$$

$$\alpha^2 = \frac{0013 9\quad 10^{-5}}{0{\cdot}242 .\ 10^{-5}}$$

$$\frac{\alpha\theta}{\rho} = \begin{array}{r} 0\cancel{7}04 \\ 4362 \\ \hline 5066 \\ -\ 7709 \\ \hline 7357 \\ +\ 3838 \\ \hline 1195 \end{array}$$

$$a = \frac{5{\cdot}440}{(0{\cdot}9)^2}$$

$$1316 : 9$$

$$\frac{\partial c_a}{\partial\theta} = \frac{0{\cdot}01462}{42} =$$

$$= \underline{0{\cdot}00035}$$

$$\begin{array}{r} 7356 \\ 0704 \\ \hline 8060 \\ 5775 \\ \hline 2285 \end{array}$$

$$c_a = \frac{5440}{0{\cdot}9} . \frac{0{\cdot}001176}{42} =$$

$$\boxed{c_a = 0{\cdot}169}$$

$$\beta = \frac{A}{B+p} \qquad \frac{A}{\cancel{B}} = 133\ B$$

$$B\,(133 - 65{\cdot}4) = 65{\cdot}4 . 950$$

$$67{\cdot}6$$

$$B = 920$$

$$\frac{1}{\rho}\frac{\partial\rho}{\partial p} = \frac{a}{b+p}$$

$$\lg\rho = a\lg(b+p)$$

$$\rho = \rho_0\,(b+p)^{\alpha}$$

$$\beta = \frac{133}{1+\frac{p}{920}}$$

$$\begin{array}{r} 77 \\ 92 \end{array}$$

$$p = 250 : \quad \frac{133 . 920}{1170} = 104{\cdot}9 \quad | \ 108{\cdot}8$$

$$\begin{array}{l} 133 : 117 \sim 114 \\ 16 \\ 4 \end{array}$$

$$p = 450 : \quad \frac{133 . 920}{1370} = 890 \quad \| \ 83{\cdot}5$$

$$p = 2750 : \quad \frac{133 . \cancel{920}}{1+3} = 33{\cdot}1 \quad \| \ 31{\cdot}7$$

~~$\cancel{6\pi}$ 5.10^{-6} cm~~

$r = 2.\,10^{-6}$ cm

$p = 8.\,10^{-18}.\,\frac{4}{3}.\,\cancel{3{\cdot}14}.\,10 = \frac{1}{3}.\,10^{-15}$ g. | $\frac{1}{2}\;10^{-7}$

$\frac{p}{p_{O_2}} = (40)^3.\,10 = 64.\,10^4$

10 km $= 10^6$ cm . 2 cm $= h$

$6\cancel{\pi}\mu\cancel{a}u = \frac{4\cancel{\pi}}{3}a^{\cancel{3}2}\rho g$

$u = \frac{2}{9}\frac{a^2\rho g}{\mu}$ | $= \frac{2}{9}\frac{4.\,10^{-12}.\,10^4}{0{\cdot}010} \neq 10^{-6}$

$$= \frac{\partial \eta}{\partial x} \left\{ \frac{v}{u^4} V^2 \frac{\partial v}{\partial \eta} - \frac{V^2}{u^3} \frac{\partial u}{\partial \eta} \right\} = \frac{V^2}{u^4} \left\{ v \frac{\partial v}{\partial \eta} - u \frac{\partial u}{\partial \eta} \right\}$$

$$\frac{\partial^2 \eta}{\partial y^2} = \frac{v^2}{u^2} \frac{\partial^2 \eta}{\partial x^2} + \frac{V^2}{u^4} \left\{ v \frac{\partial v}{\partial \eta} - u \frac{\partial u}{\partial \eta} \right\} \cdot \frac{\partial \eta}{\partial x}$$

$$v^2 \frac{\partial}{\partial \xi}\left(\frac{u}{v}\right) + 2\frac{1}{u} \frac{\partial (V^2)}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = 0$$

$$\frac{\partial v}{\partial \eta} + \frac{\partial u}{\partial \xi} = \zeta(\Delta^2)u = \zeta\left(\frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \xi^2}\right)$$

$$\frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial \eta} = \zeta(\Delta^2)v = \zeta\left(\frac{\partial^2 v}{\partial \eta^2} + \frac{\partial^2 v}{\partial \xi^2}\right)$$

$$\frac{\partial^2 v}{\partial \eta^2} + \frac{\partial^2 u}{\partial \xi \partial \eta} = \zeta\left(\frac{\partial^3 u}{\partial \eta^3} + \frac{\partial^3 u}{\partial \xi^2 \partial \eta}\right)$$

$$\frac{\partial^2 v}{\partial \xi^2} - \frac{\partial^2 u}{\partial \xi \partial \eta} = \frac{\partial^2 v}{\partial \eta^2} + \frac{\partial^2 v}{\partial \xi^2} + \zeta\left(\frac{\partial^3 v}{\partial \eta^2 \partial \xi} + \frac{\partial^3 v}{\partial \xi^3}\right)$$

+

$$\frac{\partial^3 u}{\partial \eta^3} + \frac{\partial^3 u}{\partial \xi^2 \partial \eta} + \frac{\partial^3 v}{\partial \eta^2 \partial \xi} + \frac{\partial^3 v}{\partial \xi^3} = 0$$

$$\frac{\partial^2 v}{\partial \eta \partial \xi} + \frac{\partial^2 u}{\partial \xi^2} = \left(\frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \xi^2}\right) + \zeta\left(\frac{\partial^3 u}{\partial \eta^2 \partial \xi} + \frac{\partial^3 u}{\partial \xi^3}\right)$$

$$\frac{\partial^2 v}{\partial \eta \partial \xi} - \frac{\partial^2 u}{\partial \eta^2} = \zeta\left(\frac{\partial^3 v}{\partial \eta^3} + \frac{\partial^3 v}{\partial \eta \partial \xi^2}\right)$$

$$\frac{\partial^3 u}{\partial \eta^2 \partial \xi} + \frac{\partial^3 u}{\partial \xi^3} - \left(\frac{\partial^3 v}{\partial \eta^3} + \frac{\partial^3 v}{\partial \eta \partial \xi^2}\right) = 0$$

$$\left(\frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \xi^2}\right)\left(\frac{\partial u}{\partial \eta} + \frac{\partial v}{\partial \xi}\right) = 0$$

zatem $\dfrac{\partial u}{\partial \eta} = -\dfrac{\partial v}{\partial \xi} + f(\eta, \xi)$

$R\, f(\eta \text{ or } \xi)$

φ

$$\left(\frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \xi^2}\right)\left(\frac{\partial u}{\partial \xi} - \frac{\partial v}{\partial \eta}\right) = 0$$

$$\frac{\partial u}{\partial \xi} = \frac{\partial v}{\partial \eta} + R\, f(\eta \text{ or } \xi)$$

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial \eta^2} &= -\frac{\partial^2 v}{\partial \xi \partial \eta} + \frac{\partial \varphi}{\partial \eta} \\ \frac{\partial^2 u}{\partial \xi^2} &= \frac{\partial^2 v}{\partial \eta \partial \xi} + \frac{\partial \psi}{\partial \xi} \end{aligned}\right\} \quad \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \xi^2} = \frac{\partial \varphi}{\partial \eta} + \frac{\partial \psi}{\partial \xi}$$

$$\left.\begin{aligned} \frac{\partial^2 v}{\partial \eta^2} &= \frac{\partial^2 u}{\partial \xi \partial \eta} - \frac{\partial \psi}{\partial \eta} \\ \frac{\partial^2 v}{\partial \xi^2} &= -\frac{\partial^2 u}{\partial \xi \partial \eta} + \frac{\partial \varphi}{\partial \xi} \end{aligned}\right\} \quad \frac{\partial^2 v}{\partial \eta^2} + \frac{\partial^2 v}{\partial \xi^2} = \frac{\partial \varphi}{\partial \xi} - \frac{\partial \psi}{\partial \eta}$$

$p = x^2 - y^2$

$q = 2xy$

$p = x^2 - \left(\frac{q}{2x}\right)^2$

$x^4 - px^2 = \frac{q^2}{4}$

$x = \sqrt{\frac{p}{2} \pm \sqrt{\frac{p^2}{4} + \frac{q^2}{4}}}$

$x = \sqrt{\frac{p + \sqrt{p^2+q^2}}{2}}$

$y = \sqrt{\frac{-p + \sqrt{q^2+p^2}}{2}}$

$p = \left(\frac{q}{2y}\right)^2 - y^2$

$y^4 + py^2 = \left(\frac{q}{2}\right)^2$

$x^2 + 3y^2 = \frac{p + \sqrt{\ }}{2} \quad \frac{-3p + 3\sqrt{\ }}{2}$

$= -p + 2\sqrt{p^2+q^2}$

$\frac{\partial u}{\partial x} = \frac{1}{4}\left|\frac{4y}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2}\right|$

$= \frac{1}{2}\frac{y}{x^2+y^2}\left[1 - \frac{2xy}{x^2+y^2}\right]$

$= \frac{1}{2}\frac{y(x-y)^2}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial y} = -\frac{1}{4}\left[\frac{2x}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2} - \frac{x}{x^2+y^2}\right]$

$\frac{\partial u}{\partial y} = -\frac{1}{4}\left\{\frac{2y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2}\right\} = \frac{1}{2}\frac{x^2y + y^3 - 2x^2y}{(x^2+y^2)^2}$

$= \frac{1}{2}\frac{y^3 - x^2y}{(x^2+y^2)^2}$

$p^2 + q^2 = \frac{1}{x^2+y^2}$

$x = \frac{p}{p^2+q^2}$

$y = -\frac{q}{p^2+q^2}$

$\frac{\partial v}{\partial x} = -\frac{1}{4}\left\{\frac{2y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2} + \frac{y}{x^2+y^2}\right\}$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{4}\left[\frac{x^3 + xy^2 + 2x^2y + y^3 - 8xy^2}{x^2y}\right.$

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$\frac{y}{x^2+y^2} + \frac{(y^2+x^2)2y - (y^2-x^2)2y}{(y^2+x^2)^2} \quad + y^3 + x^2y$

$= \frac{\partial u}{\partial y} = \frac{y^3 - 3x^2y}{(y^2+x^2)^2}$

$+ \frac{(y^2+x^2)2x - 2x(y^2-x^2)}{(y^2+x^2)^2} + xy^2 + x^3$

$= -\frac{3xy^2 + x^3}{(y^2+x^2)^2} \quad -\frac{\partial u}{\partial x}$

$\frac{2x}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2} - \frac{x}{x^2+y^2}$

$\frac{\partial v}{\partial y} = \frac{x^3 - 3xy^2}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial x} = \frac{3y}{x^2+y^2} - \frac{4x^2y}{(x^2+y^2)^2} + \cancel{\frac{y}{x^2+y^2}}$

$= \frac{3y^3 - x^2y}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{-2y^3 - 2x^2y}{(y^2+x^2)^2} = -\frac{2y}{y^2+x^2}$

~~$v = \sin\varphi \; \partial \xi$~~

$$v = \frac{1}{4}\left[-p\cos h p \sin\xi + \xi \sin h p \cos\xi\right]$$

$$\frac{\partial v}{\partial p} = \frac{1}{4}\left[-\cos h p \sin\xi - p \sin h p \sin\xi + \xi \cos h p \cos\xi\right] \qquad \frac{\partial v}{\partial \xi} = \frac{1}{4}\left[-p\cos h p \cos\xi + \sin h p \cos\xi - \xi \sin h p \sin\xi\right]$$

$$\frac{\partial^2 v}{\partial p^2} = \frac{1}{4}\left[-2\sin h p \sin\xi - \cancel{p\cos h p \sin\xi} + \cancel{\xi \sin h p \cos\xi}\right] \qquad \frac{\partial^2 v}{\partial \xi^2} = \frac{1}{4}\left[\cancel{+p\cos h p \sin\xi} - 2\sin h p \sin\xi - \cancel{\xi \sin h p \cos\xi}\right]$$

$$\Delta^2 v = \cancel{\tfrac{1}{4}}\left[-\sin h p \sin\xi\right.$$

$$-p \sin h p \sin\xi = \xi \cos h p \cos\xi \; ?$$

$$-p x \, \mathrm{tg}\,\xi = \xi y \, \mathrm{ctg}\,\xi \; ?$$

$$\mathrm{tg}^2\xi = -\frac{\xi y}{p x}$$

$$\frac{\partial \xi}{\partial x} = \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial x \partial y} =$$

$$= \frac{\partial p}{\partial y}$$

$$(\xi x \mp p y) = v$$

$$\frac{\partial v}{\partial x} = x\frac{\partial p}{\partial y} + \xi \mp y\frac{\partial p}{\partial x} \qquad \frac{\partial v}{\partial y} = -x\frac{\partial p}{\partial x} \mp p \mp y\frac{\partial p}{\partial y}$$

$$\frac{\partial^2 v}{\partial x^2} = 2\frac{\partial p}{\partial y} + x\frac{\partial^2 p}{\partial x \partial y} \mp y\frac{\partial^2 p}{\partial x^2} \qquad \frac{\partial^2 v}{\partial y^2} = -x\frac{\partial^2 p}{\partial x \partial y} \mp 2\frac{\partial p}{\partial y} \mp y\frac{\partial^2 p}{\partial y^2}$$

$$v = \frac{1}{4}(\xi x + p y)$$

$$v = \frac{1}{4}\left[\rho \sinh\rho \cos\xi + \xi \cosh\rho \sin\xi\right]$$

$$\frac{\partial v}{\partial \rho} = \frac{1}{4}\left[\sinh\rho\cos\xi + \rho\cosh\rho\cos\xi + \xi\sinh\rho\sin\xi\right] \qquad \frac{\partial v}{\partial \xi} = \frac{1}{4}\left[-\rho\sinh\rho\sin\xi + \cosh\rho\sin\xi + \xi\cosh\rho\cos\xi\right]$$

$$\frac{\partial^2 v}{\partial \rho^2} = \frac{1}{4}\left[2\cosh\rho\cos\xi + \rho\sinh\rho\cos\xi + \xi\cosh\rho\sin\xi\right] \qquad \frac{\partial^2 v}{\partial \xi^2} = \frac{1}{4}\left[-\rho\sinh\rho\cos\xi + 2\cosh\rho\cos\xi - \xi\cosh\rho\sin\xi\right]$$

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 v}{\partial \xi^2} = \cosh\rho\cos\xi$$

$$\underbrace{\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \xi^2}} = \frac{e^{\rho}+e^{-\rho}}{2}\,\frac{e^{i\xi}+e^{-i\xi}}{2} = \frac{e^{\rho+i\xi}+e^{-\rho+i\xi}+e^{\rho-i\xi}+e^{-\rho-i\xi}}{4}$$

$$16\frac{\partial^2 u}{\partial\alpha\,\partial\beta} = e^{\alpha}+e^{-\alpha}+e^{\beta}+e^{-\beta}$$

$$16\frac{\partial u}{\partial\beta} = \int\left[\ldots\ldots\ \ldots\right]\partial\alpha$$

$$= e^{\alpha} - e^{-\alpha} + \alpha e^{\beta} + \alpha e^{-\beta} + f.\beta$$

$$16u = \beta\left(e^{\alpha}-e^{-\alpha}\right) + \alpha\left(e^{\beta}-e^{-\beta}\right) + \Phi(\beta) + \Psi(\alpha)$$

$$16u = (\rho - i\xi)\underbrace{\left(e^{\rho+i\xi}-e^{-\rho-i\xi}\right)} + (\rho+i\xi)\underbrace{\left(e^{\rho-i\xi}-e^{-\rho+i\xi}\right)} + -$$

$$= \rho\left(e^{\rho}-e^{-\rho}\right)\cos\xi + i\left(e^{\rho}+e^{-\rho}\right)\sin\xi \qquad \left(e^{\rho}-e^{-\rho}\right)\cos\xi - i\left(e^{\rho}+e^{-\rho}\right)\sin\xi$$

$$16u = 2\left[\rho\left(e^{\rho}-e^{-\rho}\right)\cos\xi + \left(e^{\rho}+e^{-\rho}\right)\xi\sin\xi\right] + \cancel{\rho\left(e^{\rho}-e^{-\rho}\right)\cos\xi + \left(e^{\rho}+e^{-\rho}\right)\xi\sin\xi}$$

$$u = \frac{1}{4}\left[\rho\sinh\rho\cos\xi + \cosh\rho\,\xi\sin\xi\right] + \Phi(\rho) + \Psi(\xi)$$

tkr

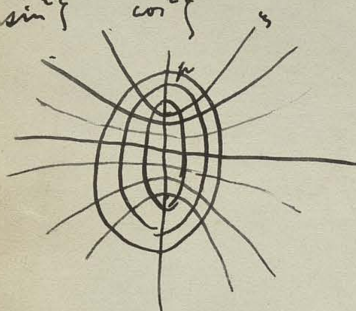
$$x+iy = \sinh(\mu+i\xi) = \frac{e^{\mu+i\xi}-e^{-\mu-i\xi}}{2}$$

$$x = \frac{e^{\mu}-e^{-\mu}}{2}\cos\xi = \sinh\mu\cos\xi$$

$$y = \frac{e^{\mu}+e^{-\mu}}{2}\sin\xi = \cosh\mu\sin\xi$$

$$\frac{x^2}{\sinh^2\mu}+\frac{y^2}{\cosh^2\mu}=1$$

$$\frac{y^2}{\sin^2\xi}-\frac{x^2}{\cos^2\xi}=1$$



$$x+iy = \cosh(\mu+i\xi) = \frac{e^{\mu+i\xi}+e^{-\mu-i\xi}}{2}$$

$$x = \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi = \cosh\mu\cos\xi$$

$$y = \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi = \sinh\mu\sin\xi$$

$$\frac{x^2}{\cosh^2\mu}+\frac{y^2}{\sinh^2\mu}=1$$

$$\frac{x^2}{\cos^2\xi}-\frac{y^2}{\sin^2\xi}=1$$

$$1 = \sinh\mu\,\frac{\partial\mu}{\partial y}\sin\xi+\cosh\mu\cos\xi\,\frac{\partial\xi}{\partial y}$$

$$0 = \cosh\mu\,\frac{\partial\mu}{\partial y}\cos\xi-\sinh\mu\sin\xi\,\frac{\partial\xi}{\partial y}$$

$y=0$: $\xi=0$

$x=0$: $y<1$: $\mu=0$

$y>1$: $\xi=-\frac{\pi}{2},+\frac{\pi}{2}$

$$\frac{\partial\mu}{\partial y} = \frac{\sinh\mu\sin\xi}{\sinh^2\mu\sin^2\xi+\cosh^2\mu\cos^2\xi}$$

$$\frac{\partial\xi}{\partial y} = \frac{\cosh\mu\cos\xi}{\cdots\cdots}$$

$$\cosh\mu = \frac{e^{\mu}+e^{-\mu}}{2}$$

$$\sinh\mu = \frac{e^{\mu}-e^{-\mu}}{2}$$

$$\cosh^2\mu = \frac{e^{2\mu}+e^{-2\mu}}{4}+\frac{1}{2} = \frac{\cosh 2\mu+1}{2}$$

$$\sinh^2\mu = \frac{e^{2\mu}+e^{-2\mu}}{4}-\frac{1}{2} = \frac{\cosh 2\mu-1}{2}$$

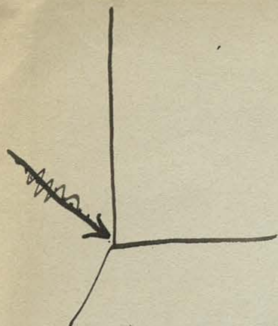
$$\cosh^2\mu-\sinh^2\mu = 1$$

$$u = \frac{1}{4}(\mu x\pm\xi y)$$

$$\frac{\partial u}{\partial x} = \frac{1}{4}\left(\mu+x\frac{\partial\mu}{\partial x}\pm y\frac{\partial\xi}{\partial x}\right) \qquad \frac{\partial u}{\partial y} = \frac{1}{4}\left(x\frac{\partial\mu}{\partial y}\pm\xi\pm y\frac{\partial\xi}{\partial x}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{4}\left(2\frac{\partial\mu}{\partial x}+x\frac{\partial^2\mu}{\partial x^2}\pm y\,\cancel{\frac{\partial^2\xi}{\partial x\partial y}}\right) \qquad \frac{\partial^2 u}{\partial y^2} = \frac{1}{4}\left(x\frac{\partial^2\mu}{\partial y^2}\mp 2\frac{\partial\xi}{\partial x}\mp y\,\cancel{\frac{\partial^2\xi}{\partial x\partial y}}\right)$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=1$$



$$A \sin \alpha\left[t - \frac{x\cos\lambda + y\cos\mu + z\cos\nu}{a}\right]$$

$\sigma = \sigma' + \sigma''$

$\sigma_1 = \sigma_1$

$x=0$: $u = u_1$

rotin $\frac{\partial u_1}{\partial t} = \frac{\partial u_1}{\partial t}$ rotin: $\boxed{a^2 \frac{\partial \sigma}{\partial x} = a_1^2 \frac{\partial \sigma_1}{\partial x}}$

$p = p_1$

$p_0(1 + k\sigma) = p_0(1 + k_1\sigma_1)$ $\boxed{k\sigma = k_1\sigma_1}$

$$kA'\sin\alpha'\left(t - y\frac{\cos\mu'}{a}\right) + kA''\sin\alpha''\left(t - y\frac{\cos\mu''}{a}\right) = k_1A_1\sin\alpha_1\left(t - y\frac{\cos\mu_1}{a_1}\right)$$

$\alpha' = \alpha'' = \alpha_1$

$\frac{\cos\mu'}{a} = \frac{\cos\mu''}{a} = \frac{\cos\mu_1}{a_1}$

$\mu = \mu''$ $\sin\lambda' : \sin\lambda_1 = a : a_1$

$a^2 \frac{\partial\sigma}{\partial x} = a_1^2 \frac{\partial\sigma_1}{\partial x}$

$aA'\cos\lambda' + aA''\cos\lambda'' = a_1A_1\cos\lambda_1$

$$\left.\begin{aligned} [A' - A'']\cos\lambda'\frac{\sin\lambda'}{\sin\lambda_1} &= A_1\cos\lambda_1 \\ k[A' + A''] &= k_1 A_1 \end{aligned}\right\}$$

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$$\frac{\partial u}{\partial t} + \cancel{u\frac{\partial u}{\partial x} + \dots} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \dots = 0$$

$$\frac{\partial v}{\partial t} + \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \cancel{u\frac{\partial \rho}{\partial x} + \dots} = 0$$

$$\frac{\partial w}{\partial t} + \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \quad \dots \quad \left.\begin{matrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{matrix}\right\} \qquad \frac{\partial \rho}{\partial t} = -\rho_0\left(\frac{\partial u}{\partial x} + \dots\right) \quad \Big| \ \frac{\partial}{\partial t}$$

$$\cancel{\frac{\partial^2 \rho}{\partial t^2} =} \quad \rho_0\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t} + \dots\right) = -\frac{\partial^2 \rho}{\partial t^2} = -\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \right)$$

$$\frac{\partial^2 u}{\partial y \partial t} - \frac{\partial^2 v}{\partial x \partial t} = 0$$

$\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \text{const} = 0$ jeśli początkowo spoczynek

zatem ruch potencyalny

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p$$

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$$

$$\cancel{\frac{1}{p_0}\frac{\partial p}{\partial x} = k\,\rho}$$

$$\log p - \log p_0 = k(\log \rho - \log \rho_0)$$

$$\frac{1}{p_0}\frac{\partial p}{\partial x} = \frac{k}{\rho_0}\frac{\partial \rho}{\partial x}$$

$$\frac{1}{p_0}\frac{\partial^2 p}{\partial x^2} = \frac{k}{\rho_0}\frac{\partial^2 \rho}{\partial x^2}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{k\,p_0}{\rho_0}\left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial z^2}\right)$$

$$\rho = \rho_0(1+\sigma)$$

$$\frac{\partial \rho}{\partial t} = \rho_0\frac{\partial \sigma}{\partial t} \quad \dots \quad \dots \text{etc.}$$

$$\frac{\partial^2 \sigma}{\partial t^2} = \frac{k\,p_0}{\rho_0}\,\Delta^2 \sigma$$

$$\frac{\partial^2 \sigma}{\partial t^2} = \frac{k\,p_0}{\rho_0}\frac{\partial^2 \sigma}{\partial x^2}$$

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$$c = \sqrt{kRT} = \sqrt{1{\cdot}4 . \frac{13{\cdot}6 . 76 . 980}{0{\cdot}001293}}$$

1461
1335
8808
9912
~~4362~~

21516
1117
~~0001~~
~~4161~~
0399

51995

331 $\frac{m}{sec}$

331·7

Wüllner (Kundt) 331·9

$$a' = a\left(1 - \frac{\gamma}{2r\sqrt{\pi n}}\right) \qquad \gamma = \sqrt{\frac{\mu}{\rho}} + \left(\sqrt{k} - \frac{1}{\sqrt{k}}\right)\sqrt{\frac{K}{c\rho}}$$

~~Kundt~~	~~λ = 180 mm~~	~~90~~	~~60~~
~~8 mm~~	~~1·0104~~	~~1·0089~~	~~1·005~~
~~13~~	~~1·0~~	~~1·~~	~~1~~
~~3·5~~	~~0·826~~	~~0·867~~	

Low

	r = 14 mm	4·675 mm
n = 256	v = 327·29	320·60
n = 1023·25	328·68	325·29

a = 330·58 $\frac{m}{sec}$

051

k vnitřní tep = 1·3947

otwarte - zamk. $N = \frac{331.8}{4(l+}$

I). Wpływ otworu $N = \frac{a}{4(l + \frac{\pi r}{4})}$

II). Tarcie wewn. i przewodn. ciepła [illegible] Kirchhoff.

III). Skończone amplitudy silne tony [illegible] (Regnault)

Earnshaw, Riemann

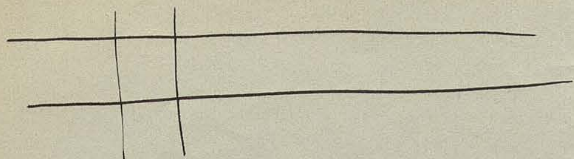
a Dla różnych gazów

[illegible] [illegible].

[illegible] dla wody 0·00005

Ag 0·0000037

$$\cancel{\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}}$$



$$\rho \frac{\partial u}{\partial t}\Delta x = p \dots - \dots \qquad \rho\,\Delta x\,\frac{\partial \rho}{\partial t}\,\Delta t = (\rho u) \dots - \dots)$$

przy ścianach $u = 0$

$$\frac{\partial u}{\partial t} = \frac{\partial p}{\partial x} = \frac{\partial \sigma}{\partial x} = 0$$

przy otworze $\sigma = 0$

$$\sigma = (A \sin \beta x + B \cos \beta x)(M \sin \alpha t + N \cos \alpha t)$$

$$= f_1(x + at) + f_2(x - at)$$

$$a = \sqrt{k\frac{p_0}{\rho_0}} = \sqrt{kR\theta}$$

$$\cancel{\sigma = 2} \qquad u = \frac{\partial \varphi}{\partial x}$$

$$\cancel{\sigma = \frac{f(x - at)}{r}}$$

$$\frac{\partial}{\partial r}(f r) = f + r\frac{\partial f}{\partial r}$$

$$\frac{\partial^2}{\partial r^2} \quad = 2\frac{\partial f}{\partial r} + r\frac{\partial^2 f}{\partial r^2}$$

$$\sigma = f(r, t)$$

$$\frac{\partial \sigma}{\partial x} = \frac{\partial f}{\partial r}\frac{x}{r}$$

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{\partial^2 f}{\partial r^2}\frac{x^2}{r^2} + \frac{\partial f}{\partial r}\frac{1}{r} - \frac{\partial f}{\partial x}\frac{x^2}{r^3}$$

$$\Delta^2 \sigma = \frac{\partial^2 f}{\partial r^2} + 2\frac{\partial f}{\partial r}\frac{1}{r} = \cancel{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right)} = \frac{1}{r}\frac{\partial^2}{\partial r^2}(f r)$$

$$\cancel{\frac{\partial^2 (f r)}{\partial t^2} = \frac{\partial^2}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)} \qquad \frac{\partial^2}{\partial t^2}(f r) = \frac{\partial^2}{\partial r^2}(f r)$$

$$f(r) = \varphi\,(r \pm at)$$

$$\sigma = \frac{\varphi\,(r \pm at)}{r}$$

$$u = \frac{\log(x^2+y^2)}{2} + \frac{2y^2}{y^2+x^2} - 1 \qquad \frac{\partial u}{\partial x} = \frac{x}{x^2+y^2} - \frac{4xy^2}{(x^2+y^2)^2}$$

$$v = -\frac{2xy}{x^2+y^2} + \operatorname{arctg}\frac{y}{x} \qquad \frac{\partial v}{\partial y} = \frac{-x}{x^2+y^2} + \frac{4xy^2}{(x^2+y^2)^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \xi$$

$$u = \frac{1}{4}\left[-\frac{\log(p^2+\xi^2)}{2} + \frac{2\xi^2}{p^2+\xi^2} - 1\right]$$

$$v = \frac{1}{4}\left[\frac{2\xi p}{p^2+\xi^2} - \operatorname{arctg}\frac{\xi}{p}\right]$$

$$\frac{\partial u}{\partial p} = \frac{1}{4}\left[-\frac{p}{p^2+\xi^2} - \frac{4p\xi^2}{(p^2+\xi^2)^2}\right]$$

$$\frac{\partial u}{\partial \xi} = \frac{1}{4}\left[\frac{3\xi}{p^2+\xi^2} - \frac{4\xi^3}{(p^2+\xi^2)^2}\right]$$

$$\frac{\partial v}{\partial p} = \frac{1}{4}\left[\frac{3\xi}{p^2+\xi^2} - \frac{4\xi p^2}{(p^2+\xi^2)^2}\right]$$

$$\frac{\partial v}{\partial \xi} = \frac{1}{4}\left[\frac{p}{p^2+\xi^2} - \frac{4p\xi^2}{(p^2+\xi^2)^2}\right]$$

$$\frac{\partial^2 u}{\partial p^2} = \frac{p^4 + 12p^2\xi^2 - 5\xi^4}{4(p^2+\xi^2)^3}$$

$$\frac{\partial^2 u}{\partial \xi^2} = \frac{3p^4 - 12p^2\xi^2 + [illegible]\xi^4}{4(p^2+\xi^2)^3}$$

$$\frac{\partial^2 u}{\partial p^2} + \frac{\partial^2 u}{\partial \xi^2} = \cancel{\cdots}$$

$$= \frac{4p^4 - 4\xi^4}{4(p^2+\xi^2)^3} = \frac{p^2-\xi^2}{(p^2+\xi^2)^2} = \frac{\frac{\partial p}{\partial x}}{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2}$$

$$\frac{\partial v}{\partial p} + \frac{\partial u}{\partial \xi} = \frac{1}{4}\left[\frac{6\xi}{p^2+\xi^2} - \frac{4\xi\cancel{(p^2+\xi^2)}}{(p^2+\xi^2)^{\cancel{2}}}\right] = \frac{1}{2}\,\frac{\xi}{p^2+\xi^2}$$

$$\frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial p} = \frac{1}{4}\left[\frac{2p}{p^2+\xi^2}\right] = \frac{1}{2}\,\frac{p}{p^2+\xi^2}$$

$$\frac{\partial^2 v}{\partial p^2} = \frac{2p^3\xi - 14p\xi^3}{4(p^2+\xi^2)^3}$$

$$\frac{\partial^2 v}{\partial \xi^2} = \frac{-10p^3\xi + 6p\xi^3}{4(p^2+\xi^2)^3}$$

$$\frac{\partial^2 v}{\partial p^2} + \frac{\partial^2 v}{\partial \xi^2} = -\frac{8p^3\xi - 8p\xi^3}{4(\quad)^3}$$

$$= -\frac{2p\xi}{(p^2+\xi^2)^2}$$

$$= \frac{-\frac{\partial p}{\partial y}}{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad\qquad \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\zeta$$

$$\begin{cases}\dfrac{\partial\varphi}{\partial x}=-\dfrac{\partial\xi}{\partial y}\\[2ex] \dfrac{\partial\varphi}{\partial y}=\dfrac{\partial\xi}{\partial x}\end{cases}$$

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial\varphi}\frac{\partial\varphi}{\partial x}+\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial\varphi^2}\left(\frac{\partial\varphi}{\partial x}\right)^2+2\frac{\partial^2 u}{\partial\varphi\partial\xi}\frac{\partial\varphi}{\partial x}\frac{\partial\xi}{\partial x}+\frac{\partial^2 u}{\partial\xi^2}\left(\frac{\partial\xi}{\partial x}\right)^2+\frac{\partial u}{\partial\varphi}\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial u}{\partial\xi}\frac{\partial^2\xi}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial\varphi^2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]+2\frac{\partial^2 u}{\partial\varphi\partial\xi}\Big[\underbrace{\cancel{\frac{\partial\varphi}{\partial x}\frac{\partial\xi}{\partial x}+\frac{\partial\varphi}{\partial y}\frac{\partial\xi}{\partial y}}}_{=0}\Big]+\frac{\partial^2 u}{\partial\xi^2}\left[\left(\frac{\partial\xi}{\partial x}\right)^2+\left(\frac{\partial\xi}{\partial y}\right)^2\right]+$$

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\frac{\partial^2 v}{\partial\varphi^2} \qquad \cancel{\frac{\partial^2 v}{\partial\varphi\partial\xi}} \qquad \frac{\partial^2 v}{\partial\xi^2}$$

$$\Delta^2 u=\left(\frac{\partial^2 u}{\partial\varphi^2}+\frac{\partial^2 u}{\partial\xi^2}\right)\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]=\frac{\partial\varphi}{\partial x}\ \Big\|=\left(\frac{\partial^2 u}{\partial\varphi^2}+\frac{\partial^2 u}{\partial\xi^2}\right)\left[\left(\frac{\partial\xi}{\partial x}\right)^2+\left(\frac{\partial\xi}{\partial y}\right)^2\right]=-\frac{\partial\xi}{\partial y}$$

$$\Delta^2 v=\left(\frac{\partial^2 v}{\partial\varphi^2}+\frac{\partial^2 v}{\partial\xi^2}\right)\left[\qquad\qquad\right]=\frac{\partial\varphi}{\partial y}=\left(\frac{\partial^2 v}{\partial\varphi^2}+\frac{\partial^2 v}{\partial\xi^2}\right)\left[\qquad\qquad\right]=\frac{\partial\xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial\varphi^2}+\frac{\partial^2 u}{\partial\xi^2}=\frac{\frac{\partial\varphi}{\partial x}}{\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2}=\frac{-\frac{\partial\xi}{\partial y}}{\left(\frac{\partial\xi}{\partial x}\right)^2+\left(\frac{\partial\xi}{\partial y}\right)^2}\ \Big\|\ \frac{\partial^2 v}{\partial\varphi^2}+\frac{\partial^2 v}{\partial\xi^2}=\frac{\frac{\partial\varphi}{\partial y}}{\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2}=\frac{\frac{\partial\xi}{\partial x}}{\left(\frac{\partial\xi}{\partial x}\right)^2+\left(\frac{\partial\xi}{\partial y}\right)^2}$$

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial\varphi}\frac{\partial\varphi}{\partial x}+\frac{\partial u}{\partial\xi}\frac{\partial\varphi}{\partial y}$$

$$\frac{\partial v}{\partial y}=\frac{\partial v}{\partial\varphi}\frac{\partial\varphi}{\partial y}-\frac{\partial v}{\partial\xi}\frac{\partial\varphi}{\partial x}\ \Big|=-\frac{\partial u}{\partial\varphi}\frac{\partial\varphi}{\partial x}-\frac{\partial u}{\partial\xi}\frac{\partial\varphi}{\partial y}\ \Big|\ -\frac{\partial\varphi}{\partial y}$$

$$\frac{\partial v}{\partial x}=\zeta+\frac{\partial u}{\partial y}=\zeta+\frac{\partial u}{\partial\varphi}\frac{\partial\varphi}{\partial y}-\frac{\partial u}{\partial\xi}\frac{\partial\varphi}{\partial x}\ \Big|=\frac{\partial v}{\partial\varphi}\frac{\partial\varphi}{\partial x}+\frac{\partial v}{\partial\xi}\frac{\partial\varphi}{\partial y}\ \Big|\ \frac{\partial\varphi}{\partial x}$$

$$\zeta\frac{\partial\varphi}{\partial x}-\frac{\partial u}{\partial\xi}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]=\frac{\partial v}{\partial\varphi}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]$$

$$\frac{\partial v}{\partial\varphi}=\zeta\,\frac{\frac{\partial\varphi}{\partial x}}{\left(\frac{\partial\varphi}{\partial x}\right)^2+(\cdot)^2}-\frac{\partial u}{\partial\xi}$$

$$U = cx\left(1+\frac{a^3}{2r^3}\right) = c\,r\cos\theta\left(1+\frac{a^3}{2r^3}\right)$$

$$\frac{\partial U}{\partial r} = c\cos\theta - \frac{c a^3}{r^3}\cos\theta \quad \Big|^{a=r} = 0$$

$$\frac{\partial U}{\partial \theta} = -c\,r\sin\theta\left(1+\frac{a^3}{2r^3}\right)$$

$$\frac{\partial U}{\partial x} = c\left(1+\frac{a^3}{2r^3}\right) - \frac{3cx^2a^3}{2r^5}$$

$$\lim \frac{\partial U}{\partial x} = c$$

$$i = \lambda\frac{\partial U}{\partial x} \qquad c = \frac{i}{\lambda}$$

$$\frac{\partial\varphi}{\partial t} = \kappa\Delta^2\varphi \qquad \frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = r^2\frac{\partial\varphi}{\partial t}$$

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\partial\varphi}{\partial t} \qquad r^2\varphi = \psi$$

$$r\frac{\partial^2\varphi}{\partial r^2} + 2\frac{\partial\varphi}{\partial r} = r\frac{\partial\varphi}{\partial t}$$

$$\frac{\partial^2}{\partial r^2}(r^2\varphi) = \frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{\partial}{\partial r}(2r\varphi)$$

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial r^2} - 2\frac{\partial}{\partial r}\left(\frac{\psi}{r}\right)$$

$$\frac{r^2}{t}$$

$$\cancel{\varphi = \mathrm{const}}$$

$$\frac{\partial}{\partial r}(r\varphi) = \varphi + r\frac{\partial\varphi}{\partial r}$$

$$\frac{\partial^2}{\partial r^2} \quad = 2\frac{\partial\varphi}{\partial r} + r\frac{\partial^2\varphi}{\partial r^2}$$

$$\frac{\partial^2(r\varphi)}{\partial r^2} = \frac{\partial(r\varphi)}{\partial t}$$

$$= \left(v \frac{\partial^2 u}{\partial \xi^2} - u \frac{\partial^2 v}{\partial \xi^2}\right) + \left(\frac{\partial \eta}{\partial x}\right)^2 \left(\frac{v}{u^2} \frac{\partial^2 u}{\partial \eta^2} - \frac{1}{u} \frac{\partial^2 v}{\partial \eta^2}\right) +$$

$$+ \left(v \frac{\partial u}{\partial \xi} - u \frac{\partial v}{\partial \xi}\right) \frac{\xi}{V^2} + \left(v \frac{\partial u}{\partial \eta} - u \frac{\partial v}{\partial \eta}\right) \left\{ \frac{1}{u^2} \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial \eta}{\partial x} \cancel{\left[\frac{1}{u}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{v^2}{u^3}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right]} \right\} =$$

$$\left[\frac{1}{u} \frac{\partial v}{\partial y} + \frac{v}{u^2} \frac{\partial v}{\partial x} - \frac{v}{u^2} \frac{\partial u}{\partial y} - \frac{v^2}{u^3} \frac{\partial u}{\partial x}\right] =$$

$$\cancel{\left[-\frac{1}{u} \frac{\partial u}{\partial x} - \frac{v^2}{u^2} \frac{\partial u}{\partial x}\right]}$$

$$= \frac{1}{u}\left[\frac{\partial v}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial v}{\partial \eta} \frac{\partial \eta}{\partial y}\right] + \frac{v}{u^2}\left[\frac{\partial v}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial v}{\partial \eta} \frac{\partial \eta}{\partial x}\right] - \frac{v}{u^2}\left[\frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y}\right] -$$

$$- \frac{v^2}{u^3}\left[\frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial x}\right] =$$

$$= \frac{\partial v}{\partial \xi}\left[-\frac{u}{u} + \frac{v^2}{u^2}\right] + \frac{\partial v}{\partial \eta}\left[\frac{v}{u^2} \frac{\partial \eta}{\partial x} + \frac{v}{u^2} \frac{\partial \eta}{\partial x}\right] - \frac{\partial u}{\partial \xi}\left[-\frac{v}{u} + \frac{v^3}{u^3}\right] - \frac{\partial u}{\partial \eta}\left[\frac{v^2}{u^3} \frac{\partial \eta}{\partial x} + \frac{v}{u^3} \frac{\partial \eta}{\partial x}\right]$$

$$v \frac{\partial u}{\partial \xi} - u \frac{\partial v}{\partial \xi} + \left(\frac{\partial u}{\partial \eta} + \frac{v}{u} \frac{\partial v}{\partial \eta}\right) \frac{\partial \eta}{\partial x} = 0$$

$$= \frac{1}{u}\left(v \frac{\partial u}{\partial \xi} - u \frac{\partial v}{\partial \xi}\right) + \frac{v^2}{u^3}\left(u \frac{\partial v}{\partial \xi} - v \frac{\partial u}{\partial \xi}\right) + 2 \frac{\partial \eta}{\partial x} \cdot \frac{v}{u^3}\left(u \frac{\partial v}{\partial \eta} - v \frac{\partial u}{\partial \eta}\right)$$

$$= \frac{u^2 - v^2}{u^3} \underbrace{\left(v \frac{\partial u}{\partial \xi} - u \frac{\partial v}{\partial \xi}\right)}_{-\frac{\partial \eta}{\partial x}\left(\frac{\partial u}{\partial \eta} + \frac{v}{u} \frac{\partial v}{\partial \eta}\right)} + 2 \frac{v}{u^3} \frac{\partial \eta}{\partial x}\left(u \frac{\partial v}{\partial \eta} - v \frac{\partial u}{\partial \eta}\right)$$

$$= \frac{\partial \eta}{\partial x}\left\{-\frac{1}{u} \frac{\partial u}{\partial \eta} - \cancel{\frac{v}{u^2} \frac{\partial v}{\partial \eta}} + \cancel{\frac{v^2}{u^3} \frac{\partial u}{\partial \eta}} + \frac{v^3}{u^4} \frac{\partial v}{\partial \eta} + 2\frac{v}{u^2} \frac{\partial v}{\partial \eta} - 2 \frac{v^2}{u^3} \frac{\partial u}{\partial \eta}\right\} =$$

$$\frac{\partial\eta}{\partial y} = -\frac{\frac{\partial\xi}{\partial x}}{\frac{\partial\xi}{\partial y}}\frac{\partial\eta}{\partial x}$$

$$\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial x} + \left(\frac{\partial u}{\partial\eta} - \frac{\partial v}{\partial\eta}\frac{\frac{\partial\xi}{\partial x}}{\frac{\partial\xi}{\partial y}}\right)\frac{\partial\eta}{\partial x} + \frac{\partial v}{\partial\xi}\frac{\partial\xi}{\partial y} = 0 \qquad \frac{v}{u} = -\frac{\frac{\partial\Psi}{\partial x}}{\frac{\partial\Psi}{\partial y}}$$

$\Psi = \text{const} =$ linii prądu

N. p. $\xi = \Psi$

$$v = +\frac{\partial\Psi}{\partial x} = \frac{\partial\xi}{\partial x} \qquad \frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$$

$$u = -\frac{\partial\Psi}{\partial y} = -\frac{\partial\xi}{\partial y} \qquad \left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2 = u^2 + v^2 = V^2$$

$$\frac{\partial\eta}{\partial y} = \frac{v}{u}\frac{\partial\eta}{\partial x}$$

$$\left(\frac{\partial\eta}{\partial x}\right)^2 + \left(\frac{\partial\eta}{\partial y}\right)^2 = \left(1 + \frac{v^2}{u^2}\right)\left(\frac{\partial\eta}{\partial x}\right)^2 = \frac{V^2}{u^2}\left(\frac{\partial\eta}{\partial x}\right)^2$$

$$\frac{\partial^2\eta}{\partial y^2} = \frac{\partial\eta}{\partial x}\frac{-v\frac{\partial u}{\partial y} + u\frac{\partial v}{\partial y}}{u^2} + \frac{v}{u}\underbrace{\frac{\partial^2\eta}{\partial x\partial y}}_{= \frac{v}{u}\frac{\partial^2\eta}{\partial x^2} + \frac{\partial\eta}{\partial x}\frac{-v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x}}{u^2}}$$

$$= \frac{\partial\eta}{\partial x}\left[\frac{1}{u}\left(\frac{\partial v}{\partial y} + \frac{v}{u}\frac{\partial v}{\partial x}\right) - \frac{v}{u^2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{v^2}{u^2}\frac{\partial^2\eta}{\partial x^2}$$

$$\frac{\partial p}{\partial\xi} = \Bigg\{\left[\frac{\partial^2 u}{\partial\xi^2}\frac{\partial\eta}{\partial y} - \frac{\partial^2 v}{\partial\xi^2}\frac{\partial\eta}{\partial x}\right]\left[\left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2\right] + \left[\frac{\partial^2 u}{\partial\eta^2}\frac{\partial\eta}{\partial y} - \frac{\partial^2 v}{\partial\eta^2}\frac{\partial\eta}{\partial x}\right]\left[\left(\frac{\partial\eta}{\partial x}\right)^2 + \left(\frac{\partial\eta}{\partial y}\right)^2\right]$$
$$+ \left[\frac{\partial u}{\partial\xi}\frac{\partial\eta}{\partial y} - \frac{\partial v}{\partial\xi}\frac{\partial\eta}{\partial x}\right]\left[\frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2}\right] + \left[\frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y} - \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial x}\right]\left[\frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial y^2}\right]\Bigg\}$$

$$\frac{\partial\xi}{\partial x}\frac{\partial\eta}{\partial y} - \frac{\partial\xi}{\partial y}\frac{\partial\eta}{\partial x}$$

$$= \frac{V^2\left(\frac{\partial^2 u}{\partial\xi^2}\frac{v}{u} - \frac{\partial^2 v}{\partial\xi^2}\right) + \left[\frac{\partial^2 u}{\partial\eta^2}\frac{v}{u} - \frac{\partial^2 v}{\partial\eta^2}\right]\frac{V^2}{u^2}\left(\frac{\partial\eta}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial\xi}\frac{v}{u} - \frac{\partial v}{\partial\xi}\right) + \left(\frac{\partial u}{\partial\eta}\frac{v}{u} - \frac{\partial v}{\partial\eta}\right)\left\{\frac{V^2}{u^2}\frac{\partial^2\eta}{\partial x^2} + \frac{\partial\eta}{\partial x}\left[\frac{1}{u}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{v^2}{u^3}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right]\right\}}{\left(\frac{v^2}{u} + u\right)}$$

$x+iy = f.(\xi+i\eta)$

$$\left(\frac{\partial F}{\partial\xi}\right)^2+\left(\frac{\partial F}{\partial\eta}\right)^2 = \left[\frac{\partial F}{\partial x}\frac{\partial x}{\partial\xi}+\frac{\partial F}{\partial y}\frac{\partial y}{\partial\xi}\right]^2+\left[\frac{\partial F}{\partial x}\frac{\partial x}{\partial\eta}+\frac{\partial F}{\partial y}\frac{\partial y}{\partial\eta}\right]^2$$

$$=\left[\left(\frac{\partial F}{\partial x}\right)^2+\left(\frac{\partial F}{\partial y}\right)^2\right]\left[\left(\frac{\partial x}{\partial\xi}\right)^2+\left(\frac{\partial x}{\partial\eta}\right)^2\right]$$

Jeśli $\left(\frac{\partial x}{\partial\xi}\right)^2+\left(\frac{\partial x}{\partial\eta}\right)^2=1$ $\quad x+iy=\cos\xi+i\sin\xi$

$\qquad\qquad \shortparallel$

$\qquad\left(\frac{\partial y}{\partial\xi}\right)^2$

~~Np. $x=\sin\xi$, $y=\sin\xi$~~

W ogólnym przypadku, podstawiając zamiast x, y, inne zmienne: ξ, η:

$$\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial x}+\frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x}=\frac{\partial^2u}{\partial\xi^2}\left[\left(\frac{\partial\xi}{\partial x}\right)^2+\left(\frac{\partial\xi}{\partial y}\right)^2\right]+\frac{\partial^2u}{\partial\eta^2}\left[\left(\frac{\partial\eta}{\partial x}\right)^2+\left(\frac{\partial\eta}{\partial y}\right)^2\right]+2\frac{\partial^2u}{\partial\xi\partial\eta}\underbrace{\left[\frac{\partial\xi}{\partial x}\frac{\partial\eta}{\partial x}+\frac{\partial\xi}{\partial y}\frac{\partial\eta}{\partial y}\right]}$$

$$+\frac{\partial u}{\partial\xi}\left[\frac{\partial^2\xi}{\partial x^2}+\frac{\partial^2\xi}{\partial y^2}\right]+\frac{\partial u}{\partial\eta}\left[\frac{\partial^2\eta}{\partial x^2}+\frac{\partial^2\eta}{\partial y^2}\right]$$

$$\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial y}+\frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y}=\frac{\partial^2v}{\partial\xi^2}\ - - - - - -$$

$$\frac{\partial u}{\partial\xi}\frac{\partial\xi}{\partial x}+\frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x}+\frac{\partial v}{\partial\xi}\frac{\partial\xi}{\partial y}+\frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial y}=0$$

Jeśli nowe współrzędne tworzą system ortogonalny:

$$\frac{\frac{\partial\xi}{\partial x}}{\frac{\partial\xi}{\partial y}}=-\frac{\frac{\partial\eta}{\partial y}}{\frac{\partial\eta}{\partial x}}$$

wtedy odpada wyrażenie

Równania wyrażone w odniesieniu do spółrzędnych p, ξ

$$f(x+iy) = \xi + ip$$

$$\frac{\partial^2\psi}{\partial p^2} + \frac{\partial^2\psi}{\partial \xi^2} = \frac{\xi}{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2}$$

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}$$

$$\Delta\varphi = 0$$

Ponieważ z φ razem wyrażone musi być χ tego rodzaju że: $\varphi + i\chi = fc.(x+iy)$ itd. zatem $\frac{\partial \varphi}{\partial x} = \frac{\partial \chi}{\partial y}$ więc dobierając do ψ odpowiednią funkcyę χ: $\Delta^2\chi = 0$ można także wprost:

$$u = -\frac{\partial(\psi+\chi)}{\partial y}$$

$$v = \frac{\partial(\psi+\chi)}{\partial x}$$

(albo też χ opuścić zaniedbać, że jako ψ bierze się najogólniejszą całkę.)

Przy ~~[illegible]~~ ścianach stałych $\boxed{\frac{\partial}{\partial x}(\psi+\chi) = 0 \; ; \; \frac{\partial}{\partial y}(\psi+\chi) = 0 \; ;}$

zatem tam $\Psi = \psi + \chi$ musi więc wartości maksymalne (minimalne)

Wogóle tylko takie funkcye ~~[illegible]~~ Ψ, które wartości maksymalne nie osiągają w pojedyńczych punktach lecz ~~[illegible]~~ na krzywych.

Jeśli Ψ wyrażone przez p, ξ: $\Psi = fc.(p, \xi)$

~~W.~~ Maksymum lub minimum Ψ musi albo dla (p_m ξ dowolne)

albo dla (p dowolne, ξ_m)

albo dla $p = \varphi(\xi)$

$$\frac{\partial F}{\partial x}\frac{\partial x}{\partial \xi}+\frac{\partial F}{\partial y}\frac{\partial y}{\partial \xi}=\cancel{\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\left(\frac{\partial x}{\partial \xi}\right)^2}$$

$$=\frac{\partial^2 u}{\partial x^2}\left[\left(\frac{\partial x}{\partial \xi}\right)^2+\text{[redacted]}\left(\frac{\partial x}{\partial \eta}\right)^2\right]+\frac{\partial^2 u}{\partial y^2}\left[\left(\frac{\partial x}{\partial \xi}\right)^2+\text{[redacted]}\left(\frac{\partial y}{\partial \eta}\right)^2\right]+$$

$$2\frac{\partial^2 u}{\partial x\partial y}\left(\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \xi}+\frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \eta}\right)+\frac{\partial u}{\partial x}\left(\frac{\partial^2 x}{\partial \xi^2}+\frac{\partial^2 x}{\partial \eta^2}\right)+\frac{\partial u}{\partial y}\left(\frac{\partial^2 y}{\partial \xi^2}+\frac{\partial^2 y}{\partial \eta^2}\right)$$

$$\frac{\partial F}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial F}{\partial y}\frac{\partial y}{\partial \eta}=\frac{\partial^2 v}{\partial x^2}\left[\left(\frac{\partial x}{\partial \xi}\right)^2+\left(\frac{\partial x}{\partial \eta}\right)^2\right]+\frac{\partial^2 v}{\partial y^2}\left[\left(\frac{\partial y}{\partial \xi}\right)^2+\left(\frac{\partial y}{\partial \eta}\right)^2\right]+$$

$$\frac{\partial u}{\partial x}\frac{\partial x}{\partial \xi}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial \xi}+\frac{\partial v}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial v}{\partial y}\frac{\partial y}{\partial \eta}=0$$

N. p. $\quad x+iy=f(\xi+i\eta)=\overset{x}{\varphi}(\xi,\eta)+i\,\overset{y}{\psi}(\xi,\eta)$

$$\left\{\begin{aligned}\frac{\partial x}{\partial \xi}&=\frac{\partial y}{\partial \eta}\\ \frac{\partial x}{\partial \eta}&=-\frac{\partial y}{\partial \xi}\end{aligned}\right\}$$

I) $\dfrac{\partial F}{\partial x}\dfrac{\partial x}{\partial \xi}-\dfrac{\partial F}{\partial y}\dfrac{\partial x}{\partial \eta}=\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}\right)\left[\left(\dfrac{\partial x}{\partial \xi}\right)^2+\left(\dfrac{\partial x}{\partial \eta}\right)^2\right]$

II) $\dfrac{\partial F}{\partial x}\dfrac{\partial x}{\partial \eta}+\dfrac{\partial F}{\partial y}\dfrac{\partial x}{\partial \xi}=\left[\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}\right]\left[\left(\dfrac{\partial x}{\partial \xi}\right)^2+\left(\dfrac{\partial x}{\partial \eta}\right)^2\right]$

III) $\cancel{\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right)\dfrac{\partial x}{\partial \xi}}+\left(\dfrac{\partial v}{\partial x}-\dfrac{\partial u}{\partial \eta}\right)\dfrac{\partial x}{\partial \eta}=0$

III) Transformacja ~~Niemożliwa~~ ogólna jeśli $\varphi=0$ lub $\dfrac{\partial x}{\partial \eta}=\dfrac{\partial y}{\partial \xi}=0$

Zatem słuszne tylko dla ruchu niewirowego ~~ściśliwego~~.

$$\frac{\partial p_1}{\partial x} = \Delta^2 u \qquad \frac{\partial p_1}{\partial y} = \Delta^2 v \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$x = \varphi(\xi, \eta) \qquad y = \psi(\xi, \eta)$$

$$\frac{\partial x}{\partial \xi} = \frac{\partial y}{\partial \eta} \qquad \frac{\partial x}{\partial \eta} = -\frac{\partial y}{\partial \xi}$$

$$1 = f'\left(\frac{\partial \xi}{\partial x} + i\frac{\partial \eta}{\partial x}\right)$$

$$\frac{\partial p_1}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial p_1}{\partial \eta}\frac{\partial \eta}{\partial x} = \frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial u}{\partial \eta}\frac{\partial^2 \eta}{\partial x^2} + 2\frac{\partial^2 u}{\partial \xi \partial \eta}\frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial^2 u}{\partial \eta^2}\left(\frac{\partial \eta}{\partial x}\right)^2 + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial u}{\partial \eta}\frac{\partial^2 \eta}{\partial x^2}$$

$$= \frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial^2 u}{\partial \eta^2}\left(\frac{\partial \eta}{\partial x}\right)^2 + 2\frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2} + 2\frac{\partial u}{\partial \eta}\frac{\partial^2 \eta}{\partial x^2} + 2\frac{\partial^2 u}{\partial \xi \partial \eta}\frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial x}$$

Jakie warunki φ, ψ ażeby w nowym układzie taki ruch był możliwy

$$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \xi}$$

$$\frac{\partial^2 u}{\partial \xi^2} = \frac{\partial^2 u}{\partial x^2}\left(\frac{\partial x}{\partial \xi}\right)^2 + \frac{\partial u}{\partial x}\frac{\partial \frac{\partial x}{\partial \xi}}{\partial x}\frac{\partial x}{\partial \xi} + 2\frac{\partial^2 u}{\partial x \partial y}\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \xi} + \frac{\partial u}{\partial y}\frac{\partial \frac{\partial y}{\partial \xi}}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial^2 u}{\partial y^2}\left(\frac{\partial y}{\partial \xi}\right)^2 + \frac{\partial u}{\partial y}\frac{\partial \frac{\partial y}{\partial \xi}}{\partial y}\frac{\partial y}{\partial \xi} + \frac{\partial u}{\partial x}\frac{\partial \frac{\partial x}{\partial \xi}}{\partial y}\frac{\partial y}{\partial \xi}$$

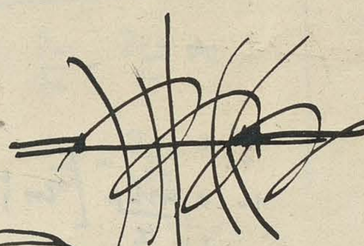
$$= \frac{\partial^2 u}{\partial x^2}\left(\frac{\partial x}{\partial \xi}\right)^2 + \frac{\partial^2 u}{\partial y^2}\left(\frac{\partial y}{\partial \xi}\right)^2 + 2\frac{\partial^2 u}{\partial x \partial y}\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \xi} + \frac{\partial u}{\partial x}\frac{\partial^2 x}{\partial \xi^2} + \frac{\partial u}{\partial y}\frac{\partial^2 y}{\partial \xi^2}$$

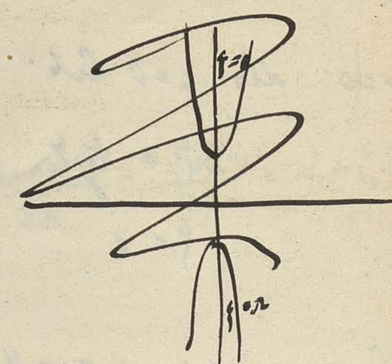
$$x+iy = \cos(\xi+i\eta) = \frac{e^{i\xi-\eta}+e^{-i\xi+\eta}}{2} = \frac{(\cos\xi+i\sin\xi)e^{-\eta}+(\cos\xi-i\sin\xi)e^{\eta}}{2}$$

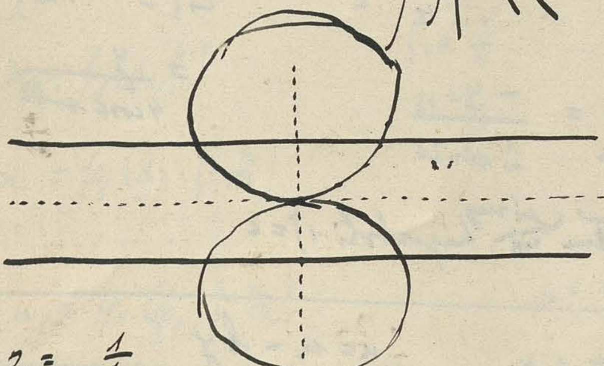
$$y = \cos\xi\cosh\eta$$

$$x = -\sin\xi\sinh\eta$$

$$\frac{y^2}{\cos^2\xi} - \frac{x^2}{\sin^2\xi} = 1$$







$$z = \frac{1}{\zeta}$$

$$x+iy = \frac{1}{\xi+i\eta} = \frac{\xi-i\eta}{\xi^2+\eta^2}$$

$$x = \frac{\xi}{\xi^2+\eta^2} \qquad \xi = \frac{x}{x^2+y^2}$$

$$y = -\frac{\eta}{\xi^2+\eta^2} \qquad \eta = -\frac{y}{x^2+y^2}$$

$$\frac{\eta}{\xi^2+\eta^2} = c$$

$$\eta = c(\xi^2+\eta^2)$$

$$d\left(\frac{\partial\eta}{\partial\xi}\right) = \frac{\partial\left(\frac{\partial\eta}{\partial\xi}\right)}{\partial}$$

$$v = \frac{\partial\varphi}{\partial y} + \underbrace{\frac{x}{4}\left[-\operatorname{tg}\zeta + 2\zeta\right]}$$

$$u = \frac{\partial\varphi}{\partial x} + \frac{\cosh\rho}{4}\left[\cos\zeta + 2\zeta\sin\zeta\right]$$

$$= \frac{\sinh\rho}{4}\left[-\sin\zeta + 2\zeta\cos\zeta\right]$$

$v = 0$ dla $\rho = 0$ lub

$$2\zeta = \operatorname{tg}\zeta$$

$$\zeta = c$$

$$u = \frac{\cosh\rho}{4}\left[\cos\zeta + \frac{\sin^2\zeta}{\cos\zeta}\right] = \frac{\cosh\rho}{4\cos\zeta} = \frac{\cosh\rho}{4\cos c}$$

$$= \frac{y}{4\cos c\sin c}$$

Superponując: $u = \dfrac{-y}{4\cos c\sin c} = \dfrac{-y}{2\sin 2c}$

otrzymuje się przyleganie zupełne na równej hiperboli $\zeta = c$

Można superponować $v = v_1 + ax$ $\qquad u = u_1 - by$

$$\zeta = \zeta_1 + a + b$$

$$v = \sinh\rho\left[\frac{-\sin(\zeta - a^{+b}) + 2(\zeta - a^{+b})\cos(\zeta - a^{+b})}{4} + a\cos(\zeta - a)\right]$$

$v = 0$ dla: $\operatorname{tg}(\zeta - (a+b)) = 2(\zeta - a) + 4a = 2(\zeta + a + b)$

$$\zeta - (a+b) = c$$

$$\frac{-x^2}{\cos^2(\zeta - (a+b))} + \frac{y^2}{\sin^2(\zeta - (a+b))} = 1$$

$$u = \frac{\cosh\rho}{4}\left[\cos(\zeta - a) + 2(\zeta - a)\sin(\zeta - a)\right]$$

$$= \frac{y}{4\sin(\zeta - a)}\left[\cos\zeta \quad \cdots \quad\right]$$

$\cancel{\lim c = \frac{\pi}{2}}$

$\cancel{\lim(a+b) = \infty}$

$\cancel{\lim b = \frac{\pi}{4}}$

$$b = \frac{\operatorname{ctg} c}{4} + \frac{c}{2}$$

$$\frac{\partial p_1}{\partial x} = \Delta^2 u_1 \qquad \frac{\partial p_2}{\partial x} = \Delta^2 u_2$$

$$\frac{\partial p_1}{\partial y} = \Delta^2 v_1 \qquad \frac{\partial p_2}{\partial y} = \Delta^2 v_2$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \qquad \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0$$

$$\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y} = \zeta_1 \qquad \frac{\partial v_2}{\partial x} - \frac{\partial u_2}{\partial y} = \zeta_2$$

$u_1 + u_2 = u$

$$\frac{\partial(v_1+v_2)}{\partial x} - \frac{\partial(u_1+u_2)}{\partial y} = \zeta_1 + \zeta_2 = \zeta$$

$\zeta_2 = \frac{\pi}{4}$

$p_2 = 0$

$x = f_1(p_1, \zeta_1)$

$x = f_1(p_1, \zeta - \frac{\pi}{4})$

. . . .

$u_1 = \varphi_1(p_1, \zeta - \frac{\pi}{4})$

$u = u_2 + \varphi_1(p_1, \zeta - \frac{\pi}{4})$

. . .

$$\frac{x^2}{\sin^2 h p} + \frac{y^2}{\cos^2 h p} = 1$$

$$\frac{-x^2}{\cos^2(\zeta - \frac{\pi}{4})} + \frac{y^2}{\sin^2(\zeta - \frac{\pi}{4})} = 1$$

$$\frac{\partial u}{\partial x} = \frac{\sinh p}{4}\left[\cos\zeta + 2\zeta\sin\zeta\right]\frac{\partial p}{\partial x} + \frac{\cosh p}{4}\left[+\sin\zeta + 2\zeta\cos\zeta\right]\frac{\partial p}{\partial y}$$

$$= \frac{1}{2}\;\frac{\sinh p\cosh p\left[\cos^2\zeta + \cancel{2\zeta\sin\zeta\cos\zeta} - \sin^2\zeta - \cancel{2\zeta\sin\zeta\cos\zeta}\right]}{\cosh 2p + \cos 2\zeta}$$

$$= \frac{1}{4}\;\frac{\sinh 2p\,\cos 2\zeta}{\cosh 2p + \cos 2\zeta}$$

$$\frac{\partial v}{\partial y} = \frac{\cosh p}{4}\left[-\sin\zeta + 2\zeta\cos\zeta\right]\frac{\partial p}{\partial y} - \frac{\sinh p}{4}\left[+\cos\zeta \mp 2\zeta\sin\zeta\right]\frac{\partial p}{\partial x}$$

$$= -\frac{\sinh p\cosh p\left[-\sin^2\zeta + \cancel{2\zeta\sin\zeta\cos\zeta} + \cos^2\zeta \mp \cancel{2\zeta\sin\zeta\cos\zeta}\right]}{2\;\;\cosh 2p + \cos 2\zeta}$$

$$= -\frac{\sinh 2p\;\;\cos 2\zeta}{4(\cosh 2p + \cos 2\zeta)} \qquad\qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial y} = \frac{\sinh p}{4}\left[\cos\zeta + 2\zeta\sin\zeta\right]\frac{\partial p}{\partial y} - \frac{\cosh p}{4}\left[\sin\zeta + 2\zeta\cos\zeta\right]\frac{\partial p}{\partial x}$$

$$= -\frac{\sin\zeta\sinh^2 p\,(\cos\zeta + 2\zeta\sin\zeta) - \cos\zeta\cosh^2 p\,(\sin\zeta + 2\zeta\cos\zeta)}{2(\cosh 2p + \cos 2\zeta)}$$

$$= \frac{-\sin\zeta\cos\zeta\overbrace{(\sinh^2 p + \cosh^2 p)}^{\cosh 2p}}{4(\cosh 2p + \cos 2\zeta)} - \frac{\zeta}{2}$$

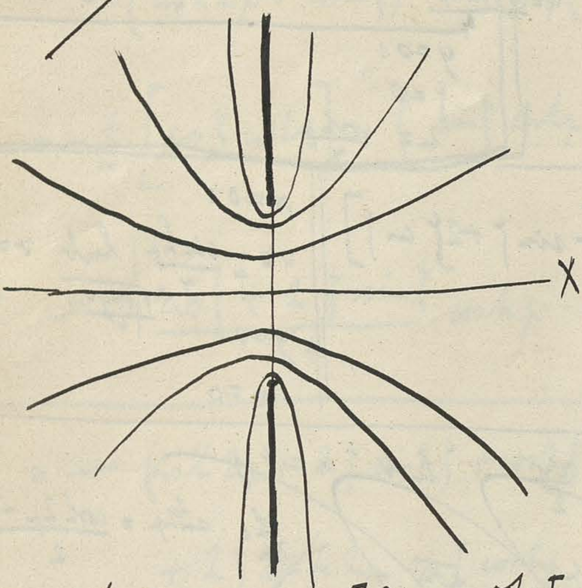
$$x+iy = \sin\left(-\frac{\pi}{4}+\zeta+i\mu\right)$$

$$x = \cos\left(-\frac{\pi}{4}+\zeta\right)\sinh\mu = \frac{\cos\zeta+\sin\zeta}{\sqrt{2}}\sinh\mu$$

$$y = \sin\left(-\frac{\pi}{4}+\zeta\right)\cosh\mu = \frac{\sin\zeta-\cos\zeta}{\sqrt{2}}\cosh\mu$$

$$u = \frac{\partial\varphi}{\partial x} + \frac{\cosh\mu}{4}\left[\cos\left(-\frac{\pi}{4}+\zeta\right) + 2\left(-\frac{\pi}{4}+\zeta\right)\sin\left(-\frac{\pi}{4}+\zeta\right)\right]$$

$$v = \frac{\partial\varphi}{\partial y} + \frac{\sinh\mu}{4}\left[-\sin\left(-\frac{\pi}{4}+\zeta\right) + 2\left(-\frac{\pi}{4}+\zeta\right)\cos\left(-\frac{\pi}{4}+\zeta\right)\right]$$



$$\frac{\partial v}{\partial x} = \frac{\cosh\mu}{4}\left[-\sin\zeta+2\zeta\cos\zeta\right]\frac{\partial\mu}{\partial x} + \frac{\sinh\mu}{4}\left[\cos\zeta-2\zeta\sin\zeta\right]\frac{\partial\mu}{\partial y}$$

$$= \cos\zeta\cosh^2\mu\left[-\sin\zeta+2\zeta\cos\zeta\right] - \sin\zeta\sinh^2\mu\left[\cos\zeta-2\zeta\sin\zeta\right]$$

$$= -\frac{\sin\zeta\cos\zeta\cosh 2\mu}{4(\cos \qquad -)} + \frac{\zeta}{2}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$$

$$\frac{\partial\psi}{\partial x} = -\frac{\sin\zeta \cosh\mu}{4} + \zeta\frac{\cos\zeta\cosh\mu}{2}$$

$$= \frac{\sinh\mu}{4}\left[-\sin\zeta + 2\zeta\cos\zeta\right] \qquad = 0 \text{ dla } \mu = 0$$

i dla $\zeta = 0$

$$u = \underbrace{\frac{\partial\varphi}{\partial x} + \frac{\cosh\mu}{4}\left[\cos\zeta + 2\zeta\sin\zeta\right]}$$

$$u = \frac{\partial\varphi}{\partial x} + \frac{y}{4}\left[\operatorname{ctg}\zeta + 2\zeta\right]$$

$x = 0$:

$\zeta = \pm\frac{\pi}{2}$ | lub $\mu = 0$, $y = \sin\zeta$

$u = \pm y\frac{\pi}{4}$ | lub: $u = \frac{\sqrt{1-y^2}}{4} + \frac{y}{2}\arcsin y$

$u = \pi\frac{\cosh\mu}{4}$

$y = 0$:

$\zeta = 0$

$u = \frac{\cosh\mu}{4}$

Zatem ~~...~~ φ

superponując $u = \mp y\frac{\pi}{4}$

$$v = \frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial x} = \frac{\partial\varphi}{\partial y} + \frac{\sinh\mu}{4}\left[-\sin\zeta + 2\zeta\cos\zeta\right]$$

$$= \frac{\partial\varphi}{\partial y} + \frac{x}{4}\left[-\operatorname{tg}\zeta + 2\zeta\right]$$

$x = 0$:

$v = \mp\frac{\sinh\mu}{4} = \frac{\sqrt{y^2-1}}{4}$ | lub $v = 0$

$y = 0$:

$v = 0$

~~$\mu x + \zeta y = \alpha$~~

~~$-\mu y + \zeta x = \beta$~~

~~$x = 0$: $\alpha = \pm\frac{\pi y}{2}$ lub $\alpha = \zeta y$~~

~~$\beta = -\mu y = -\mu\cosh\mu$~~ $\frac{1}{\sinh\mu}$

Nie czyni zadość $\Delta^2\varphi = 0$!

~~$\varphi = \cosh^2 \mu$~~

~~$\varphi = \frac{\sinh 2\mu - 2\mu}{4}$~~

$$\frac{\partial\varphi}{\partial x} = \frac{\cosh 2\mu - 1}{2}\cdot\frac{\partial\mu}{\partial x} = \sinh^2\mu\cdot\frac{\cos\zeta\cosh\mu}{\cosh^2\mu - \sin^2\zeta}$$

$(= \sinh^2\mu + \cos^2\zeta)$

~~$\frac{\partial f}{\partial\mu} = \sinh^2\mu = \frac{\cosh 2\mu - 1}{2}$~~

~~$f = \frac{\sinh 2\mu - 2\mu}{4}$~~

~~$= \frac{\sinh\mu\cosh\mu - \mu}{2}$~~

$x = 0$, $= 0$

$$\frac{\partial\varphi}{\partial y} = \sinh^2\mu\,\frac{\sin\zeta\sinh\mu}{\sinh^2\mu + \cos^2\zeta} \quad \overset{x=0}{=} \pm\sinh\mu$$

$$\frac{\partial\psi}{\partial y} = \mp\frac{1}{8}\left[\frac{+(\cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta)\cos\zeta\cosh\mu}{\cancel{\ldots - \cosh\mu}} + \right.$$

$$\left. + 2\zeta\sinh 2\mu\,\sin\zeta\sinh\mu \right]$$

$$= -\frac{1}{4\,(\cosh 2\mu + \cos 2\zeta)}\Big[(\cosh 2\mu + \cos 2\zeta)\cos\zeta\cosh\mu + 4\zeta\,(\sin\zeta\cos^2\zeta\cosh\mu + \sin\zeta\sinh^2\mu\cosh\mu) \Big]$$

$$= -\frac{1}{4}\left[\cos\zeta\cosh\mu\right] - \zeta\sin\zeta\cosh\mu\left(\frac{\cos^2\zeta + \sinh^2\mu}{\cos 2\zeta + \cosh 2\mu}\right) = \frac{1}{2}$$

$$\frac{\partial\psi}{\partial y} = -\frac{\cos\zeta + 2\zeta\sin\zeta}{4}\cdot\cosh\mu =$$

$$\frac{\partial\psi}{\partial x} = -\frac{(\cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta)\sin\zeta\sinh\mu + 2\zeta\sinh 2\mu\cos\zeta\cosh\mu}{4\,(\cosh 2\mu + \cos 2\zeta)}$$

$$= -\frac{\sin\zeta\sinh\mu}{4} + \zeta\,\frac{\sinh\mu\cosh^2\mu\cos\zeta - \sinh\mu\sin^2\zeta\cos\zeta}{\cosh 2\mu + \cos 2\zeta}$$

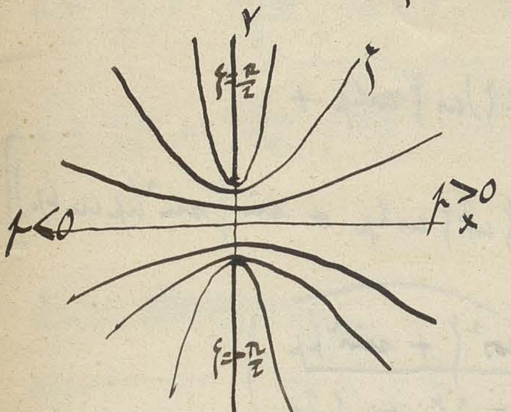
$$\frac{\partial\psi}{\partial x} = \frac{\sin^2\zeta - \cosh^2 p}{16 \sinh p}\left[\frac{-\sin^2\zeta + \cosh^2 p}{\sin\zeta} - 4\zeta\,\frac{\cos^2\zeta - \sinh^2 p}{\cos\zeta}\right]$$

$$x = \cos\zeta \sinh p \qquad \frac{x^2}{\sinh^2 p} + \frac{y^2}{\cosh^2 p} = 1$$

$$y = \sin\zeta \cosh p \qquad -\frac{x^2}{\cos^2\zeta} + \frac{y^2}{\sin^2\zeta} = 1$$

$x = 0: \quad \zeta = \pm\frac{\pi}{2}$ lub $p = 0$

$y = 0: \quad \zeta = 0,$



$$x^2 + y^2 = \cos^2\zeta \sinh^2 p + \sin^2\zeta \cosh^2 p = (1 - \sin^2\zeta)\sinh^2 p + \sin^2\zeta \cosh^2 p =$$

$$= \sinh^2 p + \sin^2\zeta = \cosh^2 p - \cos^2\zeta$$

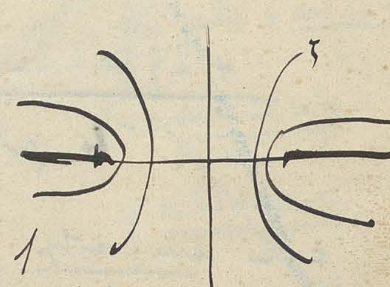
$$\frac{xy}{\frac{1}{4}} = \sin 2\zeta \sinh 2p$$

$$x + iy = \cos(\zeta + ip) = \frac{e^{i\zeta - p} + e^{-i\zeta + p}}{2}$$

$$x = \cos\zeta \cosh p$$

$$y = -\sin\zeta \sinh p$$

$$\frac{x^2}{\cosh^2 p} + \frac{y^2}{\sinh^2 p} = 1 \qquad \frac{x^2}{\cos^2\zeta} - \frac{y^2}{\sin^2\zeta} = 1$$



$$u = \frac{\partial\psi}{\partial x} - \frac{\partial\psi}{\partial y}$$

$$\sin^2\zeta\,\frac{\cosh 2\mu - 1}{2} + \cos^2\zeta\,\frac{\cosh 2\mu + 1}{2}$$

$$= \frac{\cosh 2\mu + \cos 2\zeta}{2}$$

$$= \cancel{\sinh\mu}\ \sin^2\zeta(\cancel{\sinh^2\mu}-1) + (\cancel{1-\sin^2\zeta})\cosh^2\mu$$

$$= \cosh^2\mu - \sin^2\zeta$$

$$= \sinh^2\mu + \cos^2\zeta$$

$$\frac{\partial\psi}{\partial y} = -\frac{\frac{\partial\psi}{\partial\zeta}}{\frac{\partial\mu}{\partial x}} + \frac{\frac{\partial\psi}{\partial\mu}}{\frac{\partial\zeta}{\partial y}} = \frac{(\sin^2\zeta\,\sinh^2\mu + \cos^2\zeta\cosh^2\mu)}{8}$$

$$\left[-\frac{\cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta}{\cos\zeta\cosh\mu} + \frac{2\zeta\sinh 2\mu}{\sin\zeta\sinh\mu}\right]$$

$$= -\frac{\cosh 2\mu + \cos 2\zeta}{\cos\zeta\cosh\mu} - 2\zeta\,\frac{2\sin^2\zeta\,\cancel{\cos\zeta} + 2\,\cancel{\sinh\mu}\cosh^2\mu\,\cancel{\cos\zeta}}{\sin\zeta\,\cancel{\cos\zeta\cosh\mu}\cosh\mu}$$

$$= -\frac{\cosh 2\mu + \cos 2\zeta}{16\cosh\mu}\left[\frac{\cosh 2\mu + \cos 2\zeta}{\cos\zeta} + 4\zeta\,\frac{\sin^2\zeta + \cosh^2\mu}{\sin\zeta}\right]$$

$$= \frac{\sin^2\zeta - \cosh^2\mu}{16\cosh\mu}\left[\frac{-\sin^2\zeta + \cos^2\cosh\mu}{\cos\zeta} + 4\zeta\,\frac{\sin^2\zeta + \cosh^2\mu}{\sin\zeta}\right]$$

$$\frac{\partial\psi}{\partial x} = \cancel{}\ \frac{\frac{\partial\psi}{\partial\zeta}}{\frac{\partial\mu}{\partial y}} + \frac{\frac{\partial\psi}{\partial\mu}}{\frac{\partial\mu}{\partial x}} = \frac{\sin^2\zeta - \cosh^2\mu}{16}.$$

$$\underbrace{\left[-\frac{\cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta}{\sin\zeta\sinh\mu} + \frac{2\zeta\sinh 2\mu}{\cos\zeta\cosh\mu}\right]}$$

$$\frac{-\sin^2\zeta + \cosh^2\mu}{\sin\zeta\sinh\mu} + 2\zeta\,\frac{2\sinh^2\mu\,\cancel{\cosh\mu}\,\cancel{\sin\zeta} - 2\,\cancel{\cosh\mu}\,\cancel{\sin\zeta}\cos^2\zeta}{\cancel{\sin\zeta}\cos\zeta\sinh\mu\,\cancel{\cosh\mu}}$$

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$$\psi = \int \frac{e^{\rho}+e^{-\rho}}{8i}\, d\rho \underbrace{\int (\alpha-\rho)(e^{\alpha}+e^{-\alpha})\, d\alpha}_{\alpha e^{\alpha} - e^{\alpha} - \alpha e^{-\alpha} - e^{-\alpha} - \rho e^{\alpha} + \rho e^{-\alpha}}$$

$$= \qquad e^{\alpha}(\alpha-1) - e^{-\alpha}(\alpha+1) - \rho(e^{\alpha} - e^{-\alpha})$$

$$= \frac{1}{8i}\left\{ \left[e^{\rho}-e^{-\rho}\right]\left[e^{\alpha}(\alpha-1) - e^{-\alpha}(\alpha+1)\right] - (e^{\alpha} + e^{-\alpha})\left[\rho(e^{\rho}-e^{-\rho}) - (e^{\rho}+e^{-\rho})\right]\right\}$$

$$= \frac{1}{8i}\left\{ \underbrace{(e^{\rho}-e^{-\rho})(e^{\alpha}-e^{-\alpha})}_{e^{\alpha+\rho}+e^{-\alpha-\rho}-e^{\alpha-\rho}-e^{-\alpha+\rho}}(\alpha-\rho) \underbrace{- (e^{\rho}-e^{-\rho})(e^{\alpha}+e^{-\alpha}) + (e^{\rho}+e^{-\rho})(e^{\alpha}-e^{-\alpha})}_{} \right\}$$

$$= -\cancel{e^{\alpha+\rho}} + e^{\alpha-\rho} - e^{-\alpha+\rho} + \cancel{e^{-\alpha-\rho}}$$
$$+ \cancel{e^{\alpha+\rho}} + e^{\alpha-\rho} - e^{-\alpha+\rho} - \cancel{e^{-\alpha-\rho}}$$

$$= \frac{1}{32i}\left\{ 2i\zeta\left[e^{2\mu}+e^{-2\mu}-e^{2i\zeta}-e^{-2i\zeta}\right] + 2(e^{2i\zeta}-e^{-2i\zeta})\right\}$$

$$= \frac{1}{8}\left\{ \zeta\left[\cosh 2\mu - \cos 2\zeta\right] + \sin 2\zeta \right\}$$

$$\frac{\partial\psi}{\partial\zeta} = \frac{1}{8}\left\{ \cosh 2\mu - \cos 2\zeta + 2\zeta \sin 2\zeta + 2\cos 2\zeta \right\}$$

$$= \frac{1}{8}\left\{ \cosh 2\mu + 2\zeta\sin 2\zeta + \cos 2\zeta \right\}$$

$$\frac{\partial\psi}{\partial\mu} = \frac{1}{8}\cdot 2\zeta \sinh 2\mu$$

$$x + iy = \sin(\zeta + i\mu) = \frac{e^{i\zeta-\mu} - e^{-i\zeta+\mu}}{2} = \frac{e^{-\mu}(\cos\zeta + i\sin\zeta) - e^{\mu}(\cos\zeta - i\sin\zeta)}{2}$$

$x^2+y^2=$

$$x = \cos\zeta \sinh\mu$$

$$y = \sin\zeta \cosh\mu$$

$$\begin{array}{l|l} 1 = -\sin\zeta \cdot \frac{\partial\mu}{\partial y} \cdot \sinh\mu + \cos\zeta\cosh\mu \frac{\partial\mu}{\partial x} & \sin\zeta\sinh\mu \\ 0 = \sin\zeta \frac{\partial\mu}{\partial x} \sinh\mu + \cos\zeta\cosh\mu \frac{\partial\mu}{\partial y} & \cdot\cos\zeta\cosh\mu \end{array}$$

$$-\sin\zeta\sinh\mu = \sin^2\zeta\sinh^2\mu + \cos^2\zeta\cosh^2\mu$$

$$\frac{\partial\mu}{\partial y} = -\frac{\sin\zeta\sinh\mu}{\sin^2\zeta\sinh^2\mu + \cos^2\zeta\cosh^2\mu}$$

$$\frac{\partial\mu}{\partial x} = \frac{\cos\zeta\cosh\mu}{}$$

$$\frac{1}{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2} = \sin^2\zeta\sinh^2\mu + \cos^2\zeta\cosh^2\mu = \sinh^2\mu + \cos^2\zeta$$

$$\cancel{\mu = \frac{\alpha+\beta+i\zeta}{\beta+\mu\ \alpha+i\zeta} \quad \partial + \ \ 2\zeta =}$$

$$\frac{\partial^2\Psi}{\partial\mu^2} + \frac{\partial^2\Psi}{\partial\zeta^2} = \zeta\sinh^2\mu + \zeta\cos^2\zeta = \zeta\,\frac{e^{2\mu} + e^{-2\mu} + e^{2i\zeta} + e^{-2i\zeta}}{4}$$

$$\cancel{= \frac{\alpha+\beta}{8}\left(e^{\frac{\alpha-\beta}{i}} + e^{-\frac{\alpha-\beta}{i}} + e^{(\alpha+\beta)i} + e^{-(\alpha+\beta)i}\right)}$$

$$\alpha+\beta = 2\mu$$
$$\alpha-\beta = 2i\zeta$$

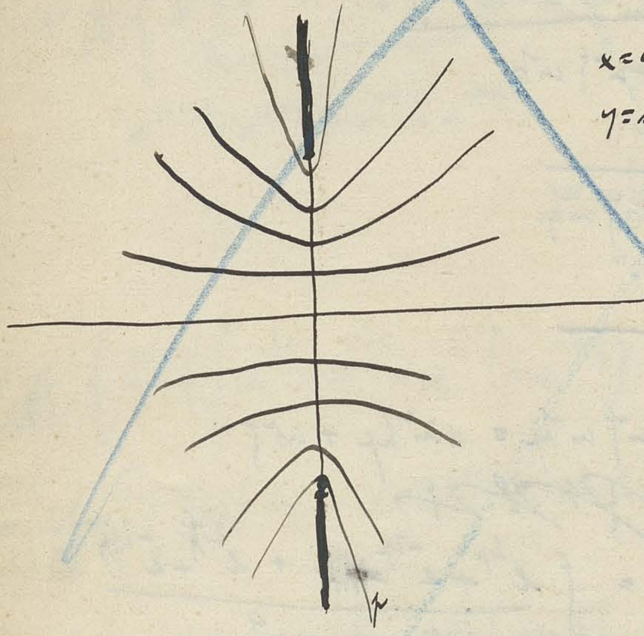
$$= \frac{\alpha-\beta}{8i}\left(e^{\alpha+\beta} + e^{-\alpha-\beta} + e^{\alpha-\beta} + e^{-\alpha+\beta}\right)$$

$$=$$

$$\frac{\partial\psi}{\partial x} = \frac{\cos^2 h\xi - \sin^2\eta}{4}\left[2\xi\left(\frac{\sinh\xi\,\cancel{\cosh\xi}}{\cancel{\cosh\xi}\,\cos\eta} + \frac{\cancel{\sin\eta}\,\cos\eta}{\cancel{\sin\eta}\,\sinh\xi}\right) - \frac{\sin^2 h\xi + \cos^2\eta}{\cos\eta\,\cosh\xi}\right]$$

$$= \frac{(\cos^2 h\xi - \sin^2\eta)}{4\cos\eta}(\sin^2 h\xi + \cos^2\eta)\left[\frac{2\xi}{\cancel{\sinh}\,\xi} - \frac{1}{\cosh\xi}\right]$$

$$= \frac{(\cos^2 h\xi - \sin^2\eta)^2}{4\cos\eta}\left[\frac{2\xi}{\sinh\xi} - \frac{1}{\cosh\xi}\right] = v$$



$x = \cos\eta\,\sinh\xi$ | $\frac{x^2}{\sin^2 h\xi} + \frac{y^2}{\cos^2 h\xi} = 1$

$y = \sin\eta\,\cosh\xi$ | $-\frac{x^2}{\cos^2\eta} + \frac{y^2}{\sin^2\eta} = 1$

$$\frac{\partial\psi}{\partial y} = \frac{\sinh^2\xi + \cos^2\mu}{2\sin\mu\,\sinh 2\xi}\left[2\xi\sinh\xi\cosh\xi + 2\xi\sinh\xi\sin\mu - \cosh^3\xi + \sin\mu\cosh\xi\right]$$

$$= \frac{\sinh^2\xi + \cos^2\mu}{4\sin\mu}\left[2\xi\left(\cosh\xi + \frac{\sin^2\mu}{\cosh\xi}\right) - \frac{\cosh^2\xi}{\sinh\xi} + \frac{\sin^2\mu}{\sinh\xi}\right]$$

$$\cancel{\frac{\xi\sin^2\mu}{\cosh\xi} + (\cosh^2\xi + \sin^2\mu)\frac{\xi}{\cosh\xi} - \frac{\xi}{\sinh\xi}}$$

$$\cosh^2\xi - \sin^2\mu = \sinh^2\xi + \cos^2\mu$$

$$\cosh^2\xi - \sin^2\mu = \frac{\cosh 2\xi + 1}{2} - \frac{1-\cos 2\mu}{2} = \frac{\cosh 2\xi + \cos 2\mu}{2}$$

$$\cancel{\cosh^2\xi - \sin^2\mu = \frac{\cosh 2\xi + 1}{2} - \frac{1-\cos 2\mu}{2}}$$

$$\cancel{\cosh 2\xi + \cos 2\mu = 2\cosh^2\xi - 1 + 1 - 2\sin^2\mu = 2(\cosh^2\xi - \sin^2\mu)}$$

$$\cancel{= 2\sinh^2\xi + 1 + 2\cos^2\mu - 1 = 2(\sinh^2\xi + \cos^2\mu)}$$

$$\cosh^2\xi + \sin^2\mu = \frac{\cosh 2\xi + 1}{2} + \frac{1 - \cos 2\mu}{2} = \cosh 2\xi - \cos 2\mu + 1$$

$$= \frac{\cosh 2\xi + \cos 2\mu}{8\sin\mu}\left[2\xi\,\frac{\cosh 2\xi - \cos 2\mu + 1}{2\cosh\xi} - \frac{\cosh 2\xi + \cos 2\mu}{2\sinh\xi}\right]$$

$$= \frac{\cosh^2\xi - \sin^2\mu}{4\sin\mu}\left[2\xi\,\frac{\cosh^2\xi + \sin^2\mu}{\cosh\xi} - \frac{\cosh^2\xi - \sin^2\mu}{\sinh\xi}\right] = -u$$

$$\psi = \frac{1}{32}\left\{ e^{2\xi}[2\xi - 2] + e^{-2\xi}[2+2\xi] - 2\xi(e^{2i\eta} + e^{-2i\eta})\right\}$$

$$= \frac{1}{16}\left\{ \xi\left[e^{2\xi} + e^{-2\xi} - (e^{2i\eta} + e^{-2i\eta})\right] - (e^{2\xi} - e^{-2\xi})\right\}$$

$$= \frac{1}{8}\left\{ \xi\left[\cosh 2\xi - \cos 2\eta\right] - \sinh 2\xi\right\} +$$

$$\frac{\partial\psi}{\partial y} = \frac{\frac{\partial\psi}{\partial\xi}}{\frac{\partial\xi}{\partial y}} + \frac{\frac{\partial\psi}{\partial\eta}}{\frac{\partial\eta}{\partial y}}$$

$$\frac{\partial\psi}{\partial\xi} = \frac{1}{8}\left\{ \cosh 2\xi - \cos 2\eta + 2\xi\sinh 2\xi - 2\cosh 2\xi\right\}$$

$$= \frac{1}{8}\left\{ 2\xi\sinh 2\xi - \underbrace{(\cosh 2\xi + \cos 2\eta)}_{=2(\sinh^2\xi + \cos^2\eta)}\right\}$$

$$= \frac{1}{4}\left\{2\xi\sinh\xi\cosh\xi - \sinh^2\xi - \cos^2\eta\right\}$$

$$\frac{\partial\psi}{\partial\eta} = \frac{1}{4}\xi\sin 2\eta = \frac{1}{2}\xi\sin\eta\cos\eta$$

$$\frac{\partial\psi}{\partial y} = \frac{[\sinh^2\xi + \cos^2\eta]}{4}\left[\frac{2\xi\sinh\xi\cosh\xi - \sinh^2\xi - \cos^2\eta}{\sin\eta\sinh\xi} + \frac{2\xi\sin\eta\cos\eta}{\cos\eta\cosh\xi}\right]$$

$$= \frac{\sinh^2\xi + \cos^2\eta}{\sin 2\eta\sinh 2\xi}\left[2\xi\sinh\xi\cosh\xi\cos\eta + 2\xi\sinh\xi\sin\eta\cos\eta - \sinh\xi\cosh\xi\cos\eta - \cos^3\eta\cosh\xi\right]$$

$\xi + i\mu = \alpha$ $\quad$ $\xi - i\mu = \beta$ $\qquad \xi = \frac{\alpha+\beta}{2} \qquad \mu = \frac{\alpha-\beta}{2i}$

$$\frac{\partial^2\psi}{\partial\mu^2} + \frac{\partial^2\psi}{\partial\xi^2} = \xi \cos^2\mu + \xi \sin^2 h \xi = \xi\, \frac{e^{2\xi} + e^{-2\xi} + e^{2i\mu} + e^{-2i\mu}}{4}$$

$$4\, \frac{\partial^2\psi}{\partial\alpha\,\partial\beta} = \frac{\alpha+\beta}{2}\; \frac{e^{\alpha+\beta} + e^{-\alpha-\beta} + e^{\alpha-\beta} + e^{-\alpha+\beta}}{4}$$

$$\frac{\partial^2\psi}{\partial\alpha\,\partial\beta} = \frac{1}{32}(\alpha+\beta)\left[e^{\beta}(e^{\alpha}+e^{-\alpha}) + e^{-\beta}(e^{\alpha}+e^{-\alpha})\right]$$

$$\psi = \iint \frac{(\alpha+\beta)}{32}(e^{\alpha}+e^{-\alpha})(e^{\beta}+e^{-\beta})\, d\alpha\, d\beta$$

$$\int x e^{x}\,dx = x e^{x} - e^{x} \qquad \int x e^{-x}\,dx = -x e^{-x} - e^{-x}$$

$$\psi = \frac{1}{32}\int (e^{\beta}+e^{-\beta})\left[\alpha e^{\alpha} - e^{\alpha} - \alpha e^{-\alpha} - e^{-\alpha} + \beta(e^{\alpha}-e^{-\alpha})\right] d\beta$$

$$= \frac{1}{32}\left\{(e^{\beta}-e^{-\beta})\left[e^{\alpha}(\alpha-1) - e^{-\alpha}(\alpha+1)\right] + \left[e^{\alpha}-e^{-\alpha}\right]\left[\beta e^{\beta} - e^{\beta} - \beta e^{-\beta} - e^{-\beta}\right]\right\}$$

$$= \frac{1}{32}\Big\{(e^{\beta}-e^{-\beta})\alpha(e^{\alpha}-e^{-\alpha}) + (e^{\alpha}-e^{-\alpha})(e^{\beta}-e^{-\beta})\beta -$$

$$- e^{\alpha+\beta} + \cancel{e^{\alpha-\beta}} \cancel{- e^{-\alpha+\beta}} + e^{-\alpha-\beta} - e^{\alpha+\beta} \cancel{+ e^{-\alpha+\beta}} \cancel{- e^{\alpha-\beta}} + e^{-\alpha-\beta}\Big\}$$

$$= \frac{1}{32}\Big\{(\alpha+\beta)\underbrace{(e^{\beta}-e^{-\beta})(e^{\alpha}-e^{-\alpha})}_{e^{\alpha+\beta}+e^{-\alpha-\beta}-e^{\alpha-\beta}-e^{-\alpha+\beta}} - 2e^{\alpha+\beta} + 2e^{-\alpha-\beta}\Big\}$$

$$x+iy = \sinh(\xi+i\mu) = \frac{e^{\xi+i\mu} - e^{-\xi-i\mu}}{2}$$

$$\cosh^2\mu = 1+\sinh^2\mu$$

$$1+\cosh 2\mu = 2\cosh^2\mu$$

$$\cosh 2\mu - 1 = 2\sinh^2\mu$$

$$x = \frac{e^{\xi}\cos\mu - e^{-\xi}\cos\mu}{2} = \cos\mu\sinh\xi$$

$$y = \frac{e^{\xi}\sin\mu + e^{-\xi}\sin\mu}{2} = \sin\mu\cosh\xi$$

$$\cancel{\cosh(\xi+i\mu)\cdot\cosh(\xi-i\mu) = \frac{(e^{\xi+i\mu} - e^{-\xi-i\mu})(e^{\xi-i\mu} - e^{-\xi+i\mu})}{4}}$$

$$\cancel{= \frac{e^{2\xi} - e^{-2i\mu} - e^{2i\mu} + e^{-2\xi}}{4}} = \cancel{\frac{\cosh(2\xi)}{2} - \frac{\cos 2\mu}{2}}$$

$$1 = -\sin\mu\sinh\xi\cdot\frac{\partial\mu}{\partial x} + \cos\mu\cosh\xi\cdot\frac{\partial\mu}{\partial y} \quad \Big|\ \sin\mu\sinh\xi$$

$$0 = -\sin\mu\sinh\xi\,\frac{\partial\mu}{\partial y} - \cos\mu\cosh\xi\cdot\frac{\partial\mu}{\partial x} \quad \Big|\ \cos\mu\cosh\xi$$

$$\sin\mu\sinh\xi = -(\sin^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi)\frac{\partial\mu}{\partial x}$$

$$\frac{\partial\mu}{\partial x} = \frac{-\sin\mu\sinh\xi}{\sin^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi}\ ; \quad \frac{\partial\mu}{\partial y} = \frac{\cos\mu\cosh\xi}{\ldots\ldots}$$

$$\frac{1}{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2} = \sin^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi$$

$$= \sin^2\mu\,\frac{\cosh 2\xi - 1}{2} + \cos^2\mu\,\frac{\cosh 2\xi + 1}{2} = \frac{\cosh 2\xi}{2} + \frac{\cos 2\mu}{2}$$

$$= \sinh^2\xi - \cos^2\mu\sinh^2\xi + \cos^2\mu\cosh^2\xi = \sinh^2\xi + \cos^2\mu$$

$$x + iy = f_0(\xi + i\mu)$$

$$\Delta^2 \psi = \zeta$$

$$\frac{\partial^2 \psi}{\partial \mu^2} + \frac{\partial^2 \psi}{\partial \xi^2} = \frac{\zeta}{\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2}$$

$$x = f_1(\mu, \xi)$$

$$y = f_2(\mu, \xi)$$

$$1 = \left(\frac{\partial \xi}{\partial x} + i \frac{\partial \mu}{\partial x}\right) f_0' = \left(\frac{\partial \mu}{\partial y} + i \frac{\partial \mu}{\partial x}\right) f_0'$$

$$\frac{1}{\frac{\partial \mu}{\partial y} + i \frac{\partial \mu}{\partial x}} = f_0'(\xi + i\mu)$$

$$\frac{1}{\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2} = f_0'(\xi + i\mu)\, f_0'(\xi - i\mu)$$

$$\psi = \dots$$

$$\Delta^2 \varphi = 0 \qquad \frac{\partial^2 \varphi}{\partial \mu^2} + \frac{\partial^2 \varphi}{\partial \xi^2} = 0$$

$$\varphi = \Re f_0(\mu \pm i\xi)$$

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} = \frac{\partial \varphi}{\partial \mu}\frac{\partial \mu}{\partial x} + \frac{\partial \varphi}{\partial \xi}\frac{\partial \xi}{\partial x} - \frac{\partial \psi}{\partial \mu}\frac{\partial \mu}{\partial y} - \frac{\partial \psi}{\partial \xi}\frac{\partial \xi}{\partial y}$$

$$= \frac{\partial \mu}{\partial y}\left(\frac{\partial \varphi}{\partial \xi} - \frac{\partial \psi}{\partial \mu}\right) + \frac{\partial \mu}{\partial x}\left(\frac{\partial \varphi}{\partial \mu} + \frac{\partial \psi}{\partial \xi}\right)$$

$$= \frac{\frac{\partial \varphi}{\partial \xi} - \frac{\partial \psi}{\partial \mu}}{\frac{\partial y}{\partial \mu}} + \frac{\frac{\partial \varphi}{\partial \mu} + \frac{\partial \psi}{\partial \xi}}{\frac{\partial x}{\partial \mu}}$$

Rozwiązanie równania: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = f(x,y)$ || Przyczem u oznacza funkcyę jakąś bądź

$$x+iy=\alpha \qquad \frac{\partial u}{\partial x}=\frac{\partial u}{\partial \alpha}\frac{\partial \alpha}{\partial x}+\frac{\partial u}{\partial \beta}\frac{\partial \beta}{\partial x}=\frac{\partial u}{\partial \alpha}+\frac{\partial u}{\partial \beta}$$

$$x-iy=\beta \qquad \frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial \alpha^2}+2\frac{\partial^2 u}{\partial \alpha\partial \beta}+\frac{\partial^2 u}{\partial \beta^2}$$

$$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial \alpha}\frac{\partial \alpha}{\partial y}+\frac{\partial u}{\partial \beta}\frac{\partial \beta}{\partial y}=i\left(\frac{\partial u}{\partial \alpha}-\frac{\partial u}{\partial \beta}\right)$$

$$\frac{\partial^2 u}{\partial y^2}=-\left[\frac{\partial^2 u}{\partial \alpha^2}-2\frac{\partial^2 u}{\partial \alpha\partial \beta}+\frac{\partial^2 u}{\partial \beta^2}\right]$$

$$\Delta^2 u = 4\frac{\partial^2 u}{\partial \alpha\partial \beta}=f\left(\frac{\alpha+\beta}{2},\frac{\alpha-\beta}{2i}\right)$$

$$u=\frac{1}{4}\int d\alpha\left[\int f\left(\frac{\alpha+\beta}{2},\frac{\alpha-\beta}{2i}\right)d\beta\right]+f_1(\alpha)+f_2(\beta)$$

$$u=\frac{1}{4}\iint f\left(\frac{\alpha+\beta}{2},\frac{\alpha-\beta}{2i}\right)d\alpha\, d\beta+\underbrace{f_1(\alpha)+f_2(\beta)}_{R\,f(x\pm iy)}$$

Transformacya na p, ξ zamiast x, y

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial p}\frac{\partial p}{\partial x}+\frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial p^2}\left(\frac{\partial p}{\partial x}\right)^2+\frac{\partial u}{\partial p}\frac{\partial^2 p}{\partial x^2}+2\frac{\partial^2 u}{\partial p\partial \xi}\frac{\partial p}{\partial x}\frac{\partial \xi}{\partial x}+\frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2+\frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2}$$

$$\Delta^2 u=\frac{\partial^2 u}{\partial p^2}\left[\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial y}\right)^2\right]+\frac{\partial u}{\partial p}\left[\cancel{\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}}\right]+2\frac{\partial^2 u}{\partial p\partial \xi}\left[\cancel{\frac{\partial p}{\partial x}\frac{\partial \xi}{\partial x}+\frac{\partial p}{\partial y}\frac{\partial \xi}{\partial y}}\right]+$$

$$+\frac{\partial^2 u}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2+\left(\frac{\partial \xi}{\partial y}\right)^2\right]+\frac{\partial u}{\partial \xi}\left[\cancel{\frac{\partial^2 \xi}{\partial x^2}+\frac{\partial^2 \xi}{\partial y^2}}\right]$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\left[\frac{\partial^2 u}{\partial p^2}+\frac{\partial^2 u}{\partial \xi^2}\right]\left[\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial y}\right)^2\right]$$

$$\text{(I)}\quad \boxed{\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ &\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \xi \end{aligned}}$$

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = -\frac{\partial \xi}{\partial y} \qquad \text{(III)}.$$

$$\frac{\partial p}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\partial \xi}{\partial x}$$

$$\text{(II)},\quad \Delta^2 p = 0 \qquad \Delta^2 \xi = 0$$

$$f(x+iy) = g(x,y) + i\,h(x,y)$$

$$f' = \frac{\partial g}{\partial x} + i\,\frac{\partial h}{\partial x} = \frac{1}{\cancel{i}}\left[\frac{\partial g}{\partial y} + i\,\frac{\partial h}{\partial y}\right]$$

$$\left|\ \frac{\partial g}{\partial x} = \frac{\partial h}{\partial y}\ \right|\ \frac{\partial g}{\partial y} = -\frac{\partial h}{\partial x}\ \Big|$$

$$g = \xi \qquad h = p$$

1). $\Delta^2 u = -\cancel{\frac{\partial g}{\partial y}}$

$$u = -J\left(\frac{\partial p}{\partial y}\right) + \varphi$$

$$v = J\left(\frac{\partial p}{\partial x}\right) + \varphi'$$

} Wtedy φ i φ' jednak nie są dowolne tylko: muszą jednak spełniać warunki (I). t.j. $\left\{\begin{aligned} &\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi'}{\partial y} = 0 \\ &\frac{\partial \varphi'}{\partial x} - \frac{\partial \varphi}{\partial y} = 0 \end{aligned}\right.$

zatem $\nabla^2 u = 0$

2). albo lepiej: znaleźć $J(h) = F$ $\quad \Delta^2 F = h$

wtedy: $u = -\dfrac{\partial F}{\partial y} + \varphi$

$$v = \frac{\partial F}{\partial x} + \varphi'$$

3). Najlepiej $\quad u = -\dfrac{\partial F}{\partial y} + \dfrac{\partial A}{\partial x}$

$$v = +\frac{\partial F}{\partial x} + \frac{\partial A}{\partial y} \qquad \Bigg\|\ \begin{aligned} &\Delta^2 A = 0 \\ &\Delta^2 F = h = p \end{aligned}$$

$$\frac{\partial v}{\partial \alpha} + \frac{\partial v}{\partial \beta} + \left[\frac{\partial u}{\partial \alpha} i - \frac{\partial u}{\partial \beta} i\right] \cdot \frac{\alpha - \beta}{2i} \quad \frac{\partial^2 u}{\partial \alpha \partial \beta}$$

$$\frac{\partial}{\partial \alpha}(v + ui) + \frac{\partial}{\partial \beta}(v - ui) =$$

$$\frac{\partial}{\partial \alpha}(u + iv) - \frac{\partial}{\partial \beta}(u + iv) = 2(\alpha - \beta) \frac{\partial u}{\partial \alpha \partial \beta}$$

$$u = px \pm fy \qquad v = -py \pm fx$$

$$\frac{\partial u}{\partial x} = p + x\frac{\partial p}{\partial x} \pm y\frac{\partial f}{\partial y} \qquad \frac{\partial v}{\partial y} = -p - y\frac{\partial p}{\partial y} \pm x\frac{\partial f}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial y} = x\frac{\partial p}{\partial y} \pm f \mp y\frac{\partial f}{\partial x} \qquad \frac{\partial v}{\partial x} = -y\frac{\partial p}{\partial x} \pm f \pm x\frac{\partial f}{\partial y}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

~~$u = py \quad fy$~~

~~$v =$~~

$$u = px + f \qquad \frac{\partial u}{\partial x} = p + x\frac{\partial p}{\partial x} \qquad \frac{\partial u}{\partial y} = x\frac{\partial p}{\partial y}$$

$$v = \qquad \frac{\partial v}{\partial y} =$$

~~$u = py$~~ ~~$\frac{\partial u}{\partial x} = y$~~

~~$v = y$~~

1

$$\frac{\partial(u+iv)}{\partial p} + i\,\frac{\partial(u+iv)}{\partial \zeta} = i\zeta\left[\frac{\partial^2(u+iv)}{\partial p^2} + \frac{\partial^2(u+iv)}{\partial \zeta^2}\right]$$

$p + i\zeta = \alpha \qquad u + iv = \omega \qquad p = \frac{\alpha+\beta}{2}$

$p - i\zeta = \beta \qquad\qquad\qquad\qquad \zeta = \frac{\alpha-\beta}{2i}$

$$\cancel{\frac{\partial\omega}{\partial\alpha}} + \frac{\partial\omega}{\partial\beta} + i\left[\cancel{\frac{\partial\omega}{\partial\alpha}}\, i - \frac{\partial\omega}{\partial\beta}\, i\right] = \frac{\alpha-\beta}{2} \cdot 4\,\frac{\partial^2\omega}{\partial\alpha\,\partial\beta}$$

$$2\,\frac{\partial\omega}{\partial\beta} = 2(\alpha-\beta)\,\frac{\partial^2\omega}{\partial\alpha\,\partial\beta}$$

$$\frac{1}{\alpha-\beta} = \frac{\partial}{\partial\alpha}\left(\log\frac{\partial\omega}{\partial\beta}\right)$$

$$\log(\alpha-\beta) + f_1(\beta) = \log\frac{\partial\omega}{\partial\beta}$$

$$\cancel{\omega = f_1(\alpha) + \int f(\beta)\,d\beta + \int \log(\alpha-\beta)\,d\beta}$$

$$\frac{\partial\omega}{\partial\beta} = e^{\log(\alpha-\beta) + f_1(\beta)} = (\alpha-\beta)\,.\,F(\beta)$$

$$\omega = f_1(\alpha) + \int(\alpha-\beta)\,F(\beta)\,d\beta$$

$$\cancel{u = \frac{1}{4}\left[\xi y + \rho x\right]}$$

$$\cancel{v = \frac{1}{4}\left[-\rho y + \xi x\right]}$$

$$\Delta^2 \psi = \xi \cdot \left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right]\left[\frac{\partial^2 \psi}{\partial \rho^2} + \frac{\partial^2 \psi}{\partial \xi^2}\right]$$

$$\frac{\partial^2 \psi}{\partial \rho^2} + \frac{\partial^2 \psi}{\partial \xi^2} = \xi\left[\sin^2 h\rho \cos^2 \xi + \cos^2 h\rho \sin^2 \xi\right]$$

$$\frac{\partial^2 \psi}{\partial \alpha\, \partial \beta} = \frac{1}{4}\,\frac{\alpha-\beta}{2i}\left[2\left[e^{2\alpha} + e^{-2\alpha} - 2 + e^{2\beta} - e^{-2\beta} - 2\right]\right]$$

$$\cancel{e^{\alpha-\beta} + e^{\beta-\alpha} - e^{-\alpha-\beta} - e^{\alpha+\beta}}$$

$$\frac{\partial v}{\partial \rho} + \frac{\partial u}{\partial \xi} = \xi\left(\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \xi^2}\right)$$

$$\frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial \rho} = \xi\left(\frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 v}{\partial \xi^2}\right)$$

$$u = \frac{1}{4}\left[\rho x + \xi y\right] + f_1(\rho + i\xi) + f_1(\rho - i\xi)$$

$$v = \frac{1}{4}\left[\xi x - \rho y\right] + f_2(\rho + i\xi)$$

$$\frac{\partial v}{\partial \rho} = \frac{1}{4}\left[\xi \frac{\partial x}{\partial \rho} - y - \rho \frac{\partial y}{\partial \rho}\right] + f_2'(\rho + i\xi) + f_2'(\rho - i\xi)$$

$$\frac{\partial u}{\partial \xi} = \frac{1}{4}\left[\rho \frac{\partial x}{\partial \xi} + y - \xi \frac{\partial y}{\partial \xi}\right] + i\left[f_1'(\rho + i\xi) - f_1'(\rho - i\xi)\right]$$

$$\frac{\partial x}{\partial \rho} = -\frac{\partial y}{\partial \xi}$$

$$\frac{\partial v}{\partial \rho} + \frac{\partial u}{\partial \xi} = \frac{1}{2}\,\xi \frac{\partial x}{\partial \rho} + (f_2' + i f_1')(\rho + i\xi) + (f_2' - i f_1')(\rho - i\xi)$$

$$\underbrace{\frac{\partial v}{\partial \rho} + i\frac{\partial v}{\partial \xi} + \frac{\partial u}{\partial \xi} - i\frac{\partial u}{\partial \rho}}_{\frac{\partial(u+iv)}{\partial \xi} + \frac{1}{i}\frac{\partial(u+iv)}{\partial \rho}} = \xi\left[\frac{\partial^2(u+iv)}{\partial \rho^2} + \frac{\partial^2(u+iv)}{\partial \xi^2}\right]$$

$$\frac{\partial(u+iv)}{\partial \xi} + \frac{1}{i}\frac{\partial(u+iv)}{\partial \rho} =$$

$$u = \frac{1}{4}\left[p x + \xi y\right] + \varphi(p, \xi)$$

$$\frac{\partial \xi}{\partial x} = \frac{\partial p}{\partial y}$$

$$v = \frac{1}{4}\left[-p y + \xi x\right] + \psi(p, \xi)$$

$$\frac{\partial \xi}{\partial y} = -\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{4}\left[p + x\frac{\partial p}{\partial x} + y\frac{\partial \xi}{\partial x}\right] + \frac{\partial \varphi}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial \varphi}{\partial \xi}\frac{\partial \xi}{\partial y}$$

$$\frac{\partial v}{\partial y} = \frac{1}{4}\left[-p - y\frac{\partial p}{\partial y} + x\frac{\partial \xi}{\partial y}\right] + \frac{\partial \psi}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \psi}{\partial \xi}\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left.\begin{aligned}\frac{\partial v}{\partial x} &= \frac{1}{4}\left[-y\frac{\partial p}{\partial x} + \xi + x\frac{\partial \xi}{\partial x}\right]\\ \frac{\partial u}{\partial y} &= \frac{1}{4}\left[x\frac{\partial p}{\partial y} + \xi + y\frac{\partial \xi}{\partial y}\right]\end{aligned}\right\} \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\cancel{u + iv =} \quad f(p + i\xi) = \cancel{\frac{1}{4}\left[p(x - iy) + \xi(x + iy)\right] +}$$

$$\varphi(p, \xi) + i\,\psi(p, \xi)$$

$$f' = \frac{\partial \varphi}{\partial p} + i\frac{\partial \psi}{\partial p} \quad \Big|\ \frac{\partial p}{\partial x}$$

$$i f' = \frac{\partial \varphi}{\partial \xi} + i\frac{\partial \psi}{\partial \xi} \quad \Big|\ \frac{\partial \xi}{\partial x}$$

$$f'\,\frac{\partial(p + i\xi)}{\partial x} = \frac{\partial \varphi}{\partial x} + i\frac{\partial \psi}{\partial x}$$

$$z = p + i\xi$$

$$f'(z) = \frac{\partial f}{\partial p} = \frac{1}{i}\frac{\partial f}{\partial \xi}$$

$$u = \frac{1}{4}\left[\xi \underbrace{\cosh\mu\cos\xi}_{y} - \mu \underbrace{\sinh\mu\sin\xi}_{-x}\right]$$

$$u = \frac{1}{4}(\xi y + \mu x)$$

$$\frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\xi^2} = -\sinh\mu\cos\xi$$

$$= -\frac{e^{\mu}-e^{-\mu}}{2}\,\frac{e^{i\xi}+e^{-i\xi}}{2} = -\frac{1}{4}\left[e^{\mu+i\xi} - e^{-(\mu+i\xi)} + e^{\mu-i\xi} - e^{-\mu+i\xi}\right]$$

$$\frac{\partial^2 v}{\partial\alpha\partial\beta} = -\frac{1}{16}\left[e^{\alpha} - e^{-\alpha} + e^{\beta} - e^{-\beta}\right]$$

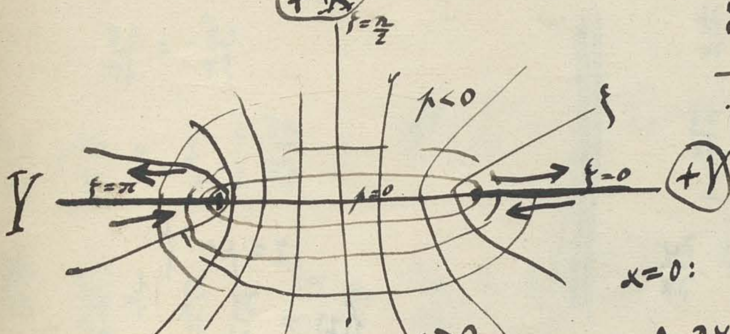
$$v = -\frac{1}{16}\left[\beta(e^{\alpha}+e^{-\alpha}) + \alpha(e^{\beta}+e^{-\beta})\right]$$

$$= -\frac{1}{16}\left[(\mu-i\xi)(\cosh\mu\cos\xi + i\sinh\mu\sin\xi) + (\mu+i\xi)(\cosh\mu\cos\xi - i\sinh\mu\sin\xi)\right]$$

$$v = -\frac{1}{4}\left[\mu\cosh\mu\cos\xi + \xi\sinh\mu\sin\xi\right]$$

$$v = \frac{1}{4}\left[-\mu y + \xi x\right]$$

$\sinh\mu = 0$	
$x=0$: $\xi = 0$	$u = 0$; $v = -\frac{\mu y}{4}$
$y=0$: $\xi = \pm\frac{\pi}{2}$	$u = \frac{\mu x}{4}$; $v = \frac{\pi x}{8}$



$x=0$:

$u = 0, \frac{\pi y}{4}$; $v = -\frac{\mu y}{4}$,

$$y - ix = \cosh(\mu + i\zeta) = \frac{e^{\mu+i\zeta} + e^{-\mu-i\zeta}}{2}$$

$$y = \cosh\mu \cos\zeta \qquad \frac{x^2}{\sinh^2\mu} + \frac{y^2}{\cosh^2\mu} = 1$$

$$x = -\sinh\mu \sin\zeta \qquad \frac{y^2}{\cos^2\zeta} - \frac{x^2}{\sin^2\zeta} = 1$$

$$1 = -\sinh\mu \cos\zeta \frac{\partial\zeta}{\partial x} - \cosh\mu \sin\zeta \frac{\partial\mu}{\partial x} \quad \left(\frac{\partial\zeta}{\partial x} = \frac{\partial\mu}{\partial y}\right)$$

$$0 = \sinh\mu \cos\zeta \frac{\partial\mu}{\partial x} - \cosh\mu \sin\zeta \frac{\partial\zeta}{\partial x} \quad \left(= \frac{\partial\mu}{\partial y}\right)$$

$$\left|\begin{matrix} \sinh\mu\cos\zeta \\ \cosh\mu\sin\zeta \end{matrix}\right| \begin{matrix} \cosh\mu\sin\zeta \\ -\sinh\mu\cos\zeta \end{matrix}$$

$$\frac{\partial\mu}{\partial y} = -\frac{\sinh\mu\cos\zeta}{\sin^2h\mu\cos^2\zeta + \cos^2h\mu\sin^2\zeta}$$

$$\frac{\partial\mu}{\partial x} = \frac{-\cosh\mu\sin\zeta}{\cos^2h\mu\sin^2\zeta + \sin^2h\mu\cos^2\zeta}$$

$$\frac{\partial^2 u}{\partial\zeta^2} + \frac{\partial^2 u}{\partial\mu^2} = -\cosh\mu\sin\zeta$$

$$= -\frac{e^\mu + e^{-\mu}}{2}\,\frac{e^{i\zeta} - e^{-i\zeta}}{2i} = \frac{1}{4i}\left[-e^{\mu+i\zeta} - e^{-\mu+i\zeta} + e^{\mu-i\zeta} + e^{-\mu-i\zeta}\right]$$

$$\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{1}{16i}\left[-e^{\alpha} + e^{-\alpha} + e^{\beta} - e^{-\beta}\right]$$

$$u = \frac{1}{16i}\left[-\beta(e^{\alpha} + e^{-\alpha}) + \alpha(e^{\beta} + e^{-\beta})\right]$$

$$= \frac{1}{8}\left[-(\mu - i\zeta)(\cosh\mu\cos\zeta + i\sinh\mu\sin\zeta) + (\mu + i\zeta)(\cosh\mu\cos\zeta - i\sinh\mu\sin\zeta)\right]$$

$$= \frac{1}{8}\left[\cancel{-\mu\cosh\mu\cos\zeta - \zeta\sinh\mu\sin\zeta}\right.$$

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$v = -\frac{1}{16}\left[\beta(e^{\alpha}-e^{-\alpha}) - \alpha(e^{\beta}-e^{-\beta})\right]$

$= -\frac{1}{16}\left[(\mu - i\zeta)(e^{\mu+i\zeta} - e^{-\mu-i\zeta}) - (\mu+i\zeta)(e^{\mu-i\zeta} - e^{-\mu+i\zeta})\right]$

$= -\frac{1}{16}\Big[\mu e^{\mu}[(\cos\zeta + i\sin\zeta) - (\cos\zeta - i\sin\zeta)]$

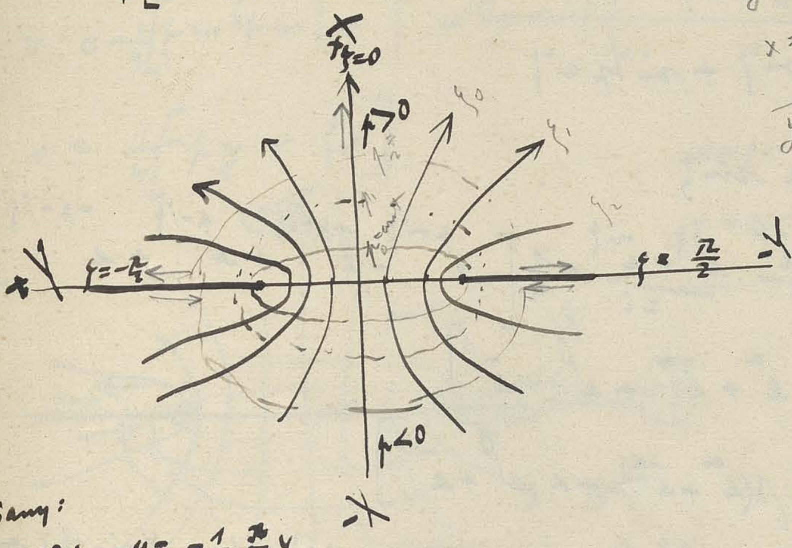
$+ \mu e^{-\mu}[-(\cos\zeta - i\sin\zeta) + (\cos\zeta + i\sin\zeta)]$

$+ i\zeta e^{\mu}[-(\cos\zeta + i\sin\zeta) - (\cos\zeta - i\sin\zeta)]$

$+ i\zeta e^{-\mu}[\cos\zeta - i\sin\zeta + \cos\zeta + i\sin\zeta]\Big]$

$v = \frac{1}{4}[-\mu \cosh\mu \sin\zeta + \zeta \sinh\mu \cos\zeta]$

$v = \frac{1}{4}[\mu y + \zeta x] + R\, f(\mu + i\zeta)$



$y = 0$ $\zeta = 0$

$x = 0$ $\mu = 0$, lub $\zeta = \frac{\pi}{2}$

$y = 0$ $u = \frac{1}{4}\mu x$

$v = 0$

$x = 0$ $u = -\frac{\zeta y}{4}$ lub $\frac{\pi}{2}\frac{y}{4}$

$v = 0$ lub $\frac{\mu y}{4}$

$\frac{e^{\mu} - e^{-\mu}}{2}$

Ściany:

$x = 0$: $u = \mp\frac{1}{4}\frac{\pi}{2}y$

$v = \frac{\mu y}{4}$

$$x - iy = \sinh(\mu + i\zeta)$$

$$x = \sinh\mu \cos\zeta \qquad \frac{x^2}{\sinh^2\mu} + \frac{y^2}{\cosh^2\mu} = 1$$

$$y = -\cosh\mu \sin\zeta \qquad \frac{y^2}{\sin^2\zeta} - \frac{x^2}{\cos^2\zeta} = 1$$

$$\begin{array}{l|c|c} 1 = \cosh\mu\cos\zeta \dfrac{\partial\mu}{\partial x} - \sinh\mu\sin\zeta \dfrac{\partial\zeta}{\partial x} & \cosh\mu\cos\zeta & \sinh\mu\sin\zeta \\ 0 = -\sinh\mu\sin\zeta \dfrac{\partial\mu}{\partial x} - \cosh\mu\cos\zeta \dfrac{\partial\zeta}{\partial x} & -\sinh\mu\sin\zeta & \cosh\mu\cos\zeta \end{array}$$

$$\cosh\mu\cos\zeta = (\cosh^2\mu\cos^2\zeta + \sinh^2\mu\sin^2\zeta)\frac{\partial\mu}{\partial x}$$

$$\frac{\partial\mu}{\partial x} = \frac{\cosh\mu\cos\zeta}{\cosh^2\mu\cos^2\zeta + \sinh^2\mu\sin^2\zeta} \quad \Bigg\| \quad \frac{\partial\mu}{\partial y} = -\frac{\sinh\mu\sin\zeta}{\sinh^2\mu\sin^2\zeta + \cosh^2\mu\cos^2\zeta}$$

$$\frac{\partial^2 u}{\partial\mu^2} + \frac{\partial^2 u}{\partial\zeta^2} = \cosh\mu\cos\zeta$$

$$= \frac{e^{\mu}+e^{-\mu}}{2}\,\frac{e^{i\zeta}+e^{-i\zeta}}{2} = \frac{1}{4}\left[e^{\mu+i\zeta} + e^{-(\mu+i\zeta)} + e^{-\mu+i\zeta} + e^{\mu-i\zeta}\right]$$

$$u = \frac{1}{4}\left[\mu\sinh\mu\cos\zeta + \zeta\cosh\mu\sin\zeta\right]$$

$$u = \frac{1}{4}\left[\mu x - \zeta y\right] + R\, f.(\mu+i\zeta)$$

$$\frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\zeta^2} = -\sinh\mu\sin\zeta = -\frac{e^{\mu}-e^{-\mu}}{2}\,\cancel{\frac{e^{i\zeta}-e^{-i\zeta}}{2} = -\frac{1}{4}\left[e^{\mu+i\zeta} + e^{-\mu-i\zeta} - e^{\mu-i\zeta} - e^{-\mu+i\zeta}\right]}$$

$$\cancel{16\frac{\partial^2 v}{\partial\alpha\partial\beta} = -\left[e^{\alpha} + e^{-\alpha} - e^{\beta} - e^{-\beta}\right]}$$

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial x}\left(-\frac{\partial v}{\partial y}\right) + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = -\frac{\partial \zeta}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial x^2} - \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\partial \zeta}{\partial x}$$

$$\left.\begin{aligned} u &= \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} \\ v &= \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x} \end{aligned}\right\} \qquad \begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= \Delta^2 \varphi = 0 \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} &= \Delta^2 \psi = \zeta \end{aligned}$$

$$\Delta^2 u = -\frac{\partial \zeta}{\partial y} = \frac{\partial p}{\partial x}$$

$$\Delta^2 v = \frac{\partial \zeta}{\partial x} = \frac{\partial p}{\partial y}$$

$$\Delta^2 p = 0 \qquad \Delta^2 \zeta = 0 \;\downarrow\; \Delta^4 \psi = 0$$

$$f(x+iy) = \varphi + i\psi$$

$$\left.\begin{aligned} f' &= \frac{\partial \varphi}{\partial x} + i\frac{\partial \psi}{\partial x} \\ i f' &= \frac{\partial \varphi}{\partial y} + i\frac{\partial \psi}{\partial y} \end{aligned}\right| \; i \;\Big\} \qquad \begin{aligned} \frac{\partial \varphi}{\partial y} &= -\frac{\partial \psi}{\partial x} \\ \frac{\partial \varphi}{\partial x} &= \frac{\partial \psi}{\partial y} \end{aligned}$$

$$f(x-iy) = \varphi + i\psi$$

$$\left.\begin{aligned} f' &= \frac{\partial \varphi}{\partial x} + i\frac{\partial \psi}{\partial x} \\ -i f' &= \frac{\partial \varphi}{\partial y} + i\frac{\partial \psi}{\partial y} \end{aligned}\right| \; i \;\Big\} \qquad \begin{aligned} \frac{\partial \varphi}{\partial x} &= -\frac{\partial \psi}{\partial y} \\ \frac{\partial \varphi}{\partial y} &= \frac{\partial \psi}{\partial x} \end{aligned}$$

$$\boxed{f(x-iy) = p + i\zeta}$$

$$\frac{\partial p}{\partial x} = -\frac{\partial \zeta}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\partial \zeta}{\partial x}$$

$f = Az$

$\rho = x$

$\zeta = y = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$

$u = -\dfrac{\frac{a y^2}{2} + \frac{b x^2}{2}}{a+b}$ $\qquad -\frac{x^2}{2} + \frac{\partial\varphi}{\partial x}$

$\psi = \frac{y^3}{6}, \quad \frac{x^2 y}{2}$ $\qquad v = \dfrac{bxy}{a+b}$ $\qquad xy + \frac{\partial\varphi}{\partial y}$

$\psi = \dfrac{a\frac{y^3}{6} + b\frac{x^2 y}{2}}{a+b}$

$\left.\begin{array}{l}\frac{\partial u}{\partial x} = -x \\ \frac{\partial v}{\partial y} = +x\end{array}\right\} = 0$ $\qquad \left.\begin{array}{l}\frac{\partial v}{\partial x} = y \\ -\frac{\partial u}{\partial y} = 0\end{array}\right\} = \zeta$ $\qquad \begin{array}{|l}1 = -1 \\ 0 = 0\end{array}$

$u = -\frac{\rho^2}{2}$

$v = \rho\zeta$

$\frac{\partial u}{\partial \rho} = -\rho \qquad \frac{\partial^2 u}{\partial \rho^2} = -1 \qquad \frac{\partial^2 u}{\partial \zeta^2} = 0$

$\partial^2 u = -1 = \frac{1}{1}$

$$-\frac{e^{p}-e^{-p}}{2}\sin\xi = \left[\left(\frac{e^{p}-e^{-p}}{2}\sin\xi\right)^2+\left(\frac{e^{p}+e^{-p}}{2}\cos\xi\right)^2\right]\frac{\partial p}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{-\sinh p\,\sin\xi}{\cos^2\xi+\sinh^2 p}$$

$\Delta u = \cosh p\,\cos\xi$ $\qquad$ $\Delta v = -\sinh p\,\sin\xi$

$$u = \frac{1}{4}\left[p\sinh p\cos\xi - \xi\cosh p\sin\xi\right] \begin{array}{l} +\Phi(p+i\xi) \\ +\Psi(p-i\xi)\end{array}$$

$$\frac{\partial u}{\partial \xi} = \frac{1}{4}\left[-p\sinh p\sin\xi - \cosh p\sin\xi - \xi\cosh p\cos\xi\right]$$

$$\frac{\partial v}{\partial p} = \xi\cosh p\cos\xi -$$

$$v = \frac{1}{4}\left[p\cosh p.\sin\xi + \xi\sinh p.\cos\xi\right]$$

$$\frac{\partial v}{\partial p} = \frac{1}{4}\left[+\cosh p\sin\xi + p\sinh p\sin\xi + \xi\cosh p\cos\xi\right]$$

$u = \frac{1}{2}p\,x$

$$\frac{\partial u}{\partial x} = \frac{1}{2}\left[p + x\frac{\partial p}{\partial x}\right] \qquad \frac{\partial u}{\partial y} = \frac{1}{2}x\frac{\partial p}{\partial y}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\left[2\frac{\partial p}{\partial x} + x\frac{\partial^2 p}{\partial x^2}\right] \qquad \frac{\partial^2 u}{\partial y^2} = \frac{1}{2}x\frac{\partial^2 p}{\partial y^2}$$

$$\Delta^2 u = \frac{\partial p}{\partial x} + x\,\cancel{\Delta p}$$

$u = -\frac{1}{2}\xi y$

$$\frac{\partial u}{\partial x} = -\frac{1}{2}y\frac{\partial \xi}{\partial x} = -\frac{1}{2}y\frac{\partial p}{\partial y} \;\Big|\; \frac{\partial u}{\partial y} = \frac{1}{2}\left(\xi - y\frac{\partial p}{\partial x}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{2}y\frac{\partial^2 p}{\partial x\partial y} \;\Big\|\; \frac{\partial^2 u}{\partial y^2} = \frac{1}{2}\left(-2\frac{\partial p}{\partial x} + y\frac{\partial^2 p}{\partial x\partial y}\right)$$

$$\Delta^2 u = \frac{\partial p}{\partial x}$$

$v = \frac{1}{2}p\,y$

$$\frac{\partial v}{\partial x} = \frac{1}{2}y\frac{\partial p}{\partial x} \qquad \frac{\partial v}{\partial y} = \frac{1}{2}\left(p + y\frac{\partial p}{\partial y}\right)$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{2}y\frac{\partial^2 p}{\partial x^2} \qquad \frac{\partial^2 v}{\partial y^2} = \frac{1}{2}\left(2\frac{\partial p}{\partial y} + y\frac{\partial^2 p}{\partial y^2}\right)$$

$$\Delta^2 v = \frac{\partial p}{\partial y}$$

$v = \frac{1}{2}\xi x$

$$\frac{\partial v}{\partial x} = \frac{1}{2}\left(\xi + x\frac{\partial p}{\partial y}\right) \;\Big\|\; \frac{\partial v}{\partial y} = -\frac{1}{2}x\frac{\partial p}{\partial x}$$

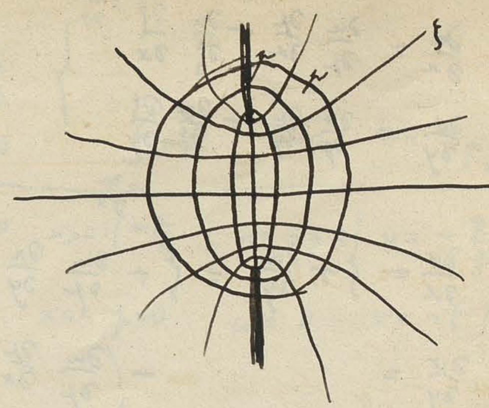
$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{2}\left(2\frac{\partial p}{\partial y} + x\frac{\partial^2 p}{\partial x\partial y}\right) \;\Big\|\; \frac{\partial^2 v}{\partial y^2} = -\frac{1}{2}x\frac{\partial^2 p}{\partial x\partial y}$$

$$\Delta^2 v = \frac{\partial p}{\partial y}$$

$$x + iy = \sinh(\mu + i\xi)$$

$$x = \sinh\mu \cdot \cos\xi$$

$$y = \cosh\mu \cdot \sin\xi$$



$$\frac{x^2}{\sinh^2\mu} + \frac{y^2}{\cosh^2\mu} = 1 \qquad \frac{y^2}{\sin^2\xi} - \frac{x^2}{\cos^2\xi} = 1$$

$y = 0 : \quad \xi = 0$

$x = 0 : \quad y < 1 : \quad \mu = 0$

$\qquad\quad y > 1 : \quad \xi = -\frac{\pi}{2}, +\frac{\pi}{2}$

$$1 = \cosh(\mu + i\xi) \cdot \left(\frac{\partial\mu}{\partial x} + i\frac{\partial\xi}{\partial x}\right) \qquad i = \cosh(\mu + i\xi) \cdot \left(\frac{\partial\mu}{\partial y} + i\frac{\partial\xi}{\partial y}\right)$$

$$1 = \left[\frac{e^{\mu}+e^{-\mu}}{2}\cos\xi + \frac{e^{\mu}-e^{-\mu}}{2} i \sin\xi\right]\left[\frac{\partial\mu}{\partial x} + i\frac{\partial\xi}{\partial x}\right] \qquad \cancel{i = e^{\mu} + e^{-\mu}}$$

$$1 = \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi \cdot \frac{\partial\mu}{\partial x} - \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi \cdot \frac{\partial\mu}{\partial y} \qquad 0 = \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi\,\frac{\partial\mu}{\partial y} + \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi\,\frac{\partial\mu}{\partial x}$$

$$0 = \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi\,\frac{\partial\mu}{\partial x} + \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi \cdot \frac{\partial\mu}{\partial y}$$

$$\frac{e^{\mu}+e^{-\mu}}{2}\cos\xi = \left[\left(\frac{e^{\mu}+e^{-\mu}}{2}\cos\xi\right)^2 + \left(\frac{e^{\mu}-e^{-\mu}}{2}\sin\xi\right)^2\right]\frac{\partial\mu}{\partial x}$$

$$\left[\cosh^2\mu\cos^2\xi + \sinh^2\mu - \sinh^2\mu\cos^2\xi\right] = \left[\cancel{\cos^2\xi + \sinh^2\mu}\right] \ldots\ldots$$

$$\frac{\partial\mu}{\partial x} = \frac{\cosh\mu\cos\xi}{\cos^2\xi + \sinh^2\mu}$$

$$\left(\frac{e^{2\mu}+e^{-2\mu}}{4}\right) \cancel{\ldots} \frac{\ldots}{2} + \frac{\cos^2\xi}{2}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \mu}\frac{\partial \mu}{\partial x} + \frac{\partial u}{\partial \xi}\underbrace{\frac{\partial \xi}{\partial x}}_{=\frac{\partial \mu}{\partial y}}$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial \mu}\frac{\partial \mu}{\partial y} + \frac{\partial v}{\partial \xi}\underbrace{\frac{\partial \xi}{\partial y}}_{= -\frac{\partial \mu}{\partial x}}$$

$$\frac{\partial \mu}{\partial x}\left(\frac{\partial u}{\partial \mu} - \frac{\partial v}{\partial \xi}\right) + \frac{\partial \mu}{\partial y}\left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \mu}\right) = 0$$

$$\frac{\partial v}{\partial x} = \xi + \frac{\partial u}{\partial y} = \xi + \frac{\partial u}{\partial \mu}\frac{\partial \mu}{\partial y} - \frac{\partial u}{\partial \xi}\frac{\partial \mu}{\partial x} = \frac{\partial v}{\partial \mu}\frac{\partial \mu}{\partial x} + \frac{\partial v}{\partial \xi}\frac{\partial \mu}{\partial y} \quad \Big| \quad \frac{\partial \mu}{\partial x} \quad \Big| \quad \frac{\partial \mu}{\partial y}$$

$$\frac{\partial v}{\partial y} = -\left(\frac{\partial u}{\partial \mu}\frac{\partial \mu}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial \mu}{\partial y}\right) = \frac{\partial v}{\partial \mu}\frac{\partial \mu}{\partial y} - \frac{\partial v}{\partial \xi}\frac{\partial \mu}{\partial x} \quad \Big| \quad \frac{\partial \mu}{\partial y} \quad \Big| \quad -\frac{\partial \mu}{\partial x}$$

$$\frac{\partial u}{\partial \mu} = \frac{1}{q}\left[\cosh\mu \sin\xi + \mu \sinh\mu \sin\xi - \cosh\mu\, \xi \cos\xi\right] \sinh\mu \sin\xi \quad \Big|$$

$$\frac{\partial u}{\partial \xi} = \frac{1}{q}\left[\mu \cosh\mu \cos\xi - \sinh\mu \cos\xi + \sinh\mu\, \xi \sin\xi\right] \cosh\mu \cos\xi \quad \Big| \; -$$

$$\frac{\partial v}{\partial y} = \frac{1}{q}\left[\cosh\mu \sinh\mu + \mu\underbrace{\left(\sin^2 h\mu \sin^2\xi - \cos^2 h\mu \cos^2\xi\right)}_{-\cos^2 h\mu + \sin^2 h\mu \ldots} - \xi \sinh\mu \cosh\mu \sin\xi \cos\xi\right]$$

$$\xi \frac{\partial \mu}{\partial x} - \frac{\partial u}{\partial \xi}\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right] = \frac{\partial v}{\partial \mu}\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right]$$

$$\xi \frac{\partial \mu}{\partial y} + \frac{\partial u}{\partial \mu}\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right] = \frac{\partial v}{\partial \xi}\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \ldots\right]$$

$$\frac{\partial v}{\partial \mu} = \xi \frac{\frac{\partial \mu}{\partial x}}{\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2} - \frac{\partial u}{\partial \xi} = +\xi (\Delta^2) u - \frac{\partial u}{\partial \xi}$$

$$\frac{\partial v}{\partial \xi} = \xi \frac{\frac{\partial \mu}{\partial y}}{(\quad)} + \frac{\partial u}{\partial \mu} = \xi (\Delta^2) v + \frac{\partial u}{\partial \mu}$$

$$u = \frac{1}{2}\left[p \cosh p \sin\xi - \sinh p \cdot \xi \cos\xi\right]$$

$$u = \frac{1}{2}\left[px - \xi y\right] \quad + ax + by +$$

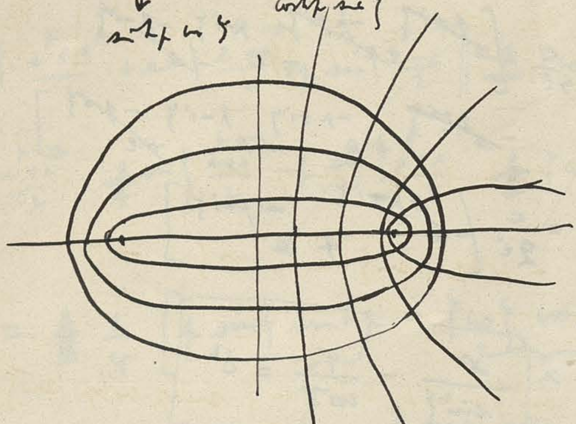
$$v = +\frac{1}{2}\left[py + \xi x\right]$$

$\sinh p \cos\xi$ $\qquad$ $\cosh p \sin\xi$

$x = 0 \quad \xi = 0 \quad u = 0 \quad v = \frac{-1}{4} p y$

$y = 0 \quad x > c : \xi = \frac{\pi}{2} \quad u = \frac{1}{4} p x \mid v = -\frac{\pi x}{8}$

$x < c : p = 0 \quad u = 0 \quad v = -\frac{1}{4}\xi x$



$$\frac{\partial p}{\partial x} = -\frac{\partial \xi}{\partial y}$$

$$\frac{\partial p}{\partial y} = \frac{\partial \xi}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{4}\left[p + x\frac{\partial p}{\partial x} - y\frac{\partial \xi}{\partial y}\right]$$

$$\frac{\partial v}{\partial y} = +\frac{1}{4}\left[p + y\frac{\partial p}{\partial y} + x\frac{\partial \xi}{\partial x}\right] \qquad \Sigma = \frac{p}{2}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = +\frac{1}{4}\left[\cancel{p} + y\frac{\partial p}{\partial x} + \cancel{\xi} + x\frac{\partial \xi}{\partial y} - x\frac{\partial p}{\partial y} + \cancel{\xi} - y\frac{\partial \xi}{\partial x}\right] \quad -\frac{\xi}{2}$$

$$\frac{\partial v}{\partial p} = +\xi \sinh p \sin\xi - \frac{1}{2}\left[p \cosh p \cos\xi - \sinh p \cos\xi + \sinh p \cdot \xi \sin\xi\right]$$

$$= \qquad -\frac{1}{2}\left[p \cosh p \cos\xi - \sinh p \cos\xi - \xi \sinh p \sin\xi\right]$$

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$$x = \cancel{\frac{e^{\xi}-e^{-\xi}}{2}\sin\mu}$$

$$y = \cancel{\frac{e^{\xi}+e^{-\xi}}{2}\cos\mu}$$

$y=0 \quad x=0$

$y=0 \quad \mu = \pm\frac{\pi}{2}$

$\cos i \quad \sin \quad e^{i}$

$\cos - i \sin \quad e^{-\xi}$

$$\frac{e^{i}-e^{-i}}{2i}$$

By machine:

$$x = c\cosh\mu \sin\xi$$

$$y = -c\sinh\mu\cos\xi$$

$$\boxed{\frac{x^2}{\cosh^2\mu} + \frac{y^2}{\sinh^2\mu} = c^2}$$

$$x+iy = c\left[\frac{e^{\mu}+e^{-\mu}}{2}\,\frac{e^{i\xi}-e^{-i\xi}}{2i} + i\,\frac{e^{\mu}-e^{-\mu}}{2}\,\frac{e^{i\xi}+e^{-i\xi}}{2}\right]$$

$$= \frac{c}{4i}\left[\cancel{e^{\mu+i\xi}} \cancel{+e^{-\mu+i\xi}} - e^{\mu-i\xi} \cancel{-e^{-\mu-i\xi}} \cancel{-e^{\mu+i\xi}} + e^{-\mu+i\xi} - e^{\mu-i\xi} \cancel{+e^{-\mu-i\xi}}\right]$$

$$= \frac{c}{2i}\left[-e^{+\mu-i\xi} + e^{-(\mu-i\xi)}\right]$$

$$\boxed{\frac{x^2}{\sin^2\xi} - \frac{y^2}{\cos^2\xi} = c^2}$$

$\varphi = \frac{\pi}{2} - \xi \ldots$

$$\left.\begin{matrix} x=0 & \xi = 0 \\ y=0 & \xi = \frac{\pi}{2}\end{matrix}\right\}$$

$$\frac{\partial\mu}{\partial x} = \frac{\sinh\mu\sin\xi}{\sinh^2\mu + \cos^2\xi} \qquad \frac{\partial\mu}{\partial y} = -\frac{\cosh\mu\cos\xi}{\sinh^2\mu+\cos^2\xi}$$

$$\frac{\partial^2 u}{\partial\mu^2} + \frac{\partial^2 u}{\partial\xi^2} \qquad \frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\xi^2}$$

$$\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{1}{2^2}\left[e^{\frac{\alpha+\beta}{2}} - e^{-\frac{\alpha+\beta}{2}}\right]\frac{\left[e^{\frac{\alpha-\beta}{2}} - e^{-\frac{\alpha-\beta}{2}}\right]}{i} = \left(e^{\alpha} - e^{-\beta} - e^{\beta} + e^{-\alpha}\right) = \frac{\cosh\alpha - \cosh\beta}{i}$$

$$\frac{\partial}{\partial\alpha}\left[\beta\sinh\alpha - \alpha\cosh\beta\right] = \beta\cosh\alpha - \sinh\beta$$

$$= \cosh\alpha - \cosh\beta$$

$$\frac{\partial}{\partial\beta}\Big[$$

$$v = (\mu-i\xi)\frac{\left(e^{\mu+i\xi}-e^{-\mu-i\xi}\right)}{i} - (\mu+i\xi)\frac{\left(e^{\mu-i\xi}-e^{-\mu+i\xi}\right)}{i} = e^{\mu}\left[(\mu-i\xi)(\cos\xi+i\sin\xi) - (\mu+i\xi)(\cos\xi - i\sin\xi)\right]$$

$$+e^{-\mu}\left[-(\mu-i\xi)(\cos\xi - i\sin\xi) + (\mu+i\xi)(\cos\xi+i\sin\xi)\right]$$

$$= \left[\mu\sin\xi - \xi\cos\xi\right]e^{\mu} + (\mu\sin\xi + \xi\cos\xi)e^{-\mu} = \mu\sin\xi\cosh\mu - \xi\cos\xi\sinh\mu$$

$$\frac{\partial^2 v}{\partial \rho^2} + \frac{\partial^2 v}{\partial \zeta^2} = - \frac{e^\rho + e^{-\rho}}{2} \sin\zeta \quad \cancel{+Ax+By}$$

$$\rho \sin\zeta \frac{e^\rho - e^{-\rho}}{2} \pm \zeta\cos\zeta \frac{e^\rho + e^{-\rho}}{2} \qquad \Big| \qquad \rho\cos\zeta \frac{e^\rho - e^{-\rho}}{2} \pm \cos\zeta \frac{e^\rho + e^{-\rho}}{2} \mp \zeta\sin\zeta \frac{e^\rho - e^{-\rho}}{2}$$

$$\sin\zeta \frac{e^\rho - e^{-\rho}}{2} + \rho\sin\zeta \frac{e^\rho + e^{-\rho}}{2} \pm \zeta\cos\zeta \frac{e^\rho - e^{-\rho}}{2}$$

$$\sin\zeta \frac{e^\rho + e^{-\rho}}{2} + \sin\zeta \frac{e^\rho + e^{-\rho}}{2} + \cancel{\rho\sin\zeta \frac{e^\rho - e^{-\rho}}{2}} \pm \cancel{\zeta\cos\zeta \frac{e^\rho + e^{-\rho}}{2}}$$

$$\mp\sin\zeta \frac{e^\rho + e^{-\rho}}{2} \mp \sin\zeta \frac{e^\rho + e^{-\rho}}{2} - \cancel{\rho\sin\zeta \frac{e^\rho - e^{-\rho}}{2}} \mp \cancel{\zeta\cos\zeta \frac{e^\rho + e^{-\rho}}{2}}$$

$$v = \pm \frac{1}{4}\left[\rho \underbrace{\sin\zeta \sinh\rho}_{y} - \zeta \underbrace{\cos\zeta \cosh\rho}_{x}\right] +$$

$$u = \frac{1}{4}\left[\rho x + y\zeta\right] + Ax + By$$

$$v = \frac{1}{4}\left[\rho y - x\zeta\right]$$

$y=0$: $x<c$: $\rho = 0 \quad \| \quad u = 0 \qquad v = -\frac{x\zeta}{4}$

$x>c$: $\zeta = 0, \pi \quad \| \quad u = \frac{\rho x}{4}$; $v = 0$; $-\frac{x\pi}{4}$

$x=0$: $\zeta = \frac{\pi}{2} \quad \| \quad u = \frac{y\pi}{8}$; $v = \frac{\rho y}{4}$

$$\rho(e^\rho + e^{-\rho})\sin\zeta \pm \zeta\cos\zeta(e^\rho - e^{-\rho})$$

$$(e^\rho + e^{-\rho})\sin\zeta + \rho(e^\rho - e^{-\rho})\sin\zeta \pm \zeta\cos\zeta(e^\rho + e^{-\rho}) \qquad \Big| \qquad \rho(e^\rho + e^{-\rho})\cos\zeta \pm \cos\zeta(e^\rho - e^{-\rho}) \mp \zeta\sin\zeta(e^\rho - e^{-\rho})$$

$$2(e^\rho - e^{-\rho})\sin\zeta + \cancel{\rho(e^\rho + e^{-\rho})\sin\zeta} \pm \cancel{\zeta\cos\zeta(e^\rho - e^{-\rho})}$$

$$\mp 2(e^\rho - e^{-\rho})\sin\zeta - \cancel{\rho(e^\rho + e^{-\rho})\sin\zeta} \mp \cancel{\zeta\cos\zeta(e^\rho - e^{-\rho})}$$

StV

$$= \frac{(\mu+i\xi)}{16}\left[e^{\mu}(\cos\xi - i\sin\xi) + e^{-\mu}(\cos\xi + i\sin\xi)\right] + (\mu - i\xi)\left[e^{\mu}(\cos\xi + i\sin\xi) + e^{-\mu}(\cos\xi - i\sin\xi)\right]$$

$$= \frac{e^{\mu}}{16}\left[2(\mu\cos\xi + \xi\sin\xi) - i\,\cancel{(\mu\sin\xi - \xi\cos\xi)}\right]$$

$$+ e^{-\mu}\left[2(\mu\cos\xi - \xi\sin\xi)\right]$$

$$u = \frac{1}{4}\left[\cos\xi \cdot \mu \cdot \cosh\mu + \sin\xi \cdot \xi \cdot \sinh\mu\right]$$

$$= \frac{1}{4}\left[\mu x \quad + y\,\xi \quad\right]$$

Dla $y=0$: albo $\mu = 0$ $\quad x<c$

albo $\xi = 0, \pi, 2\pi \ldots$ $\quad x>c$

$u = 4\mu x \operatorname{tgh}\mu$

Dla $x=0$: $\xi = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$u = 4y\frac{\pi}{2}\operatorname{ctgh}\mu$

$$\frac{\partial^2 v}{\partial\mu^2} + \frac{\partial^2 v}{\partial\xi^2} = \frac{\frac{\partial\mu}{\partial\xi}}{(\ldots)^2} = -\cosh\mu\sin\xi$$

$$8\frac{\partial^2 v}{\partial\alpha\partial\beta} = -\frac{1}{2}\left(e^{\frac{\alpha+\beta}{2}} + e^{\frac{-\alpha+\beta}{2}}\right)\left(e^{\frac{\alpha-\beta}{2}} - e^{-\frac{\alpha+\beta}{2}}\right)$$

$$= -\frac{1}{2}\left(e^{\alpha} + e^{-\beta} - e^{\beta} - e^{-\alpha}\right) = -\sinh\alpha + \sinh\beta$$

$$\frac{\partial}{\partial\alpha}\left(-\beta\cosh\alpha + \alpha\cosh\beta\right) = -\beta\sinh\alpha + \cosh\beta$$

$$\frac{\partial^2}{\partial\alpha\partial\beta} \qquad = -\sinh\alpha + \sinh\beta$$

$$v = -\frac{1}{8}\left[\beta\cosh\alpha - \alpha\cosh\beta\right] = -\frac{1}{8}\left[(\mu-i\xi)\left(e^{\mu+i\xi} + e^{-\mu-i\xi}\right) - (\mu+i\xi)\left(e^{\mu-i\xi} + e^{-\mu+i\xi}\right)\right]$$

$$= e^{\mu}\left[(\mu-i\xi)(\cos\xi + i\sin\xi) - (\mu+i\xi)(\cos\xi - i\sin\xi)\right] + e^{-\mu}\left[(\mu-i\xi)(\cos\xi - i\sin\xi) - (\mu+i\xi)(\cos\xi + i\sin\xi)\right]$$

$$= i\,e^{\mu}\,2(\mu\sin\xi - \xi\cos\xi) \qquad - 2e^{-\mu}\,i\,(\mu\sin\xi + \xi\cos\xi)$$

$$\frac{\partial \mu}{\partial x} = \frac{\sinh \mu \cos \zeta}{\sinh^2 \mu + \sin^2 \zeta} \qquad \frac{\partial \mu}{\partial y} = - \frac{\cosh \mu \sin \zeta}{\sinh^2 \mu + \sin^2 \zeta}$$

$$\frac{\frac{\partial \mu}{\partial x}}{\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2} = \frac{\sinh \mu \cos \zeta \cdot (\sinh^2 \mu + \sin^2 \zeta)}{\underbrace{\sinh^2 \mu \cos^2 \zeta + \cosh^2 \mu \sin^2 \zeta}_{= \sinh^2 \mu + \sin^2 \zeta}} = \sinh \mu \cos \zeta$$

$$\sinh \mu \cos \zeta = \frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial \zeta^2} = \frac{e^{\mu} - e^{-\mu}}{2} \cos \zeta$$

$$\mu + i\zeta = \alpha \qquad \mu = \frac{\alpha + \beta}{2} \qquad \frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial \zeta^2} = 4 \frac{\partial^2 u}{\partial \alpha \partial \beta}$$

$$\mu - i\zeta = \beta \qquad \zeta = \frac{\alpha - \beta}{2i}$$

$$8 \frac{\partial^2 u}{\partial \alpha \partial \beta} = \left[e^{\frac{\alpha+\beta}{2}} - e^{-\frac{\alpha+\beta}{2}} \right] \cos \frac{\alpha - \beta}{2i} = \frac{1}{2}\left(e^{\frac{\alpha+\beta}{2}} - e^{-\frac{\alpha+\beta}{2}}\right)\left(e^{\frac{\alpha-\beta}{2}} + e^{-\frac{\alpha-\beta}{2}}\right)$$

$$= \frac{1}{2}\left(e^{\alpha} - e^{-\beta} + e^{\beta} - e^{-\alpha}\right) = \sinh \alpha + \sinh \beta$$

$$\frac{\partial}{\partial \alpha}\left[\alpha \cosh \beta + \beta \cosh \alpha\right] = \cosh \beta + \beta \sinh \alpha$$

$$\frac{\partial}{\partial \alpha \partial \beta} \qquad = \sinh \beta + \sinh \alpha$$

$$u = \frac{1}{8}\left[\alpha \cosh \beta + \beta \cosh \alpha\right]$$

$$= \frac{1}{16}\left[(\mu + i\zeta)\left(e^{\mu - i\zeta} + e^{-\mu + i\zeta}\right) + (\mu - i\zeta)\left(e^{\mu + i\zeta} + e^{-\mu - i\zeta}\right)\right]$$

$$\frac{\sinh\mu\,\cos\xi}{\cosh^2\mu-\cos^2\xi}=\sqrt{\frac{(\cosh^2\mu-1)\,\cos^2\xi}{2\left[\left(\frac{1+x^2+y^2}{2}\right)^2-x^2\right]}}=\sqrt{\frac{x^2-\frac{1+y^2+x^2}{2}+\sqrt{\left(\frac{1+x^2+y^2}{2}\right)^2-x^2}}{2\left[\frac{1+r^2}{2}-x^2\right]}}$$

$\cancel{x+i\xi=e}$

$$x+iy=e^{\mu+i\xi}\quad+\mu+i\xi$$

$\cancel{x=e^{\mu}\cos\xi+\mu}$ $\quad \cancel{\sqrt{x^2+y^2}=e^{\mu}}$ $\quad \cancel{\operatorname{tg}\xi=\frac{y}{x}}$

$\cancel{y=e^{\mu}\sin\xi+\xi}$ $\quad \cancel{\mu=\log r}$ $\quad \cancel{\xi=\operatorname{arctg}\frac{y}{x}=0}$

$$1=\left[e^{\mu+i\xi}\quad+1\right]\left(\frac{\partial\mu}{\partial x}+i\frac{\partial\xi}{\partial x}\right)$$

$$1=(e^{\mu}\cos\xi+1)\frac{\partial\mu}{\partial x}-e^{\mu}\sin\xi\,\frac{\partial\xi}{\partial x}$$

$$0=e^{\mu}\sin\xi\,\frac{\partial\mu}{\partial x}+(e^{\mu}\cos\xi+1)\frac{\partial\xi}{\partial x}$$

$$\begin{vmatrix} e^{\mu}\cos\xi+1 & -e^{\mu}\sin\xi \\ e^{\mu}\sin\xi & e^{\mu}\cos\xi+1 \end{vmatrix}$$

$$(e^{\mu}\cos\xi+1)=\left[(e^{\mu}\cos\xi+1)^2+e^{2\mu}\sin^2\xi\right]\frac{\partial\mu}{\partial x}$$

$$\frac{\partial\mu}{\partial x}=\frac{1+e^{\mu}\cos\xi}{1+e^{2\mu}+2e^{\mu}\cos\xi}$$

$$\frac{\partial\mu}{\partial y}=\frac{-e^{\mu}\sin\xi}{1+e^{2\mu}+2e^{\mu}\cos\xi}$$

$\mu e^{\mu}\cos\xi$

$e^{\mu}\cos\xi+\mu e^{\mu}\cos\xi$

$2e^{\mu}\cos\xi+\cancel{\mu e^{\mu}\cos\xi}$

$$\frac{-\frac{\partial\mu}{\partial x}}{\left(\frac{\partial\mu}{\partial x}\right)^2+\left(\frac{\partial\mu}{\partial y}\right)^2}=-\frac{(1+e^{\mu}\cos\xi)\,\cancel{(1+e^{2\mu}+2e^{\mu}\cos\xi)}}{\cancel{1+e^{2\mu}\cos 2\mu+2e^{\mu}\cos\xi}+}$$

$$=-(1+e^{\mu}\cos\xi)=\frac{\partial^2u}{\partial\mu^2}+\frac{\partial^2u}{\partial\xi^2}$$

$u=\quad\mu e^{\mu}\cos\xi$

$$u=-\left(\frac{\mu^2}{2}+\mu e^{\mu}\cos\xi\right)=-\frac{\mu x}{2}$$

$+\xi$

$$\frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial \xi^2} = \frac{\sin h\mu \cos\xi \left(\sin^4 \xi - \sin^4 h\mu\right)}{\sin^2 h\mu \cos^2\xi \left[-\sin^2 h\mu + \sin^2\xi\right]^2 + \cos^2 h\mu \sin^2\xi \left[\sin^2 h\mu + \sin^2\xi\right]^2}$$

$$= \frac{\sin h\mu \cos\xi \left[\sin^4\xi - \sin^4 h\mu\right]}{\cancel{\sin^4 h\mu}\ \sin^4\xi\left[\sin^2\xi + \sin^2 h\mu\left(1+2\cos^2 h\mu\right)\right] + \sin^4 h\mu\left[\sin^2 h\mu + \sin^2\xi\left(1-2\cos^2\xi\right)\right]}$$

$$\frac{x^2}{\cos^2 h\mu} + \frac{y^2}{\cos^2 h\mu - 1} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2-1} = 1$$

$$a^2 = \frac{1+}{2} \pm$$

$$1 \pm a^2$$

$$x^2(\cos^2 h\mu - 1) + y^2(\cos^2 h\mu) = \cos^4 h\mu - \cos^2 h\mu$$

$$\cos^4 h\mu - \cos^2 h\mu\left[1+x^2+y^2\right] = -x^2$$

$$\cos^2 h\mu = \frac{1+x^2+y^2}{2} \pm \sqrt{\left(\frac{1+x^2+y^2}{2}\right)^2 - x^2}$$

$$\frac{x^2}{\cos^2\xi} - \frac{y^2}{1-\cos^2\xi} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{1-a^2} = 1$$

$$-x^2\left(1-\cos^2\xi\right) + y^2\cos^2\xi = -\cos^2\xi + \cos^4\xi$$

$$\frac{1+a^2}{2} - \frac{1-a^2}{2}$$

$$\cos^4\xi - \cos^2\xi\left[1+y^2+x^2\right] = -x^2$$

$$\cos^2\xi = \frac{1+y^2+x^2}{2} \pm \sqrt{}$$

$$\cos^2 h\mu - \cos^2\xi = 2\sqrt{}$$

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$$x+iy = \cosh(\mu + i\xi) = \frac{e^{\mu - i\xi} + e^{-\mu + i\xi}}{2} = \ldots$$

$$x = \cosh\mu \cos\xi = \frac{e^{\mu}+e^{-\mu}}{2}\cos\xi$$

$$y = \sinh\mu \sin\xi = \frac{e^{\mu}-e^{-\mu}}{2}\sin\xi$$

$$\frac{x^2}{\cosh^2\mu} + \frac{y^2}{\sinh^2\mu} = 1$$

$$\sinh^2\mu = y^2 + x^2 \operatorname{tgh}^2\mu$$

$$2 \sinh\cosh\mu \cdot \frac{\partial\mu}{\partial x} = 2x \operatorname{tgh}^2\mu + 2x^2 \frac{\frac{\partial\mu}{\partial x}}{\cosh^2\mu}\operatorname{tgh}\mu$$

$$\frac{\partial\mu}{\partial x}\left[\sinh 2\mu - \frac{2\operatorname{tgh}\mu}{\cos^2\xi}\right] = 2\cosh\mu\cos\xi \operatorname{tgh}^2\mu$$

$$\frac{\partial\mu}{\partial x} = \frac{2\sinh\mu \operatorname{tgh}\mu \cos\xi}{\sinh\mu\cosh\mu - \cos^2\xi \cdot \operatorname{tgh}\mu}$$

$$= \frac{4\sinh\mu \cdot \cos\xi}{\cos^2 h\mu - \cos^2\xi}$$

$$\sinh^2\mu - \cosh^2\mu = -1$$

$$\cosh^2\mu = 1 + \sinh^2\mu$$

$$x^2 + y^2 \operatorname{ctg}^2 h\mu = \cos^2 h\mu$$

$$2y \operatorname{ctg}^2 h\mu - 2y^2 \frac{\operatorname{ctg}h\mu}{\sin^2 h\mu}\frac{\partial\mu}{\partial y} = 2\cos h\mu \sin h\mu \frac{\partial\mu}{\partial y}$$

$$\frac{\partial\mu}{\partial y} = \frac{\cancel{\sinh\mu}\sin\xi \operatorname{ctg}^2 h\mu \cancel{\cosh\mu}}{-\sinh\mu\cosh\mu + \cancel{\sinh\mu}\sin^2\xi \operatorname{ctg}h\mu} = \frac{-\cos h\mu \sin\xi}{\sin^2 h\mu + \sin^2\xi}$$

$$\frac{\left(\frac{\partial\mu}{\partial x}\right)^2 + \left(\frac{\partial\mu}{\partial y}\right)^2}{\frac{\partial\mu}{\partial x}} = \frac{\sin^2 h\mu \cos^2\xi\left[\sin^2 h\mu + \sin^2\xi\right]^2 + \cos^2 h\mu \sin^2\xi\left[\cos^2 h\mu - \cos^2\xi\right]^2}{\sinh\mu\cos\xi \cdot (\sin^2 h\mu + \sin^2\xi)^2(\cos^2 h\mu - \cos^2\xi)}$$

$$= \cos^2\xi \sin^6 h\mu + \sin^2\xi \cos^6 h\mu + 2\sin^2\xi\cos^2\xi\left[\cancel{\sin^2 h\mu + \sinh\mu}\right] + \cos^2\xi\sin^2\xi\left[\sin^2 h\mu \sin^2\xi + \cos^2 h\mu \cos^2\xi\right]$$
$- \ldots$

$$x+iy = e^{\xi}(\cos\varphi + i\sin\varphi) = e^{\xi+i\varphi}$$

$$\varphi = \theta = \operatorname{arctg}\frac{y}{x}$$

$$\xi = \lg r \qquad r = e^{\xi}$$

$$u = \frac{1}{4}\left(y \lg r^{2u} - x\theta\right) \qquad = \frac{1}{4}\left[\xi e^{\xi}\sin\varphi - \varphi e^{\xi}\cos\varphi\right]$$

$$v = \frac{1}{4}\left(-x\lg r + y\left(\frac{\pi}{2}-\theta\right)\right)$$

$$\frac{\partial u}{\partial\varphi} = \frac{1}{4}\left[\xi e^{\xi}\cos\varphi - e^{\xi}\cos\varphi + e^{\xi}\varphi\sin\varphi\right]$$

$$\frac{\partial^2 u}{\partial\varphi^2} = \frac{1}{4}\left[-\xi e^{\xi}\sin\varphi + 2e^{\xi}\sin\varphi + \cancel{\ldots} + e^{\xi}\varphi\cos\varphi\right]$$

$$\frac{\partial u}{\partial\xi} = \frac{1}{4}\left[e^{\xi}\sin\varphi + \xi e^{\xi}\sin\varphi - e^{\xi}\varphi\cos\varphi\right]$$

$$\frac{\partial^2 u}{\partial\xi^2} = \frac{1}{4}\left[\xi e^{\xi}\sin\varphi + 2e^{\xi}\sin\varphi - e^{\xi}\varphi\cos\varphi\right]$$

$$\Delta^2 u = e^{\xi}\sin\varphi = y$$

$$\frac{\partial\varphi}{\partial x} = -\frac{y}{x^2+y^2} \qquad \frac{\partial\varphi}{\partial y} = \frac{x}{x^2+y^2} \qquad \frac{\frac{\partial\varphi}{\partial x}}{\left(\frac{\partial\varphi}{\partial x}\right)^2 + (\ )^2} = \frac{-\frac{y}{x^2+y^2}}{\frac{y^2+x^2}{(x^2+y^2)^2}} = -y$$

Zawsze można stawić $\mu = 1$

$$\cos^2\xi\sin^6 h\varphi + \sin^2\xi\cos^6 h\varphi + \cos^2\xi\sin^2\xi\left[\sin^2\xi\,(\cos^2 h\varphi - 1) + \cos^2 h\varphi\sin^2\xi\ \cancel{\ldots}\right]$$

$$\cos^2 h\varphi - \sin^2\xi\ \cancel{\ldots}\ \cancel{+2\sin^2 h\varphi}\ \cancel{+2\cos^2 h\varphi}$$

$$\cos^2\xi\sin^2\xi\left[-\xi\sin^2 h\varphi + \cancel{\ldots}\cos^2 h\varphi\ \cancel{+\sin^2 h\varphi} + \cos^2\xi\,(\underbrace{\cos^2 h\varphi\ \cancel{+\sin^2 h\varphi}}_{=1})\right] - \cos^2 h\varphi - 2\sin^2 h\varphi - \sin^2\xi$$

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Streamlines:

$$\frac{v}{u} = \frac{dy}{dx} \qquad u\,dy - v\,dx = 0$$

$$\frac{\partial\varphi}{\partial x}dy - \frac{\partial\varphi}{\partial y}dx - \underbrace{\left(\frac{\partial\psi}{\partial y}dy + \frac{\partial\psi}{\partial x}dx\right)}_{d\psi} = 0$$

$$x = f(\mu, \xi) \qquad y = f.(\mu, \xi)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \mu}\frac{\partial \mu}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \mu\partial x}\frac{\partial \mu}{\partial x} + \frac{\partial u}{\partial \mu}\frac{\partial^2 \mu}{\partial x^2} + \frac{\partial^2 u}{\partial \xi\partial x}\frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2}$$

$$= \frac{\partial^2 u}{\partial \mu^2}\left(\frac{\partial \mu}{\partial x}\right)^2 + 2\frac{\partial^2 u}{\partial \xi\partial \mu}\left(\frac{\partial \mu}{\partial x}\right)\left(\frac{\partial \xi}{\partial x}\right) + \frac{\partial^2 u}{\partial \xi^2}\left(\frac{\partial \xi}{\partial x}\right)^2 + \frac{\partial u}{\partial \mu}\frac{\partial^2 \mu}{\partial x^2} + \frac{\partial u}{\partial \xi}\frac{\partial^2 \xi}{\partial x^2}$$

$$\Delta^2 u = \frac{\partial^2 u}{\partial \mu^2}\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right] + 2\frac{\partial^2 u}{\partial \mu\partial \xi}\left(\frac{\partial \mu}{\partial x}\frac{\partial \xi}{\partial x} + \frac{\partial \mu}{\partial y}\frac{\partial \xi}{\partial y}\right) + \frac{\partial^2 u}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right]$$

$$+ \frac{\partial u}{\partial \mu}\cancel{\left[\frac{\partial^2 \mu}{\partial x^2} + \frac{\partial^2 \mu}{\partial y^2}\right]} + \frac{\partial u}{\partial \xi}\cancel{\left[\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right]}$$

$$= \underbrace{\cancel{\frac{\partial^2 u}{\partial \mu^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right]\left[\mu^2 + 1\right]}} + 2\frac{\partial^2 u}{\partial \mu\partial \xi}\cancel{\left[-\mu\frac{\partial \xi}{\partial y}\frac{\partial \xi}{\partial x} + \mu\frac{\partial \xi}{\partial x}\frac{\partial \xi}{\partial y}\right]}$$

$$= \left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right]\left[\mu^2\frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial \xi^2}\right] = \left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right]\left[\frac{\partial^2 u}{\partial \mu^2} + \frac{1}{\mu^2}\frac{\partial^2 u}{\partial \xi^2}\right] = -\frac{\partial \xi}{\partial y}$$

$$\frac{\partial^2 u}{\partial (\mu)^2} + \frac{\partial^2 u}{\partial \xi^2} = \frac{-\frac{\partial \xi}{\partial y}}{\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2}$$

$$\frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial (\mu\xi)^2} = \frac{1 + \frac{\partial \mu}{\partial x}}{\mu\left[\left(\frac{\partial \mu}{\partial x}\right)^2 + \left(\frac{\partial \mu}{\partial y}\right)^2\right]} = \text{[illegible]}$$

$$x^2 - y^2 = \mu$$
$$2xy = \xi$$

$$x^2 - \left(\frac{\xi}{2x}\right)^2 = \mu$$

$$\cancel{x^4} - \mu x^2 = \frac{\xi^2}{4}$$

$$x = \sqrt{\frac{\mu}{2} + \sqrt{\frac{\xi^2}{4} + \frac{\mu^2}{4}}}$$

$$y = \sqrt{-\frac{\mu}{2} + \sqrt{\frac{\xi^2}{4} + \frac{\mu^2}{4}}}$$

$$\frac{\partial \mu}{\partial x} = 2x; \quad \frac{\partial \mu}{\partial y} = -2y;$$

$$\frac{\partial^2 u}{\partial \mu^2} + \frac{\partial^2 u}{\partial (\mu\xi)^2} = 4(x^2+y^2) = 2\sqrt{\mu^2 + \xi^2}$$

$$u = -\frac{r}{2}\left[\theta\sin\theta + \frac{\cos\theta}{2}\right] + r\,\frac{(\frac{\pi}{2}-1)\cos\theta + (\frac{\pi}{2}+1)\sin\theta}{4\pi}\left(\frac{\pi}{2}+1\right)$$

$$v = \frac{r}{2}\left[\theta\cos\theta - \frac{\sin\theta}{2}\right] + r\,\frac{(\frac{\pi}{2}-1)\cos\theta + (\frac{\pi}{2}+1)\sin\theta}{4\pi}\left(\frac{\pi}{2}-1\right)$$

r = ploz r

$$u = r\cos\theta\left[\frac{\frac{\pi^2}{4}-1}{4\pi} - \frac{1}{4}\right] + r\sin\theta\left[\frac{(\frac{\pi}{2}+1)^2}{4\pi} - \frac{\theta}{2}\right]$$

$$v = r\cos\theta\left[\frac{(\frac{\pi}{2}-1)^2}{4\pi} + \frac{\theta}{2}\right] + r\sin\theta\left[\frac{\frac{\pi^2}{4}-1}{4\pi} - \frac{1}{4}\right]$$

$\theta = 0:\quad u = r\,\frac{\frac{\pi^2}{4}-1-\pi}{4\pi} \qquad v = r\,\frac{(\frac{\pi}{2}-1)^2}{4\pi}$

$\theta = \frac{\pi}{4}\quad u = 0,\ v = 0$

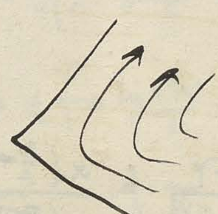
$\cancel{\theta = \frac{3\pi}{4}\quad u = -\frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}-1-\pi}{4\pi} + \frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}+\pi+1-\frac{3\pi^2}{2}}{4\pi} = \frac{\sqrt{2}}{2}\ 2\pi+2-}$

$\theta = -\frac{\pi}{4}\quad u = +\frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}-1-\pi}{4\pi} - \frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}+\pi+1+\frac{\pi^2}{2}}{4\pi} = -\frac{\sqrt{2}}{4\pi}\left(\frac{\pi^2}{4}+\pi+1\right)$

$v = \frac{\sqrt{2}}{2}\,\frac{\frac{\pi^2}{4}-\pi+1+\frac{\pi^2}{2}}{4\pi} - \frac{\sqrt{2}}{2}\left[\frac{\frac{\pi^2}{4}-1-\pi}{4\pi}\right]$

Mámdy v podlaze zpět superponovat takové posuvné úhly tak že dle

$\theta = -\frac{\pi}{4}$ bylo stejné.



$$v = \frac{x}{2}\,\mathrm{arctg}\,\frac{y}{x}$$

$$u = -\frac{y}{2}\,\mathrm{arctg}\,\frac{y}{x} + \frac{\pi}{4}\,y$$

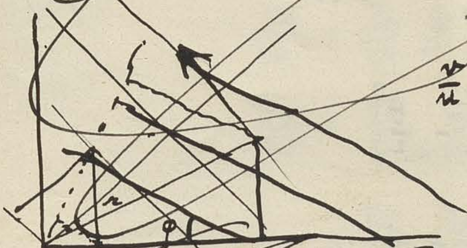
$$p = u \log r$$

$$= \frac{r}{2}\,\theta\cos\theta$$

$$= \frac{r}{2}\left[\frac{\pi}{2}\sin\theta - \theta\sin\theta\right] = \frac{r}{2}\sin\theta\left[\frac{\pi}{2} - \theta\right]$$

$\theta = 0$ $\quad v = 0$ $\quad u = 0$

$\theta = \frac{\pi}{2}$ $\quad v = 0$ $\quad u = 0$



$$\frac{v}{u} = \frac{\theta}{\frac{\pi}{2} - \theta}\,\mathrm{ctg}\,\theta$$

$$v = \frac{r}{2}\left[\theta(\cos\theta + \sin\theta) - \frac{(\sin\theta - \cos\theta)}{2}\right]$$

$$u = \frac{r}{2}\left[-\theta(\cos\theta + \sin\theta) + \frac{(\cos\theta - \sin\theta)}{2}\right]$$

$\theta = 0:$ $\quad v = \frac{r}{4}$ $\quad u = -\frac{r}{4}$

$\theta = \frac{\pi}{2}:$ $\quad v = \frac{r(\pi - 1)}{4}$ $\quad u = -\frac{r(\pi-1)}{4}$

$$\left(\frac{\pi}{2}+1\right)^2 - \left(\frac{\pi}{2}-1\right)^2 = \frac{1}{2}$$

$y = 0$

$x = y \qquad \theta = \frac{\pi}{4}:$

$$u = \frac{1}{4}\left[y(\log y + \log 2) - y\frac{\pi}{4}\right]$$

$$v = \frac{1}{4}\left[-y(\log y + \log 2) + y\frac{\pi}{4}\right]$$

$u = -v$

$$\cancel{u = -\frac{\sqrt{2}}{8}\left(\frac{\pi}{2}-1\right)\left(x + y\right)\frac{\sqrt{2}}{2}}$$

$$\mathrm{tg}\,\varphi = \frac{\frac{\pi}{2} - 1}{\frac{\pi}{2} + 1}$$

$$u = \frac{\sqrt{2}}{8}\left(\frac{\pi}{2} - 1\right)$$

$$\cos\varphi = \frac{\frac{\pi}{2}+1}{\sqrt{\frac{\pi^2}{4} + 1 - \pi + \frac{\pi^2}{4} + 1 + \pi}} = \frac{\frac{\pi}{2}+1}{\sqrt{2}\sqrt{\frac{\pi^2}{4}+1}}$$

$$\sin\varphi = \frac{\frac{\pi}{2} - 1}{\sqrt{2}\sqrt{\frac{\pi^2}{4}+1}}$$

$$v = -\frac{x\left(\frac{\pi}{2}-1\right) + y\left(\frac{\pi}{2}+1\right)}{4\pi}\left(\frac{\pi}{2}-1\right)$$

$$u = \frac{x\sin\varphi + y\sin\varphi}{\cos\left(\frac{\pi}{4}-\varphi\right)r} = \frac{x\cos\varphi + y\sin\varphi}{(\cos\varphi + \sin\varphi)\frac{\sqrt{2}}{2}} = \frac{x\left(\frac{\pi}{2}-1\right) + y\left(\frac{\pi}{2}+1\right)}{\pi\,\frac{\sqrt{2}}{2}}$$

$$u = \frac{x\left(\frac{\pi}{2}-1\right) + y\left(\frac{\pi}{2}+1\right)}{4\pi}\left(\frac{\pi}{2}+1\right)$$

$$r = \lg z$$

$$p = \lg z$$

$$\zeta = \theta = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$$

$$\psi = \frac{r^2\theta}{4}$$

$$\psi = \frac{(x^2+y^2)}{4} \operatorname{arctg}\frac{y}{x}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{x}{2}\operatorname{arctg}\frac{y}{x} - \frac{y}{x^2}\cancel{\frac{x^2+y^2}{4}\frac{x}{1+\frac{y^2}{x^2}}} = \frac{r}{2}\left[\theta\cos\theta - \frac{\sin\theta}{2}\right]$$

$$+u = -\frac{\partial\psi}{\partial y} = -\frac{y}{2}\operatorname{arctg}\frac{y}{x} \pm \cancel{\frac{x^2+y^2}{4x}\frac{1}{1+\frac{y^2}{x^2}}} = -\frac{r}{2}\left[\theta\sin\theta + \frac{\cos\theta}{2}\right]$$

$\theta = 0 \quad v = 0$, $\quad +u = -\frac{r}{4}$

$\theta = \frac{\pi}{2}: \quad v = -\frac{r}{4}$, $\quad +u = -r\frac{\pi}{4}$

$\theta = \frac{\pi}{4}: \quad v = \frac{r}{4}\frac{\sqrt{2}}{2}\left(\frac{\pi}{2} - 1\right)$, $\quad u = -\frac{r}{4}\frac{\sqrt{2}}{2}\left(\frac{\pi}{2}+1\right)$

$$\frac{\partial\psi}{\partial x} = \cancel{\frac{\partial\psi}{\partial r}\frac{x}{r} + \frac{\partial\psi}{\partial\theta}\frac{y}{1+\frac{y^2}{x^2}}}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{1}{r}\frac{\partial\psi}{\partial r}$$

$$p = \theta$$

$$\zeta = \lg z = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$$

$$-\frac{\partial\zeta}{\partial y} = \frac{y}{x^2+y^2} = \Delta^2 u$$

$$+\frac{\partial\zeta}{\partial x} = \frac{-x}{x^2+y^2} = \Delta^2 v$$

$$\psi = \frac{1}{2}\int dx \int \lg(x^2+y^2)\, dx + \int dy \int$$

$$u = -\frac{1}{4}\left\{y \lg\frac{(x^2+y^2)}{2} - x \operatorname{arctg}\frac{y}{x}\right\} +$$

$$v = \frac{1}{4}\left\{-x \lg\frac{x^2+y^2}{2} + y \operatorname{arctg}\frac{x}{y}\right\} - \frac{\pi}{8}y$$

$x^2+y^2=1:$

$$\left.\begin{aligned} u &= -\frac{1}{4}\theta\cos\theta \\ v &= -\frac{1}{4}(\theta)\sin\theta \end{aligned}\right\} V = -\frac{1}{4}\cdot\theta$$

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$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2} = \frac{A}{4}\left\{\frac{6y}{x^2+y^2} - \cancel{4}\,2y\,\frac{\cancel{(x^2+y^2)}}{(x^2+y^2)^2}\right\} = \frac{Ay}{x^2+y^2} \quad !$$

$$u = \frac{\partial \Phi_1}{\partial x} - \frac{\partial \Phi_2}{\partial y} + \frac{A}{4}\left\{\log\frac{x^2+y^2}{2} + \frac{y^2-x^2}{y^2+x^2}\right\} \qquad \frac{A}{4}\left[\log r - \cos 2\theta\right]$$

$$v = \frac{\partial \Phi_1}{\partial y} + \frac{\partial \Phi_2}{\partial x} - \frac{A}{4}\left\{\frac{2xy}{x^2+y^2} - \operatorname{arctg}\frac{y}{x}\right\} \qquad -\frac{A}{4}\left[\sin 2\theta - \theta\right]$$

$1 - \frac{2x^2}{y^2+x^2}$

$\theta = 0 \quad u = \log r - 1 \qquad \theta = \frac{\pi}{4} \quad u = \log r - \cancel{\theta}$

$v = 0 \qquad v = 1 - \frac{\pi}{4}$

$\frac{2y^2}{y^2+x^2} - 1$

$\theta = \frac{\pi}{2} \quad u = \log r + 1$

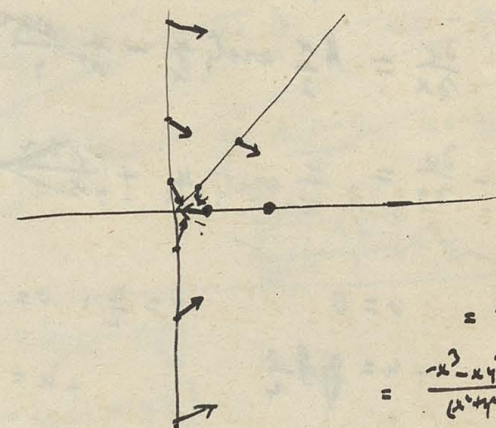
$v = \frac{\pi}{2}$

$\cancel{\theta = \pi} \quad \cancel{u = \log r - 1}$

$\cancel{v = -\pi}$

$\theta = -\frac{\pi}{2} \quad u = \log r + 1$

$v = +\frac{\pi}{2}$



$= x^3 - 3xy^2$

$= \frac{-x^3 - xy^2 - 2xy^2 + 2x^3}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial y} = -\frac{2x}{x^2+y^2} + \frac{4xy^3}{(x^2+y^2)^2} + \quad \Big| \quad \frac{\partial u}{\partial x} = \frac{-x}{x^2+y^2} - \cancel{} - \frac{2x(y^2-x^2)}{(x^2+y^2)^2}$

$= 4$

$\frac{dy}{dx} = \frac{v}{u}$

$$\left.\begin{aligned}\frac{\partial v}{\partial x} &= -\frac{2y}{x^2+y^2} + \frac{4x^2y}{(x^2+y^2)^2} - \cancel{} \\ \frac{\partial u}{\partial y} &= \frac{y}{x^2+y^2} + \frac{2y}{y^2+x^2} - \frac{2y(y^2-x^2)}{(y^2+x^2)^2}\end{aligned}\right\} \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \cancel{} + \frac{4x^2y + 2y^3 - 2x^2y - 6x^2y - 6y^3}{(y^2+x^2)^2}$$

$= \cancel{4y^3 \; 4x^2y + 4x^2y + 2y^3 - 2x^2y}$

$3y^3 + 3yx^2 - 2y^3 + 2yx^2 = y^3 + yx^2 = \frac{y}{x^2+y^2}$

$= \cancel{} \; \frac{1}{4}\,\frac{4y}{y^2+x^2}$

$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} =$

$$+\frac{A}{2i}\left[\frac{1}{u}-\frac{1}{v}\right] = 4\frac{\partial^2\psi}{\partial u\partial v}$$

$$\psi = u \log v - v \log u$$

$$\frac{\partial\psi}{\partial u} = \log v - \frac{v}{u} \qquad \frac{\partial\psi}{\partial v} = \frac{u}{v} - \log u$$

$$\frac{\partial^2\psi}{\partial u\partial v} = \frac{1}{v} - \frac{1}{u} \qquad \frac{\partial^2\psi}{\partial u\partial v} = \frac{1}{v} - \frac{1}{u}$$

$$-\frac{A}{8i}\left[\frac{1}{v}-\frac{1}{u}\right] = \frac{\partial^2\psi}{\partial u\partial v}$$

$$\psi = -\frac{A}{8i}(u\log v - v\log u) + f_1(u) + f_2(v)$$

$$= -\frac{A}{8}\left[\frac{(x+iy)\log(x-iy) - (x-iy)\log(x+iy)}{i}\right]$$

$$x+iy = re^{i\theta} \qquad \log = \log r + i\theta$$

$$x-iy = re^{-i\theta} \qquad \log = \log r - i\theta$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$\psi = -\frac{A}{8}\left\{\frac{[\log r - i\theta]re^{i\theta} - [\log r + i\theta]re^{-i\theta}}{i}\right\}$$

$$= -\frac{A}{8}\left\{r\log r\,\frac{e^{i\theta}-e^{-i\theta}}{i} - \theta\, r(e^{i\theta}+e^{-i\theta})\right\}$$

$$= -\frac{A}{4}\left\{r\log r\sin\theta - r\theta\cos\theta\right\}$$

$$\psi = -\frac{A}{4}\left\{+y\frac{\log(x^2+y^2)}{2} - x\,\mathrm{arctg}\frac{y}{x}\right\}$$

Prova: $\frac{\partial\psi}{\partial x} = -\frac{A}{4}\left\{\cancel{\frac{2xy}{x^2+y^2}} - \mathrm{arctg}\frac{y}{x} + \cancel{\frac{x\,y}{x^2}}\right\}$ $\qquad \frac{2xy}{x^2+y^2} - \mathrm{arctg}\frac{y}{x} + \frac{yx}{1+\frac{y^2}{x^2}}$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{A}{4}\left\{\cancel{\frac{2y}{x^2+y^2}} - \cancel{\frac{4x^2y}{(x^2+y^2)^2}} + \cancel{\frac{y}{x^2}}\right\}$$

$$\frac{\partial\psi}{\partial y} = -\frac{A}{4}\left\{+\log\frac{x^2+y^2}{2} + \frac{y^2}{x^2+y^2} - \frac{x^2}{x^2+y^2}\right\}$$

$$\frac{\partial^2\psi}{\partial y^2} = -\frac{A}{4}\left\{+\frac{y}{x^2+y^2} + \cancel{\frac{2y^3}{(x^2+y^2)^2}} + \frac{2x^2y}{(x^2+y^2)^2}\right\}$$

18v

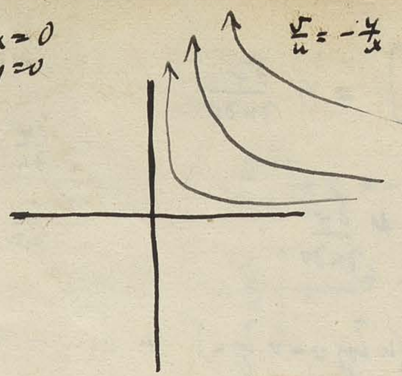
u i v znikają równocześnie dla $x=0$ $y=0$

$$u = -\frac{A}{6}(3xy^2+x^3)$$

$$v = \frac{A}{6}(y^3+3x^2y)$$

$$p = p_0 + \mu A(x^2-y^2)$$

$$\frac{v}{u} = -\frac{y}{x}\,\frac{y^2+3x^2}{3y^2+x^2}$$



$$p = 2Axy$$

$$\zeta = A(x^2-y^2) = \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} \qquad \psi = \frac{A}{12}(x^4-y^4)+\phi$$

$$u = \frac{Ay^3}{3} +$$

$$v = \frac{Ax^3}{3} -$$

$$p^2+\zeta^2 = (x^2+y^2)^2$$

$$x^2+y^2 = \frac{1}{\sqrt{p^2+\zeta^2}}$$

$$f = \frac{A}{z} = \frac{A}{r}(\cos\theta - i\sin\theta)$$

$$\int \log(z^3)\,dz$$

$$\frac{\partial\psi}{\partial y} = -\frac{2yx}{(x^2+y^2)^2} = \frac{2p\zeta}{\cancel{(\ )}}$$

$$\frac{\partial\psi}{\partial x} = \frac{1}{x^2+y^2} - \frac{2x^2}{(\ )^2} = \frac{y^2-x^2}{(x^2+y^2)^2} = (\zeta^2-p^2) \qquad 2x\,\zeta y z^2 \qquad 2y\,\zeta z^2$$

$$p = A\frac{\cos\theta}{r} = \frac{Ax}{x^2+y^2}$$

$$\zeta = -A\frac{\sin\theta}{r} = -\frac{Ay}{x^2+y^2} = \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} = +\frac{A}{2i}\left[\frac{+1}{x+iy} + \frac{-1}{x-iy}\right]$$

$$\cancel{\int \frac{2y}{z}\,dz} \qquad \cancel{\int 2yz^2\,dz} \qquad \int \cancel{2y z\,dz}$$

$$\cancel{\frac{1}{x+iy}\ 2\ \frac{1}{x+iy}\ 2\ \frac{1}{x-iy}\ 2\ \frac{1}{x-iy}}$$

$$\cancel{\frac{y}{2}\,\zeta y z^2} \qquad \cancel{\frac{x}{2}\,\zeta y z^2} \qquad \cancel{\frac{x}{2}\,\zeta y z^2}$$

$$x+iy = u \qquad x = \frac{u+v}{2}$$

$$x-iy = v \qquad y = \frac{u-v}{2}$$

$$\frac{\partial\psi}{\partial x} = \left(\frac{\partial\psi}{\partial u}+\frac{\partial\psi}{\partial v}\right) \qquad \frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi}{\partial u^2}+2\frac{\partial^2\psi}{\partial u\partial v}+\frac{\partial^2\psi}{\partial v^2}$$

$$\frac{\partial\psi}{\partial y} = i\left(\frac{\partial\psi}{\partial u}-\frac{\partial\psi}{\partial v}\right) \qquad \frac{\partial^2\psi}{\partial y^2} = -\left(\frac{\partial^2\psi}{\partial u^2}-2\frac{\partial^2\psi}{\partial u\partial v}+\frac{\partial^2\psi}{\partial v^2}\right)$$

$$d\psi = \frac{\partial\psi}{\partial x}dx+\frac{\partial\psi}{\partial y}dy = \frac{\partial\psi}{\partial u}du+\frac{\partial\psi}{\partial v}dv$$

$$= \left(\frac{\partial\psi}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial\psi}{\partial v}\frac{\partial v}{\partial x}\right)dx + \ldots$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} = 4\frac{\partial^2\psi}{\partial u\partial v}$$

Exempl. visq. slow

$f(x+iy) = \frac{p}{\mu} + i\zeta$

$\cos\varphi = \frac{x}{\cosh\psi}$ $\sin\varphi = \frac{y}{\cosh\psi}$

$\frac{x^2}{\cosh^2\psi} + \frac{y^2}{\sinh^2\psi} = c^2$

$\cosh = e^{\psi} + e^{-\psi}$

$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$p = \text{const}$ $\perp$
$\zeta = \text{const}$

$$\left.\begin{aligned}\frac{\partial p}{\partial x} &= \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = -\mu\frac{\partial \zeta}{\partial y}\\ \frac{\partial p}{\partial y} &= \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = +\mu\frac{\partial \zeta}{\partial x}\end{aligned}\right\}\quad \begin{aligned}\Delta^2 p &= 0\\ \Delta^2 \zeta &= 0\end{aligned}$$

$$u = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial y}$$

$$v = \frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 = \Delta^2\varphi = 0$$

$$-\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \zeta = \Delta^2\psi$$

$\int(u\,dy - v\,dx) =$

$\cancel{\Delta^2 u = -\frac{\partial\zeta}{\partial y}}$
$\cancel{\Delta^2 v = \frac{\partial\zeta}{\partial x}}$

Ex.: $f = Az^2$

$$p = Ar^2\cos 2\theta = A(x^2 - y^2)$$

$$\zeta = Ar^2\sin 2\theta = 2Axy$$

$$2Axy = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$$

$$\psi = \frac{1}{6}A(xy^3 + x^3y) + \Phi(x+iy)$$

$$\boxed{\begin{aligned}\frac{\partial\psi}{\partial x} &= \frac{1}{6}(Ay^3 + A3x^2y)\\ \frac{\partial^2\psi}{\partial x^2} &= Axy\end{aligned}}$$

$$u = \frac{\partial\Phi_1}{\partial x} - \frac{\partial\Phi_2}{\partial y} - \frac{A}{6}(3xy^2 + x^3)$$

$$v = \frac{\partial\Phi_1}{\partial y} + \frac{\partial\Phi_2}{\partial x} + \frac{A}{6}(y^3 + 3x^2y)$$

$\cancel{+\frac{\partial^2\Phi_1}{\partial x\partial y} + \frac{\partial^2\Phi_2}{\partial x\partial y} + \frac{\partial^2\Phi_2}{\partial y^2}}$

$\cancel{\frac{\partial v}{\partial x} = Axy}$ $\cancel{\frac{\partial u}{\partial y} = -Axy}$

$\cancel{\zeta = 2Axy}$

$-\frac{A}{2}(y^2 + x^2)$

$-\frac{1}{2}y^2x - \frac{1}{2}x^2$

$x\frac{1}{2}y^2$

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Supozycja: 6 ~~stała~~

$$\boxed{\cancel{V_i = \frac{4\pi(a^{4}+\delta)^2 - a^2}{\pi}} \; 6 + c}$$

$$\frac{4}{3}\pi a^3(\rho-\rho')g = 6\pi\mu a c$$

$$c = \frac{2}{9}\,\frac{a^2(\rho-\rho')g}{\mu}$$

$\rho-\rho' = 1$

$\mu = 0{\cdot}018$

$a = 1\mu = 10^{-4}$

$\delta = 1\mu = 10^{-4}$

$$c = \frac{2}{9}\,\frac{10^{-8}\cdot 980}{4\cdot 0{\cdot}018} = \frac{2\cdot 9{\cdot}8}{4\cdot 9\cdot 1{\cdot}8}\cdot 10^{-4}\ \frac{cm}{m} = 0{\cdot}3\cdot 10^{-4}\ cm$$

$$= \frac{1{\cdot}2}{4}\cdot 86400\cdot 10^{-4}\ \frac{cm}{d} = \frac{1}{4}\ \frac{10\,cm}{dzień}$$

Kolophonium – H_2O; $\rho = 1{\cdot}07$ $\quad c = \dfrac{0{\cdot}2\,cm}{dzień}$

$2r_1 = 10^{-7}$ cm (ein Loschm.)

$2r_2 = 10^{-4}$ cm

$$\frac{m_1}{m_2} = 10^{-9} \qquad \frac{R_1}{R_2} = 10^{9}$$

$h_1 = 10\,km = 10^{6}$ cm

$h_2 = 10^{-3}$ cm

aby to było widoczne, musiałoby być

$2r_2 = 10^{-5}\,cm = 0{\cdot}1\mu \qquad h_2 = 1\,cm$

albo jeśli $\rho = 1{\cdot}07$

$h_2 = 13$ cm

---

aż przy 10^{-6} cm!

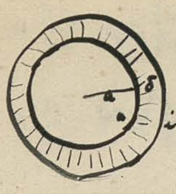
$$\text{Capp. } \overbrace{W_1 + W_2 - W_{12}}^{\Delta_C W} = \alpha\, 4\pi\left(2a^2 - a_{12}^2\right) = 4\pi a^2\alpha\left(2-\sqrt[3]{4}\right)$$

$\alpha_{water-air} = 80\,(CGS)$ $\qquad \varphi_0 - \varphi_i = 1\,Volt = \frac{1}{300}$ $\qquad \delta = 10^{-7}$ cm

$$\frac{\Delta_E W}{\Delta_C W} = \frac{\dfrac{(\varphi_0-\varphi_i)^2}{\delta}}{4\pi\alpha} = \frac{\left(\frac{1}{300}\right)^2\cdot 10^{7}}{4\pi\cdot 80} = \frac{10}{9\cdot 4\cdot \pi\cdot 0{\cdot}8} = \frac{1}{10}$$

~~ale tak~~ ale często mogą być oba wpływy równe jeśli α i emisyjna

Oliva – H_2O: 22



$V_a = \varphi_a$

$V_i = \varphi_i$

Supozycja: $\varphi_a - \varphi_i = \text{stała}$

(wynikałaby stąd supozycja że σ stała)

$$\left.\begin{aligned} \varphi_a &= \Phi\left(\frac{1}{a} - \frac{1}{a+\delta}\right) + c \\ \varphi_i &= \Phi\left(\frac{1}{r} - \frac{1}{r}\right) + c \end{aligned}\right\} \quad \varphi_a - \varphi_i = \Phi\left(\frac{1}{a} - \frac{1}{a+\delta}\right)$$

$$\Phi = \frac{(\varphi_a - \varphi_i)}{\frac{1}{a} - \frac{1}{a+\delta}} = (\varphi_a - \varphi_i)\,\frac{a(a+\delta)}{\delta}$$

$$\sigma_a = (\varphi_a - \varphi_i)\,\frac{\left(1+\frac{\delta}{a}\right)}{4\pi\delta}$$

$$\sigma_i = (\varphi_a - \varphi_i)\,\frac{\left(1+\frac{\delta}{a}\right)^{-1}}{4\pi\delta}$$

$$W = (V_a - V_i)\,\Phi = (\varphi_a - \varphi_i)^2\,\frac{a(a+\delta)}{\delta}$$

$$a_{12}^3 = 2a^3 \qquad a_{12} = a\sqrt[3]{2}$$

$$W_1 + W_2 - W_{12} = \frac{(\varphi_a - \varphi_i)^2}{\delta}\left[2(a^2 + a\delta) - \left(a^2\sqrt[3]{4} + a\delta\sqrt[3]{2}\right)\right]$$

$$\Delta_E W = (\varphi_a - \varphi_i)^2\,\frac{\left[a^2(2-\sqrt[3]{4}) + a\delta(2-\sqrt[3]{2})\right]}{\delta}$$

dodatnia w kierunku powiększania $\underline{a}$

$$\cancel{\Delta^2\varphi = \varepsilon = \frac{\partial^2\varphi}{}} \qquad \varphi = f(r) \qquad \frac{\partial\varphi}{\partial x} = f'\frac{x}{r} \qquad \frac{\partial^2\varphi}{\partial x^2} = \frac{f'}{r} - \frac{x^2}{r^3}f' + f''\frac{x^2}{r^2}$$

$$\Delta^2\varphi = \frac{2f'}{r} + f''$$

$$\varepsilon = -\frac{1}{4\pi}\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{2}{r}\frac{\partial\varphi}{\partial r}\right)$$

$$\int 4\pi(a+\xi)^2\,\varepsilon\,d\xi = -\int \underbrace{\left[r^2\frac{\partial^2\varphi}{\partial r^2} + 2r\frac{\partial\varphi}{\partial r}\right]}_{\frac{d}{dr}\left(r^2\frac{\partial\varphi}{\partial r}\right)} dr = r^2\frac{\partial\varphi}{\partial r}\Big|_{-\delta}^{+\delta} = 0$$

$$\left.\begin{aligned}\frac{\partial p}{\partial x} &= -\frac{\mu}{\rho}\frac{\partial \zeta}{\partial y}\\ \frac{\partial p}{\partial y} &= +\frac{\mu}{\rho}\frac{\partial \zeta}{\partial x}\end{aligned}\right\}\quad \begin{aligned}\Delta^2 p &= 0\\ \Delta^2 \zeta &= 0\end{aligned}$$

$$f(x+iy) = \varphi + i\psi$$

$$\Delta^2\varphi = 0 \qquad \Delta^2\psi = 0$$

$$-\frac{\frac{\partial\psi}{\partial x}}{\frac{\partial\psi}{\partial y}} = \frac{\frac{\partial\varphi}{\partial y}}{\frac{\partial\varphi}{\partial x}}$$

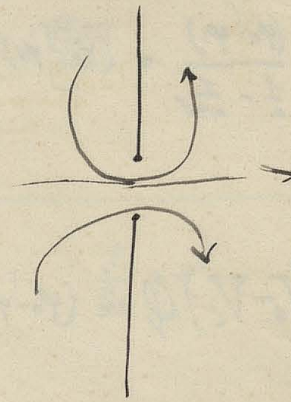
$$\cancel{\frac{\partial p}{\partial x}\frac{\partial \zeta}{\partial x} = -\frac{\mu}{\rho}\frac{\partial p}{\partial y}\frac{\partial \zeta}{\partial y}}$$

$$\frac{\partial p}{\partial x}\frac{\partial \zeta}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial \zeta}{\partial y} = 0$$

$$\underline{f(x+iy) = p + i\zeta}$$

$$\zeta = \psi(x,y) = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$$

$$\left.\begin{aligned}\frac{\partial \zeta}{\partial x} &= \frac{\partial^2 u}{\partial x\partial y} - \frac{\partial^2 v}{\partial x^2} = -\Delta^2 v\\ \frac{\partial \zeta}{\partial y} &= \qquad\qquad\qquad \Delta^2 u\end{aligned}\right\}$$



→ rotor przy ściankach $\quad \nabla^2\zeta = 0$

$$\zeta^2 = -\nabla^2\Omega$$

p, $= f_1(x, y^2)$ $\qquad p_\infty = \frac{c_1}{c_2}$ ----

$p(0, y^2) = \text{const}$

$\zeta = f_1(x^2, y)$ $\qquad \zeta(x, 0) = 0$

ζ

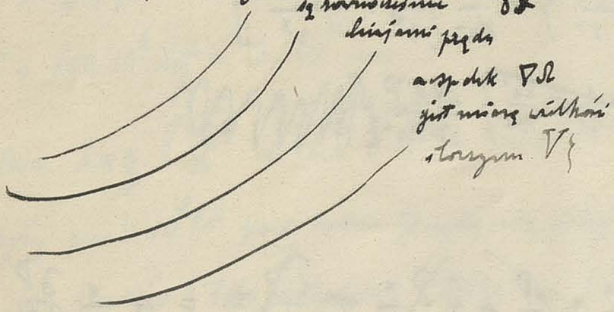
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + v\overbrace{\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)}^{\zeta}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial p}{\partial y} = u\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y}$$

$$\left.\begin{aligned} v\zeta &= \frac{\partial}{\partial x}\left(\frac{p}{\rho} - \frac{u^2+v^2}{2}\right) \\ u\zeta &= -\frac{\partial}{\partial y}\underbrace{\left(\frac{p}{\rho} - \frac{u^2+v^2}{2}\right)}_{\Omega} \end{aligned}\right\} \quad V\zeta = \left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right)(\Omega)$$

$$\Omega = \text{const}$$

$$\frac{v}{u} = -\frac{\frac{\partial}{\partial x}}{\frac{\partial}{\partial y}}$$



$$\frac{\partial \Omega}{\partial x}dx + \frac{\partial \Omega}{\partial y}dy = 0$$

$$\frac{dy}{dx} = -\frac{\frac{\partial \Omega}{\partial x}}{\frac{\partial \Omega}{\partial y}} = \frac{v}{u}$$

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \mu\left(-\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 u}{\partial y^2}\right) = \mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = \mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$v\zeta = \frac{\partial \Omega}{\partial x} + \mu\frac{\partial \zeta}{\partial y}$$

$$u\zeta = -\left[\frac{\partial \Omega}{\partial y} + \mu\frac{\partial \zeta}{\partial x}\right]$$

$$v\frac{\partial \zeta}{\partial y} + u\frac{\partial \zeta}{\partial x} = \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\zeta$$

$$v\frac{\partial \zeta}{\partial x} - u\frac{\partial \zeta}{\partial y} - \zeta^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Omega$$

$$\frac{v}{u} = \frac{\frac{\partial \Omega}{\partial x} + \mu\frac{\partial \zeta}{\partial y}}{-\frac{\partial \Omega}{\partial y} + \mu\frac{\partial \zeta}{\partial x}}$$

$$(v\nabla)\zeta = \mu\nabla^2\zeta$$

$$\Delta X - \frac{\partial F}{\partial x} = \overbrace{4\pi A^2\mu\lambda}^{a}\ \frac{\partial X}{\partial t} + \beta\frac{\partial^2 X}{\partial t^2})$$

$$X = P\frac{x}{r}\ ;\quad Y = P\frac{y}{r}\ ; \ldots$$

$$F = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 3\frac{P}{r} - \frac{P(x^2+y^2+z^2)}{r^3} + \frac{dP}{dr}\left(\frac{x^2}{r^2} + \ldots\right)$$

$$= \frac{2P}{r} + \frac{dP}{dr}$$

$$\frac{\partial X}{\partial x} = \frac{P}{r} - \frac{Px^2}{r^3} + \frac{dP}{dr}\frac{x^2}{r^2} \qquad + \frac{d^2P}{dr^2}\frac{x^3}{r^3}$$

$$\frac{\partial^2 X}{\partial x^2} = -\frac{3Px}{r^3} + \frac{3Px^3}{r^5} + \frac{dP}{dr}\left(-\frac{3x^2}{r^3}\right)\frac{x}{r} + \frac{dP}{dr}\frac{2x}{r^2}$$

$$\frac{\partial X}{\partial y} = -P\frac{xy}{r^3} + \frac{dP}{dr}\frac{xy}{r^2}$$

$$\frac{\partial^2 X}{\partial y^2} = -\frac{Px}{r^3} + 3\frac{Pxy^2}{r^5} - \frac{dP}{dr}\frac{xy^2}{r^4} + \frac{dP}{dr}\frac{x}{r^2} - 2\frac{dP}{dr}\frac{xy^2}{r^4} + \frac{d^2P}{dr^2}\frac{xy^2}{r^3}$$

$$\Delta^2 X = P\left\{-\frac{5x}{r^3} + \frac{3x^3 + 3xy^2 + 3xz^2}{r^5}\right\}$$

$$+ \frac{dP}{dr}\left\{\frac{x}{r^2} - \frac{3x^3}{r^4} + \frac{4x}{r^2} - \frac{3xy^2}{r^4} - \frac{3xz^2}{r^4}\right\} + \frac{d^2P}{dr^2}\left[\frac{x^3}{r^3} + \frac{x(y^2+z^2)}{r^3}\right]$$

$$= P\ -\frac{2x}{r^3} + \frac{dP}{dr}\left\{\cancel{\frac{1}{r} - \frac{3x^2}{r^3} + \frac{x}{r^2} + \frac{3x^3}{r^4}}\right\} + \frac{x}{r}\frac{d^2P}{dr^2}$$

$$+ \frac{2x}{r^2}\ \frac{dP}{dr}$$

$$\cancel{-\frac{2x}{r^3}P} + \cancel{\frac{2x}{r^2}\frac{dP}{dr}} + \cancel{\frac{x}{r}\frac{d^2P}{dr^2}} - \cancel{2\frac{x}{r^2}\frac{dP}{dr}} + \cancel{2P\frac{x}{r^3}} + \cancel{\frac{d^2P}{dr^2}\frac{x}{r}} = a\ \frac{x}{r}\frac{\partial P}{\partial t} + \beta\frac{x}{r}\frac{\partial^2 P}{\partial t^2} = 0$$

$$P = 1 + P_0\, e^{-\frac{4\pi\lambda}{\varepsilon}t}$$

$$P = \varphi(r) + \psi(r)\, e^{-\frac{4\pi\lambda}{\varepsilon}t}$$

$$\left.\begin{array}{lll} t=0: & P=0 & \varphi(r)+\psi(r)=0 \\ t=\infty: & P=0 & \varphi(r)=0 \end{array}\right\}\ P=0$$

1). Stati [illegible] || 1). Plattencondens. 2). [illegible] 3). [illegible] 2 Medien

Schutzring conden.

2). [illegible] $\ldots\, 2x$; [illegible]

$\Delta^2 V = 0$ $\Delta^2 \theta = 0$ $\Delta^2 \varrho = 0$

$$c\varrho \frac{\partial \theta}{\partial t} = \kappa \Delta^2 \theta \qquad \frac{\partial \varrho}{\partial t} = k \frac{\partial^2 \varrho}{\partial x^2}$$

Właściwie powinno się to nazywać przewodzeniem potencyału

$t = 86400 \cdot 365$

259200
51840
4320
$31,536.000$

$$\frac{x}{2\sqrt{kt}} = 1$$

$$x = 2\sqrt{kt}$$

$$= 2\sqrt{3 \cdot 10^{10}} = 3{,}4 \cdot 10^5 \text{ cm} = 3{,}4 \text{ km}$$

$T = 100 \cdot 10^6$ lat $= 3 \cdot 10^{15}$ sec

$$A = 2a\sqrt{\frac{kt}{\pi}} = 2a\sqrt{10^{10}} = 2a \cdot 10^5$$

Gdyby przed 10^8 lat pow. morza cząstka wody pokryta została, i gdyby woda bez stanu uwolniona (ale bez prądów) toby dotychczas tylko na 2 km głębokości została absolutnie wysycona

W 1 dniu: $A = 2\sqrt{\frac{\cancel{0.06}}{\pi}} = 1$ cm

Z tego pochodzi powolność wzrostu roślin, wogóle organizmów i długość życia.

Rozmieszczanie atmosfery i powietrze nad morzem nie jest absolutnie wysycone.

Meteoryty (stale w których niema prądów) powstają umniejszającej niezmiennie; metale obok siebie leżące mało się zmieniają.

Takie prawo Newtona daje się powolnego zastosow. do θ i do ϱ

$\frac{k}{c\varrho} = 0{,}015$ | $\sqrt{kt} = \sqrt{4{,}5 \cdot 10^{13}} = 7 \cdot 10^6$

k diff. $= 2{,}67 \cdot 10^{-5}$

$$\int^{0{,}477} = \frac{1}{2}$$

spadnie na $\frac{1}{2}$ wtedy gdy

$$\frac{x}{2\sqrt{kt}} = 0{,}477$$

1 cm

Ayrton & Perry Wied. I 574, Pr. R. S. 27 p. 219 (1878)

mult. × 10^4 / 2 = Hg = 1 / Sod

Paraffin:	46°	Widerstand = 34.000 . 10^{13} Ω
	50°	1000 . 10^{12}
	70°	10 . 10^{12}
	77.8	1.35 . 10^{12}
Guttapercha:	24°	83 . 10^{12}
	44°	8.9 . 10^{12}
	83°	0.5 . 10^{12}
Ebonit:	36°	61.000 . 10^{12}
	96.8°	9700 . 10^{12}

Kohlrausch H_2O 21.5° $\lambda = 0.71$ Hg $= 10^{10}$

Pogg. Erg. B. 8 p. 1 (1876)

Kohlr. 24 p. 48 (1885) ? Vacuum dest. $\lambda = 0.25$ Hg $= 10^{10}$

abs. Äther ~~Kohlrausch~~ Pfeiffer Wied. Ann. 26 $\ll 0.003 \cdot 10^{-10}$ (Hg = 1)

Benoist Heute Wied. Ann. 20 p. 280 (1883)

[illegible] Isolator

Heydweiller Wied. Ann. 69 p. 531 Glassorte: 2.10^{-4} C.G.S. cm.

$= 2.10^{-26}$ C.G.S. em.

Luft > 5 mm Hg. → p6 < 5 C.G.S. cm. $\lambda < \lambda_{Glas}$

=

5 – 0.1 ……

? ? [illegible] …… $\lambda = 10^{-1}$ cm.

10^{-23} em.

$V \frac{2\pi\lambda l \cdot 10^{-18}}{1} = \frac{1}{30} V \frac{r\pi}{l} \cdot 60$ r = 1 mm

$l^2 = 6.10^{17} \frac{r}{2} = 3.10^{17}$

$l = 10^8 \sqrt{30} = 5.5 . 10^8$ m

$= 5.5 . 10^{2}$ km

= 5500 km

$l^2 = \frac{1}{2 \cdot 0.00046} = 10^3$

$l = 31$ m = 3.1 cm

Analogja i różnice stałych przewodn. elektrycznych, cieplnych i dyfuzyi.

$\Delta^2\varphi = 0$ – najczęściej tak bardzo często w fizyce, oczywiście wszędzie pewna analogja

ale czasem tylko ~~przypadkowa~~ prawie n.p. fale akustyczne, pole elektrostatyczne

i ruch cieczy potenc.

Czy istnieją analogie rzeczywiste i prawne?

Ściśle biorąc, tylko różnice jakościowe, bo właściwie wszystko w fizyce teoret. jest tylko analogją; ale gdy tylko mało cech wspólnych to żadnej korzyści, i odwrotnie przykładem

Bardzo wielka analogja istnieje między owemi przypadkami równości [illegible]

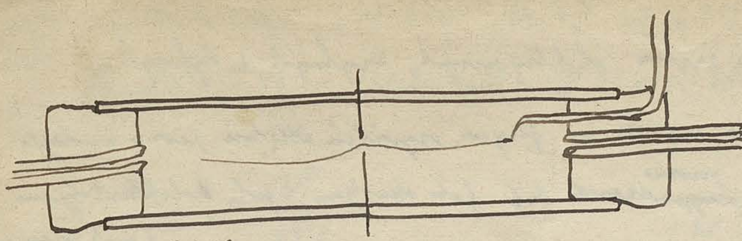
n.p. prąd w elektrolitach.

Różnice zachodzące wielkości współczynników.

	Elektr.
Ag	60
Cu	56
Hg	1
conc. NaCl	~~Na~~ 2.10^{-5}
H_2O	$\frac{1}{4}.10^{-14}$
Paraffin 460:	3.10^{-22}
powietrze	Hertz. 2.10^{-17}
vacuum	—
Eter	10^{-18}

	Ciepło
Ag	1·096 (Weber)
Cu	1·1 – 0·63
Hg	0·02 (Berget)
szkło	0·001 – 0·0023
H_2O	0·0013 (Wachsmuth)
Paraffin	0·0002 Weber
Guttaperka	0·00046 Lees
~~Hartgummi~~ Kork	0·00013 "
Vulc. Kautschuk, Wachs	0·00009 (?) Forbes
~~Horn~~	
powietrze	0·00006

	Dyf.
Caramel	$5{\cdot}4.10^{-7}$
Rohrzucker	36·[illegible] "
HCl	267.10^{-7}
NaCl	123.10^{-7}
$CuSO_4$	24.10^{-7}
H – O	0·722
O – CO_2	0·180
Au – Pb	500°: 3·19
	100°: 0·00002
	NaCl – H_2O = 1
CO_2 – H_2O	$1{\cdot}6.10^{-5}$
Kautschuk – CO_2	$0{\cdot}54.10^{-5}$



	η 20°
Glycerin	8.0
Vaselinöl	0.91
H_2O	0.0099
Methylalkohol	0.0041

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad \Big| \ \frac{\partial}{\partial y}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad \Big| \ \frac{\partial}{\partial x}$$

$$-u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y} = \cancel{u}\ v^2\frac{\partial\left(\frac{u}{v}\right)}{\partial y} =$$

$$+u\frac{\partial v}{\partial x} - v\frac{\partial u}{\partial x}$$

$$\left.\begin{array}{l} \cancel{-\frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}\frac{\partial u}{\partial y}} - u\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^2 u}{\partial y^2} \\ \cancel{-\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial x}}\left[+u\frac{\partial^2 v}{\partial x^2} - v\frac{\partial^2 u}{\partial x^2}\right] \end{array}\right\} = \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$-u\,\Delta^2 v + v\,\Delta^2 u = \frac{\mu}{\rho}\Delta^2\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$\frac{\partial c}{\partial t} = \mathrm{div}$$

us [illegible] $\frac{\partial}{\partial y} \sim$ [illegible]!

Kula w cieczy idealnej:

Quellen & Senken analog. do potenc.

Double Sources w optyce elektr.

$$\frac{\partial\varphi}{\partial t} = -\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \ldots\right) = -\rho\,\Delta^2\varphi$$

φ nie jest substancyą!

~~Pierwsze Sources~~ Pogg. Ann. 49 (1832)

Przykłady Schustera etc.

ale mimo to tylko powierzchowna

podobnie jak Seilwellen-Optik etc. Platten - u. s. w.

Więc może rozróżnimy analogię formalną i rzeczywistą?

Jednak ~~[illegible]~~ jest ta różnica tylko stopniowa; do pewnego punktu wspólność, poza tem nie | gdyby zupełna to zupełnie nie powiedzielibyśmy analogia tylko

A czasem a priori nie można rozróżnić n.p. Vector Pot. elektr. - Helm. ; jak Wärme Th. - elektr. Th.

„zawsze formalna"!

Ostatecznie w ogóle w fizyce każde wytłomaczenie = tylko analogia | Gastheorie = analogia z mechanizmem etc.

Nazwy „analogii" wtedy tylko używamy do analogii niezupełnych z pominięciem przyczyny wspólnej

Więc porzućmy stosunkowo daleko idące; ~~[illegible]~~ wspólności: substancyalizacya

może nawet kiedyś dalej pójdzie idące n.p. elektr.-dyf. = ~~w~~ elektrolytach, a elektrony? Wiedemann-Franz

Walka w fizyce której analogii więcej uwagi przypisujemy ciepło substancya, ruch?

Elektryczne masy czy istnieją? Magnetyzm?

18v

183

$$\frac{1}{x} = \frac{\cosh 2b + \cos 2a}{\sin 2a} = r\cos\theta$$

$$\frac{\cosh 2b + \cos 2a}{\sinh 2b} = r\sin\theta$$

$$r^2 = (\cosh 2b + \cos 2a)^2 \cdot \frac{\sin^2 2a + \sin^2 h 2b}{\sin^2 2a \sin^2 h 2b}$$

$$\cosh \delta = \frac{1 + \delta + \frac{\delta^2}{2} + 1 - \delta + \frac{\delta^2}{2}}{2}$$

$$= 1 + \frac{\delta^2}{2}$$

$$\lim_{r=\infty} rb = \boxed{\text{finite const.}}$$

$$\cosh 2b = 1 + \frac{\delta^2}{2}$$

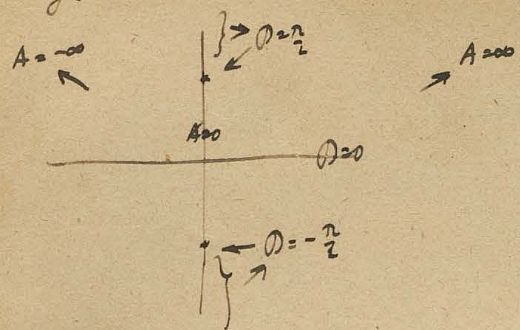
$2b$

$$e^{2b} = \frac{\delta^2}{2} + 1$$

$$b = \lg \frac{\delta^2}{2} + 1$$

$$\lim \frac{\delta^2}{\delta^2} = \text{const}$$

$\lg(z+\sqrt{1+z^2}) = \operatorname{arcsinh} z = A + i\Theta$



$$e^a \cos b = x + \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2}$$

$$e^a \sin b = y + \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2}$$

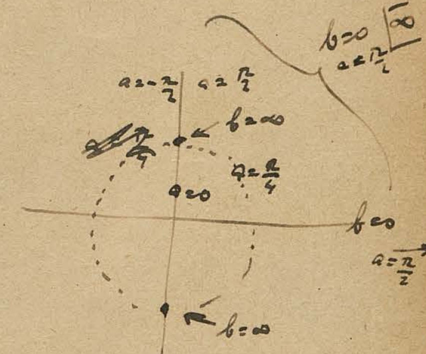
$$A = \lg\sqrt{r^2 + r_1 r_2 + 2r\sqrt{r_1 r_2}\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}$$

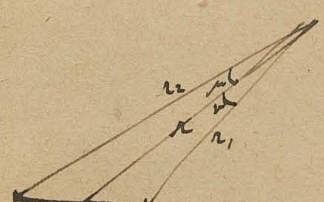
$$\Theta = \arcsin\left\{\frac{r\sin\theta + \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2}}{\sqrt{\cdots}}\right\}$$

$\operatorname{artg} z = a + ib$

$$x = \frac{\sin 2a}{\cosh 2b + \cos 2a}\,; \quad y = \frac{\sinh 2b}{\cosh 2b + \cos 2a}\,;$$

$$a = \frac{1}{2}\operatorname{artg}\frac{2x}{1-r^2} \qquad b = \frac{1}{2}\operatorname{artgh}\frac{2y}{1+r^2}$$





$$\frac{\nu-\mu}{2} = \frac{xy}{r^4}$$

$$\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right) = \cancel{1-\frac{\cdots}{r^4}}$$

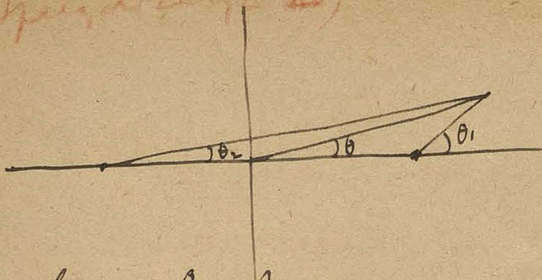
$$= \frac{1}{2}(\nu-\mu) = -\frac{\sin\theta\cos\theta}{r^2}$$

$$\frac{r}{\sqrt{r_1 r_2}} = 1 + \frac{\cos 2\theta}{2r^2}$$

$y=0$: $b=0$:

$$x = \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha} = \operatorname{tg}\alpha$$

[illegible] II)



$$\sin\theta = \theta - \frac{\theta^3}{3!} = \frac{\eta}{r} = \frac{\eta}{1+\xi}$$

$$\cos\theta = 1 - \frac{\theta^2}{2} = 1 - \frac{\eta^2}{2}$$

$$\frac{\theta}{2} = \frac{\eta}{2+\xi}$$

$$\cos\frac{\theta}{2} = 1 - \frac{\eta^2}{32}$$

linqe $\theta = \theta_2 = 0$

$$u = -\frac{2y}{\sqrt{2r_1}} \sin\frac{\theta_1}{2}$$

$$v = +\frac{2x}{\sqrt{2r_1}} \cos\frac{\theta_1}{2} - \frac{2}{\sqrt{2r_1}} \sin\frac{\theta_1}{2} = \frac{2\xi}{\sqrt{2r}} \sin\frac{\theta}{2}$$

$$\sin\theta \cos\frac{\theta_1+\theta_2}{2} - \cos\theta \sin\frac{\theta_1+\theta_2}{2} =$$

$$\sin\theta\left[\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} - \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right] - \cos\theta\left[\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2} + \cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right] =$$

$$= \eta(1-\xi)\left[\cos\frac{\theta_1}{2}\left(1-\frac{\eta^2}{32}\right) - \sin\frac{\theta_1}{2}\,\frac{\eta}{4}\left(1-\frac{\xi}{4}\right)\right] - \left(1-\frac{\eta^2}{2}\right)\left[\sin\frac{\theta_1}{2}\left(1-\frac{\eta^2}{32}\right) + \cos\frac{\theta_1}{2}\,\frac{\eta}{4}\left(1-\frac{\xi}{4}\right)\right]$$

$$= \cos\frac{\theta_1}{2}\left[\eta(1-\xi)\left(1-\frac{\eta^2}{32}\right) - \left(1-\frac{\eta^2}{2}\right)\frac{\eta}{4}\left(1-\frac{\xi}{4}\right)\right]$$

$$- \sin\frac{\theta_1}{2}\left[\left(1-\frac{\eta^2}{2}\right)\left(1-\frac{\eta^2}{32}\right) + \frac{\eta^2}{4}(1-\xi)\left(1-\frac{\xi}{4}\right)\right]$$

$$= \cos\frac{\theta_1}{2}\left[\eta - \eta\xi - \frac{\eta}{4} + \frac{\eta\xi}{16}\right] - \sin\frac{\theta_1}{2}\underbrace{\left[1 - \frac{\eta^2}{2} - \frac{\eta^2}{32} + \frac{\eta^2}{4}\right]}_{1-\frac{9\eta^2}{32}}$$

$\frac{23}{32}$

$8-16-1$

$$u = -\frac{2r\sin\theta}{\sqrt{2r}}\left(1+\frac{3\xi}{4}\right)\sin\frac{\theta}{2} = -\sqrt{2r}\,\sin\theta\,\sin\frac{\theta}{2}$$

$$= -\sqrt{\frac{2}{r}}\,\eta\,\sin\frac{\theta}{2}$$

$$\frac{r}{\sqrt{r_1 r_2}} = \frac{1+\xi}{\sqrt{r_1(2+\xi)}} = \frac{1}{\sqrt{2r_1}}(1+\xi)\underbrace{\left(1+\frac{\xi}{2}\right)^{-\frac{1}{2}}}_{1-\frac{\xi}{4}}$$

$$\frac{1}{\sqrt{r_1 r_2}} = \frac{1}{\sqrt{2r_1}}\left(1+\frac{\xi}{2}\right)^{-\frac{1}{2}}$$

$$\frac{xr}{\sqrt{r_1 r_2}} = \frac{1}{\sqrt{2r_1}}\underbrace{(1+\xi)^2\left(1-\frac{\xi}{4}\right)}_{1+\frac{7}{4}\xi}$$

$$v = \sin\frac{\theta_1}{2}\left\{+\frac{2}{\sqrt{2r_1}}\left(1+\frac{7}{4}\xi\right)\left(1-\frac{9\eta^2}{32}\right) - \frac{2}{\sqrt{2r_1}}\left(1+\frac{\xi}{4}\right)\left(1-\frac{\eta^2}{32}\right)\right]$$

$$+\cos\frac{\theta_1}{2}\left\{-\frac{2}{\sqrt{2r_1}}\left(1+\frac{7}{4}\xi\right)\frac{3\eta}{4} - \frac{2}{\sqrt{2r_1}}\left(1-\frac{\xi}{4}\right)\frac{\eta}{4}\right]$$

$$= \frac{\sqrt{2}}{\sqrt{r_1}}\left[\sin\frac{\theta_1}{2}\left(1+\frac{7}{4}\xi-1+\frac{\xi}{4}\right) - \cos\frac{\theta_1}{2}\left(\frac{3\eta}{4}+\frac{\eta}{4}\right)\right]$$

$$= \sqrt{\frac{2}{r_1}}\left[\sin\frac{\theta_1}{2}\cdot 2\xi - \cos\frac{\theta_1}{2}\cdot\eta\right]$$

$$= \sqrt{2r_1}\left[2\sin\frac{\theta_1}{2}\cos\theta_1 - \cos\frac{\theta_1}{2}\sin\theta_1\right]$$

$$= \underbrace{\sin\frac{\theta_1}{2}\cos\theta_1 - \cos\frac{\theta_1}{2}\sin\theta_1}_{-\sin\frac{\theta}{2}} + \sin\frac{\theta_1}{2}\cos\theta_1 = \sin\frac{\theta}{2}\left[\cos\theta-1\right]$$

$$v = \sqrt{2r}\,\sin\frac{\theta}{2}(\cos\theta-1) = \sqrt{\frac{2}{r}}\,\sin\frac{\theta}{2}(\xi - r)$$

Identyczne z danymi w obliczeniu rozwiązania przytem [illegible]

Obliczenie poprawki o skutek bezwładności

$u = u_0 + \frac{\rho}{\mu} u_1$

$p = p_0 + \frac{\rho}{\mu} p_1$

I). $\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} = \frac{1}{\mu}\frac{\partial p_0}{\partial x}$

II). $\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} = \frac{1}{\mu}\frac{\partial p_1}{\partial x} + \frac{\rho}{\mu}\left(u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y}\right)$

$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0$

$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} = \frac{1}{\mu}\frac{\partial p_1}{\partial y} + \frac{\rho}{\mu}\left(u_0 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial v_0}{\partial y}\right)$

$$\frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} = -\rho\left[\frac{\partial}{\partial x}\underbrace{\left(u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right)}_{u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial v_0}{\partial x} + v_0\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)} + \frac{\partial}{\partial y}\underbrace{\left(u_0\frac{\partial v_0}{\partial x} + v_0\frac{\partial v_0}{\partial y}\right)}_{u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} + u\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}\right]$$

$$= -\rho\left[\frac{\partial}{\partial y}(u\zeta) - \frac{\partial}{\partial x}(v\zeta) + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{u^2+v^2}{2}\right)\right]$$

$$= -\rho\left[u\frac{\partial \zeta}{\partial y} - v\frac{\partial \zeta}{\partial x} - \zeta^2 + \quad\right]$$

$F = \int (B^2 - S'')\, z\, dz$

$u = -\frac{\zeta y}{2}$

$v = \frac{\zeta x}{2} + 2 J F$

$= u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} - v\zeta = \frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right) - v\zeta$

$\frac{\partial u}{\partial x} = -\frac{y}{2}\frac{\partial \zeta}{\partial x}$

$\frac{\partial u}{\partial y} = -\frac{\zeta}{2} - \frac{y}{2}\frac{\partial \zeta}{\partial y}$

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\zeta y^2}{4}\frac{\partial \zeta}{\partial x} - \frac{yx}{4}\zeta\frac{\partial \zeta}{\partial y} - \frac{\zeta^2 x}{4}$

$- \left(\zeta + y\frac{\partial \zeta}{\partial y}\right) J F$

Najpierw dla prostszego przykładu: przepływ na około krawędzi:

$$u = -\frac{\sqrt{2}\,y}{\sqrt{r}}\sin\frac{\theta}{2}$$

$$v = \frac{\sqrt{2}\,x}{\sqrt{r}}\sin\frac{\theta}{2} - \sqrt{2}\sqrt{r}\sin\frac{\theta}{2}$$

$$\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{r}}\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)$$

$$\frac{1}{\sqrt{\beta}} = \frac{1}{\sqrt{r}}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$$

$$\frac{1}{\sqrt{r}}\sin\frac{\theta}{2} = \frac{1}{2i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right)$$

$$\sqrt{r}\sin\frac{\theta}{2} = \frac{1}{2i}\left(\sqrt{\alpha} - \sqrt{\beta}\right)$$

$$y = \frac{\alpha-\beta}{2i}$$

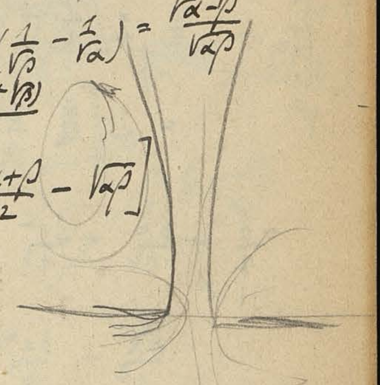
$$x = \frac{\alpha+\beta}{2}$$

$$\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) = \frac{\sqrt{\alpha}-\sqrt{\beta}}{\sqrt{\alpha\beta}}$$

$$u = -\frac{\alpha-\beta}{2i}\,\frac{1}{2i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) = \frac{\alpha-\beta}{4}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) = \frac{(\sqrt{\alpha}-\sqrt{\beta})^2(\sqrt{\alpha}+\sqrt{\beta})}{4\sqrt{\alpha\beta}}$$

$$v = \frac{\alpha+\beta}{2}\,\frac{1}{2i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right) - \frac{1}{2i}\left(\sqrt{\alpha}-\sqrt{\beta}\right) = \left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right)\frac{1}{2i}\left[\frac{\alpha+\beta}{2} - \sqrt{\alpha\beta}\right]$$

$$= \frac{1}{4i}\left(\frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\alpha}}\right)\left(\sqrt{\alpha}-\sqrt{\beta}\right)^2 = \frac{1}{4i}\,\frac{\left(\sqrt{\alpha}-\sqrt{\beta}\right)^3}{\sqrt{\alpha\beta}}$$



$$u = -y\sqrt{\frac{1-\cos\theta}{r}} = -y\frac{\sqrt{r-x}}{r}$$

$$v = \frac{x\sqrt{r-x}}{r} - \sqrt{r-x} = \left(\frac{x}{r}-1\right)\sqrt{r-x}$$

$$u^2+v^2 = (r-x)\left(\frac{y^2}{r^2} + \frac{x^2}{r^2} - \frac{2x}{r} + 1\right) = 2(r-x)\left(1-\frac{x}{r}\right) = \frac{2(r-x)^2}{r}$$

$$\zeta = \frac{2\sqrt{2}}{\sqrt{r}}\sin\frac{\theta}{2} = \frac{2\sqrt{r-x}}{r}$$ (control)

$$u\zeta = -\frac{2y(r-x)}{r^2}$$

$$\frac{\partial}{\partial y}(\ ) = -\frac{2(r-x)}{r^2} + \frac{4y^2(r-x)}{r^4} - \frac{2y^2}{r^3}$$

$$v\zeta = 2\left(\frac{x}{r}-1\right)\frac{r-x}{r} = -2\left(\frac{r-x}{r}\right)^2 = -2\left(1-\frac{x}{r}\right)^2$$

$$\frac{\partial}{\partial x}(\ ) = +4\left(1-\frac{x}{r}\right)\left(\frac{1}{r} - \frac{x^2}{r^3}\right)$$

$$= 4\,\frac{r-x}{r}\,\frac{y^2}{r^3}$$

$$-\frac{2(r-x)}{r^2} + \frac{4y^2(r-x)}{r^4} - \frac{2y^2}{r^3} \Big| -4\frac{y^2}{r^3} + \frac{4xy^2}{r^4} - \cancel{} = \cancel{-\frac{6(r-x)}{r^2} - \frac{2y^2}{r^3} - \frac{8xy^2}{r^4}}$$

$$= \cancel{-\frac{6}{r} + \frac{6x}{r^2} - \frac{2y^2}{r^3} - \frac{8xy^2}{r^4}} = -\frac{2}{r} + \frac{2x}{r^2} - \frac{2y^2}{r^3}$$

$$u^2+v^2 = \frac{2(r-x)^2}{r} = 2\left(r - 2x + \frac{x^2}{r}\right)$$

$$\frac{\partial}{\partial x} = 2\left\{\frac{x}{r} - 2 + \frac{2x}{r} - \frac{x^3}{r^3}\right\} = 2\left[-2 + \frac{3x}{r} - \frac{x^3}{r^3}\right]$$

$$\frac{\partial^2}{\partial x^2} = 2\left[\frac{3}{r} - \frac{3x^2}{r^3} - \frac{3x^2}{r^3} + 3\frac{x^4}{r^5}\right]$$

$$\frac{\partial}{\partial y} = 2\left[\frac{y}{r} - \frac{x^2 y}{r^3}\right]$$

$$\frac{\partial^2}{\partial y^2} = 2\left[\cancel{\frac{1}{r} - \frac{y^2}{r^3} - \frac{x^2}{r^3}} + \frac{3x^2y^2}{r^5}\right]$$

$$\nabla^2 = 2\left[\frac{3}{r} - \frac{6x^2}{r^3} + \frac{3x^2}{r^3}\right] = 2\left[\frac{3}{r} - \frac{3x^2}{r^3}\right]$$

$$\cancel{-\frac{6}{r} + \frac{8x}{r^2} - \frac{2y^2}{r^3} - \frac{8xy^2}{r^4} + \frac{3}{r} - \frac{3x^2}{r^3}} = \cancel{+\frac{4}{r} - \frac{4x}{r^2} - \frac{3}{r} + \frac{3x^2}{r^3}}$$

$$\cancel{\frac{6x}{r^2} - \frac{6x^2}{r^3} - \frac{2}{r} + \frac{2x^2}{r^3} - \frac{8x}{r^2} + \frac{8x^3}{r^4}} = \cancel{-\frac{2}{r} - \frac{2x}{r^2} - \frac{4x^2}{r^3} + \frac{8x^3}{r^4}}$$

$$\cancel{+ \frac{4(r-x)}{r^2} +}$$

$$\frac{3}{r} - \frac{3x^2}{r^3} - \frac{2}{r} + \frac{2x}{r^2} - \frac{2}{r} + \frac{2x^2}{r^3}$$

$$= -\frac{1}{r} + \frac{2x}{r^2} - \frac{x^2}{r^3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{(r-x)^2 x}{r^3} \qquad \Big| \qquad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{(r-x)^2 y}{r^3}$$

$$\frac{\partial}{\partial x}(\qquad) + \frac{\partial}{\partial y}(\qquad) = -\frac{(r-x)^2}{r^3} = -\frac{1}{r}\left(1-\frac{x}{r}\right)^2$$

$$= -\frac{1}{r}\left(1-\cos\theta\right)^2 = -\frac{4\sin^4\frac{\theta}{2}}{r}$$

$$\frac{\partial^2 p_1}{\partial x^2}+\frac{\partial^2 p_1}{\partial y^2}=\frac{1}{r}\left(1-\frac{x}{r}\right)^2=\frac{4\sin^4\frac{\theta}{2}}{r}$$

$$x=\frac{\alpha+\beta}{2}\qquad y=\frac{\alpha-\beta}{2i}$$

$$r^2=\frac{\alpha^2+2\alpha\beta+\beta^2}{4}-\frac{\alpha^2-2\alpha\beta+\beta^2}{4}$$

$$4\frac{\partial^2 p}{\partial\alpha\partial\beta}=\frac{\left[\sqrt{\alpha\beta}-\frac{\alpha+\beta}{2}\right]^2}{(\alpha\beta)^{3/2}}$$

$$=\frac{4\alpha\beta}{4}=\alpha\beta$$

$$\alpha^2+\beta^2=2(x^2-y^2)$$

$$=\frac{1}{4}\frac{\left[-\alpha+2\sqrt{\alpha\beta}-\beta\right]^2}{\sqrt{\alpha\beta}^{\,3}}=\frac{1}{4}\frac{\left[\sqrt{\alpha}-\sqrt{\beta}\right]^4}{\sqrt{\alpha\beta}^{\,3}}=\frac{1}{2}\left[\frac{\sqrt{\alpha}}{\sqrt{\beta^3}}-\frac{2}{\sqrt{\alpha\beta}}+\frac{\sqrt{\beta}}{\sqrt{\alpha^3}}\right]$$

$$\cancel{\int\frac{1}{\sqrt{\alpha\beta^3}}\,d\alpha\,d\beta=-4\sqrt{\frac{\alpha}{\beta}}}$$

$$\int\frac{d\alpha\,d\beta}{\alpha\beta}=\log\alpha\ldots\qquad\frac{(2i\sqrt{r}\sin\frac{\theta}{2})^4}{r^3}=+8\frac{\sin^4\frac{\theta}{2}}{r}$$

$$\int\frac{d\alpha\,d\beta}{\sqrt{\alpha\beta}}=4\sqrt{\alpha\beta}\qquad\int\sqrt{\frac{\alpha}{\beta^3}}\,d\alpha\,d\beta=-\frac{1}{3}\sqrt{\frac{\alpha^3}{\beta}}$$

$$p=8\left[-\frac{1}{3}\sqrt{\frac{\alpha^3}{\beta}}-8\sqrt{\alpha\beta}-\frac{1}{3}\sqrt{\frac{\beta^3}{\alpha}}\right]=\frac{-8}{\sqrt{\alpha\beta}}\left[\frac{1}{3}\sqrt{\alpha}^4+8(\sqrt{\alpha\beta})^2+\frac{1}{3}\sqrt{\beta}^4\right]$$

$$=\frac{-8}{\sqrt{\alpha\beta}}\left[\frac{\alpha^2}{3}+8\alpha\beta+\frac{\beta^2}{3}\right]=\frac{-8}{r}\left[\frac{2(x^2-y^2)}{3}+8r^2\right]$$

Prüfe $\quad 8r+\frac{2}{3}\left(\frac{x^2}{r}-\frac{y^2}{r}\right)$

$$8\frac{x}{r}+\frac{2}{3}\left[\frac{2x}{r}-\frac{x^3}{r^3}+\frac{y^2x}{r^3}\right]\qquad 8\frac{y}{r}+\frac{2}{3}\left(-\frac{x^2y}{r}-\frac{2y}{r}+\frac{y^3}{r^3}\right)$$

$$8\frac{1}{r}-8\frac{x^2}{r^3}+\frac{2}{3}\left[\frac{2}{r}-\frac{2x^2}{r^3}-\frac{3x^2}{r^3}+\frac{3x^4}{r^5}+\frac{y^2}{r^3}-\frac{3y^2x^2}{r^5}\right]$$

$$y^2=r^2-x^2$$
$$y^4=r^4-2r^2x^2+x^4$$

$$8\frac{1}{r}-8\frac{y^2}{r^3}+\frac{2}{3}\left[-\frac{2}{r}+\frac{2y^2}{r^3}+\frac{3y^2}{r^3}-\frac{3y^4}{r^5}-\frac{x^2}{r^3}+\frac{3x^2y^2}{r^5}\right]$$

$$\frac{8}{r}+\frac{2}{3}\left[6\left(\frac{y^2}{r^3}-\frac{x^2}{r^3}\right)+3\left(\frac{x^4}{r^5}-\frac{y^4}{r^5}\right)\right]=\frac{8}{r}+\frac{2}{3}\left[\underbrace{6\left(\frac{1}{r}-\frac{2x^2}{r^3}\right)+3\left(\frac{x^4}{r^5}-\frac{1}{r}+\frac{2x^2}{r^3}-\frac{x^4}{r^5}\right)}_{3\left(\frac{1}{r}-2\frac{x^2}{r^3}\right)}\right]$$

$x^2 - y^2$

$$X = \int \frac{1}{16} \frac{(\sqrt{\alpha}-\sqrt{\beta})^4}{\sqrt{\alpha\beta}^3}\, d\alpha\, d\beta \qquad \sqrt{\alpha} = u \quad \alpha = u^2$$
$$\sqrt{\beta} = v \quad \beta = v^2$$

$$= \frac{4}{16}\int \frac{(u-v)^4}{u^3v^3}\, uv\, du\, dv = \frac{1}{4}\int \frac{(u-v)^4}{u^2v^2}\, du\, dv =$$

$$= \frac{1}{4}\int \frac{u^4 - 4u^3v + 6u^2v^2 - 4uv^3 + v^4}{u^2v^2}\, du\, dv =$$

$$= \frac{1}{4}\int \left(\frac{u^2}{v^2} - 4\frac{u}{v} + 6 - 4\frac{v}{u} + \frac{v^2}{u^2}\right) du\, dv =$$

$$= \frac{1}{4}\left[-\frac{u^3}{3v} - 2u^2 \log v + 6uv - 2v^2 \log u - \frac{v^3}{3u}\right]$$

$$= -\frac{1}{4}\left[\frac{u^4+v^4}{3uv} - 6uv + 2(u^2 \log v + v^2 \log u)\right]$$

$$= -\frac{1}{4}\left[\frac{\alpha^2+\beta^2}{3\sqrt{\alpha\beta}} - 6\sqrt{\alpha\beta} + 2(\alpha \log \beta + \beta \log \alpha)\right]$$

$$= -\frac{1}{4}\left[\frac{2(x^2-y^2)}{3r} - 6r + \left[(x+iy)(\log r - i\theta) + (x-iy)\log(r+i\theta)\right]\right]$$
$$2(x \log r + y\theta)\Big]$$

Probe: $\frac{2x}{3r} - \frac{x^3 - xy^2}{3r^3} - 6\frac{x}{r} + 2\left(\log r + \frac{x^2}{r^2} - \frac{y^2}{r^2}\right)$

$\frac{2}{3r} \cancel{\frac{x^2}{3r^3}} - \frac{2y}{3r} - \frac{x^2y - y^3}{3r^3} - 6\frac{y}{r} + 2\left(\frac{xy}{r^2} + \theta + y\frac{x}{r^2}\right)$

$\cancel{\frac{2}{3r}} - \frac{2x^2}{3r^3} - \frac{3x^2-y^2}{3r^3} + \frac{x^4 - x^2y^2}{r^5} + \cancel{\frac{6}{r}} - \frac{6x^2}{r^3} + 2\left(\frac{x}{r^2} + \frac{2x}{r^2} - \frac{2x^3}{r^4} + \frac{2xy^2}{r^4}\right)$

$-\cancel{\frac{2}{3r}} + \frac{2y^2}{3r^3} - \frac{x^2-3y^2}{3r^3} + \frac{x^2y^2-y^4}{r^5} - \frac{6}{r} - \frac{6y^2}{r^3} + 2\left(\frac{x}{r^2} - \frac{2xy^2}{r^4} + \frac{x}{r^2} + \frac{x}{r^2} - \frac{2xy^2}{r^4}\right)$

$$= \frac{2(y^2-x^2) - 4x^2 + 4y^2 + 3x^2 - 3y^2}{3r^3} + \frac{6}{r} - 8\frac{x}{r^2}$$

$$= \frac{3y^2-3x^2}{3r^3} \dots \quad = \frac{y^2-x^2}{r^3} \dots \quad = \frac{2}{r} - \frac{4x^2}{r^3} - \frac{6}{r} + \frac{8x}{r^2}$$

$$= \cancel{-\frac{4}{r} + \frac{8x}{r^2} - \frac{2x^2}{r^3}}$$

$$-\frac{4}{r} + \frac{8x}{r^2} - \frac{4x^2}{r^3}$$

$$= \frac{(y-x)^2}{r^3} \qquad \text{(stimmt)!}$$

$$p = -\frac{1}{4}\left[\frac{2}{3}\,\frac{r^2(\cos^2-\sin^2)}{r} - 6r + 2(x\lg r + y\theta)\right]$$

$$= -\frac{1}{4}\left[\frac{2}{3}\cos 2\theta - 6 + 2\left[\cos\theta \lg r + \theta\sin\theta\right]\right]$$

Dodać można lub odjąć dowolną funkcję rozwiązań równania $\Delta^2 f = 0$

$$z\lg z = x\lg r - y\theta$$

Ale mimo wszystko nie uda się uwolnić p od przybierania nieskończ. wielkich wartości w ∞ odległości. Jest to naturalnem zupełnie ponieważ prędkość w ∞ się staje ∞, a zatem też ciśnienie hydrodynamiczne.

~~Próba~~ Gdyby odrzucić część $\Delta^2\left(\frac{u+v}{2}\right)$:

$$\Delta^2 p = -\frac{2}{r} + \frac{2x}{r^2} - \frac{2y^2}{r^3} = -\frac{4}{r} + \frac{2x}{r^2} + \frac{2x^2}{r^3}$$

$$= -\frac{4}{\sqrt{\alpha\beta}} + \frac{\alpha+\beta}{\alpha\beta} + \frac{(\alpha+\beta)^2}{2\sqrt{\alpha\beta^3}} = -\frac{4}{\sqrt{\alpha\beta}} + \frac{1}{\beta} + \frac{1}{\alpha} + \frac{1}{2}\sqrt{\frac{\alpha}{\beta^3}} + \frac{1}{\sqrt{\alpha\beta}} + \frac{1}{2}\sqrt{\frac{\beta}{\alpha^3}}$$

$$u = -2\frac{y^2 x}{r^4} \qquad \frac{\partial u}{\partial x} = -\frac{2y^2}{r^4} + 8\frac{y^2x^2}{r^6} \qquad \frac{\partial u}{\partial y} = -\frac{4yx}{r^4} + 8\frac{y^3x}{r^6}$$

$$v = -2\frac{y^3}{r^4} \qquad \frac{\partial v}{\partial x} = \cancel{\quad} \cdot 8\frac{y^3x}{r^6} \qquad \frac{\partial v}{\partial y} = -6\frac{y^2}{r^4} + 8\frac{y^4}{r^6}$$

$$\frac{4y^4x}{r^8} - \frac{16y^4x^3}{r^{10}} + \frac{8y^4x}{r^8} - \frac{16y^6x}{r^{10}} = -\frac{4y^4x}{r^8} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$-16\frac{y^5x^2}{r^{10}} + 12\frac{y^5}{r^8} - 16\frac{y^7}{r^{10}} = -\frac{4y^5}{r^8} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$-4\left[\frac{y^4}{r^8} - 8\frac{y^4x^2}{r^{10}} + \frac{5y^4}{r^8} - \frac{8y^6}{r^{10}}\right] =$$

$$-4\left[-2\frac{y^4}{r^8}\right] = 8\frac{y^4}{r^8} = -\Delta^2 p$$

$$y = \frac{\alpha-\beta}{2i} \qquad r = \sqrt{\alpha\beta}$$

$$-4\frac{\partial^2 p}{\partial\alpha\partial\beta} = \frac{1}{2}\frac{(\alpha-\beta)^4}{(\alpha\beta)^4} = \frac{1}{2}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^4 = \frac{1}{2}\left(\frac{1}{\alpha^4} - \frac{4}{\alpha^3\beta} + \frac{6}{\alpha^2\beta^2} - \frac{4}{\alpha\beta^3} + \frac{1}{\beta^4}\right)$$

$$-8p = -\frac{\beta}{3\alpha^3} + \frac{2}{\alpha^2}\lg\beta + \frac{6}{\alpha\beta} + \frac{2}{\beta^2}\lg\alpha - \frac{\alpha}{3\beta^3} \quad]$$

$$= \cancel{-\frac{1}{3r^2}\ldots + \frac{6}{r^2} \ldots}$$

$$+\frac{2}{r^2}\left[i\theta(\cos 2\theta - i\sin 2\theta) + i\theta(\ldots\right.$$

$$-2\theta\sin 2\theta$$

$$= -\frac{1}{3r^2}\left[e^{-i\theta-3i\theta} + e^{i\theta+3i\theta}\right] + \ldots$$

$$= -\frac{2}{3r^2}\cos 4\theta + \frac{6}{r^2} + \frac{2\lg r}{r^2}\left[(\cos 2\theta - i\sin 2\theta) + (\cos 2\theta + i\sin 2\theta)\right.$$

$$+2\cos 2\theta$$

$$p = -\frac{1}{4r^2}\left\{ 3 - \frac{\cos 4\theta}{3} + 2\log r \cdot \cos 2\theta - 2\theta \sin 2\theta \right\}$$

$$\left.\begin{aligned}\cos 2\left(\tfrac{\pi}{2}-\varphi\right) &= \cos(\pi - 2\varphi) = -\cos 2\varphi \\ \cos 2\left(\tfrac{\pi}{2}+\varphi\right) &= \cos(\pi + 2\varphi) = -\cos 2\varphi\end{aligned}\right\}$$

$$\frac{1}{\alpha^2}\log\alpha = \frac{1}{r^2}(\cos 2\theta - i\sin 2\theta)(\log r + i\theta)$$

$$= \frac{1}{r^2}\left[\cos 2\theta \cdot \log r + \theta \sin 2\theta\right]$$

$$\cos 4\left(\tfrac{\pi}{2}-\varphi\right) = \cos(2\pi - 4\varphi) = \cos 4\varphi = \cos 4\left(\tfrac{\pi}{2}+\varphi\right)$$

symetrycznie względem Y

$$\theta \sin 2\theta \;\Big\|\; \begin{aligned}\left(\tfrac{\pi}{2}-\varphi\right)\sin(\pi - 2\varphi) &= +\left(\tfrac{\pi}{2}-\varphi\right)\sin 2\varphi \\ \left(\tfrac{\pi}{2}+\varphi\right)\sin(\pi + 2\varphi) &= -\left(\tfrac{\pi}{2}+\varphi\right)\sin 2\varphi\end{aligned}$$

asymetrycznie, zatem to trzeba wyrugować dodając

$$p = -\frac{1}{4r^2}\left\{3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta \cdot \log r\right\}$$

$$= -\frac{1}{4\alpha\beta}\left\{3 - \frac{1}{6}\left(\frac{\beta^2}{\alpha^2} + \frac{\alpha^2}{\beta^2}\right) + 2\left(\frac{\beta}{\alpha} + \frac{\alpha}{\beta}\right)(\log\alpha + \log\beta)\right\}$$

$$= -\frac{1}{4}\left\{\frac{3}{\alpha\beta} - \frac{1}{6}\left[\frac{\beta}{\alpha^3} + \frac{\alpha}{\beta^3}\right] + 2\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)(\log\alpha + \log\beta)\right\}$$

$$\cos 4\theta = \cancel{\ldots}$$

$$\cancel{= \Re(\cos 2\theta + i\sin 2\theta)(\ldots)}$$

$$\cancel{= \Re\,\alpha^2\beta^2}$$

$$\frac{\beta^2}{\alpha^2} + \frac{\alpha^2}{\beta^2} = e^{-4i\theta} + e^{+4i\theta} = 2\cos 4\theta$$

$$\cos 2\theta = \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$$

$$\text{Pot } p = \frac{1}{16}\left[3\log\alpha\cdot\log\beta + \frac{1}{4}\frac{\beta^2}{\alpha^2} + \frac{1}{4}\frac{\alpha^2}{\beta^2} \underbrace{- \frac{2\beta}{\alpha}(\log\alpha + x) - \frac{2\alpha}{\beta}(\log\beta + x) - \frac{2\alpha}{\beta}(\log\alpha - x) - \frac{2\beta}{\alpha}(\log\beta - x)}_{= -2\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)\log\alpha\beta}\right]$$

$$\int \frac{\log x}{x^2}\,dx = -\frac{\log x}{x} + \int \frac{1}{x^2}\,dx = -\frac{1}{x}(\log x + 1)$$

$$\int \log x\,dx = x(\log x - 1)$$

$$\int \frac{\log x}{x^3}\,dx = -\frac{1}{2}\frac{\log x}{x^2} + \frac{1}{2}\int\frac{dx}{x^3} = -\frac{1}{4x^2} - \frac{1}{2}\frac{\log x}{x^2}$$

$$\cancel{\frac{1}{x^2}(\log x + 1) - \frac{1}{x}}$$

$$+\frac{1}{2x^3} - \frac{1}{2x^3} + \frac{\log x}{x^3}$$

$$\mathrm{Pot}\, p = \frac{1}{16}\left[3 \lg\alpha \lg\beta + \frac{1}{4}\left(\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}\right) - 2\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)\lg\alpha\beta\right]$$

$$= \frac{1}{16}\left[3(\log r + i\theta)(\log r - i\theta) + \frac{1}{4}\cos 4\theta - 4\cos 2\theta \log r\right]$$

$$= \frac{1}{16}\left\{\left[3(\log r)^2 + 3\theta^2\right] + \frac{\cos 4\theta}{4} - 4\cos 2\theta \log r\right\}$$

$$R(\lg\alpha)^2 = (\log r)^2 - \theta^2$$

$$\mathrm{Pot}\, p = \frac{1}{16}\left\{6(\log r)^2 + \frac{\cos 4\theta}{4} - 4\cos 2\theta \log r\right\}$$

$$u = \frac{\partial\, \mathrm{Pot}\, p}{\partial x}$$

$$p = \frac{1}{4r^2}\left\{3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta . \log r\right\}$$

$$\frac{\partial p}{\partial x} = -\frac{x}{2r^4}\left\{3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta . \log r\right\}$$

$$+ \frac{1}{4r^2}\left[\left\{-\frac{4\sin 4\theta}{3} + 8\sin 2\theta \log r\right\}\frac{\sin\theta}{r} + 4\cos 2\theta . \frac{x}{r^2}\right]$$

$$= \frac{1}{2r^3}\left\{-\left[3 - \frac{\cos 4\theta}{3} + 4\cos 2\theta . \log r\right]\cos\theta + 2\cos 2\theta . \cos\theta + \left[-\frac{2\sin 4\theta}{3} + 4\sin 2\theta \log r\right]\sin\theta\right\}$$

$$-\cos 2\theta \cos\theta + \sin 2\theta \sin\theta = -\cos 3\theta$$

$$\cos\theta \cos 4\theta - 2\sin\theta \sin 4\theta =$$

$$\frac{\partial r}{\partial x} = \frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial \beta}$$

$$\frac{\partial r}{\partial y} = i\left(\frac{\partial r}{\partial \alpha} - \frac{\partial r}{\partial \beta}\right)$$

$$x = \frac{\alpha+\beta}{2}$$

$$= \frac{1}{4}\left[-\frac{3}{r^2} + \frac{x^4-6x^2y^2+y^4}{3r^6} - \frac{4(x^2-y^2)}{r^4}\log r\right]$$

$$p = -\frac{1}{4}\left[\frac{3}{\alpha\beta} - \frac{1}{6}\left[\frac{\beta}{\alpha^3} + \frac{\alpha}{\beta^3}\right] + \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)(\log\alpha + \log\beta)\right]$$

$$\frac{\partial p}{\partial \alpha} = -\frac{1}{4}\left[-\frac{3}{\alpha^2\beta} - \frac{3}{\alpha\beta^2} + \frac{3\beta}{6\alpha^4} - \frac{1}{6\alpha^3} - \frac{1}{6\beta^3} + \frac{3\alpha}{6\beta^4} + \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right.$$
$$\left. - 2\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right)(\log\alpha + \log\beta)\right]$$

$$= -\frac{1}{4}\left[-\frac{2}{\alpha^2\beta} - \frac{2}{\alpha\beta^2} + \frac{\beta}{2\alpha^4} + \frac{\alpha}{2\beta^4} + \frac{5}{6\alpha^3} + \frac{5}{6\beta^3} + \cancel{\frac{2}{\alpha^3} + \frac{2}{\beta^3}} + \cancel{\frac{2}{\alpha\beta^2} + \frac{2}{\alpha^2\beta}}\right.$$
$$\left. - 2\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right)\log(\alpha\beta)\right]$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{4\left(\frac{\alpha-\beta}{2i}\right)^4\left(\frac{\alpha+\beta}{2}\right)}{(\alpha\beta)^4} = -\frac{1}{8}\frac{\left[\alpha^3 - 3\alpha^2\beta + 3\alpha\beta^2 - \beta^3\right]\left[\alpha^2 - \beta^2\right]}{\alpha^4\beta^4} =$$

$$= -\frac{1}{8}\,\frac{\alpha^5 - 3\alpha^4\beta + 3\alpha^3\beta^2 - \alpha^2\beta^3 - \alpha^3\beta^2 + 3\alpha^2\beta^3 - 3\alpha\beta^4 + \beta^5}{\alpha^4\beta^4} =$$

$\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$

$\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$

$$= -\frac{1}{8}\left[\frac{\alpha}{\beta^4} + \frac{\beta}{\alpha^4} - \frac{3}{\beta^3} - \frac{3}{\alpha^3} + \frac{2}{\alpha\beta^2} + \frac{2}{\alpha^2\beta}\right]$$

$-\frac{5}{3} + 3 = \frac{4}{3}$

$-\frac{5}{2} - 2 = -\frac{9}{2}$

$$+2\left(\frac{1}{\alpha\beta^2} + \frac{1}{\alpha\beta}\right)$$

$$\Delta^2 u = \frac{1}{8}\left[\cancel{-4\left(\frac{1}{\alpha\beta^2} + \frac{1}{\alpha^2\beta}\right)} - 2\left(\frac{\beta}{\alpha^4} + \frac{\alpha}{\beta^4}\right) + 3\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right) + 4\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right)\log(\alpha\beta)\right]$$

$$u = \frac{1}{8}\left[4\left(\frac{1}{\beta}\log\alpha + \frac{1}{\alpha}\log\beta\right) + \frac{5}{6}\left(\frac{\beta^2}{\alpha^3} + \frac{\alpha^2}{\beta^3}\right) - \frac{5}{2}\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) + 2\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right)\right.$$
$$\left. + 4\left(\frac{\beta\log\alpha}{\alpha^2} + \frac{\alpha\log\beta}{\beta^2}\right) + 4\left(\frac{\beta\log\beta}{\alpha^2} - \frac{\beta}{\alpha^2}\right) + 4\left(\frac{\alpha\log\alpha}{\beta^2} - \frac{\alpha}{\beta^2}\right)\right]$$

$$= \frac{1}{8}\left[-\frac{9}{2}\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) - \frac{5}{6}\left(\frac{\beta^2}{\alpha^3} + \frac{\alpha^3}{\beta^3}\right) + 4\left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right) + 4\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right)\log\alpha\beta\right]$$

$$\cancel{u = \frac{1}{8}\left[\frac{9}{r}\cos 3\theta + \frac{5}{3r}\cos 5\theta - 8\frac{(x\log r - y\theta)}{r^2} - \frac{8}{r}\cos 3\theta \log r\right]}$$

$$\Delta^2 u = \frac{1}{4}\left[\left(\frac{1}{\alpha\beta^2} + \frac{1}{\alpha^2\beta}\right) - \left(\frac{\beta}{\alpha^4} + \frac{\alpha}{\beta^4}\right) + \frac{2}{3}\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right) + 2\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right)\log\alpha\beta\right]$$

$$4\frac{\partial \dot u}{\partial\alpha\,\partial\beta} =$$

$$(u) = \frac{1}{16}\left[-\left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right) + \frac{1}{6}\left(\frac{\beta^2}{\alpha^3} + \frac{\alpha^2}{\beta^3}\right) - \frac{1}{3}\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) + 2\left[-\frac{1}{4}\left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) - \right.\right.$$

$$\left.\left. -\frac{1}{2}\left(\frac{\beta\log\alpha}{\alpha^2} + \frac{\alpha\log\beta}{\beta^2}\right) - \frac{1}{2}\left(\frac{\alpha\log\alpha}{\beta^2} + \frac{\beta\log\beta}{\alpha^2}\right) + \frac{1}{2}\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right)\right]\right]$$

$$= \frac{1}{16}\left[+\frac{1}{6}\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + \frac{1}{6}\left(\frac{\alpha^2}{\beta^3} + \frac{\beta^2}{\alpha^3}\right) - \left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right)\log\alpha\beta\right]$$

$$(u) = \frac{1}{8}\left[\frac{1}{6}\,\frac{\cos 3\theta}{r} + \frac{1}{6}\,\frac{\cos 5\theta}{r} - \frac{x\log r - y\theta}{r^2} - 2\frac{\cos 3\theta}{r}\log r\right]$$

$$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} = -\frac{1}{4}\left[\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)(1 + 2\log r) - \left(\frac{\log\alpha}{\beta^2} + \frac{\log\beta}{\alpha^2}\right)\right]$$

$$\cos 3\left(\frac{\pi}{2} - \varphi\right) = \cos\left(\frac{3\pi}{2} - 3\varphi\right) = -\sin 3\varphi$$

$$\cos 3\left(\frac{\pi}{2} + \varphi\right) = \cos\left(\frac{3\pi}{2} + 3\varphi\right) = \sin 3\varphi$$

$$\frac{1}{z} = \frac{\cos\theta - i\sin\theta}{r}$$

$$\left(\frac{\pi}{2} - \varphi\right) \cos\varphi$$

$$\left(\frac{\pi}{2} + \varphi\right) \cos\varphi \qquad -\frac{\pi}{2}\frac{\sin\theta}{r}$$

$$= -\frac{1}{4}\left[\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{\log\alpha}{\alpha^2} + \frac{\log\beta}{\beta^2}\right] + \cancel{\frac{\log\beta}{\alpha} + \frac{\log\alpha}{\beta}} - \cancel{\left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right)}$$

$$= -\frac{1}{4}\left[R\left(\frac{1}{\alpha^2}\right) + R\left(\frac{\log\alpha}{\alpha^2}\right)\right] = -\frac{1}{2}\left[\frac{\cos 2\theta}{r^2} + \frac{\log r}{r^2}\right]$$

$$= -\frac{1}{4} R\left(\frac{\log\alpha + 1}{\alpha^2}\right)$$

$$\left(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta}\right)\left[f(\alpha) + f(\beta)\right] +$$

$$\left(\frac{\partial}{\partial\alpha} - \frac{\partial}{\partial\beta}\right)\left[\varphi(\alpha) - \varphi(\beta)\right]$$

$$= f'(\alpha) + f'(\beta) + \varphi'(\alpha) + \varphi'(\beta)$$

$$f = -\frac{\log x}{x} \qquad \varphi = -\frac{1}{x}$$

$$\cancel{\psi} = -\frac{1}{4}\Big\{\frac{3}{\alpha\beta} - \frac{1}{6}\Big[\frac{\beta}{\alpha^3}+\frac{\alpha}{\beta^3}\Big] + \Big(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\Big)\log\alpha\beta\Big\} \qquad \Big(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\Big)(\log\alpha+\log\beta)$$

$$+\frac{\partial\psi}{\partial y} = -\frac{i}{4}\Big\{-\frac{3}{\alpha^2\beta}+\frac{3}{\alpha\beta^2}+\frac{\beta}{2\alpha^4}+\frac{1}{6\alpha^3}-\frac{1}{6\beta^3}-\frac{\alpha}{2\beta^4}+\frac{-2}{\alpha^3}(\log\alpha+\log\beta)+\frac{1}{\alpha^3}+\frac{1}{\alpha\beta^2}$$

$$+\frac{2}{\beta^3}(\log\alpha+\log\beta)-\frac{1}{\alpha^2\beta}-\frac{1}{\beta^3}\Big\}$$

$$= -\frac{i}{4}\Big\{4\Big(\frac{1}{\alpha\beta^2}-\frac{1}{\alpha^2\beta}\Big)+\frac{7}{6}\Big(\frac{1}{\alpha^3}-\frac{1}{\beta^3}\Big)+\frac{1}{2}\Big(\frac{\beta}{\alpha^4}-\frac{\alpha}{\beta^4}\Big)+2\Big(\frac{1}{\beta^3}-\frac{1}{\alpha^3}\Big)\log\alpha\beta\Big\}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y} = -\frac{4}{(\alpha\beta)^4}\Big(\frac{\alpha-\beta}{2i}\Big)^5 = i\,\frac{(\alpha-\beta)^5}{8(\alpha\beta)^4} = \frac{i}{8}\Big\{\frac{\alpha}{\beta^4}-\frac{5}{\beta^3}+\frac{10}{\alpha\beta^2}-\frac{10}{\alpha^2\beta}+\frac{5}{\alpha^3}-\frac{\beta}{\alpha^4}\Big\}$$

$-\frac{13}{3}+5$

$-\frac{7}{3}+5$

$$\Delta^2 v = \frac{i}{8}\Big\{\cancel{20\Big(\frac{1}{\alpha\beta^2}-\frac{1}{\alpha^2\beta}\Big)} + \frac{8}{3}\Big(\frac{1}{\alpha^3}-\frac{1}{\beta^3}\Big) - 2\Big(\frac{\beta}{\alpha^4}-\frac{\alpha}{\beta^4}\Big) + 4\Big(\frac{1}{\beta^3}-\frac{1}{\alpha^3}\Big)\log\alpha\beta\Big\}$$

$$2\Big(\frac{1}{\alpha\beta^2}-\frac{1}{\alpha^2\beta}\Big) \qquad +\frac{8}{3}$$

Proba:

$$\Delta^2\frac{\partial v}{\partial y} = -\Delta^2\frac{\partial u}{\partial x} = 2\Big(\frac{2}{\alpha^2\beta^2}+\frac{2}{\alpha\beta^3}+\frac{2}{\alpha^3\beta}\Big)+2\Big(\frac{1}{\alpha^4}+\frac{1}{\beta^4}-\frac{4\beta}{\alpha^5}-\frac{4\alpha}{\beta^5}\Big)+4\Big(\frac{1}{\alpha\beta^3}+\frac{1}{\beta\alpha^3}\Big)$$

$$-12\Big(\frac{1}{\alpha^4}+\frac{1}{\beta^4}\Big)\log\alpha\beta$$

$$\Delta^2 v = \frac{i}{4}\Big[\Big(\frac{1}{\alpha\beta^2}-\frac{1}{\alpha^2\beta}\Big)-\Big(\frac{\beta}{\alpha^4}-\frac{\alpha}{\beta^4}\Big)+\frac{4}{3}\Big(\frac{1}{\alpha^3}-\frac{1}{\beta^3}\Big)+2\Big(\frac{1}{\alpha^3}-\frac{1}{\beta^3}\Big)\log\alpha\beta\Big]$$

$$(v) = \frac{i}{16}\Big[-\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}+\frac{1}{6}\Big(\frac{\beta^2}{\alpha^3}-\frac{\alpha^2}{\beta^3}\Big)-\frac{2}{3}\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)+2\Big\{-\frac{1}{4}\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)-$$

$$-\frac{1}{2}\Big(\frac{\beta\log\alpha}{\alpha^2}-\frac{\alpha\log\beta}{\beta^2}\Big)-\frac{1}{2}\Big(\frac{\beta\log\beta}{\alpha^2}-\frac{\alpha\log\alpha}{\beta^2}\Big)+\frac{1}{2}\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)\Big\}\Big]$$

$$= \frac{i}{16}\Big[-\frac{1}{6}\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)+\frac{1}{6}\Big(\frac{\beta^2}{\alpha^3}-\frac{\alpha^2}{\beta^3}\Big)-\Big(\frac{\log\alpha}{\beta}-\frac{\log\beta}{\alpha}\Big)-\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)\log\alpha\beta\Big]$$

$$= \frac{1}{8}\Big[-\frac{\sin 3\theta}{6r}+\frac{\sin 5\theta}{6r}+\frac{yr+x\theta}{r^2}-2\frac{\sin 3\theta}{r}\log r\Big]$$

$$8\mu = \frac{\alpha^4+\beta^4}{3(\alpha\beta)^3} + \frac{6}{\alpha\beta} + 2 \,--$$

$$= \frac{2(x^4-6x^2y^2+y^4)}{3r^6} + \frac{6}{r^2} + 2\,\frac{\left[(x^2-y^2)\log r - 2xy\,\theta\right]}{r^4} \qquad + \cancel{\frac{2xy^2}{r}}$$

$$8\frac{\partial\mu}{\partial x} = \frac{8x^3-24xy^2}{3r^6} - \frac{12(x^5-6x^3y^2+xy^4)}{3r^8} - \frac{12x}{r^4} + 2\,\frac{2x\log r + \cancel{\cdots}x - 2y\theta +}{r^4}$$

$$- 8\frac{\left[(x^2-y^2)\log r - 2xy\,\theta\right]x}{r^6}$$

$$= x\left[\frac{8x^2-24y^2}{3r^6} - \frac{4(x^4-6x^2y^2+y^4)}{r^8} - \frac{12}{r^4} + \frac{24}{r^4}\right] + \log r\left[\frac{4x}{r^4} - \frac{8(x^2-y^2)x}{r^6}\right]$$

$$+\left(\frac{16x^2y}{r^6} - \frac{4y}{r^4}\right)\theta$$

$$= 4\log r\,\frac{r^2-2x^2+2y^2}{r^6}x + \frac{4\theta}{r^6}y\left[4x^2-x^2-y^2\right] + x\left[\,\cdots\,\right]$$

$$= \frac{4}{r^6}\left[x(3y^2-x^2)\log r + 4y(3x^2-y^2)\,\theta\right] + x\left[\frac{8x^2+24x^2}{3r^6} - \frac{8}{r^4} - \frac{12}{r^4} + \frac{2}{r^4}\right.$$

$$\left. - \frac{4(x^4+6x^4+x^4)}{r^8} + \frac{32x^2}{r^6} - \frac{4}{r^4}\right]$$

$$= -\frac{22}{r^4} + \frac{32\cdot 4x^2}{3r^6} - \frac{32x^4}{r^8}$$

$$\cancel{(v_y) = \frac{1}{48}\left[\frac{-\sin 3\theta + \sin 5\theta}{r}\right] + \frac{\theta\sin\theta + 2\cos\theta - 2\sin 3\theta\log r}{8r}}$$

$$4\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{\partial p}{\partial\alpha} + \frac{\partial p}{\partial\beta} + F$$

$$4\frac{\partial^2 v}{\partial\alpha\partial\beta} = i\left(\frac{\partial p}{\partial\alpha} - \frac{\partial p}{\partial\beta}\right) + G$$

$$4\frac{\partial^2 p}{\partial\alpha\partial\beta} = -\left[\frac{\partial F}{\partial\alpha} + \frac{\partial F}{\partial\beta} + i\frac{\partial G}{\partial\alpha} - i\frac{\partial G}{\partial\beta}\right]$$

$$= -\left[\frac{\partial}{\partial\alpha}(F+iG) + \frac{\partial}{\partial\beta}(F-iG)\right]$$

$$4p = -\int(F+iG)\,d\beta - \int(F-iG)\,d\alpha + \cdots$$

$$4\left(\frac{\partial p}{\partial\alpha} + \frac{\partial p}{\partial\beta}\right) = -\cancel{(F-iG)} - \cancel{(F+iG)} - \int\frac{\partial(F+iG)}{\partial\alpha}d\beta - \int\frac{\partial(F-iG)}{\partial\beta}d\alpha$$

$$4\frac{\partial^2 u}{\partial\alpha\partial\beta} = \frac{F}{2} - \frac{1}{4}\int\frac{\partial}{\partial\alpha}(F+iG)\,d\beta - \frac{1}{4}\int\frac{\partial(F-iG)}{\partial\beta}d\alpha$$

$$u = \frac{1}{8}\int F\,d\alpha\,d\beta - \frac{1}{16}\iint(F+iG)\,d\beta\,d\beta - \frac{1}{16}\iint(F-iG)\,d\alpha\,d\alpha +$$

Poprawione u, v, tak aby spełniały warunek $\operatorname{div} = 0$:

$$((u)) = \frac{1}{16}\left[\frac{1}{6}\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + \frac{1}{6}\left(\frac{\alpha^2}{\beta^3} + \frac{\beta^2}{\alpha^3}\right) - \underbrace{\left(\frac{\log\alpha}{\beta} + \frac{\log\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right)\log\alpha\beta - 2\left(\frac{\log\alpha}{\alpha} + \frac{\log\beta}{\beta}\right)}\right]$$

$$\cancel{\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right)\log\alpha\beta}$$

$$\cancel{(\alpha+\beta)\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\log\alpha\beta}$$

$$= \cancel{8\cos\theta\cdot\frac{\cos 2\theta}{r^2}\log r}$$

$$= \frac{1}{8}\left[\frac{\cos 3\theta + \cos 5\theta}{6r} - \frac{x\log r - y\theta}{r^2}\right.$$
$$\left. - \frac{2\cos 3\theta}{r}\log r + \frac{2(x\log r + y\theta)}{r^2}\right]$$

$$= \cancel{\frac{1}{48}\left[\frac{\cos 3\theta + \cos 5\theta}{r}\right] - \frac{1}{2}\frac{\cos\theta\cos 2\theta}{r}\log r}$$

$$((v)) = \frac{i}{16}\left[\frac{1}{6}\left(\frac{\alpha}{\beta^2} - \frac{\beta}{\alpha^2}\right) + \frac{1}{6}\left(\frac{\beta^2}{\alpha^3} - \frac{\alpha^2}{\beta^3}\right) - \left(\frac{\log\alpha}{\beta} - \frac{\log\beta}{\alpha}\right) - \left(\frac{\beta}{\alpha^2} - \frac{\alpha}{\beta^2}\right)\log\alpha\beta + 3\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)\right]$$

$$= \frac{1}{8}\left[\frac{3\sin\theta}{r} - \frac{\sin 3\theta - \sin 5\theta}{6r} + \frac{\theta x + y\log r}{r^2} - \frac{2\sin 3\theta}{r}\log r\right]$$

$$u = \frac{1}{8\pi}\left[\frac{\cos 3\theta + \cos 5\theta}{6} + (\cos\theta - 2\cos 3\theta)\lg r + 3\theta\sin\theta\right] \qquad + \frac{\cos\theta\lg r + \theta\sin\theta}{r}$$

$$v = \frac{1}{8\pi}\left[\frac{-\sin 3\theta + \sin 5\theta}{6} + 3\sin\theta + (\sin\theta - 2\sin 3\theta)\lg r \cancel{+ \theta\cos\theta}\right] \qquad + \frac{\sin\theta\lg r - \theta\cos\theta}{r}$$

Do tego dodać dowolny ruch potencjalny

$$u = Rf + \cancel{I\varphi}$$

$$v = -If + \cancel{R\varphi}$$

$$\frac{\lg\alpha}{\alpha} = \frac{\cos\theta\lg r + \theta\sin\theta}{r} + i\,\frac{-\sin\theta\lg r + \theta\cos\theta}{r}$$

$$u = \frac{1}{8\pi}\left[\frac{\cos 3\theta + \cos 5\theta}{6} + 2(\cos\theta - \cos 3\theta)\lg r + 4\theta\sin\theta\right]$$

$$v = \frac{1}{8\pi}\left[\frac{-\sin 3\theta + \sin 5\theta}{6} + 3\sin\theta + 2(\sin\theta - \sin 3\theta)\lg r\right]$$

Co dalej? ~~Dodajmy do~~ $\cancel{\begin{cases} u & -\frac{\sin\theta}{3r} \\ v & +\frac{\sin\theta}{3r}\end{cases}}$ Nie można!

Trzeba tak, aby się zerowały $u = v = 0$ dla ściany $\theta = \begin{cases} 0 \\ \pi \end{cases}$ i dla $r = \infty$

Ale czy to jedyne możliwe rozwiązanie?

$\theta = \frac{\pi}{2}$ $\quad u = \frac{1}{8\pi}\left[2\pi\right.$

Więc asymetria! w u

$v = \frac{1}{8\pi}\left[-\frac{1}{3} + 3 + 4\lg r\right]$

$$\int (u\cos\theta + v\sin\theta)\, r\, d\theta = \Big[1 - \underbrace{(\cos\theta\cos 3\theta + \sin\theta\sin 3\theta)}_{1 - \cos 2\theta \;=\; 2\sin^2\theta}\Big]\lg r$$

więc całkowita ilość wypływającego byłaby ∞ !? dla $\lim r = 0$

$\int_0^\pi \cancel{\ldots}$

$$1 - 2\cos 2\theta = 1 - 2(\cos^2 - \sin^2) = \cancel{\sin^2 - \cos^2 + 2\sin^2}$$
$$= 4\sin^2 - 2$$

$$\int_0^{\frac{\pi}{2}} (2\sin^2 - 1)$$

Uogólnienie $\neq 1$

$$u = 2y\, J\left(\frac{2}{\sqrt{z^2-1}}\right) = \frac{\alpha-\beta}{i}\;\frac{\frac{\alpha}{\sqrt{\alpha^2-1}} - \frac{\beta}{\sqrt{\beta^2-1}}}{2i} = \frac{1}{2}(\alpha-\beta)\left(\frac{\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}}\right)$$

$$v = -2x\, J\frac{2}{\sqrt{z^2-1}} + 2J\frac{1}{\sqrt{z^2-1}} = -(\alpha+\beta)\frac{\frac{\alpha}{\sqrt{\alpha^2-1}} - \frac{\beta}{\sqrt{\beta^2-1}}}{2i} + \frac{\frac{1}{\sqrt{\alpha^2-1}} - \frac{1}{\sqrt{\beta^2-1}}}{i}$$

$$\frac{\partial u}{\partial x} = \frac{1}{2}(\alpha-\beta)\left[\frac{1}{\sqrt{\beta^2-1}} - \frac{\beta^2}{\sqrt{\beta^2-1}^3} - \frac{1}{\sqrt{\alpha^2-1}} + \frac{\alpha^2}{\sqrt{\alpha^2-1}^3}\right]$$

$$= \frac{1}{2}(\alpha-\beta)\left[\frac{-1}{\sqrt{\beta^2-1}^3} + \frac{1}{\sqrt{\alpha^2-1}^3}\right]$$

$$\cancel{\frac{\alpha^2}{\sqrt{\alpha^2-1}} - \sqrt{\alpha^2-1}}$$

$$= \frac{\alpha-\beta}{2i}\left(\frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right) + \frac{\sqrt{\beta^2-1}-\sqrt{\alpha^2-1}}{i}$$

$$\frac{\partial v}{\partial y} = \frac{-1}{2}\left[(\alpha+\beta)\left[\frac{1}{\sqrt{\alpha^2-1}} - \frac{\alpha^2}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}} - \frac{\beta^2}{\sqrt{\beta^2-1}^3}\right] - \left[\frac{\alpha}{\sqrt{\alpha^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}^3}\right]\right]$$

$$= \frac{1}{2}(\alpha+\beta)\left[\frac{1}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}^3}\right] - \left[\frac{\alpha}{\sqrt{\alpha^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}^3}\right]$$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ stimmt

$$\frac{\partial u}{\partial y} = \frac{i}{2}\left[2\left(\frac{\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}}\right) + (\alpha-\beta)\left(\frac{-1}{\sqrt{\alpha^2-1}} + \frac{\alpha^2}{\sqrt{\alpha^2-1}^3} - \frac{1}{\sqrt{\beta^2-1}} + \frac{\beta^2}{\sqrt{\beta^2-1}^3}\right)\right]$$

$$= \frac{i}{2}\left[\cancel{\ldots + (\beta-\alpha)\left(\frac{1}{\sqrt{\alpha^2-1}} + \frac{1}{\sqrt{\beta^2-1}}\right) + (\alpha-\beta)\left(\frac{\alpha^2}{\sqrt{\alpha^2-1}^3} + \frac{\beta^2}{\sqrt{\beta^2-1}^3}\right)}\right]$$

$$\frac{\partial v}{\partial x} = \frac{1}{2i}\left[-2\left(\frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right) - (\alpha+\beta)\left(\frac{1}{\sqrt{\alpha^2-1}} - \frac{1}{\sqrt{\beta^2-1}} - \frac{\alpha^2}{\sqrt{\alpha^2-1}^3} + \frac{\beta^2}{\sqrt{\beta^2-1}^3}\right)\right]$$

$$+ \frac{1}{i}\left[\frac{-\alpha}{\sqrt{\alpha^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}^3}\right]$$

$$= \frac{1}{2i}\left\{-2\left(\frac{\alpha^3}{\sqrt{\alpha^2-1}^3} - \frac{\beta^3}{\sqrt{\beta^2-1}^3}\right) - (\alpha+\beta)\left(\frac{-1}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}^3}\right)\right\}$$

$$\frac{\partial u}{\partial y} = \frac{i}{2}\left\{2\left(\frac{\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}}\right) + (\alpha-\beta)\left(\frac{1}{\sqrt{\alpha^2-1}^3} + \frac{1}{\sqrt{\beta^2-1}^3}\right)\right\}$$

$u = -\frac{y}{2}\zeta$

$v = +\frac{x}{2}\zeta + 2\,\mathfrak{I}\,F$

$F = \int (H'' - S'')\,\alpha\, d\alpha$

$= (H' - S')\,\alpha - \int (H' - S')\, d\alpha$

$= \alpha (H' - S') - (H - S)$

$H' - S' = \varphi'(\alpha)$

$\zeta = -4\,\mathfrak{I}\varphi'(\alpha)$

$= 2\,\frac{\varphi'(\beta) - \varphi'(\alpha)}{i}$

$\frac{z^2}{\sqrt{z^2-1}} - \sqrt{z^2-1} = \frac{1}{\sqrt{z^2-1}}$

$\mathfrak{I} F =$

$\cancel{u = -\alpha\, y}$ $\quad u = -\frac{\alpha + \beta}{2i}\,\frac{\varphi'(\beta) - \varphi'(\alpha)}{i} = \frac{\alpha - \beta}{2}\left[\varphi'(\beta) - \varphi'(\alpha)\right]$

$v = \frac{\alpha + \beta}{2}\,\frac{\varphi'(\beta) - \varphi'(\alpha)}{i} + \frac{\alpha\varphi'(\alpha) - \beta\varphi'(\beta)}{i} - \frac{\varphi(\alpha) - \varphi(\beta)}{i}$

$= \frac{\alpha\varphi'(\beta) - \beta\varphi'(\alpha) + \alpha\varphi'(\alpha) - \beta\varphi'(\beta)}{2i} - \frac{\varphi(\alpha) - \varphi(\beta)}{i}$

$= \cancel{\alpha\left[\varphi'(\alpha) + \varphi'(\beta)\right] - \beta\,\varphi'(\alpha)}$

$= \frac{\alpha - \beta}{2i}\left[\varphi'(\alpha) + \varphi'(\beta)\right] - \frac{\varphi(\alpha) - \varphi(\beta)}{i}$

$u^2 = \frac{(\alpha - \beta)^2}{4}\left[\varphi'(\beta) - \varphi'(\alpha)\right]^2 = \frac{\alpha^2 - 2\alpha\beta + \beta^2}{4}\left[\varphi'(\alpha)^2 - 2\varphi'(\alpha)\varphi'(\beta) + \varphi'(\beta)^2\right]$

$2u\frac{\partial u}{\partial x} = \cancel{\frac{2\alpha - 2\beta - 2\alpha + 2\beta}{4}}\left[\cdots\right] + \frac{(\alpha - \beta)^2}{2}\Big[\varphi'(\alpha)\varphi''(\alpha) - \varphi''(\alpha)\varphi'(\beta) -$
$- \varphi'(\alpha)\varphi''(\beta) + \varphi'(\beta)\varphi''(\beta)\Big]$

$u\frac{\partial u}{\partial x} = \frac{(\alpha - \beta)^2}{4}\left[\varphi'(\alpha) - \varphi'(\beta)\right]\left[\varphi''(\alpha) - \varphi''(\beta)\right]$

$\alpha^2 - 2\alpha\beta + \beta^2$

$2\alpha - 2\beta - 2\alpha + 2\beta$

$$\frac{\partial u}{\partial y} = \frac{i}{2}\Big[2\big[\varphi'(\beta) - \varphi'(\alpha)\big] + (\alpha-\beta)\big[\varphi''(\alpha) + \varphi''(\beta)\big]\Big]$$

$$v\frac{\partial u}{\partial y} = \frac{1}{2}\,(\alpha-\beta)\big[\varphi'(\alpha)+\varphi'(\beta)\big]\big[\varphi'(\beta)-\varphi'(\alpha)\big] - \frac{(\alpha-\beta)^2}{4}\big[\varphi'(\alpha)+\varphi'(\beta)\big]\big[\varphi''(\alpha)+\varphi''(\beta)\big]$$

$$- \big[\varphi(\alpha)-\varphi(\beta)\big]\big[\varphi'(\beta)-\varphi'(\alpha)\big] + \frac{\alpha-\beta}{2}\big[\varphi(\alpha)-\varphi(\beta)\big]\big[\varphi''(\alpha)+\varphi''(\beta)\big]$$

$$u\,\cancel{\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}} = -4\, J\varphi \,.\, J\varphi' + 4y\left(J\varphi'\, R\varphi' - J\varphi\, R\varphi''\right) + 4y^2\, R\varphi'\, R\varphi''$$

$$= 4\left(-J\varphi + y\,R\varphi'\right)\left(J\varphi' + yR\varphi''\right)$$

duży mnożnik $\sim \frac{4xy^4}{28}$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = +\big[\varphi(\alpha)-\varphi(\beta)\big]\big[\varphi'(\alpha)-\varphi'(\beta)\big] + \frac{\alpha-\beta}{2}\Big\{\big[\varphi(\alpha)-\varphi(\beta)\big]\big[\varphi''(\alpha)+\varphi''(\beta)\big] -$$

$$-\big[\varphi'(\alpha)+\varphi'(\beta)\big]\big[\varphi'(\alpha)-\varphi'(\beta)\big]\Big\} + \frac{(\alpha-\beta)^2}{4}\Big\{\big[\varphi'(\alpha)-\varphi'(\beta)\big]\big[\varphi''(\alpha)-\varphi''(\beta)\big] -$$

$$\big[\varphi'(\alpha)+\varphi'(\beta)\big]\big[\varphi''(\alpha)+\varphi''(\beta)\big]\Big\}$$

$$\frac{\partial}{\partial x} \quad = \big[\varphi'(\alpha)-\varphi'(\beta)\big]^2 + \big[\varphi(\alpha)-\varphi(\beta)\big]\big[\varphi''(\alpha)-\varphi''(\beta)\big] + \frac{\alpha-\beta}{2}\Big\{\cancel{\big[\varphi'(\alpha)-\varphi'(\beta)\big]\big[\varphi''(\alpha)+\varphi''(\beta)\big]} +$$

$$+\big[\varphi(\alpha)-\varphi(\beta)\big]\big[\varphi'''(\alpha)+\varphi'''(\beta)\big] - \cancel{\big[\varphi''(\alpha)+\varphi''(\beta)\big]\big[\varphi'(\alpha)-\varphi'(\beta)\big]} - \big[\varphi'(\alpha)+\varphi'(\beta)\big]\big[\varphi''(\alpha)-\varphi''(\beta)\big]\Big\} +$$

$$+\left(\frac{\alpha-\beta}{2}\right)^2\Big\{\big[\varphi''(\alpha)-\varphi''(\beta)\big]^2 + \big[\varphi'(\alpha)-\varphi'(\beta)\big]\big[\varphi'''(\alpha)-\varphi'''(\beta)\big] - \big[\varphi''(\alpha)+\varphi''(\beta)\big]^2 - \big[\varphi'(\alpha)+\varphi'(\beta)\big]\big[\varphi'''(\alpha)+\varphi'''(\beta)\big]\Big\}$$

$x+iy$

$-x+iy$

$$v^2 = -\frac{(\alpha-\beta)^2}{4}\left[\varphi'(\alpha)+\varphi'(\beta)\right]^2 - \left[\varphi(\alpha)-\varphi(\beta)\right]^2 + \frac{(\alpha-\beta)}{}\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi(\alpha)-\varphi(\beta)\right]$$

$\cancel{\alpha^2 - 2\alpha\beta + \beta^2}$

$2\alpha - 2\beta + 2\alpha - 2\beta = 4(\alpha-\beta)$

$$\frac{1}{i} v\frac{\partial v}{\partial y} = -\cancel{(\alpha-\beta)\left[\varphi'(\alpha)+\varphi'(\beta)\right]^2} - \frac{(\alpha-\beta)^2}{4}\left[\varphi'(\alpha)\varphi''(\alpha) + \varphi''(\alpha)\varphi'(\beta) - \varphi(\alpha)\varphi''(\beta)\right.$$

$$\left. - \varphi'(\beta)\varphi''(\beta)\right] + \cancel{2\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi(\alpha)-\varphi(\beta)\right]} + \frac{(\alpha-\beta)}{2}\left[\varphi''(\alpha) - \varphi''(\beta)\right]\left[\varphi(\alpha)-\varphi(\beta)\right]$$

$$+\cancel{(\alpha-\beta)\left[\varphi'(\alpha)+\varphi'(\beta)\right]^2} - 2\varphi(\alpha)\varphi'(\alpha) + 2\varphi'(\alpha)\varphi(\beta) - \cancel{2\varphi(\alpha)\varphi'(\beta) + 2\varphi(\beta)\varphi'(\beta)}$$

$$+ \cancel{2\varphi(\alpha)\varphi'(\alpha)} - 2\varphi'(\alpha)\varphi(\beta) + 2\varphi(\alpha)\varphi'(\beta) - \cancel{2\varphi(\beta)\varphi'(\beta)}$$

$$= -\frac{(\alpha-\beta)^2}{4}\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right] + \frac{\alpha-\beta}{2}\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]$$

$$i\, u\frac{\partial v}{\partial x} = \frac{\alpha-\beta}{2}\left[\varphi'(\beta)-\varphi'(\alpha)\right]\left\{\frac{\alpha-\beta}{2}\left[\varphi''(\alpha)+\varphi''(\beta)\right] - \varphi'(\alpha)+\varphi'(\beta)\right\}$$

$$= \frac{(\alpha-\beta)^2}{4}\left[\varphi'(\beta)-\varphi'(\alpha)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right] + \frac{\alpha-\beta}{2}\left[\varphi'(\alpha)-\varphi'(\beta)\right]^2$$

$$i\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = \frac{(\alpha-\beta)^2}{4}\left\{\left[\varphi'(\beta)-\varphi'(\alpha)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right] - \left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\beta)-\varphi''(\alpha)\right]\right\}$$

$$+\frac{\alpha-\beta}{2}\left\{\left[\varphi'(\alpha)-\varphi'(\beta)\right]^2 + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]\right\}$$

$$\varphi''(\alpha)\varphi'(\beta) - \varphi''(\beta)\varphi'(\alpha)$$

$$= \frac{(\alpha-\beta)^2}{\cancel{2}}\left\{\cancel{-2\varphi'(\alpha)\varphi''(\alpha) + 2\varphi'(\beta)\varphi''(\beta)}\right\} +$$

$$+\frac{\alpha-\beta}{2}\left\{\left[\varphi'(\alpha)-\varphi'(\beta)\right]^2 - \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]\right\}$$

$\alpha^2 - 2\alpha\beta + \beta^2$

$2\alpha - 2\beta$

$\cancel{2\alpha\ 2\beta}$

stimmt

$dy_i - \frac{4y^5}{28}$

$$\cancel{\frac{\partial}{\partial y}\left[\quad\right] = (\alpha-\beta)\left\{-2\varphi'(\alpha)\varphi''(\alpha) + 2\varphi'(\beta)\varphi''(\beta)\right\} + \frac{(\alpha-\beta)^2}{2}\left\{-2\varphi'(\alpha)^2 - 2\varphi(\alpha)\varphi''(\alpha)\right.}$$

$$\cancel{\left. -2(\varphi'(\beta))^2 - 2\varphi(\beta)\varphi''(\beta)\right\} + \left[\varphi'(\alpha)-\varphi'(\beta)\right]^2 + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right] +}$$

$$\cancel{(\alpha-\beta)\left\{\varphi'(\alpha)\varphi''(\alpha) - \varphi''(\alpha)\varphi'(\beta) + \varphi'(\alpha)\varphi''(\beta) - \varphi'(\beta)\varphi''(\beta)\right.} + \cancel{\left[\varphi'(\alpha)+\varphi'(\beta)\right]\left[\varphi''(\alpha)-\varphi''(\beta)\right]}$$

$$\cancel{\left. + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\alpha)+\varphi''(\beta)\right]\right\}}$$

$\cancel{\Delta^2_\mu = [\varphi(\alpha) + \psi(\beta)]^2}$

$$\Delta^2 u = \frac{\partial \mu}{\partial x} + F \qquad \Delta^2 v = \frac{\partial \mu}{\partial y} + G \qquad \Delta^2 \mu = -\left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y}\right)$$

$\cancel{4\frac{\partial^2 \mu}{\partial\alpha\partial\beta} = 4\frac{\partial\mu}{\partial\alpha} = -F + iG}$

$$4\frac{\partial \mu}{\partial\alpha} = -\beta\left(\frac{\partial F}{\partial\alpha} + i\frac{\partial G}{\partial\alpha}\right) - F + iG - \frac{\partial}{\partial\alpha}\left[\int F\,d\beta + iG\,d\beta\right]$$

$$4\frac{\partial\mu}{\partial\beta} = -\alpha\left(\frac{\partial F}{\partial\beta} - i\frac{\partial G}{\partial\beta}\right) - F - iG$$

$$4\frac{\partial^2\mu}{\partial\alpha\partial\beta} = -\left(\frac{\partial F}{\partial\alpha} + \frac{\partial F}{\partial\beta} + i\frac{\partial G}{\partial\alpha} - i\frac{\partial G}{\partial\beta}\right)$$

$$16\frac{\partial^2 u}{\partial\alpha\partial\beta} = -\beta\frac{\partial F}{\partial\alpha} - \alpha\frac{\partial F}{\partial\beta} - i\left[\beta\frac{\partial G}{\partial\alpha} - \alpha\frac{\partial G}{\partial\beta}\right] + 2(F$$

$$16u = -\frac{\beta^2}{2}F - \frac{\alpha^2}{2}F - i\left[\frac{\beta^2}{2}G - \frac{\alpha^2}{2}G\right] + 2\iint F\,d\alpha\,d\beta$$

$\cancel{= -\frac{\alpha^2+\beta^2}{2}(F+iG) + 2\iint (F+iG)\,d\alpha\,d\beta + u_0}$

$$16\frac{\partial v}{\partial\alpha\partial\beta} = i\left[-\beta\frac{\partial F}{\partial\alpha} + \alpha\frac{\partial F}{\partial\beta} - i\left[\beta\frac{\partial G}{\partial\alpha} + \alpha\frac{\partial G}{\partial\beta}\right]\right] + 2G$$

$$16v = \frac{\beta^2}{2}G + \frac{\alpha^2}{2}G + i\left[-\frac{\beta^2}{2}F + \frac{\alpha^2}{2}F\right] + 2\iint G\,d\alpha\,d\beta$$

$$u = \frac{1}{32}\left\{-(\alpha^2+\beta^2)F + i(\alpha^2-\beta^2)G + 4\iint F\,d\alpha\,d\beta\right\} + u_0$$

$$v = \frac{1}{32}\left\{(\alpha^2+\beta^2)G + i(\alpha^2-\beta^2)F + 4\iint G\,d\alpha\,d\beta\right\} + v_0$$

$u_0 + u_{10} + u_{20}$

∇u_{20}

∇v_0

$$F = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u}{\partial\alpha}(u+iv) + \frac{\partial u}{\partial\beta}(u-iv)$$

$$G = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial v}{\partial\alpha}(u+iv) + \frac{\partial v}{\partial\beta}(u-iv)$$

$$F+iG = \frac{\partial(u+iv)}{\partial\alpha}(u+iv) + \frac{\partial(u+iv)}{\partial\beta}(u-iv)$$

$$F-iG = \frac{\partial(u-iv)}{\partial\alpha}(u+iv) + \frac{\partial(u-iv)}{\partial\beta}(u-iv)$$

$$\Delta^2\underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}_{\xi} = \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y}$$

$$\Delta^4\psi = $$

$$= \frac{\partial}{\partial\alpha}G + \frac{\partial G}{\partial\beta} - i\frac{\partial F}{\partial\alpha} + i\frac{\partial F}{\partial\beta}$$

$$\mathfrak{F} = -\frac{(\alpha-\beta)^2}{2}\left[\varphi'(\alpha)\varphi''(\beta) + \varphi'(\beta)\varphi''(\alpha)\right] + \frac{\alpha-\beta}{2}\Big\{\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\beta)+\varphi''(\alpha)\right] +$$

$$+\left[\varphi'(\beta)\right]^2 - \left[\varphi'(\alpha)\right]^2\Big\} + \left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi'(\alpha)-\varphi'(\beta)\right]$$

$$i\mathfrak{G} = -\frac{(\alpha-\beta)^2}{2}\left[\varphi'(\alpha)\varphi''(\beta) - \varphi'(\beta)\varphi''(\alpha)\right] + \frac{\alpha-\beta}{2}\Big\{\left[\varphi(\alpha)-\varphi(\beta)\right]\left[\varphi''(\beta)-\varphi''(\alpha)\right] +$$

$$\left[\varphi'(\beta)-\varphi'(\alpha)\right]^2\Big\}$$

$$\cancel{\iint i\mathfrak{G}\,d\alpha\,d\beta = \frac{(\alpha-\beta)^2}{2}\left[\varphi'(\alpha)\varphi(\beta)-\varphi'(\beta)\varphi(\alpha)\right] - (\alpha-\beta)\left[\varphi(\alpha)\Phi(\beta)-\varphi(\beta)\Phi(\alpha)\right] +}$$

$$\cancel{\Phi(\alpha)\Psi(\beta) - \Phi(\beta)\Psi(\alpha)}$$

$$\int \mathfrak{F}\,d\alpha\,d\beta = -\frac{(\alpha-\beta)^2}{2}\left[\varphi(\alpha)\varphi'(\beta) + \varphi'(\alpha)\varphi(\beta)\right] + (\alpha-\beta)\left[\Phi(\alpha)\varphi'(\beta) - \varphi'(\alpha)\Phi(\beta)\right] -$$

$$-\left[\Psi(\alpha)\varphi'(\beta) + \Psi(\beta)\varphi'(\alpha)\right] + \cdots$$

$$\iint \mathfrak{F}\,d\alpha\,d\beta = \iint dr\,d\varphi \begin{vmatrix} \frac{\partial\alpha}{\partial r} & \frac{\partial\alpha}{\partial\varphi} \\ \frac{\partial\beta}{\partial r} & \frac{\partial\beta}{\partial\varphi}\end{vmatrix} = \iint \frac{dr\,d\varphi}{\begin{vmatrix} \frac{\partial r}{\partial\alpha} & \frac{\partial\varphi}{\partial\alpha} \\ \frac{\partial r}{\partial\beta} & \frac{\partial\varphi}{\partial\beta}\end{vmatrix}} = \iint \frac{dr\,d\varphi}{\Big|}$$

$\alpha = re^{i\varphi}$

$\beta = re^{-i\varphi}$

$$\begin{vmatrix} e^{i\varphi} & ire^{i\varphi} \\ e^{-i\varphi} & -ire^{-i\varphi}\end{vmatrix} = -ir - ir \qquad = -2i\iint \mathfrak{F} r\,dr\,d\varphi$$

$$\varphi' = \frac{z}{\sqrt{z^2-1}} \qquad \varphi = \sqrt{z^2-1} \qquad F = \frac{1}{\sqrt{z^2-1}}$$

$$\varphi'' = \frac{1}{\sqrt{z^2-1}} - \frac{z^2}{\sqrt{}^3} = \frac{-1}{\sqrt{z^2-1}^3}$$

$$F = +\frac{(\alpha-\beta)^2}{2}\left[\frac{+\alpha}{\sqrt{\alpha^2-1}\sqrt{\beta^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}\sqrt{\alpha^2-1}^3}\right] + \frac{\alpha-\beta}{2}\left[\left[\sqrt{\alpha^2-1}+\sqrt{\beta^2-1}\right]\left[\frac{1}{\sqrt{\alpha^2-1}^3}+\frac{1}{\sqrt{\beta^2-1}^3}\right]\right.$$

$$\left. + \frac{\beta^2}{\beta^2-1} - \frac{\alpha^2}{\alpha^2-1}\right] + \left[\sqrt{\alpha^2-1} - \sqrt{\beta^2-1}\right]\left[\frac{\alpha}{\sqrt{\alpha^2-1}} - \frac{\beta}{\sqrt{\beta^2-1}}\right]$$

$$= \frac{(\alpha-\beta)^2}{2}\left[\frac{\alpha}{\sqrt{\alpha^2-1}\sqrt{\beta^2-1}^3} + \frac{\beta}{\sqrt{\beta^2-1}\sqrt{\alpha^2-1}^3}\right] + \frac{\alpha-\beta}{2}\left[\frac{-1}{\alpha^2-1} + \frac{\sqrt{\beta^2-1}}{\sqrt{\alpha^2-1}^3} + \frac{1}{\beta^2-1} - \frac{\sqrt{\alpha^2-1}}{\sqrt{\beta^2-1}^3}\right.$$

$$\left. + \frac{\beta^2}{\beta^2-1} - \frac{\alpha^2}{\alpha^2-1}\right] + \alpha + \beta - \beta\frac{\sqrt{\alpha^2-1}}{\sqrt{\beta^2-1}} - \alpha\frac{\sqrt{\beta^2-1}}{\sqrt{\alpha^2-1}}$$

Jednoznaczność rozwiązań ruchu spokojnego stałego

Rayleigh i Helmholtz: ~~[illegible]~~ w obrębie skończonym

rozwiązanie określone przez prędkości na granicy obrębu

[t. zn. przez $u = f(x, y)$
$v = f_2(x, y)$]

W warunkach Poiseuille'a etc. wystarczy jedna funkcya p!

Ogólnie np. naczynie z tłokiem

Stosując mój sposób rozwiązania:

Dotychczas używałem znaku X' w sensie funkcyi rzeczywistej; jeśli jednak dopuścimy (i dla tego dwa typy rozwiązań) funkcye złożone z części rzeczywistej i urojonej $Z' = X' + i Y'$

to stanie się: $\xi + i\eta = 8 Z' = 8(X' + i Y')$

otrzymamy: $\xi = 8(R X' - J Y')$

$\eta = 8(J X' + R Y')$ to znaczy właśnie superpozycye

rozwiązań I i II, zatem w dolnym szeregu:

$u_1 = -\frac{\xi_1 y}{2} + J\int 2X'' z\,dz + J f(z)$	$v_1 = \xi_1 \frac{x}{2} - R\int 2X'' z\,dz + R f(z)$
$u_2 = -\frac{\xi_2 y}{2} + R\int 2Y'' z\,dz + R f(z)$	$v_2 = \frac{\xi_2 x}{2} + J\int 2Y'' z\,dz - J f(z)$
$u_1 = -\frac{\xi_1 y}{2} + \int\frac{X''\alpha\,d\alpha - X''\beta\,d\beta}{i} + \frac{f(\alpha) - f(\beta)}{2i}$	$v_1 = \frac{\xi_1 x}{2} - \int X''\alpha\,d\alpha + X''\beta\,d\beta + \frac{f(\alpha) + f(\beta)}{2}$
$u_2 = -\frac{\xi_2 y}{2} + \int Y''\alpha\,d\alpha + Y''\beta\,d\beta + \frac{f\alpha + f\beta}{2}$	$v_2 = \frac{\xi_2 x}{2} + \int\frac{Y''\alpha\,d\alpha - Y''\beta\,d\beta}{i} - \frac{f(\alpha) - f(\beta)}{2i}$

$$\cancel{u = -\frac{\xi y}{2} + \int \frac{Z''(\alpha)\,\alpha\, d\alpha}{i}} \qquad = R\,Z'(\alpha) + i\,J\,Z'(\alpha) = A(\alpha y) + i\,B(\alpha y)$$

$$Z'(\alpha) = X'(\alpha) + i\,Y'(\alpha) = M'(x,y) + i\,N'(x,y) + i(\mathcal{M}'(x,y) + i\,\mathcal{N}'(x,y))$$

$$Z'(\beta) = X'(\beta) + i\,Y'(\beta) = M'(x,y) - i\,N'(x,y) + i(\mathcal{M}'(x,y) - i\,\mathcal{N}'(x,y))$$

$$u = -\frac{\xi y}{2} + \int \underbrace{\left(\frac{X''_{\alpha}}{i} + Y''(\alpha)\right)}_{\frac{1}{i}[iB + A]} \alpha\, d\alpha + \frac{1}{i}\int \underbrace{\left(-X'' + iY''\right)}_{\frac{1}{i}\left[\underbrace{-M + iN + i\mathcal{M}' + \mathcal{N}'}_{\frac{1}{i}[iB - A]}\right]} \beta\, d\beta$$

$$= -\frac{\xi y}{2} + \int (B + iA)\,\alpha\, d\alpha + (B - iA)\,\beta\, d\beta$$

$$= -\frac{\xi y}{2} + \frac{1}{i}\int \underbrace{(A + iB)}_{f(x+iy) = Z'(\alpha)} \alpha\, d\alpha - \underbrace{(A - iB)}_{f(x-iy)}\beta\, d\beta$$

$$\cancel{\frac{1}{i}\left[f_1(\alpha) + i f_2(\alpha) - f_1(\beta) + i f_2(\beta)\right] = \frac{1}{i}\left[\underbrace{(f_1 + i f_2)}_{f(\alpha+iy)}(\alpha) - \underbrace{(f_1 - i f_2)}_{f(x-iy)}(\beta)\right]}$$

$$= -\frac{\xi y}{2} + 2J\int Z'(\alpha)\,\alpha\, d\alpha + \underbrace{J f_1(\alpha) + R f_2(\alpha)}$$

Czy to może być zawsze wyrażone jako $R\,Z(\alpha)$?

$$R f_2(\alpha) = J[i f_2(\alpha)] \qquad \text{zatem} \quad J f_1(\alpha) + R f_2(\alpha) = J\left[(f_1 + i f_2)\alpha\right]$$

$$-X''_{\alpha} + iY''(\alpha) \quad \alpha\, d\alpha \qquad - X''(\beta) + i\,Y''(\beta)$$

$$- \underset{(+i)}{Z'}(\alpha)\ \alpha\, d\alpha \qquad - \underset{(-i)}{Z'}(\beta)\ \beta\, d\beta = -2R\,Z'(\alpha)\,\alpha\, d\alpha$$

$$R f_1(z) + J f_2(z) = R\left[(f_1 + i f_2)\alpha\right]$$

$$\zeta = 4\left[Z'(\alpha) + Z'(\beta)\right]$$

Zatem rozwiązanie:

$$\zeta + i p = 8 Z'(x+iy)$$

$$u = -\frac{\zeta y}{2} + I\int 2 Z'' z\, dz + \overbrace{I f(z)}^{u_0}$$

$$v = +\frac{\zeta x}{2} - R\int 2 Z'' z\, dz + \overbrace{R f(z)}^{v_0}$$

obejmuje przypadek I i II jeśli się Z' dopuści jako funkcyę złożoną z części rzeczywistej i urojonej

funkcya f będzie jednoznacznie określona jeśli wartości jej na obrębie będą dane

zatem wystarczy to do określenia Z'

Wogóle zatem zadanie jednoznacznie określone jeśli f i prędkości u albo v dane na obrębie. A jeśli u_0, v_0 (pochodzące od potęgi z) nieskończenie może dla ∞ ~~powiększ~~ tożsame zaś one w nieskończoności jednak dane w skutek warunków określone

tylko dla Z' = rzeczyw. funkcyi!

Do obliczenia ψ z danego Z', f

$$u = -i\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right) \qquad v + iu = 2\frac{\partial\psi}{\partial\alpha} = \frac{\zeta\beta}{2} - 2\int Z''_\beta \beta\, d\beta + f(\alpha)$$

$$v = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta} \qquad v - iu = 2\frac{\partial\psi}{\partial\beta} = \frac{\zeta\alpha}{2} - 2\int Z''_\alpha \alpha\, d\alpha + f(\beta)$$

$$\frac{\partial\psi}{\partial\alpha} = \beta\left[Z'(\alpha) + Z'(\beta)\right] - \int Z''(\beta)\,\beta\, d\beta + \tfrac{1}{2} f(\alpha)$$

$$\psi = \beta Z(\alpha) + \alpha\beta Z'(\beta) - \alpha\underbrace{\int Z''(\beta)\,\beta\, d\beta}_{=\beta Z'(\beta) - \int Z'(\beta)} + \tfrac{1}{2}\int f(\alpha)\, d\alpha$$

$$\psi = \beta Z(\alpha) + \alpha Z(\beta) + \frac{1}{2}\int f(\alpha)\, d\alpha + \frac{1}{2}\int f(\beta)\, d\beta$$

Wystarczające warunki: 1). Jeśli f dany (rozkład) w nieskończoności

2). jeśli $-\frac{\zeta y}{2} + I\int\ldots$, $\frac{\zeta x}{2} - R\int\ldots$ są skończone w nieskończ.

3). Jeśli $\lim u, v = 0$ w nieskończoności

wtedy w każdym razie zadanie jednoznacznie określone

$\Delta^2 \Delta^2 \psi = 0$

jeżeli ζ ma być nieciągłe

$$\Delta^2 \psi = f_1(\alpha) + f_2(\beta) = 4\frac{\partial^2 \psi}{\partial\alpha\partial\beta}$$

$$4\psi = \beta \underbrace{\int f_1(\alpha)\,d\alpha}_{F_1(\alpha)} + \alpha \underbrace{\int f_2(\beta)\,d\beta}_{F_2(\beta)} + g_1(\alpha) + g_2(\beta)$$

$$u = -\frac{\partial\psi}{\partial y} = i\left[F_1(\alpha) - F_2(\beta) - \beta F_1'(\alpha) + \alpha F_2'(\beta) + g_1'(\alpha) - g_2'(\beta)\right]$$

$$v = \frac{\partial\psi}{\partial x} = F_1(\alpha) + F_2(\beta) + \beta F_1'(\alpha) + \alpha F_2'(\beta) + g_1'(\alpha) + g_2'(\beta)$$

$$F'(\alpha) = M + iN \quad \Big| \; +x+iy \qquad = 2i(yM - xN) = -2iJ\left[\beta F'(\alpha)\right]$$

$$F(\beta) = M - iN \quad \Big| \; +x+iy \qquad 2(xM + yN) = 2R\left[\beta F'(\alpha)\right]$$

$$u = J\left[g_1(\alpha) - \beta F(\alpha)\right]$$

$$(M+iN)(x-iy) - (M-iN)(x+iy) = $$
$$= 2i(Nx - My)$$
$$(M+iN)(x-iy) + (M-iN)(x+iy) =$$
$$= Mx + Ny$$

$$u = J\left[-G(\alpha) + \beta F(\alpha)\right]$$

$$v = R\left[G(\alpha) + \beta F(\alpha)\right]$$

$$\psi = \beta F(\alpha) + \alpha F(\beta) + g(\alpha) + g(\beta)$$

$$u = -\frac{\partial\psi}{\partial y} = -i\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right) = \frac{1}{i}\left[\beta F'(\alpha) + F(\beta) - F(\alpha) - \alpha F'(\beta) + g'(\alpha) - g'(\beta)\right]$$

$$v = \frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta} = \left[\beta F'(\alpha) + F(\beta) + F(\alpha) + \alpha F'(\beta) + g'(\alpha) + g'(\beta)\right]$$

$$u = 2J\left[\beta F'(\alpha) - F(\alpha) + g'(\alpha)\right]$$

$$v = 2R\left[\beta F'(\alpha) + F(\alpha) + g'(\alpha)\right]$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 4\frac{\partial^2\psi}{\partial\alpha\partial\beta} = 4\left[F'(\alpha) + F'(\beta)\right] = 8RF'(\alpha) = \zeta$$

wychodzi na to samo co moją metodą.

W naszym przykładzie mamy :

3-1

$$Z' = \frac{i\,z}{2\sqrt{z^2-1}} \quad \Bigg| \quad f = -\frac{i}{\sqrt{z^2-1}} \qquad Z = \frac{i}{2}\sqrt{z^2-1}$$

$$\cancel{\int \frac{dz}{\sqrt{z^2-1}} = \lg(z+\sqrt{z^2-1})}.$$

$$\psi = \frac{i}{2}\left[\beta\sqrt{\alpha^2-1} + \alpha\sqrt{\beta^2-1} - \sqrt{\alpha^2-1} + \sqrt{\beta^2-1}\right]$$

$$= \frac{i}{2}\Big[\underbrace{r e^{-i\theta}\sqrt{r_1 r_2}\, e^{\frac{i(\theta_1+\theta_2)}{2}} + r e^{i\theta}\sqrt{r_1 r_2}\, e^{-i\frac{\theta_1+\theta_2}{2}}}_{r\sqrt{r_1 r_2}\left[e^{-i\left(\theta-\frac{\theta_1+\theta_2}{2}\right)} + e^{i\left(\theta-\frac{\theta_1+\theta_2}{2}\right)}\right]} - \sqrt{r_1 r_2}$$

$$- \sqrt{r_1 r_2}\; e$$

Próba $\frac{\partial\psi}{\partial x} = \frac{i}{2}\left[\beta\frac{\alpha}{\sqrt{\alpha^2-1}} + \sqrt{\alpha^2-1} + \sqrt{\beta^2-1} + \frac{\alpha\beta}{\sqrt{\beta^2-1}} - \frac{\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta}{\sqrt{\beta^2-1}}\right]$

$$\cancel{= \frac{i}{2}\left[\frac{(\beta-1)\alpha}{\sqrt{\alpha^2-1}} + \frac{\beta(\alpha-1)}{\sqrt{\beta^2-1}} + \sqrt{\alpha^2-1} + \sqrt{\beta^2-1}\right]}$$

$$= \frac{i}{2}\left[r\cancel{\left(\frac{1}{\sqrt{\alpha^2-1}} + \frac{1}{\sqrt{\beta^2-1}}\right)} - 2\frac{z}{\sqrt{z^2-1}} + 2\sqrt{z^2-1}\right]$$

$$= \frac{i}{2}\Big[$$

$$-\frac{z-1}{z+1} \qquad (z-1)^2$$

$$\psi = \frac{1}{2}\left[\beta\sqrt{\alpha^2-1} + \alpha\sqrt{\beta^2-1} + \frac{1}{2}\left(\lg\frac{(\alpha+1)}{(\alpha-1)} - \lg\frac{\beta+1}{\beta-1}\right)\right]$$

$$\frac{\partial\psi}{\partial x} = \frac{1}{2}\left[\frac{\beta\alpha}{\sqrt{\alpha^2-1}} + \frac{\alpha\beta}{\sqrt{\beta^2-1}} + \sqrt{\alpha^2-1} + \sqrt{\beta^2-1} + \frac{1}{\sqrt{\alpha^2-1}} + \frac{1}{\sqrt{\beta^2-1}}\right]$$

$$= \frac{1}{2}\left[\frac{\alpha(\alpha+\beta)}{\sqrt{\alpha^2-1}} + \frac{\beta(\alpha+\beta)}{\sqrt{\beta^2-1}} - \frac{2}{\sqrt{\alpha^2-1}} + \frac{2}{\sqrt{\beta^2-1}}\right]$$

$$= \frac{1}{2}\Big[2x - 2\beta - \frac{2.2}{\sqrt{r_1 r_2}} \ldots \frac{\theta_1+\theta_2}{2}$$

$p - i\zeta = 8 Z'$ $\qquad$ $\cancel{\ } + \zeta = 4\underline{(Z'\alpha + Z'\beta)}\, i$

$u = -\frac{\zeta y}{2} + R\int 2 Z''z\, dz + R f(z) \quad = -i\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right)$ $\quad i$

$v = \ \ \zeta\frac{x}{2} + J\int \qquad - J f(z) \quad = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta}$

$2\frac{\partial\psi}{\partial\alpha} = \zeta\frac{(x-iy)}{2} + (J + iR)\int 2 Z''z\, dz \ \cancel{\ } + i(R + iJ)f$

$= \zeta\frac{\beta}{2} + 2\int Z''\alpha\, d\alpha + i f(\alpha)$

$\frac{\partial\psi}{\partial\alpha} = i\left[Z'(\alpha) - Z'(\beta)\right]\beta + \alpha Z'(\alpha) - Z + \frac{i}{2} f(\alpha)$

$\psi = i\left[\beta Z(\alpha) - \alpha\beta Z'(\beta)\right] + \alpha Z(\alpha) - 2\int Z(\alpha)\, dz + \frac{i}{2}\int f(\alpha)\, d\alpha$

$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} = 0$ $\qquad$ $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$

$u = \zeta\frac{2}{2}\ldots + \cdots$ $\qquad$ $= a$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right) = 0$ $\qquad$ $\frac{\partial^2 u}{\partial x^2} = -\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right) = -\frac{\partial\zeta}{\partial y} = \frac{\partial p}{\partial x}$

$\cancel{\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} = 0}$ $\frac{\partial}{\partial y}$

$$\frac{\partial p}{\partial x} = \Delta^2 u \qquad \frac{\partial p}{\partial y} = \Delta^2 v \qquad \frac{\partial p}{\partial z} = \Delta^2 w \quad \Bigg\} \quad \Delta^2 p = 0$$

zatem rozkład p jest jednoznacznie określony w obrębie jednowartościowym przez podanie wartości p na powierzchni obrębu [także jeśli obręb ∞] (Kirchhoff p. 189 Lamb pg 47)

to nie jest wystarczające! Implicite zakładamy że przegrody nie sięgają do nieskończoności! zatem tylko przestrzeń ∞ w śród której wydzielone ciała jednowartościowe o ściankach stałych

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Tutaj jednak interesuje nas także inny przypadek: Kanał nieskończony t. j. przestrzeń jednowartościowa, sięgająca do nieskończ. i ograniczona aż do nieskończ. ~~przegrodzającą~~ ścianą. Wtedy p samo przez się równanie $\Delta p = 0$ i warunki graniczne w nieskończ. nie jest dostatecznie określone. Mimo to tylko jedno rozwiązanie możliwe,

Jeśli dwa, to różnica $p_1 - p_2 = P$ będzie nieskończenie mała w ∞

$$\frac{\partial p_1}{\partial x} = \Delta^2 u_1 \qquad \frac{\partial p_2}{\partial x} = \Delta^2 u_2$$

$$\frac{\partial (p_1 - p_2)}{\partial x} = \Delta^2 (u_1 - u_2)$$

$$\xi_1 - \xi_2 = \frac{\partial (v_1 - v_2)}{\partial x} - \frac{\partial (u_1 - u_2)}{\partial y}$$

$p_1 - p_2$, $u_1 - u_2$, $v_1 - v_2$, $w_1 - w_2$, $\xi_1 - \xi_2$ } nieskończenie małe w odległości ∞; ~~nie~~ muszą być nieskończenie małe w ∞?

Więc chodzi o to: czy pod warunkiem ~~nieskończenie małej~~ może powstać w porzyjnym naczyniu ruch skończony w skończonym przekroju?

Funkcya dyssypacyjna Φ

$$\text{Praca } W = \iint (p_{xn} u + p_{yn} v + p_{zn} w)\, dS = \iiint_{-\infty}^{\infty} \Phi \, dx\, dy\, dz$$

$$p_{xn} = p_{xx} \cos nx + p_{xy} \cos ny + p_{xz} \cos nz$$

$$= \left(p - 2\mu \frac{\partial u}{\partial x}\right) \cos nx + \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \cos ny + \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \cos nz$$

Czy p, ξ przez warunki 0 w nieskończ. oznaczone?

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=0$$

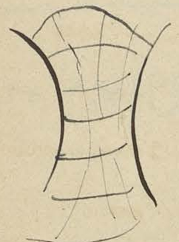
$$\frac{\partial^2 \xi}{\partial x^2}+\frac{\partial^2 \xi}{\partial y^2}=0$$

$$\frac{\partial p}{\partial x}=-\frac{\partial \xi}{\partial y}$$

Zatrzymanie: przestrzeń sięgająca do nieskończoności, (częściowo) ograniczona ścianami stałemi

p i ξ znikające małe w nieskończ. odległości

-przed ciecz odrębny $\varphi + i\psi$ określony temi warunkami jednoznacznie, $=0$ tutaj jednak różnica bo na ścianach nie ma warunku analogicznego do $\frac{\partial \varphi}{\partial n}=0$



w każdym razie: $p+i\xi = f(z) = 0$ dla $z=\infty$

więc jeśli nie ma być wszędzie $=0$ to musi mieć punkty osobliwe w skończoności (ale ewentualnie poza ścianami!)

N. p. $f(z) = \frac{1}{z_1} - \frac{1}{z_2}$ $\quad p = \frac{\cos\theta_1}{r_1} - \frac{\cos\theta_2}{r_2}$

$$\xi = \frac{\sin\theta_2}{r_2} - \frac{\sin\theta_1}{r_1}$$

$$= W = \iint p\,(u\cos nx + v\cos ny + w\cos nz)\,dS +$$

$$\cancel{-2\mu\iint\left(u\frac{\partial u}{\partial x}\,dy\,dz + v\frac{\partial v}{\partial x}\,dx\,dz + w\frac{\partial w}{\partial z}\,dx\,dy\right)dS}$$

$$\underbrace{-\mu\iint\left[u\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) + 2v\frac{\partial v}{\partial y} + w\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]dS.\cos ny}_{} + \cdots$$

$$=\mu\iiint \frac{\partial u}{\partial y}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) + 2\frac{\partial v}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right) \Bigg| + u\left(\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}\right) + 2v\frac{\partial^2 v}{\partial y^2} + w\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 v}{\partial y\partial z}\right)$$

$$\frac{\partial v}{\partial z}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right) + 2\frac{\partial w}{\partial z}\frac{\partial w}{\partial z} + \frac{\partial u}{\partial z}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \Bigg| + v\left(\frac{\partial^2 w}{\partial y\partial z}+\frac{\partial^2 v}{\partial z^2}\right) + 2w\frac{\partial^2 w}{\partial z^2} + u\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\partial z}\right)$$

$$\frac{\partial w}{\partial x}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) \Bigg| + w\left(\frac{\partial^2 u}{\partial z\partial x}+\frac{\partial^2 w}{\partial x^2}\right) + 2u\frac{\partial^2 u}{\partial x^2} + v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial y\partial x}\right)$$

$$W = \iint p(ul+vm+wn)\,dS - \mu\iiint\Big\{2\Big[\Big(\frac{\partial u}{\partial x}\Big)^2+\Big(\frac{\partial v}{\partial y}\Big)^2+\Big(\frac{\partial w}{\partial z}\Big)^2\Big]+\Big(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\Big)^2+\Big(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\Big)^2+\Big(\frac{\partial v}{\cdot}\Big)\Big\}$$

$$\left.\begin{array}{l}\underbrace{-\mu\iiint \cancel{u\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}\Big)} + u\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\Big) \\ \quad +v \\ \quad +w}\end{array}\right\} = -\mu\iiint(u\Delta^2 u+v\Delta^2 v+w\Delta^2 w)\,dx\,dy\,dz$$

$$= \iint p(ul+vm+wn)\,dS - \mu\iiint\Phi\,dx\,dy\,dz - \iiint\Big(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}\Big)\,dx\,dy\,dz$$

$$= \cancel{\iiint p\Big(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\Big)\,dx\,dy\,dz} - \mu\iiint\Phi\,d\tau$$

Z drugiej strony możemy rozdzielić całą przestrzeń w cienkie rurki prądu i podczas przepływu

stacjonarnego zatem ilości $Vq = \text{const}$ w nieskończoności

zatem $(ul+vm+wn)\,dS = V_0\,ds$

Przeprowadzając zatem przekrój w nieskończoności i rozdzielając go zupełnie takich rurek

prądu ażeby $V_0\,ds$ było dla nich jednakowe otrzymamy

$\sum V_0\,ds$ = skończone   jeżeli przekrój skończony w nieskończoności i prędkość

$$W = \sum P.V_0\,ds - \mu\underbrace{\sum\Big(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\Big)ul\,dS}+\dots$$

$$< \sum\Big(2\frac{\partial u}{\partial x}+\dots\Big)V_0\,ds+\dots$$

lepiej obierając końcowe elementa normalnie do prądu!

oznaczając największe wartości przez P_M i DV_M

$$\left.\begin{array}{l}\sum\dots < P_M\sum V_0\,ds \\ \sum\dots < DV_M\sum V_0\,ds\end{array}\right\}$$ każda z nich niech maleje

zatem $\iiint\Phi\,d\tau$ znikające zatem $u\,v\,w = 0$

Do tego ~~dowodu~~ wymagamy : $\lim_{\infty} P = 0$

$$\lim_{\infty}\left(2\frac{\partial V}{\partial n} + \cdots \right) = 0$$

zatem albo $\lim_{\infty} V = 0$ ~~prędkość końcowa w nieskończoności taka że~~ co wymaga ∞ wielkiej końcowej prędkości

albo $\lim_{\infty} V^2 = \text{const}$ co wymaga skończonych prędkości, ale ścianami gładkimi i bardzo wielkiej długości

w razie zachowania wpływu bezwładności

$$W = -\mu \iiint \Phi \, dx\,dy\,dz - \rho \iiint \underbrace{\left[u\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + v\left(u\frac{\partial v}{\partial x} + \cdots\right) + w\left(\cdots\right)\right]}_{\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\frac{u^2+v^2+w^2}{2}\right)} dx\,dy\,dz$$

$$= \iint \frac{V^2}{2}\underbrace{(ul + vm + wn)}_{V_n}\, dS - \cancel{\iiint \frac{V^2}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dx\,dy\,dz}$$

obejmując końcowe elementy $\perp$ prądu $= \iint \frac{V^3}{2}\, dS$ co znika oczywiście pod owemi założeniami

a zatem pozostaje owe rozwiązanie także ważnem przy uwzględnieniu bezwładności.

W tym razie nie można jednak wnioskować z tego o jednoznaczności ogólnego rozwiązania

ponieważ superpozycja nie możliwa!

To samo pozostaje ważnem w razie kanałów połączonych (wielokrotnie wartych sięgających do ∞)

Ale ~~zdaje się że~~ ~~przyjmuje się~~ ogólne zadanie ruchu nie jest ~~będzie~~ określone dostatecznie przez warunek zadania!

$\lim_{\infty} \mathfrak{z} = 0$ $\lim_{\infty} P = f$ dana ~~tylko porządek wielkości~~ (bo prędkość nieskończony jest)

n zupełnie dowolne!

f dowolne!

N. p. $p = -4\frac{\sin n\theta}{r^n}$

$$u = -\frac{2y\cos n\theta}{r^n} - \frac{n}{n-1}\frac{\sin(n-1)\theta}{r^{n-1}} + \mathfrak{I}!$$

$$v = \frac{2x\cos n\theta}{r^n} - \frac{n}{n-1}\frac{\cos(n-1)\theta}{r^{n-1}} + \mathfrak{R}f$$

Tak samo w ogólnym wypadku podanie nawet wartości u, v, w na powierzchni obszaru ∞ wielkiej nie jest wystarczającem do określenia ruchu

Dowód Rayleigha niestosowny

Czy to się zgadza z oczywistemi prawami? Stoi

daje $\left.\begin{matrix}\theta = 0 \\ = \frac{\pi}{2} \\ = \pi\end{matrix}\right\} u = v = 0$

$\lim_{\infty} p = 0!$

O punkt osobliwy gdzie ją skacze!

zatem tam trzeba wszystko ... rozumieć rośnie nawet do ∞

Stosując $\frac{p - i\xi}{4} = \frac{1}{z^2} = \frac{\cos 2\theta}{r^2} - i\frac{\sin 2\theta}{r^2}$ | $\frac{\xi + ip}{4} = \frac{\cos 2\theta}{r^2} - i\frac{\sin 2\theta}{r^2}$

$$\int -\frac{2}{z^2}\,dz = \frac{2}{z} = \frac{2\cos\theta}{r} - i\,\frac{2\sin\theta}{r}$$

$$u = -\,2\,\frac{\sin 2\theta \sin\theta}{r} + \frac{2\cos\theta}{r} \qquad \theta$$

$$v = 2\,\frac{\sin 2\theta\cos\theta}{r} - 2\,\frac{\sin\theta}{r}$$

$$u = -2\,\frac{\cos 2\theta \sin\theta}{r} - \frac{2\sin\theta}{r}$$

$$v = \frac{2\cos 2\theta\cos\theta}{r} - \frac{2\cos\theta}{r}$$

$$\xi = \frac{\lg r \cos 2\theta + \theta \sin 2\theta}{r^2}$$

$$u = 2\,\frac{\lg r \cos 2\theta + \theta \sin 2\theta}{r}\sin\theta + \ldots$$

$$v = 2\,\frac{\lg r \cos 2\theta + \theta\sin 2\theta}{r}\cos\theta - \frac{2(\cos\theta \lg r + \theta\sin\theta) + \cos\theta}{r}$$

$$\theta=\frac{\pi}{2};\quad p=-\frac{1}{4r^2}\left[3-\frac{1}{3}\right]$$
$$\theta=\frac{3\pi}{2}$$
$$\theta=\frac{\pi}{4};\quad p=-\frac{1}{4r^2}\left[3+\frac{1}{3}-\pi+2\pi\right]$$
$$=-\frac{1}{4r^2}\left[3+\frac{1}{3}+\pi\right]$$

$$p=-\frac{1}{4r^2}\left[3-\frac{\cos 4\theta}{3}-4\theta\sin 2\theta+2\pi\sin 2\theta\right]$$

$$2\,\frac{\cos 2\theta\log r+\theta\sin 2\theta}{r^2}=\frac{\log\alpha}{\alpha^2}+\frac{\log\beta}{\beta^2}$$

$$2\,\frac{\theta\sin 2\theta}{r^2}=\frac{\log\alpha}{\alpha^2}+\frac{\log\beta}{\beta^2}-\frac{1}{2}\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)\log\alpha\beta$$

$$4\theta\,\frac{\sin 2\theta}{r^2}=\left[\log\alpha-\log\beta\right]\left[\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right]$$

$$2\,\frac{\cos 4\theta}{r^2}=\frac{\alpha}{\beta^3}+\frac{\beta}{\alpha^3}$$

$$\frac{1}{r^2}=\frac{1}{\alpha\beta}$$

$$2\,\frac{\sin 2\theta}{r^2}=\frac{1}{i}\left[\frac{1}{\beta^2}-\frac{1}{\alpha^2}\right]$$

niezwr... co do względności p!

Biorę $\quad \frac{p-i\xi}{4}=\frac{\log z}{z^2}=\frac{\log r\cos 2\theta+\theta\sin 2\theta}{r^2}+i\,\frac{\theta\cos 2\theta-\log r\sin 2\theta}{r^2}$

$$\int\left(\frac{1}{z^2}-\frac{2\log z}{z^2}\right)dz=-\frac{1}{z}+\frac{2}{z}(\log z+1)=\frac{2\log z}{z}+\frac{1}{z}$$
$$=\frac{2(\log r\cdot\cos\theta+\theta\sin\theta)+\cos\theta}{r}+i\,\frac{2(-\sin\theta\log r+\theta\cos\theta)-\sin\theta}{r}$$

$$u=2\,\frac{\left|\theta\cos 2\theta-\log r\sin 2\theta\right|}{r^2}\sin\theta+\frac{2(\log r\cos\theta+\theta\sin\theta)+\cos\theta}{r}$$

$$v=-2\,\frac{\left[\theta\cos 2\theta-\log r\sin 2\theta\right]\cos\theta}{r}+\frac{2(-\log r\sin\theta+\theta\cos\theta)-\sin\theta}{r}$$

$$-\cancel{2\sin 2\theta\cos\theta}\cos\theta+2\cos^2\theta+\cancel{2\sin 2\theta\cos\theta}\sin\theta-2\sin^2\theta=2\int_0^{\pi}\cos 2\theta\,d\theta=\int_0^{2\pi}\cos\varphi\,d\varphi=0$$

$$\int_0^{\pi}\theta\sin 2\theta\,d\theta=-\frac{\cos 2\theta\cdot\theta}{2}\Big|_0+\frac{1}{2}\int_0^{\pi}\cos 2\theta\,d\theta=0$$

$\cancel{-\Delta^2 p = t}$

$$4\frac{\partial^2 p}{\partial\alpha\partial\beta} = -\left(\frac{\partial F}{\partial\alpha} + \frac{\partial F}{\partial\beta} + i\frac{\partial G}{\partial\alpha} - i\frac{\partial G}{\partial\beta}\right)$$

$$4\frac{\partial p}{\partial\alpha} = -F + iG - \frac{\partial}{\partial\alpha}\int(F+iG)\,d\beta$$

$$4\frac{\partial p}{\partial\beta} = -F - iG - \frac{\partial}{\partial\beta}\int(F-iG)\,d\alpha$$

$$16\frac{\partial^2 u}{\partial\alpha\partial\beta} = 2F - \frac{\partial}{\partial\alpha}\int(F+iG)\,d\beta - \frac{\partial}{\partial\beta}\int(F-iG)\,d\alpha$$

$$16u = 2\iint F\,d\alpha\,d\beta - \iint(F+iG)\,d\beta^2 - \iint(F-iG)\,d\alpha^2$$

$$16\frac{\partial^2 v}{\partial\alpha\partial\beta} = 2G - i\frac{\partial}{\partial\alpha}\int(F+iG)\,d\beta + i\frac{\partial}{\partial\beta}\int(F-iG)\,d\alpha$$

$$16v = 2\iint G\,d\alpha\,d\beta - i\iint(F+iG)\,d\beta^2 + i\iint(F-iG)\,d\alpha^2$$

$$16u = -\iint F\left[d\alpha^2 - 2\,d\alpha\,d\beta + d\beta^2\right] + i\iint G\,(d\alpha^2 - d\beta^2)$$

$$16v = \iint G\left[d\alpha^2 - 2\,d\alpha\,d\beta + d\beta^2\right] + i\iint F(d\alpha^2 - d\beta^2)$$

$$\Delta^2 u = \frac{\partial p}{\partial x} + F \qquad u = -\frac{\partial \psi}{\partial y}$$

$$\Delta^2 v = \frac{\partial p}{\partial x} + G \qquad v = \frac{\partial \psi}{\partial x}$$

$$\Delta^2\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{\partial G}{\partial y} - \frac{\partial F}{\partial x}$$

$$\Delta^2 \Delta^2 \psi = \qquad = 16 \frac{\partial^4 \psi}{\partial \alpha^2 \partial \beta^2} = \frac{\partial}{\partial \alpha}(iG - F) + \frac{\partial}{\partial \beta}(-iG - F)$$

$$F = -\frac{4y^4 x}{r^8} \qquad F - iG = +\frac{4y^4}{r^8}(x - iy) = -\frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 \beta$$

$$G = -\frac{4y^5}{r^8} \qquad F + iG = -\frac{4y^4}{r^8}(x+iy) = \frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^4 \alpha$$

$$2\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^3\left(-\frac{\beta}{\alpha^2} + \frac{\alpha}{\beta^2}\right) = 2\left(\frac{\beta-\alpha}{\alpha\beta}\right)^3 \frac{\alpha^3 - \beta^3}{\alpha^2\beta^2}$$

$$= \frac{2}{(\alpha\beta)^5} \begin{array}{l} \beta^3\alpha^3 - 3\beta^2\alpha^4 + 3\beta\alpha^5 - \alpha^6 \\ -\beta^6 + 3\beta^5\alpha - 3\beta^4\alpha^2 + \beta^3\alpha^3 \end{array}$$

$$16\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} = 2\left[\frac{2}{(\alpha\beta)^2} - 3\left(\frac{1}{\alpha\beta^3} + \frac{1}{\alpha^3\beta}\right) + 3\left(\frac{1}{\alpha^4} + \frac{1}{\beta^4}\right) - \left(\frac{\alpha}{\beta^5} + \frac{\beta}{\alpha^5}\right)\right]$$

$$\psi = \frac{1}{8}\left[2\log\alpha \cdot \log\beta - \frac{3}{2}\left[\frac{\alpha}{\beta}(\log\alpha - 1) + \frac{\beta}{\alpha}(\log\beta - 1)\right] + \frac{1}{4}\left(\frac{\beta^2}{\alpha^2} + \frac{\alpha^2}{\beta^2}\right) - \frac{1}{72}\left(\frac{\alpha^3}{\beta^3} + \frac{\beta^3}{\alpha^3}\right)\right]$$

$$= \frac{1}{8}\left[2\left[(\log r)^2 + \theta^2\right] - \frac{3}{}\left[\log r \cos 2\theta - \theta \sin 2\theta\right] - 3\cos 2\theta + \frac{1}{4}\cos 4\theta - \frac{1}{72}\cos 6\theta\right]$$

$$v=\frac{1}{8}\Big[2\Big(\frac{\log\beta}{\alpha}+\frac{\log\alpha}{\beta}\Big)-\frac{3}{2}\Big(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}\Big)-\cancel{\frac{\alpha}{\beta}\frac{1}{\alpha}}+\frac{3}{2}\Big(\frac{\alpha\log\alpha}{\beta^2}+\frac{\beta\log\beta}{\alpha^2}\Big)-\cancel{\frac{3}{2}\Big(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\Big)}$$

$$\cancel{\frac{1}{2}}\Big(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\Big)-\frac{1}{2}\Big(\frac{\beta^2}{\alpha^3}+\frac{\alpha^2}{\beta^3}\Big)-\frac{1}{24}\Big(\frac{\alpha^2}{\beta^3}+\frac{\beta^2}{\alpha^3}\Big)+\frac{1}{24}\Big(\frac{\alpha^3}{\beta^4}+\frac{\beta^3}{\alpha^4}\Big)\Big]$$

$$=\frac{1}{8}\Big[\frac{1}{2}\Big(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}\Big)+\frac{3}{2}\Big(\frac{\alpha\log\alpha}{\beta^2}+\frac{\beta\log\beta}{\alpha^2}\Big)-\Big(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\Big)-\frac{13}{24}\Big(\frac{\alpha^2}{\beta^3}+\frac{\beta^2}{\alpha^3}\Big)$$

$$+\frac{1}{24}\Big(\frac{\alpha^3}{\beta^4}+\frac{\beta^3}{\alpha^4}\Big)\Big]$$

$$\beta F(\alpha)+\alpha F(\beta)\quad+F(\alpha)+F(\beta)\quad+g(\alpha)+g(\beta)$$

$$-\frac{3}{2}\Big(\beta\frac{\log\alpha}{\alpha^2}+\alpha\frac{\log\beta}{\beta^2}\Big)+\frac{3}{2}\Big(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\Big)+\frac{3}{2}\Big(\frac{\log\alpha}{\alpha}+\frac{\log\beta}{\beta}\Big)-2\Big(\frac{\log\alpha}{\alpha}+\frac{\log\beta}{\beta}\Big)$$

$$u=-\frac{i}{8}\Big[2\Big(\frac{\log\beta}{\alpha}-\frac{\log\alpha}{\beta}\Big)-\frac{3}{2}\Big(\frac{\log\alpha}{\beta}-\frac{\log\beta}{\alpha}\Big)+\frac{3}{2}\Big(\frac{\beta\log\beta}{\alpha^2}-\frac{\alpha\log\alpha}{\beta^2}\Big)+2\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)$$

$$+\cancel{\frac{1}{2}\Big(\frac{\alpha}{\beta^2}-\frac{\beta}{\alpha^2}\Big)}-\cancel{\frac{1}{2}\Big(\frac{\beta^2}{\alpha^3}-\frac{\alpha^2}{\beta^3}\Big)}+\frac{11}{24}\Big(\frac{\alpha^2}{\beta^3}-\frac{\beta^2}{\alpha^3}\Big)+\frac{1}{24}\Big(\frac{\beta^3}{\alpha^4}-\frac{\alpha^3}{\beta^4}\Big)\Big]$$

$$+\frac{1}{i}\Big[\Big(\beta\frac{\log\alpha}{\alpha^2}-\alpha\frac{\log\beta}{\beta^2}\Big)\frac{3}{2}+\frac{3}{2}\Big(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\Big)+\frac{3}{2}\Big(\frac{\log\beta}{\beta}-\frac{\log\alpha}{\alpha}\Big)-2\Big(\frac{\log\alpha}{\alpha}-\frac{\log\beta}{\beta}\Big)\Big]$$

$$v=\frac{1}{4r}\Big[\frac{1}{2}\big(\log r.\cos\theta-\theta\sin\theta\big)+\frac{3}{2}\big(\log r\cos3\theta-\theta\sin3\theta\big)-\cos3\theta-\frac{13}{24}\cos5\theta$$

$$+\frac{1}{24}\cos7\theta$$

$$-\frac{3}{2}\big(\log r.\cos3\theta+\theta\sin3\theta\big)+\frac{3}{2}\cos3\theta+\frac{1}{2}\big(\log r.\cos\theta+\theta\sin\theta\big)$$

$$u=\frac{1}{4r}\Big[-\frac{7}{2}\big(\theta\cos\theta+\sin\theta\log r\big)-\frac{3}{2}\big(\theta\cos3\theta+\sin3\theta\log r\big)+2\sin3\theta+\frac{11}{24}\sin5\theta$$

$$-\frac{1}{24}\sin7\theta$$

$$+\frac{3}{2}\big(\sin3\theta\log r-\theta\cos3\theta\big)-\frac{3}{2}\sin3\theta-\frac{7}{2}\big(\theta\cos\theta-\sin\theta\log r\big)\Big]$$

$\theta=0\qquad u=v=0$

$\theta=\frac{\pi}{2}\qquad u=0$

$\theta=\pi\qquad u=10\pi\quad,v=0$

$$\Delta^2\Delta^2\psi = \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} = \frac{\partial}{\partial\alpha}(G - iF) + \frac{\partial}{\partial\beta}(G+iF)$$

$$G - iF = -\frac{4y^4}{r^8}(y-ix) = -\frac{1}{2}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^4\frac{\alpha}{i}$$

$$G + iF = -\frac{4y^4}{r^8}(y+ix) = +\frac{1}{2}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^4\frac{\beta}{i}$$

$$i\Delta^4\psi = -\frac{1}{2}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^4 + \frac{1}{2}\left(\quad\right)^4 + 2\frac{\alpha}{\alpha^2}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^3 + \frac{2\beta}{\beta^2}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^3$$

$$= 2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^3$$

$$= 2\left(\begin{array}{l}\frac{1}{\alpha^4} - \frac{3}{\alpha^3\beta} + \frac{3}{\alpha^2\beta^2} - \frac{1}{\alpha\beta^3} \\ \frac{1}{\alpha^3\beta} - \frac{3}{\alpha^2\beta^2} + \frac{3}{\alpha\beta^3} - \frac{1}{\beta^4}\end{array}\right)$$

$$16\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}i = 2\left[\left(\frac{1}{\alpha^4}-\frac{1}{\beta^4}\right) - 2\left(\frac{1}{\alpha^3\beta} - \frac{1}{\alpha\beta^3}\right)\right]$$

$$\psi = \frac{1}{8i}\left[\frac{1}{4}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right) - \left(\frac{\beta\log\beta}{\alpha} - \frac{\alpha\log\alpha}{\beta}\right) + \left(\frac{\beta}{\alpha}-\frac{\alpha}{\beta}\right)\right]$$

$$= -\frac{1}{8}\left[\frac{1}{2}\sin 4\theta + 2\sin 2\theta - 2(\theta\cos 2\theta + \log r\sin 2\theta)\right]$$

$$2(-\theta\cos 2\theta + \log r\sin 2\theta) = \left[\frac{\beta\log\alpha}{\alpha} - \frac{\alpha\log\beta}{\beta}\right]$$

$$+4\theta \qquad = 4i\left[\log\alpha - \log\beta\right]$$

$$(\psi) = -\frac{1}{8}\left[\frac{1}{2}\sin 4\theta + 2\sin 2\theta + 4\theta(1-\cos 2\theta)\right]$$

$$v = \frac{1}{8i}\Big[-\frac{1}{2}\left(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\right) - \frac{1}{2}\left(\frac{\beta^2}{\alpha^3}-\frac{\alpha^2}{\beta^3}\right) - \left(\frac{\log\beta}{\alpha}-\frac{\log\alpha}{\beta}\right) - \cancel{\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)} + \left(\frac{\beta\log\beta}{\alpha^2}-\frac{\alpha\log\alpha}{\beta^2}\right)$$

$$+ \cancel{\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)} - \cancel{\left(\frac{\beta}{\alpha^2}-\frac{\alpha}{\beta^2}\right)}\Big]$$

$$= \frac{1}{8r}\Big[\sin 3\theta + \sin 5\theta + 2\left[\theta\cos\theta + \log r\sin\theta\right] - 2\left[\theta\cos 3\theta + \log r\sin 3\theta\right]\Big]$$

$$u = -\frac{1}{8}\Big[-\frac{3}{2}\left(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\right) - \frac{1}{2}\left(\frac{\beta^2}{\alpha^3}+\frac{\alpha^2}{\beta^3}\right) + \left(\frac{\log\alpha}{\beta}+\frac{\log\beta}{\alpha}\right) + \cancel{\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)} + \left(\frac{\beta\log\beta}{\alpha^2}+\frac{\alpha\log\alpha}{\beta^2}\right)$$

$$- \cancel{\left(\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}\right)} - \cancel{\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)}\Big]$$

$$= \frac{1}{8r}\Big[3\cos 3\theta + \cos 5\theta + 2\left(\log r\cos\theta - \theta\sin\theta\right) - 2\left(\log r\cos 3\theta - \theta\sin 3\theta\right)\Big]$$

$$\cancel{-4\cos 2\theta}$$

$$u = \frac{\partial\psi}{\partial y} = \frac{1}{8}\left[2\cos 4\theta + \cancel{4\cos 2\theta} + 4 + 8\theta\sin 2\theta\right]\frac{\cos\theta}{r}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{8}\left[\qquad\right]\frac{\cos\theta}{r}$$

$\theta = 0$	$\theta = \frac{\pi}{2}$	$\theta = \pi$
$u = 4(1-\log r)$	$u = 0$	$u = -4(1-\log r)$
$v = 0$	$v =$	$v = 0$

$+ 2(2) - 6(5)$

$$F = -\frac{4x^5}{r^8} \qquad G - iF = -\frac{4x^4}{r^8}(y - ix) = -\frac{1}{2i}\frac{(\alpha+\beta)^4}{(\alpha\beta)^4}\,\alpha = -\frac{\alpha}{2i}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^4$$

$$G = -\frac{4x^4 y}{r^8} \qquad G + iF = -\frac{4x^4}{r^8}(y + ix) = \frac{1}{2i}\frac{(\alpha+\beta)^4}{(\alpha\beta)^4}\,\beta = \frac{\beta}{2i}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^4$$

$$\frac{2}{i}\left[+\frac{\alpha}{\alpha^2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^3 - \frac{\beta}{\beta^2}\left(-\right)\right] = \frac{2}{i}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^3 = 16\,\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}$$

$$\frac{1}{\alpha^4} + \frac{3}{\alpha^3\beta} + \cancel{\frac{3}{\alpha^2\beta^2}} + \frac{1}{\alpha\beta^3} - \frac{1}{\alpha^3\beta} - \cancel{\frac{3}{\alpha^2\beta^2}} - \frac{3}{\alpha\beta^3} - \frac{1}{\beta^4}$$

$$\frac{1}{\alpha^4} - \frac{1}{\beta^4} + 2\left(\frac{1}{\alpha^3\beta} - \frac{1}{\alpha\beta^3}\right) = 8i\,\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2}$$

$$\psi = \frac{1}{8i}\left[\frac{1}{\cancel{1}2}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right) + \left(\frac{\beta\log\beta}{\alpha} - \frac{\alpha\log\alpha}{\beta}\right) - \left(\frac{\beta}{\alpha}-\frac{\alpha}{\beta}\right)\right]$$

$+\frac{1}{3}\sin 2\theta$

$$= -\frac{1}{8}\left[+\frac{1}{\cancel{6}}\sin 4\theta \;\cancel{+}\,2\sin 2\theta + 2(\theta\cos 2\theta + \log r \sin 2\theta)\right] + \Big[\qquad\Big]$$

$(\theta\cos 2\theta - \log r \sin 2\theta)$

$+\frac{1}{3}\cos\theta$

$-2\sin 3\theta + \cancel{2\sin\theta} - \cancel{\ldots} - \frac{8}{3}\sin\theta$

$$v = \frac{1}{8r}\left[\frac{\sin 3\theta + \sin 5\theta}{3} - 2\left[\theta\cos\theta + \log r\sin\theta\right] + 2\left[\theta\cos 3\theta + \log r \sin 3\theta\right]\right]$$

$\cancel{2\cos\theta} - 2\cos 3\theta + \frac{8}{3}\cos\theta$

$$u = \frac{1}{8r}\left[\frac{\cos 3\theta + \cos 5\theta}{3} + 2\left[\log r\cos\theta - \theta\sin\theta\right] + 2\left[\log r\cos 3\theta - \theta\sin 3\theta\right]\right]$$

$\theta = 0 : \; v = 0$

$\theta = \frac{\pi}{2} : \; v = -4\log r$

$u = 0$

$\theta = -\frac{\pi}{2} : \; v = 4\log r$

$u = 0$

$$\frac{\xi + i\eta}{4} = \log\frac{2}{z^2} = \frac{\log r\cos 2\theta + \theta\sin 2\theta}{r^2} + i\,\frac{\theta\cos 2\theta - \log r\sin 2\theta}{r^2}$$

$$u = -\frac{2\left[\log r\cos 2\theta + \theta\sin 2\theta\right]\sin\theta}{r} + \frac{2(-\sin\theta\log r + \theta\cos\theta) - 2\sin\theta}{r}$$

$$v = \frac{2\left[\log r\cos 2\theta + \theta\sin 2\theta\right]\cos\theta}{r} - \frac{2(\cos\theta\log r + \theta\sin\theta) \cancel{+\cos\theta}}{r}$$

$\theta = 0:\quad u = 0,\ v = 0$	$\theta = \frac{\pi}{2}:\quad u = \frac{2\log r - 2}{r},\ v = -\frac{\pi}{r}$	$\theta = -\frac{\pi}{2}:\quad u = -\frac{2\log r + 2}{r},\ v = -\frac{\pi}{r}$

$$u = -\frac{2\sin\theta\left[(\frac{\pi}{2}-\theta)\cos 2\theta - \log r\sin 2\theta\right]}{r} + \frac{2\left(-\log r\cos\theta + (\frac{\pi}{2}-\theta)\sin\theta\right) - 2\cos\theta}{r}$$

$$v = \frac{2\cos\theta\left[-(\frac{\pi}{2}-\theta)\cos 2\theta - \log r\sin 2\theta\right]}{r} + \frac{2\left(\log r\sin\theta + (\frac{\pi}{2}-\theta)\cos\theta\right) + \cancel{\sin\theta}}{r}$$

$$u = -\frac{2\sin\theta\left[\theta\cos 2\theta - \log r\sin 2\theta\right]}{r} - \frac{2(\log r\cos\theta + \theta\sin\theta) + 2\cos\theta}{r} \qquad \cancel{+2\pi\sin\theta\cos^2\theta}$$

$$v = 2\cos\theta\left[\theta\cos 2\theta - \log r\sin 2\theta\right] + 2(\log r\sin\theta - \theta\cos\theta) + \cancel{\sin\theta} \qquad \cancel{+2\pi\cos\theta\sin^2\theta}$$

$\theta = 0:\quad u = -2\log r + 2,\ v = 0$	$\theta = \frac{\pi}{2}:\quad u = 0,\ v = 2\log r$	$\theta = -\frac{\pi}{2}:\quad u = 0,\ v = -2\log r$

$$v = \frac{1}{8r}\left[\frac{\overset{-2\sin 3\theta - \frac{8}{3}\sin\theta}{\sin 3\theta + \sin 5\theta}}{3} - 16.\theta.\sin^2\theta\cos\theta\right] \cancel{+4\pi\sin^2\theta\cos\theta}$$

$$u = \frac{1}{8r}\left[\frac{\overset{-2\cos 3\theta + \frac{8}{3}\cos\theta}{\cos 3\theta + \cos 5\theta}}{3} - 16.\theta.\sin\theta\cos^2\theta + 4\cos\theta\right] \cancel{-4\pi\sin\theta\cos^2\theta}$$

$\theta = 0, \pm\frac{\pi}{2}:\quad u = v = 0$

$$\psi = \int (v\cos\varphi - u\sin\varphi)\, r\, d\varphi =$$

$$-\frac{1}{8}\int\Big[\underbrace{(\sin 3\varphi\cos\varphi - \cos 3\varphi\sin\varphi)}_{\frac{\sin 2\varphi}{3}} - \frac{2\cos 3\varphi\sin\varphi}{3} + \underbrace{(\sin 5\varphi\cos\varphi - \cos 5\varphi\sin\varphi)}_{\frac{\sin 4\varphi}{3}} - 4\cos\varphi\sin\varphi\Big]\, d\varphi$$

$$= 2\sin 3\varphi\cos\varphi \qquad = \frac{2}{3}\sin 2\varphi \cdot 0 = \int_0^{\varphi} \qquad \frac{10}{3}\sin\frac{\varphi}{2} \qquad = \frac{\sin^2\varphi}{2}$$

$$v = \frac{1}{16i}\left[\left(\frac{\alpha}{\beta^2} - \frac{\beta}{\alpha^2}\right) + \frac{1}{3}\left(\frac{\alpha^2}{\beta^3} - \frac{\beta^2}{\alpha^3}\right) - \frac{4}{\pi}(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right] - 2\left(\frac{\alpha}{\beta^2} - \frac{\beta}{\alpha^2}\right) - \frac{8}{3}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$

$$u = \frac{1}{16}\left[\frac{1}{3}\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + \frac{1}{3}\left(\frac{\alpha^2}{\beta^3} + \frac{\beta^2}{\alpha^3}\right) + \frac{2}{\pi}(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 4\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right]$$

$$-2\left(\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}\right) + \frac{8}{3}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ stimmt!

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial \alpha}(v - iu) + \frac{\partial}{\partial \beta}(v + iu)$$

$$-2\frac{\alpha}{\beta^2} + \frac{8}{3}\frac{1}{\alpha}$$

$$v - iu = \frac{1}{8i}\left[\frac{\cancel{\ldots}\frac{\beta}{\alpha^2} + \cancel{\ldots} + \frac{\alpha^2}{\beta^3}}{3} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\frac{1}{\beta}\right]$$

$$v + iu = \frac{1}{8i}\left[\frac{-\frac{\alpha}{\beta^2}\ \cancel{\ldots} - \frac{\beta^2}{\alpha^3}}{3} - 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) - 2(\lg\alpha - \lg\beta)\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\frac{1}{\alpha}\right]$$

$$+2\frac{\beta}{\alpha^2} - \frac{8}{3}\frac{1}{\beta}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{1}{8i}\left[\frac{\cancel{\ldots} - 2\left(\frac{\beta}{\alpha^3} - \frac{\alpha}{\beta^3}\right) + 2\left(\frac{\alpha}{\beta^3} - \frac{\beta}{\alpha^3}\right)}{3} - 2\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right) - 2\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)\right.$$

$$+\frac{8}{3}\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)$$

$$\left. + \underbrace{2\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\left(\frac{1}{\alpha\beta} + \frac{1}{\beta\alpha}\right)}_{\cancel{\ldots}\ 4\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right)} + 2(\lg\alpha - \lg\beta)\left[\frac{1}{\beta^2} + \frac{1}{\alpha^2} + \frac{1}{\alpha^2} + \frac{1}{\beta^2}\right]\right]$$

$$\zeta = \frac{1}{8i}\left[\frac{20}{3}\left(\frac{1}{\beta^2} - \frac{1}{\alpha^2}\right) + \frac{4}{3}\left(\frac{\alpha}{\beta^3} - \frac{\beta}{\alpha^3}\right) + 4(\lg\alpha - \lg\beta)\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\right] = \Delta^2\varphi$$

$$= 4\frac{\partial^2\varphi}{\partial\alpha\partial\beta}$$

$$\frac{\partial^2\zeta}{\partial\alpha\partial\beta} = \frac{1}{8i}\left[\cancel{4}\left(\frac{1}{\alpha^3} - \frac{1}{\beta^3}\right) + 8\left(\frac{1}{\beta\alpha^3} - \frac{1}{\alpha\beta^3}\right)\right]$$

$$\cancel{\psi = \frac{1}{8i}\left[\frac{1}{3}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)+\frac{1}{12}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right)+(\log\beta-\log\alpha)\left(\frac{\beta}{\alpha}+\frac{\alpha}{\beta}\right)\right]}$$

$$\cancel{= \frac{1}{4}\left[\frac{1}{3}\sin 2\theta - \frac{1}{12}\sin 4\theta + 2\theta\cos 2\theta\right]}$$

$$\psi = -\frac{1}{8}\left[\frac{1}{6}\sin 4\theta - \frac{5}{3}\sin 2\theta + 4\,\theta\cos 2\theta\right]$$

$$= -\frac{1}{8i}\left[\frac{1}{12}\left(\frac{\alpha^2}{\beta^2}-\frac{\beta^2}{\alpha^2}\right)-\frac{5}{6}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)+(\log\alpha-\log\beta)\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)\right]$$

$$\psi = \frac{1}{8i}\left[\frac{1}{12}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right)+\frac{5}{6}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)+\left(\frac{\alpha\log\alpha}{\beta}+\beta\frac{\log\beta}{\alpha}-\beta\frac{\log\alpha}{\alpha}+\frac{\alpha\log\beta}{\beta}\right)\right]$$

$$\psi_0 = \frac{1}{8i}\left[\frac{1}{12}\left(\frac{\beta^2}{\alpha^2}-\frac{\alpha^2}{\beta^2}\right)+\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)+\left(-\frac{\alpha\log\alpha}{\beta}+\beta\frac{\log\beta}{\alpha}\right)\right]$$

$$\psi-\psi_0 = \frac{1}{8i}\left[-\frac{1}{6}\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)+\left(\alpha\frac{\log\beta}{\beta}-\beta\frac{\log\alpha}{\alpha}\right)\right]$$

$$= \frac{1}{8i}\left[\alpha\left(\frac{\log\beta}{\beta}-\frac{1}{6\beta}\right)-\beta\left(\frac{\log\alpha}{\alpha}-\frac{1}{6\alpha}\right)\right]$$

$$v = +\frac{1}{8}\left[\frac{2}{3}(\cos 4\theta+\cos 2\theta)-8\theta\sin 2\theta\right]\frac{\sin\theta}{r}$$

$$u = +\frac{1}{8}\left[\frac{2}{3}(\cos 4\theta+\cos 2\theta)-8\theta\sin 2\theta\right]\frac{\cos\theta}{r}$$

$$V_r = v\sin\theta + u\cos\theta = +\frac{1}{8r}\left[\frac{2}{3}(\cos 4\theta+\cos 2\theta)-8\theta\sin 2\theta\right] = \frac{1}{6r}(\cos 3\theta - 12\theta\sin\theta)\cos\theta$$

$$V_{\perp r} = \cancel{v\cos\theta - u\sin\theta = \frac{1}{8}\left[\frac{2}{3}(\cos 4\theta+\cos 2\theta)-8\theta\sin 2\theta\right]\frac{\sin 2\theta}{r}} = 0$$

$$V_r = 0: \quad \frac{2}{3}\underbrace{(\cos 4\theta+\cos 2\theta)} = 8\theta\sin 2\theta$$

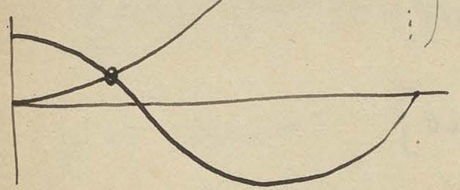
$$\cos 3\theta\cos\theta = 6\theta\sin 2\theta = 12\,\theta\sin\theta\cos\theta$$

$$\cos 3\theta = 12\theta\sin\theta$$

$$V_r = \frac{\cos\theta}{6r}\left[\cos 3\theta - 12\,\theta \sin\theta\right] \Big\} 6r$$

$y = \theta \sin\theta$

$\frac{\partial y}{\partial \theta} = \sin\theta + \theta\cos\theta$



Jeden průsečík v prvním kvadrantu $< \frac{\pi}{6} = 30^\circ$

	$\cos 3\theta$	$12\theta \sin\theta$
20°	0·5	1·2
15°	0·707 ~~[illegible]~~	0·811
14°	0·743	0·703

12. 0·342 . 0·349

12. 0·259 . 0·262

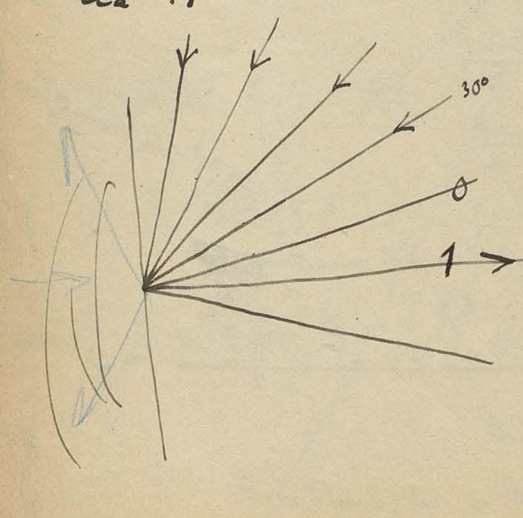
přibližně při: $\cos 3\theta = 12 \sin^2\theta$

12 . 0·0676
135
0·811

12 . $0·242^2$ = 586 . 12
117
703

12 . $\frac{1}{2}$ 0·524
3·144 . 0·866
2515
188
19
2·822

cca $14^\circ 20'$



$$\int_{-\infty}^{+\infty} e^{-\alpha x^2-\beta x^4} dx = \int_{-\infty}^{+\infty} e^{-\beta\left(x^2+\frac{\alpha}{2\beta}\right)^2+\frac{\alpha^2}{4\beta}} dx$$

$$= e^{\frac{\alpha^2}{4\beta}}$$

$$\int e^{-\alpha(x^2+b)^2} dx$$

$$x^2+b=\sqrt{y}$$

$$2x\,dx=\frac{1}{2}\frac{dy}{\sqrt{y}}$$

$$dx=\frac{1}{4}\frac{dy}{\sqrt{y}\sqrt{\sqrt{y}-b}}$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2-\beta x^4} dx = F(\alpha,\beta) = F(\alpha,0)+\beta\left(\frac{\partial F}{\partial\beta}\right)_0+\cdots$$

$$= \int_{-\infty}^{+\infty} e^{-\alpha x^2}dx - \beta\int x^4 e^{-\alpha x^2}(-\beta x^4)\,dx + \frac{\beta^2}{1.2}\int x^8 e^{-\alpha x^2}dx - \cdots$$

$$\frac{\beta}{\alpha^2}\ll 1$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$$

$$= \sqrt{\frac{\pi}{\alpha}}\left[1-\frac{3\beta}{4\alpha^2}+\frac{\beta^2}{1.2}\cdot\frac{3.5.7}{2^4\alpha^4}-\frac{\beta^3}{3!}\frac{1.3.5.7.9.11}{2^6.\alpha^6}\cdots\right]$$

$$\int x^2 e^{-\alpha x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\neq \sqrt{\frac{\pi}{\alpha}}\left[1-\frac{3\beta}{4\alpha^2}\right]$$

$$\int x^4 \ldots = \frac{3}{4}\sqrt{\frac{\pi}{\alpha^5}}$$

$$-\int x^2 e^{-\alpha x^2-\beta x^4}dx = -\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}\left[1-\frac{3\beta}{4\alpha^2}\right]+\sqrt{\frac{\pi}{\alpha}}\frac{3\beta}{2\alpha^3}$$

$$\int x^6 = \frac{3.5}{8}\sqrt{\frac{\pi}{\alpha^7}}$$

$$= \sqrt{\frac{\pi}{\alpha}}\left[\frac{3\beta}{2\alpha^3}+\frac{3\beta}{8\alpha^3}-\frac{1}{2\alpha}\right] = \sqrt{\frac{\pi}{\alpha}}\left[15\right.$$

$$\int x^8 = \frac{3.5.7}{16}\sqrt{\frac{\pi}{\alpha^9}}$$

$$\int x^2 e^{-\alpha x^2-\beta x^4}dx = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}\left[1-\frac{15\beta}{8\alpha^2}\right]$$

$$\int x^4 e^{-\alpha x^2-\beta x^4} = \sqrt{\frac{\pi}{\alpha}}\left[\frac{3}{4\alpha^2}-\beta\cdot\frac{3.5.7}{16\alpha^4}\right] = \frac{3}{4\alpha^2}\sqrt{\frac{\pi}{\alpha}}\left[1-\frac{35\beta}{4\alpha^2}\right]$$

$$\int e^{-\alpha x^2-\beta x^4}dx = \underset{\int e^{-\beta x^4}dx}{F(0\beta)} + \alpha\left(\frac{\partial F}{\partial \alpha}\right)_0 + \dots$$

$$= \frac{1}{4\sqrt[4]{\beta}}\Gamma\left(\frac{1}{4}\right) + \alpha\int x^2 e^{-\beta x^4}dx + \frac{\alpha^2}{2}\int x^4 e^{-\beta x^4}dx - \dots$$

$$\frac{\alpha}{\sqrt{\beta}} \ll 1 \qquad = \frac{\alpha^2}{\beta} \ll 1 \qquad + \frac{(-1)^n}{n!}\alpha^n \int x^{2n} e^{-\beta x^4}dx$$

$$\int x^{2n} e^{-\beta x^4}dx = \qquad \cancel{x\sqrt[4]{\beta} = y} \qquad \cancel{dx = \frac{dy}{\sqrt[4]{\beta}}}$$

$$\beta x^4 = z \qquad x = \left(\frac{z}{\beta}\right)^{\frac{1}{4}}$$

$$4\beta x^3 dx = dz$$

$$dx = \frac{dz\,\beta^{3/4}}{4\beta\, z^{3/4}} = \frac{dz}{4 z^{3/4}\beta^{1/4}}$$

$$\cancel{= \int y^{2n} e^{-y}}$$

$$= \int \frac{z^{\frac{n}{2}}}{\beta^{\frac{n}{2}}} e^{-z} \frac{dz}{4\beta^{1/4} z^{3/4}} = \frac{1}{4\beta^{\frac{2n+1}{4}}}\int e^{-z} z^{\frac{2n-3}{4}} dz$$

$$= \frac{1}{4\beta^{\frac{2n+1}{4}}}\Gamma\left(\frac{2n+1}{4}\right)$$

$$\int e^{-\alpha x^2-\beta x^4}dx =$$

$$\int V_r^2 \, r \, d\varphi = \frac{1}{8}\int_0^{\frac{\pi}{2}} \frac{2}{3}(\cos 4\theta + \cos 2\theta)\, d\theta - \int_0^{\frac{\pi}{2}} \theta \cos 2\theta \, d\theta =$$

$$= \frac{1}{12}\left[\frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2} + \theta \frac{\cos 2\theta}{2} - \underbrace{\int \frac{\cos 2\theta}{2} d\theta}_{-\frac{\sin 2\theta}{4}} \right]$$

$$= \frac{1}{12}\left[\frac{\sin 4\theta + \sin 2\theta}{4} + \frac{\theta \cos 2\theta}{2}\right]_0^{\frac{\pi}{2}} = -\frac{1}{24}\,\frac{\pi}{2} = -\frac{\pi}{48}$$

ujemny wypływ, zatem całkowita przy danym spadku ciśnienia istniejąca ilość wypływu zostaje zmniejszona wskutek bezwładności!

$$\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta = \frac{\pi}{4}$$

$$(V_r^2) = \frac{2\cos^2\theta}{r} + \frac{\varrho}{\mu}\,\frac{\cos\theta}{6r}(\cos 3\theta - 12\theta \sin\theta)$$

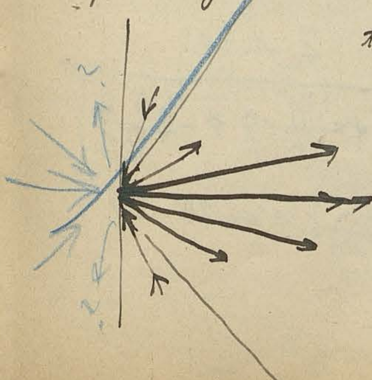
$$= \frac{\cos\theta}{r}\left[2\cos\theta + \frac{\varrho}{6\mu}\underbrace{(\cos 3\theta - 12\theta\sin\theta)}\right]$$

dla $\frac{\pi}{2} > \theta > 15^\circ$ ujemne skończone

zatem zawsze istnieje (dla każdego $\frac{\varrho}{\mu}$) taki obszar, gdzie prąd przeciwny!

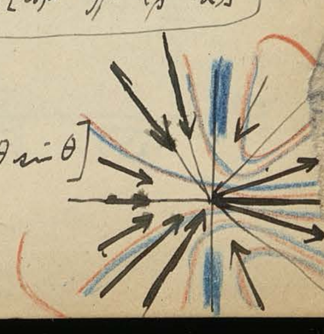
Można otrzymać rozkład prądu bez przyjęcia że taka ilość $-\frac{\pi}{48}$ znika w źródle 0, superponując równocześnie

czyli $\Delta\psi = \frac{1}{6}\left[\lg\alpha - \lg\beta + \left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)\right]$



$$V_r = + \frac{\cos^2\theta}{12 r}\,,$$ zatem:

$$V_r = \frac{\cos\theta}{6r}\left[\frac{\cos\theta}{2} + \cos 3\theta - 12\theta\sin\theta\right]$$



Prova: $\psi = \frac{1}{2i}\left[\alpha \frac{\log\beta}{\beta} - \beta \frac{\log\alpha}{\alpha}\right] + \cancel{\ldots}$

$$= (\log r - i\theta)(\cos 2\theta + i \sin 2\theta) - \ldots$$

$$\psi = -\theta \cos 2\theta + \sin 2\theta \log r$$

$$-u = \frac{\partial\psi}{\partial y} = \left[-\cos 2\theta + 2\theta \sin 2\theta + 2\cos 2\theta \log r\right]\frac{\cos\theta}{r} + \frac{\sin 2\theta}{r}\sin\theta$$

$$v = \frac{\partial\psi}{\partial x} = -\left[-\cos 2\theta + 2\theta\sin 2\theta + 2\cos 2\theta \log r\right]\frac{\sin\theta}{r} + \frac{\sin 2\theta}{r}\cos\theta$$

$\theta = 0$	$u =$ $v = 0$
$\theta = \pm\frac{\pi}{2}$	$u = 0$ $v = \pm\left(\frac{-1 + 2\log r}{r}\right)$

$$-\sqrt{z^2+1} + \frac{z^2}{\sqrt{z^2+1}} = -\frac{1}{\sqrt{z^2+1}}$$

$\cancel{\psi = \alpha \frac{\log\alpha}{\alpha} \ldots \frac{\ldots}{\beta}}$ x

$$F' = -\frac{z}{\sqrt{z^2+1}} \qquad F = -\sqrt{z^2+1} \qquad g' = \frac{1}{\sqrt{z^2+1}}$$

$$u = \frac{4r^2}{\sqrt{r_1 r_2}} \sin\theta \sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right) - \frac{4}{\sqrt{r_1 r_2}} \cos\frac{\theta_1+\theta_2}{2}$$

$$v = -\frac{4r^2}{\sqrt{r_1 r_2}} \cos\theta \sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right)$$

$$F'' = -\frac{1}{\sqrt{z^2+1}} + \frac{z^2}{\sqrt{z^2+1}^3} = \frac{-1}{\sqrt{z^2+1}^3}$$

$$F = \frac{1}{z} \quad F' = -\frac{1}{z^2} \quad F'' = \frac{2}{z^3}$$

$$g' = -\frac{2}{z}$$

$$u = \frac{2\cos^3\theta}{r}$$

$$v = \frac{2\sin\theta\cos^2\theta}{r}$$

(2 sorgenti)

$$\psi = i\left[\frac{\beta}{\alpha} - \frac{\alpha}{\beta} \cancel{\psi} - 2\log\frac{\alpha}{\beta}\right]$$

$$\cancel{F.\,F'' = \frac{1}{\alpha^2+1}}$$

$$\cancel{\alpha F(\beta)\, F''(\alpha) = \frac{\alpha\beta}{\sqrt{\beta^2+1}\sqrt{\alpha^2+1}^3}} = -\sin 2\theta - 2\theta$$

$$g(\beta)\, F''(\alpha) =$$

$$\Delta^2\Delta^2\psi = \cancel{\frac{\partial}{\partial\alpha}(\mathcal{G}-i\mathcal{F})} + \frac{\partial}{\partial\beta}(\mathcal{G}+i\mathcal{F}) = \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)$$

$$\cancel{\mathcal{F}+i\mathcal{G} = (u+iv)\frac{\partial}{\partial\alpha}(u+iv) + (u-iv)\frac{\partial(u+iv)}{\partial\beta}}$$

$$\cancel{\mathcal{F}-i\mathcal{G} = (u+iv)\frac{\partial}{\partial\alpha}(u-iv) + (u-iv)\frac{\partial(u-iv)}{\partial\beta}}$$

$$= \cancel{\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}}$$

$$\cancel{-\frac{\partial u}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\frac{\partial u}{\partial y}}$$

$$+u\frac{\partial^2 v}{\partial x^2} + v\frac{\partial^2 v}{\partial x\partial y} - u\frac{\partial^2 u}{\partial x\partial y} - v\frac{\partial^2 u}{\partial y^2}$$

$$= u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

$$= \left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta$$

$$= \left(u\frac{\partial}{\partial\alpha} + u\frac{\partial}{\partial\beta} + iv\frac{\partial}{\partial\alpha} - iv\frac{\partial}{\partial\beta}\right)\zeta$$

$$= (u+iv)\frac{\partial\zeta}{\partial\alpha} + (u-iv)\frac{\partial\zeta}{\partial\beta}$$

$$\psi_0 = \frac{1}{i}\left[\beta\,\mathcal{F}(\alpha) - \alpha\,\mathcal{F}(\beta) + g(\alpha) - g(\beta)\right]$$

$$u = -\frac{\partial\psi}{\partial y} = -i\left(\frac{\partial\psi_0}{\partial\alpha} - \frac{\partial\psi_0}{\partial\beta}\right) = -\left[\beta\,\mathcal{F}'(\alpha) - \mathcal{F}(\beta) + g'(\alpha) - \mathcal{F}(\alpha) + \alpha\,\mathcal{F}'(\beta) + g'(\beta)\right]$$

$$v = \frac{\partial\psi}{\partial x} = \frac{\partial\psi_0}{\partial\alpha} + \frac{\partial\psi_0}{\partial\beta} = \frac{1}{i}\left[\beta\,\mathcal{F}'(\alpha) - \mathcal{F}(\beta) + g'(\alpha) + \mathcal{F}(\alpha) - \alpha\,\mathcal{F}'(\beta) - g'(\beta)\right]$$

$$u+iv = 2\left[\mathcal{F}(\alpha) - \alpha\,\mathcal{F}'(\beta) - g'(\beta)\right]$$

$$u-iv = +2\left[\mathcal{F}(\beta) - \beta\,\mathcal{F}'(\alpha) - g'(\alpha)\right]$$

$$\zeta = \Delta^2\psi = \frac{4}{i}\left[\mathcal{F}'(\alpha) - \mathcal{F}'(\beta)\right]$$

$$\frac{\partial\zeta}{\partial\alpha} = \frac{4}{i}\,\mathcal{F}''(\alpha)$$

$$\frac{\partial\zeta}{\partial\beta} = -\frac{4}{i}\,\mathcal{F}''(\beta)$$

$$Z = \frac{8}{i}\left\{\left[\mathcal{F}(\alpha)\mathcal{F}''(\alpha) - \mathcal{F}(\beta)\mathcal{F}''(\beta)\right] - \left[\alpha\,\mathcal{F}'(\beta)\mathcal{F}''(\alpha) - \beta\,\mathcal{F}'(\alpha)\mathcal{F}''(\beta)\right] - \left[g'(\beta)\mathcal{F}''(\alpha) - g'(\alpha)\mathcal{F}''(\beta)\right]\right\}$$

$$\mathcal{F}(\alpha) - \alpha\,\mathcal{F}'(\beta) - g'(\beta) = -\sqrt{\alpha^2+1} + \frac{\alpha\beta}{\sqrt{\beta^2+1}} - \frac{1}{\sqrt{\beta^2+1}} = \frac{r^2 - r_1r_2 - 1}{\sqrt{r_1r_2}}\underbrace{\left(\cos\frac{\theta_1+\theta_2}{2} + i\sin\frac{\theta_1+\theta_2}{2}\right)}_{e^{i\frac{\theta_1+\theta_2}{2}}}$$

$$\frac{\partial\zeta}{\partial\alpha} = -\frac{4}{i\sqrt{\alpha^2+1}^3} = -\frac{4}{i\sqrt{r_1r_2}^3}e^{-\frac{3}{2}(\theta_1+\theta_2)i}$$

$$\boxed{Z = 8\,\frac{r^2 - r_1r_2 - 1}{(r_1r_2)^2}\sin(\theta_1+\theta_2) = \frac{\partial\mathcal{G}}{\partial\beta} - \frac{\partial\mathcal{F}}{\partial\alpha}}$$

$$\frac{1}{8}\int Z\,d\alpha\,d\beta = \left[\beta\int F(\alpha)\,F''(\alpha)\,d\alpha - \right]$$

$$-\left[F(\beta)\int\alpha\,F''(\alpha)\,d\alpha - \right]$$

$$-\left[g(\beta)\;F(\alpha) - \right]$$

$$Z = 16\,\frac{\partial^4\psi}{\partial\alpha^2\partial\beta^2} = \frac{8}{i}\,[\ldots$$

$$\psi = \frac{1}{2i}\iint d\alpha^2\,d\beta\ldots$$

$$\int F F''\,dz = \int\frac{dz}{z^2+1} = \operatorname{arctg} z$$

$$\iint F F''\,dz^2 = \int \operatorname{arctg} z\,dz = z\operatorname{arctg} z - \int\frac{z\,dz}{1+z^2} = z\operatorname{arctg} z - \frac{1}{2}\lg(1+z^2)$$

$$\int F\,dz = -\int\sqrt{z^2+1}\,dz = -z\sqrt{z^2+1} + \int\frac{z^2}{\sqrt{z^2+1}}\,dz$$

$$= -\lg\left(z+\sqrt{1+z^2}\right) + \int\frac{z^2}{\sqrt{z^2+1}}$$

$$\int\frac{z^2}{\sqrt{}} = \frac{1}{2}z\sqrt{z^2+1} - \frac{1}{2}\lg\left(z+\sqrt{1+z^2}\right)$$

$$\cancel{\ldots} - \frac{1}{2}\sqrt{1+z^2} + \frac{1}{2}\frac{z^2}{\sqrt{1+z^2}} - \frac{1}{2}\frac{1}{\sqrt{1+z^2}}$$

$$\cancel{\frac{1}{2}\left[-\sqrt{1+z^2} + \frac{z^2}{\sqrt{1+z^2}} - \frac{1}{\sqrt{1+z^2}}\right]} = \frac{-1-z^2-z^2-1}{2\sqrt{1+z^2}} = -\sqrt{1+z^2}$$

$$\int F\,dz = -\frac{z\sqrt{z^2+1}}{2} - \frac{1}{2}\lg\left(z+\sqrt{1+z^2}\right)$$

$$\int\alpha F''\,d\alpha = -\int\frac{z\,dz}{\sqrt{1+z^2}^{\,3}} = \frac{1}{\sqrt{1+z^2}} \qquad \iint\alpha F'' = \lg\left(z+\sqrt{1+z^2}\right)$$

$$g = \int g'\,dz = \int\frac{dz}{\sqrt{1+z^2}} = \lg\left(z+\sqrt{1+z^2}\right) \qquad \int g\,dz = z\lg\left(z+\sqrt{1+z^2}\right) - \int\frac{z\,dz}{\sqrt{1+z^2}}$$

$$= z\lg\left(z+\sqrt{1+z^2}\right) - \sqrt{1+z^2}$$

$$\psi = \frac{1}{2i}\Bigg\{\frac{\beta\alpha}{2}\left[\beta\operatorname{arctg}\alpha - \alpha\operatorname{arctg}\beta\right] - \frac{1}{4}\left[\beta^2\lg(1+\alpha^2) - \alpha^2\lg(1+\beta^2)\right]$$

$$+ \cancel{\ldots}\lg\left(\alpha+\sqrt{1+\alpha^2}\right)\left[\frac{\beta\sqrt{\beta^2+1}}{2} + \cancel{\frac{1}{2}\lg\left(\beta+\sqrt{1+\beta^2}\right)}\right] - \lg\left(\beta+\sqrt{1+\beta^2}\right)\left[\frac{\alpha\sqrt{\alpha^2+1}}{2} + \cancel{\frac{1}{2}\lg\left(\alpha+\sqrt{1+\alpha^2}\right)}\right]$$

$$+ \cancel{\lg\left(\beta+\sqrt{1+\beta^2}\right)\cdot\sqrt{1+\alpha^2} - \sqrt{1+\beta^2}\lg\left(\alpha+\sqrt{1+\alpha^2}\right)}\Bigg\}$$

$$+ \sqrt{1+\alpha^2}\;\beta\lg\left(\beta+\sqrt{1+\beta^2}\right) - \sqrt{1+\beta^2}\;\alpha\lg\left(\alpha+\sqrt{1+\alpha^2}\right)$$

$$\cancel{\pm\sqrt{1+\alpha^2}\sqrt{1+\beta^2} + \sqrt{1+\alpha^2}\sqrt{1+\beta^2}}$$

$$\Psi = \frac{1}{2i}\left\{ \frac{\beta\alpha}{2}\left[\beta \operatorname{arctg}\alpha - \alpha \operatorname{arctg}\beta\right] - \frac{1}{2}\left[\beta^2 \lg\sqrt{1+\alpha^2} - \alpha^2 \lg\sqrt{1+\beta^2}\right]\right.$$

$$\left. + \lg(\alpha+\sqrt{1+\alpha^2})\left[\frac{\beta\sqrt{\beta^2+1}}{2} - \alpha\sqrt{1+\beta^2}\right] - \lg(\beta+\sqrt{1+\beta^2})\left[\frac{\alpha\sqrt{1+\alpha^2}}{2} - \beta\sqrt{1+\alpha^2}\right]\right\}$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left\{ \frac{\beta^2}{2}\operatorname{arctg}\alpha + \frac{\alpha^2}{2}\operatorname{arctg}\beta + \cancel{\frac{\beta^2\alpha}{2(1+\alpha^2)} + \frac{\alpha^2\beta}{2(1+\beta^2)}} - \alpha\beta(\operatorname{arctg}\alpha + \operatorname{arctg}\beta)\right.$$

$$- \left[\cancel{\frac{\beta^2\alpha}{2(1+\alpha^2)} + \frac{\alpha^2\beta}{2(1+\beta^2)}} - \beta\alpha\lg\sqrt{1+\beta^2} - \alpha\beta\lg\sqrt{1+\alpha^2}\right] +$$

$$+ \frac{\beta\sqrt{\beta^2+1}}{2\sqrt{1+\alpha^2}} + \frac{\alpha\sqrt{1+\alpha^2}}{2\sqrt{1+\beta^2}} - \frac{\alpha\sqrt{1+\beta^2}}{\sqrt{1+\alpha^2}} - \beta\frac{\sqrt{1+\alpha^2}}{\sqrt{1+\beta^2}} -$$

$$- \sqrt{1+\beta^2}\lg(\alpha+\sqrt{1+\alpha^2}) - \sqrt{1+\alpha^2}\lg(\beta+\sqrt{1+\beta^2}) +$$

$$+ \frac{\sqrt{1+\beta^2}}{2}\lg(\alpha+\sqrt{1+\alpha^2}) + \frac{\sqrt{1+\alpha^2}}{2}\lg(\beta+\sqrt{1+\beta^2})$$

$$+ \frac{\beta^2}{2\sqrt{1+\beta^2}}\lg(\alpha+\sqrt{1+\alpha^2}) + \frac{\alpha^2}{2\sqrt{1+\alpha^2}}\lg(\beta+\sqrt{1+\beta^2})$$

$$\left. + \frac{\alpha\beta}{\sqrt{1+\beta^2}}\lg\alpha+\sqrt{\ } + \frac{\alpha\beta}{\sqrt{1+\alpha^2}}\lg\ldots\right\}.$$

$$\frac{-3(1+\beta^2) - \beta^2 + 2\alpha\beta}{2\sqrt{1+\beta^2}} = \cancel{\frac{-3 - 4\beta^2}{2\sqrt{1+\beta^2}}}$$

$$= \frac{1}{2}\left\{\frac{r^2}{2}\left[a\cos 2\theta + b\sin 2\theta - 2a\right] + \frac{r^3}{r_1 r_2}\left[\cancel{\cos 2\theta\cos(\theta-(\theta_1+\theta_2)) + \sin 2\theta\sin(\theta-(\theta_1+\theta_2))}\right]\right.$$

$$+ \frac{r}{2}\left[\cos\theta . \lg r_1 r_2 + \sin\theta . (\theta_1+\theta_2)\right] +$$

$$+ \frac{r}{2}\cos(\theta+\theta_1+\theta_2) - r\cos[\theta-(\theta_1+\theta_2)] -$$

$$-\left\{\frac{3}{2}\sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{2\sqrt{r_1 r_2}}\left[\cos\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) - 2\cos\frac{\theta_1+\theta_2}{2}\right]\right\} A -$$

$$\left. -\left\{\frac{3}{2}\sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{2\sqrt{r_1 r_2}}\left[\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) + 2\sin\frac{\theta_1+\theta_2}{2}\right]\right\} B\right\}$$

$$\theta: \theta_2 = \pm\frac{\pi}{2} : \cancel{-\frac{3\pi y^2}{4}}$$

$$-u = \frac{\pi y}{2} - \frac{3}{2}\sqrt{y^2-1} \cdot \frac{\pi}{2}$$

$$-\frac{3y^2}{2\sqrt{y^2-1}} \cdot \frac{\pi}{2}$$

$$= \frac{\pi}{2}\left[y - \frac{3(2y^2-1)}{2\sqrt{y^2-1}}\right]$$

$a = \mathcal{R}\, \mathrm{arctg}\, \alpha = \frac{1}{2} \mathrm{arctg} \frac{2x}{1-r^2}$ $\quad b = \mathcal{I}\, \mathrm{arctg}\, \alpha = \frac{1}{2} \mathrm{arctgh} \frac{2y}{1+r^2}$

$$A = \lg \sqrt{r^2 + r_1 r_2 + 2r\sqrt{r_1 r_2} \cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}$$

$$B = \arcsin \frac{r \sin\theta + \sqrt{r_1 r_2} \sin \frac{\theta_1+\theta_2}{2}}{\sqrt{r^2 + r_1 r_2 + 2r\sqrt{r_1 r_2}\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)}}$$

$\underline{\cancel{\theta}\ y = 0 \ ; \quad \theta = 0 \ ; \quad \theta_2 = -\theta_1 \ ;}$

$a = \frac{1}{2} \mathrm{arctg} \frac{2x}{1-x^2} \quad ; \quad b = 0 \ ;$

$$A = \lg \sqrt{x^2 + x^2 + 1 + 2x\sqrt{1+x^2}} = \lg\left(x + \sqrt{1+x^2}\right)$$

$B = \arcsin 0 = 0$

$x = \pm 0 \ ; \quad \theta = \theta_1 = \theta_2 = \pm \frac{\pi}{2}$

$a = \cancel{\frac{\pi}{2}} \ ; \quad b = \pm \frac{1}{2} \mathrm{arctgh} \frac{2y}{1+y^2} \ ;$

$\theta = 0 : \frac{2y}{\sqrt{y^2-1}}$ $\quad v = \frac{\pi}{2}$

$$A = \lg \sqrt{y^2 + y^2 - 1 + 2y\sqrt{y^2-1}} = \pm \lg\left(y + \sqrt{y^2-1}\right)$$

$$B = \pm \arcsin \frac{y + \sqrt{y^2-1}}{y + \sqrt{y^2-1}} = \pm\frac{\pi}{2}$$ $\quad \boxed{= \mathrm{arctg} \frac{y + \sqrt{y^2-1}}{0} = \infty}$

$$v = \frac{r^2}{2}\left[b \cos 2\theta - a \sin 2\theta + \cancel{2} b\right] - r\left[\frac{\theta_1+\theta_2}{2} \cos\theta - \frac{1}{2} \lg r_1 r_2 \cdot \sin\theta\right]$$

$$- \frac{y}{2} \sin(\theta + \theta_1 + \theta_2) - \frac{r}{\cancel{2}} \sin(\theta - \theta_1 - \theta_2) - \frac{1}{2\sqrt{r_1 r_2}}\left[A \sin \frac{\theta_1+\theta_2}{2} + B \cos \frac{\theta_1+\theta_2}{2}\right]$$

$$\cancel{- \frac{\sqrt{r_1 r_2}}{2}\left[B \cos \frac{\theta_1+\theta_2}{2} - A \sin \frac{\theta_1+\theta_2}{2}\right]} \qquad - \frac{r^2}{\sqrt{r_1 r_2}}\left[A \sin \frac{\theta_1+\theta_2}{2} + B \cos \frac{\theta_1+\theta_2}{2}\right]$$

$\theta = \pm\frac{\pi}{2}$:

$$v = \pm \frac{r^2}{2} \frac{1}{2} \mathrm{arctgh} \frac{2y}{1+y^2} \pm \frac{y}{2} \lg(y^2-1) \pm \frac{y}{2} \pm y \mp \frac{\left(\frac{1}{2} + r^2\right)}{\sqrt{r_1 r_2}} \lg\left(y + \sqrt{y^2-1}\right) \cancel{\pm \frac{\sqrt{y^2-1}}{2} \lg\left(y + \sqrt{y^2-1}\right)}$$

$$\psi = \frac{1}{2i}\left[\beta\,\alpha^2 \operatorname{arctg}\alpha - \alpha\,\beta^2 \operatorname{arctg}\beta\right]$$

$$\begin{matrix}-u\\ v\end{matrix} = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[2\alpha\beta(\operatorname{arctg}\alpha \pm \operatorname{arctg}\beta) + \beta\frac{\alpha^2}{1+\alpha^2} \pm \frac{\alpha\beta^2}{1+\beta^2} \mp (\alpha^2 \operatorname{arctg}\alpha \pm \beta^2 \operatorname{arctg}\beta)\right]$$

$$-u = \left[2r^2 a - r^2\cos 2\theta \cdot a + r^2 \sin 2\theta \cdot b + \frac{r^3}{r_1 r_2}\cos\left(\theta - (\theta_1+\theta_2)\right)\right]$$

$$v = \left[2r^2 b + r^2(a\sin 2\theta + b\cos 2\theta) + \frac{r^3}{r_1 r_2}\sin(\theta - (\theta_1+\theta_2))\right]$$

1).

$\theta = \pm\frac{\pi}{2}$: $-u = \frac{3\pi y^2}{2}$		$\theta = 0$:
$\cancel{-u = \theta^2}$	$v = \pm\frac{r^2}{2}\operatorname{arctgh}\frac{2y}{1+y^2} \pm \frac{y^3}{y^2-1}$	$v = 0$

$$\psi = \frac{1}{2i}(\alpha^3 \operatorname{arctg}\alpha - \beta^3 \operatorname{arctg}\beta)$$

$$\begin{matrix}-u\\ v\end{matrix} = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[3(\alpha^2 \operatorname{arctg}\alpha \pm \beta^2 \operatorname{arctg}\beta) + \frac{\alpha^3}{1+\alpha^2} \pm \frac{\beta^3}{1+\beta^2}\right]$$

2).

$$= \left[3r^2(a\cos 2\theta - b\sin 2\theta) + \frac{r^3}{r_1 r_2}\cos(3\theta - (\theta_1+\theta_2))\right].$$

$$v = 3r^2(a\sin 2\theta + b\cos 2\theta) + \frac{r^3}{r_1 r_2}\sin(3\theta - (\theta_1+\theta_2))$$

$\theta = \pm\frac{\pi}{2}$:		$\theta = 0$:
$-u = \frac{-3y^2\pi}{2}$	$v = \mp\frac{3}{2}r^2 \operatorname{arctgh}\frac{2y}{1+y^2} \pm \frac{y^3}{y^2-1}$	$v = 0$

$$\psi = \frac{1}{2i}\left[\beta\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) - \alpha\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{1+\beta^2})\right]$$

3).

$$\begin{matrix}-u\\ v\end{matrix} = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\frac{\alpha\beta}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{1+\alpha^2}) \pm \frac{\alpha\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{1+\beta^2}) \pm \alpha + \beta \mp \sqrt{1+\alpha^2}\,\lg(\alpha + -) - \sqrt{\ } -\right.$$

$$-u = \frac{r^2}{\sqrt{r_1 r_2}}\left(A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_1+\theta_2}{2}\right) + r\cos\theta - \sqrt{r_1 r_2}\left[A\cos\frac{\theta_1+\theta_2}{2} - B\sin\frac{\theta_1+\theta_2}{2}\right]$$

$$v = \frac{r^2}{\sqrt{r_1 r_2}}\left(B\cos\frac{\theta_1+\theta_2}{2} - A\sin\frac{\theta_1+\theta_2}{2}\right) - r\sin\theta + \sqrt{r_1 r_2}\left[A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right]$$

$\cancel{\theta = 0: \quad v = \frac{y^2}{\sqrt{y^2-1}}\frac{\pi}{2} + \sqrt{y^2-1}\frac{\pi}{2}}$

$-u = \theta$

$v = \frac{\pi}{2}\left(\frac{y^2}{\sqrt{1-y^2}} + \sqrt{1-y^2}\right) = \frac{\pi}{2}\frac{1}{\sqrt{1-y^2}} - y$

$\theta = \pm\frac{\pi}{2}$:	
$-u = \frac{y^2}{\sqrt{y^2-1}}\cdot\frac{\pi}{2} + \sqrt{y^2-1}\,\frac{\pi}{2}$; $= \frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}$	$v = -\frac{y^2}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1}) - y + \sqrt{y^2-1}\,\lg(y+\sqrt{y^2-1})$ $= -y - \frac{1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$

$$\psi = \frac{1}{2i}\left[\alpha\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) - \beta\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{1+\beta^2})\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\Big[\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) + \frac{\alpha^2}{\sqrt{1+\alpha^2}}\lg\,\alpha+\sqrt{1+\alpha^2} + \alpha$$
$$+\sqrt{1+\beta^2}\,\lg\ldots \quad + \ldots \quad + \beta\Big]$$

$$-u = \sqrt{r_1 r_2}\left[A\cos\frac{\theta_1+\theta_2}{2} - B\sin\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[A\cos\left(2\theta-\frac{\theta_1+\theta_2}{2}\right) - B\sin\left(2\theta-\frac{\theta_1+\theta_2}{2}\right)\right]$$
$$+ r\cos\theta$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\Big[\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{}) \qquad (4)$$
$$-\sqrt{1+\beta^2}\,\lg\ldots$$

$$v = \sqrt{r_1 r_2}\left[A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[A\sin\left(2\theta-\frac{\theta_1+\theta_2}{2}\right) + B\cos\left(2\theta-\frac{\theta_1+\theta_2}{2}\right)\right]$$
$$+ r\sin\theta$$

$\boxed{\theta = \pm\frac{\pi}{2}}$ $\cancel{\theta = 0:\ v = \frac{\pi}{2}\left(\sqrt{y^2-1}+\frac{y^2}{\sqrt{y^2-1}}\right) = \frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}}$

$$-u = -\frac{\pi}{2}\sqrt{r_1 r_2} - \frac{r^2}{\sqrt{r_1 r_2}}\,\frac{\pi}{2} = -\frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}$$

$\theta_1 = \frac{\pi}{2},\ \theta_2 = -\frac{\pi}{2}$
$u = 0$
$v = \frac{\pi}{2}\left(\sqrt{1-y^2} - \frac{y^2}{\sqrt{1-y^2}}\right) = -\frac{\pi}{2}\frac{1-2y^2}{\sqrt{y^2-1}}$

$$v = \sqrt{r_1 r_2}\,\lg(y+\sqrt{y^2-1}) + \frac{r^2}{\sqrt{r_1 r_2}}\lg(y+\sqrt{y^2-1}) + r = \frac{2y^2-1}{\sqrt{y^2-1}}\lg(y+\sqrt{}) + y$$

(4a)	(4b)	(3)
$-u = -\frac{\pi}{2}\sqrt{y^2-1}$	$-u = -\frac{y^2}{\sqrt{y^2-1}}\frac{\pi}{2}$	$-u = \frac{\pi}{2}\frac{2y^2-1}{\sqrt{y^2-1}}$
$v = \sqrt{y^2-1}\,(\lg y+\sqrt{y^2-1})$	$v = \frac{y^2}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$	$v = -y - \frac{2y^2-1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$

$(4a-4b)$: $-u = +\frac{\pi}{2}\frac{1}{\sqrt{y^2-1}}$	$(3-4a+4b)$: $-u = 2\sqrt{y^2-1}$	$(3+4b+4a)$: $-u = 0$
$v = \frac{-1}{\sqrt{y^2-1}}\lg(y+\sqrt{y^2-1})$	$v = -y$	$v = 2\sqrt{y^2-1}\,\lg(y+\ldots)$

$$\psi = \frac{1}{2i}\left[\beta\alpha \lg\sqrt{1+\alpha^2} - \alpha\beta \lg\sqrt{1+\beta^2}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\beta \lg\sqrt{1+\alpha^2} + \alpha\lg\sqrt{1+\beta^2} - \alpha\lg\sqrt{1+\alpha^2} - \beta\lg\sqrt{1+\beta^2} + \frac{\beta\alpha^2}{1+\alpha^2} + \frac{\alpha\beta^2}{1+\beta^2}\right]$$

$$= (\theta_1+\theta_2)\, r\sin\theta + \frac{r^3}{r_1 r_2}\underbrace{\left[\cos\theta\cos\frac{\theta_1+\theta_2}{2} + \sin\theta\sin\frac{\theta_1+\theta_2}{2}\right]}_{\cos\left(\theta - \frac{(\theta_1+\theta_2)}{2}\right)}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\beta\lg\sqrt{1+\alpha^2} - \alpha\lg\sqrt{1+\beta^2} + \frac{\beta\alpha^2}{1+\alpha^2} - \frac{\alpha\beta^2}{1+\beta^2} + \alpha\lg\sqrt{1+\alpha^2} - \beta\lg\sqrt{1+\beta^2}\right]$$

$$= (\theta_1+\theta_2)\, r\cos\theta + \frac{r^3}{r_1 r_2}\sin(\theta-(\theta_1+\theta_2)) \qquad (5)$$

$$\boxed{\theta = \pm\frac{\pi}{2}}$$

$-u = y\pi + \cancel{\frac{y^3}{y^2+1}}$	$v = \cancel{\pm}\pm\frac{y^3}{y^2-1}$

$$\psi = \frac{1}{2i}\left[\alpha^2\lg\sqrt{1+\alpha^2} - \beta^2\lg\sqrt{1+\beta^2}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[2\alpha\lg\sqrt{1+\alpha^2} + 2\beta\lg\sqrt{1+\beta^2} + \frac{\alpha^3}{1+\alpha^2} + \frac{\beta^3}{1+\beta^2}\right]$$

$$= r\left[\cos\theta\lg r_1 r_2 - \sin\theta\cdot(\theta_1+\theta_2)\right] + \frac{r^3}{r_1 r_2}\cos(3\theta-(\theta_1+\theta_2)) \qquad (6)$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[2\alpha\lg\sqrt{1+\alpha^2} - 2\beta\lg\sqrt{--} + \frac{\alpha^3}{1+\alpha^2} - \frac{\beta^3}{1+\beta^2}\right]$$

$$= r\left[\sin\theta\lg r_1 r_2 + \cos\theta\cdot(\theta_1+\theta_2)\right] + \frac{r^3}{r_1 r_2}\sin(3\theta-(\theta_1+\theta_2))$$

$$\boxed{\theta = \pm\frac{\pi}{2}}$$

$-u = -\pi y$	$v = \pm\left(y\lg(y^2-1) + \frac{y^3}{y^2-1}\right)$	

$$\psi = \frac{1}{2i}\left[-\beta\sqrt{\alpha^2+1} + \alpha\sqrt{\beta^2+1}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2i}\left[\sqrt{\beta^2+1} + \sqrt{\alpha^2+1} - \frac{\alpha\beta}{\sqrt{\alpha^2+1}} - \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] =$$

$$= \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} - \frac{r^2}{\sqrt{r_1 r_2}}\cos\frac{\theta_1+\theta_2}{2}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\sqrt{\beta^2+1} - \sqrt{\alpha^2+1} - \frac{\alpha\beta}{\sqrt{\alpha^2+1}} + \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] =$$

$$= -\sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\sin\frac{\theta_1+\theta_2}{2}$$

$\theta = \frac{\pi}{2}$:

$u = 0$

$$v = -\sqrt{y^2-1} + \frac{y^2}{\sqrt{y^2-1}} = \frac{1}{\sqrt{y^2-1}}$$

$\theta_1 = \frac{\pi}{2},\ \theta_2 = -\frac{\pi}{2};$

$$\cancel{-u = \sqrt{1-y^2} - \frac{y^2}{\sqrt{1-y^2}} + \frac{1-2y^2}{\sqrt{1-y^2}}}$$

$v = 0$

$$\psi = \frac{1}{2}\left[\beta\sqrt{\alpha^2+1} + \alpha\sqrt{\beta^2+1}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{i}{2}\left[\sqrt{\beta^2+1} - \sqrt{\alpha^2+1} + \frac{\alpha\beta}{\sqrt{\alpha^2+1}} - \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] = \frac{1}{2i}\left[\sqrt{\alpha^2+1} - \sqrt{\beta^2+1} + \frac{\alpha\beta}{\sqrt{\beta^2+1}} - \frac{\alpha\beta}{\sqrt{\alpha^2+1}}\right]$$

$$= \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\sin\frac{\theta_1+\theta_2}{2}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2}\left[\sqrt{\alpha^2+1} + \sqrt{\beta^2+1} + \frac{\alpha\beta}{\sqrt{\alpha^2+1}} + \frac{\alpha\beta}{\sqrt{\beta^2+1}}\right] =$$

$$= \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\cos\frac{\theta_1+\theta_2}{2}$$

$\theta = \pm\frac{\pi}{2}$:

$$u = \sqrt{y^2-1} + \frac{y^2}{\sqrt{y^2-1}} = \pm\frac{2y^2-1}{\sqrt{y^2-1}}$$

$v = 0$

$$\cancel{\psi = \frac{1}{2i}\left[\beta\sqrt{\alpha^2+1} - \beta\sqrt{\alpha^2+1}\right]}\ \frac{1}{2i}\left[\cancel{\lg(\alpha+\sqrt{\alpha^2+1}) + \lg(\beta+\sqrt{\beta^2+1})}\right]$$

$$\cancel{\psi = \frac{1}{2i}\left[-\beta\sqrt{\alpha^2+1} + \alpha\sqrt{\beta^2+1} + 2\lg(\alpha+\sqrt{1+\alpha^2}) - 2\lg(\beta+\sqrt{1+\beta^2})\right]}$$

$$-u = \frac{1}{2}\ \sqrt{\alpha^2+1} + \sqrt{\beta^2}$$

$$\psi = \frac{1}{2i}\left[\lg(\alpha + \sqrt{1+\alpha^2}) - \lg(\beta + \sqrt{1+\beta^2})\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\frac{1}{\sqrt{1+\alpha^2}} + \frac{1}{\sqrt{1+\beta^2}}\right] = \frac{1}{2\sqrt{r_1 r_2}}\cos\frac{\theta_1+\theta_2}{2}$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\qquad - \qquad\right] = \frac{1}{2\sqrt{r_1 r_2}}\sin\frac{\theta_1+\theta_2}{2}$$

$\theta = \frac{\pi}{2}$	$\theta_1 = \frac{\pi}{2} \mid \theta_2 = -\frac{\pi}{2}$
$u = 0$	$-u = \frac{1}{2\sqrt{1-y^2}}$
$v = \frac{1}{2\sqrt{y^2-1}}$	$v = 0$

$$\psi = \frac{1}{2i}\left[\alpha\sqrt{\alpha^2+1} - \beta\sqrt{\beta^2+1}\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\sqrt{\alpha^2+1} + \sqrt{\beta^2+1} + \frac{\alpha^2}{\sqrt{\alpha^2+1}} + \frac{\beta^2}{\sqrt{\beta^2+1}}\right]$$

$$= \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\cos\left(2\theta - \frac{\theta_1+\theta_2}{2}\right)$$

$$v = \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \frac{r^2}{\sqrt{r_1 r_2}}\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right)$$

$\theta = \frac{\pi}{2}$, $\theta_1 = \frac{\pi}{2}$, $\theta_2 = -\frac{\pi}{2}$ (a)	(b)
$-u = \sqrt{1-y^2}$	$\frac{-y^2}{\sqrt{1-y^2}}$
$v = 0$	$v = 0$

$$\psi = \frac{1}{2i}\left[[\lg(\alpha + \sqrt{\alpha^2+1})]^2 - [\lg(\beta + \sqrt{1+\beta^2})]^2\right]$$

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[\frac{1}{\sqrt{1+\alpha^2}}\lg(\alpha + \sqrt{\cdots}) + \frac{1}{\sqrt{1+\beta^2}}\lg \cdots\right] = \frac{1}{\sqrt{r_1 r_2}}\left[A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_1+\theta_2}{2}\right]$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\left[\qquad - \qquad\right] = \frac{1}{\sqrt{r_1 r_2}}\left[-A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right]$$

$\theta = \frac{\pi}{2}$: $\quad -u = \frac{\pi}{2}\frac{1}{\sqrt{y^2-1}}$

$$v = \frac{-1}{\sqrt{y^2-1}}\lg(y + \sqrt{y^2-1})$$

$= (\psi a) - (\psi b)$

$\theta_1 = \frac{\pi}{2}$, $\theta_2 = -\frac{\pi}{2}$, $\theta = \frac{\pi}{2}$
$-u = 0$
$v = \frac{\pi}{2}\frac{1}{\sqrt{1-y^2}}$

$$\psi = \frac{1}{2i}\left[\sqrt{1+\alpha^2}\,\lg\alpha \quad - \sqrt{\cdot}\,\lg\beta\right]$$

$$-u = \frac{1}{2}\left[\sqrt{1+\alpha^2} + \ldots \quad + \sqrt{1+\alpha^2}\,\lg\alpha + \frac{\alpha}{\sqrt{1+\alpha^2}}\lg\alpha\right.$$

$$-u = \sqrt{r_1 r_2}\cos\frac{\theta_1+\theta_2}{2} + \sqrt{r_1 r_2}\left[\lg r\cdot\cos\frac{\theta_1+\theta_2}{2} - \theta\sin\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[\lg r\cdot\cos\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) - \theta\sin(\quad)\right]$$

$$\theta_1 = \theta_2 = \theta = \frac{\pi}{2}: \quad -u = -\frac{\pi}{2}\sqrt{y^2-1} - \frac{\pi}{2}\frac{y^2}{\sqrt{y^2-1}}$$

$$v = \frac{1}{2i}\left[\sqrt{1+\alpha^2}\cdot \; ----\quad\right]$$

$$= \sqrt{r_1 r_2}\sin\frac{\theta_1+\theta_2}{2} + \sqrt{r_1 r_2}\left[\lg r\sin\frac{\theta_1+\theta_2}{2} + \theta\cos\frac{\theta_1+\theta_2}{2}\right] + \frac{r^2}{\sqrt{r_1 r_2}}\left[\lg r\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) + \theta\cos(\ldots)\right]$$

$$v = \sqrt{y^2-1} + \sqrt{y^2-1}\,\lg r + \frac{y^2}{\sqrt{y^2-1}}\lg r$$

$$\psi = \frac{1}{2i}\left[\beta\sqrt{1+\alpha^2}\,\lg\alpha - \alpha\sqrt{1+\beta^2}\,\lg\beta\right]$$

$$-u = \frac{1}{2}\left[\frac{\beta\sqrt{1+\alpha^2}}{\alpha} + \frac{\beta\alpha}{\sqrt{1+\alpha^2}}\lg\alpha - \sqrt{1+\alpha^2}\,\lg\alpha\right.$$

$$= \sqrt{r_1 r_2}\cos\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) + \frac{r^2}{\sqrt{r_1 r_2}}\left[\lg r\cdot\cos\frac{\theta_1+\theta_2}{2} + \theta\cdot\sin\frac{\theta_1+\theta_2}{2}\right] - \sqrt{r_1 r_2}\left[\lg r\cos\frac{\theta_1+\theta_2}{2} - \theta\sin\frac{\theta_1+\theta_2}{2}\right]$$

$$\theta = \frac{\pi}{2}: \quad -u = \frac{y^2}{\sqrt{y^2-1}}\frac{\pi}{2} + \sqrt{y^2-1}\,\frac{\pi}{2}$$

$$v = \frac{1}{2i}\left[\beta\frac{\sqrt{1+\alpha^2}}{\alpha} - \ldots + \sqrt{1+\alpha^2}\,\lg\alpha - \ldots + \frac{\beta\alpha\lg\alpha}{\sqrt{1+\alpha^2}} - \ldots\right]$$

$$= -\sqrt{r_1 r_2}\sin\left(2\theta - \frac{\theta_1+\theta_2}{2}\right) + \frac{r^2}{\sqrt{r_1 r_2}}\left[-\lg r\sin\frac{\theta_1+\theta_2}{2} + \theta\cos\frac{\theta_1+\theta_2}{2}\right] + \sqrt{r_1 r_2}\left[\lg r\cdot\sin\frac{\theta_1+\theta_2}{2} + \theta\cos\frac{\theta_1+\theta_2}{2}\right]$$

$$\theta = \frac{\pi}{2}: \quad v = -\sqrt{y^2-1} + \frac{y^2}{\sqrt{y^2-1}}\lg r + \sqrt{y^2-1}\,\lg r$$

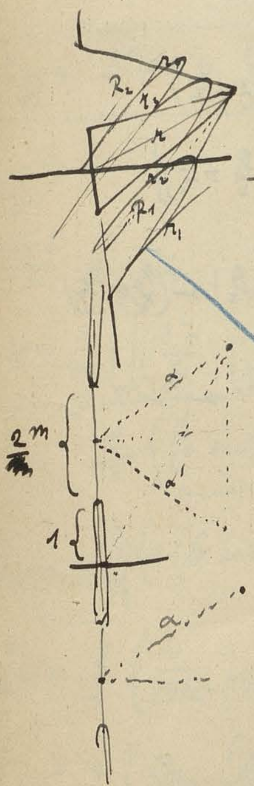
$$\psi = \frac{1}{2i}\left[(\alpha+\beta)\sqrt{1+\alpha^2}\,\lg(\alpha+\sqrt{1+\alpha^2}) - (\alpha+\beta)\sqrt{1+\beta^2}\,\lg(\beta+\sqrt{1+\beta^2})\right]$$

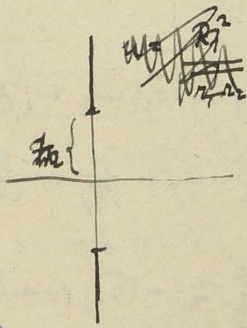
$\theta = \pm\frac{\pi}{2}$: $\quad u = v = 0$

$$\Delta^2\psi = \frac{1}{2i}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{1+\alpha^2}) + \cancel{1} - \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{1+\beta^2}) - \cancel{1}\right] = \{$$

$$p = \frac{1}{2}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{}) + \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{})\right] \qquad p=\infty \text{ dla } r=0$$

$$= \frac{r}{\sqrt{r_1 r_2}}\left[A\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right) - B\sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right)\right]$$





$$\psi = \frac{1}{i}\left[\alpha\sqrt{\beta^2+m^2} - \beta\sqrt{\alpha^2+m^2} + \lg(\alpha+\sqrt{\alpha^2+m^2}) - \lg\ldots\right]$$

$\alpha + 1 + m = \alpha'$ $\quad|\quad$ $\alpha - 1 - m = \alpha''$

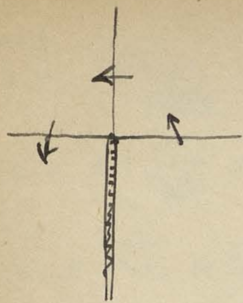
$\beta + 1 + m = \beta'$

$$\psi = \frac{1}{i}\left[(\alpha-1-m)\sqrt{(\beta-1-m)^2+m^2} - (\beta-1-m)\sqrt{(\alpha-1-m)^2+m^2} + \lg\ldots\right.$$

$$\left. + (\alpha+1+m)\sqrt{(\beta+1+m)^2+m^2} - (\beta+1+m)\sqrt{\ldots} \quad + \ldots\right]$$

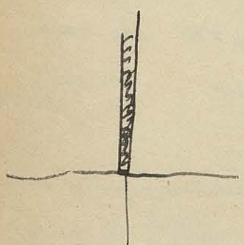
$$= \frac{1}{i}\; m^2\left[\left(\frac{\alpha-1}{m}-1\right)\sqrt{\left[\frac{\beta-1}{m}-1\right]^2+1} - \left(\frac{\beta-1}{m}-1\right)\sqrt{} \quad + \lg\left[\frac{\alpha-1}{m}-1+\sqrt{\left(\frac{\alpha-1}{m}-1\right)^2\ldots}\right]\right.$$

$$+\left(\frac{\alpha+1}{m}+1\right)\sqrt{\left[\frac{\beta+1}{m}+1\right]^2+1} \;-\; \ldots$$



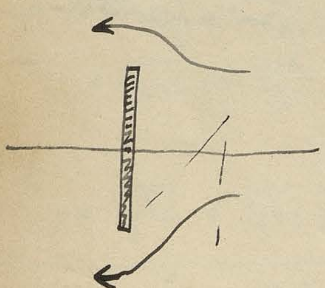
$$v = \sqrt{r}\,\cos\theta\,\sin\left(\frac{\theta}{2}+\frac{\pi}{4}\right)$$

$$u = -\sqrt{r}\left[\sin\theta + 1\right]\sin\left(\frac{\theta}{2}+\frac{\pi}{4}\right)$$



$$v = \sqrt{r}\,\cos\theta\,\sin\left(\frac{\theta}{2}-\frac{\pi}{4}\right)$$

$$u = -\sqrt{r}\left[\sin\theta - 1\right]\sin\left(\frac{\theta}{2}-\frac{\pi}{4}\right)$$



$$v = \sqrt{r_1}\cos\theta_1\sin\left(\frac{\theta_1}{2}-\frac{\pi}{4}\right) + \sqrt{r_2}\cos\theta_2\sin\left(\frac{\theta_2}{2}+\frac{\pi}{4}\right)$$

$$= r\cos\theta\left[\frac{1}{\sqrt{r_1}}\sin\left(\frac{\theta_1}{2}-\frac{\pi}{4}\right) + \frac{1}{\sqrt{r_2}}\sin\left(\frac{\theta_2}{2}+\frac{\pi}{4}\right)\right]$$

$$u = \sqrt{r_1}\left(1-\sin\theta_1\right)\sin\left(\frac{\theta_1}{2}-\frac{\pi}{4}\right) - \sqrt{r_2}\left(1+\sin\theta_2\right)\sin\left(\frac{\theta_2}{2}+\frac{\pi}{4}\right)$$

$$= \frac{\left(r_1 - 1 - y\right)}{\sqrt{r_1}}\sin\left(\frac{\theta_1}{2}-\frac{\pi}{4}\right) - \frac{\left(r_2 + y - 1\right)}{\sqrt{r_2}}\sin\left(\frac{\theta_2}{2} -\right.$$

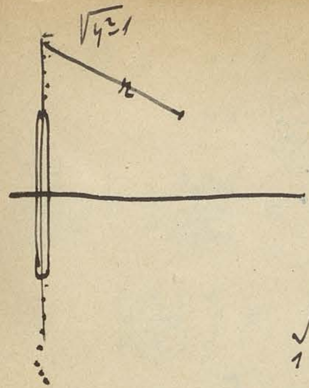
$\theta = 0$: $v = 0$

$$u = 2\sqrt{r}\,\left(1+\sin\theta_2\right)\sin\left(\frac{\theta_2}{2}+\frac{\pi}{4}\right)$$

$\theta = \theta_1 = \theta_2 = \frac{\pi}{2}$: $v = 0$

$$u = -2\sqrt{y-1}$$

$$p = \frac{2}{\sqrt{r_1}}\cos\left(\frac{\theta_1}{2}-\frac{\pi}{4}\right) + \frac{2}{\sqrt{r_2}}\cos\left(\frac{\theta}{2}+\frac{\pi}{4}\right)$$



$$u = \frac{\cos^3\theta}{r}$$

$$v = \frac{\sin\theta\cos^2\theta}{r}$$

$$\int_1^Y \sqrt{y^2-1}\,\frac{\cos^3\theta}{r}\,dy = \int_1^Y \frac{\sqrt{\eta^2-1}\cdot x^3}{\left[x^2+(y-\eta)^2\right]^2}\,d\eta$$

$$+ \int_1^Y \frac{\sqrt{\eta^2-1}}{\left[x^2+(y+\eta)^2\right]^2}\,d\eta$$

$$\xi = \Delta^2\psi = 4\,\frac{\partial^2\psi}{\partial\alpha\,\partial\beta} = \frac{2}{i}\left\{ \cdots\cdots \right\} \qquad \rho = \cdots$$

$$\frac{(2)-(1)}{8} = \qquad v = -\frac{x^2}{4}\,\mathrm{arcth}\,\frac{2y}{1+y^2} \qquad u = 0$$

$$\frac{5-6}{2} = \qquad v = -\frac{1}{2}\lg(y^2-1) \qquad -u = \pi y$$

$$\psi = \frac{1}{2i}\left\{\left[\left(\frac{\beta^2\alpha}{2} + \frac{\alpha^3}{8} - \frac{\beta\alpha^2}{8}\right)\mathrm{arctg}\,\alpha - \cdots\cdots\right] - \frac{1}{2}\left[\left(\beta^2 + 2\alpha\beta - \frac{\alpha^2}{2}\right)\lg\sqrt{1+\alpha^2} - \cdots\right]\right.$$

stimmt

$$\xi = \frac{2}{i}\left\{\underbrace{\left[\beta\,\mathrm{arctg}\,\alpha - \alpha\,\mathrm{arctg}\,\beta\right]} + \left[\frac{\sqrt{1+\beta^2}}{\sqrt{1+\alpha^2}} - \frac{\sqrt{1+\alpha^2}}{\sqrt{1+\beta^2}}\right] + \left[\frac{\alpha}{\sqrt{\alpha^2+1}}\lg(\beta+\sqrt{1+\beta^2}) - \frac{\beta}{\sqrt{\beta^2+1}}\lg(\alpha+\sqrt{\cdots})\right]\right.$$

$$4\left\{r(\beta\cos\theta - \alpha\sin\theta) - \sin(\theta_1+\theta_2) + \frac{r^{\cdots}}{\sqrt{r_1 r_2}}\left[A\sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right) - B\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)\right]\right\}$$

$$\lim_{\infty} = \infty!$$

$$\xi_{(3)} = \frac{2}{i}\left\{\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\cdots}) - \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\cdots})\right\}$$

$$= 4\left\{\frac{r}{\sqrt{r_1 r_2}}\left[A\sin\left(\theta - \frac{\theta_1+\theta_2}{2}\right) + B\cos\left(\theta - \frac{\theta_1+\theta_2}{2}\right)\right]\right\}$$ inne ξ takiego rodzaju nie istnieją?

zatem w każdym razie musi być (3) odjęte, ażeby ∞ zniknęło

$$\psi = \frac{1}{2i}\left[\frac{\beta}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) - \frac{\alpha}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ })\right]$$

(7)

$$-u = \frac{\partial\psi}{\partial y} = \frac{1}{2}\left[-\frac{\alpha\beta}{\sqrt{1+\alpha^2}^3}\lg(\alpha+\sqrt{\ }) + \frac{\beta}{1+\alpha^2} - \frac{1}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) \ldots\right.$$

$$v = \frac{\partial\psi}{\partial x} = \frac{1}{2i}\Big[\qquad + $$

$$-u = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[A\cos 3\frac{\theta_1+\theta_2}{2} + B\sin 3\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\cos(\theta - \theta_1+\theta_2) - \frac{1}{\sqrt{r_1 r_2}}\left[A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_2+\theta_1}{2}\right]$$

$$v = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[-A\sin 3\frac{\theta_1+\theta_2}{2} + B\cos 3\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\sin(\theta+\theta_1+\theta_2) + \frac{1}{\sqrt{r_1 r_2}}\left[-A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right]$$

$\theta = \frac{\pi}{2}$:

$$-u = \left(\frac{y^2}{\sqrt{y^2-1}^3} - \frac{1}{\sqrt{y^2-1}}\right)\frac{\pi}{2} = \frac{y^2-y^2+1}{\sqrt{\ }^3} = \frac{1}{\sqrt{y^2-1}^3}\frac{\pi}{2}$$

$$v = -\frac{y^2}{\sqrt{y^2-1}^3}\lg(y+\sqrt{\ }) + \frac{y}{y^2-1} - \frac{1}{\sqrt{y^2-1}}\lg(y+\sqrt{\ }) = \frac{y}{y^2-1} - \frac{2y^2-1}{\sqrt{y^2-1}^3}\lg(y+\sqrt{y^2-1})$$

$$\psi = \frac{1}{2i}\left[\frac{\alpha}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) - \frac{\beta}{\sqrt{1+\beta^2}}\lg(\beta+\sqrt{\ })\right]$$

(8)

$$-u = \frac{1}{2}\left[\frac{-\alpha^2}{\sqrt{1+\alpha^2}^3}\lg(\alpha+\sqrt{\ }) + \frac{1}{\sqrt{1+\alpha^2}}\lg(\alpha+\sqrt{\ }) + \frac{\alpha}{1+\alpha^2}\right.$$

$$v = \frac{1}{2i}\Big[$$

$$-u = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[A\cos\left(2\theta - 3\frac{\theta_1+\theta_2}{2}\right) - B\sin\left(2\theta - 3\frac{\theta_1+\theta_2}{2}\right)\right] + \frac{1}{\sqrt{r_1 r_2}}\left[A\cos\frac{\theta_1+\theta_2}{2} + B\sin\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\cos(\theta-\theta_1-\theta_2)$$

$\pi - 3\frac{\pi}{2}$ $\qquad$ $\pi -$

$$v = -\frac{r^2}{\sqrt{r_1 r_2}^3}\left[A\sin\left(2\theta - 3\frac{\theta_1+\theta_2}{2}\right) + B\cos\left(\qquad\right)\right] + \frac{1}{\sqrt{r_1 r_2}}\left[-A\sin\frac{\theta_1+\theta_2}{2} + B\cos\frac{\theta_1+\theta_2}{2}\right] + \frac{r}{r_1 r_2}\sin(\theta-\theta_1-\theta_2)$$

$$-u = -\frac{y^2}{\sqrt{y^2-1}^3}\frac{\pi}{2} + \frac{1}{\sqrt{y^2-1}}\frac{\pi}{2} = \frac{-y^2+y^2-1}{\sqrt{\ }^3} = \frac{-1}{\sqrt{y^2-1}^3}\frac{\pi}{2}$$

$$v = +\frac{y^2}{\sqrt{y^2-1}^3}\lg(y+\sqrt{\ }) - \frac{1}{\sqrt{y^2-1}}\lg(y+\sqrt{\ }) - \frac{y}{y^2-1} = -\frac{y}{y^2-1} + \frac{1}{\sqrt{y^2-1}^3}\lg(y+\sqrt{y^2-1})$$

$(7+8)$: $u=0$

$$v = \frac{-2}{\sqrt{y^2-1}} \log\left(y+\sqrt{y^2-1}\right)$$

$$\cancel{\;\;} + \frac{1}{2}\frac{2y^2-1}{\sqrt{y^2-1}} = \cancel{\;\;}\sqrt{y^2-1}\,a + \frac{y^2}{\sqrt{y^2-1}}\,b - \frac{c}{\sqrt{y^2-1}} - \frac{2d}{\sqrt{y^2-1}}$$

$$\frac{3}{2}\left[\frac{2y^2-1}{\sqrt{y^2-1}}\right] = -\sqrt{y^2-1}\,a - \frac{y^2}{\sqrt{y^2-1}}\,b + \frac{2y^2-1}{\sqrt{y^2-1}}\,c$$

$$2\,\frac{2y^2-1}{\sqrt{y^2-1}} = \frac{2y^2-2}{\sqrt{y^2-1}}\,c - \frac{2d}{\sqrt{y^2-1}} \qquad c=2 \qquad d=-1$$

$$\zeta_7 = \Delta(\psi_7) = \frac{2}{i}\left\{\frac{1}{1+\alpha^2} + \frac{\alpha}{\sqrt{1+\alpha^2}^{\,3}}\log(\alpha+\sqrt{\;}) \;-\,-\,-\,-\,-\right.$$

$$= 2\left\{\frac{1}{r_1 r_2}\sin(\theta_1+\theta_2) - \underbrace{\frac{r}{\sqrt{r_1 r_2}^{\,3}}}_{\text{rozważ } \lim_{x=\infty}\frac{\log x}{x}=0}\left[A\sin\left[\theta-\tfrac{3}{2}(\theta_1+\theta_2)\right] + B\cos\left[\theta-\tfrac{3}{2}(\theta_1+\theta_2)\right]\right]\right\}$$

~~[illegible]~~ rozważ ζ_3: $\lim_{r=\infty} \cancel{\;\;} \log r \, \underbrace{\sin\left(\theta-\frac{\theta_1+\theta_2}{2}\right)}_{=\frac{\sin 2\theta}{r^2}} = 0$

Pytanie tylko, czy p nie stanie się ∞!!

Inaczej dadzą się spełnić warunki wszystkie za pomocą $(7+8)$ 1, 2, 3, 5, 6

Inny sposób: $\bar{X} = \dfrac{\iint_{-\infty}^{\infty} - \int_0^{\infty} \frac{\partial U}{\partial x} e^{-hU} dx \, dy \, dz}{\iint_{-\infty}^{\infty} dy\, dz \int_0^{\infty} e^{-hU} dx}$

To nie naprawdę, tylko

$$\frac{\partial U}{\partial x} = 2\varphi x \left[4 - 3\delta\right] + 2\varphi' x \left[2 + 3\delta\right] + 2\varphi'' x \delta$$

$$= 2x \left\{ \varphi(4-3\delta) + \varphi'(2+3\delta) + \varphi'' \delta \right\}$$

= siła wypadkowa (tak długo wytężania ściany przyciągania)

$$U = \alpha + \gamma(x^2 + y^2 + z^2)$$

$$\frac{\int_0^{\infty} x e^{-h(\alpha + \gamma x^2)} dx}{\int_0^{\infty} e^{-h(\alpha + \gamma x^2)} dx} = \frac{\frac{1}{2\gamma h} e^{-\gamma x^2} \Big|_{\infty}^{0}}{\frac{1}{2}\sqrt{\frac{\pi}{h\gamma}}}$$

$$= \frac{1}{\sqrt{\gamma h \pi}}$$

= przeciętne jednostronne wychylenie

W stanie zerowym $(x = y = z = 0)$

$$X = \frac{4}{\sqrt{2}} \frac{1+\delta}{\sqrt{1+\delta}} \cdot \varphi(l\sqrt{1+\delta}) = \frac{4}{\sqrt{2}}\left(1 + \frac{\delta}{2}\right)\left[\varphi + l\frac{\delta}{2}\varphi'\right] = \frac{4}{\sqrt{2}}\varphi\left(1 + \frac{\delta}{2}\right)$$
$$+ \frac{2}{\sqrt{2}}\varphi' l \delta$$

$$Y = \frac{2}{\sqrt{2}\sqrt{1+\delta}}\varphi(l\sqrt{1+\delta}) + \frac{2}{\sqrt{2}}\varphi = \frac{2}{\sqrt{2}}\left[\left(1 - \frac{\delta}{2}\right)\left[\varphi + \frac{l\delta}{2}\varphi'\right] + \varphi\right] = \frac{2}{\sqrt{2}}\varphi\left(2 - \frac{\delta}{2}\right)$$
$$+ \frac{1}{\sqrt{2}}\varphi' l \delta$$

$$r_1 = \sqrt{\left(\frac{1+\delta}{\sqrt2} - x\right)^2 + \left(-\frac{1}{\sqrt2} - y\right)^2 + z^2} = \sqrt{(1+\delta) - (1+\delta)x\sqrt2 + y\sqrt2 + x^2 y^2 z^2}$$

$$+x_1 \qquad +y_1 \qquad (z_1 - z)^2$$

$$r_{10} = \sqrt{\left(-\frac{1+\delta}{\sqrt2} - x\right)^2 + \cdots} = \sqrt{1+\delta + (1+\delta)x\sqrt2 + y\sqrt2 + x^2y^2z^2}$$

$$r_2 = \sqrt{\left(\frac{1+\delta}{\sqrt2} - x\right)^2 + \left(\frac{1}{\sqrt2} - y\right)^2 + z^2} = \sqrt{(1+\delta) - (1+\delta)x\sqrt2 - y\sqrt2 + x^2y^2z^2}$$

$$r_9 = \sqrt{\cdots} \qquad \sqrt{(1+\delta) + (1+\delta)x\sqrt2 - y\sqrt2 + x^2y^2z^2}$$

$$r_3 = \sqrt{\left(\frac{1+\delta}{\sqrt2} - x\right)^2 + y^2 + \left(-\frac{1}{\sqrt2} - z\right)^2} = \sqrt{(1+\delta) - (1+\delta)x\sqrt2 + z\sqrt2 + x^2y^2z^2}$$

$$r_4 = \sqrt{\left(\frac{1+\delta}{\sqrt2} - x\right)^2 + y^2 + \left(\frac{1}{\sqrt2} - z\right)^2} = \sqrt{(1+\delta) - (1+\delta)x\sqrt2 - z\sqrt2 + \cdots}$$

---

r_1	$=\sqrt{(1+\delta)}$ $-(1+\delta)x\sqrt2$	$+y\sqrt2$		$+x^2y^2z^2$
2	$-$	$-$		″
3	$-$		$+z\sqrt2$	″
4	$-$		$-$	″

---

r_1	$=\sqrt{(1+\delta)}$	$-(1+\delta)x\sqrt2$	$+y\sqrt2$		$+\cdots$
10	$(1+\delta)$	$+$	$+$		$+\cdots$
5	1		$+$	$-z\sqrt2$	$+x^2y^2z^2$
7	1		$+$	$+$	$+\cdots$

$$\cancel{4} - 2\rho x\sqrt2 - \cancel{2y\sqrt2} + \cancel{2(x^2y^2z^2)} \qquad -1 + \frac{\rho}{2}$$

$$-4\Big[1 - \frac{x\rho}{2\sqrt2} - \cancel{\frac{y}{\sqrt2}} + \cancel{\frac{x^2 \cdots}{2}} - \frac{x^2}{4} - \frac{y^2}{4} + \frac{xy\rho}{4} - \frac{x([illegible])\rho}{4\sqrt2} + \frac{y([illegible])}{2\sqrt2} - \frac{x^3\rho}{8\sqrt2}$$

$$- \frac{y^3}{4\sqrt2} - \frac{3xy^2}{4\sqrt2} + \frac{9xy^2\rho}{8\sqrt2}\Big\}$$

$$+ \frac{x^2y\rho}{\sqrt2} \pm \frac{3}{2}\frac{xy^2\rho}{\sqrt2}$$

$$\frac{(r_1-l)^3}{2}+(r_{10}-l)^3 = \frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)+\frac{3}{4}x^2\delta+\frac{3}{4}y^2\delta+\frac{3x^2y}{2\sqrt2}\left(1+\frac{\delta}{2}\right)-\frac{3x^2y\delta}{2\sqrt2}$$

$$+\frac{3y(x^2+y^2+z^2)}{2\sqrt2}\delta-\frac{3x^2y}{4\sqrt2}\delta-\frac{3y^3}{4\sqrt2}\delta$$

$$=2\left[\frac{3x^2\delta}{4}+\frac{3y^2\delta}{4}+\frac{3x^2y}{2\sqrt2}+\frac{y^3}{2\sqrt2}+\cancel{\frac{3y^3\delta}{4\sqrt2}}+\frac{3yz^2\delta}{2\sqrt2}\right]-\cancel{\frac{3y^3\delta}{4\sqrt2}}$$

$$(r_5-l)^3+r_2 \quad = 2\left[\frac{3z^2\delta}{4}+\frac{3y^2\delta}{4}+\frac{3z^2y}{2\sqrt2}+\frac{y^3}{2\sqrt2}+\frac{3y^3\delta}{4\sqrt2}+\frac{3yx^2\delta}{2\sqrt2}\right]=2\left(\frac{3z^2y}{2\sqrt2}+\frac{y^3}{2\sqrt2}\right)$$

$$\frac{\partial}{\partial y}=2\left\{\frac{3y\delta}{2}+\frac{3(x^2+2y^2+z^2)}{2\sqrt2}+\frac{3(\cancel{x^2}+\cancel{3y^2}+z^2)\delta}{2\sqrt2}\right\}$$

$$-Y=2\varphi\left\{\sqrt2-\frac{\delta}{2\sqrt2}+y+\frac{y\delta}{4}-\frac{x^2}{4\sqrt2}-\frac{3y^2}{2\sqrt2}-\frac{z^2}{4\sqrt2}+\frac{3x^2\delta}{8\sqrt2}+\frac{3y^2\delta}{8\sqrt2}+\frac{3z^2\delta}{4\sqrt2}\right\}$$

$$+\varphi'\left\{\frac{\delta}{\sqrt2}+2y-\frac{y\delta}{2}+\frac{x^2}{2\sqrt2}+\frac{3y^2}{\sqrt2}+\frac{z^2}{2\sqrt2}-\frac{(x^2+y^2+2z^2)\delta}{4\sqrt2}\right\}$$

$$+\varphi''\left\{\frac{y\delta}{2}+\frac{x^2+2y^2+z^2}{2\sqrt2}+\frac{(\ldots+z^2)\delta}{2\sqrt2}\right\}$$

$$-\bar{Y}=4\varphi\left(\frac{1}{\sqrt2}-\frac{\delta}{4\sqrt2}\right)+\varphi'\frac{\delta l}{\sqrt2}+$$

$$+\iiint x^2\left[\varphi\left(-\frac{1}{2\sqrt2}+\frac{3\delta}{4\sqrt2}\right)+\varphi'\left(-\frac{1}{\sqrt2}-\frac{3\delta}{4\sqrt2}\right)+\varphi''\left(\frac{1}{2\sqrt2}+\frac{\delta}{2\sqrt2}\right)\right]$$

$$+y^2\left[\varphi\left(-\frac{3}{\sqrt2}+\frac{3\delta}{4\sqrt2}\right)+\varphi'\left(\frac{3}{\sqrt2}-\frac{3\delta}{4\sqrt2}\right)+\varphi''\left(\frac{1}{\sqrt2}+\frac{3\delta}{2\sqrt2}\right)\right]$$

$$+z^2\left[\varphi\left(-\frac{1}{2\sqrt2}+\frac{3\delta}{2\sqrt2}\right)+\varphi'\left(-\frac{1}{\sqrt2}-\frac{3\delta}{2\sqrt2}\right)+\varphi''\left(\frac{1}{2\sqrt2}+\frac{\delta}{2\sqrt2}\right)\right]$$

$$\frac{9}{2}-6=-\frac{3}{2}$$

$$-\frac{7}{2}+1=-\frac{5}{2}$$

$$2\left\{-\frac{\beta}{\sqrt2}+x-\frac{y\beta}{2}+\frac{9x^2\beta}{4\sqrt2}+\frac{xy}{\sqrt2}-y^2\frac{7\beta}{4\sqrt2}+\frac{z^2\beta}{2\sqrt2}\right.$$

$$\left.+\frac{y\beta}{2}-\frac{3x^2\beta}{\sqrt2}-\frac{2xy}{\sqrt2}+\frac{\beta y^2}{2\sqrt2}+\frac{\beta z^2}{2\sqrt2}\right\}$$

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V celu sterymania V:

$$r_1 \atop r_2 \Big\} 1+\delta - (1+\delta)x\sqrt2 \; {+ \atop -}\; y\sqrt2 + x^2+y^2+z^2$$

$$r_3 \atop r_4 \Big\} 1+\delta - (1+\delta)x\sqrt2 \; {- \atop +}\; z\sqrt2 + x^2+y^2+z^2$$

$$r_1 \atop r_{10} \Big\} 1+\delta - (1+\delta)x\sqrt2 \; {+ \atop +}\; y\sqrt2 + x^2+y^2+z^2$$

$$r_5 \atop r_7 \Big\} 1 + y\sqrt2 \; {- \atop +}\; z\sqrt2 + (x^2+y^2+z^2)$$

$$+ {r_1 - l \atop r_{10}-l}\Big\} = \frac{\delta}{2} + \frac{y}{\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\delta}{2}\right) - \frac{y(x^2+y^2+z^2)}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)$$
$$+ \frac{y^3}{4\sqrt2}\left(1-\frac{5\delta}{2}\right) + \frac{3x^2y}{4\sqrt2}\left(1-\frac{\delta}{2}\right)$$

$$+ {r_5 \atop r_7} - 1\Big\} = \frac{y}{\sqrt2} + \frac{x^2+y^2+z^2}{2} - \frac{z^2}{4} - \frac{y^2}{4} - \frac{y(x^2+y^2+z^2)}{2\sqrt2}$$
$$+ \frac{y^3}{4\sqrt2} + \frac{3z^2y}{4\sqrt2}$$

$$r_1+r_{10}+r_5+r_7 - l = 2\Big\{\frac{\delta}{2} + \frac{y}{\sqrt2}\left(2-\frac{\delta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(2-\frac{\delta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{y^2}{4}\left(2-\frac{3\delta}{2}\right) - \frac{z^2}{4}$$
$$-\frac{y(x^2+y^2+z^2)}{2\sqrt2}\left(2-\frac{3\delta}{2}\right) + \frac{y^3}{4\sqrt2}\left(2-\frac{5\delta}{2}\right) + \frac{3x^2y}{4\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{3z^2y}{4\sqrt2}\Big\}$$

$$\frac{\partial}{\partial y}(\quad) = 2\Big\{\frac{1}{\sqrt2}\left(2-\frac{\delta}{2}\right) + y\left(2-\frac{\delta}{2}\right) - \frac{y}{2}\left(2-\frac{3\delta}{2}\right) - \frac{x^2+3y^2+z^2}{2\sqrt2}\left(2-\frac{3\delta}{2}\right)$$
$$+ \frac{3y^2}{4\sqrt2}\left(2-\frac{5\delta}{2}\right) + \frac{3x^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{3z^2}{4\sqrt2}\Big\}$$
$$= 2\Big\{\sqrt2 - \frac{\delta}{2\sqrt2} + y + \frac{y\delta}{4} - \frac{x^2}{4\sqrt2} - \frac{3y^2}{2\sqrt2} - \frac{z^2}{4\sqrt2} + \frac{3x^2\delta}{8\sqrt2} + \frac{3y^2\delta}{8\sqrt2} + \frac{3z^2\delta}{4\sqrt2}\Big\}$$

$$(r_1-l)^2+(r_{10}-l)^2 = 2\Big\{\frac{x^2}{2}(1+\delta) + \frac{y^2}{2}(1-\delta) + \frac{y\delta}{\sqrt2} + \frac{x^2+y^2+z^2}{2}\delta - \frac{x^2}{4}\delta - \frac{y^2\delta}{4} - \frac{y(x^2+y^2+z^2)}{2\sqrt2}\delta + \frac{y^3\delta}{4\sqrt2} +$$
$$+ \frac{3x^2y\delta}{4\sqrt2} - \frac{3x^2y}{2\sqrt2} + \frac{y(x^2+y^2+z^2)}{\sqrt2}(1-\delta) - \frac{y^3}{2\sqrt2}(1-2\delta)\Big\}$$

$$\text{-----} = 2\Big\{\frac{x^2}{2} + \frac{y^2}{2} - \frac{3x^2y}{2\sqrt2} + \frac{y(x^2+y^2+z^2)}{\sqrt2} - \frac{y^3}{2\sqrt2}\Big\}$$

$$\frac{\partial}{\partial y} = 2\Big[\frac{\delta}{\sqrt2} + 2y - \frac{y\delta}{2} + \frac{x^2}{2\sqrt2} + \frac{3y^2}{\sqrt2} + \frac{z^2}{2\sqrt2} - \frac{3(x^2+y^2+2z^2)\delta}{4\sqrt2}\Big]$$

Ażeby uwzględnić wpływ temperatury : $l = l_0[1+\alpha\, dT]$

$$\varphi = \varphi_0 + l_0 \alpha\, dT\, \varphi'$$
$$= \varphi_0 + \varphi' l \alpha t$$

$$X = \frac{4\varphi_0}{\sqrt2}\left(1+\frac{\delta}{2}\right) + 2\frac{\varphi_0' l\delta}{\sqrt2} - \frac{m^2R^2T_0^2}{4\sqrt2}(A_0+B_0\delta) +$$
$$+\left[\frac{4\varphi' l}{\sqrt2}\left(1+\frac{\delta}{2}\right) + \frac{2\varphi'' l^2\delta}{\sqrt2} - \frac{m^2R^2T_0^2}{4\sqrt2}(A_0'+B_0'\delta)l\right]\alpha t$$
$$-\frac{2m^2R^2t}{4\sqrt2}(A_0+B_0\delta)$$

$$\overbrace{\frac{4\varphi_0}{\sqrt2} - \frac{m^2R^2T_0^2A_0}{4\sqrt2}}^{X_{00}} + \left\{\left[\frac{4\varphi' l}{\sqrt2} - \frac{m^2R^2T_0^2A_0' l}{4\sqrt2}\right]\alpha - \frac{2m^2R^2A_0}{4\sqrt2}\right\}t =$$
$$= X_{00}(1-6\alpha t)$$

$$\left[\frac{4\varphi'}{\sqrt2} - \frac{m^2R^2T_0^2A_0'}{4\sqrt2}\right]\alpha l - \frac{m^2R^2A_0}{2\sqrt2} = -6\left(\frac{4\varphi_0}{\sqrt2} - \frac{m^2R^2T_0^2A_0}{4\sqrt2}\right)\alpha$$

$$\left[\frac{4(\varphi' l + 6\varphi_0)}{\sqrt2} - \frac{m^2R^2T_0^2(A_0' l + 6A_0)}{4\sqrt2}\right]\alpha = \frac{m^2R^2A_0}{2\sqrt2}$$

$$\alpha = \frac{\dfrac{m^2R^2A_0}{2\sqrt2}}{\dfrac{4(6\varphi_0+\varphi' l)}{\sqrt2} - \dfrac{m^2R^2T_0^2(6A_0+A_0' l)}{4\sqrt2}}$$

$$2\left\{\frac{\delta}{\sqrt2} + 2y - \cancel{y\delta} - \frac{y\delta}{2} - \frac{x^2+3y^2+z^2}{2\sqrt2}\delta + \frac{3y^2\delta}{4\sqrt2} + \frac{3x^2\delta}{4\sqrt2} - \frac{3x^2}{2\sqrt2} + \frac{x^2+3y^2+z^2}{\sqrt2}(1-\delta) - \frac{3y^2}{2\sqrt2}(1-2\delta)\right.$$
$$\left. - \frac{3z^2}{2\sqrt2} + \frac{x^2+3y^2+z^2}{\sqrt2} - \frac{3y^2}{2\sqrt2}\right\}$$

$-\frac12 + \frac34 - 1$ $\quad -\frac12 - 1 \quad -3+1+1$

$-\frac32 + \frac34 - 3 + 3$

$-\frac32 + 1 + 1 \quad 1 - \frac32 + 1$

$$= -\frac{1}{4\sqrt{2}\,h^2}\left\{\frac{2(\varphi-\varphi' l)-\varphi'' l^2}{[2\varphi+\varphi' l]^2} + \delta\,\frac{2\varphi^2+23\varphi'^2 l^2+3\varphi''^2 l^4-35\varphi\varphi' l-27\varphi\varphi'' l^2+8\varphi'\varphi'' l^3}{4[2\varphi+\varphi' l]^3}\right\}$$

$$h = \frac{3N}{2(E-U_0)} = \frac{1}{mRT}$$

$A<0$ (inaczej term. Druckcoeffizient nie byłby dodatni!)

$$\bar{X} = \frac{4\varphi}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{2\varphi' l\,\delta}{\sqrt{2}} - \underbrace{\frac{m^2R^2T^2}{4\sqrt{2}}\left\{A+B\delta\right\}}_{\text{To stanowi różnicę względem dawniejszego przypadku}}$$

$$\bar{X}_0 = a\rho_0^2$$

$$\bar{X} + \underset{\substack{\| \\ E\delta}}{\Xi} = a\rho_0^2(1-\delta)^2 = \bar{X}_0(1-2\delta)$$

$$\Xi = \frac{E}{1+\mu}\left[\frac{\partial u}{\partial x} + \frac{\mu}{1-2\mu}\nabla\right]$$

gdy $\frac{\partial u}{\partial x} = \nabla = \delta$

$$\Xi = \frac{E}{1+\mu}\,\frac{1-\mu}{1-2\mu}\,\delta)$$

$$= E\delta$$

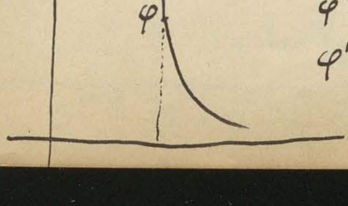
$$E = \frac{\bar{X}_0(1-2\delta)-\bar{X}}{\delta} = -\left(2\bar{X}_0 + \frac{\bar{X}-\bar{X}_0}{\delta}\right)$$

$$= -\left[\frac{8}{\sqrt{2}}\varphi - \frac{m^2R^2T^2}{2\sqrt{2}}A + \frac{2\varphi}{\sqrt{2}} + \frac{2\varphi' l}{\sqrt{2}} - \frac{m^2R^2T^2}{4\sqrt{2}}B\right]$$

$$E = -\left[\frac{10}{\sqrt{2}}\varphi + \frac{2\varphi' l}{\sqrt{2}} - \frac{m^2R^2T^2}{4\sqrt{2}}[2A+B]\right]$$

przy tem jednak należy podstawić każdorazowo $\varphi(l)$ dla odpowiedniej temp. T

$$\frac{\partial E}{\partial T} = -\left[\frac{10}{\sqrt{2}}\varphi' + \frac{2\varphi'' l}{\sqrt{2}}\right]\alpha + \frac{m^2R^2T}{2\sqrt{2}}[2A+B] - \frac{m^2R^2T^2}{4\sqrt{2}}(2A'+B')$$



$\varphi' < 0$

$\varphi'' > 0$

$$\frac{\partial}{\partial x}\left(\frac{u}{v^2}\right) = \frac{v\frac{\partial u}{\partial x} - 2u\frac{\partial v}{\partial x}}{v^3}$$

$$-\frac{1}{\sqrt{2}\,l^2}\left\{\frac{3\varphi\left(1-\frac{3\delta}{2}\right) - 3\varphi' l\left(1-\frac{3\delta}{2}\right) - \varphi'' l^2(1+3\delta)}{2\frac{h}{l}\left[\varphi(4-3\delta) + \varphi' l(2+3\delta) + \varphi'' l^2\delta\right]} + \right.$$

$$\left. + \frac{\frac{\varphi}{2}\left(1+\frac{5\delta}{2}\right) - \frac{\varphi' l}{2}\left(1+\frac{5\delta}{2}\right) - \frac{\varphi'' l^2}{2}}{2\frac{h}{l}\left[\varphi\left(4-\frac{\delta}{2}\right) + \varphi' l\left(2+\frac{\delta}{2}\right) + \varphi'' l^2\frac{\delta}{2}\right]}\right\} =$$

$$= -\frac{1}{2\sqrt{2}\,h^2}\left\{\frac{3(\varphi-\varphi' l) - \varphi'' l^2 + \varphi - \varphi' l - \varphi'' l^2}{\left[4\varphi + 2\varphi' l\right]^2} + \cancel{\delta} \ldots\right.$$

$$\delta\,\frac{\left[4\varphi+2\varphi' l\right]\left[-\frac{9\varphi}{2} + \frac{9\varphi' l}{2} - 3\varphi'' l^2 + \frac{5\varphi}{2} - \frac{5\varphi' l}{2}\right] - 2\left[3(\varphi-\varphi' l) - \varphi'' l^2\right]\left[\right.}{\left[4\varphi+2\varphi' l\right]^3}$$

$$\left.\frac{\left[-3\varphi + 3\varphi' l + \varphi'' l^2\right] - 2\left[\varphi - \varphi' l - \varphi'' l^2\right]\left[-\varphi + \frac{\varphi' l}{2} + \frac{\varphi'' l^2}{2}\right]}{\left[4\varphi+2\varphi' l\right]^3}\right\}$$

$$= -\frac{1}{2\sqrt{2}\,h^2}\left\{\frac{4(\varphi-\varphi' l) - 2\varphi'' l^2}{\left[4\varphi+2\varphi' l\right]^2} + \delta\,\frac{\left[4\varphi + 2\varphi' l\right]\left[-2\varphi + 2\varphi' l - 3\varphi'' l^2\right] -}{}\right.$$

$$-2\left[-10\varphi^2 - \frac{19}{2}\varphi'^2 - \frac{3}{2}\varphi''^2 + \frac{39}{2}\varphi\varphi' + \frac{15}{2}\varphi\varphi'' - 7\varphi'\varphi''\right]$$

$$\bar{X} = \iiint \frac{X\, e^{-hU}\, dx\, dy\, dz}{\iiint e^{-hU}\, dx\, dy\, dz}$$

$$X = A + B(x, y, z) + C(x^2, y^2, z^2)$$

$$U = \alpha + \cancel{\quad} + \gamma(x^2, y^2, z^2)$$

$$\bar{X} = A + \iiint \frac{\left[B(x,y,z) + C(x^2 y^2 z^2)\right] e^{-h[A + \cancel{\quad} + \gamma(x^2 y^2 z^2)]}\, dx\, dy\, dz}{\iiint e^{-h[A + \cancel{\quad} + \gamma(x^2 y^2 z^2)]}\, dx\, dy\, dz}$$

$$= A + \iiint \frac{C(x^2 y^2 z^2)\, e^{-h\gamma}}{\iiint e^{-h\gamma}}$$

$$= 4\varphi\left\{\frac{1}{\sqrt{2}} + \frac{\delta}{2\sqrt{2}}\right\} + \frac{2\varphi' l \delta}{\sqrt{2}} \cancel{\quad} -$$

$$\frac{-\iiint \left\{\frac{x^2}{\sqrt{2}\, l}\left[3\varphi\left(1 - \frac{3\delta}{2}\right) + 3\varphi' l\left(-1 + \frac{3\delta}{2}\right) + \varphi'' l^2 (1 + 3\delta)\right] + \frac{y^2+z^2}{\sqrt{2}\, l^2}\left[\frac{\varphi}{2}\left(1 + \frac{5\delta}{2}\right) - \frac{\varphi' l}{2}\left(1 + \frac{5\delta}{2}\right) - \frac{\varphi'' l^2}{2}\right]\right\} e^{-h \cdots}}{\int e^{-\frac{h}{l^2}\left[x^2 \left(\varphi[4 - 3\delta] + \varphi' l\,[2 + 3\delta] + \varphi'' l^2 \delta\right)\right.} dx \cdot \iint e^{-\frac{h}{l^2}\left.(y^2+z^2)\left(\varphi\left[4 - \frac{\delta}{2}\right] + \varphi' l\left[2 + \frac{\delta}{2}\right] + \varphi'' l^2 \frac{\delta}{2}\right)\right]} dy\, dz}$$

$$+X = 4\varphi\left\{-\frac{1}{\sqrt2}-\frac{\delta}{2\sqrt2}+\frac{x}{2}-\frac{3x\delta}{4}+\frac{3x^2}{4\sqrt2}+\frac{y^2+z^2}{8\sqrt2}-\frac{9\delta x^2}{8\sqrt2}+\frac{5(y^2+z^2)\delta}{16\sqrt2}\right\}$$

$$+2\varphi' l\left\{-\frac{\delta}{\sqrt2}+x+\frac{3x\delta}{2}-\frac{3x^2}{2\sqrt2}-\frac{y^2+z^2}{4\sqrt2}+\frac{9x^2\delta}{4\sqrt2}-5\frac{(y^2+z^2)\delta}{8\sqrt2}\right\}$$

$$+\frac{2\varphi'' l^2}{\cancel{\ }}\left\{\frac{\cancel{\ }x\delta}{2}-\frac{\cancel{\ }x^2}{2\sqrt2}-\frac{\cancel{\ }(y^2+z^2)}{4\sqrt2}-\frac{3x^2\delta}{2\sqrt2}\right\}$$

$$\cancel{2\varphi\sqrt2+2\varphi+\varphi l} \quad \sum_1^{12}(r-l) = 4\delta + 4(x^2+y^2+z^2) - \frac{3x^2\delta}{\cancel{\ }} - \frac{(y^2+z^2)\delta}{2}$$

$$\sum_1^{12}(r-l)^3 = 6x^2\delta + 3(y^2+z^2)\delta$$

$$U = 12\Phi(l) + \varphi l\left[4\delta + 4\frac{(x^2+y^2+z^2)}{l^2} - 3x^2\delta - \frac{y^2+z^2}{2}\delta\right]$$

$$+\varphi' l^2\left[2\frac{(x^2+y^2+z^2)}{l^2} + 3x^2\delta + \frac{y^2+z^2}{2}\delta\right]$$

$$+\varphi'' l^3\left[x^2\delta + \frac{y^2+z^2}{2}\delta\right]$$

~~[illegible]~~

$$\varphi l\left[4\delta + \left(4-\frac{\delta}{2}\right)(x^2+y^2+z^2) - \frac{5x^2\delta}{2}\right]$$

$$\left(2+\frac{\delta}{2}\right)(x^2+y^2+z^2) + \frac{5x^2\delta}{2}\Big]$$

$$U_0 = 12\Phi(l) + 4\varphi\frac{r^2}{l} + 2\frac{\varphi' r^2}{l}$$

φ dodatnie dla

$$-\frac{\partial U}{\partial r} = -4(2\varphi+\varphi')\frac{r}{l}$$

$$(2\varphi+\varphi') > 0$$

$$-2 + 6\left(1+\frac{\delta}{3}\right) + x^2\left(2-\frac{3\delta}{2}\right) + (y^2+z^2)\left(2-\frac{\delta}{4}\right)$$

$$-6\left[1+\frac{2\delta}{3} + x^2 \qquad + y^2+z^2\right]$$

$$+2\left[1+\delta\right] + 3(x^2+y^2+z^2)\left(1+\frac{\delta}{3}\right) + x^2\left(1+\frac{3\delta}{2}\right) + (y^2+z^2)\left(1-\frac{\delta}{4}\right)$$

$$\cancel{= 2\delta - 4\delta + 2\delta} \quad x^2\left(2-\frac{3\delta}{2}-\cancel{6}+\cancel{6}+2\delta+2+3\delta\right) \;\Big|\; (y^2+z^2)\left(2-\frac{\delta}{4}+1-\frac{\delta}{4}-6\right)$$

$$x^2\left(4+\frac{7\delta}{2}\right)$$

$$(r_1-l)^3 = -\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right)+\frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)\cdot +3\left[\frac{x^2(\cancel{\;})}{2}\frac{\delta}{2}+\frac{y^2}{2}\frac{\delta}{2}+\frac{x^2y(1+\frac{\delta}{2})}{2\sqrt2}-\frac{xy^2(1-\frac{\delta}{2})}{2\sqrt2}\right]$$

$$+6\left[-\frac{xy\delta}{4}-\frac{x(x^2+y^2+z^2)}{4\sqrt2}\delta+\frac{x^3\delta}{8\sqrt2}+\frac{xy^2\delta}{8\sqrt2}-\frac{x^2y\delta}{4\sqrt2}+\frac{y(x^2+y^2+z^2)}{4\sqrt2}\delta-\right.$$

$$\left.-\frac{x^2y}{8\sqrt2}\delta-\frac{y^3\delta}{8\sqrt2}+\frac{xy^2}{4\sqrt2}\delta\right]$$

$$r_1+r_2+r_3+r_4 = 4\left\{1+\frac{\delta}{2}-\frac{x}{\sqrt2}\left(1+\frac{\delta}{2}\right)+\frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right)-\frac{x^2}{4}\left(1+\frac{\delta}{2}\right)-\frac{y^2}{8}\left(1-\frac{3\delta}{2}\right)-\frac{z^2}{8}\left(1-\frac{3\delta}{2}\right)\right.$$

$$\left.+\frac{x(x^2+y^2+z^2)}{2\sqrt2}\left(1-\frac{\delta}{2}\right)-\frac{x^3}{4\sqrt2}\left(1+\frac{\delta}{2}\right)-\frac{3xy^2}{8\sqrt2}\left(1-\frac{3\delta}{2}\right)-\frac{3xz^2}{8\sqrt2}\left(1-\frac{3\delta}{2}\right)\right\}$$

$$-\frac{z^2\delta}{8}$$

$$\sum_1^4(r_1-l)^2 = 4\left\{\frac{x^2}{2}(1+\delta)+\frac{y^2}{4}(1-\delta)+\frac{z^2}{4}(1-\delta)-\frac{x\delta}{\sqrt2}+\frac{x^2+y^2+z^2}{2}\delta-\frac{x^2\delta}{4}-\frac{y^2\delta}{8}+\frac{x(x^2+y^2+z^2)}{2\sqrt2}\delta\right.$$

$$-\frac{x^3\delta}{4\sqrt2}-\frac{3xy^2\delta}{8\sqrt2}-\frac{3xz^2\delta}{8\sqrt2}-\frac{x(x^2+y^2+z^2)}{\sqrt2}+\frac{x^3}{2\sqrt2}(1+\delta)+\frac{3xy^2}{4\sqrt2}(1-\delta)$$

$$\left.+\frac{3xz^2}{4\sqrt2}(1-\delta)+\cancel{\frac{xy^2}{2\sqrt2}(1-\delta)}+\cancel{\frac{xz^2}{2\sqrt2}(1-\delta)}\right\}$$

$$\sum_{1\,9}^{4\,12}(r_1-l)^2 = 8\left\{\frac{x^2}{2}(1+\delta)+\frac{y^2}{4}(1-\delta)+\frac{z^2}{4}(1-\delta)+\frac{x^2+y^2+z^2}{2}\delta-\frac{x^2\delta}{4}-\frac{y^2+z^2}{8}\delta\right\}$$

$$\sum_{5678}(r-l)^2 = 4\left\{\qquad+\frac{y^2}{2}\qquad+\frac{z^2}{2}\qquad\right\}$$

$$\sum_1^{12}(r-l)^2 = 4(x^2+y^2+z^2)+(6x^2+y^2+z^2)\delta$$

$$\sum_1^4(r_1-l)^3 = 4\left\{-\frac{x^3}{2\sqrt2}\cancel{\left(1+\frac{3\delta}{2}\right)}+\frac{3x^2\delta}{4}+\frac{3y^2\delta}{8}+\frac{3z^2\delta}{8}-\frac{3x(y^2+z^2)\left(1-\frac{\delta}{2}\right)}{4\sqrt2}-\frac{3x(x^2+y^2+z^2)\delta}{2\sqrt2}\right.$$

$$\left.+\cancel{\frac{3x^3\delta}{4\sqrt2}}+\cancel{\frac{3xy^2\delta}{8\sqrt2}}+\cancel{\frac{3xz^2\delta}{8\sqrt2}}+\frac{9x(y^2+z^2)\delta}{8\sqrt2}\right\}$$

$$=4\left\{\frac{3x^2\delta}{4}+\frac{(y^2+z^2)3\delta}{8}-\frac{3x(y^2+z^2)}{4\sqrt2}-\frac{x^3}{2\sqrt2}-\frac{3x^3\delta}{2\sqrt2}\right\}$$

$$\Phi(l)+(r-l)\varphi+\frac{(r-l)^2}{2}\varphi'+\frac{(r-l)^3}{2\cdot 3}\varphi''$$

$$r_1 = \sqrt{1+\delta}\,\sqrt{1+\alpha} = \left(1+\frac{\delta}{2}\right)\left[1+\frac{\alpha}{2}-\frac{\alpha^2}{8}+\frac{\alpha^3}{16}\right]$$

$$\sqrt{1+\delta}\,\sqrt{1+\alpha+\beta} = \left(1+\frac{\delta}{2}\right)\left[1+\frac{\alpha}{2}+\left(\frac{\beta}{2}-\frac{\alpha^2}{8}\right)-\frac{\alpha\beta}{4}+\frac{\alpha^3}{16}\right]$$

$$\alpha = -x\sqrt{2}+\frac{y\sqrt{2}}{1+\delta} \qquad \beta = \frac{x^2+y^2+z^2}{1+\delta}$$

$$r_1 = \left(1+\frac{\delta}{2}\right)\left[1-\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}(1+\delta)}+\frac{x^2+y^2+z^2}{2(1+\delta)}-\frac{x^2}{4}-\frac{y^2}{4(1+\delta)^2}+\frac{xy}{2(1+\delta)}+\frac{x(x^2+y^2+z^2)}{2\sqrt{2}(1+\delta)}\right.$$

$$\left.-\frac{y(x^2+y^2+z^2)}{2\sqrt{2}(1+\delta)^2}+\frac{2\sqrt{2}}{16}\left(-x^3+\frac{y^3}{(1+\delta)^3}+\frac{3x^2y}{1+\delta}-\frac{3xy^2}{(1+\delta)^2}\right)\right]$$

$$r_1 = 1+\frac{\delta}{2}-\frac{x}{\sqrt{2}}\left(1+\frac{\delta}{2}\right)+\frac{y}{\sqrt{2}}\left(1-\frac{\delta}{2}\right)+\frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right)-\frac{x^2}{4}\left(1+\frac{\delta}{2}\right)-\frac{y^2}{4}\left(1-\frac{3\delta}{2}\right)+\frac{xy}{2}\left(1-\frac{\delta}{2}\right)$$

$$+\frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\left(1-\frac{\delta}{2}\right)-\frac{y(x^2+y^2+z^2)}{2\sqrt{2}}\left(1-\frac{3\delta}{2}\right)-\frac{x^3}{4\sqrt{2}}\left(1+\frac{\delta}{2}\right)+\frac{y^3}{4\sqrt{2}}\left(1-\frac{5\delta}{2}\right)+\frac{3x^2y}{4\sqrt{2}}\left(1-\frac{\delta}{2}\right)-$$

$$-\frac{3xy^2}{4\sqrt{2}}\left(1-\frac{3\delta}{2}\right)$$

$$(r_1-l)^2 = \frac{x^2}{2}(1+\delta)+\frac{y^2}{2}(1-\delta)-\frac{x\delta}{\sqrt{2}}+\frac{y\delta}{\sqrt{2}}+\frac{x^2+y^2+z^2}{2}\delta-\frac{x^2}{4}\delta-\frac{y^2}{4}\delta+\frac{xy}{2}\delta-xy+$$

$$+\frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\delta-\frac{y(x^2+y^2+z^2)}{2\sqrt{2}}\delta-\frac{x^3\delta}{4\sqrt{2}}+\frac{y^3\delta}{4\sqrt{2}}+\frac{3x^2y}{4\sqrt{2}}\delta-\frac{3xy^2}{4\sqrt{2}}\delta$$

$$-\frac{x(x^2+y^2+z^2)}{\sqrt{2}}+\frac{x^3}{2\sqrt{2}}(1+\delta)+3\frac{xy^2}{2\sqrt{2}}(1-\delta)-\frac{3x^2y}{2\sqrt{2}}+\frac{y(x^2+y^2+z^2)}{\sqrt{2}}(1-\delta)-$$

$$-\cancel{\frac{x^2y}{2\sqrt{2}}}-\frac{y^3}{2\sqrt{2}}(1-2\delta)+\cancel{\frac{xy^2}{\sqrt{2}}(1-\delta)}$$

$$+\frac{2z^2}{8}(1-$$

$$r_1^3+r_2^3+r_3^3+r_4^3 = 4\Big\{1+\frac{3\delta}{2}-\frac{3x}{\sqrt2}\Big(1+\frac{3\delta}{2}\Big)+\frac{3(x^2+y^2+z^2)}{2}\Big(1+\frac{\delta}{2}\Big)+\frac{3x^2}{4}\Big(1+\frac{3\delta}{2}\Big)+\frac{3y^2}{8}\Big(1-\frac{\delta}{2}\Big)$$

$$-\frac{3x(x^2+y^2+z^2)}{2\sqrt2}\Big(1+\frac{\delta}{2}\Big)+\frac{x^3}{4\sqrt2}\Big(1+\frac{3\delta}{2}\Big)+\frac{3xy^2}{8\sqrt2}\Big(1-\frac{\delta}{2}\Big)+\frac{3xz^2}{8\sqrt2}\Big(1-\frac{\delta}{2}\Big)+\frac{3z^2}{8}\Big(1-\frac{\delta}{2}\Big)\Big\}$$

$$+\frac{3x^3\delta}{16\sqrt2}+\frac{3(y^2+z^2)x\delta}{32\sqrt2}$$

$$\frac{\partial}{\partial x} = 4\Big[\Big\{-\frac{3}{\sqrt2}\Big(1+\frac{3\delta}{2}\Big)+3x\Big(1+\frac{\delta}{2}\Big)+\frac{3x}{2}\Big(1+\frac{3\delta}{2}\Big)-\frac{3(3x^2+y^2+z^2)}{2\sqrt2}\Big(1+\frac{\delta}{2}\Big)+\frac{3x^2}{4\sqrt2}\Big(1+\frac{3\delta}{2}\Big)+$$

$$+\frac{3y^2}{8\sqrt2}\Big(1-\frac{\delta}{2}\Big)+\frac{3z^2}{8\sqrt2}\Big(1-\frac{\delta}{2}\Big)\Big\}$$

$$-3\Big\{-(1+\delta)\sqrt2+2x\Big\}$$

$$+3\Big\{-\frac{1}{\sqrt2}\Big(1+\frac{\delta}{2}\Big)+\frac{x}{2l}-\frac{3x\delta}{4l}+\frac{6x^2-y^2-z^2}{8\sqrt2\, l^2}-\frac{18x^2-y^2-z^2}{16\sqrt2}\delta\Big\}\Big]$$

$$=4\Big[\frac{3x\delta}{2}-\frac{3x^2}{2\sqrt2}-\frac{3}{2\sqrt2}y^2-\frac{3z^2}{2\sqrt2}-\frac{9x^2\delta}{2\sqrt2}$$

$$\bar{X} = -\left\{\left[\varphi - l\varphi' + \frac{l^2\varphi''}{2}\right]\left[-\frac{4}{\sqrt2}\left(1+\frac{\delta}{2}\right) + \frac{2x}{l} - \frac{3x\delta}{l} + \frac{3x^2-y^2+2z^2}{4\sqrt2\, l^2} - 9\frac{x^2-7y^2+2z^2}{8\sqrt2\, l^2}\delta\right]\right.$$

$$+\left[\varphi' - l\varphi''\right] 2l\left[-(1+\delta)\sqrt2 + \frac{2x}{l}\right] +$$

$$+ \cancel{\quad}$$

$$+ \cancel{\quad}$$

$$\left. + \frac{2\varphi''}{3} l^2\left[-\frac{3}{\sqrt2}\left(1+\frac{3\delta}{2}\right) + \frac{9}{2}x + \frac{15}{4}x\delta - \frac{30x^2+9x^2\delta+y^2+\frac{3y^2\delta}{2}+7z^2+\frac{5z^2\delta}{2}}{8\sqrt2}\right]\right\}$$

$$\bar{X} = \iiint \frac{X\, e^{-k\left[\left(\varphi - l\varphi' + \frac{l^2\varphi''}{2}\right)\Sigma r + \left(\frac{\varphi' - l\varphi''}{2}\right)\Sigma r^2 + \frac{\varphi''}{6}\Sigma r^3\right]}\, dx\,dy\,dz}{\int e^{-k[\,.\quad -\quad -\quad]}\, dx\,dy\,dz}$$

$$r^3_{5,7} = 1 + \frac{3}{\sqrt2}y \mp \frac{3}{\sqrt2}z + \frac{3}{2}(x^2+y^2+z^2) + \frac{3}{4}(y^2+z^2) \mp \frac{3}{2}yz + \frac{3}{2\sqrt2}(y\mp z)(x^2+y^2+z^2) - \frac{(y\mp z)^3}{4\sqrt2}$$

$$\cancel{\quad}$$

$$r^3_5 + r^3_7 + r^3_6 + r^3_8 = 1 + \frac{3}{2}(x^2+y^2+z^2) + \frac{3}{4}(y^2+z^2) = 4\left[1 + \frac{9}{4}(x^2+y^2+z^2) - \frac{3}{4}x^2\right]$$

$$= 4 + 6(x^2+y^2+z^2) + 3(y^2+z^2)$$

$$\Sigma r^3 = 12 + 12\delta + 24(x^2+y^2+z^2) + \frac{9}{2}\delta(x^2+y^2+z^2) + \frac{21}{2}x^2\delta$$

$$e^{-hU} = e^{-h\left[(x^2+y^2+z^2)\left\{\left(\frac{\varphi}{l}-\varphi'\right)\left(4+\frac{\delta}{2}\right)+6\varphi'\right\} - \frac{5}{2}x^2\delta\left\{\frac{\varphi}{l}-\varphi'\right\}\right]}$$

$$= \frac{(\varphi - l\varphi')}{4\sqrt{2}\,l^2}\left\{\frac{\left(3-\frac{9\delta}{2}\right)}{2h^2\left[\left(\frac{\varphi}{l}-\varphi'\right)\left(4+\frac{\delta}{2}\right)+6\varphi'-\frac{5}{2}\delta\left(\frac{\varphi}{l}-\varphi'\right)\right]^2} + \frac{-\left(1-7\frac{\delta}{2}\right) + (2-\delta)}{2h^2\left[\left(\frac{\varphi}{l}-\varphi'\right)\left(4+\frac{\delta}{2}\right)+6\varphi'\right]^2}\right\}$$

$$r^3_{1,2} = 1+\frac{3\delta}{2} - \frac{3x}{\sqrt{2}}\left[1+\frac{3\delta}{2}\right] \pm \frac{3y}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] + \frac{3(x^2+y^2+z^2)}{2}\left[1+\frac{\delta}{2}\right] + \frac{3x^2}{4}\left[1+\frac{3\delta}{2}\right] +$$

$$+\frac{3y^2}{4}\left[1-\frac{\delta}{2}\right] \mp \frac{3xy}{2}\left[1+\frac{\delta}{2}\right] - \frac{3x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1+\frac{\delta}{2}\right] \pm \frac{3y(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^3}{4\sqrt{2}}\left[1+\frac{3\delta}{2}\right]$$

$$\mp \frac{y^3}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right] \mp \frac{3x^2y}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] + \frac{3xy^2}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right] \pm \cancel{\ldots}$$

$$r^3_{3,4} = 1+\frac{3\delta}{2} - \frac{3x}{\sqrt{2}}\left[1+\frac{3\delta}{2}\right] \mp \frac{3z}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] + \frac{3(x^2+y^2+z^2)}{2}\left[1+\frac{\delta}{2}\right] + \frac{3x^2}{4}\left[1+\frac{3\delta}{2}\right] +$$

$$+\frac{3z^2}{4}\left[1-\frac{\delta}{2}\right] \pm \frac{3xz}{2}\left[1+\frac{\delta}{2}\right] - \frac{3x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1+\frac{\delta}{2}\right] \mp \frac{3z(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^3}{4\sqrt{2}}\left[1+\frac{3\delta}{2}\right] +$$

$$\pm \frac{z^3}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right] \pm \frac{3x^2z}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] - \frac{3xz^2}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right]$$

$$\left.\begin{matrix} r_1^3+r_2^3+r_3^3+r_4^3 \\ +r_5^3 \end{matrix}\right\} = 8\left\{1+\frac{3\delta}{2} + \frac{3(x^2+y^2+z^2)}{2}\left(1+\frac{\delta}{2}\right) + \frac{3x^2}{4}\left(1+\frac{3\delta}{2}\right) + \frac{3y^2}{8}\left(1-\frac{\delta}{2}\right) + \frac{3z^2}{8}\left(1-\frac{\delta}{2}\right)\right\}$$

9 10 11 12 4 $+9(x^2+y^2+z^2)$ $-3x^2$

~~$r^3 = 12\,l^3(1+\delta) + 2(x^2+y^2+z^2)\,l(7+3\delta) + 6x^2\left(1+\frac{3\delta}{2}\right) + y^2\left(3+\frac{\delta}{2}\right) + z^2\left(3+\frac{\delta}{2}\right)$~~

~~$= 12\,l^3(1+\delta) + (x^2+y^2+z^2)\,l\ldots$~~

$$\frac{\partial}{\partial x}(r_1+r_2+r_3+r_4) = 4\left\{-\frac{\cancel{\;}}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{x}{2l} - \frac{3x\delta}{4l} + \frac{3x^2-y^2+2z^2}{4\sqrt{2}\,l^2} - \frac{9x^2-7y^2+2z^2}{8\sqrt{2}\,l^2}\delta\right.$$

$$\frac{\cancel{1}6x^2+y^2+z^2}{8\sqrt{2}} - \frac{18x^2-5y^2-5z^2}{16\sqrt{2}}\delta$$

$$\frac{\partial}{\partial x}(r_1^2+r_2^2+r_3^2+r_4^2) = 4l\left[-(1+\delta)\sqrt{2} + \frac{2x}{l}\right]$$

$$X = -\left\{(\varphi - l\varphi')\frac{\partial}{\partial x}(r_1+r_2+\ldots) + \frac{\varphi'}{2}\frac{\partial}{\partial x}(r_1^2+r_2^2+\ldots)\right\}$$

$$\bar{X} = \frac{\iiint_{-\infty}^{+\infty} X\, e^{-h\left[(\varphi-l\varphi')\Sigma r + \frac{\varphi'}{2}\Sigma r^2\right]}\,dx\,dy\,dz}{\iiint_{-\infty}^{+\infty} e^{-h\left[(\varphi-l\varphi')\Sigma r + \frac{\varphi'}{2}\Sigma r^2\right]}\,dx\,dy\,dz}$$

$$= 2\sqrt{2}\left[\varphi\left(1+\frac{\delta}{2}\right) + l\varphi'\frac{\delta}{2}\right] +$$

$$-\frac{\iiint(\varphi-l\varphi')\left[\frac{3x^2-y^2+2z^2}{4\sqrt{2}\,l^2} - \frac{9x^2-7y^2+2z^2}{8\sqrt{2}\,l^2}\delta\right]e^{-h[\quad]}\,dx\,dy\,dz}{\iiint e^{-h[\quad]}\,d\ldots}$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2}dx = \sqrt{\frac{\pi}{\alpha}} \qquad \frac{\int_{-\infty}^{+\infty} x^2 e^{-\alpha x^2}dx}{\int e^{-\alpha x^2}dx} = \frac{1}{2\alpha^2}$$

$$\int_{-\infty}^{+\infty} x^2 e^{-\alpha x^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$r_{5,7}^3 = 1 + \frac{3y}{\sqrt{2}} + \frac{3z}{\sqrt{2}} + 3\frac{x^2+y^2+z^2}{2} + \frac{z(\ldots)}{2\sqrt{2}} - \frac{y^3}{4\sqrt{2}} \pm \frac{z^3}{4\sqrt{2}} \pm \frac{3y^2z}{4\sqrt{2}} - \frac{3yz^2}{4\sqrt{2}} + \ldots$$

$$+\frac{y^3}{4\sqrt{2}}$$

$$r_{6,8}^3 = 1 - \frac{y}{\sqrt{2}} \ldots$$

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$$\frac{r_{1,2}}{l} = 1+\frac{\delta}{2} - \frac{x}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] \pm \frac{y}{\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{y^2}{4}\left[1-\frac{3\delta}{2}\right] +$$

$$\pm \frac{xy}{2}\left[1-\frac{\delta}{2}\right] + \frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] \mp \frac{y(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{3\delta}{2}\right] - \frac{x^3}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] +$$

$$\pm \frac{y^3}{4\sqrt{2}}\left[1-\frac{5\delta}{2}\right] \pm \frac{3x^2y}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right] - \frac{3xy^2}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right]$$

$$\cancel{\frac{r_2+r_9+r_{10}}{l} = 1+\frac{\delta}{2}}$$

$$\frac{r_1+r_2+r_9+r_{10}}{l} = 4\left\{1+\frac{\delta}{2} + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{y^2}{4}\left[1-\frac{3\delta}{2}\right]\right\}$$

$$\frac{r_{3,4}}{l} = 1+\frac{\delta}{2} - \frac{x}{\sqrt{2}}\left[1+\frac{\delta}{2}\right] \mp \frac{z}{\sqrt{2}}\left[1-\frac{\delta}{2}\right] + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{z^2}{4}\left[1-\frac{3\delta}{2}\right] \pm$$

$$\mp \frac{xz}{2}\left[1-\frac{\delta}{2}\right] + \frac{x(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{\delta}{2}\right] \pm \frac{z(x^2+y^2+z^2)}{2\sqrt{2}}\left[1-\frac{3\delta}{2}\right] - \frac{x^3}{4\sqrt{2}}\left[1+\frac{\delta}{2}\right] -$$

$$\mp \frac{z^3}{4\sqrt{2}}\left[1-\frac{5\delta}{2}\right] \mp \frac{3x^2z}{4\sqrt{2}}\left[1-\frac{\delta}{2}\right] - \frac{3xz^2}{4\sqrt{2}}\left[1-\frac{3\delta}{2}\right]$$

$$\frac{r_3+r_4+r_{11}+r_{12}}{l} = 4\left\{1+\frac{\delta}{2} + \frac{x^2+y^2+z^2}{2}\left[1-\frac{\delta}{2}\right] - \frac{x^2}{4}\left[1+\frac{\delta}{2}\right] - \frac{z^2}{4}\left[1-\frac{3\delta}{2}\right]\right\}$$

$$\frac{r_{5,7}}{l} = 1 + \frac{y}{\sqrt{2}} \mp \frac{z}{\sqrt{2}} + \frac{x^2+y^2+z^2}{2} - \frac{y^2}{4} - \frac{z^2}{4} \pm \frac{yz}{2} + \frac{y(x^2+y^2+z^2)}{2\sqrt{2}} \pm \frac{z(x^2+y^2+z^2)}{2\sqrt{2}} -$$

$$+ \frac{y^3}{4\sqrt{2}} \mp \frac{z^3}{4\sqrt{2}} \mp \frac{3y^2z}{4\sqrt{2}} + \frac{3yz^2}{4\sqrt{2}}$$

$$r_5+r_7+r_6+r_8 = 4\left\{1 + \frac{x^2+y^2+z^2}{2} - \frac{y^2}{4} - \frac{z^2}{4}\right\}$$

$$\sum r = 12l + 4\delta l + \frac{(x^2+y^2+z^2)}{l}\left(4 - \frac{\delta}{2}\right) - \frac{5}{2}\frac{x^2\delta}{l}$$

$$\sum r^2 = 4l^2\left\{3 + 2\delta + \frac{3(x^2+y^2+z^2)}{l^2}\right\}$$

$$X = -\frac{\partial}{\partial x}\left[\Phi(l) + (r-l)\,\varphi(l) + \frac{(r-l)^2}{2}\varphi'(l) + \frac{(r-l)^3}{2.3}\varphi''(l)\right]$$

$$= -\frac{\partial}{\partial x}\left[\varphi\, r + \frac{r^2 - 2lr}{2}\varphi' + \frac{r^3 - 3r^2 l + 3 r l^2}{2.3}\varphi''\right]$$

$$= -\left[\left(\varphi - l\varphi' + \frac{l^2}{2}\varphi''\right)\frac{\partial r}{\partial x} + \left(\frac{\varphi'}{2} - \frac{\varphi''}{2}\right)\frac{\partial (r^2)}{\partial x} + \frac{\varphi''}{6}\frac{\partial (r^3)}{\partial x}\right]$$

$$\varphi\Big[-\frac{1}{\sqrt2}\left(1+\frac{\delta}{2}\right) + x\left(1-\frac{\delta}{2}\right) - \underset{-\frac{3x^2}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)}{\frac{x}{2}\left(1+\frac{\delta}{2}\right)} + \cancel{\frac{y}{2}\left(1-\frac{\delta}{2}\right)} + \frac{6x^2+y^2+z^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right) - \cancel{\frac{2xy}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)} - \frac{3x^2}{4\sqrt2}\left(1+\frac{\delta}{2}\right)$$

$$+ \cancel{\frac{6xy}{4\sqrt2}\left(1-\frac{\delta}{2}\right)} - \frac{3y^2}{8\sqrt2}\left(1-\frac{3\delta}{2}\right)\Big] + \frac{\varphi'}{2}\Big[2x(1+\delta) - \frac{\delta}{\sqrt2} + \frac{x\delta}{2} - \cancel{\frac{y}{}} + \cancel{\frac{y\delta}{2}} + \frac{3x^2+y^2+z^2}{2\sqrt2}\delta$$

$$- \cancel{\frac{4xy\delta}{\sqrt2}} - \frac{3x^2\delta}{4\sqrt2} + \cancel{\frac{3xy\delta}{2\sqrt2}} - \frac{3y^2\delta}{4\sqrt2} - \cancel{\delta} - \frac{3x^2+y^2+z^2}{\sqrt2} + \frac{3x^2(1+\delta)}{2\sqrt2} + 3\frac{y^2(1-\delta)}{2\sqrt2} - \cancel{\frac{3y}{\sqrt2}}\Big]$$

$$+ \cancel{\frac{2yx(1-\delta)}{\sqrt2}} - \cancel{\frac{2xy}{2\sqrt2}} + \cancel{\frac{y^2}{\sqrt2}(1-\delta)}\Big]$$

$$+ \frac{\varphi''}{6}\frac{\partial}{\partial x}\Big[-\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right) + \cancel{\frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)} + 3\Big[\cancel{\frac{x^2y}{2\sqrt2}}\left(1+\frac{\delta}{2}\right) + \frac{xy^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right)\Big] + \frac{3x^2\delta}{4} + \frac{3y^2\delta}{8} + \frac{3z^2\delta}{8}\Big]$$

$$= \varphi\Big[\cancel{\frac{1}{\sqrt2}\left(1+\frac{\delta}{2}\right) + \frac{x}{2}\left(1+\frac{3\delta}{2}\right)} + \cancel{\frac{3x^2-y^2+z^2}{4\sqrt2}} + \frac{3x^2 + \frac{\delta}{2}y^2 + z^2}{4\sqrt2}\delta\Big] - \frac{3xz^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right)$$

$$+ \frac{\varphi''}{6}\Big[-\frac{3x^2}{2\sqrt2}\left(1+\frac{3\delta}{2}\right) + \frac{3y^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right) + \frac{3x\delta}{2}\Big] - \frac{3z^2}{4\sqrt2}\left(1-\frac{\delta}{2}\right)$$

$$\int \left(\chi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \chi}{\partial n}\right) ds = \iint \left(\chi \nabla^2 \psi - \quad\right) dx\,dy$$

$\psi = \frac{\partial \chi}{\partial n} = 0$ $\qquad$ $\chi = \nabla^2 \psi$ = vorticity

$$\iint (\nabla^2\psi)^2\, dx\,dy = \iint \psi \nabla^4 \psi \; dx\,dy$$

$\overset{\shortparallel}{0}$ bo $\nabla^4 \psi = 0$

therefore $\nabla^2 \psi = 0$

$\therefore\ \psi = 0$ because on contour $\psi = 0$

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... at Jerusalem. The eager
... people to build a beautiful
..., thus helping to replace the magnificent cathedrals of their own country, now deserted and desecrated, was shown by the nature of the gifts, which included rings, bracelets, and promises of cement, gravel, and free labour by masons, carpenters, and others.

Every week a little progress has been made, and the church is already a beautiful building, typically Armenian.

... cycles
13 trucks, 101 boats, and 445

They police towns and they ... prairies; they are to be found ... Arctic outposts of civilisation. When, in 1929, an Inspector made a tour of these outposts he travelled 2000 miles by dog team, visiting most of the islands to the north of Canada and reaching Bache Peninsula, on Ellesmere Island, within 700 miles of the North Pole.

So romance has not yet departed from these Mounties, nor is it likely to while they have the patrolling of these far north wastes.

THE PENNY-IN-THE-SLOT PAPER

Newspapers are being sold at London stations through the agency of penny-in-the-slot machines.

Each machine has three sections, holding 40 newspapers, and each section has a space in which can be seen the front page of the paper.

As in the case of other automatic machines, the idea permits considerable saving of human labour, as bookstalls and shops can be closed as soon as the last deliveries of the evening papers have arrived.

One wonders what effect, if any, this mechanical supply of news of the world's happenings will have on the readers. We have a phrase "hot from the press," but will not news come cold to us from these steel boxes? There has ever been a glamour in snatching our paper from the flying newsboy, and there is something romantic about the newspaper stall or counter with all the day's events set out in a variety of guises before us.

...
beath...
which is ... pla...

It is on ...

Thanks to their handic... Mr Robb, who has constru... sided hive, they will soon kno... the habits of bees.

They have watched the Queen ... from her cell and all the excitement in the hive before the bees swarmed. Now the bees have settled down to everyday domesticity, unperturbed by the small faces constantly on the watch.

SOMETHING WE FORGOT

We forgot something the other day: the fact that the West Kent Staghounds hunt only carted stags.

And so, when we talked of the British stag which lately died at Dunkirk crossing the Channel to escape being torn to pieces by the hounds, we were wrong. He would not have been killed, but put back into the cart and saved for another day.

Our minds must have been wandering to our old friends down in Somerset.

THE NEW PATIENT

A young blue-tit, found abandoned and nearly dead, was put to bed on some cotton-wool in the Royal Hospital for Incurables at Putney.

Now he is quite well again, but refuses to go. He seems to prefer human beings to his relations outside, and certainly the patients enjoy his company.

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Z okresu wojny

bo 1ª kartka rosyjsku
a zacząć wtedy się
uczyć po rosyjsku

Господинъ Мюллеръ пришёлъ ~~въ~~ дому господина Небохатова и
отдалъ свою визитную карточку слугѣ. Небохатовъ вышёлъ
къ нѣму, попросилъ его садиться и спросилъ ~~что~~ чѣмъ ему
послужить. Мюллеръ показалъ рекомандательное
письмо отъ ихъ общаго прiятеля Булыгина и попросилъ
о мѣсто приказчика въ торговомъ домѣ Небохатова.
Этотъ былъ очень радъ этому пребложенiю, такъ
какъ было у него такое мѣсто незанято, и прiнялъ
его въ службу. Узнавъ Мюллера имя, отчество и адресъ
прощался съ нимъ ~~его~~ а тотъ пошёлъ домой.

$$\rho^2 = y^2 + (x-\xi)^2$$

$$F_2 = 2\pi \left\{ \int_{y=0}^{\infty}\int_{\xi=x}^{\infty} y\,dy\,d\xi \cdot n_\xi \cdot f\left(\sqrt{(x-\xi)^2+y^2}\right) \cdot \frac{\xi - x}{\sqrt{(x-\xi)^2+y^2}} - \int\int_{\xi=0}^{x} \cdots\cdots \frac{x-\xi}{\sqrt{\cdots}} \right\}$$

$$= 2\pi \int_{\xi=x}^{\infty} d\xi \cdot n_\xi \int_{\rho=\xi-x}^{\infty} d\rho \; f(\rho) \cdot (\xi - x) - \cdots \int_{\xi=0}^{x} d\xi \cdot n_\xi \int_{\rho=x-\xi}^{\infty} f(\rho)\, d\rho\, (x-\xi)$$

$$z\int_z^{\infty} f(\rho)\,d\rho = F(z)$$

$$\frac{1}{2h}\frac{1}{n}\frac{\partial n}{\partial x} = 2\pi \left\{ \int_{\xi=x}^{\infty} n_\xi \cdot F(\xi - x)\, d\xi - \int_{\xi=0}^{x} n_\xi \cdot F(x-\xi)\, d\xi \right\} = \cancel{\cdots}$$

$$= 2\pi \left[\int_0^{\infty} - \int_0^{x} n_\xi\, F(x-\xi)\, d\xi \right]$$

$$F'(z) = \int_z^{\infty} f(\rho)\, d\rho - z\, f(z)$$

$$\frac{1}{2h}\frac{dn}{dx} = \frac{2\pi A}{\alpha} n_x \left[\int_x^{\infty} e^{-\alpha(\xi - x)} n_\xi\, d\xi - \int_{\xi=0}^{x} e^{-\alpha(x-\xi)} n_\xi\, d\xi \right]$$

$$\frac{1}{2h}\frac{d}{dx}\left(\frac{1}{n}\frac{dn}{dx}\right) = \frac{2\pi A}{\alpha}\left[-2 n_x + \alpha \int_x^{\infty} e^{-\alpha(\xi - x)} n_\xi\, d\xi + \alpha \int_{\xi=0}^{x} e^{-\alpha(x-\xi)} n_\xi\, d\xi \right]$$

$$\frac{1}{2h}\frac{d^2}{dx^2}\left(\frac{1}{n}\frac{dn}{dx}\right) = \frac{2\pi A}{\alpha}\left\{ -2\frac{dn}{dx} \right\} + \frac{\alpha^2}{2h}\cdot\left(\frac{1}{n}\frac{dn}{dx}\right)$$

$$\underbrace{\frac{1}{2h}\left\{ \frac{d^2}{dx^2}(\log n) \right\}} - \frac{\alpha^2}{2h}\log n = -\frac{4\pi A}{\alpha} n + \text{const}$$

für Punkte, wo $= 0$, also im ∞ ist $\quad \frac{4\pi A}{\alpha}(n - n_0) \quad \frac{\alpha^2}{2h}\log\frac{n}{n_0}$

also ist $\quad n_0\, e^{\frac{8h\pi A}{\alpha^3}(n-n_0)} = n$

$$\frac{\partial \Phi}{\partial x} = 2\pi\left[-2 n_x \cdot \overset{=0}{F(0)} + \int_{\xi=x}^{\infty} n_\xi\, F'(\xi - x)\, d\xi - \int_{\xi=0}^{x} n_\xi\, F'(x-\xi)\, d\xi \right] = \cancel{\cdots}$$

$$= \frac{1}{2h}\frac{d}{dx}\left(\frac{1}{n}\frac{dn}{dx}\right)_x$$

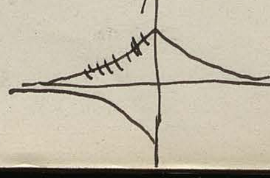
Damit $n_w \frac{mc^2}{3} = \frac{n\, mc^2}{3} - n^2 a$ wird, muss gelten:

$$n_w = n - \frac{2h}{3} n^2 a$$

$$\frac{1}{n}\frac{\partial n}{\partial x} = -\int_0^{\infty} n_\xi\, \Psi(\xi - x)\, d\xi$$

$$\log\frac{n}{n_0} = \int_0^{\infty} n_\xi \cdot X(\xi - x)\, d\xi$$

$$X = \int \Psi\, dx$$



$$n - n_w = -\int_{x=0}^{x}\int_{\xi=0}^{\infty} n_x\, n_\xi\, \Psi(\xi - x)\, d\xi\, dx$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - b^2 u$$

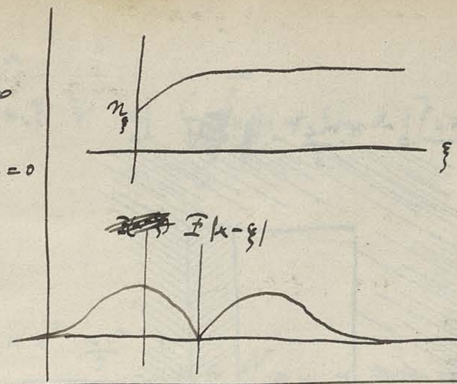
$$u = T \cdot \varphi(x)$$

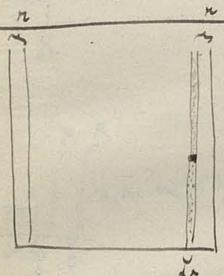
$$\frac{1}{T}\frac{\partial T}{\partial t} = \frac{1}{\varphi}\left[\frac{\partial^2 \varphi}{\partial x^2} - b^2\varphi\right] = -c$$

$$T = e^{-ct}$$

$$\frac{d^2\varphi}{dx^2} - b^2\varphi = -c\varphi$$

$$\frac{d^2\varphi}{dx^2} + (c - b^2)\varphi = 0$$





$$n_w = n\, e^{-2h\overbrace{\int_0^\infty F(r)\,dr}^{U}}$$

$$\frac{n}{\alpha}\,dx \cdot e^{-2h\int_r^\infty F(r)\,dr}$$

$$p_1 = n_w \frac{mc^2}{3} \qquad ?$$

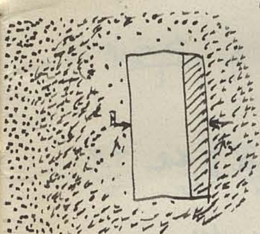
~~$F(r) \sim \frac{n}{v}$~~

~~ist darin nur die Anziehung zu berücksichtigen und die Adhäsionskräfte nicht?~~

$F(r)$ ist Funktion von n und h und zwar komplizierten Inhalt!

es ist nicht $F(r) = n f(h)$!!

sondern



$$p_2 = n\frac{mc^2}{3} - n\underbrace{\int_0^\infty F_1(r)\,dr}_{n^2 a}$$

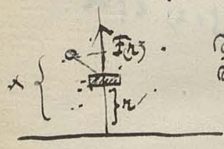
$$F_1(r) \sim n$$

$$\rho^2 = x^2 + y^2$$

$$\left(\frac{mc^2}{3}\right)\frac{\partial n}{\partial x} = n F(x)$$

$$\cancel{n = n_0 e^{-2h\int}}$$

$$n = n_w e$$



$$\frac{\partial p}{\partial r} = \overbrace{\frac{mc^2}{3}}^{=\frac{1}{2h}} \cdot \frac{\partial n}{\partial r} = n F(r)$$

$$n = A\, e^{2h\int_0^r F(r)\,dr}$$

$$n = n_w\, e^{2h\int_0^r F(r)\,dr}$$

$$n_0 = n_w\, e^{2h\int_0^\infty \cdots}$$

$$n = n_0\, e^{-2h\int_r^\infty F(r)\,dr}$$

$$n_w = n_0\, e^{-2h\int_0^\infty F(r)\,dr}$$

$$F(r) = \int n_x \cancel{2\pi R^2\, dR\, d\alpha \cos\alpha}\; f(R)$$

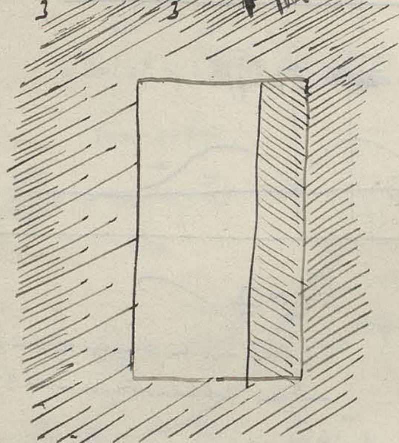
$$= 2\pi \int_{x=0}^{\infty}\int_{y=0} y\,dy\,dx \cdot n_x \cdot \underbrace{f(\sqrt{x^2+y^2})\frac{x}{\sqrt{x^2+y^2}}} - \int$$

z. B. $f = A e^{-\alpha\sqrt{x^2+y^2}}$

$$= 2\pi A \int n_x\,dx \int_x^\infty \rho\,\frac{d\rho\, e^{-\alpha\rho}}{\rho} = \cancel{\frac{2\pi A}{\alpha}\int\int n_x\,dx}$$

$$= \frac{2\pi A}{\alpha}\left[\int_x^\infty e^{-\alpha x}\, n_x\,dx - \int\right.$$

$$n_w \frac{mc^2}{3} = n \frac{mc^2}{3} - \cancel{\ldots} \; 2_n$$



$$\lg \frac{n_0}{n} = 2h \int_x^\infty F(\xi)\, d\xi$$

$$\frac{1}{n} \frac{\partial n}{\partial x} = -2h \; F(x) = -\frac{N}{RT} \; F(x)$$

$$n \frac{RT}{N} = \frac{n\mu}{N} \frac{c^2}{3} = n m \frac{c^2}{3} \quad ||| \quad v = \frac{1}{\rho} = \frac{1}{nm}$$

$$A n^2 = \frac{A}{v^2} \frac{1}{m^2} = A$$

$$e^{-\frac{NA}{RT}} = 1 - \frac{an^2}{\frac{RT}{v}}$$

$$A = -\frac{RT}{N} \lg \left(1 - \frac{an^2}{\frac{RT}{v}}\right)$$

$$p = \frac{n_w mc^2}{3} = n \frac{mc^2}{3} - an^2$$

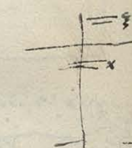
$$n_w = n_0 - \frac{an^2}{\frac{RT}{v}}$$

$$n_w = n_0 - 2h\, an^2$$

$$a = \int_{x=-\infty}^{x=b} \int_{\xi=b}^{\infty} \Psi(\xi - x)\, d\xi\, dx$$

$$\frac{\xi - b}{\ldots} + \frac{b-x}{\ldots}$$

$$= \int_{z=-\infty}^{0} \int_{\xi=0}^{\infty} \Psi(\xi + z)\, d\xi\, dz$$



$$\int_0^X \int_0^\infty n(x)\, n(\xi)\, \psi(\xi - x)\, dx\, d\xi = \int_0^X n(x)\, dx \left[\int_0^x n(\xi)\, \psi \quad d\xi + \int_x^\infty n(\xi)\, \psi(\xi - x)\, d\xi \right]$$

$$n_x - \cancel{\frac{n}{v}} = 2n \int_0^X \int_X^\infty n(x)\, n(\xi)\, \Psi(\xi - x)\, d\xi\, dx$$

$$\frac{\alpha n^2 \frac{mc^2}{3}}{\frac{RT}{N}} = \frac{\alpha n N}{\mu} \cdot \rho = \alpha\, n m \frac{N}{\mu m} \rho$$

$$= \alpha \rho^2 \cdot \frac{N}{\mu m}$$

$$\int_0^x \int_0^x n(x)\, n(\xi)\, \Psi(\xi - x)\, dx\, d\xi =$$

$$p = \rho RT - \rho^2 \underbrace{\alpha \frac{N}{\mu m}}_{\frac{\alpha}{m^2}} \qquad \frac{\mu}{N} = m$$

$$n_0 \frac{mc^2}{3} = \rho^{RT}$$

$$\frac{RT}{v} - \frac{a}{v^2} = \frac{RT}{v} e^{-\frac{N}{HT} A}$$

$$e^{-\frac{N}{HT} A} = 1 - \frac{a}{vRT}$$

$$A = -\frac{HT}{N} \lg \left(1 - \frac{a}{vRT}\right)$$

$\cancel{E \dfrac{\partial \xi}{\partial x}}$

$\xi = a \sin \alpha(x \cdot v)$

$\xi_1 = A_1 \sin \alpha_1 \left(t - \frac{x}{c}\right) \qquad \left(\xi_1 + \xi_2\right)\Big|_{x = x_0 - vt} = 0$

$\xi_2 = A_2 \sin \alpha_2 \left(t + \frac{x}{c} + \varepsilon\right)$

$$A_1 \sin \alpha_1 \left(t - \frac{x_0}{c} + \frac{v}{c} t\right) + A_2 \sin \alpha_2 \left(t + \frac{x_0}{c} - \frac{v}{c} t + \varepsilon\right) = 0$$

$$\alpha_1 \left(1 + \frac{v}{c}\right) = \alpha_2 \left(1 - \frac{v}{c}\right) \qquad \varepsilon + \frac{x_0}{c} = -\frac{x_0}{c}$$

$$A_1 = -A_2$$

$$\alpha_2 = \alpha_1 \frac{c+v}{c-v}$$

$$\frac{1}{2} \rho \dot{\xi}_1^2 = \frac{1}{2} \rho A_1^2 \alpha_1^2 \cos^2 \alpha_1 (t - -)$$

$$\frac{1}{2} \rho \overline{\dot{\xi}_1^2} = \frac{1}{4} \rho A_1^2 \alpha_1^2$$

$$c \frac{\rho}{4} A^2 (\alpha_2^2 - \alpha_1^2) - v \frac{\rho}{4} A^2 (\alpha_1^2 + \alpha_2^2) = P v$$

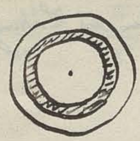
$$P = \frac{\rho A^2}{2} \alpha_1^2 \frac{\left[\left(\frac{c+v}{c-v}\right)^2 - 1\right] c - \left[\left(\frac{c+v}{c-v}\right)^2 + 1\right] v}{v \qquad (c-v)^2}$$

$$\frac{4c^2 v - 2(c^2 + v^2) v}{v} = \frac{2(c^2 - v^2)}{(c-v)^2}$$

$$P = \rho A^2 \alpha_1^2 \frac{(c^2 - v^2)}{(c-v)^2} = \rho A^2 \alpha_1^2 \frac{c+v}{c-v}$$

$$\overline{E} = \rho \frac{A^2}{2} \alpha_1^2 \left[1 + \left(\frac{c+v}{c-v}\right)^2\right] = \rho \frac{A^2 \alpha_1^2}{4} \, 4 \frac{c^2 + v^2}{(c-v)^2}$$

$$P = \overline{E} \, \frac{c^2 - v^2}{c^2 + v^2}$$



$$\frac{4}{3}\pi r\rho = F = -\frac{\partial U}{\partial r}$$

$$U = -\frac{2}{3}\pi\rho r^2 + \text{const} \qquad = 2\pi\rho a^2 - \frac{2}{3}\pi\rho r^2$$

$$U_a = -\frac{2}{3}\pi\rho a^2 + \text{const} = \frac{4}{3}\pi\rho a^2$$

$$\frac{1}{2}\sum U\varphi = \cancel{4}\; 2\pi\sigma^2\int_0^a r^2\,dr\left(2\pi\rho a^2 - \frac{2}{3}\pi\rho r^2\right) = 4\pi^2\sigma^2\left(a^2\cdot\frac{a^3}{3} - \frac{a^5}{15}\right) = \frac{16}{15}a^5\pi^2\sigma^2$$

$$= \left(\frac{4}{3}\pi a^3\sigma\right)^2\cdot\frac{9}{15}\,\frac{1}{a} = \frac{3}{5}\,\frac{e^2}{a}$$

Energetische Masse eines Elektrons sollte betragen: $\frac{3}{5}\frac{e^2}{ac^2}$
(infolge elektrostatischer Energie)

während die aber elektrodynamische Masse ist $m = \frac{e^2}{6\pi a c^2}$

Dasselbe für Schallwellen in einem Gas

$$\frac{\partial\rho}{\partial t} = -\rho\frac{\partial u}{\partial x} \qquad \rho = \rho_0(1+\sigma) \qquad \frac{\partial\sigma}{\partial t} = -\frac{\partial u}{\partial x}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = -\rho\frac{\partial u}{\partial t} \qquad p = p_0(1+k\sigma) \qquad a^2\frac{\partial\sigma}{\partial x} = -\frac{\partial u}{\partial t}$$

$$T = \rho\frac{u^2}{2} \qquad U = \frac{1}{2}\lambda\sigma^2 = a^2\rho\cdot\frac{\sigma^2}{2}$$

$$\cancel{U = \int\frac{1}{\rho}dp = a^2\int d\sigma = a^2\sigma}$$

$$\frac{dp}{d\sigma} = \lambda = p_0 k$$

$$u = A_1\alpha\cos\alpha\left(t - \frac{x}{c}\right)$$

$$\frac{\partial\sigma}{\partial t} = -\frac{A_1\alpha^2}{c}\sin\alpha\left(t - \frac{x}{c}\right)$$

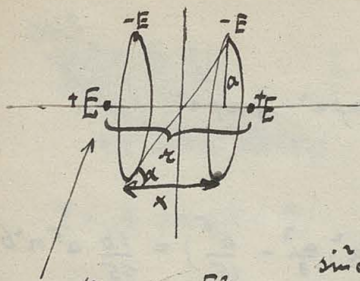
$$\sigma = \frac{A\alpha}{c}\cos\alpha\left(t - \frac{x}{c}\right)$$

$$\overline{T} = \overline{U}$$

Es sollte also dieselbe Formel $P = \overline{E}\,\frac{c\pm v}{c\mp v}$ gültig bleiben! Dagegen Rayleigh?

Ebenso in Flüssigkeit; Reflexion an der freien Oberfläche

Und wie in festen Körpern?



Falls so viele Elektronen auf d. Ring, dass man die Ladung als gleichmässig verteilt ansehen kann.

$$m \frac{d^2 x}{dt^2} = + \frac{E^2}{r^2}$$

$$\sin^2\alpha = \frac{4a^2}{4a^2+x^2} \qquad \sin\alpha = \frac{2a}{\sqrt{4a^2+x^2}}$$

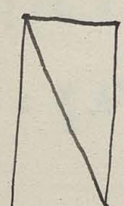
$$\frac{2a}{x} = \operatorname{tg}\alpha \qquad \cos\alpha\, d\alpha = -\frac{2a\, x\, dx}{\left(\sqrt{4a^2+x^2}\right)^3} = -\sin^3\alpha\ \frac{x\,dx}{4a^2}$$

$$-\frac{2a\,dx}{x^2} = \frac{d\alpha}{\cos^2\alpha} = -\frac{\operatorname{tg}^2\alpha}{2a}\,dx$$

$$d\alpha = -\frac{\sin^2\alpha}{2a}\,dx = -\frac{2a}{4a^2+x^2}\,dx \qquad \Big\| \qquad dx = -\frac{d\alpha}{\sin^2\alpha}\,\frac{1}{2a}$$

$$F_x = +\frac{\partial M}{\partial x} = +\frac{\partial M}{\partial \alpha}\,\frac{d\alpha}{dx}\left(\frac{E\omega}{c}\right)^2 \qquad \underline{M} = 4\pi a\, y$$

Elektrostatische Abstossung geladener Kreislinien:



$$W = \frac{1}{2}\Sigma \varphi e = \frac{E^2}{2}\,\frac{1}{2\pi a}\int_{\varphi=0}^{2\pi} \frac{a\,d\varphi}{\sqrt{x^2 + a^2(1-\cos\varphi)^2 + a^2\sin^2\varphi}}$$

$$W = \frac{E^2}{4\pi}\int_0^{2\pi} \frac{d\varphi}{\sqrt{x^2+2a^2(1-\cos\varphi)}} = \frac{E^2}{2\pi}\int_0^{\pi}\frac{d\varphi}{\sqrt{x^2+4a^2\sin^2\left(\frac{\varphi}{2}\right)}}$$

$$= \frac{E^2}{\pi x}\int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1+\underbrace{\left(\frac{2a}{x}\right)^2}_{k}\sin^2\varphi}} = \underbrace{\frac{E^2}{\pi\sqrt{x^2+4a^2}}\int_0^{\frac{\pi}{2}}\frac{d\varphi}{\sqrt{1-\underbrace{\frac{4a^2}{x^2+4a^2}}_{\sin^2\alpha}\sin^2\varphi}}}_{=K(\sin\alpha)}$$

$$\mathcal{E}_x = -\frac{\partial W}{\partial x}$$

$$= \frac{E^2 \sin\alpha}{2a\pi}\,K(\sin\alpha)$$

$$m\frac{d^2r}{dt^2} = \frac{E^2}{r^2} - \frac{E^2\cdot\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} - \frac{E^2\cdot\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2+a^2\right]^{3/2}}$$

dr

$$F_x = \mathcal{E}_x + \frac{E^2\cdot\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} - \frac{E^2\cdot\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2+a^2\right]^{3/2}}$$

dx

$$\cancel{\frac{m}{2}\left(\frac{dr}{dt}\right)^2 = -\frac{E^2}{r} + \frac{E^2}{\sqrt{\left(\frac{r-x}{2}\right)^2+a^2}} + \frac{E^2}{\sqrt{\left(\frac{r+x}{2}\right)^2+a^2}} + fc(x)}$$

$\cancel{\frac{m}{2}v_\infty^2}$?

$$\cancel{M\left(\frac{E\omega}{c}\right)^2 + W - \frac{E^2}{\sqrt{\left(\frac{r-x}{2}\right)^2+a^2}} - \frac{E^2}{\sqrt{\left(\frac{r+x}{2}\right)^2+a^2}} = \text{konst} = 0}$$

$fc.(r)$?

5.10^{-15}

$$\cancel{\frac{m}{2}v_\infty^2 = \frac{E^2}{r} + M\left(\frac{E\omega}{c}\right)^2 - W}$$

8.10^{-11} $\qquad \frac{2}{3}.10^4.4.2.10^{-19}$ $\qquad 10^{-16}$

$$\frac{m}{2}\left(\frac{dr}{dt}\right)^2 + \frac{E^2}{r} - \frac{E^2}{\sqrt{\left(\frac{r+x}{2}\right)^2+a^2}} - \frac{E^2}{\sqrt{\left(\frac{r-x}{2}\right)^2+a^2}} + W - M\left(\frac{E\omega}{c}\right)^2 = \text{konst} = \frac{m}{2}v_\infty^2$$

Falls $v_\infty = 0$ und es werden nur die Ringe berücksichtigt:

Unsinn

Gleichgewicht für:

$$\omega = \frac{h}{2\pi}\frac{1}{\mu a^2}$$

$$E^2\frac{\omega^2}{c^2}M = \frac{E^2}{2a\pi}\sin\alpha\, K$$

$$\frac{e}{\mu c} = 1.87.10^{+7} \qquad \mu = \frac{4.7.10^{-10}}{1.87.10^{7}.c}$$

$$\left(\frac{h}{2\pi}\right)^2\frac{2a\pi}{\mu^2a^4c^2} = \sin\alpha\frac{K}{M}$$

$$\frac{1}{\mu c} = \frac{1.87.10^7}{4.7.10^{-10}} = 0.4.10^{+17}$$

$$\frac{h^2}{\mu a^3 c} = 2\pi\sin\alpha\frac{K}{M} = \frac{6.27.10^{-27}.0.4.10^{17}}{a^3} = \frac{2.5.10^{-10}}{a^3}$$

Falls nur zwei Elektronen $\quad E = 2e$

$$i = \frac{E}{2a\pi}\,\frac{a\omega}{c} = \frac{E\omega}{2\pi c}$$
(elektromagn.)

$$W = \frac{4e^2}{\sqrt{x^2 + 2a^2}}$$

$$\omega = \frac{h}{2\pi}\,\frac{1}{a^2\mu}$$

$$F_x = \left(\frac{E\omega}{2\pi c}\right)^2 \cdot \frac{2a}{4a^2+x^2}\,\frac{\partial M}{\partial \alpha}$$

$$\left(\frac{\omega}{2\pi c}\right)^2 = \left(\frac{4{,}2\,.\,10^{16}}{2\pi\,.\,3\,.\,10^{10}}\right)^2 = 4\,.\,10^{10}$$

$$\frac{\partial W}{\partial x} = \frac{4e^2 x}{\sqrt{x^2+2a^2}^{\,3}}$$

Gleichgewichtslage:

$$\frac{1}{r^2} = \frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2 + a^2\right]^{3/2}} + \frac{\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2 + a^2\right]^{3/2}}$$

$$\left(\frac{\omega}{2\pi c}\right)^2 \frac{2a}{4a^2+x^2}\,\frac{\partial M}{\partial \alpha} = \frac{x}{\sqrt{x^2+2a^2}^{\,3}} + \frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2 + a^2\right]^{3/2}} - \frac{\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2 + a^2\right]^{3/2}}$$

$$r - x = \delta$$

$$\frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} = \frac{\frac{\delta}{2}}{\left[a^2 + \frac{\delta^2}{4}\right]^{3/2}} = \frac{\delta}{2a^3}\left[1 + \frac{\delta^2}{4a^2}\right]^{-3/2}$$

$$= \frac{x + \frac{\delta}{2}}{\left[\left(x+\frac{\delta}{2}\right)^2 + a^2\right]^{3/2}} = \frac{x}{[x^2+a^2]^{3/2}}\left[1 + \frac{\delta}{2x} - \frac{3x\frac{\delta}{2}}{x^2+a^2}\cdot\right]$$

$$\frac{\frac{r+x}{2}}{\cdots}\Big] = \frac{r - \frac{\delta}{2}}{\left[\left(r-\frac{\delta}{2}\right)^2 + a^2\right]^{3/2}} = \frac{r}{[r^2+a^2]^{3/2}}\left[1 - \frac{\delta}{2r} + \frac{3r\frac{\delta}{2}}{r^2+a^2}\cdots\right]$$

$$\frac{1}{r^2} - \frac{r}{[r^2+a^2]^{3/2}} = \frac{\delta}{2a^3} - \frac{\delta}{2}\left[\frac{1}{[r^2+a^2]^{3/2}} - \frac{3r^2}{[r^2+a^2]^{5/2}}\right]$$

$$\frac{\delta}{2} = \frac{\frac{1}{r^2} - \frac{2}{[r^2+a^2]^{3/2}}}{\frac{1}{a^3} - \frac{1}{[r^2+a^2]^{3/2}} + \frac{3r^2}{[r^2+a^2]^{5/2}}}$$

$$\frac{1}{x^2}\left[1 - 2\frac{\delta}{x}\right] = \frac{\delta}{2a^3} + \frac{x}{[x^2+a^2]^{3/2}}\left[1 + \frac{\delta}{2x} - \frac{3x\frac{\delta}{2}}{x^2+a^2}\right]$$

$$\frac{1}{x^2} - \frac{x}{(x^2+a^2)^{3/2}} = \frac{2\delta}{x^3} + \frac{\delta}{2a^3} + \frac{\delta}{2(x^2+a^2)^{3/2}} - \frac{3\delta}{2}\frac{x^2}{(x^2+a^2)^{5/2}}$$

$$a^2\omega = \frac{h}{2\pi\mu}$$

$$\frac{\omega^2}{(2\pi c)^2}\frac{\partial M}{\partial x} = \frac{\omega^2}{4\pi^2c^2}\cdot 4\pi a\frac{\partial y}{\partial x} = \frac{\omega^2 a}{\pi c^2}\frac{\partial y}{\partial x} = \cancel{\frac{h}{2\pi\mu c^2}\frac{\partial y}{\partial x}}$$

$$= \cancel{\frac{h}{2\pi^2}\left(\frac{k}{\mu}\right)\frac{1}{ac^2} = \frac{6{\cdot}5.10^{-27}}{20}\;\frac{1{\cdot}8{\cdot}10^{7}}{3.10^{10}\cdot 4{\cdot}7.10^{-10}}}$$

$$= \cancel{\quad}$$

$$= \frac{(4{\cdot}2.10^{+6})^2\cdot 10^{-8}}{\pi\cdot 9\cdot 10^{20}} = \frac{(4{\cdot}2)^2\cdot 10^4}{9\pi} = \frac{17{\cdot}6}{28{\cdot}3}\cdot 10^4 = \frac{2}{3}\cdot 10^4\cdot\frac{\partial y}{\partial x}$$

$$= -\frac{\partial y}{\partial x}\,\frac{2a}{4{\cdot}7x^2}$$

$$\cancel{\lim\frac{\partial M}{x} = \lim\frac{\pi}{a}\,x\,\pi\alpha\left[2K - \left(1 + \frac{1}{\omega^2\alpha}\right)E\right]}$$

$$\cancel{K = \int_0^{\frac{\pi}{2}} d\varphi}$$

$$y = \frac{2E - (1+\omega^2\alpha)K}{\pi\alpha}$$

$$E = \int_0^{\frac{\pi}{2}} d\varphi\,\sqrt{1 - \sin^2\alpha\,\sin^2\varphi} \qquad K = \int_0^{\frac{\pi}{2}}\frac{d\varphi}{\sqrt{1-\sin^2\alpha\,\sin^2\varphi}}$$

$$\lim E = \frac{\pi}{2} - \frac{\sin^2\alpha}{2}\frac{\pi}{4} \qquad \lim K = \frac{\pi}{2} + \frac{\sin^2\alpha}{2}\frac{\pi}{4}$$

$$\lim_{x\to 0} y = \frac{\left(\pi - \sin^2\alpha\frac{\pi}{4}\right) - \left(\frac{\pi}{2} + \sin^2\alpha\frac{\pi}{4}\right)}{\pi\alpha} = -\frac{\pi}{4}\sin\alpha = -\frac{\pi}{4}\frac{2a}{x} = -\frac{a\pi}{2x}$$

Bei Vernachlässigung der Kraft $\frac{\partial \psi}{\partial a}$:

$r = x + \delta$

$$\frac{x}{\sqrt{x^2+2a^2}^3} + \frac{\frac{r-x}{2}}{\left[\left(\frac{r-x}{2}\right)^2+a^2\right]^{3/2}} - \frac{\frac{r+x}{2}}{\left[\left(\frac{r+x}{2}\right)^2+a^2\right]^{3/2}} = 0$$

$$\left.\begin{aligned} &\frac{x}{\sqrt{x^2+2a^2}^3} + \frac{\delta}{2a^3} - \frac{x}{(x^2+a^2)^{3/2}}\left[1+\frac{\delta}{2x} - \frac{3}{2}\frac{\delta x}{x^2+a^2}\right] = 0 \\ &\frac{1}{x^2}\left(1-\frac{2\delta}{x}\right) - \frac{\delta}{2a^3} - \frac{x}{(\quad)^{3/2}}\left[\qquad\qquad\right] = 0 \end{aligned}\right\}$$

$$\frac{x}{\sqrt{x^2+2a^2}^3} - \frac{1}{x^2}\left(1-\frac{2\delta}{x}\right) + \frac{\delta}{a^3} = 0$$

$$\delta = \frac{\frac{1}{x^2} - \frac{x}{(x^2+2a^2)^{3/2}}}{\frac{1}{a^3} + \frac{2}{x^3}}$$

Elektrostatische Kraft bei großer Entfernung:

$$\frac{x}{(x^2+2a^2)^{3/2}} + \frac{\delta}{2a^3} = \frac{\left(x+\frac{\delta}{2}\right)}{\left[\left(x+\frac{\delta}{2}\right)^2+a^2\right]^{3/2}} = \frac{x}{\left[x^2+a^2\right]^{3/2}}\left[1+\frac{\delta}{2x} - \frac{3}{2}\frac{\delta x}{a^2+x^2}\right]$$

$$\frac{x}{\left[x^2+a^2\right]^{3/2}} - \frac{x}{\left[x^2+2a^2\right]^{3/2}} = \frac{\delta}{2}\left[\frac{1}{a^3} - \frac{1}{\left[x^2+a^2\right]^{3/2}} + \frac{3x^2}{\left[x^2+a^2\right]^{5/2}}\right]$$

$$\frac{1}{x^2}\underbrace{\left\{\left[1-\frac{3}{2}\frac{a^2}{x^2} + \frac{15}{8}\frac{a^4}{x^4}\right] - \left[1-3\frac{a^2}{x^2} + \frac{15}{8}\frac{4a^4}{x^4}\right]\right\}}_{\frac{3}{2}\frac{a^2}{x^2} - \frac{45}{8}\frac{a^4}{x^4}} = \frac{\delta}{2}\left\{\frac{1}{a^3} - \frac{1}{x^3}\left(1-\frac{3}{2}\frac{a^2}{x^2}\right) + \frac{3}{x^3}\left(1-\frac{5}{2}\frac{a^2}{x^2}\right)\right\}$$

$$\left(1-\frac{15}{4}\frac{a^2}{x^2}\right)\frac{3}{2}\frac{a^2}{x^4} = \frac{\delta}{2a^3}\left\{1+\cancel{\frac{2a^3}{x^3}}\right\}$$

$$\delta = \frac{3a^5}{x^4}\left(1-\frac{15}{4}\frac{a^2}{x^2}\right)$$

$$\frac{1}{E^2}\,\mathcal{E}_x = \underbrace{\frac{1}{r^2} - \frac{2}{[r^2+a^2]^{3/2}}} - \frac{\delta}{2a^3} + \frac{\delta}{2}\left[\frac{1}{(r^2+a^2)^{3/2}} - \frac{3r^2}{(r^2+a^2)^{5/2}}\right]$$

$$\frac{1}{r^2}\left[1 - \left(1 - \frac{3}{2}\frac{a^2}{r^2} + \frac{15}{8}\frac{a^4}{r^4}\right)\right]$$

$$\frac{1}{x^4} = \frac{1}{(r-\delta)^4} = \frac{1}{r^4}\left(1 + 4\frac{\delta}{r}\right) =$$

$$= \frac{1}{r^4}\left(1 + \cancel{12\frac{a^2}{r^2}}\right)$$

$$= \frac{1}{r^2}\left(\frac{3}{2}\frac{a^2}{r^2} - \frac{15}{8}\frac{a^4}{r^4}\right) - \frac{3}{2}\frac{a^2}{x^4}\left(1 - \frac{15}{4}\frac{a^2}{x^2}\right)$$

$$= \frac{a^4}{r^6}\,\frac{15}{4}$$

$$\lim \mathcal{E}_x = \frac{15}{4}\,E^2\,\frac{a^4}{r^6}$$

Während elektromagnetische Kraft: $\lim F_x = \frac{\omega^2 a E^2}{r c^2}\,\frac{a r}{2 r^2} = \frac{\omega^2 a^2 E^2}{2c^2}\,\frac{1}{r^2}$

Wo werden sie gleich?

$$\frac{15}{4}\,\frac{a^4}{r^6} = \frac{\omega^2 a^2}{2c^2}\,\frac{1}{r^2}$$

$$r^4 = \frac{15}{2}\,\frac{a^2 c^2}{\omega^2}$$

$$r = \sqrt[4]{\frac{15}{2}}\sqrt{\frac{ac}{\omega}} = \sqrt[4]{\frac{15}{2}}\cdot\sqrt{\frac{10^{-8}\cdot 3\cdot 10^{10}}{4{,}2\cdot 10^{16}}}$$

$$= \sqrt[4]{\frac{15}{2}}\,\sqrt{\frac{1}{1{,}4}}\cdot 10^{-7} = 1{,}4\cdot 10^{-7}$$

~~Die Wenn das Atom aus n Elektronenringen besteht, wird~~ $\cancel{\lim \mathcal{E}_x \sim \frac{a^{2+2n}}{r^{4+2n}}}$

Arbeit $\int_x^\infty \mathcal{E}_x\,dx = \frac{3}{4}\,E^2\,\frac{a^4}{r^5} = 4\,\frac{3}{4}\;2\cdot 10^{-19}\cdot\frac{10^{-32}}{(1{,}4)^5\cdot 10^{-35}} = 4\cdot\frac{3}{2}\,\frac{1}{5{,}6}\cdot 10^{-16} = \cdot 10^{-16}$

Arbeit $\int_x^\infty F_x\,dx = \frac{\omega^2 a^2 E^2}{2c^2 r} = \left(\frac{4{,}2\cdot 10^{16}\cdot 10^{-8}}{3\cdot 10^{10}}\right)^2 4\cdot\frac{2\cdot 10^{-19}}{2\cdot 1{,}4\cdot 10^{-7}} = 4\cdot 1{,}4\cdot 10^{-4-12} = 5{,}6\cdot 10^{-16}$

$c = 0$

$$D \frac{\partial W}{\partial x} = \rho W f(x) + c \quad \Big| \; W_0$$

$$D \frac{\partial W_0}{\partial x} = \rho W_0 f(x) \quad \Big| \; W$$

$$D \frac{\left(W_0 \frac{\partial W}{\partial x} - W \frac{\partial W_0}{\partial x} \right)}{W_0^2} = \frac{c}{W_0} = +D \frac{\partial}{\partial x}\left(\frac{W}{W_0}\right)$$

$$W = W_0 \cdot \left(\frac{c}{D} \int \frac{dx}{W_0} + c' \right) = W_0 c' + \frac{c}{D}\left(x + W_0 \int x \frac{W_0'}{W_0^2} dx \right)$$

$$e^{-\frac{\beta}{D} U} \cdot \int e^{+\frac{\beta}{D} U} dx$$

$$= x - e^{-\frac{\beta}{D} U} \int \frac{\beta}{D} U' . x \, dx \; e^{+\frac{\beta}{D} U}$$

lim

$$r \frac{\partial u}{\partial t} = \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial r} + r \frac{\partial^2 u}{\partial r^2}$$

falls für $t=0$ $u=1$ bis $r=a$

~~sonst~~ für $r=a$ $u=0$

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$

$$u = U e^{-\alpha t}$$

$$r \sqrt{\frac{\alpha}{D}} = x$$

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} + \frac{\alpha}{D} U = 0$$

$$\frac{\partial^2 U}{\partial x^2} + \frac{1}{x} \frac{\partial U}{\partial x} + U = 0$$

$$U = J_0(x)$$

$$u = \sum_\alpha J_0\left(r \sqrt{\frac{\alpha}{D}} \right) e^{-\alpha t}$$

$$\cancel{= \sum_\alpha K_\alpha\left(r \sqrt{\frac{\alpha}{D}} \right) e^{-\alpha t}}$$

$t = 0$

$$u = \sum_\alpha J_0\left(r \sqrt{\frac{\alpha}{D}} \right) = \sum_\beta J_0(r\beta) . A_\beta$$

$$J(\beta) = 0$$

$$A_\beta = \frac{2 \int_0^1 J(\beta x) x \, dx}{J'(\beta)^2}$$

$$\int J\,x\,dx = - [redacted]\,J - \underbrace{\int x \frac{\partial J}{\partial x^2}\,dx} \qquad = -2J - x\frac{\partial J}{\partial x}$$

$$x\frac{\partial J}{\partial x} - \int \frac{\partial J}{\partial x}\,dx$$

$$\frac{\partial}{\partial x}(Jx^2) = 2xJ + x^2\frac{\partial J}{\partial x}$$

$$\int Jx\,dx = +\frac{Jx^2}{2} - \int$$

$$\int_0^1 J(\beta x)\,x\,dx = -2\left.\frac{J(\beta x)}{\beta^2}\right|_0^1 - \frac{x}{\beta}\,\cancel{\quad}\,J'(\beta x)\Big|_0^1$$

$$= \cancel{-2\frac{J(\beta)}{\beta^2}} + \frac{2}{\beta^2} - \frac{J'(\beta)}{\beta}$$

$$\underline{\underline{\int_0^1 x\,J(\alpha x)\,dx = -\frac{J'(\alpha)}{\alpha}}} \qquad (p. 190)$$

$$A_\beta = \frac{4}{[\beta J'(\beta)]^2} - \frac{2 [redacted]}{\beta J'(\beta)}$$

$$\beta = \sqrt{\frac{\alpha}{D}} \qquad \alpha = D\beta^2$$

$$u = \sum_{\beta_1, \beta_2 \ldots} A_\beta\, J(r\beta)\, e^{-D\beta^2 t}$$

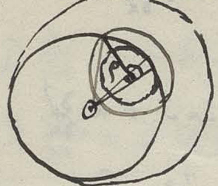
Degeym falls (Riem.-Weber I p. 183)

$$f(x) = \int_0^\infty J(\xi x)\, e^{-\frac{\xi^2 Dt}{\beta\lambda^2}} \cdot \xi\,d\xi \int_0^\infty f(\lambda)\, J(\lambda\xi)\,\lambda\,d\lambda$$

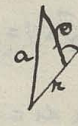
$$\left.\begin{array}{ll} f(\lambda) = 1 & \text{von } \lambda = 0 \text{ bis } \lambda = 1 [redacted] \\ f(\lambda) = 0 & \lambda = 1 \text{ bis } \lambda = \infty \end{array}\right\}$$

$$\int_0^\infty f(\lambda)\, J(\lambda\xi)\,\lambda\,d\lambda = \cancel{\frac{2}{\xi^2}\left[1 - J(\xi)\right]} - \frac{J'(\xi)}{\xi}$$

$$f(x) = \int_0^\infty e^{-\xi^2 Dt}\left\{\cancel{\frac{2J(\xi x)}{\xi}\left[1 - J(\xi)\right]} - J(\xi x)\,J'(\xi)\right\} d\xi = -\int_0^\infty J'(\xi)\,J(\xi x)\,e^{-\xi^2 Dt}\,d\xi$$



$$P = \frac{1}{a^2\pi}\int_0^a 2r\pi\,dr \cdot \frac{1}{2Dt}\left\{\int_{a-r}^{a+r} e^{-\frac{\rho^2}{4Dt}}\rho\,d\rho \cdot \frac{\arccos}{\pi}\left(\frac{a^2-r^2-\rho^2}{2r\rho}\right) + \int_{a+r}^{\infty} e^{-\frac{\rho^2}{4Dt}}\rho\,d\rho\right\}$$



$$a^2 = r^2 + \rho^2 + 2r\rho\cos\varphi$$

$$\varphi = \arccos\frac{a^2-r^2-\rho^2}{2r\rho}$$

$$\left(\frac{1}{2\sqrt{\pi Dt}}\right)^2\int e^{-\frac{x^2+y^2}{4Dt}}dx\,dy = \frac{\cancel{4}\pi r}{4\pi Dt}e^{-\frac{r^2}{4Dt}}dr = \frac{r\,dr}{2Dt}e^{-\frac{r^2}{4Dt}} =$$ [illegible]

$$\rho^2 = a^2 + r^2 - 2ar\cos\theta$$

$$\rho\sin\varphi = a\sin\theta$$

$$\varphi = \arcsin\left(\frac{a}{\rho}\sin\theta\right)$$

$$+\int_0^\pi e^{-\frac{(a^2+r^2-2ar\cos\theta)}{4Dt}}\, ar\sin\theta\,d\theta \cdot \underbrace{\frac{\arcsin}{\pi}\left(\frac{a\sin\theta}{\sqrt{a^2+r^2-2ar\cos\theta}}\right)}$$

$$\frac{1}{\pi}\left[\frac{\pi}{2} - \arccos\sqrt{\frac{a^2-a^2\cos^2\theta}{a^2+r^2-2ar\cos\theta}}\right]$$

$$= \int_{-a}^{a} e^{-\frac{a^2+r^2-2r\xi}{4Dt}}\, r\,d\xi\left[\frac{1}{2} - \frac{1}{\pi}\arccos\sqrt{\frac{a^2-\xi^2}{a^2+r^2-2r\xi}}\right]$$

$$\frac{a^2-r^2-\rho^2}{2r\rho} = \cos\varphi$$

$$\rho^2 + 2r\rho\cos\varphi = a^2 - r^2$$

$$\rho = -r\cos\varphi \pm \sqrt{a^2-r^2+r^2\cos^2\varphi}$$

$$= -r\cos\varphi \mp \sqrt{a^2-r^2\sin^2\varphi}$$

$$\cancel{2}\rho\,d\rho + \cancel{2}r\cos\varphi\,d\rho - \cancel{2}r\rho\sin\varphi\,d\varphi = 0$$

$$d\rho = d\varphi\,\frac{r\rho\sin\varphi}{\rho + r\cos\varphi}$$

$$= \int e^{-\frac{\left[-r\cos\varphi + \sqrt{a^2-r^2\sin^2\varphi}\right]^2}{4Dt}}$$

$$\frac{r^2}{4Dt} = z$$

$$\frac{r\,dr}{2} = Dt\cdot dz$$

$$\overline{r^2} = \int_0^\infty \frac{r^3\,dr}{2Dt}e^{-\frac{r^2}{4Dt}} = 4Dt\int_0^\infty \underbrace{z\,e^{-z}dz} = 4Dt$$

Dagegen bei Schichte: $\lim P = 1 - \frac{2}{\sqrt{\pi}}\left[\beta - \frac{\beta^3}{3} + \frac{\beta^5}{10}\right] + \frac{1}{\beta\sqrt{\pi}}\left[\beta^2 - \frac{\beta^4}{2} + \cdot\right]$

$$= 1 - \frac{\beta}{\sqrt{\pi}} + \frac{1}{6\sqrt{\pi}}\beta^3 = 1 - \frac{h}{2\sqrt{\pi Dt}}$$

Wenn man annimmt, dass gleichmässig verbreitet auf $(4Dt)\pi$ so Prozentsatz welcher übrig bleibt: $\frac{a^2\pi}{4Dt\pi} = \frac{a^2}{4Dt}$

lim

$$\frac{1}{2Dt\, r}\int_{a-r}^{a+r} e^{-\frac{\rho^2}{4Dt}}\, \rho\, d\rho \cdot \varphi = \frac{1}{r}\left[\varphi \cdot e^{-\frac{\rho^2}{4Dt}}\Big| - \int e^{-\frac{\rho^2}{4Dt}} \cdot d\varphi\right]$$

$$d\varphi = -\frac{1}{\sqrt{1-\left(\frac{a^2-r^2-\rho^2}{2\rho r}\right)^2}}\left[-\frac{a^2-r^2-\rho^2}{2r\rho^2} - \frac{1}{r}\right] d\rho$$

Im Grenzfall für große t: so dass $e^{-y^2} \neq 1$

$$\int_{a-r}^{a+r} \rho\, d\rho \cdot \frac{\varphi}{r} = \frac{1}{r}\left[\frac{(a+r)^2}{2} r - \frac{a^2 r}{2}\right] = \frac{2ar + r^2}{2}$$

$$P = \frac{1}{a^2 \cancel{\pi}}\int_0^a 2r\cancel{\pi}\, dr \left[\frac{2ar+r^2}{4Dt} + e^{-\frac{(a+r)^2}{4Dt}}\right] = \frac{1}{2a^2Dt}\int_0^a (2ar^2 + r^3)\,dr + \underbrace{\frac{2}{a^2}\int_0^a e^{-\frac{(a+r)^2}{4Dt}}\, r\,dr}_{\frac{2}{a^2}\int_a^{2a} e^{-\frac{\xi^2}{4Dt}}(\xi-a)\,d\xi}$$

$$= \frac{1}{2a^2Dt}\left\{\frac{2a^4}{3} + \frac{a^4}{4}\right\} + \frac{2}{a^2}\left\{-e^{-\frac{\xi^2}{4Dt}}\Big|_a^{2a}\, 2Dt - a\int_a^{2a} e^{-\frac{\xi^2}{4Dt}}\, d\xi\right\}$$

$$= \frac{11}{12}\frac{a^2}{2Dt} + 4\frac{Dt}{a^2}\, e^{-\frac{a^2}{4Dt}} - \frac{4Dt}{a^2}\, e^{-\frac{a^2}{Dt}} - \frac{4\sqrt{Dt}}{a}\int_{\sqrt{\frac{a^2}{4Dt}}}^{\sqrt{\frac{4a^2}{4Dt}}} e^{-y^2}\, dy$$

$\frac{a^2}{4Dt} = y^2$ = klein

$$1 - y^2 + \frac{y^4}{2}$$
$$-\left[1 - 4y^2 + \frac{16y^4}{2}\right]$$

$$P = \frac{11}{6}y^2 + \frac{e^{-y^2} - e^{-4y^2}}{y^2} - \frac{2}{y}\underbrace{\int_y^{2y} e^{-y^2}\, dy}$$

$$\int (1-y^2)\, dy = y - \frac{y^3}{3} + \frac{y^5}{10}\Big| = y - \frac{7y^3}{3} + \cdots$$

$$= \frac{11}{6}y^2 + 3 - \frac{15}{2}y^2 - \left[2 - \frac{14}{3}y^2\right] = 1 + \left(\frac{11}{6} + \frac{14}{3} - \frac{15}{2}\right)y^2$$

~~unmöglich~~

~~denn P immer < 1 sein!~~

$$\left.\begin{array}{r} 11 \\ +28 \\ -45 \end{array}\right\} = -6$$

$$\lim\left(P = 1 - y^2\right) = 1 - \frac{a^2}{4Dt}$$

$$P = \cancel{\frac{1}{a^2}\int_0^a r\,dr \; \frac{1}{Dt}\left\{\int_{a-r}^{a+r} \rho\, d\rho \cdot \varphi + \int\right.}$$

$$\left.\frac{\partial u}{\partial x}\right| = -\frac{1}{2\sqrt{\pi D t}}$$

$$\int_0^t D\frac{\partial u}{\partial x}\,dt = -\sqrt{\frac{Dt}{\pi}}$$

Für Grenzfall kurzer Zeiten:

$$P = \frac{2a\pi\sqrt{\frac{Dt}{\pi}}}{a^2\pi} = 2\sqrt{\frac{Dt}{a^2\pi}}$$

$$h\cdot P = 1 - \frac{2}{\sqrt{\pi}}\,\frac{e^{-\beta^2}}{2\beta} + \frac{1}{\beta\sqrt{\pi}} - \frac{e^{-\beta^2}}{\beta\sqrt{\pi}}$$

$$= \cancel{1} - \cancel{1} + \cancel{\frac{2}{\sqrt{\pi}}\frac{e^{-\beta^2}}{2\beta}} + \frac{1}{\beta\sqrt{\pi}} - \cancel{\frac{e^{-\beta^2}}{\beta\sqrt{\pi}}} = \frac{2}{h}\sqrt{\frac{Dt}{\pi}}$$

$$\iiiint e^{-\frac{(\xi-x)^2+(\eta-y)^2}{4Dt}}\,dx\,dy\,d\xi\,d\eta$$

$$u = A\xi - \frac{\xi}{3}P^3 A\frac{\xi}{\rho^3} - \frac{\xi}{6}AP^3\frac{\partial^3\rho}{\partial\xi^3} - \cdots P^5.$$

$$-\frac{\xi}{6}B P^3\frac{\partial^3\rho}{\partial\xi\partial\eta^2}$$

$$= A\xi - P^3 A\left[\frac{\xi}{\rho^3}\underbrace{\left(\frac{5}{3}-\frac{5}{2}\right)}_{-\frac{5}{6}} + \frac{5}{2}\frac{\xi^3}{\rho^5}\right]$$

$$\frac{\partial\rho}{\partial\xi} = \frac{\xi}{\rho}$$

$$\frac{\partial^2\rho}{\partial\xi^2} = \frac{1}{\rho} - \frac{\xi^2}{\rho^3} \qquad \frac{\partial^2\rho}{\partial\eta^2} = \frac{1}{\rho} - \frac{\eta^2}{\rho^3}$$

$$\frac{\partial^3\rho}{\partial\xi^3} = -\frac{3\xi}{\rho^3} + \frac{3\xi^3}{\rho^5} \qquad \frac{\partial^3\rho}{\partial\xi\partial\eta^2} = -\frac{\xi}{\rho^3} + \frac{3\xi\eta^2}{\rho^5}$$

$$u = A\xi + P^3 A\left[\cancel{\frac{5}{6}\frac{\xi}{\rho^3}} - \frac{5}{2}\frac{\xi^3}{\rho^5}\right]$$

$$+ P^3 B\left[\cancel{\frac{5}{6}\frac{\xi}{\rho^3}} - \frac{5}{2}\frac{\xi\eta^2}{\rho^5}\right] \qquad (6a)$$

$$+ P^3 C\left[\cancel{\frac{5}{6}\frac{\xi}{\rho^3}} - \frac{5}{2}\frac{\xi\zeta^2}{\rho^5}\right]$$

$$\frac{\partial\left(\frac{1}{\rho}\right)}{\partial\xi} = -\frac{\xi}{\rho^3}$$

$$\frac{\partial^2\left(\frac{1}{\rho}\right)}{\partial\xi^2} = -\frac{1}{\rho^3} + \frac{3\xi^2}{\rho^5}$$

$$p = \frac{5}{\ }kP^3\left\{\cancel{(A+B+C)\frac{1}{\rho^3}}\left\{\frac{A\xi^2 + B\eta^2 + C\zeta^2}{\rho^5}\right\}\right.$$

$$-2kA + 5kP^3\left\{A\left(\frac{3\xi^2}{\rho^5} - \frac{5\xi^4}{\rho^7}\right) + B\left(\frac{\eta^2}{\rho^5} - \frac{5\xi^2\eta^2}{\rho^7}\right) + C\left(\frac{\zeta^2}{\rho^5} - \frac{5\xi^2\zeta^2}{\rho^7}\right)\right.$$

$$X_\xi = -2kA + 10kP^3\left\{A\frac{\xi^2}{\rho^5}\right\} - 25kP^3(A\xi^2 + B\eta^2 + C\zeta^2)\frac{\xi^2}{\rho^7}$$

$$X_\eta = +k\frac{5}{2}P^3\left\{(A\xi^2 + B\eta^2 + C\zeta^2)\left(-\frac{5\xi\eta}{\rho^7} - \frac{5\xi\eta}{\rho^7}\right) + 4B\frac{\eta\xi}{\rho^5}\right\}$$

$$= -kP^3\left[25\frac{\xi\eta(A\xi^2 + B\eta^2 + C\zeta^2)}{\rho^7} - 10B\frac{\eta\xi}{\rho^5}\right\}.$$

$$X_n = +2kA\frac{\xi}{\rho} - 10kP^3\frac{(A\xi^3 + B\xi\eta^2 + C\xi\zeta^2)}{\rho^6} + 25kP^3(A\xi^2 + B\eta^2 + C\zeta^2)\frac{(\xi^3 + \xi\eta^2 + \xi\zeta^2)}{\rho^8}$$

$$= 2kA\frac{\xi}{\rho}. + 15kP^3(A\xi^2 + B\eta^2 + C\zeta^2)\frac{\xi}{\rho^6}$$

$$u = A\xi - \frac{5}{2}\frac{P^3}{\rho^5}\xi(A\xi^2 + B\eta^2 + C\zeta^2)$$

$X_n u + Y_n v + Z_n w =$

$$= \frac{2k(A^2\xi^2 + B^2\eta^2 + C^2\zeta^2)}{\rho} + 15kP^3\frac{(A\xi^2+B\eta^2+C\zeta^2)^2}{\rho^6} - 5kP^3(A\xi^2+B\eta^2+C\zeta^2)^2$$

$$\sum \rho^4 = \sum 4\pi R^6 = \sum(3\xi^4 + 6\xi^2\eta^2) = 3\frac{4}{5}\pi R^6 + 6\frac{4}{15}\pi R^6$$

$$4 = \frac{12}{5} + \cancel{\tfrac{24}{15}}\frac{8}{5} = \frac{20}{5} \text{ stimmt}$$

$$A^2\xi^4 + B^2\eta^4 + C^2\zeta^4 + 2AB\,\overline{\xi^2\eta^2} + 2AC\,\xi^2\zeta^2 + 2BC\,\eta^2\zeta^2$$

$$= \frac{4}{5}\pi R^6\left\{A^2+B^2+C^2 + \frac{2}{3}(AB + AC + BC)\right\}$$

$$A^2+B^2+C^2 + 2(AB + AC + BC) = 0$$

$$\sum(A\xi^2 + \cdots C\zeta^2)^2 = \frac{8}{15}\pi R^6\left[A^2+B^2+C^2\right]$$

$$A^2\xi^2 + B^2\eta^2 + C^2\zeta^2 = \frac{4}{3}\pi R^4\left[A^2+B^2+C^2\right]$$

$$\sum(X_n u + Y_n v + Z_n w) = \frac{2k}{R}\sum(A^2\xi^2 + B^2\eta^2 + C^2\zeta^2) + 10\frac{kP^3}{R^6}\sum(A\xi^2+B\eta^2+C\zeta^2)^2$$

$$= \frac{4}{3}\pi R^4\left[A^2+B^2+C^2\right]\left\{\frac{2k}{R} + 10\frac{kP^3}{R^6}\cdot\frac{2}{5}R^2\right\}$$

$$W = \left\{\frac{8}{3}\pi R^3 k + \frac{16}{3}\pi k P^3\right\}\delta^2$$

$$W = 2\delta^2 k\left[V + 2\Phi\right]$$

$$X_\xi\frac{\partial u}{\partial x} + X_\eta\frac{\partial w}{\partial y} + Z_\zeta\frac{\partial w}{\partial z} = \sum\begin{bmatrix}-2kA + 10kAP^3\frac{\xi^2}{\rho^5} \pm -25kP^3(A\xi^2+B\eta^2+C\zeta^2)\frac{\xi^2}{\rho^7}\\ A - 5AP^3\frac{\xi^2}{\rho^5} - \frac{5}{2}\frac{P^3}{\rho^3}(A\xi^2+B\eta^2+C\zeta^2)\left(1-5\frac{\xi^2}{\rho^2}\right)\end{bmatrix}$$

$$= -2k(A^2+B^2+C^2) + \frac{kP^3}{\rho^5}\left\{10(A^2\xi^2+\ \ ) - 25(\ \ )^2 + 10(A^2\xi^2 - \ \ ) - 25(\ \ )^2\right\}$$

$$A\,\frac{4}{5}\pi R^6 + \underbrace{(B+C)}_{=-A}\,\frac{4}{15}\pi R^6 = \frac{8}{15}A\pi R^6$$

$$\lim_{R=\infty}\iiint(u^2+v^2+w^2)\,dv = \infty\,?$$

$$u^2+v^2+w^2 = A^2\xi^2 + B^2\eta^2 + C^2\zeta^2$$

$$-5\frac{P^3}{\rho^5}(A\xi^2+B\eta^2+C\zeta^2)^2 - \frac{25}{4}\frac{P^6}{\rho^8}(A\xi^2+B\eta^2+C\zeta^2)^2$$

$$u = A\xi - \frac{5}{2}P^3(A\xi^2+B\eta^2+C\zeta^2)\frac{\xi}{\rho^5}$$

$$\frac{\partial u}{\partial x} = A - \frac{5}{2}\frac{P^3}{\rho^5}(A\xi^2+B\eta^2+C\zeta^2)\left(1-\frac{5\xi^2}{\rho^2}\right) - 5P^3A\frac{\xi^2}{\rho^5}$$

$$\sum\left(\frac{\partial u}{\partial x}\right)^2 = (A^2+B^2+C^2) + 25\frac{P^3}{\rho^7}(A\xi^2+B\eta^2+C\zeta^2)^2 - 10\frac{P^3}{\rho^5}(A^2\xi^2+B^2\eta^2+C^2\zeta^2)$$

$$\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} = \frac{25}{\cancel{2}}P^3(A\xi^2+B\eta^2+C\zeta^2)\frac{\xi\eta}{\rho^7} - 5P^3\frac{\xi\eta}{\rho^5}(A+B)$$

$$= \frac{5P^3\xi\eta}{\rho^5}\left\{C + 5\frac{(A\xi^2+B\eta^2+C\zeta^2)}{\rho^2}\right\}$$

$$(\quad)^2 = 25P^6\ldots$$

$$\cancel{\Phi = 2\mu\sum\frac{1}{2}(A^2+B^2+C^2)\,4\pi R^2}$$

$$\int\frac{\xi^2}{\rho^7}\,ds\,d\rho = \frac{4}{3}\pi\int\frac{\rho^4}{\rho^7}\,d\rho = \frac{2}{3}\pi\frac{1}{\rho^2}$$

$$\int\frac{\xi^2}{\rho^5}\,ds\,d\rho = \frac{4}{3}\pi\int\frac{d\rho}{\rho^3} = \frac{2}{3}\pi\frac{1}{\rho^2}$$

$$\int(X_n u + Y_n v + Z_n w)\,dS = \int(X_x\cos nx + X_y\cos ny + X_z\cos nz)u\,dS$$
$$+(Y_x\cos nx + Y_y\cos ny + Y_z\cos nz)v\,dS$$
$$+(Z_x\cos nx + Z_y\cos ny + Z_z\cos nz)w\,dS =$$

$$= \int(X_x u + Y_x v + Z_x w)\,dy\,dz + \ldots$$

$$= \iint\left[\frac{\partial}{\partial x}(X_x u + Y_x v + Z_x w) + \frac{\partial}{\partial y}(Y_x u + Y_y v + Y_z w) + \frac{\partial}{\partial z}(X_z u + Z_y v + Z_z w)\right]dx\,dy\,dz$$

$$\iiint \Phi\, dv = \iiint 2 \cancel{\pi} k (A^2 + B^2 + C^2) - 10 (A^2 \xi^2 + B^2 \eta^2 + C^2 \zeta^2) \frac{k P^3}{\rho^5} + 25 (A \xi^2 + B \eta^2 + C \zeta^2)^2 \frac{k P^3}{\rho^7}$$

$$\int dv = \frac{4}{3} R^3 \pi$$

$$\int \frac{\xi^2}{\rho^5} dv = \cancel{\int \frac{4}{3} \pi \rho^3 \, d\varphi} = \int \frac{\cos^2 \varphi}{\rho^3} 2\pi \sin\varphi \, d\varphi \, \rho^2 d\rho$$

$$i = \frac{K \Delta\varphi}{4\pi} \quad \frac{p\, \delta}{\mu} \quad \frac{q}{l} \quad \delta \qquad W_1 = \left(\frac{K \Delta\varphi}{4\pi}\right)^2 \frac{p^2 \delta q}{\mu^2 l}$$

$$W_0 = \cancel{\pi} \cdot \frac{p^2 R^4 \pi}{8 l \mu}$$

$$\frac{W_1}{W_0} = \left(\frac{K \Delta\varphi}{4\pi}\right)^2 \frac{6 \cancel{\pi}}{\mu} \cdot \frac{8 \cdot R^2 \pi}{R^4 \pi} = \left(\frac{K \Delta\varphi}{4\pi}\right)^2 \frac{6}{\mu} \cdot \frac{8}{R^2}$$

$$\alpha = \sqrt{\frac{\beta}{D}}$$

$$\cancel{\pi}\ u_r = \sum_\beta A_\beta e^{-\beta t} J\left(r \sqrt{\tfrac{\beta}{D}}\right) = \sum_\alpha \frac{2 e^{-\alpha^2 D t}}{J'(\alpha)^2} \int_0^1 f(\lambda)\, J(\alpha r)\, J(\alpha \lambda)\, \lambda\, d\lambda$$

$$f\left(\tfrac{\mu}{h}\right) = \varphi(\mu)$$

$$= \sum_\alpha A_\alpha e^{-\alpha^2 D t} J(\alpha r) = \sum \frac{2 e^{-\alpha^2 D t}}{h^2 J'(\alpha)^2} \int_0^a \varphi(\mu)\, J\left(\frac{\alpha z}{h}\right) J\left(\frac{\alpha \mu}{h}\right) \mu \, d\mu$$

$$= \sum \frac{2 \delta e^{-h^2 \xi_\nu^2 D t}}{\cancel{r h}\ J'(h \xi_\nu)^2} \int_0^a \varphi(\mu)\, J(\xi_\nu r)\, J(\xi_\nu \mu)\, \mu\, d\mu$$

$$\int_0^1 J(\alpha\lambda)\, \lambda\, d\lambda = \frac{J_1(\alpha)}{\alpha}$$

$$\sum A_\alpha J(\alpha r) = \begin{matrix} 1 & \quad r < a \\ 0 & \quad r \geqq a \end{matrix}$$

$$u_r = \sum_\alpha \frac{2 e^{-\alpha^2 D t}}{[J'(\alpha)]^2} \int_0^1 J(\alpha r)\, J(\alpha \lambda)\, \lambda\, d\lambda = -\sum_{\alpha_1 \alpha_2 \ldots} \frac{2 e^{-\alpha^2 D t} J(\alpha r)}{\alpha J'(\alpha)}$$

$$f(x) = +\int_0^\infty J_1(\xi)\, J(\xi x)\, e^{-\xi^2 Dt}\, d\xi$$

$$= \cancel{\int \ldots} = \int \underbrace{\frac{x}{2}\left[1 - \frac{\left(\frac{x}{2}\right)^2}{1.2} + \frac{\left(\frac{x}{2}\right)^4}{2!\,3!} - \frac{\left(\frac{x}{2}\right)^6}{3!\,4!} + \cdots\right]}_{J_1 x}\underbrace{\left[1 - \frac{\left(\frac{x}{2}\right)^2}{(1!)^2} + \frac{\left(\frac{x}{2}\right)^4}{(2!)^2} - \frac{\left(\frac{x}{2}\right)^6}{(3!)^2} \cdots\right]}_{J(x)} dx$$

$$= \int_0^\infty \frac{\xi}{2}\left[1 - \frac{\xi^2}{8} + \frac{\xi^4}{16.12}\right]\left[1 - \frac{(x\xi)^2}{4} + \frac{(x\xi)^4}{64}\right] e^{-\xi^2 Dt}\, d\xi$$

$$1 - \frac{\xi^2}{4}\left(x^2 + \frac{1}{2}\right) + \frac{\xi^4}{64}\left(x^4 + 2x^2 + \frac{1}{3}\right)$$

$$= \int\left[\frac{\xi}{2} - \frac{x^2 + \frac{1}{2}}{8}\,\xi^3 + \frac{x^4 + 2x^2 + \frac{1}{3}}{128}\,\xi^5\right] e^{-\xi^2 Dt}\, d\xi$$

$$\int_0^\infty \xi\, e^{-\xi^2 Dt}\, d\xi = \frac{1}{2Dt}\, e^{-\xi^2 Dt}\Big|_0^\infty = \frac{1}{2Dt}$$

$$\int \xi^3 e^{-\xi^2 Dt}\, d\xi = \frac{1}{2(Dt)^2}$$

$$\int \xi^5 e^{-\xi^2 Dt}\, d\xi = \frac{1}{(Dt)^3}$$

$$\frac{\partial}{\partial t}\int_0^r r u\, dr = r\frac{\partial u}{\partial r}$$

$$\int_0^a r u\, dr \Big|_t^{t=\infty} = \int_t^\infty dt \cdot a \frac{\partial u}{\partial r}\Big|_a$$

$$\int_0^a r u_t\, dr = -\int_t^\infty a\left(\frac{\partial u}{\partial r}\right)_{a,t} dt$$

$$P = 1 - \frac{2}{a^2}\int_0^a r u\, dr$$

$$\int_0^{a=1} J(\alpha r)\, r\, dr = \frac{J_1(\alpha)}{\alpha} = -\frac{J'(\alpha)}{\alpha}$$

$$P = 1 - 2\sum_{\alpha_1, \alpha_2, \ldots} 2\, e^{-\alpha^2 Dt}\frac{J(\alpha r)}{\alpha^2}$$

$$W(n) = \left(\frac{v}{V}\right)^{n}\left(1-\frac{v}{V}\right)^{N-n}\frac{N!}{n!\,N-n!}$$

$$\lg\sqrt{\frac{2N\pi}{(2\pi)^2\left(\frac{N}{2}\right)^2(1-\delta^2)}} = \lg\sqrt{\frac{2}{N\pi(1-\delta^2)}}$$

Wenn: $\frac{v}{V} = \frac{1}{2}$ $\qquad W(n) = \left(\frac{1}{2}\right)^{N}\frac{N!}{n!\,N-n!}$

$$\lg W = N\lg\left(\tfrac{1}{2}\right) + N\lg N \cancel{-N} + \tfrac{1}{2}\lg 2N\pi \qquad \cancel{-N+n+(N-n)}$$

$$- n\lg n - (N-n)\lg(N-n) - \tfrac{1}{2}\lg 2n\pi - \tfrac{1}{2}\lg 2(N-n)\pi$$

$$= -N\lg 2 + N\lg N - n\lg n - (N-n)\lg(N-n) + \frac{1}{2}\lg\underbrace{\frac{N}{n(N-n)}}_{\frac{4}{N(1-\delta^2)}} - \lg\sqrt{2\pi}$$

$$n = \frac{N}{2}(1+\delta)$$

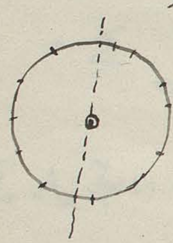
$$\lg W = -N\lg 2 + N\lg N - \frac{N}{2}(1+\delta)\left[\lg\frac{N}{2} + \delta - \frac{\delta^2}{2}\right] - \lg 2\pi + \lg\sqrt{4}$$

$$\underbrace{\qquad\qquad} \quad \underbrace{-\frac{N}{2}(1-\delta)\left[\lg\frac{N}{2} - \delta - \frac{\delta^2}{2}\right]} - \lg\sqrt{N} + \frac{\delta^2}{2} - \frac{\delta^4}{4}$$

$$= \cancel{-}N\lg\frac{N}{2} \qquad - N\left[\lg\frac{N}{2} - \frac{\delta^2}{2}\right] - N\delta^2 - \lg\sqrt{N} - \lg\sqrt{2\pi} + \frac{\delta^2}{2} - \frac{\delta^4}{4} + \lg 2$$

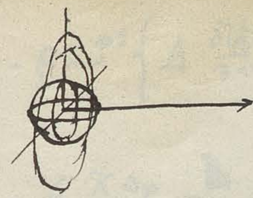
$$= -N\frac{\delta^2}{2} - \lg\sqrt{\frac{N\pi}{2}} + \delta^2 - \frac{\delta^4}{2}$$

$$W = \sqrt{\frac{2}{N\pi}}\, e^{-\frac{N\delta^2}{2}} = \sqrt{\frac{2}{N\pi}}\, e^{-\frac{(n_1-n_2)^2}{2(n_1+n_2)}}$$



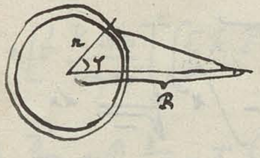
$$n_1 - n_2 = N\delta$$

$$\bar{W} =$$



$$u = u_0\left(1 + \alpha n T \tfrac{t}{\tau} + \underbrace{\beta n^2 T^2}\right) \qquad T = R^3 \qquad n = \frac{N}{R^3}$$

Einfluss der Zusammenstöße d. Kugeln



$$u_R = \frac{2}{4\pi D t} \int_0^a \int_0^\pi e^{-\frac{R^2 + r^2 - 2Rr\cos\varphi}{4Dt}} \frac{r\,d\varphi\,dr}{a^2 \pi} \qquad \Bigg\| \qquad R'^2 = \frac{R^2}{4Dt}$$

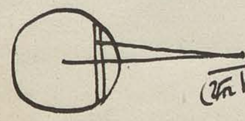
$$\cos\varphi = \xi$$

$$-\sin\varphi\, d\varphi = d\xi$$

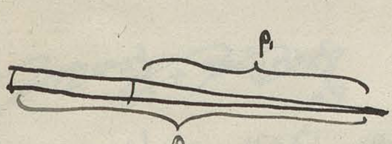
$$d\varphi = \frac{-d\xi}{\sqrt{1-\xi^2}}$$

$$= \frac{1}{2\pi^2 a^2 D t} \int_0^a r\,dr \int_{-1}^{+1} \frac{e^{-\frac{R^2 + r^2 - 2Rr\xi}{4Dt}}}{\sqrt{1-\xi^2}}\, d\xi$$

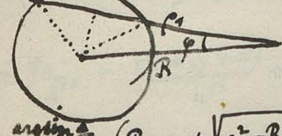
$$P = \frac{1}{2a^2\pi^2 D t} \int_a^\infty 2R\pi\, dR \int_0^a r\,dr \int_{-1}^{+1} \frac{e^{-\frac{R^2 + r^2 - 2Rr\xi}{4Dt}}}{\sqrt{1-\xi^2}}\, d\xi = \frac{16 D t}{a^2 \pi} \int_{\frac{a}{2\sqrt{Dt}}}^{\infty} R\,dR \int_0^{\frac{a}{2\sqrt{Dt}}} r\,dr \int_{-1}^{+1} \frac{e^{-(R^2 + r^2 - 2Rr\xi)}}{\sqrt{1-\xi^2}}\, d\xi$$



$$\frac{1}{(2\sqrt{\pi}\sqrt{Dt})^2} \int_{x=-a}^{x=+a} 2 \int_{y=0}^{\sqrt{a^2-x^2}} e^{-\frac{(R-x)^2 + y^2}{4Dt}}\, dx\, dy = \frac{2}{\pi} \int_{x=-a}^{+a} \frac{e^{-\frac{(R-x)^2}{4Dt}}}{2\sqrt{Dt}}\, dx \cdot \int_0^{z = \sqrt{\frac{a^2-x^2}{4Dt}}} e^{-z^2}\, dz$$



$$\int_{\rho_1}^{\rho_2} e^{-\frac{\rho^2}{4Dt}} \frac{\rho\, d\rho\, d\varphi}{2Dt} = -e^{-\frac{\rho^2}{4Dt}} \Big|_{\rho_1}^{\rho_2} \cdot d\varphi$$



$$\cancel{\rho_1, \rho_2 = R\cos\varphi \pm} \qquad a^2 = R^2 + \rho_1^2 - 2R\rho_1\cos\varphi$$

$$\rho = R\cos\varphi \pm \sqrt{a^2 - R^2 + (R\cos\varphi)^2}$$

$$2\int_0^{\arcsin\frac{a}{R}} \left[e^{-\frac{\left(R\cos\varphi + \sqrt{a^2 - R^2\sin^2\varphi}\right)^2}{4Dt}} - e^{-\frac{\left(R\cos\varphi - \sqrt{a^2 - R^2\sin^2\varphi}\right)^2}{4Dt}} \right] d\varphi \qquad \Bigg\| \qquad 0 = 2\rho\, d\rho - 2R\cos\varphi\, d\rho + 2R\rho\sin\varphi\, d\varphi$$

$$d\varphi = \frac{R\cos\varphi - \rho}{2R\rho\sin\varphi}\, d\rho$$

$$=$$

$$\int_{-1}^{+1} \frac{e^{-\alpha x}}{\sqrt{1-x^2}}\,dx = J(\alpha) = \overset{?}{J_0}(i\alpha)$$

$$\frac{\partial J}{\partial \alpha} = -\int x\frac{e^{-\alpha x}}{\sqrt{1-x^2}}\,dx = \underbrace{+\sqrt{1-x^2}\;e^{-\alpha x}\Big|_{-1}^{+1}}_{0\,?} + \alpha\underbrace{\int e^{-\alpha x}\sqrt{1-x^2}\,dx}_{=\int \frac{e^{-\alpha x}dx}{\sqrt{1-x^2}} - \int x^2 \frac{e^{-\alpha x}}{\sqrt{}}\,dx}$$

$$+\frac{\partial J}{\partial \alpha} = +\alpha J \mp \alpha\frac{\partial^2 J}{\partial \alpha^2}$$

$$\frac{\partial^2 J}{\partial \alpha^2} + \frac{1}{\alpha}\frac{\partial J}{\partial \alpha} - J = 0$$

$$= -\frac{18}{r^8} - 18\frac{x^2}{r^{10}} + 45\left\{\underline{x^4+y^4+z^4 + 2(x^2y^2+x^2z^2+y^2z^2)}\right\} = -\frac{18}{r^8} - 18\frac{x^2}{r^{10}} + 45\left(x^4 + x^2y^2 + x^2z^2 + y^2z^2\right)$$

$$= -\frac{18}{r^8} - 18\frac{x^2}{r^{10}} + \frac{45x^2}{r^{10}} + 45\frac{y^2z^2}{r^{12}} = -\frac{18}{r^8} + 27\frac{x^2}{r^{10}} + 45\frac{y^2z^2}{r^{12}}$$

$$p = -\frac{5}{3} k P^3 \left\{ A \frac{\partial^2 (\frac{1}{r})}{\partial x^2} + B \frac{\partial^2}{\partial y^2}(\tfrac{1}{r}) + C \frac{\partial^2}{\partial z^2}(\tfrac{1}{r}) \right\} *$$

$$\begin{matrix} u \\ v \\ w \end{matrix} = 6 . \frac{K \Delta\varphi}{4\pi\mu} . \nabla p$$

$$\Phi' = \left[6 . \frac{K\Delta\varphi}{4\pi\mu} \right]^2 \left[\frac{5}{3} k P^3 \right]^2 \iiint dx\,dy\,dz.$$

$$\left[A \frac{\partial^3}{\partial x^3} + B \frac{\partial^3}{\partial x \partial y^2} + C \frac{\partial^3}{\partial x \partial z^2} \right]^2 + \left[A \frac{\partial^3}{\partial x^2 \partial y} + B \frac{\partial^3}{\partial y^3} + C \frac{\partial^3}{\partial y \partial z^2} \right]^2 + \left[A \frac{\partial^3}{\partial x^2 \partial z} + - \right]^2$$

$$= \sum \left\{ A^2 \left[\left(\frac{\partial^3}{\partial x^3}\right)^2 + \left(\frac{\partial^3}{\partial x^2 \partial y}\right)^2 + \left(\frac{\partial^3}{\partial x^2 \partial z}\right)^2 \right] + 2 BC \left[\frac{\partial^3}{\partial x \partial y^2} \frac{\partial^3}{\partial x \partial z^2} + \frac{\partial^3}{\partial y^3} \frac{\partial^3}{\partial y \partial z^2} + \frac{\partial^3}{\partial y^2 \partial z} \frac{\partial^3}{\partial z^3} \right] \right\}$$

$$\frac{1}{r} \quad -\frac{x}{r^3} \quad -\frac{1}{r^3} + \frac{3x^2}{r^5} \Big/ \frac{\partial^3}{\partial x^3} = \frac{9x}{r^5} - \frac{15 x^3}{r^7} \qquad 81 \frac{x^2}{r^{10}} - 9.30 \frac{x^4}{r^{12}} + (15)^2 \frac{x^6}{r^{14}}$$

$$\frac{\partial^3}{\partial x^2 \partial y} = \frac{3y}{r^5} - \frac{15 x^2 y}{r^7} \qquad 9 \frac{y^2}{r^{10}} - 3.30 \frac{x^2 y^2}{r^{12}} + (15)^2 \frac{x^4 y^2}{r^{14}}$$

$$\frac{\partial^3}{\partial x^2 \partial z} = \frac{3z}{r^5} - \frac{15 x^2 z}{r^7} \qquad \frac{9z^2}{r^{10}} - 3.30 \frac{x^2 z^2}{r^{12}} + (15)^2 \frac{x^4 z^2}{r^{14}}$$

$$= \frac{9}{r^8} + 72 . \frac{x^2}{r^{10}} - 180 . \frac{x^4}{r^{12}} - 90 . \frac{x^2}{r^{10}} + 225 . \frac{x^4}{r^{12}}$$

$$= \frac{9}{r^8} - 18 \frac{x^2}{r^{10}} + 45 . \frac{x^4}{r^{12}}$$

$$\cancel{\frac{27}{r^{10}} - 18.15 \frac{x^3}{r^{12}} + (15)^2 \frac{x^5}{r^{14}}}$$

$$\left(\frac{3x}{r^5} - \frac{15 x y^2}{r^7}\right)\left(\frac{3x}{r^5} - \frac{15 x z^2}{r^7}\right) + \left(\frac{9y}{r^5} - \frac{15 y^3}{r^7}\right)\left(\frac{3y}{r^5} - \frac{15 z^2 y}{r^7}\right) + \left(\frac{3z}{r^5} - \frac{15 y^2 z}{r^7}\right)\left(\frac{9z}{r^5} - \frac{15 z^3}{r^7}\right) =$$

$$\left.\begin{aligned} &= 9\frac{x^2}{r^{10}} - \frac{45 x^2 (y^2+z^2)}{r^{12}} + (15)^2 \frac{x^2 y^2 z^2}{r^{14}} \\ &+ 27 \frac{y^2}{r^{10}} - 45 y^4 - 9.15 . z^2 y^2 + (15)^2 y^4 z^2 \\ &+ 27 \frac{z^2}{r^{10}} - 45 z^4 - 9.15 . y^2 z^2 + (15)^2 y^2 z^4 \end{aligned}\right\} = \frac{27}{r^8} - 18 \frac{x^2}{r^{10}} - \frac{45 (x^2 y^2 + x^2 z^2 + y^4 + z^4) + 9.30 y^2 z^2}{r^{12}} + (15)^2 \frac{y^2 z^2}{r^{12}}$$

$$= \frac{27}{r^8} - 18 \frac{x^2}{r^{10}} - \frac{45 (x^2 y^2 + x^2 z^2 + y^2 z^2 + y^4 + z^4)}{r^{12}}$$

$$\frac{W'}{\cancel{\ }} = \cancel{\ } q.M \iiint \sum \left\{ A^2\left[\frac{1}{r^8} - \frac{2x^2}{r^{10}} + \frac{5x^4}{r^{12}}\right] + 2BC\left[-\frac{R}{r^8} + \frac{3x^2}{r^{10}} + 5\frac{y^2z^2}{r^{12}}\right]\right\}$$

$$A^2 \frac{\cancel{\ }\left[4r^2\pi - \frac{8}{3}\pi r^2 + \cancel{3}4\pi r^2\right]}{r^8} + \cancel{\frac{2BC}{r^8}\left[-2.4r^2\pi + 4r^2\pi + 4r\pi\right]}$$

$$= \underbrace{\qquad}_{\frac{16}{3}\pi\frac{r^2}{r^8}}$$

$$= qM\,(A^2+B^2+C^2)\,\frac{16}{3}\pi\int_P^\infty \frac{dr}{r^6} = \cancel{\ }\frac{48\pi\,(A^2+B^2+C^2)}{5\,P^5}\cdot\frac{25}{9}\frac{P^6}{16\pi^2}(6K\Delta\varphi)^2$$

$$= \frac{5}{3}\frac{(A^2+B^2+C^2)(6K\Delta\varphi)^2\cdot P}{\pi}$$

$$W_0 = 2(A^2+B^2+C^2)\mu \qquad = \frac{5}{4}(A^2+B^2+C^2)\left(\frac{6K\Delta\varphi}{\pi}\right)^2\frac{1}{P^2}\cdot\Phi$$

$$\mu = \mu_0\left[1 + 2{\cdot}5\,\varphi + \frac{5}{8}\left\{\frac{6(K.\Delta\varphi)^2}{\mu\pi^2}\right\}\frac{1}{P^2}\,\varphi\right]$$

$$= \mu_0\left[1 + \left(2{\cdot}5 + 10\frac{6}{P^2\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2\right)\varphi\right]$$

$$K\Delta\varphi = \frac{2}{300}$$

$$6 = \cancel{\ }\frac{10^6}{9.10^{11}} = \frac{1}{9}.10^{-5}$$

$$10\cdot\frac{10^{-5}}{9.10^{-12}.0{\cdot}02}\cdot\frac{4}{9.10^4.150}$$

$$\mu = 0{\cdot}02$$

$$P = 10^{-6}$$

$$= \cancel{\ }\frac{10^4.4}{81.30} = \frac{400\cancel{0}}{24\cancel{0}} = 16$$

2·5 für $P = \sqrt{6}.10^{-6}$

$$2P = 5.10^{-6} = \frac{1}{20}\mu$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{u}{4x^2}$$

$$u = \sqrt{x}\iint_{-\infty}^{+\infty} e^{-a^2}\,da \int \varphi(x\cos\omega + 2a\sqrt{t})\,d\omega + \sqrt{x}\iint_{-\infty}^{+\infty} e^{-a^2}\,da \int \psi(x\cos\omega + 2a\sqrt{t})\lg(x\sin\omega)\,d\omega$$

$$\sqrt{x}\,\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}\sqrt{x} + \frac{1}{\sqrt{x}}\frac{\partial v}{\partial x} + \cancel{\frac{v}{4x\sqrt{x}}} - \cancel{\frac{v}{4x\sqrt{x}}}$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \frac{1}{x}\frac{\partial v}{\partial x} \qquad v = \frac{u}{\sqrt{x}}$$

$$x\cos\omega + 2a\sqrt{t} = \alpha$$

$$v = \iint_{-\infty}^{+\infty} e^{-\left(\frac{\alpha - x\cos\omega}{2\sqrt{t}}\right)^2} \frac{d\alpha}{2\sqrt{t}} \int \varphi(\alpha)\,d\omega + \cancel{\iint e^{-\frac{(\alpha - x\cos\omega)^2}{4t}} \frac{d\alpha}{2\sqrt{t}} \lg(x\sin^2\omega)\,d\omega\,.\,\psi(\alpha)}$$

$$\underline{\underline{\int_0^\infty e^{-r\xi^2}\cos x\xi\,d\xi = \frac{1}{2}\sqrt{\frac{\pi}{r}}\; e^{-\frac{x^2}{4r}}}}$$

$$e^{-\frac{(\alpha - x\cos\omega)^2}{4t}} = 2\sqrt{\frac{t}{\pi}}\int_0^\infty e^{-t\xi^2}\,\underbrace{\cos(\alpha - x\cos\omega)\xi}_{\cos\alpha\xi\,\cos(x\xi\cos\omega) + \cancel{\sin\alpha\xi\,\sin(x\xi\cos\omega)}}\,d\xi$$

$$v = \cancel{\frac{1}{\sqrt{\pi}}\iiint \varphi(\alpha)\,d\alpha\; e^{-t\xi^2}\cos x\xi\,d\xi}$$

$$= \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} d\alpha \int_{\xi=0}^{\infty} d\xi \int e^{-t\xi^2}\varphi(\alpha)\cos\alpha\xi \int_0^{\pi}\cos(x\xi\cos\omega)\,d\omega$$

$$v = \int_{-\infty}^{+\infty}\int_0^{\pi} e^{-a^2} da\ \varphi(x\cos\omega + 2a\sqrt{t})\, d\omega$$

$$\frac{\partial v}{\partial t} = \frac{1}{\sqrt{t}}\iint a\, e^{-a^2} da\ \varphi'(x\cos\omega + 2a\sqrt{t})\, d\omega$$

$$\frac{1}{x}\frac{\partial v}{\partial x} = \iint e^{-a^2} da\ \underline{\cos\omega\ \varphi'(\quad)}_{x}\ d\omega \quad \Big\| \quad \frac{\partial^2 v}{\partial x^2} = \iint e^{-a^2} da .\ \cos^2\omega\ \varphi''(\quad)\, d\omega$$

$$\frac{\partial v}{\partial t} = \frac{1}{\sqrt{t}}\int d\omega \left\{ -\frac{e^{-a^2}}{2}\varphi'(x\cos\omega + 2a\sqrt{t})\Big|_{-\infty}^{+\infty} + \sqrt{t}.\ e^{-a^2}\varphi''(x\cos\omega + 2a\sqrt{t})\, da \right\}$$

$$= \iint e^{-a^2} da .\ \varphi''(x\cos\omega + 2a\sqrt{t})\, d\omega$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} - \frac{1}{x}\frac{\partial v}{\partial x} =$$

$$\cancel{\frac{\partial^2 v}{\partial x^2} = \iint e^{-a^2} da .\ \varphi''\, d\omega - \iint e^{-a^2} da\ \varphi'' \sin^2\omega\, d\omega}$$

$$\varphi' \frac{\cancel{\sin\omega}}{x}$$

$$\iint \frac{e^{-a^2} da}{x}\Big[\underbrace{\varphi''(x\cos\omega + 2a\sqrt{t})\, x \sin^2\omega\, d\omega}_{-\cancel{\varphi' \sin\omega}\big|_0^{\pi} + \int \varphi' \cos\omega\, d\omega} - \varphi'(\ldots)\cos\omega\, d\omega\Big]$$

$$= 0 \qquad \underline{\text{stimmt.}}$$

$t=0$:

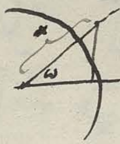
$$v = \iint e^{-a^2} da\ \varphi(x\cos\omega)\, d\omega = \int e^{-a^2} da \int_0^{\pi}\varphi(x\cos\omega)\, d\omega = \sqrt{\pi}\int_0^{\pi}\varphi(x\cos\omega)\, d\omega$$

$$\varphi(\xi) = \sum A_n \sin\frac{n\pi\xi}{a} + \sum B_n \cos\frac{n\pi\xi}{a}$$

20.

$\varphi(\xi) = 1 \qquad 0<\xi<a$ $\qquad\qquad \varphi(x\cos\omega) = \begin{matrix}1 \ldots & x\cos\omega \le a \\ 0 \ldots & > a\end{matrix} \ldots\ldots \omega > \arccos\frac{a}{x}$

$\varphi(\xi) = 0 \qquad a<\xi$



$$P = \frac{16 Dt}{a^2\pi}\int_{\frac{a}{2\sqrt{Dt}}}^{\infty} e^{-R^2} R\,dR \int_0^{\frac{a}{2\sqrt{Dt}}} e^{-r^2} r\,dr \int_{-1}^{+1} \frac{e^{+2Rr\xi}}{\sqrt{1-\xi^2}}\,d\xi$$

$$\int_{-1}^{+1}\frac{\xi^n}{\sqrt{1-\xi^2}}\,d\xi = \int_0^{\pi}\cos^n\varphi\,d\varphi = \cancel{\frac{1}{2}\int_0^{\pi}\left[e^{in\varphi}+e^{-in\varphi}\right]d\varphi = \frac{1}{2}\,\frac{e^{in\varphi}-e^{-in\varphi}}{in}\Bigg| = \frac{\sin n\varphi}{n}\Bigg|_0^{\pi}}$$

$$\int_0^{\pi}\cos^{2m}\varphi\,d\varphi = \left(\frac{1}{2}\right)^{2m}\int\left[e^{i\varphi}+e^{-i\varphi}\right]^{2m} = \left(\frac{1}{2}\right)^{2m}\int\Big\{e^{2mi\varphi}+e^{-2mi\varphi}+\binom{2m}{1}\left[e^{(2m-2)i\varphi}+e^{-(2m-2)i\varphi}\right]+\ldots$$

$$= \left(\frac{1}{2}\right)^{2m}\int_0^{\pi}\Big[\cancel{\cos 2m\varphi}+\binom{2m}{1}\cancel{\cos 2(m-1)\varphi}+\binom{2m}{2}\cancel{\cos 2(m-2)\varphi}+\ldots\quad \underline{\binom{2m}{m}\cos 0}\Big]\,d\varphi$$

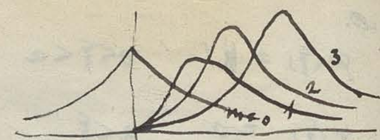
$$= \left(\frac{1}{2}\right)^{2m}\binom{2m}{m}\pi =$$

$$\int_{-1}^{+1}\frac{e^{2Rr\xi}}{\sqrt{1-\xi^2}}\,d\xi = \sum_{n=0}^{\infty}\int\frac{(2Rr\xi)^n}{n!\sqrt{1-\xi^2}}\,d\xi = \sum_{n=0}\frac{(2Rr)^{2m}}{(2m)!}\left(\frac{1}{2}\right)^{2m}\binom{2m}{m}\pi = \sum\frac{(Rr)^{2m}}{(2m)!}\underbrace{\binom{2m}{m}}\pi$$

$$\frac{2m(2m-1)(2m-2)\cdots(m+1)}{1.2.3\cdots m}\;\frac{}{1.2.3\cdots m(m+1)\cdots(2m-1)2m} = \left(\frac{1}{m!}\right)^2 \qquad = \sum\frac{(Rr)^{2m}}{(m!)^2}\pi$$

$$\int_{\frac{a}{2\sqrt{Dt}}}^{\infty} e^{-R^2}R^{2m+1}\,dR\,.\int_0^{\frac{a}{2\sqrt{Dt}}} e^{-r^2}r^{2m+1}\,dr = \frac{1}{4}\int_{\frac{a^2}{4Dt}}^{\infty} e^{-x}x^m\,dx\,.\int_0^{\frac{a^2}{4Dt}} e^{-x}x^m\,dx$$

$$P = \frac{4 \cdot Dt}{a^2} \sum \cdot \frac{1}{(m!)^2} \cdot \int_{\frac{a^2}{4Dt}}^{\infty} e^{-x} x^m \, dx \cdot \left(\int_0^{\frac{a^2}{4Dt}} e^{-x} x^m \, dx \right)^{=\cancel{2x}}$$



$$-x^m + m x^{m-1} = 0$$

$$x = m$$

$n = 0$:

$$P_0 = \frac{4 \cdot Dt}{a^2} e^{-\frac{a^2}{4Dt}} \left[1 - e^{-\frac{a^2}{4Dt}} \right] = e^{-\frac{a^2}{4Dt}} \left[1 - \left(\frac{a^2}{4Dt}\right) \frac{1}{2!} + \left(\frac{a^2}{4Dt}\right)^2 \frac{1}{3!} - \dots \right]$$

$n = 1$:

$$\int e^{-x} x \, dx = -e^{-x} x + \int e^{-x} dx = -e^{-x}(x+1)$$

$$P_1 = \frac{1}{\alpha} e^{-\alpha}(\alpha+1) \cdot \left[1 - e^{-\alpha}(\alpha+1) \right] = \frac{\alpha+1}{\alpha} e^{-\alpha} \left[1 - (1+\alpha)\left(1 - \alpha + \frac{\alpha^2}{2} - \frac{\alpha^3}{3!} - \right) \right]$$

$$= 1 - \left(\begin{matrix} 1 - \alpha + \frac{\alpha^2}{2} - \frac{\alpha^3}{6} \\ + \alpha - \alpha^2 + \frac{\alpha^3}{2} \end{matrix} \right)$$

$$= \frac{\alpha+1}{\alpha} e^{-\alpha} \left(\frac{\alpha^2}{2} - \frac{\alpha^3}{3} \right) = e^{-\alpha} (\alpha+1)\left(\frac{\alpha}{2} - \frac{\alpha^2}{3} \right) = e^{-\alpha} \left(\frac{\alpha^2}{2} - \frac{\alpha^3}{3} + \frac{\alpha}{2} - \frac{\alpha^2}{3} \right)$$

$$= \left(\frac{\alpha}{2} + \frac{\alpha^2}{6} \right) e^{-\alpha}$$

$$P_0 + P_1 = e^{-\alpha} \left[1 - \cancel{\frac{\alpha}{2}} + \frac{\alpha^2}{6} + \cancel{\frac{\alpha}{2}} + \frac{\alpha^2}{6} \right]$$

$$\neq 1 - \alpha = 1 - \frac{a^2}{4Dt}$$

$$\frac{\partial}{\partial \alpha} \Big| \int e^{-\alpha x} dx = -\frac{e^{-\alpha x}}{\alpha} \qquad -e^{-x}$$

$$-\int x e^{-\alpha x} dx = + \frac{e^{-\alpha x}}{\alpha^2} + \frac{e^{-\alpha x} \cdot x}{\alpha} \qquad -e^{-x}(1+x)$$

$$\int x^2 e^{-\alpha x} dx = -2 \frac{e^{-\alpha x}}{\alpha^3} - 2 \frac{x e^{-\alpha x}}{\alpha^2} - \frac{e^{-\alpha x} \cdot x^2}{\alpha} \qquad -e^{-x}(2 + 2x + x^2)$$

$$-\int x^3 e^{-\alpha x} dx = 6 \frac{e^{-\alpha x}}{\alpha^4} + 6 \frac{x e^{-\alpha x}}{\alpha^3} + \frac{3x^2 e^{-\alpha x}}{\alpha^2} + x^3 \frac{e^{-\alpha x}}{\alpha} \Big\| -e^{-x}(6 + 6x + 3x^2 + x^3)$$

$$-e^{-x}(24 + 24x + 12x^2 + 4x^3 + x^4)$$

$n=0: \quad e^{-x}(1-e^{-x}) \qquad = e^{-x}\left[x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!}\right]$

$n=1: \quad e^{-x}(1+x)\left[1-(1+x)e^{-x}\right] \qquad = e^{-x}\left[\frac{x^2}{2} + \frac{x^3}{6} - \frac{5}{24}x^4\right]$

$n=2: \quad \frac{1}{4}e^{-x}(2+2x+x^2)\left[2-(2+2x+x^2)e^{-x}\right] \qquad = e^{-x}\left[\frac{x^3}{6} + \frac{x^4}{24}\right]$

$n=3: \quad \frac{1}{36}e^{-x}(6+6x+3x^2+x^3)\left[6-(6+6x+3x^2+x^3)e^{-x}\right] = e^{-x}\left[-\frac{x^4}{8}\right]$

$\left(\frac{1}{24}\right)^2 e^{-x}(24+24x \cdots) \qquad = e^{-x}\left[1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}\right]\left[1-(\quad)e^{-x}\right]$

$\cancel{\Sigma = e^{-x}\left[1+(1+x)+\frac{1}{4}(2+2x+x^2)+\frac{1}{36}(6+6x+3x^2+x^3)\right]}$

$\cancel{-e^{-2x}\left[1+(1+x)^2+\frac{1}{4}(2+2x+x^2)^2+\frac{1}{36}(\quad)^2\right]}$ $\frac{8}{24}-\frac{2}{24}$

$= e^{-x}\left\{\begin{array}{l} x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \\ -x + x^2 - \frac{x^3}{2} + \frac{x^4}{6} \end{array}\right\} \left(\frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{8}\right) \left.\begin{array}{l} \\ + \frac{x^3}{2} - \frac{x^4}{3} \end{array}\right\} \frac{x^2}{2} + \frac{x^3}{6} - x^4\cdot\frac{5}{24}$

$\left[1+x+\frac{x^2}{2}\right]\left[1-e^{-x}\left(1+x+\frac{x^2}{2}\right)\right]$

$\left.\begin{array}{l} \cancel{x} - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} \\ -\cancel{x} + \cancel{x^2} - \frac{x^3}{2} + \frac{x^4}{6} \\ -\frac{x^2}{2} + \frac{x^3}{2} - \frac{x^4}{4} \end{array}\right\} \frac{x^3}{6} + \frac{x^4}{24}(4-6-1) = \left(\frac{x^3}{6} - \frac{x^4}{8}\right)(1+x) = \frac{x^3}{6} + \frac{2\cdot x^4}{48}$

$+\frac{x^4}{6}$

$\left[1+x+\frac{x^2}{2}+\frac{x^3}{6}\right]\left[1-e^{-x}\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\right] \qquad = -\frac{x^4}{8}(1+ \cdots)$

$P = e^{-x}\left\{1+\frac{x^2}{2}-\frac{x^3}{6}\right\} = \left.\begin{array}{l} 1 + \frac{x^2}{2} - \frac{x^3}{6} \\ -x \quad - \frac{x^3}{2} \\ + \frac{x^2}{2} - \frac{x^3}{6} \end{array}\right\} = 1 - x + x^2 - \frac{5}{6}x^3$

$\Sigma = e^{-x}\left[1 + (1+x) + \left(1+x+\frac{x^2}{2}\right) + \left(1+x+\frac{x^2}{2}+\frac{x^3}{3!}\right) + \cdots\right.$

$-\left(e^{-x}\right)^2\left[1+(1+x)^2+\left(1+x+\frac{x^2}{2}\right)^2+(\qquad)^2+\right.$

$= e^{-2x}\left[1\left[e^{x}-1\right] + (1+x)\left[e^{x}-(1+x)\right] + \left(1+x+\frac{x^2}{2}\right)\left[e^{x}-\left(1+x+\frac{x^2}{2}\right)\right] + \cdots\right]$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{6!} + \frac{x^7}{7!}$$

$$x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720} = x + x^2 + [redacted]x^3 + \frac{1}{2}x^4 + \frac{x^5}{4} + \frac{x^6}{12} + \frac{x^7}{36}$$

$$\frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720}$$

$$+\frac{x^3}{2} + \frac{x^4}{6} + \frac{x^5}{24} + \frac{x^6}{120} + \frac{x^7}{720}$$

$$\frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720}$$

$$\frac{x^4}{6} + \frac{x^5}{24} + \frac{x^6}{120} + \frac{x^7}{720}$$

$$\frac{x^5}{12} + \frac{x^6}{48} + \frac{x^7}{240}$$

$$\frac{6}{720} + \frac{4}{120} + \frac{2}{48} = \frac{5}{120} + \frac{2}{48} = [redacted]\ \frac{1}{12}$$

$$\frac{7}{7.720} + \frac{5}{720} + \frac{2}{240} + \frac{1}{6.24} = \frac{5}{240} + \frac{1}{6.24} = \frac{1}{48} + \frac{1}{3.48} = \frac{1}{36}$$

$$\frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{7.720}$$

$$\frac{x^5}{24} + \frac{x^6}{120} + \frac{x^7}{720}$$

$$\frac{x^6}{48} + \frac{x^7}{240}$$

$$\frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{6.24}$$

$$\frac{x^6}{120} + \frac{x^7}{7.720}$$

$$\frac{x^7}{720} + \frac{x^7}{240} + \frac{x^6}{720} + \frac{x^7}{7.720} + \frac{x^7}{720} + \frac{x^7}{7.720}$$

$$\left(1 + x + [redacted]x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{x^5}{12} + \frac{x^6}{36}\right)\left(1 + 2x + \frac{4x^2}{2} - \frac{8x^3}{6} + \frac{16x^4}{24}\right) - \frac{32x^5}{120} + \frac{64x^6}{720}$$

$$\left(1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 - \frac{4}{15}x^5 + \frac{4}{45}x^6\right)$$

$$1 + x + [redacted]x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \frac{1}{12}x^5 + \frac{1}{36}x^6$$

$$-2x - 2x^2 - 2x^3 - x^4 - \frac{1}{2}x^5 - \frac{1}{6}x^6$$

$$+2x^2 + 2x^3 + 2x^4 + x^5 + \frac{1}{2}x^6 \equiv 1 - x + [redacted]x^2 - \frac{5x^3}{6} + \frac{5}{12}x^4$$

$$-\frac{4}{3}x^3 - \frac{4}{3}x^4 - \frac{4}{3}x^5 - \frac{2}{3}x^6$$

$$+\frac{2}{3}x^4 + \frac{2}{3}x^5 + \frac{2}{3}x^6$$

$$-\frac{4}{15}x^5 - \frac{4}{15}x^6$$

$$+\frac{4}{45}x^6$$

$$+\frac{1}{3} + \frac{1}{4} = \frac{7}{12} \quad \Big\| \quad \frac{1}{12} + \frac{1}{2} - \frac{2}{3} - \frac{4}{15} = \frac{5+30-40-16}{60} = -\frac{21}{60} = -\frac{7}{20}$$

$$\frac{1}{36} + \frac{1}{3} - \frac{4}{15} - \frac{4}{45} = \frac{5+60-48-16}{4.5.9} = \frac{1}{9.20}$$

$$\frac{1}{2} - \frac{4}{3} = \frac{3-8}{6} = -\frac{5}{6}$$

$$\frac{5}{12}\ \ -64\ \ +65$$

$$\boxed{P = 1 - \alpha + \alpha^2 - \frac{5}{6}\alpha^3 + \frac{7}{12}\alpha^4 - \frac{7}{20}\alpha^5 + \frac{1}{180}\alpha^6} \quad \alpha = \frac{a^2}{4Dt}$$

$$P = \frac{1}{a^2\pi Dt}\int_a^\infty R\,dR\int_0^a r\,dr\int_{-1}^{+1}\frac{e^{-\frac{R^2+r^2-2Rr\xi}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi$$

$$\int e^{-\frac{R^2}{4Dt}} R\,dR \iint r\,\frac{e^{-\frac{(r-R\xi)^2-R^2\xi^2}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi\,dr$$

$$= \int e^{-\frac{R^2}{4Dt}(1-\xi^2)} R\,dR \iint r\,\frac{e^{-\frac{(r-R\xi)^2}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi\,dr$$

$$\frac{(r-R\xi)\,e^{-\frac{(r-R\xi)^2}{4Dt}} + R\xi\cdot e^{-\frac{(r-R\xi)^2}{4Dt}}}{\sqrt{1-\xi^2}}$$

$R = a+z$

$r = a-y$

$$P = \frac{1}{a^2\pi Dt}\int_{z=0}^{\infty}(a+z)\,dz\int_{y=z}^{y=\alpha} dy.(a-y).\int_{-1}^{+1}\frac{e^{-\frac{\quad}{4Dt}}}{\sqrt{1-\xi^2}}\,d\xi$$

$a^2+2az+z^2$

$+a^2-2ay+y^2$

$$\underbrace{-2\xi(a^2+a(z-y)-zy)}_{2a^2-2a^2\xi\, +\, 2a(z-y)(1-\xi)\, +\, z^2+y^2+2zy\xi}$$

$a-y = a-z+\underbrace{z-y}$

$= a-z+x$

$z-y=x$

$y = z-x$

$$P = \frac{1}{a^2\pi Dt}\int_{z=0}^{\infty}(a+z)\,dz\int_{z-\alpha}^{z}(a-z+x)\,dx\int e^{-2a^2(1-\xi)+2ax(1-\xi)+x^2+(z-x)^2-2z(z-x)\xi}$$

$$\underbrace{\pm 2z(z-x)}_{x^2+2z(z-x)(1-\xi)}$$

$$e^{x^2-2(1-\xi)[a^2+ax+z(z-x)]}$$

$$\int dz\; a^2+ax\;\cancel{\quad}+zx-z^2$$

$$\int_0^{\pi} e^{+\frac{Rr\cos\varphi}{2Dt}}\,d\varphi \qquad \lim_{\alpha=\infty}\int_0^{\pi} e^{\alpha\cos\varphi}\,d\varphi = ?$$

$$= \int_{-1}^{+1} e^{+\alpha\xi}\frac{d\xi}{\sqrt{1-\xi^2}}$$

$$(1-\xi^2)^{-1/2} = 1 + \frac{1}{2}\xi^2 + \frac{\frac{1}{2}\frac{3}{2}}{2}\xi^4 + \frac{\frac{1}{2}\frac{3}{2}\frac{5}{2}}{1.2.3}\xi^6$$

$$= 1 + \frac{1}{1}\frac{\xi^2}{2} + \frac{1.3}{1.2}\left(\frac{\xi^2}{2}\right)^2 + \frac{1.3.5}{1.2.3}\left(\frac{\xi^2}{2}\right)^3 +$$

$$\int e^{\alpha\xi}\xi^n\,d\xi = n!\,\frac{e^{\alpha\xi}}{\alpha^{n+1}}\left[1+\frac{\alpha}{1}\xi+\frac{\alpha^2\xi^2}{2!}+\frac{\alpha^3}{3!}\xi^3+\cdots\frac{\alpha^n\xi^n}{n!}\right]$$

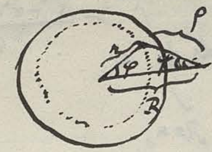
$$\cancel{\mathcal{P} = \frac{1}{a^2\pi Dt}\int e^{-\frac{R^2}{4Dt}}R\,dR\int e^{-\frac{r^2}{4Dt}}r\,dr\sum_{n=0}\underbrace{\frac{1.3.5.\cdot(2n-1)}{1.2.3\cdots n}}_{=[1.3.5\cdots(2n-1)]^2}\left(\frac{1}{2}\right)^n (2n!)\, e^{+\frac{2rR}{4Dt}}\Big[}$$

$$\cancel{\left[\left(\frac{2Dt}{rR}\right)^{2n+1} + \frac{1}{1!}\left(\frac{2Dt}{rR}\right)^{2n}\cdots + (2n)!\,\frac{2Dt}{rR}\cdots\right\}}$$

$$\cancel{\int_{-1}^{+1} e^{\alpha\xi}\xi^n\,d\xi = \frac{n!}{\alpha^{n+1}}\left\{e^{\alpha}\left[1+\alpha+\frac{\alpha^2}{2!}+\frac{\alpha^3}{3!}+\cdots\frac{\alpha^n}{n!}\right]-e^{-\alpha}\left[1-\alpha+\frac{\alpha^2}{2}-\frac{\alpha^3}{3!}\cdots\right]\right\}}$$

$$\cancel{\int_a^{\infty}\int_0^a e^{-\frac{R^2+r^2\pm 2rR}{4Dt}}R\,dR\,r\,dr\left(\frac{2Dt}{rR}\right)^m = (2Dt)^m\int dR\int \frac{e^{-\frac{(R\pm r)^2}{4Dt}}}{(rR)^{m-1}}\,dr}$$

$$P = \frac{1}{2a^2\pi^2 Dt}\iint Rr\,dR\,dr\int e^{-\frac{R^2+r^2-2Rr\cos\varphi}{4Dt}}\,d\varphi$$



$$\cancel{R = a+x \quad r = a+\xi}$$

$$\int_{\varphi=0}^{\pi} e^{-\frac{\rho^2}{4Dt}}\,d\varphi$$

$$\rho : r = \sin\varphi : \sin\psi$$

$$\sin\psi\cdot\rho = r\sin\varphi$$

$$\cancel{\rho\cos\psi\,d\psi = r\cos\varphi\,d\varphi}$$

$$\cancel{d\varphi = d\psi\,\frac{\rho\cos\psi}{r\cos\varphi} = \frac{d\psi\cdot\rho\cos\psi}{\sqrt{r^2-\rho^2\sin^2\psi}}}$$

$$\rho^2 = r^2+R^2-2rR\cos\varphi$$

$$rR\cos\varphi = \frac{r^2+R^2-\rho^2}{2}$$

$$2\rho\,d\rho = +2rR\sin\varphi\,d\varphi$$

$$d\varphi = \frac{\rho\,d\rho}{rR\sin\varphi} = \frac{\rho\,d\rho}{\sqrt{(rR)^2-\frac{(r^2+R^2-\rho^2)^2}{4}}}$$

$$\rho^2 = v$$

$$\int_{\varphi=0}^{\pi} e^{-\frac{\rho^2}{4Dt}}\,d\varphi = \frac{1}{2}\int_{v=(R-r)^2}^{(R+r)^2}\frac{e^{-\frac{v}{4Dt}}\,dv}{\sqrt{(rR)^2-\frac{(r^2+R^2-v)^2}{4}}} = \cancel{\frac{1}{2rR}\int\frac{e^{-\frac{v}{4Dt}}\,dv}{\Big[1-\underbrace{\Big(\frac{r^2+R^2-v}{2rR}\Big)^2}_{u}\Big]^{1/2}}}$$

$$(r^2+R^2).$$

$$= \int\frac{e^{-\frac{v}{4Dt}}\,dv}{\sqrt{\underbrace{4(rR)^2-(r^2+R^2)^2}_{-(R^2-r^2)^2}+2v(r^2+R^2)-v^2}}$$

$$R^2-r^2 = u$$

$$= \frac{1}{(R^2-r^2)}\int\frac{e^{-\frac{v}{4Dt}}\,dv}{\sqrt{-1+2v\,\frac{r^2+R^2}{(R^2-r^2)^2}-\frac{v^2}{(R^2-r^2)^2}}}$$

$$P = \frac{1}{2a^2\pi^2 Dt} \iint r\,dr\,d\varphi \int_{R=a}^{\infty} e^{-\frac{R^2+r^2-2rR\cos\varphi}{4Dt}} R\,dR$$

$$\frac{\sqrt{\pi}}{2}\left[\cancel{1} + \frac{e^{-x^2}}{x\sqrt{\pi}}\left(1-\frac{1}{2x^2}\right) \cdots\right]$$

$$= e^{-x^2}\left(\frac{1}{2x} - \frac{1}{4x^3}\right)$$

$$\int_a^\infty e^{-x^2+\beta x} \cdot x\,dx$$

$$\int_a^\infty e^{-\alpha x^2+\beta x}\,dx = \int_a^\infty e^{-\left(\sqrt{\alpha}x - \frac{\beta}{2\sqrt{\alpha}}\right)^2 + \frac{\beta^2}{4\alpha}}\,dx = \frac{e^{\frac{\beta^2}{4\alpha}}}{\sqrt{\alpha}} \int_{\sqrt{\alpha}a - \frac{\beta}{2\sqrt{\alpha}}}^{\infty} e^{-z^2}\,dz$$

$$\int_a^\infty e^{-x^2+\beta x}\,dx = e^{\frac{\beta^2}{4}} \int_{a-\frac{\beta}{2}}^{\infty} e^{-z^2}\,dz$$

$$\int e^{-x^2+\beta x}\,x\,dx = \frac{\beta}{2}\, e^{\frac{\beta^2}{4}} \int_{a-\frac{\beta}{2}}^{\infty} e^{-z^2}\,dz + \frac{1}{2} e^{\frac{\beta^2}{4}} e^{-\left(a-\frac{\beta}{2}\right)^2}$$

$$= \frac{e^{\frac{\beta^2}{4}}}{2}\left\{\frac{\beta}{2}\left[\frac{1}{2\left(a-\frac{\beta}{2}\right)} - \frac{1}{4\left(a-\frac{\beta}{2}\right)^3}\right] + 1\right\} e^{-\left(a-\frac{\beta}{2}\right)^2}$$

$$\frac{2\beta}{2a-\beta} - \frac{2\beta}{(2a-\beta)^3}$$

$$= e^{-a^2+a\beta}\left[\frac{a}{2a-\beta} - \frac{\beta}{(2a-\beta)^3}\right]$$

δ I $\Delta\omega$

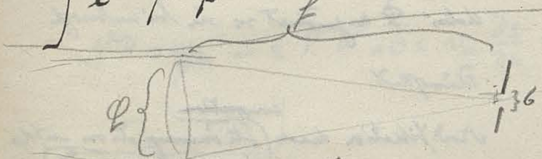
$$-x^2 + 2x\cdot\frac{\beta}{2}$$

$$= \frac{1}{2a^2\pi^2 Dt}\int r\,dr\, e^{-\frac{r^2}{4Dt}}\int d\varphi \int_{R=a}^{\infty} e^{-\left(\frac{R}{2\sqrt{Dt}}\right)^2 + 2\frac{R}{2\sqrt{Dt}}\cdot\frac{r\cos\varphi}{\sqrt{Dt}\cdot 2} \pm \frac{r^2\cos^2\varphi}{4Dt}}\, R\,dR$$

$$= \frac{1}{2a^2\pi^2 Dt}\int r\,dr\, e^{-\frac{r^2}{4Dt}\left[1-\cos^2\varphi\right]}\int d\varphi\int e^{-\left(\frac{R}{2\sqrt{Dt}} - \frac{r\cos\varphi}{2\sqrt{Dt}}\right)^2} R\,dR$$

$$= \frac{2}{a^2\pi^2}\iint e^{-\frac{r^2}{4Dt}}\, r\,dr\,d\varphi\; e^{-\frac{a^2 - 2ar\cos\varphi}{4Dt}}\left[\frac{a}{2(a-r\cos\varphi)} - \cdot\right]$$

$$\int e^{-\rho^2}\rho\,d\rho$$



Flächenhelligkeit $\varepsilon = \frac{\Delta F}{\Delta\omega}\frac{1}{S}$

$\Delta F = \varepsilon S \Delta\omega$

Für Sonne: $r^2\Delta\omega = \varphi$

$S = r^2\Omega$

$\Delta F_\odot = \varepsilon\varphi\Omega$

$\Omega f^2 = \sigma$

$\Delta F_\odot = \varepsilon\frac{\varphi\sigma}{f^2}$

$\varphi = l_g^2\Delta\omega$

$S = \sigma\left(\frac{l_g}{l_\varphi}\right)^2$

$\Delta F = \varepsilon\frac{\sigma\varphi}{(l_b)^2}$

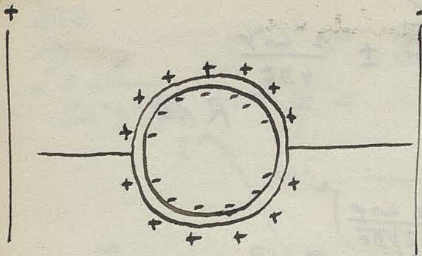
Energiefluss $\left\{\begin{array}{l}\text{Objekt}\\ \text{Sonne}\end{array}\right. = \frac{\varepsilon_B}{\varepsilon_\odot}\frac{f^2}{l_b^2}$

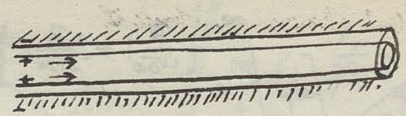
$\frac{f}{l_b} \neq 2$

ε_B (4 Amp.) $= 44.000$

$\varepsilon_\odot = 160.000$

$\Big\} = \frac{44}{160} = \frac{1}{4}$





Oberflächenleitung pro cm d. Umfanges:

$$J = \int \varepsilon v \, dn \qquad \varepsilon = \frac{1}{4\pi}\frac{\partial^2 U}{\partial n^2}$$

$$X = \varepsilon \frac{\partial \Phi}{\partial x}$$

$$\mu \frac{\partial^2 v}{\partial n^2} = \varepsilon \frac{\partial \Phi}{\partial x} + \left(\frac{\partial p}{\partial x}\right.$$

$$\mu v = \frac{\partial \Phi}{\partial x}\iint \varepsilon \, dn^2 = \frac{1}{4\pi}\frac{\partial \Phi}{\partial x} U + \text{const} = \frac{1}{4\pi}(\varphi_1 - \varphi_0)\frac{\partial \Phi}{\partial x}$$

Elektr. Endosm.: $\left.\begin{matrix} v_\xi \\ v_\eta \end{matrix}\right\} = -K\frac{\varphi_1-\varphi_0}{4\pi\mu}\left\{\begin{matrix} \frac{\partial\Phi}{\partial\xi} \\ \frac{\partial\Phi}{\partial\eta}\end{matrix}\right.$

dabei Φ angenähert so wie bei unbewegl. Flüssigkeit

Modifikation durch (convectiven Strömungsstrom infolge Oberflächenleitung und) der v_ξ, v_η

Strömungsstrom

$$V_2 - V_1 = \frac{K(\varphi_1 - \varphi_0)}{4\pi\mu}\,\sigma\,(p_2 - p_1)$$

bei Vernachlässig. d. elektrosmot. durch $V_2 - V_1$ hervorgebrachten Druckes

(und d. Oberfl. leitg. ?)

$$\frac{J_1}{J_2} = \frac{9}{16}\,\frac{K^2(\varphi_1-\varphi_0)^2}{16\cdot\delta\cdot\pi^2}\,\frac{\sigma}{\mu}\left(\frac{3\pi}{4}\right)^{2/3}\frac{\varphi^{2/3}}{a}$$

$\varphi = 0.001$ $\quad \sigma = \frac{10^{6}}{9\cdot 10^{11}} \neq 10^{-6}$

$K(\varphi_1-\varphi_0) = \frac{4}{300}$

$\delta = 4\cdot 10^{-7}$ $\quad \mu = 0.02$ $\quad a = 0.1\mu = 10^{-5}$

$$= \frac{9\pi}{16\cdot 16\cdot 10}\,\frac{16}{9\cdot 10^4\cdot 4\cdot 10^{-7}}\,\frac{10^{-6}}{0.02}\,\frac{0.01}{10^{-5}} = \frac{10}{64} \neq \frac{1}{6}$$

also erst bei ~~viel~~ kleineren Teilchen erheblich (und grösser $\sim$ 1/Teilch.)

$$\int_0^{\pi/2}\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^4 d\theta = \int_0^{\pi/2}\frac{e^{4i\theta}+e^{-4i\theta}-4\left(e^{2i\theta}+e^{-2i\theta}\right)+6}{16}\,d\theta = \frac{3}{8}\,\frac{\pi}{2}$$

$$\frac{1}{16}\left[\cancel{\frac{e^{4i\theta}-e^{-4i\theta}}{4i}} - 4\,\cancel{\frac{e^{2i\theta}-e^{-2i\theta}}{2i}} + \right.$$

Elektr. Änderungen bei Berücks. d. Oberfl. L $\quad \frac{\partial \Phi}{\partial \xi}$ etc. verändern sich

Es gilt $\quad \mathrm{div}\, c = 0$

(20) $\quad \mathrm{div}\left[\frac{\nabla \Phi}{\sigma} + \varepsilon \mathfrak{v}\right] = 0$

(21) $\quad \therefore \quad \Delta^2 \cancel{\Phi} - \sigma\left[u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} + w \frac{\partial \varepsilon}{\partial z}\right] \qquad \Delta^2 \Phi = 0$

$= -\sigma v_n \frac{\partial \varepsilon}{\partial n}$ unter Annahme dass Doppelschicht überall gleich

$$\begin{cases} \frac{\partial P}{\partial \xi} = \mu \Delta^2 v_\xi - \varepsilon \frac{\partial(\Phi + V)}{\partial \xi} \\ \frac{\partial P}{\partial \xi} = \mu \Delta^2 v_\xi \mp \varepsilon \frac{\partial(\Phi + V)}{\partial \xi} \quad = \cancel{\ldots} \end{cases}$$

$$\frac{\partial^2 V}{\partial n^2} = \cancel{\ldots} - \sigma v_n \frac{\partial^3 \varphi}{\partial n^3}$$

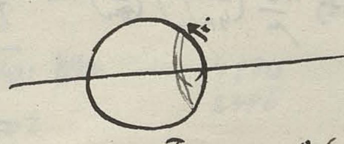
$$\frac{\partial V}{\partial n} = -\sigma \int \frac{\partial v_n}{\partial n} \frac{\partial^2 \varphi}{\partial n^2}\, dn$$

$$= -\sigma \int \frac{\partial^2 v_n}{\partial n^2} \frac{\partial \varphi}{\partial n}\, dn$$

$$\Delta^2 P = \cancel{\ldots} - \varepsilon \nabla^2 V = \sigma \varepsilon v_n \frac{\partial \varepsilon}{\partial n} = \frac{\sigma v_n}{2} \frac{\partial(\varepsilon^2)}{\partial n} + \sigma \frac{\partial \varepsilon}{\partial \xi}\left(\frac{\partial \Phi}{\partial n} + \frac{\partial V}{\partial n}\right)$$

?

Angenäherter Einfluss d. Oberfl. Leitung



pro Kugel $\quad \cancel{\ldots}$ (bei festgehaltener Kugel)

$$i = \underbrace{\left(\frac{K}{4\pi}\right)^2 \frac{1}{\mu} \int_0^\infty \left(\frac{\partial \varphi}{\partial n}\right)^2 dn}_{A} \cdot \frac{\partial \Phi}{\partial s}$$

$$\Phi = -c \cos\theta \left(r + \frac{a^3}{2r^2}\right)$$

$$\left.\frac{\partial \Phi}{\partial s}\right|_a = \frac{3}{2} c \sin\theta$$

$$u = A \frac{3}{2} c \sin^2\theta \cdot 2a\pi \sin\theta$$

$$\bar{u} = \int \frac{u\, dx}{a} = \frac{3 A a c \pi}{\cancel{a}} \int_0^{\frac{\pi}{2}} \sin^3\theta \sin\theta\, d\theta = 3 A a c \pi \int_0^{\frac{\pi}{2}} \sin^4\theta\, d\theta = \frac{9}{16} A a c \pi$$

n Kugeln pro Vol. Einheit $= \sqrt[3]{n}$ pro Längeneinheit

$$\varphi = \frac{4}{3} a^3 \pi n \quad \Big\| \quad \Sigma \bar{u} = \frac{9}{16} A a c \pi n^{2/3} = \frac{9}{16} A c \pi a \left(\frac{3\varphi}{4\pi}\right)^{2/3} \frac{1}{a^2} = \frac{9}{16} A c \left(\frac{3\varphi\pi}{4}\right)^{2/3} \frac{1}{a}$$

$$\frac{J_s}{J_\varphi} = \frac{9}{16} A \left(\frac{3\varphi\pi}{4}\right)^{2/3} \sigma \frac{1}{a}$$

Dagegen falls Ladungstheorie richtig wäre:

$eX = 6\pi\mu a u$

$$\frac{J}{J_\varphi} = \frac{e\, n u}{\lambda X} = \frac{e^2}{\lambda} n \frac{1}{6\pi\mu a} = \frac{6\pi\mu a u^2 n}{\lambda X^2}$$

Kontaktpot. $u = \frac{K(\varphi_i - \varphi_a)}{4\pi}\frac{X}{\mu}$

$$= 6\pi\mu a \frac{6}{\mu} n \left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2$$

$$\underbrace{= \frac{9}{2}\frac{\varphi}{a^2}\frac{6}{\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2} = 4\cdot 10^{-4} \text{ !!!}$$

Für Oberfl. Leitung

~~mittl. Oberflächenstrom für eine Kugel~~ (in X Kompon.):

$\bar{J} =$

Wenn aneinandergereiht in X, so dass Anzahl pro 1 cm ist $N = \frac{1}{2a}$

dann sind im Querschnitt pro cm²: $2an$ Kugeln vorhanden, also ~~...~~ Oberflächenstrom

$$\frac{J_1}{J_q} = \frac{2an\, \frac{9}{16} A a \varphi \pi\, 6}{\varphi}$$

~~$n = \varphi$~~

$$= \frac{9}{8} a^2\pi \frac{n 6}{\mu}\left(\frac{K}{4\pi}\right)^2 \int \left(\frac{\partial\varphi}{\partial n}\right)^2 dn \neq \frac{9}{8}\frac{a^2\pi}{\delta} n \frac{6}{\mu}\frac{K^2(\varphi_i-\varphi_a)^2}{(4\pi)^2}$$

$\left(n = \frac{3}{4}\frac{\varphi}{a^3\pi}\right)$

$$= \frac{9}{8}\frac{a^2\pi}{\delta}\frac{3}{4}\frac{\varphi}{a^3\pi}\frac{6}{\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2$$

$$= \frac{27}{32}\frac{\varphi}{a\delta}\frac{6}{\mu}\left(\frac{K(\varphi_i-\varphi_a)}{4\pi}\right)^2$$

Z.B. $\varphi = 0.001$ $\quad 6 = 10^{-6}$ (abs. elektrost.) $\neq 10^6\,\Omega$

$a = 10^{-5}$ $\quad K(\varphi_i-\varphi_a) = \frac{4}{300}$

$\delta = 4\cdot 10^{-7}$

$$= \frac{\cancel{27}\,3}{32}\frac{10^{-3}}{4\cdot 10^{-12}}\frac{10^{-6}}{0.02}\frac{\cancel{16}}{\cancel{9}\cdot 10^4}\frac{1}{\cancel{16}\cdot 10}$$

$$= \frac{3}{32\cdot 0.08}10^{-2} = 10^{-2} \text{ !!!}$$

$\int p\,dv = RT \lg\frac{n}{\nu}$

auf 1 gr. entfallen: $\frac{1}{m}$ Teilchen

also Arbeit pro 1 Teilchen: $m RT \lg\frac{n}{\nu} = m\frac{HT}{\mu}\lg\frac{n}{\nu} = \frac{HT}{N}\lg\frac{n}{\nu}$

$$1-\frac{2}{\sqrt{\pi}}\int_0^\beta\left(1-y^2+\frac{y^4}{2!}-\frac{y^6}{2.3!}\right)dy+\frac{1}{\beta\sqrt{\pi}}\left(\beta^2-\frac{\beta^4}{2!}+\frac{\beta^6}{3!}-\right)$$

$$=1-\frac{2}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{1!3}+\frac{\beta^5}{2!5}-\frac{\beta^7}{3!7}\right]+\frac{1}{\sqrt{\pi}}\left(\beta-\frac{\beta^3}{2!}+\frac{\beta^5}{3!}-\frac{\beta^7}{4!}+\frac{\beta^9}{5!}\right)$$

$$=1-\frac{1}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{6}+\frac{\beta^5}{5.6}-\frac{\beta^7}{3.7.8}+\frac{\beta^9}{12.9.10}\right.$$

$$-\frac{1}{24}+\frac{1}{21}=\frac{+1}{3.7.8}$$

$$+\frac{1}{10.12}-\frac{1}{12.9}=\frac{1}{120.9}$$

$$\frac{\beta^3}{2!3}\quad\frac{\beta^5}{3!5}\quad\frac{\beta^7}{4!7}\quad\frac{\beta^9}{5!9}$$

$$=1-\frac{1}{\sqrt{\pi}}\left[\beta-\frac{\beta^3}{2!3}+\frac{\beta^5}{3!5}-\frac{\beta^7}{4!7}+\frac{\beta^9}{5!9}--\right]\quad -2\frac{\beta^n}{\frac{n-1}{2}!\,n}+\frac{\beta^n}{\frac{n+1}{2}!}=$$

$$n=2m+1$$

$$\beta^{2m+1}\left[\frac{-2}{(2m+1)m!}+\frac{1}{m+1!}\right]$$

$$-\frac{2}{2m+1}+\frac{1}{m+1}$$

$$=\frac{-2m-2+2m+1}{(2m+1)\,m+1}\qquad=\beta^{2m+1}\frac{-2(m+1)+2m+1}{(2m+1)(m+1)!}$$

$$=-\beta^{2m+1}\frac{1}{(2m+1)\,m+1!}=-\frac{\beta^n}{n.\frac{n+1}{2}!}$$

$$\beta=\frac{1}{4}=0{\cdot}25$$

$$\frac{\beta^5}{30}=\frac{1}{16}\,\frac{1}{64}\,\frac{1}{30}=\frac{(0{\cdot}0625)^2}{120}$$

$$\frac{\beta^3}{6}=\frac{0{\cdot}0625}{24}=0{\cdot}02604$$

0·25
0·026
0·2240:

4$\not{8}$715
2486

7959
3802
4157

$$\frac{\beta^2}{5}=\frac{1}{800}$$

3502
2486
1016

$$P=\frac{0{\cdot}127}{0{\cdot}873}.31$$

2619
87
2·606

$$\frac{1}{W(n)}=e^{\mu}\frac{n!}{\mu^n}=\frac{e^{\mu}}{\mu^n}\left(\frac{n}{e}\right)^n\sqrt{2n\pi}=e^{\mu-n}\left(\frac{n}{\mu}\right)^n\sqrt{2n\pi}\qquad\delta^2=$$

$$p=(n)\frac{HT}{V}$$

$$\mu=mN$$

$$p v_\mu=R\theta$$

$$p=\frac{HT}{\mu}\rho=\frac{HT}{\mu}\qquad\rho=\frac{(n)}{V}\mu=\frac{\nu.m}{V}$$

$$(n)=\nu.m$$

$$=(n)\frac{mN}{V}\qquad \nu=(n)N$$

$$=(n)\frac{HT}{V}\log\frac{n}{\nu}=\frac{\nu}{N}\frac{HT}{V}\log\frac{n}{\nu}=\frac{HT}{N}$$

$$\underline{2.3.4.5.6.7}$$
$$\underline{720.7}$$
$$5040$$

$$0'4343.\ 1'55$$
$$21715$$
$$\underline{2172}$$
$$0'6732$$
$$\underline{3'7024}$$
$$4'3756$$
$$\underline{-1'3321}$$
$$3'0435$$

$$\underline{0'1903.\ 7}$$
$$1'3321$$

$$1105$$

$t=0 \qquad u=x$

$$u=c\,\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\sin\frac{n\pi x}{c} \qquad \frac{\pi n}{c}=y \qquad \frac{\pi}{c}=dy$$

$$\lim_{c=\infty} u = 2\sum(-1)^{n+1}\frac{1}{y}e^{-a^2y^2t}\sin yx$$

$$\frac{\partial v}{\partial t}=a^2\frac{\partial v}{\partial r^2} \qquad v=v_1+v_2$$

$$\left.\begin{array}{ll} t=0 & v_1=R\,\varepsilon \\ r>0 & v_2=r\,\varepsilon \end{array}\right\} \qquad \left.\begin{array}{l} r=0 \\ 0<t<\infty \end{array}\right\} \begin{array}{l} v_1=0 \\ v_2=0 \end{array}$$

$$v_2=r\varepsilon$$
$$v_1=\frac{2R\varepsilon}{\sqrt{\pi}}\int_0^{\frac{r}{2a\sqrt{t}}}e^{-\beta^2}d\beta$$

$$r^2\frac{\partial u}{\partial r}=r^2\frac{\partial\frac{v}{r}}{\partial r}=\underbrace{-v+r\frac{\partial v}{\partial r}=0}_{\text{für } r=\infty}$$

Verschiebung d. Anfangspunktes nach 0:

$$u=\frac{1}{r}\left[\frac{2R}{\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}}e^{-z^2}dz+(r-R)\right]$$

$$\frac{\partial u}{\partial r}=-\frac{u}{r}+\frac{1}{r}\left\{\frac{2R}{\sqrt{\pi}}\frac{1}{2a\sqrt{t}}e^{-\frac{(r-R)^2}{4a^2t}}+1\right\}$$

$$r^2\frac{\partial u}{\partial r}=\frac{R}{a\sqrt{\pi t}}e^{-\frac{(r-R)^2}{4a^2t}}\cdot r+r-\frac{2R}{\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}}-r+R\Bigg|^{r=\infty}=0$$

$$r^2\frac{\partial u}{\partial r}\Big|_{r=R} = \frac{R^2}{a\sqrt{\pi t}} + R \cancel{\;} = R\left[1+\frac{R}{a\sqrt{\pi t}}\right]$$

$$u = \frac{R u_R}{r}\left[\frac{2}{\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}} e^{-z^2}dz + \frac{r}{R} - 1\right] = u_R\left[\frac{2}{r\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}} e^{-z^2}dz + \frac{1}{R} - \frac{1}{r}\right]$$

$$\frac{\partial u}{\partial r} = R u_R\left[\frac{1}{r^2} - \frac{2}{r^2\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}} \sim + \frac{1}{r a\sqrt{\pi t}}\, e^{-\frac{(r-R)^2}{4a^2t}}\right]$$

$$r^2\frac{\partial u}{\partial r} = R u_R\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{r-R}{2a\sqrt{t}}} \sim + \frac{r}{a\sqrt{\pi t}}\, e^{-\frac{(r-R)^2}{4a^2t}}\right]$$

$$r^2\frac{\partial u}{\partial r}\Big|_{r=R} = R u_R\left[1+\frac{R}{a\sqrt{\pi t}}\right] = \frac{J}{4\pi\kappa} \qquad a^2 = \frac{\kappa}{c\rho}$$

$$\int_0^t J\,dt = 4\pi\kappa\, u_R R\left[t + \frac{2R\sqrt{t}}{a\sqrt{\pi}}\right]$$

für lim $t=0$:

$$J = 4R^2\pi\cdot\kappa\, u_R\cdot\frac{2\sqrt{t}}{a\sqrt{\pi}}$$

Aufgabe: Begrenztes Kugelvolumen von $r = R$ bis $r = A$

$t=0 \quad u = u_R = c \quad \| \quad r = R:\ u=0 \quad \| \quad r = A:\ \frac{\partial u}{\partial r} = 0$

$R < r < A$

$v = r\frac{\partial v}{\partial r}$

$v = 0$

$v = rc$

Anstatt dessen Verschiebung $r' = r - R$:

$t=0 \quad v = r'c + Rc \quad \| \quad r'=0 \quad u=0 \quad \| \quad r' = A' \quad \frac{\partial v}{\partial r'} - \frac{v}{A} = 0$

$0 < r' < A' \qquad = r' F(r')$

$F(r') = c\left(1+\frac{R}{r'}\right)$

$P = 1 - \frac{1}{\sqrt{\pi}}\left[\beta - \frac{\beta^3}{6} + \frac{\beta^5}{30}\right]$ $\beta = 0{\cdot}25$ $P = 0{\cdot}873$

$\beta = 0{\cdot}501$
$- \quad 52$
$0{\cdot}449$

$\beta^5 = \frac{1}{32}$

$\frac{\beta^3}{6} = \cancel{\frac{1}{32 \cdot 6}} = \cancel{\frac{1}{192}}$

$\frac{1}{48} = 20$

$\beta = 0{\cdot}5$ $P = 0{\cdot}729$

$\beta = 0{\cdot}505$ $P = 0{\cdot}726$

$\frac{\beta_0}{\sqrt{2}} = \beta = 0{\cdot}357$ $P = 0{\cdot}803$

6522
-2486
4036

0·2533
0·7467

$\beta = \sqrt{\pi}(1-P) + \frac{\beta^3}{6}$ $P = 0{\cdot}726$
0·274

4378
2486
0·6864 −1
0·0592 −1
−0·7782
0·2810 −2

$\beta = 0{\cdot}4858$
0·0191

Experimentell

$\bar{\Delta}^2$		
	für 1τ	$P = 0{\cdot}726$
2·62	2τ	~~0·878~~ 0·845
2·59	3τ	0·835
2·80	4τ	0·903
2·95	5τ	0·951
3·00	6τ	0·968

$\beta = 0{\cdot}501$
$- \quad 21$
0·480

6812
2486
4326

2707

7033
1505
0·5528 −1
0·6584 −2
0·7640 −3

0·3571
0·0455
0·0058

$\beta = 0{\cdot}3571$
0·0002
0·3573
−0·0076
0·3497

7033
3010
0·4023 −1
0·2069 −2
0161

2524
−0027
2497

4771
4914
9857

4133
4914
9219

0	112
1	168
2	130
3	69
4	32
5	5
6	1
7	1

168
520
621
32
192
125
36
49
2031 518

2077
7143
5934

5437
2486
2951

1973
0·8027

168
260
207
128
25
13

3974
2486
1488

1411
0·8589

9036
8107 7143
1893

8519
7143
1376

801

$\nu = 1546$ $\frac{1}{\nu} = 0{\cdot}647$

[illegible]

	0	1	2	3	4	5	6	7
	25	17	19	7	7			~~[illegible]~~
	18	22	13	5	~~[illegible]~~			
	19	19	7	5	1			
	38	56	30+4	20	3	—	—	2
	25	53	29	11	4	2		
Total	125	187	108	48	15	2		2

487

$$\frac{\Sigma(n-\nu)^2}{N} = \frac{\Sigma n_i^2}{N} - \nu^2$$

1903
3806
5921
2402
1.519

432
432
240
50
98
1439

1581
6875
4706
4914
9782

2955

$\bar{s}^2 =$ 1.52 + ν

1818
3786
8032
0.636

~~1818~~
~~3806~~
~~8012~~
633

7803
8097
645

1	√2	√3	√4	√5	√6
726	845	835	903	951	968
·274	·155	·165	·097	·049	·032

772

4378 1903 2175 9868 6902 5051
2486 2486 2486 2486 2486 2486
1505 2385 3010 3495 3891

6864 | 4389 | 4661 | 2354 | 9388 | 7537
·0592 -1 3167 3983 -2 7062 8164 -4

$\beta = 0·486$ ·275 ·292 172 0869 0567

3 4 1 1

$\beta_{corr} = 0·505$ 0·278 0·296 0·173 0·087 0·057

4440 4713 2380 -1 | 9395 -2 | 7559 -2
1505 2385 3010 3495 | 3891

7033 | 5945 | 7098 | 5390 | 2890 | 1450 -1
4314 2041
505 393 512 346 461 140

27 15 16 10 5 3

4378 | 1903 | 2175 | 9868 | 6902 | 5051

1411 | 7848 | 9273 | 6258 | 9792 | 6507

1384
6093
8458
3356
953
447
33147

5204
8876
6328

6328 3010
·6328 -1 3318
8884 -2 9954
0·0791 -

6328 6328 6328 6328
1505 2385 3495 3891
4823 | 3943 | 2833 | 2437
·4469 -1 | 1829 | 8499 | 7311
·280 ·152 | 099 | 071 | 054

$\bar{\beta}_1 = ·4293$

~~$\beta_2 = 0·2788$~~ || ~~$\beta_3 = 0·1908$~~ || ~~$\beta_4 = 0·1283$~~ || ~~$\beta_5 = 0·0798$~~

$\beta_1 = 0·429|3$ || $\beta_2 = 0·303|6$ || $\beta_3 = 0·248$ || $\beta_4 = 0·214|6$ || $\beta_5 = 0·192$ || $\beta_6 = 0·175$

1/2 1/2

132 43 2/5

~~0·216~~ ~~0·193~~ ~~0·176~~

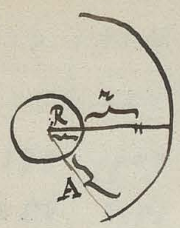
~~0·425~~ ~~0·308~~ ~~0·250~~

0·213 0·191 0·174

0·416 0·299 0·243

~~0·584~~ ~~0·701~~ ~~0·757~~

6191	4757	3856	3284	2810	2405
2486	2486	2486	2486	2486	2486
3705	2271	1370	0798	0324	9919
235	169	137	120	108	0981
2295	2493	2589	264	2676	2757
0·766	0·831	0·863	0·880	0·892	0·919
726	845	835	903	951	968
~~2488~~	~~2535~~	~~2505~~	~~2709~~	~~2853~~	~~2904~~
237	258	268	273	277	285



$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial r^2}$$

$$\left\{ t=0 \qquad v = (r+R)c = \Phi(r) \right.$$

$$\left\{ \begin{array}{ll} r=0 & v=0 \\ r=A & \dfrac{\partial v}{\partial r} - \dfrac{v}{A+r} = 0 \end{array} \right.$$

$$u = \frac{v}{R+r}$$

$$\left.\frac{\partial u}{\partial r}\right|_{r=0} = \frac{1}{R+r}\left(\frac{\partial v}{\partial r} - \frac{v}{R+r}\right)_0$$

$$= \frac{1}{R}\left(\frac{\partial v}{\partial r} - \frac{v}{R}\right)_{r=0}$$

$$v = V e^{-\gamma^2 t}$$

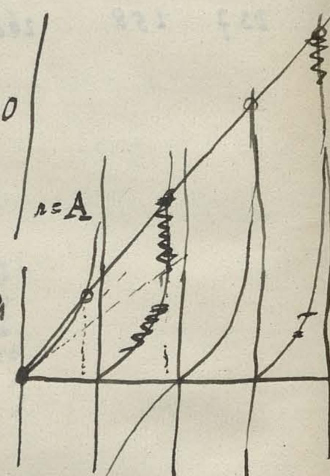
$$-\gamma^2 V = a^2 \frac{\partial^2 V}{\partial r^2}$$

$$V = \sum A \sin\left(\frac{\gamma}{a} r\right)$$

$$\frac{\gamma}{a} \cos \frac{\gamma}{a} r = \frac{\sin \frac{\gamma r}{a}}{A+R}$$

$$\operatorname{tg} \frac{\gamma r}{a} - (A+R)\frac{\gamma}{a} = 0$$

$$\operatorname{tg} \beta r - (A+R)\beta = 0 \qquad r = A$$



$$V = A \sin \beta r$$

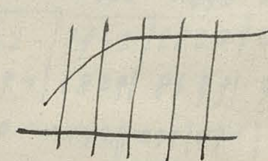
$$\operatorname{tg} \beta A = (A+R)\beta$$

$$v = \sum A_n e^{-a^2 \beta_n^2 t} \sin \beta_n r$$

$$\lim_{A=\infty} \beta_1 A = 0$$

$$\lim_{A=\infty} \beta_2 \neq \frac{3\pi}{2A}$$

$$\cancel{\lim_{A=\infty} v = A_2 e^{-\frac{\pi^2}{4} a^2 t} \sin \frac{\pi r}{2}}$$



$$\lim_{A=\infty} v = \sum A_n e^{-a^2\left[(2n+1)\frac{\pi}{2}\right]^2 \frac{t}{A^2}} \qquad (2n+1)\frac{\pi}{2}\frac{2}{A}$$

$$v = \sum A_n e^{-\frac{(2n+1)^2\pi^2}{4}\frac{a^2 t}{A^2}} \sin\left[\frac{(2n+1)\pi}{2}\frac{r}{A}\right] \cdot \frac{\pi r}{A} \cdot \frac{A}{\pi r} \qquad \frac{n\pi r}{A} = x$$

$$= \frac{A}{\pi r}\int f\left(\frac{Ax}{\pi r}\right) e^{-\frac{x^2 a^2 t}{r^2}} \sin x \, dx \qquad \frac{x}{r} = \xi$$

$$= A \int_0^\infty f(A\xi)\, e^{-a^2\xi^2 t} \sin(r\xi)\, d\xi$$

$$f(r) = \frac{2}{\pi}\int_0^\infty\!\!\int_0^\infty f(\xi) \sin \alpha r \sin \alpha \xi \, d\alpha\, d\xi = \frac{2}{\pi}\iint f(\lambda) \sin r\xi \sin \lambda\xi \, d\xi\, d\lambda$$

$$v = \iint f(\lambda)\, e^{-a^2\xi^2 t} \sin r\xi \sin \lambda\xi \, d\xi \, d\lambda$$

$$\frac{\partial v}{\partial r} = \iint f(\lambda)\, d\lambda \;\; \xi e^{-a^2\xi^2 t} \cos r\xi \sin \lambda\xi \, d\xi$$

$$\xi e^{-a^2\xi^2 t}$$

$$v = \sum_n B_n \, e^{-a^2\beta_n^2 t} \sin\beta_n r$$

$$J = -4\pi k R^2 \frac{\partial u}{\partial r}\Big|_{r=0} = 4\pi k R \left\{ \frac{v}{R} - \frac{\partial v}{\partial r} \right\}\Big|_{r=0} = 4\pi k \left\{ v - R \frac{\partial v}{\partial r} \right\}_{r=0}$$

$$= -4\pi k R \sum B_n \, e^{-a^2\beta_n^2 t} \beta_n$$

$$\int \cancel{\cos\beta r}\, dr = \cancel{\frac{\sin\beta r}{\beta}}$$

$$\int r \sin\beta r \, dr = \frac{\sin\beta r}{\beta^2} - \frac{r\cos\beta r}{\beta}$$

$$Q_t = 4\pi \sum B_n \int_{r=0}^{A} (R+r) \sin\beta_n r \, dr$$

$$= 4\pi \sum B_n \Big\{ \underbrace{-R\left(\frac{\cos\beta_n A - 1}{\beta_n}\right) - A\frac{\cos\beta A}{\beta_n} + \frac{\cos\beta A}{\beta_n^2}}_{\frac{1}{\beta}\left\{-R\cos\beta A + R - A\cos\beta A + (A+R)\cos\beta A\right\} = \frac{R}{\beta}} \Big\}$$

$$\sin\beta A = (A+R)\beta\cos\beta A$$

$$\boxed{Q_t = 4\pi R \sum{}' \frac{B_n}{\beta_n} e^{-a^2\beta_n^2 t} \qquad J = -\frac{\partial Q}{\partial t} = R\cdot 4\pi a^2 \sum{}' B_n \beta_n e^{-a^2\beta_n^2 t}}$$

$$R + r = R \sum{}' \frac{\sin\beta_n r}{\beta_n}$$

$t=0$: $\quad 0<r<A$

$$\sum_n B_n \sin\beta_n r = \Phi(r) \qquad \operatorname{tg}\beta A = (A+R)\beta$$

$$\sum B_n \underbrace{\int_0^A \sin\beta_n r \cos\beta_m r \, dr} = \int_0^A \Phi(r) \cos\beta_m r \, dr$$

$$\frac{\sin(\lambda_m-\lambda_n)A}{2(\lambda_m-\lambda_n)} - \frac{\sin(\lambda_m+\lambda_n)A}{2(\lambda_m+\lambda_n)} = \cos\beta_m A \cos\beta_n A \underbrace{\frac{\beta_n \operatorname{tg}\beta_n A - \beta_m \operatorname{tg}\beta_m A}{\beta_m^2-\beta_n^2}}_{\cancel{(A+R)\beta A} \;=0} = 0$$

$$\int_0^A \sin^2\beta_n r \, dr = \frac{A}{2} - \frac{\sin 2\beta_n A}{4\beta_n} = \frac{A}{2} - \frac{\operatorname{tg}\beta_n A}{2(1+\operatorname{tg}^2\beta_n A)\beta_n}$$

$$= \frac{A}{2} - \frac{(A+R)\cancel{\beta_n}}{2\cancel{\beta_n}\left[1+(A+R)^2\beta_n^2\right]} = \frac{1}{2}\,\frac{A(A+R)^2\beta_n^2 - R}{(A+R)^2\beta_n^2+1}$$

$$B_n = 2\,\frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R}\int_0^A \Phi(r)\sin\beta_n r \, dr \qquad \Bigg\| \qquad \lim_{\frac{R}{A}=0} B_n = \frac{2}{A}\left(1+\frac{1}{(A+R)^2\beta_n^2}\right)\int_0^A \Phi(r)\sin\beta_n r \, dr$$

$$\Phi(r) = (r+R)c$$

$$\int_0^A (R+r)\sin\beta_n r \, dr = \frac{R}{\beta_n} \qquad \int_0^A \Phi(r)\cos\beta_n r \, dr = \frac{cR}{\beta_n}$$

$$J = 4\pi a^2 R \sum B_n \beta_n e^{-a^2\beta_n^2 t}$$

$$= 8\pi a^2 R^2 c \cdot \sum \frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R} \cdot e^{-a^2\beta_n^2 t}$$

(mit Ausnahme der ersten Wurzel! Siehe später!)!!

$$\beta_n \neq (2n+1)\frac{\pi}{2}\frac{1}{A}$$

Nach genügend langer Zeit ist angenähert: $J = 8\frac{R^2\pi a^2 c}{A} \cdot \frac{(A+R)^2 \frac{9\pi^2}{4A^2}+1}{(A+R)^2\frac{9\pi^2}{4A^2} - \frac{R}{A}}\, e^{-a^2\frac{9\pi^2 t}{4A^2}}$

$$\boxed{\text{für große } \frac{A}{R}: \quad J = 8\frac{R^2\pi a^2 c}{A}\left(1+\frac{4}{9\pi^2}\right) e^{-\frac{9\pi^2}{4A^2}a^2 t}}$$

Falls nun Anzahl der Kondensationskerne die gleiche, so ist: $A^3 c = \text{const}$

Koeffizienten B_n: $\frac{1}{A}\,\frac{\frac{(A+R)^2}{A^2}\frac{(2n+1)^2\pi^2}{4}+1}{\left(\frac{A+R}{A}\right)^2\frac{(2n+1)^2\pi^2}{4}-\frac{R}{A}} \neq \frac{1}{A}$ (sehr angenähert)

also ist Größenordnung der Zeit wo obige Annäherung erlaubt ist: $\frac{9\pi^2 a^2 \tau}{4A^2} = 1$

$$\tau = \frac{4}{9\pi^2}\frac{A^2}{a^2}$$

also wenn $a^2 = D = 10^{-10}$

für $A = 10^{-4}$ ist $\tau = 5$ sek.

$= 1\mu$

$$\int_{t_1}^{\infty} J\,dt = 8\frac{R^2\pi a^2 c}{A}\,\frac{4A^2}{9\pi^2 a^2}\, e^{-\frac{9\pi^2}{4A^2}a^2 t_1} = \frac{8}{9\pi} R^2 A c\, e^{-\frac{9\pi^2 a^2}{4A^2}t_1}$$

also wird dadurch nur ein minimaler Bruchteil kondensiert

$$u = \frac{2cR^2}{r}\sum_{n=0}^{\infty}\frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2-R}\,\frac{1}{\beta_n}\,e^{-D\beta_n^2 t}\sin\beta_n r$$

Verschieden

Zwei Grenzfälle:

I). Gegebene Anzahl von Kondensationskernen, an welchen sich (durch Diffusion) Alles anlagert. Theorie umstehend.

II). Kondensation tritt nur ~~[illegible]~~ relativ selten und jedesmal nur in bestimmten geringen Beträge ein, so dass Alles durch Diffusion wieder ausgeglichen wird.

Dann gilt chem.-kinetisches Gesetz für Reaktion d. zweiten Ordnung

$$\frac{d\varphi}{dt} = -a\varphi^2 \qquad \varphi = \text{Konz. d. unkondensierten Agens}$$

$$\left.\begin{aligned} \frac{1}{\varphi} &= at + \text{const} \\ \frac{1}{\varphi_0} &= \text{const} \end{aligned}\right\} \quad \frac{1}{\varphi} = \frac{1}{\varphi_0} + at$$

$$\varphi = \frac{\varphi_0}{1 + \varphi_0 a t}$$

Dabei muss kondensiertes Produkt unwirksam werden.

III) Falls angenommen wird, dass die Weiter-Anlagerung an einem kondensierten Doppel-Molekül so vor sich geht wie an den Einzel-Bestandteilen, kann (I) auch vielleicht auch zur Theorie der Fällung verwendet werden, wenn Anzahl d. Kond. Kerne = Gesamtzahl d. Teilchen, $A = \frac{1}{2}$ Distanz bis zum nächsten Teilchen und wenn Resultat halbiert wird.

IV) Andere Auffassung: Von einem bestimmten Moment an werden alle Zusammenstöße angekreidet und es wird die Zahl der Teilchen erwogen, welche noch keinen Zusammenstoss erlitten haben. Wie bestimmt sich diese?

In Gasen analoge Aufgabe: $N = N_0 e^{-\frac{s}{\lambda}} = N_0 e^{-\frac{ct}{\lambda}}$ ‖ $\lambda = \frac{1}{\sqrt{2}\pi N \sigma^2}$

Das wäre hier unrichtig, viel zu groß, denn Wahrsch. eines Zusam.stosses ist auf Brown'schen

Zickzackweg viel geringer als wie wenn der Weg ausgebreitet wird.

Also vielleicht so:

$$\overline{\Delta s^2} = 3\overline{\Delta x^2} = 6Dt$$

$$s = \sqrt{6Dt}$$

$$\frac{ds}{dt} = \sqrt{\frac{3D}{2t}}$$

$$dN = -N\frac{ds}{\lambda_0} = -\frac{N}{\lambda_0}\sqrt{\frac{3D}{2t}}\,dt$$

$$\frac{dN}{N} = -\sqrt{\frac{3D}{2t}}\,\frac{dt}{\lambda}$$

$$\lg N = -\frac{\sqrt{6Dt}}{\lambda_0}$$

$$N = N_0\, e^{-\frac{\sqrt{6Dt}}{\lambda_0}} \qquad N = N_0\left[1 - e^{-\frac{\sqrt{6Dt}}{\lambda_0}}\right]$$

~~Das ist aber auch unrichtig denn~~

$$6Dt = \lambda_0^2 = \frac{1}{2\pi^2 N^2 \sigma^4}$$ = Zeit wenn nicht zusammengestoßen mehr bilden $\frac{1}{e}$ der Anfangszahl

$$t = \frac{1}{12\,D\pi^2 N^2\sigma^4} = \frac{\sigma^2}{12D\,\alpha^2\cdot 36.}$$

$\frac{\pi N\sigma^3}{6} = \alpha$ = Volumkonzentration

$$= \frac{1}{3.(12)^2}\,\frac{\sigma^2}{\alpha^2 D}$$

z.B. $\sigma = 38\,\mu\mu = 38.\,10^{-7}$

$D = 10^{-7}$

$\alpha = 10^{-4}$ $\quad\|\quad t = \frac{(38)^2.\,10^{-6}}{3.(12)^2.\,10^{-7}} = 30$ sek.

$1\,cm^3$ wenn gerade undurchsichtig:

$$\frac{\pi N\sigma^2}{4} = 1$$

$$\therefore\ \alpha = \frac{2\sigma}{3}$$

$$t = \frac{1}{3.(12)^2}\,\frac{4}{9D} = \frac{1}{4.\,3^5}\,\frac{1}{D}$$

$$\neq \frac{10^3}{D} = 10^{10}$$

Auch in der Form:

Teilchenabstand δ $\qquad N = \frac{1}{\delta^3}$

$$t = \frac{1}{12\pi^2}\cdot\frac{\delta^6}{D\sigma^4} = \frac{1}{12\pi^2}\cdot\frac{\delta^2}{D}\left(\frac{\delta}{\sigma}\right)^4$$

$$\cancel{\frac{mc^2}{2} = \frac{3RT}{2N} = \frac{\pi}{6}R^3\rho\frac{c^2}{2}} \qquad D = \frac{RT}{N}\,\frac{1}{6\pi R\mu}$$

$$\cancel{c = \sqrt{\frac{RT}{N}\cdot\frac{18}{\pi\rho R^3}}}$$

$$\cancel{\frac{ct}{\sqrt{6Dt}} = \sqrt{t}\sqrt{\frac{RT}{N}\cdot\frac{9}{4\pi\rho R^3}\cdot\frac{N}{RT}\,6\pi R\mu} = \sqrt{t}\sqrt{\frac{\mu}{\rho R^2}\cdot\frac{3}{2}}} \qquad \frac{t\sqrt{D\pi}}{2R\sqrt{t}}$$

$$= \sqrt{t}\,\frac{1}{R}\sqrt{\frac{\mu}{\rho}}$$

$$\sqrt{6Dt}\left[1 + \frac{\sqrt{\pi Dt}}{2R}\right]$$

$$= \left[\frac{Dt}{R}\sqrt{\frac{3\pi}{2}} + \sqrt{6Dt}\right]$$

Kondensations-Aufgabe ist eigentlich direkt zu behandeln:

1). Diffusionsaufgabe

Kugel R, umgeben von Kugelfläche A, erfüllt mit Medium von d. Dichte c

$\int_0^t J(t)\,dt = \varphi_t$ = ~~[illegible]~~ in die Kugel eingeströmtes Quantum

Wahrsch., dass ein in A befindl. Teilchen an die Kugel bis zur Zeit t zum ersten Mal

gestorben sei $= \dfrac{\varphi}{\frac{4}{3}\pi(A^3-R^3)c} = U_t$

Wahrsch. dass es nicht daran gest. sei $= 1-U_t$

Falls N Teilchen drin vorhanden sind: $W = (1-U_t)^N = e^{-NU}$

∴ Wahrsch. dass innerhalb unendlich ausgedehnten Mediums mit Teilchenzahl n pro $1\,cm^3$

bis t kein Teilchen angelagert sei $= \lim\limits_{A=\infty} W \Big|_{N=n\cdot\frac{4\pi}{3}(A^3-R^3)}$ mittl. Abstand $= \delta$ $N = \frac{4\pi}{3}\left(\frac{A}{\delta}\right)^3$

Wahrsch., dass ein (angelagert) Teilchen angelagert sei $= 1-e^{-NU} = \dfrac{\text{Anzahl d. Teilchen welche kondensiert sind}}{\text{Anzahl aller Teilchen}}$

Für Grenzfall langer Zeiten:

$\cancel{\lim e^{-NU} = \lim e^{-\frac{4\pi}{3}\frac{A^3}{\delta^3}}}$

$$U = 1 - \int_t^\infty \frac{8\pi a^2 R^2 c}{\frac{4\pi}{3} c A^3}\,\frac{1}{A}\, e^{-\frac{9\pi^2}{4A^2}a^2 t}\,dt$$

$$= 1 - \frac{8\pi a^2 R^2}{\frac{4\pi}{3}A^4}\,\frac{4A^2}{9a^2\pi^2}\, e^{-\frac{9\pi^2}{4A^2}a^2 t} = 1 - \frac{8}{3\pi^3}\frac{R^2}{A^2}\, e^{-\frac{9\pi^2}{4A^2}a^2 t}$$

Anwendbar nur wenn Exponent eine sehr kleine Grösse

Dann ist näherungsweise: $U =$

$$\boxed{\frac{\partial v}{\partial t} = D\,\Delta v \;\Big|\; v = e^{-D\lambda^2 t}\,u}$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \lambda^2 u = 0$$

$$u = \frac{y}{\sqrt{r}}$$

$$\frac{du}{dr} = \frac{1}{\sqrt{r}}\frac{dy}{dr} - \frac{1}{2\sqrt{r^3}}\,y$$

$$\frac{d^2u}{dr^2} = \frac{1}{\sqrt{r}}\frac{d^2y}{dr^2} - \frac{1}{\sqrt{r^3}}\frac{dy}{dr} + \frac{3}{4\sqrt{r^5}}\,y$$

$$+ \frac{2}{\sqrt{r^3}}\frac{dy}{dr} - \frac{1}{\sqrt{r^5}}\,y + \lambda^2\frac{y}{\sqrt{r}} = 0$$

$$\frac{d^2y}{dr^2} + \frac{1}{r}\frac{dy}{dr} - \frac{1}{4r^2}\,y + \lambda^2 y = 0$$

$$y'' + \frac{1}{r}y' + \left(\lambda^2 - \frac{1}{4r^2}\right)y = 0 \qquad \Big\| \qquad J_m'' + \frac{1}{x}J_m' + \left(1 - \frac{m^2}{x^2}\right)J_m = 0$$

$$y = J_{\frac{1}{2}}(r\sqrt{\lambda}) = \sqrt{\frac{2}{\pi\, r\sqrt{\lambda}}}\,\sin(r\sqrt{\lambda})$$

$$u = \sqrt{\frac{2}{\pi\sqrt{\lambda}}}\;\frac{1}{r}\sin(r\sqrt{\lambda})$$

$r = R \quad u = 0$ also $R\lambda = n\pi$

$$\lambda = \frac{n\pi}{R}$$

$$u = \frac{1}{r}\sum_n B_n\, e^{-D\lambda^2 t}\sin\left(\frac{n\pi r}{R}\right) = \frac{1}{r}\sum_n B_n\, e^{-n^2\frac{D\pi^2 t}{R^2}}\sin\left(\frac{n\pi r}{R}\right)$$

$$B_n = -\frac{2cR}{n\pi}\cos n\pi = (-1)^{n+1}\cdot\frac{2cR}{n\pi}$$

$t = 0 \quad u = c$

$$c\,r = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi r}{R}$$

$$B_n = \frac{2c}{R}\int_0^R r\sin\frac{n\pi r}{R}\,dr$$

$$= \frac{2c}{R}\left[\cancel{\left(\frac{R}{n\pi}\right)^2\sin\frac{n\pi r}{R}} - \frac{R}{n\pi}\,r\cos\frac{n\pi r}{R}\right]_0^R$$

$$\int_0^R \sin^2\frac{n\pi r}{R}\,dr = \frac{1}{2}\int_0^R\left[1 - \cos\frac{2n\pi r}{R}\right]dr = \frac{R}{2}$$

$$\int_0^R \sin\frac{n\pi r}{R}\sin\frac{m\pi r}{R}\,dr = \frac{1}{2}\int_0^R\left[\cos(n-m)\frac{\pi r}{R} - \cos(n+m)\frac{\pi r}{R}\right]dr = \frac{1}{2}\,\frac{\sin(n-m)\frac{\pi r}{R}}{\cdots}\Bigg|_0^R - \frac{\sin}{\cdots}\Bigg|_0^R = 0$$

$$u = \frac{2cR}{\pi r} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} e^{-n^2 \frac{D\pi^2 t}{R^2}} \cdot \sin \frac{n\pi r}{R}$$

$$\left.\frac{\partial u}{\partial r}\right|_{r=R} = -\left.\frac{u}{r}\right|_{R} + \frac{2c}{R} \sum_{n=1}^{\infty} (-1)^{n+1} e^{-\frac{n^2 D\pi^2 t}{R^2}} \underbrace{\cos n\pi}_{(-1)^n}$$

$$= - \frac{2c}{R} \sum_{n=1}^{\infty} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

$$J = -D \cdot 4\pi R^2 \left.\frac{\partial u}{\partial r}\right|_{r=R} = 4\pi R^2 D \, \frac{2c}{R} \sum_{1}^{\infty} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

$$= 8\pi c R D \sum_{n=1}^{\infty} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

Exponent Größenordnung: $D = 10^{-7}$

Sonstig $R = 19 \cdot 10^{-7}$ $\frac{D\pi^2}{R^2} = \frac{10^{-7} \cdot 10}{381 \cdot 10^{-14}} = \frac{10^7}{381} \neq \frac{1}{4} 10^6$

Somit bleibt nach sehr kurzer Zeit nur das erste Glied übrig:

$$\int J \, dt = \frac{8\pi c R^3 D}{D\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{n^2 D\pi^2 t}{R^2}}$$

~~bleibt endlich für $t=\infty$~~

~~unmöglich~~!

Diese ganze Berechnung ~~[illegible]~~ entspricht der Aufgabe: Diffusion einer Kugel (an die äussere Kugelfläche)

Dagegen unsere Aufgabe: $\sum_{n=1}^{\infty} \mathfrak{O}_n \sin\frac{n\pi r}{R} = cr$ für: $R<r<\infty$

Das ist unmöglich

$$u = \frac{1}{r}\sum_n \mathfrak{O}_n e^{-\frac{n^2 D\pi^2 t}{R^2}} \sin\frac{n\pi r}{R} = \frac{v}{r}$$

Dies gibt also im ∞ eine endliche Gesamtströmung!

$$\frac{1}{4\pi D} J_A = -\frac{v}{A} + \frac{\pi A}{R}\sum_n \mathfrak{O}_n e^{--} \cos\frac{n\pi r}{R}$$

Allgemeine Lösung:

$$u = \frac{1}{r}\left[A\sin\lambda r + B\cos\lambda r\right] \qquad r = R \quad u = 0$$

$$= \frac{1}{r}B_\lambda\left[\sin\lambda r - \mathrm{tg}\,\lambda a \cdot \cos\lambda r\right] \qquad r^2\frac{\partial u}{\partial r}\Big|_{r=A} = 0 = r^2\left(-\frac{v}{r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right)$$

$$v - r\frac{\partial v}{\partial r} = 0$$

Vielleicht besser so:

$$u = \frac{v}{r} = \frac{1}{r}\left[M\sin\lambda r + N\cos\lambda r\right]$$

$$\left\{\begin{aligned} &M\sin\lambda R + N\cos\lambda R = 0 \\ &M\sin\lambda A + N\cos\lambda A - A\lambda\left[M\cos\lambda A - N\sin\lambda A\right] = 0 \end{aligned}\right\}$$

$$M\left[\sin\lambda A - \lambda A\cos\lambda A\right] + N\left[\cos\lambda A + \lambda A\sin\lambda A\right] = 0$$

Oder auch:

$$u = \frac{1}{r}C\sin\lambda(r-a) \qquad \mathrm{tg}\,\lambda\overbrace{(r_\infty - R)}^{A} = r_\infty\lambda$$

$$\sin\lambda(r_1 - a) - r\lambda\cos\lambda(A-a) = 0$$

~~$\mathrm{tg}\,\lambda(A-a) = A\lambda$ Gleichung finden~~ $\mathrm{tg}\,\lambda A = (A+R)\lambda$

Bei großem A angenähert: $\lambda A = (2n+1)\frac{\pi}{2}$

Annäherung erste Wurzel:

$$\operatorname{tg} AR = \frac{AR - \frac{(AR)^3}{6}}{1 - \frac{(AR)^2}{2}} = AR\left[1 + \frac{(AR)^2}{2} - \frac{(AR)^2}{6}\right] = AR\left[1 + \frac{(AR)^2}{3}\right] = AR$$

$$\operatorname{tg} \lambda A = A\lambda\left[1 + \frac{(A\lambda)^2}{3}\right] = A\lambda + R\lambda$$

$$\frac{(A\lambda)^2}{3} = R\lambda$$

$$\lambda = \sqrt{\frac{3R}{A^3}} = \frac{1}{A}\sqrt{\frac{3R}{A}}$$

$$B_n = 2\,\frac{(A+R)^2\lambda_n^2 + 1}{A(A+R)^2\lambda_n^2 - R}\int_0^A (r+R)\,c\,\sin\lambda_n r\,dr \qquad (A+R)\lambda_n = \operatorname{tg}(A\lambda_n)$$

$$B_0 = 2\,\frac{\operatorname{tg}^2(A\lambda_0) + 1}{A\operatorname{tg}^2(A\lambda_0) - R}\int \dots = \lim \frac{2}{\underbrace{A\sin^2(A\lambda_0)}_{3R} - R\cos^2(A\lambda_0)}\int \dots$$

$$3R \quad -R$$

$$= \frac{1}{R}\int_0^A (r+R)\,c\,\sin\lambda_0 r\,dr = \frac{1}{R}\,\frac{cR}{\lambda_0} = \frac{c}{\lambda_0} = cA\sqrt{\frac{A}{3R}}$$

$$J = 4\pi DR\left[\underbrace{B_0\lambda_0}_{c}\,e^{-D\lambda_0^2 t} + B_1\lambda_1 e^{-D\lambda_1^2 t} + B_2\lambda_2 e^{-D\lambda_2^2 t} + \dots\right]$$

$$= 4\pi DRc\left[e^{-\frac{D\cdot 3R}{A^3}t} + 2R\sum_{n=1}^{\infty}\underbrace{\frac{(A+R)^2\lambda_n^2 + 1}{A(A+R)^2\lambda_n^2 - R}}_{=\frac{1}{A\sin^2(A\lambda_n) - R\cos^2(A\lambda_n)}}\,e^{-D\lambda_n^2 t}\right]$$

$$J = 4\pi DRc\left[e^{-\frac{3DRt}{A^3}} + 8\pi DRc\sum_{n=1}^{\infty}\left[\frac{1}{A\sin^2(A\lambda_n) - R\cos^2(A\lambda_n)}\,\underbrace{e^{-\left[\frac{(2n+1)\pi}{2}\right]^2\frac{Dt}{A^2}}}\right]\right.$$

$\frac{4\pi A^3}{3}n = 1$ falls für jedes Teilchen Kugelraum

$$= \underbrace{\frac{4\pi A^3}{3}c}\; e^{-4\pi nDRt}$$

falls dann zuerst lim $A = \infty$ so $\sum \frac{\pi}{A} = \infty$

$$\int_t^{\infty} J\,dt = \frac{4\pi A^3}{3}c\; e^{-\frac{3DRt}{A^3}} + \sum \cdots$$

falls dagegen zuerst lim $n = \infty$ so ist die $\frac{1}{A}\left[\frac{\vartheta}{3}\Theta - \vartheta\Theta\right]_{\vartheta=1}$

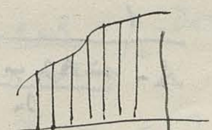
$$v = 2cR\sum_{n=}^{\infty}\frac{1}{A\sin^2\lambda_n A - R\cos^2\lambda_n A}\;\frac{1}{\lambda_n}\;e^{-D\lambda_n^2 t}\;\sin\lambda_n r$$

$$= 2cR\sum\frac{1}{\underbrace{A\lambda\sin^2 A\lambda - \cos^2\lambda A\left[\operatorname{tg}A\lambda - A\lambda\right]}}\;e^{--}$$

$$\frac{1}{A\lambda_n\cos 2A\lambda_n - \frac{1}{2}\sin 2A\lambda_n} = \frac{2}{2A\lambda_n\cos 2A\lambda_n - \sin 2A\lambda_n}$$

$$v = 2cR\left\{\frac{1}{2R}\,\frac{1}{\lambda_0}\,e^{-D\lambda_0^2 t}\sin\lambda_0 r + \underbrace{\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n+1}\,e^{-D\frac{(2n+1)^2\pi^2}{4}\frac{t}{A^2}}\sin\frac{(2n+1)\pi}{2}\frac{r}{A}}\right\}$$

$$\frac{(2n+1)\pi}{2A} = x$$



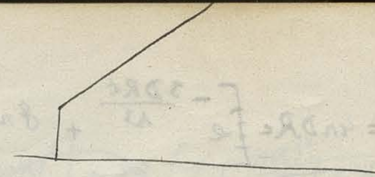
$$= \underbrace{\frac{1}{A}\sum_{\Delta x = \frac{\pi}{A}}\frac{1}{x}\,e^{-Dx^2t}\,\sin rx}$$

$$\Delta x = \frac{\pi}{A}$$

$$= \frac{1}{\pi}\int_0^{\infty} e^{-Dx^2t}\,\frac{\sin rx}{x}\,dx$$

$$u = \frac{2cR}{\pi(r+a)}\int_0^{\infty} e^{-Dx^2t}\,\frac{\sin rx}{x}\,dx + e^{-D\lambda_0^2 t}\,c\,\frac{\sin\lambda_0 r}{\lambda_0 r}$$

$$u = \frac{2cR}{\pi r}\int_0^{\infty} e^{-Dx^2t}\,\frac{\sin(r-R)x}{x}\,dx$$



$$\Phi(r) = \sum_0 D_n \sin\beta_n r = c(R+r)$$

$$= 2\sum \frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R} \int_0^A \Phi(r)\sin\beta_n r\,dr \cdot \sin\beta_n r$$

$$\tan(A\beta_n) = (A+R)\beta_n$$

$$c(R+r) = 2cR \sum_{n=1}^{\infty} \frac{(A+R)^2\beta_n^2+1}{A(A+R)^2\beta_n^2 - R} \cdot \frac{\sin\beta_n r}{\beta_n}$$

$$\sin A\beta_n = \cancel{(A+R)\beta_n \cos A\beta_n}$$

$$\cancel{(\sin V\beta_n \cos VR - \cos V\beta \sin VR)}$$

$$= V\beta$$

$$\cancel{r+R = y}$$

$$\cancel{A+R = V}$$

$$\cancel{cy = 2cR \sum \frac{V^2\beta_n^2+1}{AV^2\beta_n^2 - R}\frac{1}{\beta_n}\left[\sin\beta_n y \cos\beta_n R - \cos\beta_n y \sin\beta_n R\right]}$$

$$= 2cR\sum_{n=1}^{\infty} \underbrace{\frac{1}{A\sin^2 A\beta_n - R\cos^2 A\beta_n}}\frac{\sin\beta_n r}{\beta_n}$$

$$\frac{1}{A-(A+R)\cos^2 A\beta_n} = \frac{1}{A - \frac{\sin A\beta_n \cos A\beta_n}{\beta_n}}$$

$$c(R+r) = \cancel{2cR\sum \frac{\sin\beta_n r}{A\beta_n}} \qquad \boxed{\lim_{A=\infty}\beta_1 = \frac{1}{A}\sqrt{\frac{3R}{A}}}$$

$$= \lim_{A=\infty} 2cR\left\{\frac{\sin\beta_1 r}{A\beta_1\sin^2 A\beta_1 - R\beta_1\cos^2 A\beta_1} + \sum_{n=2}\right.$$

$$= \lim \underbrace{\frac{\sin\beta_1 r}{\beta_1}c}_{=cr} + 2cR\left[\lim \underbrace{\sum_{n=2}^{\infty}}\right] = \overbrace{\frac{1}{\pi}\int_0^\infty \frac{\sin rx}{x}\,dx}^{\frac{\pi}{2}}$$

$$= c(r+R)$$

$$v = c r e^{-\beta_1^2 D t} + \frac{2cR}{\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx$$

$$u = \frac{c}{r+R}\left[r + \frac{2R}{\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx\right] = c\left[1 - \frac{R}{R+r}\left(1 \mp \frac{2}{\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx\right)\right]$$

$$\frac{\partial u}{\partial t} = -\frac{2cR}{\pi(r+R)}\, D \underbrace{\int_0^\infty e^{-Dx^2 t}\, x \sin rx\,dx}$$

$$\cancel{-e^{-x^2 D t}\frac{\sin rx}{2Dt}}\Big|_0^\infty + \frac{r}{2Dt}\int e^{-Dx^2 t}\cos rx\,dx$$

$$= -\frac{cRr}{\pi(r+R)\,t}\int_0^\infty e^{-Dx^2 t}\cos rx\,dx$$

$$\frac{\partial u}{\partial r} = +\frac{cR}{(R+r)^2} - \frac{2cR}{(r+R)^2\pi}\int_0^\infty e^{-Dx^2 t}\frac{\sin rx}{x}\,dx + \frac{2cR}{\pi(r+R)}\int_0^\infty e^{-Dx^2 t}\cos rx\,dx$$

$$\frac{\partial^2 u}{\partial r^2} = -\frac{2cR}{(R+r)^3} + \frac{4cR}{(r+R)^3\pi}\int e^{-Dx^2 t}\frac{\sin rx}{x}\,dx - \frac{4cR}{(r+R)^2\pi}\int e^{-Dx^2 t}\cos rx\,dx$$

$$\mp \frac{2cR}{\pi(r+R)}\int e^{-Dx^2 t}\, x \sin rx\,dx$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - D\frac{\partial u}{\partial t} = \frac{cR}{(R+r)^2}\left[\frac{2}{r} - \frac{2}{R+r}\right] + \frac{2cR}{(r+R)^3\pi}\int e^{-Dx^2 t}\frac{\sin rx}{x}\left[2 - 2\frac{r+R}{r+R}\right]dx$$

$$-\frac{2cR}{(r+R)^2\pi}\int e^{-Dx^2 t}\cos rx\,[2-2] = 0$$ stimmt

$$\cancel{= -\frac{cR}{(R+r)^3} + \frac{2cR}{(R+r)^3\pi}\int e^{-Dx^2 t}\frac{\sin rx}{x}\,dx - \frac{2cR}{(r+R)^2\pi}\underbrace{\int e^{-Dx^2 t}\cos rx\,dx}_{\frac{1}{2}\sqrt{\frac{\pi}{Dt}}\,e^{-\frac{r^2}{4Dt}}}}$$

$$\int_0^\infty e^{-p\alpha^2} \cos q\alpha \, d\alpha = \frac{1}{2}\sqrt{\frac{\pi}{p}}\, e^{-\frac{q^2}{4p}}$$

$$\int_0^\infty e^{-p\alpha^2} \frac{\sin q\alpha}{\alpha}\, d\alpha = \frac{1}{2}\sqrt{\frac{\pi}{p}} \int_0^q e^{-\frac{q^2}{4p}}\, dq = \frac{\sqrt{\pi}}{2} \int_0^{\frac{q}{2\sqrt{p}}} e^{-z^2}\, dz$$

$$\boxed{u = \frac{c}{r+R}\left[r + \frac{2R}{\sqrt{\pi}} \int_0^{\frac{r}{2\sqrt{Dt}}} e^{-z^2}\, dz\right]}$$

Bei Einführung der vom Mittelpunkt aus gerechneten r:

$$u = \frac{c}{r}\left[r - R + \frac{2R}{\sqrt{\pi}} \int_0^{\frac{r-R}{2\sqrt{Dt}}} e^{-z^2}\, dz\right] = c\left[1 - \frac{R}{r} + \frac{2R}{\sqrt{\pi}\, r} \int_0^{\frac{r-R}{2\sqrt{Dt}}} e^{-z^2}\, dz\right]$$

$$\frac{\partial u}{\partial r} = c\left[\frac{R}{r^2} + \frac{2R}{\sqrt{\pi}} \frac{1}{2\sqrt{Dt}} e^{-\left(\frac{r-R}{2\sqrt{Dt}}\right)^2} - \frac{2R}{r^2\sqrt{\pi}} \int_0^{\frac{r-R}{2\sqrt{Dt}}} e^{-z^2}\, dz\right]$$

$$J_R = 4\pi D R^2 \left.\frac{\partial u}{\partial r}\right|_R = 4\pi D R^2 c \left[\frac{1}{R} + \frac{R}{\sqrt{\pi D t}}\right] = 4\pi D c R\left[1 + \frac{R}{\sqrt{\pi D t}}\right]$$

$$Q = \int_0^t J\, dt = 4\pi D c R\left[t + \frac{2R\sqrt{t}}{\sqrt{\pi D}}\right]$$

Das gilt aber

$$J_A = 4\pi D R c\left[1 + \frac{A^2}{\sqrt{\pi D t}}\, e^{-\frac{(A-R)^2}{4Dt}} - \frac{2}{\sqrt{\pi}} \underbrace{\int_0^{\frac{A-R}{2\sqrt{Dt}}} e^{-z^2}\, dz}_{\frac{\sqrt{\pi}}{2}}\right]$$

~~also entspricht es einer fortwährenden Zufuhr in jenem Betrage von aussen so dass $\lim_{r=\infty} u$ = konst erhalten wird.~~

$$\lim_{A=\infty} J_A = 0$$

also ist wirklich die Zufuhr von Aussen $= 0$

Zeitdauer von der stationären Zustand merklich erreicht ist:

$$\sqrt{t} = \frac{2R}{\sqrt{\pi D}} \qquad t = \frac{4R^2}{\pi D} = \frac{4R^3 \cdot 6\pi\mu}{\pi \frac{HT}{N}} =$$

von dunklen Entstehung wie Stationarisierung innerhalb Kugel

also ganz verschwindend

$$U_t = \frac{3}{4\pi}\frac{1}{A^3_c}\, 4\pi D c R t = \frac{3DRt}{A^3}$$

Wahrsch. dass noch kein Teilchen angelagert sei:

$$\lim_{A=\infty} W\Big|_{N=n\frac{4\pi}{3}A^3} = \lim e^{-NU_t} = \lim e^{-\frac{4\pi}{3}n\, 3DRt} = e^{-4\pi n DRt}$$

Maßzahl für Anlagerungszeit: $\tau_1 = \frac{1}{4\pi n DR} = \frac{\delta^3}{4\pi DR}$ $\Bigg\|$ $\frac{(38)^2 \cdot 10^{-12}}{4\pi \cdot 10^{-7}\, 38 \cdot 10^{-6}} = \frac{38 \cdot 10^{-6}}{4\pi \cdot 10^{-7}} = 2$ sek.

Im ersten Stadium dagegen, für äusserst kurze t:

$$\lim W = \cancel{e^{-\frac{4\pi}{3}n}} \lim_{t=0} e^{-4\pi n DR\left[t + \frac{2R\sqrt{t}}{\sqrt{\pi D}}\right]} = e^{-8\pi n\sqrt{D}R^2\sqrt{t}} = e^{-8nR^2\sqrt{\pi D t}}$$

Anlagerungszeit wäre in diesem Falle: $\tau_2 = \frac{\delta^6}{64\pi D R^4}$ also fast dieselbe Zeit wie nach mittl. Weglänge-„Methode"

$= \tau_1 \cdot \frac{\delta^3}{16 R^3}$

Was wirklich zustande kommt, hängt ab von:

$\frac{\tau_1}{\tau_2} = 16\frac{R^3}{\delta^3}$ also in Praxi immer $\tau_1 \ll \tau_2$ falls R = Teilchenradius

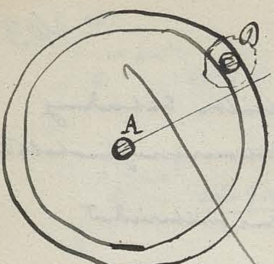
also überwiegt der Prozess (1)

$n = 10^{10}$

$R = 10^{-9}$

$\frac{\cancel{\ldots}}{\sqrt{\pi D t}} = \frac{4R\sqrt{nR}}{} = 4\sqrt{\left(\frac{R}{\delta}\right)^3}$! also ist das zweite Glied ganz verschwindend

$4\pi n DRt\left[1 + 10^{-?}\right]$



Wahrsch. dass ein Teilchen B zur Zeit t in Entf. $r - dr$ sich befinde
(n Teilchen pro cm³ n. d. gestellt)

$$W_1 = 4\pi r^2 dr \cdot n$$

Wahrsch., dass das Teilchen A ~~...~~ in der Zeit t (bis in Entf. r gekommen sei: in der sich befinde

$$W_2 = \frac{1}{(\sqrt{\pi D t})^3} e^{-\frac{r^2}{4Dt}} \int dx\,dy\,dz = \frac{4\pi r^2}{(\sqrt{\pi D t})^3} e^{-\frac{r^2}{4Dt}} dr$$

~~Wahrsch. eines Zusammenst. in der Kugelschale~~

Die Bewegung kann ersetzt werden durch Verschiebungen im Betrage von δ im Intervalle τ (in X, Y Z)

$$D = \frac{\delta^2}{2\tau}$$

~~Wahrsch. dass Teilchen A sich zur Zeit~~

Somit „Teilchendichte" A zur Zeit $t = \frac{1}{\sqrt{\pi D t}^3} e^{-\frac{r^2}{4Dt}}$

Also Wahrsch. eines Zusammenstosses zwischen $t \ldots t+\tau$

$$\int_{r=0}^{\infty} 4\pi r^2 dr\, n \frac{1}{(\sqrt{\pi D t})^3} e^{-\frac{r^2}{4Dt}} \Big| .4R^2\pi.\delta.3 = 12R^2\pi n\delta = \varepsilon$$

Wahrsch. dass zwischen $t \ldots t+\tau$ kein Zusammenstoss stattfinde: $1 - 12R^2\pi n\delta$

" " " 0 t $= (1-\varepsilon)^{\frac{t}{\tau}} = e^{-\frac{\varepsilon t}{\tau}} = e^{-12R^2\pi\frac{n\delta t}{\tau}}$

Wahrsch. dass der erste Zusam- zwischen $t - t+\tau = \varepsilon e^{-\varepsilon\frac{t}{\tau}}$

.. .. .. $t \ldots t+dt = \frac{\varepsilon}{\tau} e^{-\varepsilon\frac{t}{\tau}} dt$

" " " " über $t \ldots \infty$: $\int_t^{\infty} = e^{-\frac{\varepsilon t}{\tau}} =$

Dabei Grundfehler: Voraussetzung d. Unabhängigkeit d. Wahrsch. d. Z.

$$\Big\| \quad e^{-\frac{12R^2\pi D t n}{\delta}}$$

Richtiger vielleicht so:

Falls O in $r \ldots dr$: Wahrsch., dass zwischen $0 \ldots t$ kein Zusammenstoss stattgefunden habe:

$$= e^{-\frac{t}{\tau}\cdot\frac{12R^2 n\delta}{(\sqrt{\pi D t})^3}}\, e^{-\frac{r^2}{4Dt}}$$

Das Allgemeine, wenn Zeit von Bombardement:

$$\left.\frac{dN}{dt}\right|_{t=0} = -N_0^2\beta$$

Verschiebung um $2R$:

$$(2R)^2 = 6D\tau$$

$$\frac{ds}{dt} = \frac{2R}{\tau} = \frac{6D}{2R} = \frac{3D}{R}$$

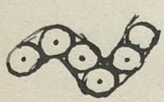
$$dN = -N\frac{ds}{\lambda_0} = -\frac{N}{\lambda_0}\frac{3D}{R}dt = -N\frac{3D}{R}n \ldots dt$$

dies konstant betrachtet!?

$$\lg N = -\frac{3D}{R\lambda_0}t$$

$$N = N_0 e^{-\frac{3D}{R\lambda_0}t} = N_0 e^{-\frac{3D}{R}t\cdot\sqrt{2}\pi n \sigma^2} = N_0 e^{-n\beta t}$$

$$= N_0 e^{-12\sqrt{2}\pi n R t D}$$



Denn: pro Zeiteinheit durchstrichenes ~~Zurückgelegtes~~ Volumen wird der Grössenordnung nach entsprechen der Grösse $\sigma^2\pi\cdot\frac{2R}{\tau}$

Anzahl d. Kondensationszeit $n\beta t = 1$

Tatsächlich stimmt das Resultat der Form nach überein mit dem früher abgeleiteten

---

Wenn man die allmähliche Abnahme der Zahl der noch freien O und damit Zusammenstösse in Betracht zieht:

$$dN = -\boxed{\frac{3D}{R}\sqrt{2}\,\pi\sigma^2}^{\,=\beta} N^2 dt$$

Anzahl der Kondensationszeit: $n\beta t = 1$

Allerdings ist dann deren Einfluss übertrieben, denn die Abnahme hat vor Allem in dem schon durchstrichenen Raum stattgefunden, aber ~~nicht~~ weniger in dem noch zu durchstreichenden

$$\frac{1}{N} = \beta t + \text{konst}$$

$$\frac{1}{N} = \frac{1}{N_0} + \beta t$$

$$N_0 = N + N N_0\beta t$$

$$N = \frac{N_0}{1 + N_0\beta t}$$

$$\left.\frac{dN}{dt}\right|_{t=0} = -\beta N_0^2$$

Wenn wir zur Lösung mit begrenztem Kugelraum zurückkehren und annehmen:

1). Kugelraum so gross, dass er dem für ein Teilchenpaar entfallenden Raum entspricht

$$\therefore \quad \frac{4\pi}{3}A^3 \cdot 2n = 1$$

2). Da der Aggregationskern nicht unbeweglich ist, sondern sich ebenso bewegt, wie die angreifenden Teilchen selbst, kann dies durch Vergrösserung des D berücksichtigt werden

Wenn man 2D setzt, so bleibt die früheren Formel unverändert

Einfluss des ersten Stadiums ; Unterschied, welcher durch Vernachlässigung desselben hervorgebracht wird β_2

$$W_{1+2} - W_1 = e^{-4\pi n R D t}\left[e^{\overbrace{-8\sqrt{\pi n}\sqrt{D}\,R^2\sqrt{t}}^{\beta_1}} - 1\right]$$

Falls $\beta_1 \ll \beta_2$: $\mp -8\sqrt{\pi D}\, n R^2 \sqrt{t}$ $\qquad z = x^2, \; dz = 2x\,dx$

$$\int_0^\infty \sqrt{t}\, e^{-\alpha t}\,dt = \int_0^\infty \frac{1}{\alpha^{3/2}}\sqrt{z}\,e^{-z}\,dz = \frac{2}{\alpha^{3/2}}\underbrace{\int_0^\infty x^2 e^{-x^2}dx}_{\frac{\sqrt{\pi}}{4}} = \frac{\sqrt{\pi}}{2\alpha^{3/2}}$$

$$\Delta W = \frac{\sqrt{\pi}}{2}\,\frac{8\sqrt{\pi D}\, n R^2}{(4\pi n R D)^{3/2}} = \cancel{\frac{1}{16\pi^2 n^2 D^{3/2} R}} = \frac{\sqrt{R}}{2\sqrt{\pi}\,D\sqrt{n}} \qquad n = \frac{1}{\delta^3} = (5\cdot 5)^{\frac{1}{2}}10^{-12}$$

z.B. $\dfrac{\sqrt{3.8\cdot 10^{-6}}\cdot 3.8\cdot 10^{-6}}{2\sqrt{\pi}\;10^{-7}} = 2\cdot 10^{-2} = 2\%$ wird aber in verdünnterer Lösung recht merklich sein können

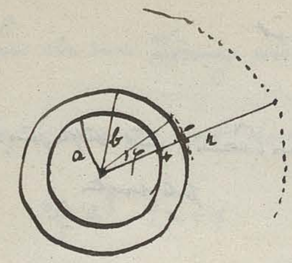
$$\Delta W_{\text{für Halbzeit } t_1} = \frac{8\sqrt{\pi D}\; nR^2}{2\sqrt{\pi}\sqrt{nRD}} = 4\sqrt{nR^3} = 4\sqrt{\left(\frac{R}{\delta}\right)^3}$$

$= 4\sqrt{\text{Volumkonzentration}}$

~~Nachdem~~ Wenn alle Teilchen sich zu Teilchenpaaren vereinigt haben ist

1). $n' = \frac{n}{2}$

2). $D' \neq \frac{D}{2}$ aber so dass $DR = \text{const}$ also würde weiterer Prozess mit Hülfe jener Sache fortschreiten



Wenn ein + Elektron auf innerer Kugelfläche, so wird das entsprechende −e auf der äusseren ~~mit~~ nach d. Wahrscheinlichkeitsgesetz

$$W(\varphi)\,d\varphi = A\, e^{-\frac{N}{RT}\chi} \sin\varphi\, d\varphi$$

$$\chi = \frac{e^2}{K\sqrt{a^2+b^2-2ab\cos\varphi}} = \frac{e^2}{Kr}$$

$$2ab\sin\varphi\, d\varphi = 2r\,dr$$

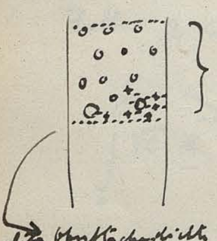
$$\int_0^\pi \sin\varphi\, e^{-\frac{N}{RT}\chi}\, d\varphi = \int_{r=b-a}^{a+b} \frac{e^{-\frac{\varepsilon}{r}}\, r\,dr}{ab}$$

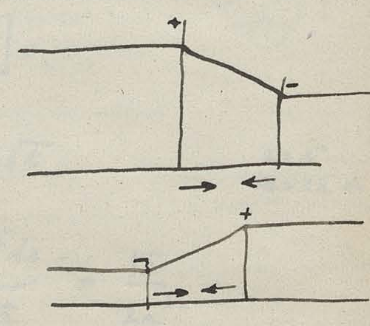
$$\frac{\varepsilon}{r} = u$$

$$-\frac{\varepsilon}{r^2}\,dr = du = -\frac{u^2\,dr}{\varepsilon}$$

$$dr = -\varepsilon\frac{du}{u^2}$$

$$= -\int_{\frac{\varepsilon}{b-a}}^{\frac{\varepsilon}{b+a}} \frac{\varepsilon^2 e^{-u}\, u^{-3}\, du}{ab}$$





Gewicht pro 1 cm³

$$\frac{\Delta E}{\Delta x} = \underbrace{\varphi\,\frac{\delta'-\delta_0}{\delta'}\, g}_{\frac{\Delta p}{\Delta x}}\; \frac{K(\varphi_1-\varphi_0)}{4\pi}\,\frac{6}{\mu}$$

Oberflächendichte $= -\frac{K}{4\pi}\frac{\Delta E}{\Delta x}$

(Kataphoret. Geschw. infolge Einzelkraft: $u = \frac{K(\varphi_1-\varphi_0)}{4\pi\mu}\frac{\Delta E}{\Delta x} = \varphi\,\frac{\delta'-\delta_0}{\delta'}\left(\frac{K(\varphi_1-\varphi_0)}{4\pi\mu}\right)^2 6\, g$

Geschw. infolge Stokes: $v = \frac{2}{9}\,\frac{a^2\,(\delta'-\delta_0)}{\mu}\, g$

$$u = \frac{1}{4}\,\frac{1.4}{2.6}\left(\frac{1}{300\cdot 4\pi\cdot 0.02}\right)^2 \frac{10^7}{9\cdot 10^{11}}\cdot g = \frac{0.7}{1.3\cdot 4\cdot 10\cdot 36}\cdot\frac{10^{-4}}{9}\, g = \frac{10^{-7}}{2\cdot(3.6)^2} = \cancel{\tfrac{1}{2}10^{-8}} = \frac{1}{2}\cdot 10^{-5}$$

$v = \frac{2}{9}\cdot\frac{10^{-8}\cdot 1.4}{0.02}\cdot 980 = 1.5\cdot 10^{-4}$ während Stock findet für die schärfste Emuls.:

(?) $v = \frac{5}{60}\frac{cm}{sek}$ bis $0.05\,\frac{cm}{sek}$!!!

elektr. Oberflächendichte $\frac{K}{4\pi}\ \frac{K(\varphi_1-\varphi_0)}{4\pi\mu}\ \varphi.\frac{\delta'\,\delta_0}{\delta'}\,6 = \beta.\frac{4}{300.4\pi.0{\cdot}02}.\frac{1}{4}\ \frac{1}{7}\ \frac{10^7}{9.10^{11}}$

$= \frac{10^{-4}}{36.\pi.6.} = \frac{1}{7}\ 10^{-6}$ (elektrost. abs.)

$$P = \frac{16\,Dt}{a\pi}\int_{\frac{a}{2\sqrt{Dt}}}^{\infty} e^{-R^2} R\,dR \int_0^{\frac{a}{2\sqrt{Dt}}} e^{-r^2} r\,dr \int_{-1}^{+1} \frac{e^{2Rr\xi}}{\sqrt{1-\xi^2}}\,d\xi$$

$R = \beta + u$

$r = \beta - v$

$$= \frac{16\,Dt}{a\pi}\int_0^{\infty} e^{-(\beta+u)^2}(\beta+u)\,du \int_\beta^0 e^{-(\beta-v)^2}(\beta-v)\,dv \int_{-1}^{+1} \frac{e^{2(\beta^2+(u-v)\beta - uv)\xi}}{\sqrt{1-\xi^2}}\,d\xi$$

$$\int_0^\infty e^{-(\beta+u)^2}(\beta+u)\,e^{+2u(\beta-v\xi)}\,du = e^{-\beta^2}\int_0^\infty e^{-u^2-2uv\xi}(\beta+u)\,du$$

$$= e^{-\beta^2+v^2\xi^2}\int_0^\infty e^{-(u+v\xi)^2}(\beta+u)\,du$$

$$\int_0^\infty e^{-(u+v\xi)^2}(\beta+u)\,du = \int_{v\xi}^\infty e^{-z^2}(z+\beta-v\xi)\,dz = \frac{e^{-z^2}}{2}\Big|^{v\xi} + (\beta-v\xi)\int_{v\xi}^\infty e^{-z^2}\,dz$$

$$= \frac{e^{-v^2\xi^2}}{2} + (\beta-v\xi)\left[\frac{\sqrt{\pi}}{2} - \int_0^{v\xi}\left[1-z^2+\frac{z^4}{2}-\frac{z^6}{6}\cdots\right]dz\right]$$

$$= \frac{e^{-\beta^2}}{2} + (\beta-v\xi)\,e^{-\beta^2+v^2\xi^2}\left[\frac{\sqrt{\pi}}{2} - v\xi + \frac{(v\xi)^3}{1!\,3} - \frac{(v\xi)^5}{2!\,5} + \frac{(v\xi)^7}{3!\,7} - \cdots\right]$$

$$\frac{\partial}{\partial t}(2\pi r u) = +D\frac{\partial}{\partial r}\left(2\pi r\frac{\partial u}{\partial r}\right)$$

$$\frac{\partial}{\partial t}(ru) = +D\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)$$

$$u=\sum v_n e^{-\mu_n t}$$

$$D\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right)+\mu v=0$$

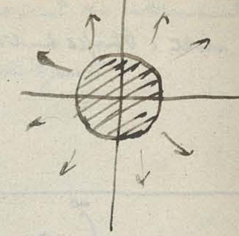
$$\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\mu}{D}v=0$$

$$\int_0^\infty r u\,dr = \text{const}$$

$$\int_0^\infty r\frac{\partial u}{\partial t}\,dr=0$$

$$\sum e^{-\mu_n t}\mu_n\int_0^\infty r\,v_n\,dr=0$$

$$\frac{\partial^2 v}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial v}{\partial \rho}+\frac{a^2\mu}{D}v=0$$



$n>0$ $\quad\displaystyle\int_0^\infty r\,v_n\,dr=0$

$$\int_0^\infty r\,e^{-p^2r^2}J_\alpha(mr)\,dr=\frac{1}{2p^2}e^{-\frac{m^2}{4p^2}}$$

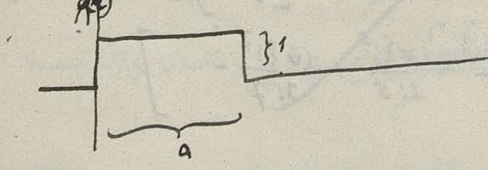
Matthews p. 77

$$\int_0^\infty r\,J_0(r)\,dr = 0 \quad (?)(?)$$

$t=0$:

$$\sum A_n J_0\left(r\sqrt{\frac{\mu_n}{D}}\right)=1 \qquad r<a$$

$$\sum A_n J_0\left(r\sqrt{\frac{\mu_n}{D}}\right)=0 \qquad r>a$$



$$\int_0^\infty d\alpha\int_0^a \lambda\rho\,\varphi(\rho)\,J_0(\lambda\rho)\,J_0(\lambda r)\,d\rho=\varphi(r) \qquad 0<r<a$$
$$=0 \qquad r>a$$
$$r<0$$

$$\int_0^\infty d\alpha\int_0^a \lambda\rho\,J_0(\lambda\rho)\,J_0(\lambda r)\,d\rho=1 \quad \| \quad 0<r<a$$
$$=0 \quad \| \quad r>a$$

$$\cancel{\frac{\partial^2 J_0}{\partial r^2}+\frac{\lambda}{r}\frac{\partial J_0}{\partial r}+\frac{\mu}{D}J_0}$$

$$\frac{\partial^2 J_0(\lambda r)}{\partial r^2}+\frac{1}{r}\frac{\partial J_0}{\partial r}+\lambda^2 J_0=0$$

$$\lambda=\sqrt{\frac{\mu}{D}}$$

$$u = \int_0^\infty e^{-D\lambda^2 t} d\lambda \int_0^a \lambda J_0(\lambda\rho) J_0(\lambda r) \rho \, d\rho$$

$$I_n(t) = i^{-n} J_n(it)$$

$$\int_0^\infty r e^{-p^2 r^2} J_n(\lambda r) J_n(\mu r) \, dr = \frac{1}{2p^2} e^{-\frac{\lambda^2+\mu^2}{4p^2}} I_n\left(\frac{\lambda\mu}{2p^2}\right)$$

Gray & Mathews p. 78

$$\cancel{u = \sum e^{-\gamma_n t} J_0\left(r\sqrt{\frac{\gamma_n}{D}}\right)}$$

$$\alpha \frac{J'(\alpha)}{J(\alpha)} = \beta \frac{J'(\beta)}{J(\beta)}$$

$t=0$

$$\cancel{\int r u J_0}$$

$$\cancel{\frac{1}{2}\left[J(\alpha)^2 + J'(\alpha)^2\right] =}$$

$$\cancel{\frac{1}{2} J(\alpha_n)^2 + J'(\alpha_n)^2 = \int_0^1 J(\alpha_n x) \, dx}$$

$$\frac{\partial^2 v}{\partial\left(\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}}\right)^2} + \frac{1}{\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}}} \frac{\partial v}{\partial\left(\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}}\right)} + v = 0$$

$$\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}} = x'$$

$$\cancel{\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho}}$$

$$\frac{\partial^2 v}{\partial x'^2} + \frac{1}{x'}\frac{\partial v}{\partial x'} + v = 0$$

$$\frac{r}{a} = x$$

$$\sqrt{\frac{\gamma a^2}{D}} = \xi$$

$$v = J_0(x') = J_0\left(\frac{r}{a}\sqrt{\frac{\gamma a^2}{D}}\right) = J_0(x\xi)$$

$$u = v e^{-\gamma t} = \sum v_n e^{-\frac{D\xi_n^2 t}{a^2}}$$

$t=0$

$f = 1, \quad 0 < x < 1$

$f = 0 \quad 1 < x < \infty$

$$f(x) = \int_0^\infty J(\xi x) \, \xi \, d\xi \underbrace{\int_0^\infty f(\lambda) J(\xi\lambda) \lambda \, d\lambda}$$

$$\int_0^1 \lambda J(\xi\lambda) \, d\lambda = \frac{J_1(\xi)}{\xi}$$

$$f(x) = \int_0^\infty \left[J_1(\xi) J(\xi x) \, d\xi \cdot e^{-\frac{D\xi^2 t}{a^2}}\right]$$

$$\frac{\partial f}{\partial r} = \frac{1}{a}\frac{\partial f}{\partial x}\Big| = \frac{1}{a}\int_0^\infty J_1(\xi)\, J_1'(x\xi)\, \xi\, d\xi \,.\, e^{-\frac{D\xi^2 t}{a^2}}$$

$$P = \frac{D}{a^2\pi}\int_t^\infty 2\pi r \frac{\partial f}{\partial r}\, dt = \frac{D}{\cancel{a^2\pi}}\,\frac{\cancel{2a\pi}}{\cancel{a}}\int_t^\infty dt \int_0^\infty J_1(\xi)\, J_1'(\xi)\, \xi\, d\xi\; e^{-\frac{D\xi^2 t}{a^2}}$$

$$= 2\int_0^\infty \frac{J_1(\xi)\, J_1'(\xi)\, d\xi}{\xi} . e^{-\frac{D\xi^2 t}{a^2}} = \underbrace{2\int_0^\infty [J_1(\xi)]^2 \frac{d\xi}{\xi}}\, . e^{-\frac{D\xi^2 t}{a^2}}$$

$$J_0 = \frac{dJ_1}{dx} + \frac{J_1}{x}$$

$t=0$

$$= 2\int_0^\infty J_1(\xi)\, J_0(\xi)\, d\xi - 2\int_0^\infty J_1 \frac{dJ_1}{d\xi} . d\xi$$

$$\downarrow$$

$$-J_0'(\xi)$$

$$= -[J_0(\xi)]^2 - [J_1(\xi)]^2 \Big|_0^\infty = 1$$

$$\underbrace{2\int_0^\infty J_0(\xi)\, J_1(\xi)\, e^{-\frac{D\xi^2 t}{a^2}} d\xi} - \underbrace{2\int_0^\infty J_1 \frac{dJ_1}{d\xi}\, e^{-\frac{D\xi^2 t}{a^2}} d\xi}$$

$$\underbrace{-[J_0(\xi)]^2 e^{-\frac{D\xi^2 t}{a^2}}\Big|_0^\infty}_{=1} - \frac{2Dt}{a^2}\int_0^\infty J_0^2\, \xi\, e^{-\frac{D\xi^2 t}{a^2}} d\xi \;=\; \cancel{-J_1^2\, e^{-\frac{D\xi^2 t}{a^2}}\Big|_0^\infty} - \int_0^\infty \frac{2Dt}{a^2} J_1^2\, \xi\, e^{-\frac{D\xi^2 t}{a^2}}$$

$$P = \frac{2Dt}{a^2}\left\{\underbrace{\int_0^\infty J_0^2\, \xi\, e^{-\frac{Dt\xi^2}{a^2}} d\xi} + \int_0^\infty J_1^2\, \xi\, e^{-\frac{Dt\xi^2}{a^2}} d\xi\right\}$$

$$\frac{a^2}{2Dt}\, e^{-\frac{a^2}{2Dt}}\left[I_0\left(\frac{a^2}{2Dt}\right) + I_1\left(\frac{a^2}{2Dt}\right)\right] \qquad = e^{-\frac{a^2}{2Dt}}\left[I_0\left(\frac{a^2}{2Dt}\right) + I_1\left(\frac{a^2}{2Dt}\right)\right]$$

$$I_0(\beta) = 1 + \frac{\beta^2}{(2)^2} + \frac{\beta^4}{(2.4)^2} + \frac{\beta^6}{(2.4.6)^2}$$

$$I_1(\beta) = \frac{\beta}{2} + \frac{\beta^3}{2.2.4} + \frac{\beta^5}{2.2.4.4.6} \cdots \quad = I_0'(\beta)$$

$$= 1 - \frac{a^2}{4Dt} + \frac{a^4}{16(Dt)^2}$$

$$\left(1-\beta+\frac{\beta^2}{2}\right)\left(1+\frac{\beta}{2}+\frac{\beta^2}{4}\right) = 1 - \frac{\beta}{2} + \cancel{\frac{\beta^2}{2}} - \cancel{\frac{\beta^2}{2}} + \frac{\beta^2}{4}$$

$$\frac{1}{6} - \frac{1}{16} = \frac{8-3}{48}$$

$$\left(1-\beta+\frac{\beta^2}{2}-\frac{\beta^3}{6}+\frac{\beta^4}{24}\right)\left(1+\frac{\beta}{2}+\frac{\beta^2}{4}+\frac{\beta^3}{16}+\frac{\beta^4}{64}\right)$$

$$-\frac{1}{24} + \frac{1}{8} - \frac{1}{16} + \frac{1}{64}$$

$$+\frac{\beta}{2} - \frac{\beta^2}{2} + \frac{\beta^3}{4} - \frac{\beta^4}{12}$$

$$+\frac{\beta^2}{4} - \frac{\beta^3}{4} + \frac{\beta^4}{8} \qquad = 1 - \frac{\beta}{2} + \frac{\beta^2}{4} - \frac{5}{48}\beta^3 + \frac{7}{3.64}\beta^4$$

$$\frac{-8+24-12+3}{3.64}$$

$$+\frac{\beta^3}{16} - \frac{\beta^4}{16}$$

$$\beta = 2\alpha$$

$$+\frac{\beta^4}{64} \qquad = 1 - \alpha + \alpha^2 - \frac{5}{6}\alpha^3 + \frac{7}{12}\alpha^4$$

stimmt mit früherem Resultat

$\cancel{I_0(\beta) = I_0(i\beta)}$

Gray & Mathews p. 227: $\mathcal{I}$

$$I_n(t) = e^t \frac{t^n}{2^n . n!}\left\{1 - t + \frac{2n+3}{2(2n+2)}t^2 - \frac{2n+5}{2.3.(2n+2)}t^3 + \frac{(2n+5)(2n+7)}{2.3.4.(2n+2)(2n+4)}t^4 - \frac{(2n+7)(2n+9)}{2.3.4.5\,(2n+2)(2n+4)}t^5 + \cdots\right\}$$

p. 68

for large t:

$$I_n(t) = \frac{1}{\sqrt{2\pi t}}e^t\left\{1 - \frac{4n^2-1}{8t} + \frac{(4n^2-1)(4n^2-9)}{2!\,(8t)^2} - \frac{(4n^2-1)(4n^2-9)(4n^2-25)}{3!\,(8t)^3} + \cdots\right\}$$

$$\therefore\; e^{-\beta}\left[I_0(\beta)+I_1(\beta)\right] = \frac{1}{\sqrt{2\pi\beta}}\left\{1 + \frac{1}{8\beta} + \frac{9}{2!(8\beta)^2} + \frac{9.25}{3!(8\beta)^3} + 1 - \frac{3}{8\beta} - \frac{3.5}{2!(8\beta)^2} - \frac{3.5.21}{3!(8\beta)^3} \cdots\right\}$$

$$P = \frac{1}{\sqrt{2\pi\beta}}\left\{2 - \frac{2}{8\beta} - \frac{\cancel{8}\,3}{\cancel{8}(8\beta)^2} - \frac{15\overbrace{(21-15)}^{6}}{6.(8\beta)^3}\right.$$

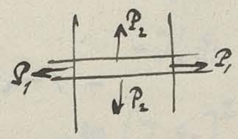
$$= \sqrt{\frac{2}{\pi\beta}}\left\{1 - \frac{1}{8\beta} - \frac{3}{128.\beta^2} - \frac{15}{2.8^3\beta^3}\right.$$

$$= \sqrt{\frac{1}{\alpha\pi}}\left\{1 - \frac{1}{16\alpha} - \frac{3}{4.128.\alpha^2} - \frac{15}{2.8^4.\alpha^3}\right.$$

stimmt

$$= \sqrt{\frac{1}{\alpha\pi}}\left\{1 - \frac{1}{16\alpha} - \frac{3}{2}\left(\frac{1}{16\alpha}\right)^2 - \frac{15}{2}\left(\frac{1}{16\alpha}\right)^3 - \dots\right\} \qquad \alpha = \frac{a^2}{4Dt}$$

$$1 - P = (1-P_1)(1-P_2)$$



$$P = P_1 + P_2 - P_1 P_2$$

$$u = \left(\frac{K(\varphi_1-\varphi_0)}{4\pi}\right)^2 6\pi$$

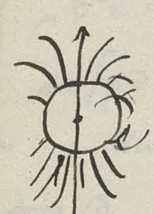
$$\frac{\partial V}{\partial s} = \frac{K(\varphi_1-\varphi_0)}{4\pi}\ \frac{3}{2}\, c\, 6 \frac{\cos\theta}{A^2}$$

$$c = \frac{9}{2}\frac{A^2 \Delta\delta\, g}{\mu}$$

$$v = \frac{K(\varphi_1-\varphi_0)}{4\pi}\ \frac{\cancel{\ }}{\mu}\ \frac{\partial V}{\partial s} = \left[\frac{K(\varphi_1-\varphi_0)}{4\pi}\right]^2 \frac{3}{2}\,\frac{6}{A^2}\,\frac{9}{2}\,\frac{A^2\Delta\delta\, g}{\mu^2}$$

$$= \left[\frac{K(\varphi_1-\varphi_0)}{4\pi}\right]^2 \frac{6}{\mu^2}\cdot g\cdot\Delta\delta\cdot\frac{27}{4}$$

$$\left[\frac{\frac{1}{9}}{4\pi\cdot 300}\right]^2 \frac{10^7\ \cdot 10^3}{9\cdot 10^{11}\cdot(0{\cdot}02)^2}\cdot 1{\cdot}2\cdot\frac{27}{4} = \frac{\cancel{1}\ 10^4\cdot 8}{9\cdot 10^7\cdot 4} = 2\cdot 10^{-4}$$



$$\cancel{V = \alpha\frac{\cos\theta}{r^2} = \alpha\frac{x}{r^3}} \qquad \cancel{X = \alpha\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right)}$$

$$\sqrt{Y^2+Z^2} = -\frac{3\alpha\, x\,\omega}{r^5}$$

$$F^2 = \alpha\left(\frac{1}{r^6} + \frac{3x^2}{r^8}\right) = \frac{\alpha}{r^6}\left(1 + 3\cos^2\theta\right)$$

$$V = \frac{\alpha}{a^2}\left(x + \frac{a^3 x}{2r^3}\right) \qquad X = \frac{\alpha}{a^2}\left[1 + \frac{a^3}{2r^3} - \frac{3a^3x^2}{2r^5}\right]_{x=a} = 0$$

$$\Omega = -\frac{3\alpha\, x\,\omega}{2a^4}$$

$$\int \frac{\partial^2}{\partial x^2}(\varphi v)\,dx = \int v\frac{\partial^2\varphi}{\partial x^2}dx + \underbrace{2\int\frac{\partial v}{\partial x}\frac{\partial\varphi}{\partial x}dx}_{\cancel{v\frac{\partial\varphi}{\partial x}} - \int v\frac{\partial^2\varphi}{\partial x^2}dx} + \int\varphi\frac{\partial^2 v}{\partial x^2}dx = 0$$

$$-\int v\frac{\partial^2\varphi}{\partial x^2}dx + \int\varphi\frac{\partial^2 v}{\partial x^2}dx = 0$$

$$\frac{1}{4\pi}\int v\frac{\partial^2\varphi}{\partial x^2}dx = \frac{1}{4\pi}\int\varphi\frac{\partial^2 v}{\partial x^2}dx$$

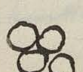
$$\int v\,\varepsilon\,dx = -\frac{1}{4\pi\mu}\underbrace{\int\varphi\,\varepsilon\,dx}\cdot\frac{\partial\Phi}{\partial x}$$

$$= +\frac{\frac{\partial\Phi}{\partial x}}{16\pi^2\mu}\underbrace{\int\varphi\frac{\partial^2\varphi}{\partial x^2}dx}_{\varphi\frac{\partial\varphi}{\partial x} - \int\left(\frac{\partial\varphi}{\partial x}\right)^2 dx}$$

$$\mp\left[\frac{K}{4\pi}(\varphi_1-\varphi_0)\right]^2\frac{1}{\delta}\,\frac{1}{\mu}\,\frac{\partial\Phi}{\partial x}$$

$a = 3\mu\mu = 3\cdot 10^{-7}$ // $\frac{A}{a} = 300$

$A = 10^{-4}$



$$\frac{I_6}{I_2} = \frac{2\cancel{\pi}}{\cancel{\ }}\left[\frac{K}{4\pi}(\varphi_1-\varphi_0)\right]^2\frac{6}{\mu}\,\frac{1}{\delta}$$

$\cancel{n_1 u + n_2 v_2}$

$$\operatorname{div}(n_1 u + n_2 v) = 0$$

$$\boxed{\bar{u} = \frac{\beta_2 u_1 + \beta_1 u_2}{\beta_1 + \beta_2}}$$

$$(\beta_2 + \beta_1)\bar{u} = \beta_2 u_1 + \beta_1 u_2$$

$$\cancel{u_1 \frac{\partial n_1}{\partial x} + v_1 \frac{\partial n_1}{\partial y} + w_1 \frac{\partial n_1}{\partial z} + \left(u_2 \frac{\partial n_2}{\partial x} + v_2 \frac{\partial n_2}{\partial y} + w_2 \frac{\partial n_2}{\partial z}\right) + n_1 \operatorname{div} \mathfrak{v}_1 + n_2 \operatorname{div} \mathfrak{v}_2 = 0}$$

$$\left.\begin{aligned} u_1 - \bar{u} &= -\beta_1 \frac{\partial U}{\partial x} \\ u_2 - \bar{u} &= \beta_2 \frac{\partial U}{\partial x} \end{aligned}\right\}$$

$$\cancel{\operatorname{div} \mathfrak{v}_1 = \operatorname{div} \bar{\mathfrak{v}} + \beta_1 \nabla^2 U}$$

$$\cancel{\operatorname{div} \mathfrak{v}_2 = \operatorname{div} \bar{\mathfrak{v}} + \beta_2 \nabla^2 U}$$

$$\rho = (n_1 - n_2)\varepsilon = -\frac{1}{4\pi} \nabla^2 U$$

$$-\rho \nabla U = \mathfrak{f}$$

$$\varepsilon(n_1 v_1 - n_2 v_2) = \underbrace{\varepsilon(n_1 - n_2)\bar{v}}_{\rho\bar{v}} + \underbrace{\varepsilon n_1(v_1 - \bar{v})}_{-\varepsilon n_1 \beta_1 \nabla U} + \underbrace{\varepsilon n_2(\bar{v} - v_2)}_{-\varepsilon n_2 \beta_2 \nabla U}$$

$$\operatorname{div}\left[\rho\bar{v} - \underbrace{\varepsilon(n_1\beta_1 + n_2\beta_2)}_{\lambda} \nabla U\right] = 0$$

$$= \operatorname{div}[\rho \bar{v} + \dot{\mathfrak{E}}]$$

wenn überall $\rho = 0$

oder $\bar{v} = 0$

so gilt $\operatorname{div}\left(\lambda \frac{\partial U}{\partial x}\right) = 0$

$$\operatorname{curl}(\nabla U \, \nabla^2 U)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial U}{\partial x} \cdot \nabla^2 U\right) - \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial y} \cdot \nabla^2 U\right) = \frac{\partial U}{\partial x} \cdot \nabla^2\left(\frac{\partial U}{\partial y}\right) - \frac{\partial U}{\partial y} \cdot \nabla^2\left(\frac{\partial U}{\partial x}\right) \neq 0$$

$$\mu \nabla^2 \bar{v} = \mathfrak{f} - \nabla p$$

$$= -\rho \nabla U - \nabla p$$

$$\operatorname{div} \bar{v} = 0$$

$$\operatorname{div}\left[\rho\bar{v} - \varepsilon(n_1\beta_1 + n_2\beta_2)\nabla U\right] = 0$$

~~div ρ ... Φ~~ Modification in der Doppelschicht:

~~$u_1 - \bar{u} = -\beta_1 \frac{\partial(\Phi-\Phi)}{\partial x}$~~

$$\left.\begin{aligned} v_1 - \bar{v} &= -\beta_1 \nabla(U-\Phi) \\ \bar{v} - v_2 &= -\beta_2 \nabla(U-\Phi) \end{aligned}\right\} \quad \bar{v} = \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1+\beta_2}$$

$$\rho = (n_1 - n_2)\varepsilon = -\frac{1}{4\pi}\nabla^2 U$$

~~$\rho \nabla U = \nabla p$~~

$$\left\{\begin{aligned} &\mu \nabla^2 \bar{v} = -\rho \nabla U - \nabla p \qquad \text{oder vielleicht: } = -\rho \nabla(U-\Phi) - \nabla p \ (?) \\ &\operatorname{div}\left[\rho \bar{v} - \varepsilon(n_1\beta_1 + n_2\beta_2)\nabla(U-\Phi)\right] = 0 \\ &\operatorname{div} \bar{v} = 0 \end{aligned}\right.$$

~~Und~~ Bewegungslosigkeit nur falls:

$$\rho \nabla U = -\nabla p$$

$$\therefore \operatorname{curl}(\rho \nabla U) = 0 \qquad \frac{\partial U}{\partial x}\frac{\partial(n_1-n_2)}{\partial y} = \frac{\partial U}{\partial y}\frac{\partial(n_1-n_2)}{\partial x}$$

etc. also falls $(n_1 - n_2) =$ const-Flächen zusammenfallen mit Äquipotentialflächen.

~~Eigentlich ist Alles zu erklären durch Betrachtung d. verschiedenen Zustände~~

Falls geschlossenes Gefäss, in welches die Elektr. durch (unauflösliche) metallische Elektr. ein- oder austritt, wird n_1 oder n_2 mit der Zeit ganz erschöpft werden, falls nicht Neu-Erzeugung von Ionen. Gleichgewichtszustand erfordert Berücksichtigung der letzteren (sowie der Recombination).

Pro Zeiteinheit werden pro Vol. in entstehendem H_2O γ ± Ionen erzeugt:

Rekombination:
$$\left.\begin{aligned} \frac{dn_1}{dt} &= \gamma - \alpha n_1 n_2 \\ \frac{dn_2}{dt} &= \gamma - \alpha n_1 n_2 \end{aligned}\right\} \text{ ohne Feld}$$

Zunahme in Zeit Δt:

$$\Delta t\left[n_{1x} u_{1x} - (n_1 u_1)_{x+\Delta x}\right] + (\gamma - \alpha n_1 n_2)\Delta x$$

$$\Delta t \iint n_1 v_{1n}\, dS + \iiint \gamma \ldots$$

Im stationären Zustand:

$$\operatorname{div}(n_1 v_1) = \gamma - \alpha n_1 n_2 = -\operatorname{div}(n_2 v_2)$$

$$\cancel{v_1 - \bar v = v_1 - \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1+\beta_2} = \frac{\beta_1(v_1 - v_2)}{\beta_1+\beta_2} = -\beta_1 \nabla(U - \Phi)}$$

$$v_1 - v_2 = -(\beta_1+\beta_2)\nabla(U - \cancel{\Phi})$$

$$\bar v = \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1+\beta_2}$$

$$\rho = (n_1 - n_2)\varepsilon = -\frac{1}{4\pi}\nabla^2(U - \Phi)$$

$$\mu \nabla^2 \bar v = -(n_1 - n_2)\varepsilon \cdot \nabla(U) - \nabla p$$

$$\operatorname{div} \bar v = 0$$

$$\beta_2 \operatorname{div} v_1 = -\beta_1 \operatorname{div} v_2$$

$$\cancel{\operatorname{div}\left[\rho \bar v - \varepsilon(n_1\beta_1 + n_2\beta_2)\nabla(U - \Phi)\right] = 0}$$

(schon in $\operatorname{div}(n_1 v_1)$ – enthalten)

$$\operatorname{div}\left[(n_1 - n_2)\frac{(\beta_2 v_1 + \beta_1 v_2)}{\beta_1+\beta_2}\right] = \operatorname{div}\left[(n_1\beta_1 + n_2\beta_2)\nabla(U)\right]$$

$$\begin{cases} \operatorname{div}(n_1 v_1) = -\operatorname{div}(n_2 v_2) = \gamma - \alpha\, n_1 n_2 \\ v_1 - v_2 = -(\beta_1+\beta_2)\,\nabla(U - [illegible]) \\ n_1 - n_2 = -\frac{1}{4\pi\varepsilon}\,\nabla^2(U-\Phi) \\ \beta_2 \operatorname{div} v_1 = -\beta_1 \operatorname{div} v_2 \\ \cancel{\operatorname{div}\left[(n_1-n_2)\beta_2 v_1 + \beta_1 v_2) - (\beta_1+\beta_2)(n_1\beta_1 + n_2\beta_2)\nabla(U-\Phi)\right] = 0} \\ \frac{\mu}{\beta_1+\beta_2}\left[\beta_2 \nabla^2 v_1 + \beta_1 \nabla^2 v_2\right] = -\nabla p - (n_1 - n_2)\,\varepsilon\cdot\nabla(U-\Phi) \end{cases}$$

$$\therefore \operatorname{div} v_2 = \operatorname{div} v_1 + (\beta_1+\beta_2)\nabla^2(U-\Phi) = -\operatorname{div} v_1 \cdot \frac{\beta_2}{\beta_1}$$

$$\therefore \operatorname{div} v_1 = -\beta_1 \nabla^2(U-\Phi)$$

$$\therefore \operatorname{div} v_2 = \beta_2 \nabla^2(U-\Phi)$$

$$\therefore \underbrace{\cancel{\operatorname{div}\left[(n_1-n_2)(\beta_2 v_1 + \beta_1 v_2) + (n_1\beta_1 + n_2\beta_2)(v_1 + v_2)\right]}}$$

$$n_1 v_1(\beta_2-\beta_1) + n_1 v_2(\beta_2-\beta_1) + \cancel{n_2 v_1(\beta_2-\beta_1)} + n_2 v_2(-2\beta_2)$$

$$= n_1(v_1+v_2)(\beta_2-\beta_1) - 2 n_2 v_2 \beta_2 \qquad = n_1(\beta_2-\beta_1)v_1$$

$$= n_1 v_1(\beta_2+\beta_1) - n_2 v_2(\beta_1+\beta_2) + n_1 \cancel{v_2(\beta_1-\beta_1) + n_2 v_1(-\beta_2+\beta_2)}$$

Wenn Erzeugung und Rekombination vernachlässigt wird (z.B. $CuSO_4$ Ionen zwischen den Elektroden)

$$\left.\begin{aligned} \operatorname{div}(n_1 v_1) = 0 &= \operatorname{div}(n_2 v_2) = 0 \\ \operatorname{div} v_1 &= -\beta_1 \nabla^2(U - [illegible]) \\ \operatorname{div} v_2 &= \beta_2 \nabla^2(U - [illegible]) \\ n_1 - n_2 &= -\frac{1}{4\pi\varepsilon}\nabla^2(U-\Phi) \end{aligned}\right\} \quad v_1 - v_2 = -(\beta_1+\beta_2)\nabla U \quad \text{(nicht erlaubt!)}$$

$$\nabla p = (n_2 - n_1)\,\varepsilon\cdot\nabla U - \frac{\mu}{\beta_1+\beta_2}\left[\beta_1 \nabla^2 v_2 + \beta_2 \nabla^2 v_1\right]$$

Unter vereinfachender Annahme, dass $\beta_1 = \beta_2$:

$$u_1 \operatorname{div} v_1 + u_1 \frac{\partial n_1}{\partial x} + v_1 \frac{\partial n_1}{\partial y} + w_1 \frac{\partial n_1}{\partial z} = 0$$

$$n_2 \operatorname{div} v_2 + u_2 \frac{\partial n_2}{\partial x} + v_2 \frac{\partial n_2}{\partial y} + w_2 \frac{\partial n_2}{\partial z} = 0$$

$\operatorname{div} v_1 = - \operatorname{div} v_2 =$

$(n_1 - n_2)\, d$ An Stellen wo $\Phi = 0$, falls: $v_1 = - v_2$

$$\operatorname{div} v_1 = - \operatorname{div} v_2 = 4\pi\varepsilon\beta(n_2 - n_1)$$

$$(n_1 - n_2) \operatorname{div} v_1 + u_1 \frac{\partial(n_1 - n_2)}{\partial x} + v_1 \frac{\partial(n_1 - n_2)}{\partial y} + w_1 \frac{\partial(\;)}{\partial z} = 0$$

$$4\pi\varepsilon\beta (n_1 - n_2)^2 = \left(u_1 \frac{\partial}{\partial x} + v_1 \frac{\partial}{\partial y} + w_1 \frac{\partial}{\partial z}\right)(n_1 - n_2)$$

$n_1 = n_2$

$$\left(u_1 \frac{\partial}{\partial x} + v_1 \frac{\partial}{\partial y} + w_1 \frac{\partial}{\partial z}\right)\left(\frac{1}{n_1 - n_2}\right) = -4\pi\varepsilon\beta = \text{const}$$

An Stellen wo $\Phi = 0$ gibt es eine mögliche Lösung:

$n_1 = n_2 = \text{const}$ $\quad\quad \operatorname{div} v_1 = \operatorname{div} v_2 = 0 \quad\quad \nabla_p = 0$

vorausgesetzt, dass diese Bedingungen an den Grenzen des betrachteten Raumes erfüllt sind!

Dabei braucht aber $v_1 \gtrless v_2$ sein

$$\frac{\cancel{\mu}}{\cancel{\beta_1+\beta_2}}\left[\beta_1 \alpha_2 c - \beta_2 \alpha_1 c\right] = -\frac{1}{4\pi}\frac{(\varphi_i - \varphi_a)}{\mu} \frac{\alpha_1 + \alpha_2}{\cancel{\beta_1+\beta_2}}$$

$v_{1\infty} = -\alpha_1 c + \gamma$

$v_{2\infty} = \alpha_2 c + \gamma$

$$\nabla_\infty U = \frac{\alpha_1 + \alpha}{\beta_1 + \beta_2} c = \frac{v_{2\infty} - v_{1\infty}}{\beta_1 + \beta_2} c$$

Durch Integration über Grenzschichte:

$$\frac{\mu}{\beta_1+\beta_2}\left[\beta_1 v_{2s} + \beta_2 v_{1s}\right] = -\frac{1}{4\pi}(\varphi_i - \varphi_a)\frac{\partial U}{\partial s}$$ | für Kugeloberfläche

Im Aussenraum: $\nabla^2 U = 0$ $\qquad n_1 = n_2$ $\qquad$ || $v_1 = v_2 = c$ für $r = \infty$

$$\operatorname{div} v_1 = \operatorname{div} v_2 = 0$$

$$\nabla p = -\frac{\mu}{\beta_1+\beta_2}\left[\beta_1 \nabla^2 v_2 + \beta_2 \nabla^2 v_1\right] \qquad v_1 - v_2 = -(\beta_1+\beta_2)\nabla U$$

$$\therefore \nabla^2 p = 0$$

$$\cancel{v_n \Delta x - \int (v_s - v_s')\,dy = 0}$$

$$\cancel{v_n = \frac{\partial}{\partial x}\int v_s\,dy}$$

$v_{1n} = v_{2n} = 0$ für Kugeloberfläche ?

Annahme:

$$v_1 = -\nabla\left[\alpha_1\, cx\left(1+\frac{a^3}{2r^3}\right)\right] + \ldots + \gamma_1 v_0 \quad \text{Stokes}$$

$$v_2 = \nabla\left[\alpha_2 \qquad "\qquad\right] + \ldots + \gamma_2 v_0$$

$$\cancel{u_1 = -\alpha_1 c\left(1+\frac{a^3}{2r^3} - \frac{3x^2a^3}{2r^5}\right)}$$

$$\cancel{v_1 = -\alpha_1 c\left(-\frac{3xya^3}{2r^5}\right)}$$

$$\cancel{w_1 = -\alpha_1 c\left(-\frac{3xza^3}{2r^5}\right)}$$

$$\cancel{\frac{\partial\left(\frac{1}{r}\right)}{\partial x}} \quad \cancel{\frac{\partial^2}{\partial x\partial y}} \quad \cancel{\frac{\partial^2}{\partial x\partial z}}$$

$$\nabla^2 v_1 = \gamma_1 \nabla^2 v_0 = \ldots$$

$$p = -\frac{\mu}{\beta_1+\beta_2}\,\frac{3}{2}\frac{ax}{r^3}\left[\beta_2\gamma_1 + \beta_1\gamma_2\right] \leftarrow = -\frac{3}{2}\mu\frac{ax}{r^3}\gamma_2$$

$$-\nabla\left[(\alpha_1+\alpha_2)\, cx\left(1+\frac{a^3}{2r^3}\right)\right] + (\gamma_1-\gamma_2)v_0 = -(\beta_1+\beta_2)\nabla U \qquad \| \; \gamma_1 = \gamma_2$$

$$U = \frac{\alpha_1+\alpha_2}{\beta_1+\beta_2}\, cx\left(1+\frac{a^3}{2r^3}\right)$$

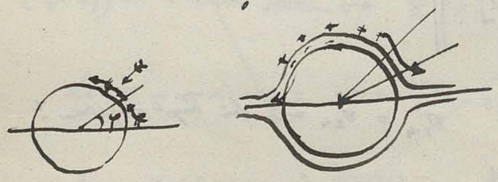
$$\mu \frac{\partial^2 v_\xi}{\partial \xi^2} = \frac{+K}{4\pi}\frac{\partial \Phi}{\partial \xi}\cdot\frac{\partial^2\varphi}{\partial\xi^2}$$

$$v_\xi = \frac{K}{4\pi\mu}\frac{\partial\Phi}{\partial\xi}(\varphi-\varphi_a)$$

Oberflächen-Konvektionsstrom: $i_\xi = \int_0^\infty \varepsilon v_\xi \, d\xi = -\left(\frac{K}{4\pi}\right)^2\frac{1}{\mu}\frac{\partial\Phi}{\partial\xi}\underbrace{\int_0^\infty \frac{\partial^2\varphi}{\partial\xi^2}(\varphi-\varphi_a)\, d\xi}$

$$= \int_0^\infty \left(\frac{\partial\varphi}{\partial\xi}\right)^2 d\xi \quad = A\frac{\partial\Phi}{\partial\xi}$$

$$A = \left(\frac{K}{4\pi}\right)^2\frac{1}{\mu}\int_0^\infty \left(\frac{\partial\varphi}{\partial\xi}\right)^2 d\xi$$



Kontinuitätsgleichung: $2\pi a^2 \sin\varphi\, d\varphi \cdot i_n = d(2\pi a \sin\varphi \; i_\xi)$

$$i_n = \cancel{\ldots}\frac{1}{a\sin\varphi}\frac{\partial}{\partial\varphi}(i_\xi \sin\varphi)$$

$$= \frac{A}{a\sin\varphi}\frac{3}{2}c\frac{\partial}{\partial\varphi}(\sin^2\varphi)$$

$$i_n = \frac{3Ac}{a}\cos\varphi$$

$$\Phi = -c\cos\varphi\left(r+\frac{a^3}{2r^2}\right)$$

$$\left.\frac{\partial\Phi}{\partial\xi}\right|_{r=a} = \frac{3}{2}c\sin\varphi$$

Es ist aber noch zu untersuchen ob die übrig bleibenden v_ξ nicht einen Konvektionsstrom begleitenden Strömung ergeben!

Das entspricht einer ~~…~~ superponierten Potentialverteilung:

$$\cancel{\Phi} = \Phi'_\mu = \cancel{\ldots}\; b\cos\varphi\frac{a^3}{2r^2}$$

$$i_n = -\frac{1}{\sigma}\left.\frac{\partial\Phi'}{\partial n}\right|_\varphi = +\frac{3b}{\sigma}\cos\varphi = \frac{3Ac}{a}\cos\varphi$$

$$\therefore \; b = \frac{Ac\sigma}{a} = \left(\frac{K}{4\pi}\right)^2\frac{\sigma c}{\mu a}\int_0^\infty\left(\frac{\partial\varphi}{\partial\xi}\right)^2 d\xi$$

$$\Phi' =$$

Anwenden auf ~~[illegible]~~ radiale Komponente der Stärke:

$$v_\xi = -\frac{K(\varphi_i-\varphi_a)}{4\pi\mu}\, c\cos\varphi\left(1-\frac{a^3}{r^3}\right) \qquad r = a+\xi$$

$$\left[1-\frac{1}{\left(1+\frac{\xi}{a}\right)^3}\right] \neq \frac{3\xi}{a}$$

$\operatorname{div}(\varepsilon v) = v_\xi \frac{\partial\varepsilon}{\partial\xi}$

pro dS: $\lambda\frac{\partial\psi}{\partial n} = \int_0^\infty v_\xi \frac{\partial\varepsilon}{\partial\xi}\, d\xi = \int v_\xi \frac{\partial^3\psi}{\partial\xi^3}\, d\xi =$

$= \int \frac{\partial v_\xi}{\partial\xi}\frac{\partial^2\psi}{\partial\xi^2}\, d\xi = \int \frac{\partial^2 v}{\partial\xi^2}\frac{\partial\varphi}{\partial\xi}$

Dieselbe trägt aber zum konvektiven Radialstrom nicht bei, falls Fläche ausserhalb Doppelschicht, wo $\varepsilon = 0$.

→ Somit im äusseren Raum:

$$\Sigma\Phi = -c\cos\varphi\left(r+\frac{a^3}{2r^2}\right) + \underbrace{\frac{a^2 c\cos\varphi}{2r^2}\cdot\left(\frac{K}{4\pi}\right)^2\frac{\sigma}{\mu}\int_0^\infty\left(\frac{\partial\psi}{\partial\xi}\right)^2 d\xi}_{}$$

kann aufgefasst werden als Potentialänderung infolge Oberflächenleitung

desto grösser je grösser σ!

also so als ob noch Superposition einer Oberflächenladung

Kräfte solcher Dipole aufeinander

$-+$ $\quad$ $-+$

$$W = E^2\left(\frac{x}{r^3} - \frac{x'}{r'^3}\right) = -E^2\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = -E^2\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right)$$

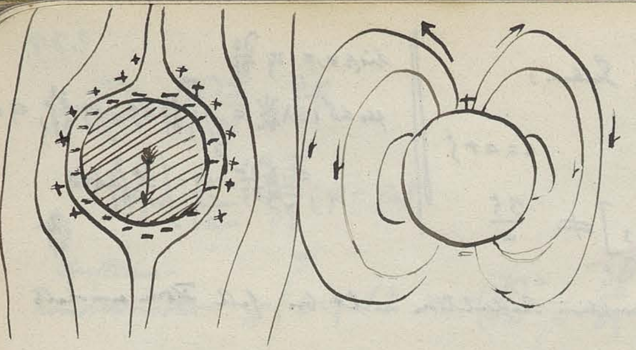
also bei Anziehung in X Achse: $W = \frac{2E^2}{x^3}$

$F_x = -6\frac{E^2}{x^4}$

Anziehung bei Berührung:

$$6\left[\left(\frac{K}{4\pi}\right)^2\frac{\sigma}{\mu}\right]^2 c^2\, \frac{\frac{a_1^2}{2}\cdot\frac{a_2^2}{2}}{(a_1+a_2)^4}\left[\int_0^\infty -\ \right]^2$$

$$\left(\frac{K}{4\pi}\right)^2\frac{\sigma}{\mu}\frac{(\varphi_i-\varphi_a)^2}{d} = \left(\frac{4}{300\cdot 4\pi}\right)^2\frac{10^7}{9\cdot 10^{11}\cdot 0{,}02\cdot 5\cdot 10^7} = \frac{10^{-6}\cdot 10^{14}}{9\cdot 10^{10}} = \frac{1}{900}$$



In erster Annäherung $\varepsilon = -\frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\xi^2}$

aber ~~es~~ es wird eben durch die Bewegung geändert.

$$\frac{1}{1+x} = 1 - x + x^2 \ldots$$

$$\frac{1}{(1+x)^3} = 1 - 3x + \frac{-3\cdot -4}{1\cdot 2}$$
$$+6x^2$$

Einfluss der Tangentialströmung:

Ruhende Komp. in der Nähe der Kugel: (gegenüber ruhender Kugel)

$$u = c\left(\frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right) + \frac{3}{4}\frac{c a x^2}{r^3}\left(1 - \frac{a^2}{r^2}\right)$$

$$v = -\frac{3}{4}\frac{c a x y}{r^3}\left(1 - \frac{a^2}{r^2}\right)$$

$$w =$$

$$v_\omega = -\frac{3}{4}\frac{c a x \omega}{r^3}\left(1 - \frac{a^2}{r^2}\right)$$

$$v_r = \frac{ux + v\omega}{r} = c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)\cos\varphi - \frac{3}{4}\frac{ca}{r}\left(1 - \frac{a^2}{r^2}\right)\cos\varphi = c\left(1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right)\cos\varphi$$

$$v_{r\varphi} = -\frac{u\omega + vx}{r} = -c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)\sin\varphi + \cancel{\frac{3}{4}\frac{ca}{r}\left(1 - \frac{a^2}{r^2}\right)\cos^2\varphi\sin\varphi}$$

$$= -c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)\sin\varphi$$

In Entfernung ξ: $r = a + \xi$

$$v_r = c\cos\varphi\left(1 - \frac{3}{2}\left(1+\frac{\xi}{a}\right)^{-1} + \frac{1}{2}\left(1+\frac{\xi}{a}\right)^{-3}\right) = c\cos\varphi\left[\frac{3}{2}\left(\frac{\xi}{a} - \frac{\xi^2}{a^2}\right) - \frac{1}{2}\left(3\frac{\xi}{a} - 6\frac{\xi^2}{a^2}\right)\right]$$

$$= +\frac{3}{2}c\cos\varphi\cdot\frac{\xi^2}{a^2}$$

$$v_{r\varphi} = -c\sin\varphi\left(1 - \frac{3}{4}\left(1+\frac{\xi}{a}\right)^{-1} - \frac{1}{4}\left(1+\frac{\xi}{a}\right)^{-3}\right) = -c\sin\varphi\left[-\frac{3}{4}\left(\frac{\xi}{a} - \frac{\xi^2}{a^2}\right) - \frac{1}{4}\left(3\frac{\xi}{a} - 6\frac{\xi^2}{a^2}\right)\right] =$$

$$= \frac{3}{2}c\frac{\xi}{a}\sin\varphi$$

$$p = \frac{3}{2}\frac{c\mu a\cos\varphi}{r^2}$$

$$\left(\frac{\partial p}{\partial r}\right)_{r=a} = -3\frac{c\mu\cos\varphi}{a^2}$$

$$\mathrm{div}\,(\varepsilon\mathfrak{v}) = -\frac{1}{\cancel{2\pi} r^2 \sin\varphi}\frac{\partial}{\partial\varphi}\cancel{2\pi}\left[r\sin\varphi \cdot v_\varphi\right] + \underbrace{\frac{1}{r^2}\frac{\partial(v_r\, r^2)}{\partial r}}_{\frac{\partial v_r}{\partial r} + \frac{2}{r}v_r}$$

$$= \frac{-1}{r}\left(\frac{\partial v_\varphi}{\partial\varphi} + \frac{\cos\varphi}{\sin\varphi}\, v_\varphi\right) + \frac{\partial v_r}{\partial r} + \cancel{\frac{2}{r} v_r}$$

$$= \underbrace{\frac{-3c}{2r}\left[\frac{\xi}{a}\cos\varphi + \frac{\cos\varphi}{\sin\varphi}\sin\varphi\,\frac{\xi}{a}\right]}_{\neq\; -\frac{3c}{a}\,\frac{\xi}{a}\cos\varphi} + \frac{3c}{a}\cos\varphi\,\frac{\xi}{a^2} = 0$$

$$\cancel{\mathrm{div}(\varepsilon\mathfrak{v}) = \frac{2}{r}\frac{\partial}{\partial\varphi}\left[\frac{3}{2}c\cos\varphi\,\frac{\xi^2}{a^2}\,\frac{\partial\varphi}{\partial\xi}\right]}$$

Solange ε nur als Funktion ξ angesehen wird, solange also die Dicke der Doppelschicht nicht ungleichförmig wird, kommt für div (ε𝔳) nur die Normalkomponente v_r in Betracht

$$\mathrm{div}\,(\varepsilon\mathfrak{v}) = u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

$$= v_r\frac{\partial\varepsilon}{\partial r}$$

also bleibt frühere Theorie ganz streng, solange Dichte d. Doppelsch. konzentrisch geschichtet, also Rückwirkung d. Beweg. auf ε unberücksichtigt.

$U = V - \varphi \leftarrow$ aufgeprägtes Potential

$$\varepsilon = -\frac{1}{4\pi}\nabla^2 V = -\frac{1}{4\pi}\nabla^2(U+\varphi)$$

$$U = \frac{\epsilon}{4\pi}\iiint \frac{1}{r}\left(u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}\right)d\omega$$

$$U + \varphi = \iiint \frac{\varepsilon}{r}\,d\omega$$

$$\frac{\partial\varepsilon}{\partial\xi} = -\frac{1}{4\pi}\nabla^2\left[\frac{\partial(\varphi + U)}{\partial\xi}\right]$$

$$\frac{\partial\varepsilon}{\partial\eta} = -\frac{1}{4\pi}\nabla^2\left[\frac{\partial U}{\partial\eta}\right]$$

Korrekte Behandlung, wenn tangentiale Variabilität des ε berücksichtigt

$\nabla\varphi =$ eingeprägte Kraft, welche die gestörte Doppelschicht wiederherstellt

$V =$ ~~Pot~~ Gesamt Potential; im Normalzustand $V = \varphi$

$U = V - \varphi$ = Pot. der Zusatzkräfte, welche Leitungsströme erzeugen

$$\operatorname{div}\left[\frac{1}{\sigma}\nabla U + \varepsilon v\right] = 0$$

Vernachlässigung der Unterschied in σ ist das in der Doppelschicht erlaubt?

gewiss nicht denn es gilt:

$$\lambda = \frac{1}{\sigma} = e(n_1\beta_1 + n_2\beta_2)\,!$$

$$\frac{1}{\sigma}\nabla^2 U = -u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

$$\varepsilon = -\frac{1}{4\pi}\nabla^2 V = -\frac{1}{4\pi}\nabla^2(U+\varphi)$$

$$+\frac{4\pi\varepsilon}{\sigma} + \frac{\nabla^2\varphi}{\sigma} = u\frac{\partial\varepsilon}{\partial x} + v\frac{\partial\varepsilon}{\partial y} + w\frac{\partial\varepsilon}{\partial z}$$

$$\varepsilon - \frac{\sigma}{4\pi}\left[v_r\frac{\partial\varepsilon}{\partial\xi} + v_\xi\frac{\partial\varepsilon}{\partial\xi}\right] = -\frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\xi^2}$$

I). Näherungsweise:

$$\varepsilon = = \frac{1}{4\pi}\frac{\partial^2\varphi}{\partial\xi^2} - \frac{\sigma}{4\pi}\left[v_r\frac{\partial\varepsilon}{\partial\xi} + v_\xi\frac{\partial\varepsilon}{\partial\xi}\right]$$

$$= -\frac{1}{4\pi}\left[\frac{\partial^2\varphi}{\partial\xi^2} - \frac{\sigma}{4\pi}\left[v_r\frac{\partial^3\varphi}{\partial\xi^3}\right]\right]$$

II) Näherung:

$$\varepsilon = -\frac{1}{4\pi}\left[\frac{\partial^2\varphi}{\partial\xi^2} - \frac{\sigma}{4\pi}\left[v_r\left(\frac{\partial^3\varphi}{\partial\xi^3} - \frac{\sigma}{4\pi}\frac{\partial}{\partial\xi}\left(v_r\frac{\partial^3\varphi}{\partial\xi^3}\right)\right) - v_\xi\cdot\frac{\sigma}{4\pi}\cdot\frac{\partial v_r}{\partial\xi}\frac{\partial^3\varphi}{\partial\xi^3}\right]\right]$$

Ordng: ξ^2 $\quad \xi^3 \quad \xi^2$

$$\therefore\ \varepsilon = -\frac{1}{4\pi}\left[\frac{\partial^2\varphi}{\partial\xi^2} + \frac{\sigma}{(4\pi)^2}\left[v_r\frac{\partial^3\varphi}{\partial\xi^3} - \frac{\sigma}{4\pi}v_\xi\frac{\partial v_r}{\partial\xi}\frac{\partial^2\varphi}{\partial\xi^3}\right]\right]$$

$$\frac{3}{2}c\cos\varphi\,\frac{\xi^2}{a^2} \qquad \frac{3}{2}c\sin\varphi\,\frac{\xi}{a} \qquad \frac{3}{2}c\cos\varphi\,\frac{\xi}{a^2}$$

$$4\pi\varepsilon = -\frac{\partial\varphi}{\partial\xi^2} + \frac{6}{4\pi}\,\frac{3c}{2}\,\frac{\cos\varphi}{a^2}\,\xi^2\frac{\partial^3\varphi}{\partial\xi^3} - \frac{6^2}{(4\pi)^2}\,\frac{9}{4}\,\frac{c^2\sin\varphi\cos\varphi}{a^3}\,\xi^2\frac{\partial^3\varphi}{\partial\xi^3}$$

$$\cancel{4\pi\int_0^\infty \varepsilon\, d\xi = \frac{6}{4\pi}\,\frac{3c\cos\varphi}{2a^2}(\varphi_i-\varphi_a) - \left(\frac{6}{4\pi}\right)^2\frac{9c^2\sin\varphi\cos\varphi}{4a^3}\underbrace{\int_0^\infty \xi\,\frac{\partial\varphi}{\partial\xi}\,d\xi}_{=\int_0^\infty \varphi\, d\xi}}$$

~~wird ∞, also ist jene Entwicklung nach ξ nicht gestattet~~

~~Man muss die Form beibehalten:~~

$$\cancel{4\pi\int_0^\infty \varepsilon\, d\xi = \frac{6}{4\pi}\,\frac{3c\cos\varphi}{2a^2}(\varphi_i-\varphi_a) - \left(\frac{6}{4\pi}\right)^2\int^\infty v_\xi\,\frac{\partial v_\xi}{\partial\xi}\,\frac{\partial^2\varphi}{\partial\xi^2}\,d\xi}$$

$$4\pi\int_0^\infty \varepsilon\, d\xi = \frac{6}{4\pi}\cdot\frac{3c\cos\varphi}{a^2}\left[1-\underbrace{\frac{6}{4\pi}\,\frac{3c\sin\varphi}{2a}}\right](\varphi_i-\varphi_a) =$$

Korrektionsglied im Allgemeinen unbedeutend

Bei Vernachlässigung derselben

$$\frac{6}{4\pi}(\varphi_i-\varphi_a)\underbrace{\frac{3c\cos\varphi}{a^2}}_{\frac{1}{\mu}\frac{\partial\varphi}{\partial\xi}\big|_{\xi=0}} = \frac{\partial U}{\partial\xi}\Big|_{\xi=a} = \cancel{\frac{6}{4\pi}(\varphi_i-\varphi_a)\cdot\frac{3c x}{2r^3}\Big|_{r=a}}$$

$$U = \frac{6}{4\pi}\,\frac{(\varphi_i-\varphi_a)}{\mu}\,\varphi$$

Das alles unter Voraussetzung dass die Änderungen in der ursprünglichen Geschwindigkeit v verschwindend gering sind und dass Leitfähigkeits Unterschied vernachlässigt wird.

Wenn Leitfähigkeits Unterschied berücksichtigt werden sollen:

zu beschränken sich natürlich auf den Bereich der Doppelschichte, wo $\varepsilon \gtrless 0$

$$\left\{\begin{aligned} v_1 - \bar{v} &= -\beta_1 \nabla(\underbrace{V - \Phi}_{U = \text{Potential für Leitungsstrom}}) \\ \bar{v} - v_2 &= -\beta_2 \nabla(V - \Phi) \end{aligned}\right.$$

wobei Φ die eingeprägten Kräfte mißt, welche die Doppelschichtbildung bewirken

$$\varepsilon = (n_1 - n_2)\, e = -\frac{1}{4\pi} \nabla^2 V$$

Gesamtstrom: $i = e(n_1 v_1 - n_2 v_2) = e(n_1 - n_2)\bar{v} + e\, n_1(v_1 - \bar{v}) + e\, n_2(\bar{v} - v_2)$

$$= \varepsilon \bar{v} - \underbrace{e(n_1\beta_1 + n_2\beta_2)\nabla(V - \Phi)}_{\text{od } \frac{\partial U}{\partial x}}$$

$$\bar{v} = \frac{\beta_2 v_1 + \beta_1 v_2}{\beta_1 + \beta_2}$$

$$\boxed{\operatorname{div} i = 0}$$

$$\therefore \operatorname{div}(\varepsilon \bar{v}) = \operatorname{div}\left[e(n_1\beta_1 + n_2\beta_2)\nabla(V - \Phi)\right]$$

$$\frac{\beta_1(v_1 - v_2)}{\beta_1 + \beta_2} = -\beta_1 \nabla(V - \Phi)$$

$$= \frac{1}{\beta_1 + \beta_2}\left[\beta_2 \operatorname{div} v_1 + \beta_1 \operatorname{div} v_2\right]$$

$$v_1 - v_2 = -(\beta_1 + \beta_2)\nabla(V - \Phi)$$

$$= \cancel{e[\operatorname{div} n_1 \bar{v}} = \cancel{e \operatorname{div}[(n_1 - n_2)\bar{v}]}$$

$\operatorname{div} \bar{v} = 0$ also ~~$\beta_1 \operatorname{div} v_1 = -\beta_1 \operatorname{div} v_2$~~

$$\left\{\begin{aligned} \operatorname{div} v_1 &= -\beta_1 \nabla^2(V - \Phi) &= \beta_1\left[4\pi\varepsilon + \nabla^2\Phi\right] \\ \operatorname{div} v_2 &= \beta_2 \nabla^2(V - \Phi) &= \beta_2\left[4\pi\varepsilon - \nabla^2\Phi\right] \end{aligned}\right.$$

Annahme dass keine Ionen neu erzeugt werden, also: $\operatorname{div}(n_1 v_1) = \operatorname{div}(n_2 v_2) = 0$

$$\therefore \quad \operatorname{div}(n_1 \bar{v}) = \beta_1 \operatorname{div}\left[n_1 \nabla(V-\Phi)\right]$$

$$\operatorname{div}(n_2 \bar{v}) = -\beta_2 \operatorname{div}\left[n_2 \nabla(V-\Phi)\right]$$

~~in dem Falle [illegible]~~ $\cancel{\operatorname{div}(\Sigma v) = 0}$

$$\beta_2 \operatorname{div}(n_1 v_1) + \beta_1 \operatorname{div}(n_1 v_2) = \beta_1 (\beta_1+\beta_2) \operatorname{div}\left[n_1 \nabla(V-\Phi)\right]$$

Dann gelten als unabhängige Gleichungen für Auffindung der: $n_1\ n_2\ v_1\ v_2\ V$

$$\left\{\begin{array}{l} v_1 - \bar{v} = -\beta_1 \nabla(V-\Phi) \\ \bar{v} - v_2 = -\beta_2 \nabla(V-\Phi) \\ n_1 - n_2 = -\frac{1}{4\pi e}\nabla^2 V \\ \operatorname{div} n_1 v_1 = 0 \\ \operatorname{div} n_2 v_2 = 0 \end{array}\right.$$

falls $\bar{v}$ als gegeben angesehen wird

somit kommt dann noch $\operatorname{div} \bar{v} = 0$

und die mechanische Druckgleichung:

$$\nabla p = (n_2 - n_1) e\, \nabla(V-\Phi) - \underbrace{\frac{\mu}{\beta_1+\beta_2}\left[\beta_1 \nabla^2 v_2 + \beta_1 \nabla^2 v_2\right]}_{\mu \nabla^2 \bar{v}}$$

$$n_1 \operatorname{div} v_1 + u_1 \frac{\partial n_1}{\partial x} + v_1 \frac{\partial n_1}{\partial y} + w_1 \frac{\partial n_1}{\partial z} = 0$$

jedenfalls ist $\left.\frac{\partial \mathcal{U}}{\partial \varsigma}\right|_{r=a} = 0$

$$\left.\begin{array}{l} +\beta_1 \nabla^2 \mathcal{U} = u_1 \frac{\partial \log n_1}{\partial x} + v_1 \frac{\partial \log n_1}{\partial y} + w_1 \frac{\partial \log n_1}{\partial z} \\ -\beta_2 \nabla^2 \mathcal{U} = u_2 \frac{\partial \log n_2}{\partial x} + v_2 \frac{\partial \log n_2}{\partial y} + w_2 \frac{\partial \log n_2}{\partial z} \end{array}\right\}$$

$$\cancel{u \frac{\partial}{\partial x}(\beta_2 \log n_1 + \beta_1 \log n_2) + v \frac{\partial}{\partial y}(\beta_2 \log n_1 + \beta_1 \log n_2) + w \frac{\partial}{\partial z} \cdots = 0}$$

$$n_1 v_1 - n_2 v_2 = -(\beta_1 n_1 + n_2 \beta_2) \nabla(V-\Phi) \cancel{+ (n_1 - n_2)\bar{v}}$$

$$\operatorname{div}(n_1 \bar{v}) = -\beta_1 \operatorname{div}\left[n_1 \nabla(\overbrace{V-\Phi}^{\mathcal{U}})\right] = \bar{u} \frac{\partial n_1}{\partial x} + \bar{v} \frac{\partial n_1}{\partial y} + \bar{w} \frac{\partial n_1}{\partial z}$$

Grössenordnung von: $n_1 - n_2 = -\frac{K}{4\pi e}\frac{\partial^2 \varphi}{\partial n^2} \# -\frac{K(\varphi_i - \varphi_a)}{4\pi e\, d^2}$

$$= \frac{4}{300\cdot 4\pi\cdot 4.7\cdot 10^{-10}\cdot (4\cdot 10^{-7})^2} = \frac{10^{22}}{3\cdot\pi\cdot 4.7\cdot 16} = \frac{4}{3}\,10^{21}\ !!$$

$$\frac{28\,2}{75\,2}$$

Also Ionenzahl in der Doppelschicht kolossal gross!
(pro cm²)

Stock

$$\cancel{E = \frac{54.4}{4\pi\cdot 1.28}\cdot\frac{1}{300} = \frac{K(\varphi_i-\varphi_a)}{4\pi d} = 10^{-2}\,\frac{1.4}{\pi\cdot 10^{23}} = 2\cdot 10^{-3}}$$

$$D = \frac{\rho-\rho'}{\rho'}\,\frac{16\pi^2 \mu g}{6\cdot s} \quad \cancel{159}$$

$$\frac{19.8\cdot 16\cdot\pi^2\cdot 0.02\cdot 9\cdot 10^{11}}{23450\cdot 4.29\cdot 10^{7}} \qquad \underset{\text{elektrost.}}{\sigma} = \frac{4.29\cdot 10^{7}}{9\cdot 10^{11}}$$

$$= \frac{1.98\cdot 1.6\cdot\pi^2\cdot 2}{2.34\cdot 4.29}\cdot 9 \neq 59$$

$A = 0.0063$ $\qquad E = \frac{59}{4\pi\cdot 0.0063} = 700$ (elektrost.) $= \frac{A}{d} = \frac{K(\varphi_i - \varphi_a)}{4\pi d}$

$$A = \frac{K(\varphi_i - \varphi_a)}{4\pi}$$

$$l = \frac{6.3^2\cdot 10^{-6}}{59} = 7.3\cdot 10^{-7}$$

Dagegen Anzahl d. Ionen pro cm² der Grenzschichte: $\frac{4}{3}\cdot 10^{21}\cdot 4\cdot 10^{-7} = 5.3\cdot 10^{14}$

Somit mittl. Abstand in der Ebene $\frac{10^{-7}}{\sqrt{5.3}}$ von derselben Grössenordnung wie Dicke der Doppelschicht

Es ist eine wirklich ~~e~~ „eine ionige" Schichte!

allerdings kolossal angereichert im Vergleich zum Flüssigkeits-Innern

Jedenfalls ist also $n_1 - n_2$ von höherer Ordnung als n_1 und n_2 im Innern d. Flüssigkeit

---

Im Falle lamellarer Strömung [in constantem Gefälle] längs ebener Wand:

$n_1 = f_0(\xi) \qquad n_2 = f_0(\xi)$

$V = ax + \varphi(\xi) \qquad\qquad v_1 - \bar{v} = -\rho_1 a$

$\Phi = \varphi(\xi) \qquad\qquad \bar{v} - v_2 = -\rho_2 a$

$\rho = 0$

$$(n_2 - n_1)\, e\, a = \frac{\mu}{\rho_1 + \rho_2}\left[\rho_1 \frac{\partial^2 v_2}{\partial \xi^2} + \rho_2 \frac{\partial^2 v_1}{\partial \xi^2}\right]$$

$$= \mu \frac{\partial^2 \bar{v}}{\partial \xi^2}$$

$$n_1 - n_2 = -\frac{1}{4\pi e}\frac{\partial^2 V}{\partial \xi^2} = -\frac{1}{4\pi e}\frac{\partial \varphi}{\partial \xi^2} \qquad\qquad \bar{v} = -\frac{a}{4\pi\mu}(\varphi - \varphi_0)$$

$$\bar{v}_i = -\frac{a}{4\pi\mu}(\varphi_i - \varphi_0)$$

$$v_1 = -\rho_1 a - \frac{(\varphi - \varphi_0)a}{4\pi\mu}$$

$$v_2 = \rho_2 a - \frac{(\varphi - \varphi_0)a}{4\pi\mu}$$

Alles in Übereinstimmung mit früherer Theorie

Also Komplikationen erst bei Strömung längs gekrümmter Flächen

---

$$\Psi = 2\pi r^2 \int_0^\varphi \sin\varphi\, d\varphi \cdot \frac{\partial \Phi}{\partial r} = 2\pi r^2 c\left(1 - \frac{a^3}{r^3}\right)\int_0^\varphi \sin\varphi\cos\varphi\, d\varphi = 2\pi c\, r^2\left(1 - \frac{a^3}{r^3}\right)\frac{\sin^2\varphi}{2}$$

$$= \pi c\, \omega^2\left(\frac{1}{r} - \frac{a^3}{r^3}\right)$$

$$\cancel{\Delta\Psi_{r\,\mathrm{const}} = 2\omega\pi c\left(\frac{1}{r} - \frac{a^3}{r^3}\right)\Delta\omega = 2\pi c \sin\varphi\left(1 - \frac{a^3}{r^3}\right)\Delta\omega}$$

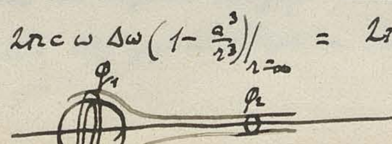
Für $\varphi = \frac{\pi}{2}$: $\quad \cancel{\Delta\psi = \pi c\left(1 + \frac{2a^3}{r^3}\right)\Delta r \Big|_{r=a} = \frac{3}{2}\pi c \Delta r}$

$$\psi = \pi c\left(r^2 - \frac{a^3}{r}\right)$$

$$\Delta\psi = \pi c\left(2r + \frac{a^3}{r^2}\right)\Delta r\Big|_{r=a} = \frac{3}{2} a\pi c\, \Delta r \qquad = \pi c\, \omega^2$$

Für r = const

$$\Delta\psi = 2\pi c\, \omega\, \Delta\omega\left(1 - \frac{a^3}{r^3}\right)\Big|_{r=\infty} = 2\pi c\, \omega\, \Delta\omega$$

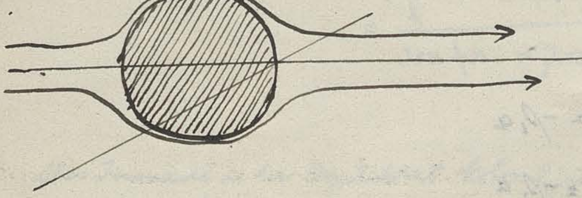
also 

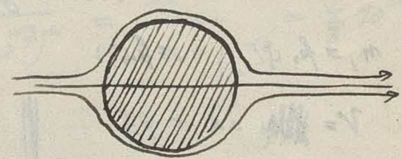
$\Phi_2 = \cancel{2a\pi\Delta r \frac{3}{2}} = \frac{3}{4}\Phi_1$

Im Fall der Elektrosmose:

Elektr. Stromlinien richtiger v_1 Linien

$\bar{v}$ Linien



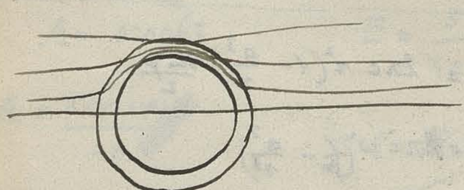


Unmöglich

In ∞ Abstand muss das Feld ungestört sein, also Querschnitt der tangierenden Strömungsröhre von derselben Grössenordnung wie Meridian-Querschnitt durch Doppelschicht, daher müsste auch v_1 in der ganzen Länge derselben von gleicher Grössenordnung sein!

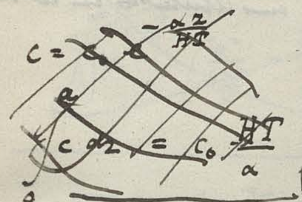
Wenn nicht so muss man $\operatorname{div}(n, v_1) = 0$ aufgeben!

Andere Möglichkeit: dass das Feld in der Doppelschichten-Randgegend entsprechend geschwächt ist. Dasselbe wäre der Fall, wenn z. B. anstatt Doppelsch. eine Schichte besser leitenden Materials vorhanden wäre. Auch dann würde überall $\operatorname{div}(n, v_1) = 0$ sein.



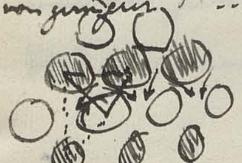
Danaben

φ	0·09	0·24	0·33	0·53	0·66	1·05	2·11
$\Delta\mu \cdot 10^5$	2	7 (22)	9	13 (24)	17 (38)	28	58 (148)
	2·2	2·9	2·7	2·5	2·6	2·7	2·7

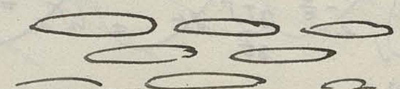


Idee zur Erklärung d. Viskositätsanomalie binärer Flüssigkeitsgemische im krit. Lösungspunkt:

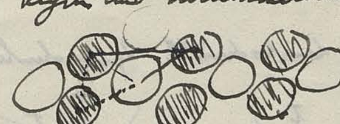
Bei Annäherung an krit. Lösungspunkt (von oben) erfolgt graduelle Trennung in 2 Phasen, so zwar dass diejenige von höherem Kohäsionsspannung kugelförmige Aggregate bildet, und die Zwischenräume von derjenigen von geringerer ↑ .. ausgefüllt werden, also ähnlich wie:



Würde Deformation der Flüssigkeit in homogener Weise vor sich gehen:



so würde Viskosität nur einen Mittelwert der beiden Visk. der Teile betragen. Infolge der Kohäsion setzen aber die inhomogenen Teile einer Formänderung Widerstand entgegen und suchen sich so zu bewegen wie Gemisch starrer Kugeln und intermediärer Flüssigkeit was natürlich eine kolossale Steigerung bedingt.



Vergleichsdaten:

Friedländer Zeitschr. f. ph. Ch. 38, 404, 1901 — μ_{H_2O} 1·12
Rothmund 63, 54, 1908 — μ Orthotoluidin 1·46 } μ krit. Gemisch = 3·68!
Wo. Ostwald Koll. Z. 12, 219, 1913

prozentuale Visk. Zunahme ↑ pro 1°: 34·3%!

Dabei ist es wohl denkbar, dass bei langsamem Vorgang die Kugeln sich starr verhalten, aber bei raschem sich deformieren. Also Abhängigkeit d. Visk. von Schergeschw. ist nicht ausgeschlossen.

$W(t)$ Wahrsch. dass vor Zeit t keine grössere Entfernung als a: (in der Ebene)

$$W = \frac{1}{4\pi D t}\int_0^a e^{-\frac{r^2}{4Dt}}\, 2\pi r\, dr = \int_0^{\frac{a^2}{4Dt}} e^{-z}\, dz = \left[1 - e^{-\frac{a^2}{4Dt}}\right]$$

Wahrsch., dass war zur Zeit t .. $r<a$
aber zur Zeit $t+dt$ $r>a$ $\Big\}$ = Wahrsch. eines Übertrittes (Eintrittes) zur Zeit $t - t+dt$

$$= \frac{dW}{dt}dt = -\frac{a^2}{4Dt^2}\, e^{-\frac{a^2}{4Dt}}\, dt$$

Erinnerungsfunction: $e^{-\frac{t}{\tau}}$

~~Durchschn. erinnerter Aufenthaltszeit~~

$$\cancel{\int_0^\infty e^{-\frac{t}{\tau}} \frac{\partial W}{\partial t}\, dt = W e^{-\frac{t}{\tau}} + \frac{1}{\tau}\int W e^{-\frac{t}{\tau}} dt}$$

Falls also seit t anwesend, so kommt dies mit $\int_0^t e^{-\frac{t}{\tau}} dt = \tau\left[1 - e^{-\frac{t}{\tau}}\right]$

in Rechnung; dies ist die äquivalente Zeitdauer

Also wird durchschnittl. äquivalente Zeitdauer:

$$-\int_0^\infty \frac{\partial W}{\partial t} dt \int_0^t e^{-\frac{t}{\tau}} dt = \cancel{-W\int_0^t e^{-\frac{t}{\tau}} dt\Big|_0^\infty} + \int_0^\infty e^{-\frac{t}{\tau}}\, W\, dt$$

$$= +\int_0^\infty e^{-\frac{t}{\tau}}\left[1 - e^{-\frac{a^2}{4Dt}}\right] dt = \tau - \underbrace{\int_0^\infty e^{-\left[\frac{t}{\tau} + \frac{a^2}{4Dt}\right]} dt}_{=\,?}$$

$$\frac{t}{\tau} + \frac{a^2}{4Dt} = z$$

$$t^2 - t\tau z + \frac{a^2\tau}{4D} = 0 \qquad t = \frac{\tau z}{2} \pm \sqrt{\left(\frac{\tau z}{2}\right)^2 - \frac{a^2\tau}{4D}}$$

$$dt = \frac{\tau}{2}\,dz \pm \frac{\frac{\tau^2}{4}\,z\,dz}{\sqrt{\frac{\tau^2 z^2}{4} - \frac{a^2\tau}{4D}}}$$

~~Im Grenzfall kleiner~~ $\cancel{\frac{a^2}{D}}$:

$$\cancel{\int_0^\infty e^{-\frac{t}{\tau}}\left[\frac{a^2}{4Dt} - \cdots\right]dt = \frac{a^2}{4D}\int_0^\infty e^{-z}\,\frac{dz}{z}}$$

$$\Sigma\left(e^{-\frac{t_1}{\tau}}\Delta t_1 + e^{-\frac{t_2}{\tau}}\Delta t_2 + \ldots\right)$$

Kriterium für Verschwinden des Teilchens infolge Br. B.:

Falls Teilchen an gewisser Stelle: Wahrsch. dass eine äquivalente Aufenthaltsdauer deutlich innerhalb gewisser Grenzen liege = ?

Integral dieser Wahrsch. bis zur Sichtbarkeitsdauer = Wahrsch. der Unsichtbarkeit

0·01279	0·01714	0·03702	0·01083
1004	1004	1004	1004
275	710	2698	79

$79 = \mu_0 [1 + 0·0787]$

$\widetilde{73\varphi}$

$\frac{5\,gr}{100\,cm^3}$

$\varphi\delta + (1-\varphi)\delta_0 = \bar{\delta}$

$\varphi\delta = 0·05$

$\varphi = \frac{0·05}{2} = 0·025$

ode

$\varphi = \frac{0·05}{1·90} = 0·0263$

$\frac{275 \cdot 2}{12·51} = \frac{11 \cdot 2}{5} = 4·4 \qquad \varphi = 0·0625$

$\frac{710 \cdot 2}{25·02} = 5·7$

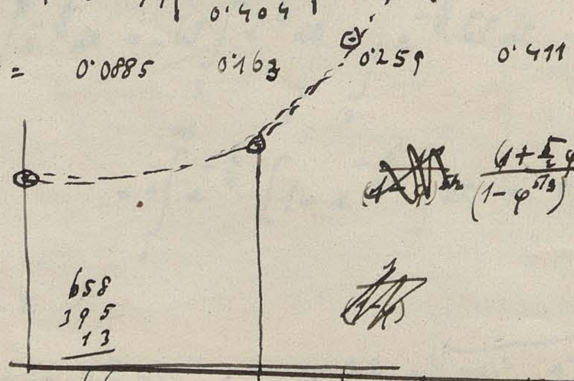
0·125

$\frac{2688 \cdot 2}{5003} = 10·8$

0·25

$\mu = \mu_0 (1 + 2\varphi)$

Schalt:	5	12·5	25	50
$\varphi =$	0·0263	$0·0658_3$	0·13165	$0·2633_3$
$\varepsilon =$	~~3·16~~	~~4·4~~	~~5·7~~	~~10·8~~
	2·99	4·16	5·37	10·2
$\frac{a}{b} =$	0·297	~~0·396~~ 0·404	0·509	0·641
$\left(\frac{a}{b}\right)^3 =$	0·0885	0·163	0·259	0·411



658
395
13
1066
533

$+ \frac{5}{2}\varphi + \frac{\frac{5}{2}\frac{3}{2}}{1 \cdot 2}\varphi^2$

$\frac{35}{8}$

$\frac{(1 + \frac{5}{2}\varphi)}{(1 - \varphi^{5/3})^2}$

$\varphi^{5/3} = \left(\frac{a}{b}\right)^5 = \varphi\left(\frac{a}{b}\right)^2$

8960
−4202
4758

6990
2788
$\lg\varphi$ 4202 −3

7378
−8184
6194

4200
0·3670 −3
0·4734 −1
0·9468 −2

2·0969
0·699

0972
2788
1·8184
0·0305 −2
0·6061
1·2122

2·3979
0·7993

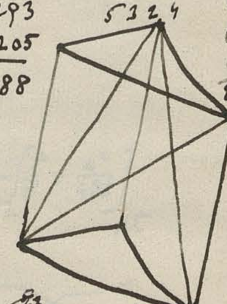
8494
−1194
7300

3982
2788
2·1194
7065
4130
5324

6993
2788
2·4205
8068
6136
0341
1761
2102

2687

4293
−4205
0088

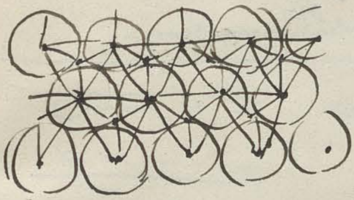


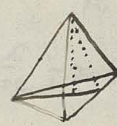
0·033

1052
26
3
108

Der Kugelsatz
Dieser grad ist wohl von Steffen

$\infty\ \infty\ \infty$ ganz anders als $\circ\ \circ\ \circ\ \circ\ \circ\ \circ$!





$\sqrt{a^2-\left(\frac{2}{3}\frac{a}{2}\sqrt{3}\right)^2} = a\sqrt{1-\frac{1}{3}} = a\sqrt{\frac{2}{3}}$

$a\cdot\frac{a}{2}\sqrt{3}\cdot a\sqrt{\frac{2}{3}}$

$\frac{x}{a}\ \frac{y}{\frac{a}{2}\sqrt{3}}\ \frac{z}{a\sqrt{\frac{2}{3}}}\cdot\frac{4}{3}\left(\frac{a}{2}\right)^3\pi : xyz =$

$\cancel{0.49715}$
$\cancel{-0.3891}$
$\cancel{0.10805}$
$\cancel{0.89195}$

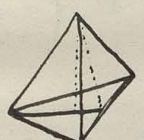
4771
1505
0.6276

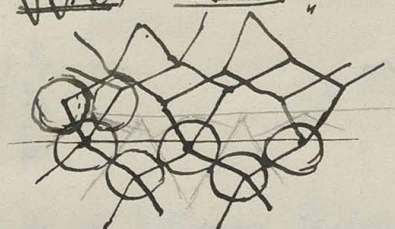
0.49715
0.6276
0.86955

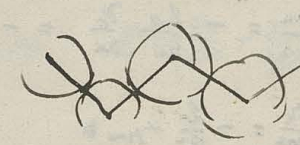
$\frac{\pi}{3\sqrt{2}} : 1 = \frac{\pi}{3\sqrt{2}} : 1$

$\boxed{0.13045}$

~~0.7298~~ 0.7406 „closest packing" $\frac{3\sqrt{2}}{\pi} = 1.3505$







$h = a\sqrt{\frac{2}{3}}$ || $Vol_{Tetr.} = \frac{a}{2}\,\frac{a}{2}\sqrt{3}\cdot\frac{a}{3}\sqrt{\frac{2}{3}} = \frac{a^3}{6\sqrt{2}}$

$Vol_{eingeschr.\ Kugel} = \frac{4}{3}\pi\left(\frac{1}{4}a\sqrt{\frac{2}{3}}\right)^3 = a^3\pi\cdot\frac{4}{3}\ \frac{2\sqrt{2}}{3\sqrt{3}}\ \frac{1}{64} = \frac{a^3\pi}{9\sqrt{3}\cdot4\sqrt{2}}$

$\frac{Vol_{Kugel}}{Vol_{Tetr.}} = \frac{\pi\cdot6\sqrt{2}}{9\sqrt{3}\ 4\cdot\sqrt{2}} = \frac{\pi}{6\sqrt{3}} = \frac{\pi}{2\sqrt{27}}$

0.49715
0.7157
0.78145 −1

~~= 0.6046~~

$= 0.3023$ „loosest packing"

Erörterung d. Effektes der Ungleichförmigkeit d. Geschwindigkeitsgefälles

$$\mu \frac{\partial}{\partial r}\left(\cancel{2\pi} r \frac{\partial u}{\partial r}\right) = -\cancel{2\pi} r \frac{\partial p}{\partial x} = \mu\left(\frac{\partial u}{\partial r} + r\frac{\partial^2 u}{\partial r^2}\right)$$

$$-\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = -c$$

$$u = \alpha(R^2 - r^2) \qquad \mu(-2\alpha - 2\alpha) = -c$$

$$= \frac{1}{4\mu}\frac{\partial p}{\partial x}\cdot(R^2 - r^2) \qquad \alpha = \frac{c}{4\mu}$$

$$\cancel{\frac{\partial u}{\partial r}} \quad u_{Max} = \frac{R^2}{4\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial r} = -\frac{2r}{4\mu}\frac{\partial p}{\partial x} = -\frac{2r}{R^2}u_{Max}$$

$$\frac{\partial^2 u}{\partial r^2} = -\frac{2}{R^2}u_{Max}$$

Erörterung d. relat. Translationsgeschw. $a^2\frac{\partial^2 u}{\partial r^2}$

Widerstandsarbeit pro sek.: $6\pi a\mu\left(a^2\frac{\partial^2 u}{\partial r^2}\right)^2 N$ Anzahl d. Kugeln pro cm³
pro cm³

~~Normale Widerstandsarbeit:~~ $\frac{\partial p}{\partial x}$

Zeitdauer d. Bewegung durch die Röhre. $\tau = \frac{l}{u}$

∴ Widerstandsarbeit bei Durchgang durch die Röhre: $\int \frac{6\pi a\mu\left(a^2\frac{\partial^2 u}{\partial r^2}\right)^2 N l}{\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2)}\cdot Vol$

Gesamte

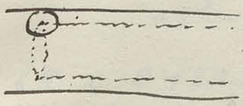
Normale Widerstandsarbeit : $Vol \cdot \frac{\partial p}{\partial x}\cdot l$

$$\frac{W_1}{W_0} = \frac{6\pi a \mu a^4 \left(\frac{\partial p}{\partial x}\right)^2 N l}{\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)^2 l\,(R^2 - r^2)} \qquad \left(\frac{1}{\pi R^2}\int_0^R \frac{2\pi r\,dr}{R^2 - r^2} = -\frac{1}{R^2}\lg(R^2 - r^2)\Big|_0^{R-a}\right.$$

$$= \frac{1}{R^2}\lg\left(\frac{2aR}{R^2}\right)$$

$$\frac{W_1}{W_0} = \frac{6\pi a^5 N}{R^2} \lg \frac{R}{2a} = \frac{9}{2}\varphi \cdot \frac{a^2}{R^2} \log\left(\frac{R}{2a}\right)$$

Somit verschwindend gering im Vergleich zur Einstein'schen Korrektion



Einengung d. Lösung

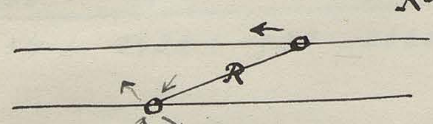
$$\varphi' = \varphi\left(\frac{R^2\pi - 2aR\pi}{R^2\pi}\right) = \varphi \frac{R}{R-2a} = \varphi \frac{1}{1-\frac{2a}{R}}$$

Ad Einsteins Rechnung:

Komponenten der durch Gegenwart der Kugel bedingten Zusatzgeschwindigkeit:

$$\left.\begin{matrix} u \\ v \\ w \end{matrix}\right\} \frac{5}{3} P^3 \cdot \begin{matrix} A \\ B \\ C \end{matrix} \left\{\begin{matrix} \frac{\xi}{\rho^3} \\ \frac{\eta}{\rho^3} \\ \frac{\zeta}{\rho^3} \end{matrix}\right. - \cdots \quad (\text{in Richtung } \pm 45^\circ) \quad \mathfrak{C} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \text{ etc.}$$

diese muss also auf die in Entfernung R befindliche Kugel P einen Druck ausüben: $\sim 6\pi P\mu \cdot \frac{5}{3}\frac{P^3}{R^3}\xi A$ und pro Zeiteinheit eine Arbeit leisten: $6\pi P\mu\left(\frac{5}{3}\frac{P^3}{R^3}\xi A\right)^2$



$$\int \frac{1}{R^2} \cos\varphi \; 2\pi R^2 \sin\varphi \, d\varphi \, dR$$

falls aber Kugeln überall gleich (dicht) verteilt wären würden sich alle jene Zusatz-Komp. kompensieren. In Wirklichkeit kommen die zufälligen Überschüsse zur Geltung.

Wahrsch., dass nur eine Kugel in der Schichte: $\frac{4R^2\pi\, dR}{dV}$: $W(R\cdot) = N \cdot dV\, e^{-\cdots}$ // $N = \frac{1}{\delta^3}$

$$N\int_{R=P} P^7 \frac{A^2\xi^2}{R^6} R^2\, dR = \frac{P^6}{\delta^3} \cdots A^2$$

wenn dies für alle Kugeln summiert wird, gibt es einen Ausdruck von Ordng φ^2 für $\Delta\mu$

auf grössten Distanzen sind bei Integrand an sich immer kleiner und überdies wird es immer unwahrscheinlicher dass keine Kompensation eintritt

Fremdlich Z. Kol. 12, 230, 1913 $\quad$ Al(OH)$_3$ Dichte 2'42 | Zandvoort p 170

49 $\quad$ 52

Al_2O_3 $\quad$ $Al_2O_3 + 3H_2O$

$$\begin{array}{r} 54 \\ \underline{48} \\ 102 \end{array} \qquad \begin{array}{r} 102 \\ 6 \\ \underline{48} \\ 156 \end{array}$$

$$\frac{0'187}{2'42} \cdot \frac{156}{102} = \qquad \begin{array}{r} 2718 \\ \underline{1931} \\ 4649 \\ \underline{3924} \\ 0725 \end{array} \qquad \begin{array}{r} 3838 \\ \underline{0086} \end{array}$$

$$0'1182 \ \frac{cm^3}{Liter} \quad \| \quad \varphi = 0'0000118$$

$$\frac{\Delta p}{p} = \frac{8}{52} = 0'154$$

$$k = \frac{0'154}{0'118} \cdot 10^3 = \underline{\underline{1'3 . 10^3}}$$

$$h = \frac{32}{3} \pi^3 \frac{(n-1)^2}{N \lambda^4} = \frac{32}{3} \pi^3 \frac{(2'944)^2 . 10^{-8}}{3 . 10^{19} . 5^4 . 10^{-20}} = \frac{32 . \pi^3 \, 2'944}{25 . 225} . 10^{-7}$$

$$= 1'53 . 10^{-7}$$

$$\begin{array}{r} '49715 . 3 \\ 1'49145 \\ '9378 \\ \underline{1'5051} \\ \underline{3'9344} \\ \underline{3'7501} \\ 0'1843 \end{array} \qquad 4689 \,\Big|\, \begin{array}{r} 1'3979 \\ \underline{2'3522} \\ 3'7501 \end{array}$$

$$\varepsilon' = \frac{\varepsilon \, q \, \omega . \cancel{x}}{2\pi \cancel{x} . 4\pi} h = \frac{\varepsilon \, r^2 \pi \, \omega}{8 \pi \pi} h = \frac{\varepsilon \, r \omega}{8} h \quad \Bigg| \quad \omega = \frac{3^2 \pi}{4 . 4 . 29^2 \pi} = \left(\frac{3}{4 . 29}\right)^2 =$$

$$\varepsilon = 15.000$$

$$= \frac{1'5 . 10^4 . 0'2}{8} . \left(\frac{3}{4 . 29}\right)^2 . 1'5 . 10^{-7}$$

$$= \frac{2'4}{8 . (40)^2} . 10^{-4} = \frac{0'3}{1'6} . 10^{-7} = 2 . 10^{-8} \quad \Bigg| \quad \varphi = 4\pi\varepsilon' . 2\pi x = 8\pi . 0'2 . \varepsilon'$$

$$= 314 . 16 . \varepsilon'$$

$$\begin{array}{r} 188 \\ \hline 502 \end{array} \qquad = 5 . \varepsilon'$$

$$= 10^{-7}$$

mur blanc à la plus lune = 45° : 10^{-6}

~~$\varphi : 1 = 20^2 : (12 . 10^5)^2$~~ $\qquad \alpha = \frac{(12 . 10^5)^2}{20^2} 10^{-7} = \frac{1}{4} 10^3$

$H = \dfrac{13{\cdot}6 \cdot [illegible] \cdot 76}{0{\cdot}001293 \cdot [illegible]}$

$= 8\,km$

$lH = 0{\cdot}1222$

$e^{-lH} = 0{\cdot}8856$

1·1335
1·9912
1·8808
6·0055
− 1116 +3
8·8939
0·1843 −7
2·0782

0·4343
8686
869
87
−·0530|7²
0·94693

1·1335
1·8808
3·0143
−0·1116 +3
5·9027
0·1843 −7
0·0870 −1
~~−0·9120.~~ 0·4343
3648
2736
365
27
−0·3960Z
= 0·6039 −1

$\frac{1}{e}$

Wouldn't k?

$\frac{13}{93}$ $\frac{21}{190}$ $\frac{45}{288}$ $\frac{57}{337}$ $\frac{98}{385}$ $\frac{457}{490}$ $\cdot 10^{-4} \cdot 10^{5}$ / 10^{-6}.

1139	3222	6532	7559	9912	6599
9685	2788	4594	5276	5855	6902
1454-1	0434-1	1938	2283	4057	9697

Koagulationsproblem:

Irgend eines der Teilchen kann man herausfassen und die relativen Bewegungen der übrigen in Bezug auf jenes ins Auge fassen

$$\overline{\Delta\xi^2} = \overline{(\Delta x_1 - \Delta x_2)^2} = \overline{\Delta x_1}^2 + \overline{\Delta x_2}^2 - 2\,\cancel{\overline{\Delta x_1 \Delta x_2}} = 2\,\overline{\Delta x^2}$$

Also finden die relativen Bewegungen so statt als ob die Diff-Konstante verdoppelt wäre.

Nun Frage, wie gross ist die W., dass eines der ~~t~~ beweglichen T. an das ruhende zum ersten Male in der Zeit t — $t+dt$ stösst?

Wahrsch., dass bis zur Zeit t keines angeklebt sei, berechnet sich nach Diffusionstheorie, wie früher dargelegt, falls dabei nur das Ankleben an dem herausgefassten und nicht der beweglichen untereinander berücksichtigt wird.

(1) $W = \ldots e^{-\ldots nt}$

In Wirklichkeit vermindert sich sowohl die Anzahl wie die Beweglichkeit der bewegl. T., durch die ~~da~~ Doppelteilchenbildung etc.

Also gibt Formel (1) einen zu raschen Koagulationsverlauf für die späteren Stadien

(und zwar desto mehr wenn nur Bildung von Doppelteilchen gemessen wird)

Wenn also ~~nur~~ ausschliesslich die Zusammenstösse der ursprünglichen Teilchen ins Auge gefasst werden so vielleicht so: Anzahl n der beweglichen verringert sich in gleichem Masse wie W

also $W = \ldots e^{-\ldots n_0 W t}$

Das sollte aber nicht im Zeitgesetz sondern in der Diff Gleichg. geschehen!

ob das wahr ist?

$$W(\xi)\,d\xi = \int W(x_1)\,dx_1\, W(x_2)\,dx_2 = \int_{x_1=-\infty}^{+\infty} W(x_1)\,dx_1 \int W(\xi - x_1)\,d\xi$$

$$= \frac{1}{(\sqrt{2\pi Dt})^2}\int e^{-\frac{(\xi-x)^2}{2Dt} - \frac{x^2}{2Dt}}\,d\xi\,dx = \frac{d\xi}{2\pi Dt}\cdot e^{-\frac{\xi^2}{2Dt}}\int e^{-\frac{x^2}{Dt}+\frac{x\xi}{Dt}}\,dx$$

$$= \frac{e^{-\frac{\xi^2}{2Dt}}\,d\xi}{2\pi Dt}\int_{-\infty}^{\infty} e^{-\frac{(x-\frac{\xi}{2})^2}{Dt}+\frac{\xi^2}{4Dt}}\,dx = \frac{e^{-\frac{\xi^2}{2Dt}+\frac{\xi^2}{4Dt}}}{2\sqrt{\pi Dt}}\,d\xi = \frac{e^{-\frac{\xi^2}{4Dt}}}{\sqrt{4\pi Dt}}\,d\xi$$

also wirklich so als ob $2D$ anstatt D Warum

Eindimensionale Diffusion gegen feste Wand

Wahrsch. dass bis t nicht angeklebt, falls ein Teilchen auf Schichte $0 \ldots H$

$$W' = 1 - \frac{2}{H}\sqrt{\frac{Dt}{\pi}}$$

Falls n Teilchen auf jener Schichte, Wahrsch. dass bis t keines derselben angeklebt:

$$W_0 = (W')^n = e^{-2\nu\sqrt{\frac{Dt}{\pi}}}$$

$$\lg W = -2\nu\sqrt{\frac{Dt}{\pi}}$$

$$\frac{1}{W}\frac{dW}{dt} = -\nu\sqrt{\frac{D}{t\pi}}$$

Wahrsch. dass bis t $(n-1)$ nicht angeklebt aber einer angeklebt (irgend einer unter den n)

$$W_1 = (W')^{n-1}\,\frac{2}{H}\sqrt{\frac{Dt}{\pi}}\binom{n}{1} = 2\nu\sqrt{\frac{Dt}{\pi}}\;e^{-2\nu\sqrt{\frac{Dt}{\pi}}}$$

Wahrsch. dass bis t irgend welche zwei unter den n angeklebt, die übrigen aber nicht

$$W_2 = \frac{1}{2}\left(2\nu\sqrt{\frac{Dt}{\pi}}\right)^2 e^{-2\nu\sqrt{\frac{Dt}{\pi}}}\;*$$

Tatsächlich ist dann $\Sigma(W_0 + W_1 + W_2 + \ldots) = 1$

und durchschnittl. Anzahl der bis t angefallenen Teilchen ist:

$$2\nu\sqrt{\frac{Dt}{\pi}} = \beta$$

$$0\,e^{-\beta} + 1\beta e^{-\beta} + 2\frac{\beta^2}{2}e^{-\beta} + 3\frac{\beta^3}{3!}e^{-\beta} + 4\frac{\beta^4}{4!}e^{-\beta} + \cdots \Big]$$

$$= e^{-\beta}\beta\left[1 + \beta + \frac{\beta^2}{2} + \frac{\beta^3}{3!} + \cdots\right] = \beta = 2\nu\sqrt{\frac{Dt}{\pi}}$$

Allgemeine Form

$$W_{(n)} = \frac{e^{-\nu\varphi}(\nu\varphi)^n}{n!}$$ Falls: $\overline{n W_{(n)}} = \nu\varphi$ und falls Unabhängigkeit der Einzelereignisse (für jedes Molekül)

Falls Abgang der Teilchen durch Diff. Sukzessive Änderung?

$$\frac{\partial}{\partial t}(We^{\nu\varphi}) = D\frac{\partial^2}{\partial x^2}(We^{\nu\varphi})$$

$$W e^{\nu\varphi}\cdot\nu\frac{\partial\varphi}{\partial t} + e^{\nu\varphi}\frac{\partial W}{\partial t} = D e^{\nu\varphi}\frac{\partial^2 W}{\partial x^2}$$

$$\frac{\partial W}{\partial t} + W\nu\frac{\partial\varphi}{\partial t} = D\frac{\partial^2 W}{\partial x^2}$$

Andere Änderung: $W_0 = e^{-\nu\varphi}$

$$\frac{\partial W}{\partial t} = e^{-\nu\varphi}\cdot\nu\frac{\partial\varphi}{\partial t}$$ $\nu = \nu_0 W_0$

$$\frac{\partial W}{\partial t} = e^{-\nu_0 W\varphi}\cdot\nu_0 W\cdot\frac{\partial\varphi}{\partial t}$$

Oder auch:

$\nu = e^{-\nu\varphi}$

$\lg \nu = -\nu\varphi$

$\varphi = -\frac{\lg\nu}{\nu}$

Voraussetzungen:

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial r^2}$$

$\underline{t=0}$: überall $=0$ mit Ausnahme $\underline{r=b}$

$$v = \lim \alpha (r+R)\, e^{-\frac{(r+R-b)^2}{\varepsilon}}$$

$$= \lim \frac{r+R}{\cancel{\varepsilon b^2 \sqrt{\pi}}}\, \cancel{e^{-\frac{(r+R-b)^2}{\varepsilon^2}}}$$

$$= \Phi(r) \quad = \lim \frac{r+R}{\varepsilon (b+R)^2 \sqrt{\pi}}\, e^{-\frac{(r-b)^2}{\varepsilon^2}}$$

$$\int v (r+R)\, dr = \int_0^\infty \rho^2 e^{-\frac{(\rho-b)^2}{\varepsilon^2}} d\rho$$

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$$= \varepsilon b^2 \underbrace{\int e^{-\frac{\xi^2}{\varepsilon^2}} d\xi}_{\sqrt{\pi}} = 1$$

$$\alpha\, \varepsilon\, b^2 \sqrt{\pi} = 1$$

$\underline{r=0}$ $\quad v=0$

$\underline{r=A}$ $\quad \frac{\partial v}{\partial r} = \frac{v}{R+A}$

$$v = \sum B_n\, e^{-a^2 \beta_n^2 t} \sin \beta_n r \qquad B_n = 2\, \frac{(A+R)^2 \beta_n^2 + 1}{A(A+R)^2 \beta_n^2 - R} \int_0^A \Phi(r) \sin \beta_n r\, dr$$

$$\lim \int_0^A \frac{(r+R)\, e^{-\frac{(r+R-b)^2}{\varepsilon^2}}}{\varepsilon b^2 \sqrt{\pi}} \sin \beta_n r\, dr = \cancel{\frac{1}{b} \sin \beta_n (b-R)}$$

$$= \frac{1}{b+R} \sin \beta_n b \qquad \beta_n = \frac{2n+1}{2} \pi \frac{1}{A} = x$$

$$v_n = \frac{2}{b+R} \sum \frac{(A+R)^2 \beta_n^2 + 1}{A(A+R)^2 \beta_n^2 - R} \sin \beta_n r \sin \beta_n b\; e^{-D^2 \beta_n^2 t}$$

$$\underbrace{\frac{1}{A} \sum_{\Delta x = \frac{\pi}{A}} \sin rx \sin bx\; e^{-Dx^2 t}}$$

$$= \frac{1}{\pi} \int_0^\infty e^{-Dx^2 t} \sin rx \sin bx\, dx$$

$$\lim B_0 = \frac{2}{b+R}\, \frac{(A+R)^2 \beta_0^2 + 1}{A(A+R)^2 \beta_0^2 - R} \sin \beta_0 b \qquad \beta_0 = \frac{1}{A} \sqrt{\frac{3R}{A}}$$

$$= \cancel{\frac{2}{b+R}\, \frac{A}{2R} \sin \beta_0 b} = \frac{2b}{b+R}\, \frac{1}{A} \sqrt{\frac{3R}{A}}\, \frac{(A+R)^2 \cdot 3R + A^3}{A(A+R)^2 \cdot 3R - A^3 R} = 0$$

$$\int_0^\infty e^{-Dx^2 t}\underbrace{\sin bx \sin rx}_{\frac{1}{2}\left[\cos(b-r)x - \cos(b+r)x\right]}\, dx = \frac{1}{4}\sqrt{\frac{\pi}{Dt}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$

$$v = \frac{\alpha\sqrt{\frac{1}{4Dt\pi}}}{b+R}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$

Wenn nun gebildet wird:

$$V = c\int_{b=0}^{\infty} 4\pi(b+R)^2 u\, db = c\cdot 4\pi\int_{b=0}^{\infty}(b+R)\, v\, db$$

so sollte dies dem früheren Beispiel entsprechen, wo $\left. v = c(r+R)\right|^{t=0}$ war

$$V = \frac{4\pi c}{\sqrt{\pi}}\int_0^\infty \underbrace{\frac{(b+R)}{\sqrt{4Dt}}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]\frac{db}{\sqrt{4Dt}}\cdot\sqrt{4Dt}$$

$\cancel{b = r + R = r}$

$$= \frac{4c}{\sqrt{\pi}}\sqrt{4Dt}\int_0^\infty(\beta+P)\left[e^{-(\beta-\rho)^2} - e^{-(\beta+\rho)^2}\right]d\beta$$

$$\left\{\left[\beta-\rho+P+\rho\right]e^{-(\beta-\rho)^2} - \left[\beta+\rho+P-\rho\right]e^{-(\beta+\rho)^2}\right\}d\beta$$

$$\cancel{= \frac{1}{2}\left[-e^{-(\beta-\rho)^2} + e^{-(\beta+\rho)^2}\right]_0^\infty} + \cancel{(P-\rho)\int e^{-(\beta-\rho)^2} - e^{-\beta}}$$

$$+(P+\rho)\int_0^\infty e^{-(\beta-\rho)^2}d\beta - (P-\rho)\int_0^\infty e^{-(\beta+\rho)^2}d\beta$$

$$= (P+\rho)\left[\frac{\sqrt{\pi}}{2} + \int_0^\rho e^{-z^2}dz\right] - (P-\rho)\left[\frac{\sqrt{\pi}}{2} - \int_0^\rho e^{-z^2}dz\right] = \rho\sqrt{\pi} + 2P\int_0^\rho e^{-z^2}dz$$

Kontaktbestimmung:

$$= 4\pi(R+b)^2\, db\cdot c$$

$$\alpha\int_{r=0}^{\infty}\underbrace{(R+r)^2 u\, dr} = \alpha\int(R+r)\, v\, dr$$

$$= \alpha(R+b)$$

$$\alpha = (R+b)\, db\cdot c\cdot 4\pi$$

$$v = \frac{(R+b)\, db\cdot c}{\sqrt{4Dt\pi}}\left[e^{--} - --\right]4\pi$$

$$V = \sum_{b=0}^{\infty} v$$

$$= \frac{c\cdot 4\pi}{\sqrt{4\pi Dt}}\int(b+R)\left[e^{-\frac{(b-r)^2}{4Dt}} - \ldots\right]db$$

$$V = \frac{4c\pi}{\sqrt{\pi}}\frac{\sqrt{4Dt}}{}\left[\rho\sqrt{\pi} + 2\rho\int_0^{\rho} e^{-z^2}dz\right] = 4c\pi\left[r + 2\frac{R}{\sqrt{\pi}}\int_0^{\frac{r}{2\sqrt{Dt}}} e^{-z^2}dz\right]$$

$$u = \frac{V}{r+R}$$ stimmt mit früheren Resultat mit Ausnahme des Faktor 4π

---

also ist u für eine in Entfernung $b+R$ befindliche Quelle ~~...~~ wahre:

$$u = \frac{v}{\cancel{...}(R+b)} = \cancel{\frac{\alpha}{R+b}\frac{}{\sqrt{4Dt\pi}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]}$$

und für einen in Entfernung $b+R$ befindlichen Quellpunkt:

$$u = \left(\frac{v\,\cancel{...}}{4\pi(R+b)^3\,db}\right) = \frac{c}{4\pi(R+b)^2\sqrt{4Dt\pi}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$

$$\left.\frac{\partial u}{\partial r}\right|_{r=0} = \frac{c}{4\pi(R+b)^2\sqrt{4Dt\pi}}\left[\frac{b-r}{2Dt}\cdot e^{-\frac{(b-r)^2}{4Dt}} + \frac{b+r}{2Dt}\cdot e^{-\frac{(b+r)^2}{4Dt}}\right]_{r=0} = \frac{c}{(R+b)^2\sqrt{4Dt\pi}}\frac{b}{Dt}\frac{e^{-\frac{b^2}{4Dt}}}{4\pi}$$

(Konstante γ so bestimmen, dass $\int 4\pi u(R+b)^2 db = c = 4\pi c\gamma$ somit $\gamma = \frac{1}{4\pi}$)

$$J_R = 4\pi\left.\frac{\partial u}{\partial r}\right|_{r=0} R^2 D = \frac{cR^2D}{(R+b)^2\sqrt{4Dt\pi}}\frac{b}{Dt}e^{-\frac{b^2}{4Dt}}$$

$$\frac{b}{2\sqrt{Dt}} = z \qquad -\frac{b\,dt}{4\sqrt{Dt^3}} = dz$$

Gesammte mit $t=\infty$ eingeflossene Menge:

$$\varphi = D\frac{cR^2}{(R+b)^2\sqrt{\pi}}\int_0^t \frac{b}{Dt\sqrt{4Dt}}e^{-\frac{b^2}{4Dt}}dt = D\frac{cR^2}{(R+b)^2\sqrt{\pi}}\cdot\frac{2}{D}\cdot\int_{\infty}^{\frac{b}{2\sqrt{Dt}}} e^{-z^2}dz$$

bis $t=\infty$ würde also einfliessen: $\varphi_\infty = \frac{cR^2}{(R+b)^2}$

Daraus für den Fall einer bis $R=\infty$ homogenen Raumerfüllung

$$J = \frac{cR^2\,4\pi}{t\sqrt{\pi}}\int_{b=0}^{\infty}\frac{b}{\sqrt{4Dt}}e^{-\frac{b^2}{4Dt}}db = \frac{4cR^2\sqrt{\pi}}{\sqrt{t}}$$

$$v = \frac{\alpha}{\sqrt{4Dt\pi}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$ für Quellsphärenschale

~~Also für in Entfernung b+R befindlichen Quellpunkt:~~

$$\cancel{u = \frac{1}{r+R}\,\frac{\alpha}{4\pi(R+b)^2 db}\,\frac{1}{\sqrt{4Dt\pi}}\left[e^{\cdots} - \right]}$$

~~Bedingung für α:~~ $\cancel{\int 4\pi(R+b)^2\,\alpha\,db}$

$$\int_0^\infty 4\pi(R+b)^2\,db\cdot u = \frac{1}{r+R}\left[r + \frac{2R}{\sqrt{\pi}}\int_0^{\frac{r}{2\sqrt{Dt}}} e^{-z^2}\,dz\right]$$

$$= \frac{\alpha}{r+R}\int_0^\infty \frac{(R+b)}{\sqrt{4Dt\pi}}\left[e^{\cdots} - \cdots\right]db$$

$\alpha = 1$

Bedingung für α: $\qquad u = \frac{v}{r+R}$

$$\lim_{t=0}\left[\int_0^\infty 4\pi(r+R)^2\,u\,dr = 1 = \int_0^\infty 4\pi(r+R)\,v\,dr\right.$$

$$= 4\pi(R+b)\,\alpha$$

Also: für Quellsphärenschale:

$$v = \frac{1}{4\pi(R+b)}\,\frac{1}{\sqrt{4\pi Dt}}\left[e^{-\frac{(b-r)^2}{4Dt}} - e^{-\frac{(b+r)^2}{4Dt}}\right]$$

$\boxed{\text{stimmt vollständig}}$

für Quellpunkt dasselbe!

Also für homogene Raumerfüllung:

$$v = \int_0^\infty 4\pi c(R+b)^2\,db\,\frac{1}{4\pi(R+b)}\,\frac{1}{\sqrt{4\pi Dt}}\left[e^{\cdots}\right] = c\overbrace{\int_0^\infty \frac{(R+b)}{\sqrt{4\pi Dt}}\left[e^{\cdots}\right]db}^{= r + \frac{2R}{\sqrt{\pi}}\int e^{-z^2}dz}$$

Für Quellpunkt:

$$J_R = +4\pi D R^2 \frac{\partial u}{\partial r}\Big|_{r=0} = \cancel{\ldots} = \cancel{4\pi D\left(r\frac{\partial v}{\partial r} + v\right)\Big|_{r=0} = -4\pi D v_{r=0}}$$

$$= +4\pi D R^2 \frac{\partial}{\partial r}\left[\frac{v}{(r+R)}\right]\Big|_{r=0} = +4\pi D R^2\left[\frac{1}{(r+R)}\frac{\partial v}{\partial r} + v\right] = +4\pi D R^2\left[\frac{1}{R}\frac{\partial v}{\partial r}\right]_{r=0}$$

$$\frac{\partial v}{\partial r}\Big| = \frac{1}{4\pi(R+b)}\frac{1}{\sqrt{4\pi D t}}\left[\frac{b-r}{2Dt}e^{-\frac{(b-r)^2}{4Dt}} + \frac{b+r}{2Dt}e^{-\frac{(b+r)^2}{4Dt}}\right]_{r=0} = \frac{1}{4\pi(R+b)}\frac{1}{\sqrt{4\pi D t}}\frac{b}{Dt}e^{-\frac{b^2}{4Dt}}$$

$$v\Big| = \frac{1}{4\pi(R+b)}\frac{1}{\sqrt{4\pi D t}} = 0$$

$$J_R = \frac{R}{R+b}\frac{b}{\sqrt{4\pi D t^3}}e^{-\frac{b^2}{4Dt}}$$

Für homogene Raumerfüllung:

$$J = 4\pi c\int_{b=0}^{\infty} J_R (R+b)^2\, db = 4\pi c R\int_0^{\infty}\frac{(R+b)\,b}{\sqrt{4\pi D t^3}}e^{-\frac{b^2}{4Dt}}\,db$$

$$= \frac{4\pi c}{\sqrt{\pi}} R\left[\frac{R}{\sqrt{t}}\frac{b\,e^{-\frac{b^2}{4Dt}}}{4Dt}db\cdot\sqrt{4D} + \frac{b^2 e^{-\frac{b^2}{4Dt}}db}{(4Dt)^{3/2}}\cdot 4D\right]$$

$$= \frac{4\pi c}{\sqrt{\pi}}\left[\frac{R^2}{\sqrt{t}}\frac{\sqrt{4D}}{2} + 4DR\int_0 \underbrace{z^2 e^{-z^2}dz}_{\frac{\sqrt{\pi}}{4}}\right]$$

$$= 4\pi c D R\left[1 + \frac{R}{\sqrt{Dt\pi}}\right] \quad \boxed{\text{stimmt}}$$

$$\frac{b^2}{4Dt} = z^2 \qquad -\frac{b^2}{4Dt^2}dt = 2z\,dz \qquad \frac{dt}{t} = \frac{2\,dz}{z}$$

Gesammte seit $t=0$ eingeflossene Quantität:

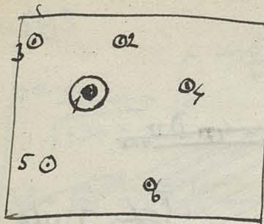
$$Q = \int J_R\,dt = \frac{R}{R+b}\frac{1}{\sqrt{\pi}}\int_0^t \frac{b}{\sqrt{4Dt}}\frac{e^{-\frac{b^2}{4Dt}}}{t}dt = -\frac{R}{R+b}\frac{1}{\sqrt{\pi}}\cdot 2\int_{\infty}^{\frac{b}{2\sqrt{Dt}}}\cancel{z}\,e^{-z^2}\frac{dz}{\cancel{z}}$$

nach langer Zeit fließt ein:

$$\boxed{Q_\infty = \frac{R}{R+b}}\,!$$

also Unterschied gegen 2 dimensionalen Fall, wo für $t=\infty$; $Q_\infty = 1$

In der früheren Rechnung war



Wahrsch. dass bis zur Zeit t keine Anlagerung stattgefunden habe

W_t = Wahrsch. dass $\underbrace{\text{weder (2) noch (3) noch (4)}}$ ... an (1) angelagert seien

wobei Anzahl der Partner ($n_0 - 1$)

$= W_{t2} \cdot W_{t3} \cdot W_{t4} \cdots = (W)^{n_0-1}$

Dagegen in Wirklichkeit, falls es sich um die Bildung von Doppelteilchen handelt, ist

W_{t2} = Wahrsch. dass sich (2) nicht angelagert habe und zwar weder falls es einfach ist noch falls es

Man muss dann unterscheiden: ~~die Wa~~ die Bildungsgeschw. der Doppelteilchen

d.h. den Teilbetrag der Abnahme der Zahl der Einzelteilchen, welcher von Bildung von Doppelteilchen herrührt:

erhält man aus

$$\frac{dW_1}{dt} \qquad \text{wobei } W_1 = W^{n-1}$$

dabei ist W = Wahrsch. dass

$$n_0 = n_1 + 2n_2 + 3n_3 + \cdots$$

$$\frac{d}{dt}n_1 = -2\frac{dn_2}{dt} - 3\frac{dn_3}{dt} - \cdots$$

Damit sich ein Doppelteilchen bildet, müssen die zusammentreffenden einfach sein, ~~also~~

~~Wahrsch. dass~~ Wahrsch. für eine Doppelbildung in Zeitmoment dt:

$$W_{(1+1)}\,dt$$

$$-\frac{dn_1}{dt} = n_1\left[W_{(1+1)} + W_{(1+2)} + W_{(1+3)} + \cdots\right] =$$

$$\frac{dn_2}{dt} = \frac{1}{2} n_1 W_{(1+1)} - n_2\left[W_{(2+1)} + W_{(2+2)} + W_{(2+3)} + \cdots\right]$$

$$\frac{dn_3}{dt} = n_1 W_{(1+2)} - n_3\left[W_{(3+1)} + W_{(3+2)} + \cdots\right]$$

Dabei ist allgemein $n_{k} W_{k+i} = n_{i} W_{i+k}$

$$-\frac{dn_1}{dt} = n_1\left[W_{1+1} + W_{1+2} + W_{1+3} + \cdots\right] = n_1 W_{1+1} - 2n_2\left[W_{2+1} + W_{2+2} + \cdots\right]$$
$$+ 3n_1 W_{1+2} - 3n_3\left[W_{3+1} + W_{3+2} + \cdots\right]$$
$$+ 4n_1 W_{1+3} + 2n_2 W_{2+2} -$$
$$- 4n_4\left[W_{4+1} + W_{4+2} + \cdots\right]$$

$$= n_1 W_{1+1} + 3n_1 W_{1+2} - 2n_2 W_{2+1}$$
$$+ 4n_1 W_{1+3} + \cancel{2n_2 W_{2+2}} - 3n_3 W_{3+1} - \cancel{2n_2 W_{2+2}}$$

$$= n_1 W_{1+1} + n_1 W_{1+2} + n_1 W_{1+3} + \cdots \quad \boxed{\text{stimmt}}$$

Der bimolekularen chemischen Reaktion ist

$\frac{dn_1}{dt} = n_1 W_{1+1}$ und: $W_{1+1} = (n_1 - 1)\alpha$ falls Voraussetzung, dass auf viele Zusammenstösse nur eine günstige stattfindet, so dass jeden

Moment die Flüssigkeit als vollständig durchgemischt angesehen werden kann, dass also momentaner Vorgang unabhängig von der Art der Vorgeschichte

Charakteristikum der üblichen chemischen Kinetik: jeder Folgezustand kann als momentaner Anfangszustand angesehen werden.

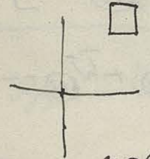
Bei Prozessen, für welche das wahr ist, müsste Intensität der Durchmischung ohne Einfluss sein auf den Reaktionsverlauf

Versuche bei rasch reagierenden Substanzen in sehr grosser Verdünnung!

Im Gegenteil dazu müsste Koagulationsverlauf in hohem Grade abhängen von Rührgeschwindigkeit

Diffusion in einem in Scherungsbewegung befindlichen Medium

$$\frac{\partial c}{\partial t} = D \frac{\partial c}{\partial x}\Big|_{x+\Delta x} + (uc)_{x+\Delta x} - D \frac{\partial c}{\partial x}\Big|_{x} + (uc)_x + \cdots$$



$$= D \nabla^2 c - \left[\frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial z}(wc)\right]$$

$w = 0$

$v = 0$

$u = \beta y$

$$\frac{\partial c}{\partial t} + \beta y \frac{\partial c}{\partial x} = D \nabla^2 c$$

Allgemeine Ähnlichkeitsgesetze

1). Abhängigkeit von D resp. μ:

Koagulationsgeschw. in beliebigem Stadium $\sim \frac{\Theta}{\mu}$

Koagulationszeit (für beliebigen Teilprozess) $\sim \frac{\mu}{\Theta}$

2). Abhängigkeit von Lineardimensionen

Falls alle Lineardim. im Verh. α verringert, wird $R \cdots \to \alpha R_0$

$n \cdots \to \frac{n_0}{\alpha^3}$

dann ist

$D \cdots \frac{x^2}{t}$ $\qquad \underline{\underline{t \sim \alpha^2}}$

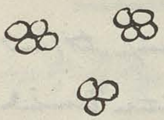
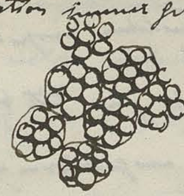
z.B. $t = \frac{1}{4\pi n D R} = \frac{\alpha^3}{4\pi n_0 D \alpha R_0} = \alpha^2 t_0$ $\boxed{\text{stimmt}}$

für bei $t=0$

$n\sqrt{D} \cdot R^2 \sqrt{t} = \cdots$

$t = \frac{1}{D n^2 R^4} \quad \frac{\alpha^6}{n_0^2 R_0^4 \alpha^4} = \alpha^2 t_0$ $\boxed{\text{stimmt}}$

Raumerfüllung bei fortschreitender Koagulation immer geringer:

Anfang  Später 

Denn die Zwischenräume bilden immer einen grösseren Prozentsatz des entstehenden Aggregats

Zwischenraum von Größenordnung der koagulierenden Teilchen

Darum dürfte Raumerfüllung auch abhängen von Konzentr. d. koagulierenden Subst.

Wenn man so argumentiert:

$$W = e^{-4\pi n D R t}$$

$$\frac{dW}{dt} = -4\pi n D R\, W$$ = relat. Anzahl der T zum ersten Mal sich anlegenden

darin sollte aber nicht die Anzahl n_0 der ursprünglich vorhandenen Teilchen, sondern nur mehr die Anzahl $\underbrace{n_0 W}_{n}$ der zur Zeit t noch freien Teilchen stehen,

also:

$$\frac{dW}{dt} = -4\pi n_0 W^2 DR$$ oder auch $$\frac{dn}{dt} = -4\pi n^2 DR$$

$$\frac{dW}{W^2} = -4\pi n_0 DR\, dt$$ $$\frac{dn}{n^2} = -4\pi DR\, dt$$

also entsprechend der chemischen Kinetik einer Reaktion zweiter Ordnung

$$n = \frac{n_0}{1 + 4\pi n_0 DRt}$$

infolge Einflusses der Relativbewegung ist aber D zu verdoppeln, also

$$n = \frac{n_0}{1 + 8\pi n_0 DRt}$$

Dies wäre folgendermaßen zu begründen: Für die Anzahl der noch zur Zeit t anlegenden Teilchen kann man die ursprüngliche Formel verwenden, wenn darin die Zahl n_0 durch n ersetzt wird, denn es wird auf dasselbe hinauskommen ob unter den n_0 ein grösserer Prozentsatz erfolge vorherige Anlagerung an andere Teilchen ausgeschaltet wurde oder aber ob von vornherein für die zur Zeit t anlagernden nur die zu jener Zeit bestehende Dichte eingesetzt (und die übrigen von vornherein von der Betrachtung ausgeschlossen wurden) und ihre weitere Verminderung ignoriert wird.

Dabei ist allerdings vorausgesetzt, dass die Verteilung jener n im Raume gleichmässig ist.

Ob das genau richtig?

Aber anderseits: Verminderung erfolgt nicht nur durch Anlagerung einfacher sondern auch mehrfacher Teilchen, also ist die D. Gl. zu ergänzen durch Berücksichtigung derselben

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Unter den früheren vereinfachten Annahmen (nur Ankleben an die herausgehobene Kugel)

Wahrsch. dass bis t gerade	0	Teilchen angeklebt und die übrigen n	Teilchen nicht angeklebt waren	$= e^{-\nu\varphi}$
	1	$n-1$		$\nu\varphi\, e^{-\nu\varphi}$
	2	$n-2$		$\frac{(\nu\varphi)^2}{2}\, e^{-\nu\varphi}$

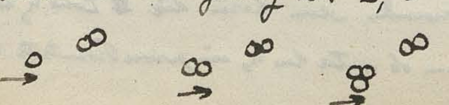
also Zuwachsgeschw. der Doppelteilchen:

$$= \frac{1}{2}\frac{d}{dt}\left(\nu\varphi\, e^{-\nu\varphi}\right) = \underbrace{\nu\frac{d\varphi}{dt}\, e^{-\nu\varphi}}_{\text{sich vereinigende Paare}} - \underbrace{\nu^2\varphi\,\frac{d\varphi}{dt}\, e^{-\nu\varphi}}_{\text{entstehende Dreieraggregate}} = \nu\frac{d\varphi}{dt}\left(1-\nu\varphi\right)e^{-\nu\varphi}$$

$$= -\frac{d}{dt}\left(e^{-\nu\varphi}\right)$$

In Wirklichkeit ist nun der erste Teil (Zuwachs durch Vereinigung einzelner T.) ebenso wie früher zu ersetzen durch $\cancel{\frac{d(nW)}{dt}}$ $-\frac{d}{dt}\left[e^{-n_0 W\varphi}\right]\cdot n_0 W$

dagegen erfolgt Abnahme der Doppelteilchen nicht nur infolge Übergang in Dreier sondern auch durch Anlagerung von 2, 3 .... an ein Doppelteilchen



etc. und hiefür sind verschiedene $\overset{R}{D}$ Werte gültig

Wenn also ν_0 ν_1 ν_2 ν_3 --- die augenblicklichen Anzahlen von

einzelnen | doppelten | dreifachen | ... n ... Teilchen bedeuten, so ist

$$\frac{d\nu_1}{dt} = -4\pi\nu_1^2 D_{11} R_1 - 4\pi\nu_1\nu_2 D_{12} R_{12} - 4\pi\nu_1\nu_3 D_{13} R_1 - \cdots$$

$$\frac{d\nu_2}{dt} = \frac{1}{2}\left(4\pi\nu_1^2 D_{11} R_1\right) - 4\pi\nu_2\nu_1 D_{21} R_{21} - \left(4\pi\nu_2^2 D_{22} R_{22}\right) - 4\pi\nu_2\nu_3 D_{23} R_{23} - - -$$

$$\frac{d\nu_3}{dt} = 4\pi\nu_1\nu_2 D_{12} R_{12} - 4\pi\nu_3\nu_1 D_{13} R_{13} - 4\pi\nu_3\nu_2 D_{23} R_{23} - \left(4\pi\nu_3^2 D_{33} R_{33}\right) - -$$

$$\frac{d\nu_4}{dt} = 4\pi\nu_1\nu_3 D_{13} R_{13} + \frac{1}{2}\left(4\pi\nu_2^2 D_{22} R_{22}\right) - 4\pi\nu_4\nu_1 D_{14} R_{14} - -$$

Relative Bewegung von Teilchen ungleicher Grösse:

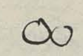
$\cancel{\overline{\Delta\xi^2} = 2Dt_1}$

$$W(\xi)\,d\xi = \frac{1}{2\pi t\sqrt{D_1 D_2}}\int e^{-\frac{(\xi+x)^2}{4D_1 t} - \frac{x^2}{4D_2 t}}\,d\xi\,dx = \frac{e^{-\frac{\xi^2}{4D_1 t}}\,d\xi}{\cdot\,-}\int e^{-\frac{x^2}{4t}\left(\frac{1}{D_1}+\frac{1}{D_2}\right) - \frac{\xi x}{2D_1 t}}\,dx$$

$$= \cdots\int e^{-\frac{x^2(D_1+D_2) + 2x\xi D_2 \pm \frac{\xi^2 D_2^2}{D_1+D_2}}{4D_1 D_2 t}} = \frac{e^{-\frac{\xi^2}{4D_1 t} + \frac{\xi^2 D_2}{4D_1(D_1+D_2)t}}}{2\pi t\sqrt{D_1 D_2}}\,d\xi\int e^{-(x-\cdots)^2\frac{D_1+D_2}{2D_1 D_2 t}}\,d-$$

$$= \frac{e^{+\frac{\xi^2 D_2 - (D_1+D_2)\xi^2}{4D_1(D_1+D_2)t}}}{2\sqrt{\pi t}\,\sqrt{D_1 D_2}}\cdot\sqrt{\frac{D_1 D_2}{D_1+D_2}} = \frac{e^{-\frac{\xi^2}{4(D_1+D_2)t}}}{2\sqrt{\pi(D_1+D_2)t}}$$

also gilt für die relative Bewegung die Summe der Diff. Konstanten



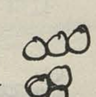
$$D_{12} = D_1 + D_2 = DR\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$D_{12} R_{12} = D_{12}\frac{(R_1 + R_2)}{2} = \frac{D_1 R_1}{2}$$

$$= DR\frac{(R_1+R_2)^2}{2R_1 R_2}$$

z.B. $D_{11} R_{11} = 4D\frac{R}{2}$

$D_{(12)} R_{(12)} = \frac{3}{2}D\cdot 3 = \frac{9}{2}\frac{DR}{2}$



$D_{(22)} R_{22} = D\cdot\frac{4R}{2} = 4\frac{DR}{2}$

$D_{(13)} R_{(13)} = \frac{4}{3}D\cdot 4 = \frac{16}{3}\frac{DR}{2}$

also unwesentlicher Unterschied

Wenn also $4\pi D_{11} R_{11} = \alpha$ gesetzt wird, ist annähernd:

$$\frac{dv_1}{dt} = -\alpha(v_1^2 + v_1 v_2 + v_1 v_3) + v_1 v_4 + - \quad = -\alpha v_1 (v_1 + v_2 + v_3 + \dots)$$

$$\frac{dv_2}{dt} = -\alpha\left(-\frac{v_1^2}{2} + v_1 v_2 + v_2^2\right) + v_2 v_3 + - \quad = \alpha \frac{v_1^2}{2} - \alpha v_2 (v_1 + v_2 + v_3 \dots)$$

$$\frac{dv_3}{dt} = -\alpha(-v_1 v_2 + v_1 v_3) + v_2 v_3 + v_3^2 + \dots = \alpha v_1 v_2 - \alpha v_3 (v_1 + v_2 + v_3 + \dots)$$

$$\frac{dv_4}{dt} = \alpha\left(v_1 v_3 + \frac{1}{2} v_2^2\right) - \alpha v_4 (v_1 + v_2 + v_3 + \dots)$$

$$\frac{dv_5}{dt} = \alpha(v_1 v_4 + v_2 v_3) - \alpha v_5 (v \dots)$$

$$\frac{d\sum v}{dt} = \alpha\left[\frac{v_1^2}{2} + v_1 v_2 + \frac{v_2^2}{2} + v_1 v_3 + v_1 v_4 + v_2 v_3 + v_1 v_5 + v_2 v_4 + \frac{v_3^2}{2} + \dots\right]$$

$$-\alpha\left(\sum v\right)^2$$

$$= -\frac{\alpha}{2}\left(\sum v\right)^2$$

$$\frac{d\Sigma}{\Sigma^2} = -\frac{\alpha}{2}$$

$$D_{11} \neq 2D$$

$$\frac{\alpha v_0}{2} = \beta = 4\pi v_0 D . R_{11}$$

$$\frac{1}{\Sigma(v)} = +\frac{\alpha}{2} t + \frac{1}{v_0}$$

$$\Sigma(v) = \frac{1}{\frac{1}{v_0} + \frac{\alpha t}{2}} = \frac{v_0}{1 + \frac{\alpha t v_0}{2}} =$$

$$\frac{dv_1}{dt} = -\alpha v_1 \Sigma =$$

$$\frac{1}{v_1}\frac{dv_1}{dt} = -\alpha \Sigma = -\frac{\alpha v_0}{1 + \alpha t \frac{v_0}{2}}$$

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$$\log \nu_1 = -\int \frac{\alpha \nu_0\, dt}{1+\frac{\alpha}{2}\nu_0 t} = -\frac{2}{\cancel{\ }} \log\left(1+\frac{\alpha}{2}\nu_0 t\right) + \log \nu_0$$

für $\lim t = 0$

$\lim \nu_1 = \nu_0(1-\alpha\nu_0 t)$

$= \nu_0 e^{-\alpha\nu_0 t}$ (wie früher)

$$\nu_1 = \frac{\nu_0}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^2} = \frac{\left(\Sigma'\right)^2}{\nu_0}$$

$$\underbrace{\frac{d\nu_2}{dt} + \alpha\nu_2 \Sigma' = \alpha\frac{\nu_1^2}{2}}$$

$$\nu_2+\nu_3+\nu_4+\dots = \Sigma - \nu_1 = \frac{\nu_0}{1+\frac{\alpha\nu_0 t}{2}}\left[1-\frac{1}{1+\frac{\alpha\nu_0 t}{2}}\right]$$

$$= \frac{\alpha\frac{\nu_0^2 t}{2}}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^2}$$

$$\frac{d\nu_2}{dt} + \frac{\alpha\nu_0\nu_2}{1+\frac{\alpha\nu_0 t}{2}}$$

$$\frac{d}{dt}\left[\nu_2\left(1+\frac{\alpha\nu_0 t}{2}\right)^2\right] = \frac{d\nu_2}{dt}\left(1+\frac{\alpha\nu_0 t}{2}\right)^2 + \cancel{2}\frac{\alpha\nu_0}{\cancel{2}}\left(1+\frac{\alpha\nu_0 t}{2}\right)\nu_2$$

$$= \left(1+\frac{\alpha\nu_0 t}{2}\right)^2 \cdot \alpha\frac{\nu_1^2}{2} = \frac{\alpha\nu_0^2}{2\left[1+\frac{\alpha\nu_0 t}{2}\right]^2}$$

$$\nu_2\left(1+\frac{\alpha\nu_0 t}{2}\right)^2 = -\frac{\nu_0}{1+\frac{\alpha\nu_0 t}{2}} + \text{const} \quad \rightarrow \quad = \nu_0\left(1-\frac{1}{1+\frac{\alpha\nu_0 t}{2}}\right)$$

$$0 = -\nu_0 + \text{const} \qquad = \frac{\alpha\frac{\nu_0^2 t}{2}}{1+\frac{\alpha\nu_0 t}{2}}$$

$$\therefore \quad \nu_2 = \frac{\frac{\alpha\nu_0^2}{2}t}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^3}$$

$$\nu_3+\nu_4+\nu_5+\dots = \frac{\alpha\nu_0^2 t}{2}\left[\frac{1}{(1+\varepsilon)^2} - \frac{1}{(1+\varepsilon)^3}\right] = \nu_0\frac{\left(\frac{\alpha\nu_0 t}{2}\right)^2}{\left[1+\frac{\alpha\nu_0 t}{2}\right]^3}$$

Maximum für ν_2:

$$\frac{d}{dt}\left[\frac{\varepsilon}{(1+\varepsilon)^3}\right] = \frac{1}{(1+\varepsilon)^3}\frac{d\varepsilon}{dt} - \frac{3\varepsilon\frac{d\varepsilon}{dt}}{(1+\varepsilon)^4} = 0$$

$$3\varepsilon = 1+\varepsilon$$

$$\varepsilon_M = \frac{1}{2}$$

$$t_M = \frac{1}{\alpha\nu_0}$$

$$\nu_{2\,max} = \frac{\frac{\nu_0}{2}}{\left(\frac{3}{2}\right)^3} = \frac{4\nu_0}{27}$$

$$\frac{dv_3}{dt} + \frac{\alpha v_0 v_3}{1+\frac{\alpha v_0 t}{2}} = \frac{\alpha v_0 \frac{\alpha v_0^2}{2} t}{\left(1+\frac{\alpha v_0 t}{2}\right)^5}$$

$$\frac{d}{dt}\left[v_3\left(1+\frac{\alpha v_0 t}{2}\right)^2\right] = \frac{\alpha v_0 \cdot \frac{\alpha v_0^2 t}{2}}{\left(1+\frac{\alpha v_0 t}{2}\right)^3} = \cancel{\alpha} = \frac{\alpha v_0^2}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}\left[1-\frac{1}{1+\frac{\alpha v_0 t}{2}}\right]$$

$$v_3\left(1+\frac{\alpha v_0 t}{2}\right)^2 = \int \dots\ dt = \frac{v_0}{\left(1+\frac{\alpha v_0 t}{2}\right)^2} - \frac{2v_0}{1+\frac{\alpha v_0 t}{2}}$$

$$= \frac{v_0}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}\left[1-2\left(1+\frac{\alpha v_0 t}{2}\right)\right] = -v_0\frac{(1+\alpha v_0 t)}{\left(1+\frac{\alpha v_0 t}{2}\right)^2} + c$$

$\cancel{v_3 = -v_0}$

$$= v_0\left\{\frac{\left(1+\frac{\alpha v_0 t}{2}\right)^2 - (1+\alpha v_0 t)}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}\right\}$$

$$= v_0\frac{\left(\frac{\alpha v_0 t}{2}\right)^2}{\left(1+\frac{\alpha v_0 t}{2}\right)^2}$$

$$\therefore v_3 = \frac{v_0\left(\frac{\alpha v_0 t}{2}\right)^2}{\left(1+\frac{\alpha v_0 t}{2}\right)^4}$$

$$\underset{\Sigma v}{\frac{1}{1+\varepsilon}} = \underset{v_1}{\frac{1}{(1+\varepsilon)^2}} + \underset{v_2}{\frac{\varepsilon}{(1+\varepsilon)^3}} + \underset{v_0}{\frac{\varepsilon^2}{(1+\varepsilon)^4}}$$

$$= \frac{1}{(1+\varepsilon)^2}\left[1+\frac{\varepsilon}{1+\varepsilon}+\left(\frac{\varepsilon}{1+\varepsilon}\right)^2 + \dots\right]$$

$\cancel{1+\varepsilon = x}$

$$\underbrace{\frac{1}{x} = \frac{1}{x^2} + \cancel{\frac{x-1}{x^3}} + \frac{(x-1)^2}{x^4} + \dots}$$

$$\frac{1}{1-\frac{\varepsilon}{1+\varepsilon}} = 1+\varepsilon$$

$\boxed{\text{Stimmt}}$

also wahrscheinlich ist

$$\boxed{v_n = \frac{v_0\left(\frac{\alpha v_0 t}{2}\right)^{n-1}}{\left[1+\frac{\alpha v_0 t}{2}\right]^{n+1}}}$$

Maximum für v_3: $\frac{2\varepsilon}{(1+\varepsilon)^4} = \frac{4\varepsilon^2}{(1+\varepsilon)^5}$

$$1 = \frac{2\varepsilon}{1+\varepsilon}$$

$\varepsilon = 1$

$$\boxed{t_M = \frac{2}{\alpha v_0} \qquad v_{3\,Max} = \frac{v_0}{16}}$$

$$\frac{(n-1)\varepsilon^{n-2}}{(1+\varepsilon)^{n+1}} = \frac{(n+1)\varepsilon^{n-1}}{(1+\varepsilon)^{n+2}} \quad (1+\varepsilon)(n-1) = (n+1)\varepsilon$$

$$\varepsilon_M = \frac{n-1}{2}$$

$$v_{n\,Max} = 4v_0\frac{(n-1)^{n-1}}{(n+1)^{n+1}}$$

Wie rasch ändern sich die Amplituden der Normalkoordinaten?

Raschheit der Schwankungen?

Wie schnell ändert sich (die durchschnittliche) kinet. Energie eines (Gas-)Moleküls?

abhängig nur von mittl. Weglänge. (resp. Stosszahl)

$$\nu_i = 2c \sin\frac{i\pi}{2(n+1)} = 2c\sin\theta$$

$$= 2\sqrt{\frac{\alpha}{m}}\sin\frac{\varphi}{2}$$

$$\nu_n = \sqrt{\frac{\alpha}{m}}\, p_m$$

$$\downarrow$$

$$4\sin^2\frac{\varphi}{2}$$

$$\nu_n = 2\sqrt{\frac{\alpha}{m}}\sin\frac{\varphi}{2} = \nu_0 \sin\frac{\varphi}{2}$$

„man kann zeigen, dass:

$$N(\varphi)\,d\varphi = C\,d\varphi$$

$$C = \frac{N\bar{L}}{2\pi}$$

Ges. Anzahl $\int_{\varphi}^{\varphi+d\varphi}$

$$p = 4\sin^2\frac{\varphi}{2} = \nu^2\frac{m}{\alpha} = \frac{4\nu^2}{\nu_0^2} \qquad \| \; \nu_0 = 2\sqrt{\frac{\alpha}{m}}$$

$$dp = 4\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\,d\varphi = 2\sin\varphi\,d\varphi$$

$$\therefore N(p)\,dp = \frac{N\bar{L}}{4\pi\sin\varphi}\,dp = N(\nu)\,d\nu$$

oder auch im Maßstabe ν: $\cancel{N(\nu)\,d\nu = \frac{N\bar{L}}{4\pi\sin\varphi}\cdot\frac{\nu_0}{2}\cos\frac{\varphi}{2}\,d\varphi = N\bar{L}}$

$$N(\nu)\,d\nu = \frac{N\bar{L}}{4\pi\sin\varphi}\,2\sin\varphi\,d\varphi = \frac{N\bar{L}}{2\pi}\,d\varphi$$

Das ist wohl auch unmittelbar ersichtlich aus obiger Form

indem $\sum \nu = n$

$$\Delta\theta = \frac{\pi}{2(n+1)}$$

$$\nu_1 + 2\nu_2 + 3\nu_3 \cdots = \frac{1}{(1+\varepsilon)^2}\left[1 + 2\frac{\varepsilon}{1+\varepsilon} + 3\left(\frac{\varepsilon}{1+\varepsilon}\right)^2 + \cdots\right]$$

$$= \frac{1}{(1+\varepsilon)^2}\,\frac{1}{\left(1-\frac{\varepsilon}{1+\varepsilon}\right)^2} = 1$$

$$\nu_1 + 1.2\,\nu_2 + 2.3\,\nu_3 + \dots = \frac{1}{(1+\varepsilon)^2}\left[1 + 1.2\frac{\varepsilon}{1+\varepsilon} + \dots\right] = \frac{1}{(1+\varepsilon)^2}\,\frac{1+\frac{\varepsilon}{1+\varepsilon}}{\left(1-\frac{\varepsilon}{1+\varepsilon}\right)^3} = 1+2\varepsilon$$

$$x + x^2 + x^3 + \dots = \frac{x}{1-x}$$

$$1 + 2x + 3x^2 + \dots = \frac{1}{1-x} + \frac{x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$1 + 1.2x + 2.3x^2 + \dots = \frac{2x}{(1-x)^3} + \frac{1}{(1-x)^2} = \frac{1+x}{(1-x)^3}$$

$$\begin{vmatrix} 1 & 0 & -(2+\frac{\gamma}{\alpha}) \\ (2+\frac{\gamma}{\alpha}) & 0 & -1 \\ 1 & -\frac{\gamma}{\alpha} & 1 \end{vmatrix} = 0$$

$$\begin{matrix} 1 & 0 & \dots \\ (\quad) & 0 & -1 \end{matrix}$$

$$x + \frac{\gamma}{\alpha}\left(2+\frac{\gamma}{\alpha}\right)^2 - x$$

$$+\left(2+\frac{\gamma}{\alpha}\right) - \frac{\gamma}{\alpha} - \left(2+\frac{\gamma}{\alpha}\right) = 0$$

$$\left(2+\frac{\gamma}{\alpha}\right)^2 = 1$$

$$2 + \frac{\gamma}{\alpha} = \pm 1$$

$$\begin{cases} \frac{\gamma_1}{\alpha} = 0 \\ \frac{\gamma_2}{\alpha} = -1 \\ \frac{\gamma_3}{\alpha} = -3 \end{cases}$$

$$\xi_3 = x_2 - x_1$$

$$\xi_1 = x_3 - x_2$$

$$\xi_2 = x_1 - x_3$$

$$x_1 = A_1 \sin(\sqrt{\alpha\gamma_1}\, t + \varepsilon_1)$$

$$\begin{vmatrix} (1+\gamma) A_1 & -A_2 & 0 \\ -A_1 & (2+\gamma)A_2 - A_3 & \\ 0 & -A_2 & A_3(1+\gamma) \end{vmatrix} = 0$$

$$\begin{vmatrix} 1+\gamma & -1 & 0 \\ 1 & -(2+\gamma) & 1 \\ 0 & -1 & 1+\gamma \end{vmatrix}$$

$$\begin{matrix} 1+\gamma & -1 & 0 \\ 1 & -(2+\gamma) & 1 \end{matrix}$$

$$-(1+\gamma)^2(2+\gamma) + 2(1+\gamma) = 0$$

$$(1+\gamma)(2+\gamma) = 2$$

$$\cancel{2} + 3\gamma + \gamma^2 = \cancel{2}$$

$$\gamma = -1$$

$$\gamma = 0$$

$$\gamma = -3$$

Normale Vorgang:

$0 < k < \frac{n}{2}$

$$y_k = \sum_{i=1}^{\frac{n-1}{2}} \varphi_i \sin \frac{ik\pi}{\frac{n+1}{2}} \qquad \Bigg\| \qquad \frac{n}{2} < k < n+1 \qquad y_k = \sum_{i=1}^{\frac{n-1}{2}} \psi_i \sin \frac{ik\pi}{\frac{n+1}{2}}$$

$$\Phi_i = \frac{2}{n+1}\left\{ \sum_{k=1}^{\frac{n-1}{2}} \sin\frac{ik\pi}{n+1} \sum_{i=1}^{\frac{n-1}{2}} \varphi_i \sin \frac{ik\pi}{\frac{n+1}{2}} + \sum_{k=\frac{n+1}{2}}^{n} \sin\frac{ik\pi}{n+1} \sum_{i=1}^{\frac{n-1}{2}} \psi_i \ldots \right\}$$

$$\frac{d^2x_1}{dt^2} = \alpha(x_1 - x_2)$$

$$\frac{d^2x_2}{dt^2} = \alpha(x_2 - x_1) + x_2 - x_3 = \alpha(2x_2 - x_1 - x_3)$$

$$\frac{d^2x_3}{dt^2} = \alpha(x_3 - x_2)$$

$$\frac{d^2(x_2 - x_1)}{dt^2} = \alpha\left[x_2 - x_3 + 2(x_2 - x_1)\right]$$

$$\frac{d^2(x_3 - x_2)}{dt^2} = \alpha\left[2(x_3 - x_2) + (x_1 - x_2)\right]$$

$$\frac{d^2(x_1 - x_3)}{dt^2} = \alpha\left[(x_1 - x_2) - (x_3 - x_2)\right]$$

$$\cancel{(\ddot\xi_1 + \ddot\xi_3) = \alpha(\xi_1 + \xi_3) = -\ddot\xi_2}$$

$$\ddot\xi_3 = \alpha\left[2\xi_3 - \xi_1\right] = -\gamma\xi_3$$

$$\ddot\xi_1 = \alpha\left[2\xi_1 - \xi_3\right] = -\gamma\xi_1$$

$$\ddot\xi_2 = \alpha\left[-\xi_3 - \xi_1\right] = -\gamma\xi_2$$

$$\cancel{\xi_4 - \frac{\gamma}{\alpha}\xi_3 = 0}$$

$$\cancel{\xi_1}$$

$$\xi_1 - \left(\frac{\gamma}{\alpha} + 2\right)\xi_3 = 0$$

$$\xi_1\left(2 + \frac{\gamma}{\alpha}\right) - \xi_3 = 0$$

$$\xi_1 - \frac{\gamma}{\alpha}\xi_2 + \xi_3 = 0$$

oder bei Annahme der Quantentheorie:

$$W(U_k, \dot{U}_k)\, dU_k\, d\dot{U}_k = A\, e^{-\left[\frac{\mu}{2}\dot{U}_k^2 + \frac{\alpha p_k}{2} U_k^2\right]\cdot\frac{1-e^{-\frac{h\nu_k}{kT}}}{h\nu_k}}\, dU_k\, d\dot{U}_k$$

Also Wahrscheinlichkeit eines ~~...~~ Wertes der Normalkoordinate U_k bei beliebigem $\dot{U}_k$:

$$W(U_k)\, dU_k = \int_{-\infty}^{+\infty} \dots\, d\dot{U}_k = C\, e^{-\frac{\alpha}{2} p_k U_k^2 \frac{N}{HT}}\, dU_k$$

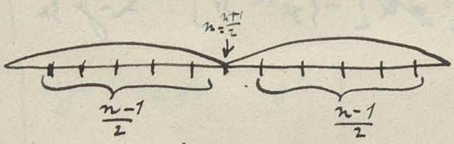
Somit mittlerer Betrag der Arbeit pro Zeiteinheit:

$$\overline{\left[\frac{\alpha}{2} y_k (y_{k+1} - y_{k-1})\right]} = \frac{\alpha}{2}\sum \sin\frac{ik\pi}{n+1}\cos\frac{ik\pi}{n+1}\sin\frac{i\pi}{n+1}\underbrace{\int_{-\infty}^{\infty}\Phi_i^2\, W(\Phi_i)\, d\Phi_i}_{\frac{1}{2}\frac{2}{\alpha}\frac{1}{p_k}\frac{HT}{N}}$$

Falls aber Temperaturdifferenz zu beiden Seiten von k?

Wie hängen die U_k der Partialsysteme mit den U_k des ganzen Systems zusammen?

Annahme zwei gleich lange Punktreihen



$$y_m = \sum \Phi_i \sin\frac{i\pi\frac{n+1}{2}}{n+1} = \sum_{i=1}^{n}\Phi_i \sin\frac{i\pi}{2}$$
$$= \Phi_1 - \Phi_2 + \Phi_3 - \dots$$

$$\Phi_i = \frac{2}{n+1}\sum_{k=1}^{n} y_k \sin\frac{ik\pi}{n+1}$$

$$\Phi_{n-i+1} = \frac{2}{n+1}\sum_{k=1}^{n} y_k \underbrace{\sin\left[\frac{(n+1-i)k\pi}{n+1}\right]}_{\sin\left(k\pi - \frac{ik\pi}{n+1}\right)} = \frac{2}{n+1}\sum_{k=1}^{n} y_k \sin\frac{ik\pi}{n+1}(-1)^{k+1}$$

$\cancel{\varphi_{2i}+\psi_{2i} = \Phi_i}$ $\quad \Phi_{2m} = \varphi_m + \psi_m \qquad \varphi_m = \frac{2}{\frac{n-1}{2}+1}\sum_{k=1}^{\frac{n-1}{2}} y_k \sin\frac{mk\pi}{\frac{n+1}{2}}$

$\cancel{\sin}\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right) = \cancel{\sin}\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + \cancel{\sin}\frac{\alpha-\beta}{2}\cos$

~~Wenn Elektron eindringt in Flüssigkeitstropfen, Betrag von gewonnener Arbeit?~~

$$\cancel{W = \frac{D-1}{4\pi}\int E^2\,dv = \frac{D-1}{4\pi}\,4\pi e^2\int_0^R \frac{r^2\,dr}{r^4} = \infty}$$

$\cancel{\cos(\alpha+\beta) - \cos(\alpha-\beta) = \ldots}$

$\sin\alpha - \sin\beta = \cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$$T = \frac{m}{2}\left[\dot u_1^2 + \dot u_2^2 + \dot u_3^2 \cdots\right] = \frac{m}{2}\left[\dot U_1^2 + \dot U_2^2 + \dot U_3^2 + \cdots\right]$$

$$V = \frac{\alpha}{2}\left[(u_1-u_0)^2 + (u_2-u_1)^2 + \cdots\right] = \frac{\alpha}{2}\left[p_1 U_1^2 + p_2 U_2^2 + \cdots\right]$$

$$F_{01} = \alpha(u_1 - u_0)$$

$$W_{ik}\,dt = \dot u_i F_{ik}\,dt = \alpha\,\dot u_i(u_k - u_i)$$

$\cancel{\alpha\, u_i(u_k - u_i)}$

vielleicht besser den symmetrischen Ausdruck

$$\alpha\left[\frac{u_i(u_{i+1}-u_i) + u_i(u_i - u_{i-1})}{2}\right] = \frac{\alpha}{2}\,u_i(u_{i+1} - u_{i-1})$$

$$y_k = \sum_{i=1}^{n}\Phi_i \sin\frac{ik\pi}{n+1} = \cancel{\int\Phi}$$

$$y_{k+1} - y_{k-1} = \sum \Phi_i \cos\frac{ik\pi}{n+1}\sin\frac{i\pi}{n+1}$$

$$\alpha\cdot\frac{y_k(y_{k+1}-y_{k-1})}{2} = \frac{\alpha}{2}\cdot\sum_{i=1}^{n}\Phi_i\sin\frac{ik\pi}{n+1}\cdot\sum_i \dot\Phi_i\cos\frac{ik\pi}{n+1}\sin\frac{i\pi}{n+1}$$

Wahrscheinlichkeit eines Wertes $y \ldots y+dy$ $\quad y = \sum_1^n \Phi_i \sin\frac{ik\pi}{n+1}$

$$W(y)\,dy = A\,e^{-}$$

Wahrscheinlichkeit eines Amplitudenwertes einer Normalkoordinate $\quad \Phi_i = A_i\sin(\nu_i t + \varepsilon_i)$

$$W(A_i) = B\,e^{-A}$$

Wahrscheinlichkeit der Werte $\left\{\begin{matrix} U_k \ldots U_k + dU_k \\ \dot U_k \ldots \dot U_k + d\dot U_k\end{matrix}\right\}$ $\quad W(U_k, \dot U_k)\,dU_k\,d\dot U_k =$

$$= A\,e^{-\left[\frac{m}{2}\dot U_k^2 + \frac{\alpha}{2}p_k U_k^2\right]\frac{N}{HT}}\,dU_k\,d\dot U_k$$

$$y_k = \sum_{i=1}^{n} \alpha_{ik} \cdot C_i \sin(\nu_i t + \varepsilon_i) = \sum_{i=1}^{n} \alpha_{ik} \Phi_i = \sum_{i=1}^{n}{}' \Phi_i \sin\frac{ik\pi}{n+1}$$

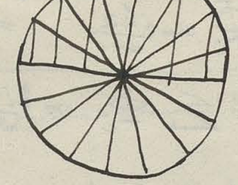
$$y_1 = \sin\frac{\pi}{n+1} \cdot C_1 \sin(\nu_1 t + \varepsilon_1) + \sin\frac{2\pi}{n+1} C_2 \sin(\nu_2 t + \varepsilon_2) + \cdots \sin\frac{i\pi}{n+1} C_i \sin(\nu_i t + \varepsilon_i)$$

$$y_2 = \sin\frac{2\pi}{n+1} C_1 \sin(\nu_1 t + \varepsilon_1) + \sin\frac{2.2\pi}{n+1} C_2 \sin(\nu_2 t + \varepsilon_2) + \cdots \sin\frac{2i\pi}{n+1} C_i \sin(\nu_i t + \varepsilon_i)$$

$$y_3 = \sin\frac{3\pi}{n+1} C_1 \sin(\nu_1 t + \varepsilon_1) + \sin\frac{3.2\pi}{n+1} C_2 \sin(\nu_2 t + \varepsilon_2) + \cdots \sin\frac{3i\pi}{n+1} C_i \sin(\nu_i t + \varepsilon_i)$$

$$2\sum_{k=1}^{n}{}' \sin\frac{km\pi}{n+1} \sin\frac{ki\pi}{n+1} = \sum_{k=1}^{n} \left[\cos\frac{k(m-i)\pi}{n+1} - \cos\frac{k(m+i)\pi}{n+1}\right] = 0$$

$$2\sum_{k=1}^{n} \left(\sin\frac{ki\pi}{n+1}\right)^2 = \sum_{k=1}^{n} \left(1 - \cos\frac{2ki\pi}{n+1}\right) = \sum_{k=0}^{n+1} = n+1$$



$$\sum{}' y_k \sin\frac{ki\pi}{n+1} = \frac{n+1}{2} C_i \sin(\nu_i t + \varepsilon_i) = \frac{n+1}{2} \Phi_i$$

$$\Phi_i = \frac{2}{n+1} \sum_{k=1}^{n} y_k \sin\frac{ki\pi}{n+1}$$

$$E_p = c^2 \frac{m}{2} \sum_{k=1}^{n} (y_k)^2 = \frac{m}{2} c^2 \frac{n+1}{2} \sum_{k=1}^{n} C_k^2 \sin^2(\nu_k t + \varepsilon_k) = c^2 \frac{m}{2} \frac{n+1}{2} \sum_{k=1}^{n} \Phi_k^2 \quad (?)$$

$$T = \frac{m}{2} \sum y_k'^2 = \frac{m}{2} \frac{n+1}{2} \sum_{k=1}^{n} (\Phi_k')^2$$

$$\Phi_i = C_i \cos\varepsilon_i \sin\nu_i t + C_i \sin\varepsilon_i \cos\nu_i t = (\Phi_i)_0 \cos\nu_i t + \frac{1}{\nu_i} (\Phi_i')_0 \sin\nu_i t$$

$$\Phi_i' = (\Phi_i')_0 \cos\nu_i t - \nu_i (\Phi_i)_0 \sin\nu_i t$$

$\alpha_{ik} = \beta_{ik} = \sin \frac{ik\pi}{n+1}$

$$y_k = \alpha_{1k} U_1 + \alpha_{2k} U_2 + \alpha_{3k} \cdot U_k + \dots \quad \alpha_{nk} U_k + \beta_{1k} V_1 + \beta_{2k} V_2 + \dots \quad \beta_{nk} V_n$$

$$y_k = \alpha_{1k} A_1 \sin \nu_1 t + \alpha_{2k} A_2 \sin \nu_2 t + \dots \qquad + \beta_{1k} B_1 \cos \nu_1 t + \beta_{2k} B_2 \cos \nu_2 t + \dots$$

$$y'_k = \alpha_{1k} \nu_1 A_1 \cos \nu_1 t + \alpha_{2k} \nu_2 A_2 \cos \nu_2 t + \dots \qquad - \beta_{1k} \nu_1 B_1 \sin \nu_1 t - \beta_{2k} \nu_2 B_2 \sin \nu_2 t \dots$$

$$\sum y_k \alpha_{ik} = \sin \nu_1 t \, . \, A_1 \sum \alpha_{1k} \alpha_{ik} + \cos \nu_1 t \, . \, A_1 \nu_1 \sum \alpha_{1k} \alpha_{ik} + \dots$$

$$\sin \nu_i t \, . \, A_i \underbrace{\sum (\alpha_{ik})^2}_{\frac{1}{2}(n+1)} + \dots$$

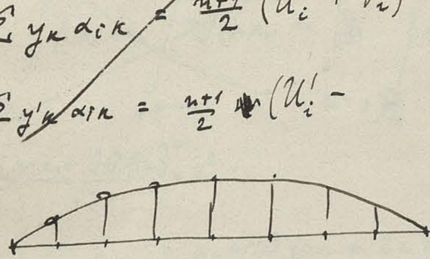
$$\sum y_k \alpha_{ik} = \frac{1}{2}(n+1) \left[A_i (\sin \nu_i t) + B_i (\cos \nu_i t) \right]$$

$$\sum y'_k \alpha_{ik} = \frac{1}{2}(n+1) \nu_i \left[A_i \cos \nu_i t - B_i \sin \nu_i t \right]$$

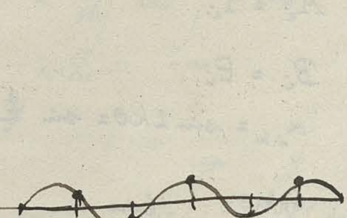
$$\sum y_k \alpha_{ik} = \frac{n+1}{2} (U_i + V_i)$$

$$\sum y'_k \alpha_{ik} = \frac{n+1}{2} (U'_i -$$

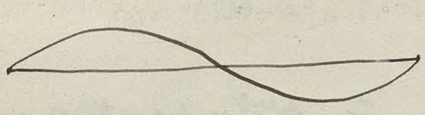
Φ_1



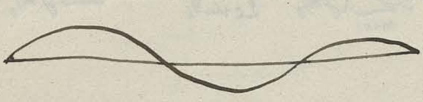
Φ_n



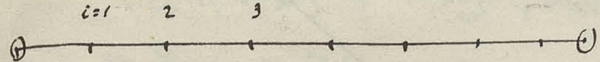
Φ_2



Φ_3



Wie wird infolge der elektrostatischen Druckkräfte die Gestalt eines Flüssigkeitströpfchens durch ein angehängtes Elektron geändert  oder  ?



$$y_k = \sum_{i=1}^{i=n} E_i \sin 2k\theta \cos(2ct\sin\theta) + \sum F_i \sin 2k\theta \sin\overbrace{(2ct\sin\theta)}$$

$$\boxed{\theta = \frac{i\pi}{2(n+1)}}$$

Allgemeine Form:

$$U_i = A_i \sin \nu_i t \quad \cancel{x}$$
$$V_i = B_i \cos \nu_i t$$

~~$y_k =$ ...~~

$$y_k = \alpha_{1k}\cdot U_1 + \alpha_{2k}\cdot U_2 + \alpha_{3k}\cdot U_3 + \dots \alpha_{nk}\cdot U_n + \beta_{1k} V_1 + \dots \quad \beta_{nk} V_n$$

$$\cancel{\nu_i =} \quad \nu_i = 2c\sin\theta = 2c\sin\frac{i\pi}{2(n+1)}$$

$$A_i = F_i$$

$$B_i = E_i$$

$$\alpha_{ik} = \sin 2k\theta = \sin\left(\frac{ik\pi}{n+1}\right) = \beta_{ik}$$

Dabei ist ~~...~~: ~~...~~

$$E_i = \frac{2}{n+1}\sum_{k=1}^{k=n} (y_k)_{t=0} \sin 2k\theta$$

$$F_i = \frac{1}{(n+1)c\sin\theta}\sum_{k=1}^{k=n} (y'_k)_{t=0} \sin 2k\theta$$

$$\Phi_j = \frac{2}{n+1}\sum_{k=1}^{n} \sin kj\pi \left\{\sum_{i=1}^{n} \frac{2}{n+1} \sin 2k\theta_i \left[\cos(2ct\sin\theta_i)\sum_{m=1}^{n} (y_m)_0 \sin 2m\theta_i + \frac{\sin(2ct\sin\theta_i)}{2c\sin\theta_i}\sum (y'_m)_0 \sin 2m\theta_i\right]\right\}$$

$$= \frac{2}{n+1}$$

$$\frac{d}{d\vartheta}\int\limits_{x=-\infty}^{x} W(x\,x_0)_\vartheta\,dx$$

$$\lim \frac{1}{\vartheta}\left\{ \int\limits_{x_0=X_0}^{\infty}(A + Bx_0 + Cx_0^2)\,dx_0 \cdot \frac{d}{d\vartheta}\int\limits_{x=-\infty}^{X_0} W(x\,x_0)_\vartheta\,dx - \int\limits_{x_0=-\infty}(A+Bx_0)\,dx_0\,\frac{d}{d\vartheta}\int\limits_{x=X_0}^{\infty} W(x\,x_0)\,dx \right.$$

$$= \frac{d}{d\vartheta}\left\{ \underbrace{A\left[\int\limits_{x_0=X_0}^{\infty} dx_0 \int\limits_{x=-\infty}^{X_0} - \int\limits_{x_0=-\infty}^{X_0} dx_0 \int\limits_{X_0}^{\infty}\right]}_{} + B\left[\int x_0\,dx_0 - \right]\right\}$$

$$= A\frac{d}{d\vartheta}\int\limits_{-\infty}^{\infty} dx_0 \int\limits_{x=-\infty}^{X_0} W(x\,x_0)_\vartheta\,dx + B\,\frac{d}{d\vartheta}\int\limits_{-\infty}^{\infty} x_0\,dx_0 \int W\ldots$$

$$\cancel{\int\limits_{x=X_0}^{\infty} f(x_0)\,dx_0 \int\limits_{x=-\infty}^{X_0} W(x\,x_0)\,dx} \quad \int\limits_{x_0=+X_0}^{\infty} dx_0 \int\limits_{-\infty}^{X_0} W(x\,x_0)_\vartheta\,dx = \frac{1}{\sqrt{\pi}}\int\limits_{x_0=X_0}^{\infty} dx_0 \int\limits_{-\infty}^{\frac{X_0-x_0-f(x_0)}{2\sqrt{D\vartheta}}} e^{-z^2}\,dz$$

$$\frac{[x-x_0-\beta\vartheta f(x_0)]^2}{4D\vartheta} = z^2$$

$$x = x_0 + \beta\vartheta f(x_0) + 2z\sqrt{D\vartheta}$$

$$dx = 2\sqrt{D\vartheta}\,dz$$

$$= -\frac{1}{2\sqrt{D\vartheta}}\frac{1}{\sqrt{\pi}}\underbrace{\int\limits_{-\frac{f(x_0)}{2\sqrt{D\vartheta}}}^{-\infty} dy \int\limits_{-\infty}^{y} e^{-z^2}\,dz}$$

$$\frac{l^2 m}{t^2 l^3} = \frac{lm}{t^2}\frac{1}{l^2}$$

$= 200 \cdot 4 \cdot 10^7$

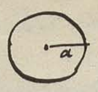
$= 8000$ Atm!

$$y \cdot \int\limits_{-\infty}^{y} e^{-z^2}\,dz - \int y\,e^{-y^2}\,dy$$

$$\frac{3 \cdot 4 \cdot 10^7}{0.74 \cdot 10^{-12}} = 2.5 \cdot 10^{-5}$$

Arbeit: $10^{10} \cdot 5 \cdot 10^{-5} = 5 \cdot 10^5$

de facto $4.8 \cdot 10^5$



Mit welcher „Dichtigkeit" (Wahrscheinlichkeit) ~~…~~ hat zur Zeit $(-t)$ das Teilchen ~~…~~ die Stelle O ~~…~~ überdeckt?

falls sehr oft von derselben Anfangslage ausgegangen, wie oft hatte es (zu jener Zeit) eine Entfernung $< a$

$$= \frac{1}{2Dt}\int_0^a e^{-\frac{r^2}{4Dt}}\, r\, dr = \left[1 - e^{-\frac{a^2}{4Dt}}\right] = \text{Wahrsch., dass } r < a$$

Wie lange Zeit hindurch wird durchschnittlich jede Stelle überdeckt?

Was ist die Wahrsch., dass die „gesammte Überdeckungsdauer" $\tau \ldots \tau + d\tau$ betrage?

" " die „relative Überdeckungsdauer" zur Zeit $(-t)$ den Wert z gehabt habe.

Grössenordnung: $\overline{\Delta r^2} = 4Dt = a^2$ $\qquad D = \frac{H\theta}{N}\frac{1}{6\pi\mu a}$

$$t = \frac{a^2}{4D} = \underbrace{\frac{3\pi\mu a^3}{2}\frac{N}{H\theta}}$$

das ~~…~~ ist maassgebend für die photographische Helligkeit

Während welchen Bruchteiles des Zeitraumes $-t$ bis $-(t+dt)$ war der O Punkt (durchschnittlich) überdeckt?

Was ist die ~~…~~ Wahrscheinlichkeit einer Überdeckungsdichtigkeit $z \ldots z+dz$ zur Zeit $(-t)$?

Andrerseits kann man die Sache auffassen, als ob die Dauer des Sichtbarbleibens einer Intensitätsänderung nach Maassgabe einer Wahrscheinlichkeitsfunktion $e^{-\gamma t}$ ~~…~~ gleichkommt. Infolge dessen wäre statt der eigentlichen Zickzack Kurve eine teilweise ausgeglichene Curve zu sehen.

$$c\int_0^\infty \frac{1}{2\sqrt{\pi D t}}\, e^{-\frac{x^2}{4Dt}}\, \cancel{dt}\, dt \qquad \frac{x}{2\sqrt{Dt}} = z \qquad \frac{x\, dt}{4\sqrt{Dt^3}} = dz$$

$$\frac{dt}{2\sqrt{Dt}} = \frac{2dz \cdot t}{x}$$

$$= \int \frac{2t}{x\sqrt{\pi}}\, e^{-z^2}\, dz \qquad t = \frac{x^2}{4Dz^2}$$

$$= \int_0^\infty \frac{1}{2D\sqrt{\pi}}\, e^{-z^2} \frac{dz}{z^2} = \infty$$

Aber trotzdem kann die früher erhaltene Verteilung endlich sein, weil zweidimensionaler Fall

Superposition von Verteilungen:

$$\sum_t \frac{1}{4\pi D t}\, e^{-\frac{r^2}{4Dt}}\, r\, dr\, d\varphi \qquad \text{in Zeitintervallen} \quad \tau \neq \frac{2a}{c_{xy}}$$

$$c\int \frac{1}{2Dt}\, e^{-\frac{r^2}{4Dt}}\, 2a\, dt$$

10/10 15

An B. Wie gross ist die Wahrsch., dass, falls ein Teilchen sich momentan im Punkte 0 befindet, dasselbe sich während des (verflossenen) Zeitraumes $-t \ldots -(t+dt)$ (ununterbrochen?) in einer Entfernung $r \ldots r+dr$ aufgehalten habe?

$$\int_{r=0}^{\infty} W(-t, r)\, dr = 1 \qquad \int_0^\infty\int_0^\infty W\, dr\, dt = 1$$

$= W(-t, r).\, dr\, dt$

Wenn man annimmt, dass der (physiologische) Gesichtseindruck nach Massgabe einer (Erinnerungs-) Funktion φ sich zusammensetzt aus den verflossenen Eindrücken, so wird die (~~mittl~~ durchschnittl.) resultierende Stärke desselben in einem gewissen Moment:

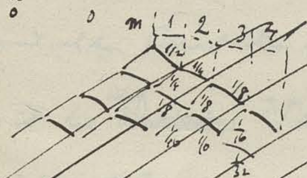
$$S = \int_0^\infty \varphi(t)\, dt \int_0^a W(-t, r)\, dr$$

wobei a = Teilchenradius

$\varphi(t)$ wahrscheinlich von der Form $\varphi(-t) = e^{-\gamma t}$

Dagegen wäre die durchschnittl. photographische Stärke (Summenwirkung) nach ∞ langer Zeit

$$F = \int_0^\infty dt \int_0^a W(-t, r)\, dr \quad (= \text{Wahrsch. normaler Belichtung})$$



$$F_m = \left(\frac{1}{2}\right)^m \Big[\underbrace{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots}_{=2}\Big] = \frac{1}{2^{m-1}}$$

$$2\sum_1^N F_m = 2\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \frac{1}{2^{N-1}}\right]$$

(für ungerade m)

$$\sum_m F_m = \left(\frac{1}{2}\right)^m 2\left[\cancel{\binom{m-1}{0}} + \binom{m-1}{1} + \binom{m-1}{2} + \binom{m-1}{3} + \cdots\right] = 1$$

$$\sum_m F_{mn} = \left(\frac{1}{2}\right)^m \left[1 + \frac{1}{2} + \frac{1}{2^3}\binom{m+1}{1} + \frac{1}{2^3}\binom{m+2}{1} + \frac{1}{2^4}\binom{m+3}{2} + \frac{1}{2^5}\binom{m+4}{2} + \cdots\right] = F$$

Wenn Oberfläche = Ω

Stosszahl in der Zeit t = $N = \frac{\nu c}{\sqrt{6\pi}}\,\Omega\, t$ $= \left.\begin{matrix}\text{eintretende}\\ = \text{austretende}\end{matrix}\right\}$ Anl. B

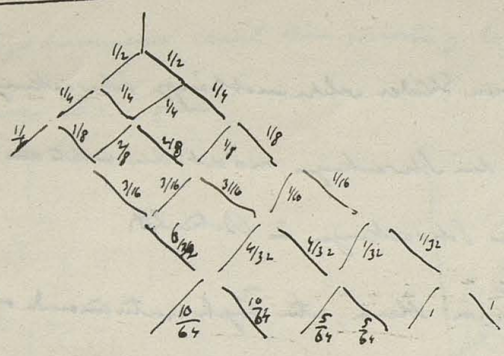
Dann wird die zufällige Schwankung

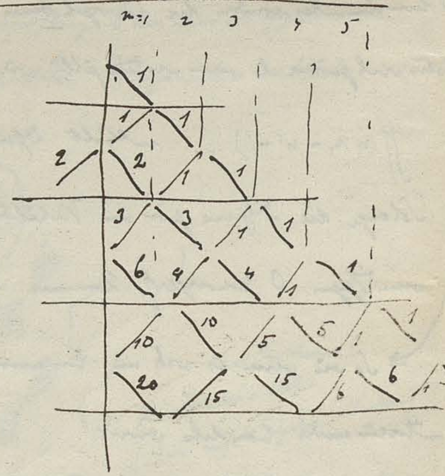
$$\overline{\Delta_t^2} = \frac{2\nu c}{\sqrt{6\pi}}\,\Omega t$$

Im Falle planparalleler Platte von Fläche F:

$$\overline{\Delta_t^2} = \frac{4\nu c}{\sqrt{6\pi}}\, F t$$

gültig für sehr kurze Zeiten t
so dass nur äusserste Schichte in Betracht kommt





Über gewisse
Mängel im Aufbau d. klassischen Thermodynamik
und d. Boltzmann'schen Gastheorie

Automat! nicht wenn Mensch mitwirkt oder nachhilft, unveränderliche Einrichtung

Angeblich aufgebaut auf Unmöglichkeit d. Perp. mob. II — dies soll aber auch nach Schwankungsth. gelten

De facto noch eine Reihe weiterer Voraussetzungen:

Determinationspostulat : Vorgang eindeutig abhängig von d. makroskopischen Variablen || dies ist hauptsächlich

Klass. Th. lässt willkürl. Eingreifen d. Menschen zu

Im Stosszahlansatz gibt es Schwankungen, entsprechend nicht der ganzen Zahl ($n\,d\omega$) des betreffenden Volumelementes, sondern der hervorgehobenen Art von Molekülpaaren $n\,n'\,f\,f'\,d\omega\,d\omega' \ldots$

(also viel grösser als wenn ersteres gültig wäre).

$\iint (n\,n_1 - n'\,n_1') \ldots$ enthält Differenz zweier Glieder welche unabhängige Schwankungen ausführen.

Solange die Differenz gross im Verhältnis zu den Schwankungen, macht dies nichts aus, aber wenn sie gegen 0 konvergiert, kommen eben die Schwankungen in Betracht.

So wie wenn es sich um Zusammenstösse von blauen, roten Kugeln untereinander oder aller untereinander handeln würde.

Stossgleichung in der Boltzm. Form ist streng gültig für d. Mittelwert aber so vieler gleichartiger Gase, aber nicht für ein individuelles Gas.

Allgemeine Form:

Schwankungsglied

$$\frac{\partial f}{\partial t} + \int v_x \cancel{\frac{\partial f}{\partial x}} + F\frac{\partial f}{\partial \ldots} = \iint + \Phi_t$$

Wenn Integration über alle Geschwind.

$$\frac{\partial n}{\partial t} = -\int v_x \frac{\partial n}{\partial x}\, dv$$

falls alle örtlichen Ungleichheiten vernachlässigt, aber gerade darauf kommt es an

$$\overline{\left[\frac{\partial}{\partial t}(n-\nu)\right]^2} = ?$$

$$= \overline{\Delta^2_t}$$

Die physikalische Wahrsch. eines Ereignisses muss ganz unabhängig davon sein, was wir über dasselbe wissen

---

Andere Fassung der Zufälligkeit:

$y = f(x)$ ~~Dabei~~

y sind zufällig, wenn einer beliebigen Verteilung der (diskreten Punktmenge (mit Ausnahme prozentuell unendlich

wenig zahlreicher singulärer Ausnahmsfälle) der x, immer annähernd eine und dieselbe Verteilung der Punktmenge y entspricht.

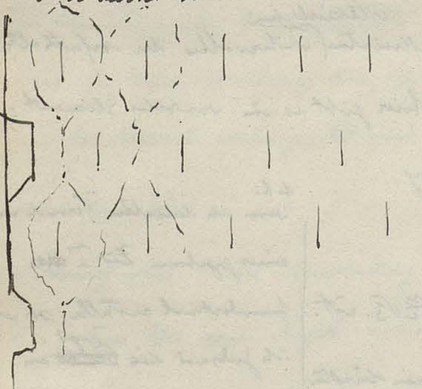
(und zwar mit wachsendem n immer grösserer Annäherung)

Allgemeine Definition eines Diffusionsexponenten bei jeder Zufälligkeit

---

Konstruktion d. Galton'schen Brettes: es müssen die Nägelentfernung d nur sehr wenig grösser sein als der Kugeldurchmesser, damit keine persistency der horizontalen Komponente verbleibe.

Oder auch so:



Zwei Probleme:
~~Der eine Teil~~:
1). wie Zufall regelm. Wirkung haben könne
2). wie regelm. Änderung den „Zufall" erzeugen könne
z.B. Ein Galton'scher Stift, Zersplitterung d. Instrumentgebrauchs (Borel)

Wodurch ist der regelmässige Effekt d. Zufalls.

unbegrenzten
Bereich des φ z.B. Lefcäs mit Kugel, Gartlwein = höheres Stadium der Abstraktion von $\varphi(x)$

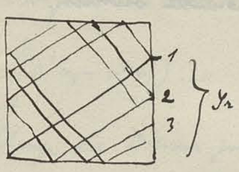
---

304 Momenzerfall = Beispiel objektiven Wahrsch.

~~AD Korrektur~~ Eingeständnis:
„Es fehlt ein ganz wesentliches Moment .... ohne welches man allenfalls eine Statistik aber nicht eine Wahrscheinlichkeitsrechnung begründen könnte".

„Zufall" wenn solcher Zusammenhang zwischen x und y dass zwar im Allg. für verschiedene x [auch wenig]
ganz andere y auftreten, jedoch so dass immer einer [darstellbaren] / x Punktmenge (mit hinreichend grosser
Punktzahl n) ein solcher y Punkt Komplex entspricht, dass er sich für lim n=∞ [entsprechend beliebig grossen] einer
[Häufigkeits] Funktion φ(y) [beliebig] nähert – unabhängig davon wie die x Punkte gewählt werden.

Nehmen wir ein Quadrat und werfen von einem Eckpunkt unter ∡ 30° eine elastische Kugel hinein, so wird die Fläche des Quadrates ~~mit der~~ im Laufe der Zeit von der Bahn derselben gleichförmig überdeckt, also kann man sagen „dass im Laufe der Zeit jeder ~~[illegible]~~ [Aufenthalts] Bereich innerhalb jenes Quadrates gleich wahrscheinlich wird?"



Nur in der Bedeutung, dass eine lange Zeit verstreicht und dann innerhalb eines nicht näher definierten [willkürlichen] Intervalles der Aufenthaltsort untersucht wird. Also auch hier gibt es ein variables Element: diesen Moment der Beobachtung.

~~Aber~~ die Ordinaten y_r sind definiert durch ~~[illegible]~~
$(\frac{n}{2}\sqrt{3} - m)$ wo m die grösste ganze Zahl bedeutet welche $< \frac{n}{2}\sqrt{3}$ ist.
~~in $\frac{n}{2}\sqrt{3}$ enthalten~~. Dass dieselben gleichförmig verteilt sein dürften ~~[illegible]~~ scheint ebenso wahrscheinlich wie die gleichförmige Verteilung in Poincaré's Beispiel (der ~~[illegible]~~ Logarithmen)

d.h. wenn ich denselben Versuch mit einer gegebenen Zeit T ~~[illegible]~~ hundertmal anstelle, so werde ich jedesmal die [Kugel] ~~[illegible]~~ an derselben Stelle finden

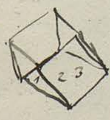
an welcher? das weiss ich nicht, wenn ich die Rechnung nicht angestellt habe, und die subjektive Wahrsch., sie in einem bestimmten Gebiete vorzufinden, ist gleich der Grösse des Gebietes : Grösse d. Quadrates

der „Vermutungsgrad"

aber objektiv physikalisch ist dies wertlos: entweder ist sie drinnen oder nicht.
Doch wenn ich jenen Versuch hundertmal ~~[illegible]~~ [in] verschiedenen jedesmal sehr langen Intervallen T wiederhole (also wenn T variabel ist)
so erhalte ich die objektive ~~[illegible]~~ Häufigkeit des Auftreten der Kugel

(Kleine Ursache – grosse Wirkg. Aber das genügt nicht!)

Würfel auf einer Ecke aufgestellt wird umfallen und (wenn weicher Stoss) eine der drei Zahlen 1, 2, 3 ergeben



doppel Poincaré p. 4.!

Dabei kann von Zufall nicht die Rede sein; der Effekt hängt nur von der Anfangslage ab und zwar ob Schwerpunkt in einer der drei Richtungen (wenn auch unmerklich ~~nur unmerklich~~ wenig) überhängt.

Dagegen wenn der Würfel aus einer Höhe von 1 m herabfallen gelassen wird, ändert sich die Sache vollständig, denn er wird wieder aufspringen dabei rotieren, wieder auffallen usw. und es wird ~~so~~ einer allmähligen Änderung der Anfangslage ein rascher Wechsel ~~aller~~ der schliesslich ~~die Wahrsch. einer schliesslichen Endlage~~ erreichten Endlagen entsprechen, so dass letztere alle ungefähr gleich wahrsch. sein werden, (falls der (Variations-) Bereich der Anfangslage nicht gar zu beschränkt ist!)

Bei den Beispielen der vorigen Seite

ist 1). keine Erklärung der „objektiven Wahrscheinlichkeit" weil eben eine solche schon bezüglich der Ursache vorausgesetzt wird

wohl aber 2).

Analog wäre: bei Anwendung des seitlich begrenzten Galton'schen Brettes:

wenn ~~alle~~ eine Kugel rollen gelassen wird, und die sukzessive bei Passieren der Stiftreihen auftretenden x notiert werden, so wird bei genügend grosser Zahl n der Stiftreihen eine annähernd gleichförmige Verteilung der x statthaben

---

Letztere Art (2) nur dort möglich, wo eben derselbe Vorgang automatisch sich wiederholt, jedesmal mit Anfangsbedingungen welche von d. vorhergehenden abhängen und den Bedingungen des ... unterliegen.

In jedem Falle aber kann für jedes x, ... x+Δx ein so großes α_n gefunden werden dass

$\frac{x}{\alpha_n} = n\pi$ n = ganze Zahl $\left(\alpha_n = \frac{x}{n\pi}\right.$

und gleichzeitig

$\frac{x+\Delta x}{\alpha_n} = n\pi + \delta$ $\delta < \vartheta\pi$ ϑ = beliebig kleiner echter Bruch

$\frac{\Delta x}{\alpha_n} = \delta$

$\frac{\Delta x}{\alpha_n} < \vartheta\pi$

$\frac{\Delta x}{x} n < \vartheta$

Wenn in ein Gefäss von mathematisch genau bekannter Gestalt (und mit genau reflektierenden Wänden) ein Punkt von bestimmter Stelle xyz in bestimmter Richtung uvw mit gegebener Anfangsgeschw. hineingeschleudert wird, so kann man dessen Lage und Geschw. richtung für einen gegebenen Zeitpunkt T vorausberechnen und zwar eindeutig.

1). Wird in (x y z) oder in (u v w) eine Variation zugelassen, so entsteht schon ein Wahrscheinlichkeitsproblem.) 2). Ebenso falls ein ganzer Zeitraum T ... T+t betrachtet wird. In letzterem Falle kommt eine Mannigfaltigkeit diskreter Werte von u v w etc in Betracht, welche mit Wachsen von t sich einer kontinuierl. Mannigfalt. nähert, so dass für jedes Bereich u+Δu, v+Δv, ... eine Zeit t angebbar ist ....

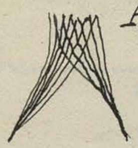
So wohl nicht, denn sonst wäre jedes $y\,dy = f(x)\,dx$ ein Wahrscheinl. problem, sondern falls t so groß ist, und der Homogenität Verschiebt bereich Δy Δx Δz .. so groß dass Endverteilung annähernd unabhängig von der Grösse ΔxΔyΔz.

Im Falle 1). stützt man sich schon auf den Begriff d. kontinuierlichen Variationsbereiches, also wird schon eine Wahrscheinlichkeitsverteilung der Ursache vorausgesetzt

" " 2). ist dies nicht der Fall; da kommt der Wahrsch. begriff durch Grenzübergang einer diskreten Wertreihe zustande.

~~Apos~~

(Dabei vorausgesetzt, dass die im Variabilitätsbereich herrschende Funktion eine „reguläre" ist, also Rekursion auf Wahrscheinlichkeitsbegriff)



(Begriff d. Zufalls) Bedingungen

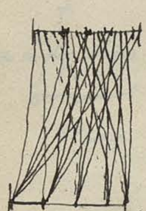
unbekannte (oder absichtlich ignorierte) Teilursache

1). Kleine ~~unterscheidbare~~ Änderung d. A erzeugt grosse Änderung d. B. (Das ist der populäre Begriff d. Zufalls! Aber dies allein ermöglicht keine „Wahrscheinlichkeit")

2). Innerhalb d. Variabilitätsbereiches von A gibt es ~~...~~ für jeden

Endzustand B eine Menge (ihn hervorbringende) diskrete Anfangszustände A
B+ΔB)

Dieser Punkt (2) schliesst notwendig (1) in sich ein!



Vielleicht besser so: innerhalb d. Variabilitätsbereiches von A gibt es kein zusammenhängendes endliches Gebiet keinen Teil der so beschaffen wäre, dass nicht ~~...~~

3). Die Häufigkeit der ~~...~~ B.– B+ΔB hervorbringenden Zustände ist im ganzen Variabilitätsbereich von A überall dieselbe.

ΔB ← das ist eine ~~simple~~ Lage, im Allgem. wird nur annähernd derselbe Teil erzeugt.

(selbstverständlich begrenzten)
Beim Galton'schen Brett:

für jeden noch so kleinen Bereich der Anfangsbedingungen A – A+ΔA ist eine Zahl n der Nagelreihen angebbar, für welche die grösste Schwankung der B (bei homogener Verteilung ~~...~~ ~~...~~ innerhalb ΔA) kleiner gemacht werden kann als eine beliebig kleine vorgegebene Zahl ε

Ebenso bei Gastheorie: Zu jedem Variab. bereich Δv eine Zeit T wo (Zersplitterung der Zustandsbereiche, Bendl) alle möglichen Zustände mit Schwankung < ε

Natürlich ist $n = f_c(\Delta A, \varepsilon)$

Grenzfall z.B.:

$$y = \lim_{\alpha=0} \sin\left(\frac{x}{\alpha}\right)$$

dies ergibt eine ungleichförmige Verteilung; hängt mit gleichf. derart zusammen,

dass ~~...~~$\left(\frac{x}{\alpha}\right) = n\pi + z$ / n = grösst mögliche ganze Zahl

$\lim_{\alpha=0}\left(\frac{x}{\alpha}\right)$ eine gleichförmige Verteilung von $\frac{z}{2}$ ergibt

$0 + \xi = ...$

$x = (2n\pi + \xi)\alpha$

arcsin

falls x, .. x+Δx commensurabel sind kann (für jedes x, x+Δx) ~~...~~ immer ein solches α_n gefunden werden dass ~~für ein bestimmtes α_n~~

$\frac{x}{\alpha} = n\pi + z$ $\qquad$ $\frac{x}{\alpha_n} = n\pi$ $\qquad$ n, z = ganze Zahl

$\frac{x+\Delta x}{\alpha} = n\pi + z + \Delta z$ $\qquad$ $\frac{x+\Delta x}{\alpha_n} = (n+m)\pi$

$\frac{\Delta x}{\alpha} = \Delta z$

g = Wärmekapazität eines Moleküls =

gn = " eines cm³ Gas $= c\rho_0$

$$\therefore \quad \theta_1 - \theta_0 = w\sqrt{\frac{3\pi}{8}}\,\frac{1}{Cc\rho_0}$$

Verhältnis zum Eigengewicht:

$$\frac{F}{\varphi} = \frac{\cancel{2\pi}\,\cancel{\ }\,g\,\frac{a^2\lambda}{2}\,\frac{w}{c\theta_0}\,\frac{1}{C}\sqrt{\frac{3\pi}{8}}}{\frac{4}{3}a^3\pi\,\rho_1\,g} = \frac{3}{4}\sqrt{\frac{3\pi}{8}}\,\frac{\lambda}{a}\,\frac{w}{c\rho_1\theta_0}\,\frac{1}{C}$$

Annahme (Jäger) $w = 1\,\frac{\text{kal gr.}}{\text{cm}^2\ \text{sek}}$ $\qquad \rho_1 = 10 \qquad c = 0.27 \qquad \theta_0 = 300^\circ$

$C = 4.10^4$

Somit ganz verschwindend!

Andre Schätzungsart, für entgegengesetztes Extrem, d. i. unter Voraussetzung d. gewöhnl. Wärmeleitung:

$$-r^2\frac{\partial\theta}{\partial r} = \text{const} = \alpha$$

$$\left.\begin{aligned}\theta &= \frac{\alpha}{r} + \cancel{\ }\,\theta_0\\ \theta_1 &= \frac{\alpha}{a} + \theta_0\end{aligned}\right\}\quad \theta - \theta_0 = (\theta_1 - \theta_0)\frac{a}{r}$$

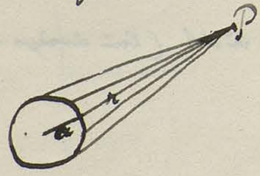
$$F = \rho_0 g\int_{r=a}^{\infty}\frac{\theta_1-\theta_0}{\theta_0}\,\frac{4r^2\pi a}{r}\,dr = \cancel{\ldots}$$

$$\cancel{= \rho_0 g \ldots \frac{\theta_1-\theta_0}{\theta_0^2}}$$

~~also muss die Abweichung von der obigen Verteilung ganz wesentlich in Betracht kommen!~~

Erwärmtes Kügelchen in Luft; Auftrieb infolge Konvektionsströmung?

Voraussetzung so klein dass $\frac{a}{\lambda}$ klein



Temperatur im Punkte P

$$\theta = \frac{2r^2\pi\left(1-\frac{\sqrt{r^2-a^2}}{r}\right)\theta_1 + \left[4r^2\pi - 2r^2\pi\left(1-\frac{\sqrt{r^2-a^2}}{r}\right)\right]\theta_0}{4r^2\pi}$$

$$2\pi a^2 \underbrace{\int_0^{\varphi} d\varphi \sin\varphi}_{1-\cos\varphi}$$

$$= \theta_0 + \frac{1}{2}\left[\theta_1-\theta_0\right]\left[1-\sqrt{1-\left(\frac{a}{r}\right)^2}\right]$$

$r \geqq a$

Auftrieb im gesammten Raum

$$\cancel{\frac{1}{2}\int \ldots \, dr = \frac{1}{4}a^2}$$

$$F = \rho_0 g \int_{r=a}^{\infty} \frac{\theta-\theta_0}{\theta_0}\, 4r^2\pi\, dr = \rho_0 g\, \frac{1}{2}\, \frac{\theta_1-\theta_0}{\theta_0}\, 4\pi \int_a^{\infty} \underbrace{\left[1-\sqrt{1-\left(\frac{a}{r}\right)^2}\right]}_{\neq \frac{1}{2}\frac{a^2}{r^2}} r^2\, dr$$

also $= \infty$!

Rationeller Weise ist aber die obere Grenze nicht ∞ sondern $a+\lambda$ zu machen (womit der Effekt wohl unterschätzt wird)

Somit:

$$F = 2\pi \rho_0 g\, \frac{\theta_1-\theta_0}{\theta_0}\, \frac{a^2\lambda}{2}$$

Berechnung von θ_1:

Zugeführte Wärmemenge (durch Strahlung $w \frac{cal}{cm^2}$) $\quad a^2\pi w$

Abgeführte " (durch Molekularstösse) $\quad 4a^2\pi \frac{nC}{\sqrt{6\pi}}(\theta_1-\theta_0)\gamma$

Somit: $\quad \theta_1-\theta_0 = w\,\frac{\sqrt{6\pi}}{4}\,\frac{1}{nC\gamma}$

Also tritt die maximale Wahrsch. [illegible] $\sim \xi^2$

(aber nicht $t = \frac{\xi^2}{2D}$ wie nach gewöhnl. Br. F. zu erwarten wäre)

Also Wahrsch. dass bis zur Zeit t die Elongation ξ noch nicht (kein einziges Mal) erreicht worden ist:

$$W = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

Wahrsch. dass bis zur variabeln Zeit t die ([illegible]) festgehaltene Elongation ξ erreicht wurde:

$$W(t) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

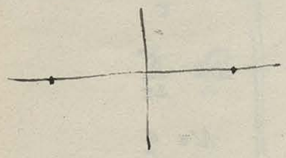
Wahrsch. dass in der (festen) Zeit t die (variable) Elongation (0 bis ξ) nicht überschritten wird

$$W(\xi) = \int_0^{\xi} \frac{1}{2\sqrt{D\pi t}}\, e^{-\frac{\xi^2}{4Dt}}\,d\xi \qquad \frac{\xi^2}{4Dt} = z^2 \qquad \xi = z\, 2\sqrt{Dt}$$

$$= \frac{1}{\sqrt{\pi}} \int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

$$\nu\Delta = \frac{3\pi c_0^2}{\frac{e^2}{m}\cdot q} \qquad q = \frac{3\pi\cdot(3\cdot10^{10})^{\cancel{2}}}{4'77\cdot10^{-10}\cdot 1'77\cdot10^{7}\cdot \cancel{2\cdot10^{..}}}$$

Kann man nicht dasselbe machen mit Diffusionsmethode?



$$\Phi(\xi)\,d\xi = \frac{1}{2\sqrt{D\pi t}}\left[e^{-\frac{(x-\xi)^2}{4Dt}} - e^{-\frac{(x+\xi)^2}{4Dt}}\right]d\xi$$

$$U = \int_0^\infty \Phi \, \cancel{(x)}\, d\xi$$

$$\frac{x-\xi}{\sqrt{4Dt}} = z$$

$$\frac{dx}{2\sqrt{Dt}} = dz$$

$$W_{(t)}\,dt = -\frac{\partial U}{\partial t}\,dt$$

$$U = \frac{1}{\sqrt{\pi}}\left[\int_{-\frac{\xi}{2\sqrt{Dt}}}^{\infty} e^{-z^2}\,dz - \int_{\frac{\xi}{\cdot}}\right]$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\frac{\xi}{2\sqrt{Dt}}} e^{-z^2}\,dz$$

$$W_{(t)}\,dt = +\frac{2}{\sqrt{\pi}}\,e^{-\frac{\xi^2}{4Dt}}\cdot\frac{\xi}{4\sqrt{Dt^3}}$$ stimmt ✓.

Wann ist ein Maximum d. Wahrscheinlichkeit?

$$\cancel{\frac{\xi^2}{4Dt} = z} \quad \frac{1}{\sqrt{t^3}}\,e^{-\frac{\xi^2}{4Dt}}$$

$$\cancel{e} \quad \frac{1}{\sqrt{t^3}}\,\frac{\xi^2}{4Dt^2} - \frac{3}{2\sqrt{t^5}} = 0$$

$$\frac{\xi^2}{4Dt^2} = \frac{3}{2t}$$

$$t_m = \frac{\xi^2}{6D}$$

Falls genaue Schwebespannung herrscht, Wahrsch. der Überschreitung von ξ in vorgegebener Zeit

Wahrsch. eines Überschusses n beim m ten Wurf

$$\overline{W}(n) = \frac{1}{2^m}\binom{m}{\frac{m-n}{2}} \mp \sqrt{\frac{2}{\pi m}}\, e^{-\frac{n^2}{2m}}$$

$$m = \frac{t}{\tau} \qquad D = \frac{\delta^2}{2\tau} \qquad N = \frac{1}{\delta}$$

Wahrsch. eines **erstmaligen** Übersch. n beim m ten Wurf

$$\overline{W}'(n) = \frac{n}{m}\cdot\frac{1}{2^m}\binom{m}{\frac{m-n}{2}}$$

$$n = \frac{\xi}{\delta}$$

$$\overline{W}(\xi)\, d\xi = \frac{1}{2\sqrt{D\pi t}}\, e^{-\frac{\xi^2}{4Dt}}\, d\xi$$

$$\underbrace{\sqrt{\frac{2\tau}{\pi t}}\, e^{-\frac{\xi^2\tau}{2\delta^2 t}}} = e^{-\frac{\xi^2}{4Dt}}\cdot\frac{\delta}{\sqrt{D\pi t}}$$

$$\frac{n}{m} = \frac{\xi\tau}{\delta t} = \frac{\xi}{Vt} \quad (?)$$

$$d\xi = \delta .\, dn$$

ξ viel zu ungenau

Somit wäre

$$\lim \overline{W}'(\xi, t)^{dt} = \frac{\xi}{Vt}\,\frac{\tau}{2\sqrt{D\pi t}}\, e^{-\frac{\xi^2}{4Dt}}$$

$$\frac{\delta}{\tau} = V$$

$$\frac{\xi^2}{4Dt} = z$$

$$t = \frac{\xi^2}{4Dz}$$

$$dt = -\frac{\xi^2}{4Dz^2}\, dz$$

$$\frac{\xi\, dt}{2t\sqrt{D\pi t}}\, e^{-\frac{\xi^2}{4Dt}} = \sqrt{\frac{z}{\pi}}\cdot\frac{4Dz}{\xi^2}\cdot e^{-z}\cdot\frac{\xi^2}{4Dz^2}\, dz$$

$$= \frac{1}{\sqrt{\pi}}\,\frac{e^{-z}}{\sqrt{z}}\, dz$$

$$\int_0^\infty \frac{e^{-z}}{\sqrt{z}}\, dz = \int_0^\infty 2\,\frac{e^{-x^2}}{x}\, x\, dx = \sqrt{\pi}$$

Stimmt

also: $$\int_{t=0}^{\infty} \frac{\xi\, e^{-\frac{\xi^2}{4Dt}}\, dt}{2t\sqrt{D\pi t}} = 1$$

Wie rasch rotiren solche Teilchen ? (Wirkliche Rot. geschw.)

Ungefähr $K = m\frac{a^2}{2} = \frac{4}{3}\frac{\pi a^5 \rho}{2} = 2a^5\rho$ für $a = 10^{-5}$ $\rho = 1$

$$K\frac{(2\pi n)^2}{2} = 10^{-14} = 2.10^{-25}.2\pi^2 n^2$$

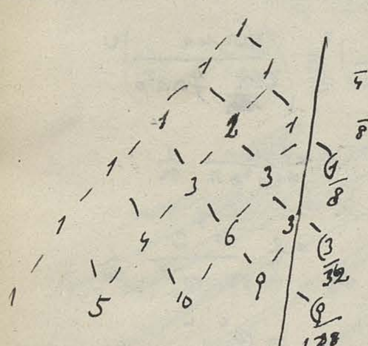
$$n = 10^5$$

$$A_{cent} : A_{capp} =$$

$$4\pi^2 n^2 a \frac{a}{2} . m : 30\ 4a^2 n$$

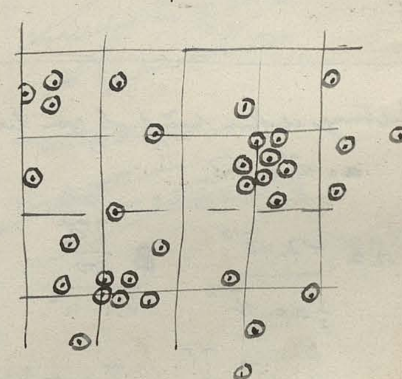
$$\frac{m}{2}\pi n^2 : 30$$

$$10^{-15}$$



$\frac{7}{8}$

$\frac{25}{32}$



~~Anzahl~~ Anzahl der Zusammenstösse im Volumen ω:

$$\frac{n^2}{\omega}\cdot\overbrace{\sigma\sqrt{\frac{2\pi}{hn}}}^{A} = \frac{\nu^2(1+\delta)^2}{\omega}\ldots$$

?

$$\overline{n^2} = \nu^2(1+\overline{\delta^2}) = \nu^2+\nu$$

$$\sum\frac{n^2}{\omega}\cancel{A} = \cancel{A}\sum\frac{\nu^2+\nu}{\omega} = \cancel{A}\left[N^2+\frac{N}{\omega}\right]$$

Otóż w tem że powinno dokładniej być:

$\frac{n(n-1)}{\omega}\ldots$ stąd jest $\sum\frac{n^2-n}{\omega} = N^2+\frac{N}{\omega}-\frac{N}{\omega} = N^2$

$h = 6{\cdot}4 \,.\, 10^{-27}$

ε (Rutherf. & Robins.) $= 0{\cdot}4284 \,.\, 10^{13}\, e \quad = h\nu = 6{\cdot}4 \,.\, 10^{-27} \cdot \frac{3.10^{10}}{\lambda}$

$$\frac{4{\cdot}78 \,.\, 10^{-10}}{3.10^{10}}$$

$$\lambda = \frac{3 \,.\, 6{\cdot}4 \,.\, 10^{-17}}{0{\cdot}428 \,.\, 4{\cdot}78 \,.\, 10^{3}} \,.\, 3.10^{10}$$

$4{\cdot}78 \,.\, 0{\cdot}1428 \,.\, 10^{-7}$
191
14
$0{\cdot}683 \,.\, 10^{-7}$

$$= \frac{9 \,.\, 6{\cdot}4}{0{\cdot}428 \,.\, 4{\cdot}78} \,.\, 10^{-10} = \frac{576}{4{\cdot}78 \,.\, 0{\cdot}428} \,.\, 10^{-9}$$

$$\lambda = 3 \,.\, 10^{-9}$$

$$\nu = \frac{0{\cdot}683 \,.\, 10^{-7}}{6{\cdot}4 \,.\, 10^{-27}} = \frac{c}{\lambda}.$$

Reibungseinfluss hängt ab von der Grösse von
$a = 10^{-4}\,\mathrm{cm} = 1\,\mu$

$$A = \frac{4{\cdot}7 \,.\, 10^{-10}}{\frac{4}{3}\pi\rho \,.\, 10^{-12}} \quad \frac{\frac{V}{300}}{4\pi^2 n^2}$$

$$= 10^{-2}\,\frac{V}{n^2}$$ || V, n von Grössenordnung 1–10

($\rho = 1$)

$a = 0{\cdot}1\,\mu = 10^{-5}$

$$A = 10 \,.\, \frac{V}{n^2}$$

$$\left(\frac{\beta}{\alpha m}\right)^2 = \left(\frac{6\pi\mu a}{2\pi n \cdot \frac{4}{3}\pi a^3 \rho}\right)^2$$

$$= \frac{9}{4}\,\frac{\mu}{a^2\rho\pi}\,\frac{1}{n}$$

$$= \frac{9}{4}\,\frac{10^{-4}.2}{\pi.\,10^{-8}}\,\frac{1}{n}$$

$$\left(= \frac{10^4}{n}\right)^2$$

also ist Reibungseinfluss kolossal

Auch so allgemein: $\quad v = \frac{mg}{\beta}$ = Fallgeschwind.

$$\frac{\beta}{\alpha m} = \frac{g}{v\, 2\pi n} = \frac{10^3}{10^{-2}.\,2\pi n}$$

~~[illegible]~~

Wie wird also Kirchhoffs Formel?

Trägheitsvermehrung (prozentuelle) $\quad 1 + \frac{\rho_{Luft}}{\rho_{Kugel}}\left(\frac{1}{2} + \frac{9}{4.10^{-4}\sqrt{\frac{1}{2}\frac{2\pi n}{\mu}\rho_{Luft}}}\right)$

$$= 1 + \frac{\rho'}{\rho_0}\left(\frac{1}{2} + \frac{3.10^3}{\sqrt{n}}\right)$$

Falls die Messung etwas Brauchbares liefern soll, muss $\frac{\beta}{\alpha m}$ klein sein $= \frac{9}{4}\,\frac{10^{-4}.2}{\pi.a^2\rho n} = \frac{3}{2}\,\frac{10^{-4}}{a^2.\rho.n}$

für $a = 10^{-5}$ $\quad \rho = 10$ $\quad n = 10^5$!!!

Praktisch nicht ausführbar! Bei so kleinen Teilchen ist Trägheitseinfluss ganz verschwindend!

Falls bei Millikan-Ehrenhaft Versuchen ein Wechselstromfeld angewendet wird mit horizontaler Axe:

$$m\frac{d^2x}{dt^2} = eE\cos\alpha t$$

$$x = -A\sin\alpha t$$

$$mA\alpha^2 = eE$$

$$A = \frac{e}{m}\frac{E}{\alpha^2}$$

$$= \frac{e}{m}\frac{E}{(2\pi n)^2}$$

Dies gibt also eine Relation, welche an und für sich durch Messung von A das Verhältnis $\frac{e}{m}$ ergibt

(unter Voraussetzung dass die Hemmung zu vernachlässigen ist)

~~Also würden Messungen überhaupt ausreichen um $\frac{e}{m}$ zu bestimmen als Kleinheitsmass~~

$$\cancel{mA(\alpha+\varepsilon)^2 = b}$$

$$m\frac{d^2x}{dt^2} - \overset{\beta}{6\pi\mu a}\frac{dx}{dt} = eE\cos\alpha t$$

$$\cancel{x = -A\sin(\alpha+\varepsilon)t}$$

$$mA\alpha^2\cos\varepsilon t - \beta A$$

$$\tau = \frac{2\pi}{6} = \frac{1}{n}$$

$$\alpha = 2\pi n$$

$$\cancel{A(\alpha+\varepsilon)^2\cos(\alpha+\varepsilon)t + \beta A(\alpha+\varepsilon)\cos(\alpha+\varepsilon)t = eE\cos\alpha t}$$

$$\cancel{A(\alpha+\varepsilon)^2 - \beta A(\alpha+\varepsilon)\operatorname{tg}\varepsilon t =}$$

$$x = -A\sin(\alpha t+\varepsilon)$$

$$A\alpha^2 m\cos\varepsilon + A\beta\alpha\sin\varepsilon = eE$$

$$\cancel{A}\alpha^{\cancel{2}} m\sin\varepsilon + \cancel{A}\beta\cancel{\alpha}\cos\varepsilon = 0$$

$$\operatorname{tg}\varepsilon = -\frac{\beta}{\alpha m}$$

$$A\left(\alpha^2+\frac{\beta^2}{m^2}\right)m\cos\varepsilon = eE$$

$$A = \frac{eE}{m\left(\alpha^2+\frac{\beta^2}{m^2}\right)}\sqrt{1+\frac{\beta^2}{\alpha^2 m^2}} = \frac{eE}{\alpha\sqrt{\alpha^2+\frac{\beta^2}{m^2}}\,m} = \frac{eE}{m\alpha^2}\left(1-\frac{\beta^2}{2\alpha^2 m^2}\right)$$

$t_1 = \frac{x}{v}$ $\quad \cancel{t' = t +}$

$x +$

$c\tau = x_0 + u\tau$

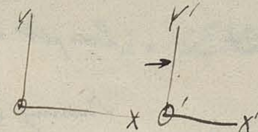
$\tau = \frac{x_0}{c-u}$

$t_1' = t_1 + \frac{x_0}{c-u}$

$t_2' = t_2 + \frac{x_0 + ut}{c-u}$

$\quad t_1 + t$

$t_2' - t_1' = t\left(1 + \frac{u}{c-u}\right) = t\frac{c}{c-u}$



$\frac{x}{c} - \frac{x\sqrt{1-\frac{v^2}{c^2}}}{c+v}$

$= \frac{x}{c}\left[1 - \sqrt{\frac{c^2-v^2}{c\pm v}}\right] \cdot \frac{x}{c}\left[1 - \sqrt{\frac{c+v}{c-v}}\right]$

$$\begin{cases} t' = \beta\left(t - \frac{v}{c^2}x\right) \\ x' = \beta(x - vt) \\ y' = y \\ z' = z \end{cases} \qquad \beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Anfangspunkt: O' $\left.\begin{matrix} \\ x'=0 \end{matrix}\right\}$ $x = vt$

$x = \beta v t'$

$\left.\begin{matrix} O \\ x=0 \end{matrix}\right\}$: $x' = -\beta v t$

$x' = -vt'$

$$\begin{cases} t = \beta\left(t' + \frac{v}{c^2}x'\right) \\ x = \beta(x' + vt') \\ y = y' \\ z = z' \end{cases}$$

$\cancel{t + \frac{x}{c} = t + \frac{v}{}}$

Zur Zeit $t = t' = 0$ $\quad x = x'$

Nach Ablauf von t wird von O ein Zeichen nach O' geschickt, dort reflektiert

$vt + v\tau = c\tau \qquad \tau = \frac{vt}{c-v}$

$T = 2\tau$

$t + \tau = t\left(1 + \frac{v}{c-v}\right) = t\frac{c}{c-v}$

$=$

$2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right) = 2\pi\left(nt - \frac{x}{\lambda}\right) = \cancel{n\beta t' + n\beta\frac{v}{c^2}x' - \frac{\beta x'}{\lambda} - \frac{\beta v t'}{\lambda}}$

$= \left(n - \frac{v}{\lambda}\right)\beta t' + \beta x'\left(\frac{nv}{c^2} - \frac{1}{\lambda}\right)$

$n\left(t - \frac{x}{c}\right) = n\beta t' - n\beta\frac{v}{c^2}x' - \frac{n}{c}\beta x' - \frac{n\beta v t'}{c} = n\beta t'\left(1 - \frac{v}{c}\right) - n\beta\frac{x'}{c}\left(1 - \frac{v}{c}\right)$

$= \left(1 - \frac{v}{c}\right) n\beta\left(t' - \frac{x'}{c}\right)$

$= \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\; n\left(t' - \frac{x'}{c}\right) \qquad \neq \left(1 - \frac{v}{c}\right)\ldots$

$d\omega = \frac{\pi r^2}{4\pi D^2} = \frac{r^2}{4D^2}$

$dS_1 = D^2\alpha^2\pi$

$F = \mathcal{E}_1 \cdot \frac{r^2\alpha^2\pi}{4}$

$\frac{1{,}5^2}{4}\pi \cdot \left(\frac{1}{120}\right)^2 \cdot 300000 \;\|\; \frac{10^4}{200} \cdot 0{,}04$

$= 3.10 = 30 \;\|\; 2$

$\mathcal{E} \rightarrow \boxed{} d \quad \mathcal{E}' = \frac{\mathcal{E} h \Delta x}{4\pi d \Delta x} = \mathcal{E}\frac{h}{4\pi d} = \mathcal{E}'$

$F = \mathcal{E}_1\, dS_1\, d\omega_1 \qquad \mathcal{E}_2 = \mathcal{E}_1 \frac{dS_1\, d\omega_1 \cdot h\,\cancel{\Delta x}}{4\pi d \cdot \cancel{\Delta x}}$

$dS_1 = \frac{d^2}{4}$

$d\omega_1 = \frac{1}{200}$

$h = 10^{-7}$

$d =$

$= \mathcal{E}_1 \frac{d}{4} \frac{d\omega_1 h}{4\pi}$

$= 10^4 \cdot \frac{0{,}1}{200 \cdot 10} \cdot 10^{-7} = \frac{10^{-4}}{2.10^3} = \frac{1}{2} \cdot 10^{-7}$

mur blanc à pleine lune à 45° : 10^{-6}

$h = \frac{32}{3} \frac{\pi^3}{n\lambda_F^4} (n_0 - 1)^2$

$0{,}000273$

$\lambda_F = 0{,}486 \cdot 10^{-4}$

$n = 3 \cdot 10^{19}$

$0{,}6866 \cdot 10^{-5}$

$0{,}7464 - 18$

1,49145

1,5051

0,8724 − 8

0,8690 − 5

− 2,7006

0,1684 − 7

0,954247 − 19

0,7464 − 18

2,7006

$h = 1{,}47 \cdot 10^{-7}$

$L x + 2\tau v = \tau c$

$x + 2\tau v = \tau c$

$\tau = \frac{Lx}{c - 2v}$

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…nificant Emblems

…HY NOT FOLLOW …TTER EXAMPLES?

…body has been saying once more …r British postage-stamps are …e most insignificant and un-… the world.

…N. has often said it; and we are …d that, if we have regard to the …ence of the nation that issues …they must be awarded the palm …verty. Their design is of the lowest …r of art, and they amount to the …rest advertisement of our country.

…hat a pity it is! This is not a matter … stamp-collector alone; it is …l and imperial importance. …f people outside these islands … see our … by

A Brave Bengali

How To Hold a Tiger

The courage with which natives of Africa go lion-hunting, armed only with spears, is well known.

Now comes a story concerning the bravery of a young man in India who tackled a tiger with his hands.

In the district of Bankura a man-eating tiger had become a constant terror, until at last the whole manhood of the village turned out to hunt it with clubs and poles.

They found the tiger, which fled before the crowd as a bird of prey flies before a flock of small ones; but when the beast had climbed a palmyra tree it was a different matter, because only one man could tackle it there.

Such a man was found in a young Bengali named Banshi Mukherji, who went to the tree, took steady aim, and hurled a spear.

With one bound the tiger was upon him. But the unarmed youth did not lose his head. Instantly he thrust his …d into the tiger's open mouth, and …d its tongue. He held on till the …rs had rushed in and killed it …dgels.

…sured sev… …alf

Tracking Down … Influenza

The Ferret To … of M…

The first step to… human beings from influen… successfully made.

Three doctors in the research… tories of the National Institute … Hill have proved that the ferret cat… influenza from man and hands it on … contact to other ferrets. A ser… obtained from infected ferrets … tralises the virus, or filter-passing … which has for some years been held … at any rate one of the causes of influ…

The cause having now been fou… discoverers are hoping to find a …

The Return of the Plague Rat

The black rat, dreaded carrier … bubonic plague, is incr…ing its numbe… in England.

Perhaps our succ… …erminating its natural enemy, … rat of the sewers and basem… …n it fresh oppor… …ck rat … …erse to … …ngerous … …orts … … it…